

Barometric Fluctuations in Wells Tapping Deep Unconfined Aquifers

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Water levels in wells screened only below the water table in unconfined aquifers fluctuate in response to atmospheric pressure changes. These fluctuations occur because the materials composing the unsaturated zone resist air movement and have capacity in stress air with a change in pressure. Consequently, the translation of any pressure change at land surface is slowed as it moves through the unsaturated zone to the water table, but it reaches the water surface in the well instantaneously. Thus a pressure imbalance is created that results in a water level fluctuation. Barometric effects on water levels in unconfined aquifers can be computed by solution of the differential equation governing the flow of gas in the unsaturated zone subject to the appropriate boundary conditions. Solutions to this equation for two sets of boundary conditions were applied to compute water level response in a well tapping the Ogallala Formation near Lubbock, Texas from simultaneous microbarograph records. One set of computations, based on the step function unit response solution and convection, resulted in a very good match between computed and measured water levels. A second set of computations, based on analysis of the amplitude ratio of simultaneous cyclic microbarograph and water level fluctuations, gave inconsistent results in terms of the unsaturated zone pneumatic properties but provided useful insights on the nature of unconfined-aquifer water level fluctuations.

INTRODUCTION

The water level in a well screened only below the water table in an unconfined aquifer fluctuates in response to barometric changes at land surface. The magnitude of these fluctuations may be significant if the overlying unsaturated materials are thick or have relatively low permeability to air at their prevailing moisture content. Consequently, it is sometimes desirable to correct water level data collected during tests on unconfined aquifers for such effects. In addition, it is worthwhile to understand the nature and causes of these fluctuations even when they are not significant on a practical basis. This paper describes the phenomenon responsible for barometrically induced fluctuations and presents procedures to predict them using simultaneous water level and microbarograph records.

PREVIOUS WORK

The mechanism by which water levels in wells tapping deep unconfined aquifers respond to changes in atmospheric pressure has not been previously described in detail. However, Buckingham (1906) has described the movement of air into or out of the unsaturated zone due to changes in atmospheric pressure, with particular reference to the effects of such movement on the diffusion of oxygen into the soil. He presented the differential equation governing such movement and derived an analytical equation describing the phase lag and attenuation of periodic soil gas pressure changes at the water table due to periodically varying barometric fluctuations at land surface. Stallman (1967) suggested that such air movement and the attendant pressure lag could be measured and exploited to determine the hydraulic properties of materials in the unsaturated zone. Weeks (1978) developed and tested the methodology for making such determinations but did not discuss the implications of his results with regard to barometric effects on water levels in wells tapping underlying unconfined aquifers.

Other workers, apparently unfamiliar with Buckingham's paper, have also described the mechanics of barometric fluctuations in wells tapping unconfined aquifers, although their descriptions are incomplete. Von Eimern (1950) explored the effects of barometric fluctuations on water levels in a well tapping a shallow unconfined aquifer in northwest Germany. He attributed the barometrically induced water level fluctuations to the fact that the 'friction' of the unsaturated zone materials prevented the barometric pressure from being transmitted in its entirety to the water table, whereas the pressure was transmitted instantaneously down the well bore. Although his explanation is qualitatively correct; he did not present an equation governing the fluctuations. He also does not appear to have recognized the dependence of the magnitude of the water level fluctuations on the rate, as well as magnitude, of the barometric fluctuations. Thiazand (1954, p. 36) challenged von Eimern's explanation and erroneously stated, in defiance of Darcy's law, that no decline in pressure can occur with depth as air moves downward through an unsaturated porous medium. Finally, Gilliland (1969, p. 244) briefly mentions the resistance to air flow in the unsaturated zone as being important in explaining barometrically induced water level fluctuations. He does not present an equation, nor does he describe the mechanism in detail.

Several authors [e.g., Peck, 1960; van Hylckama, 1968; Turk, 1975] have described barometric effects on water levels in wells tapping shallow unconfined aquifers. These authors have dealt with situations in which the depth to water below land surface is less than 1 or 2 m, and they postulate the presence of entrapped air below the water table as a mechanism for producing such effects. These effects are entirely different from those described in this paper. Furthermore, the mechanism described here generally would not produce measurable effects in such shallow aquifers.

THEORY

Water levels in wells tapping unconfined aquifers are affected by changes in barometric pressure because air must move into or out of the overlying unsaturated zone in order to transmit the pressure change to the water table. This movement is slowed by the finite permeability of the unsaturated materials and by their capacity to store or release soil gas as the pressure changes. Consequently, the change in soil gas

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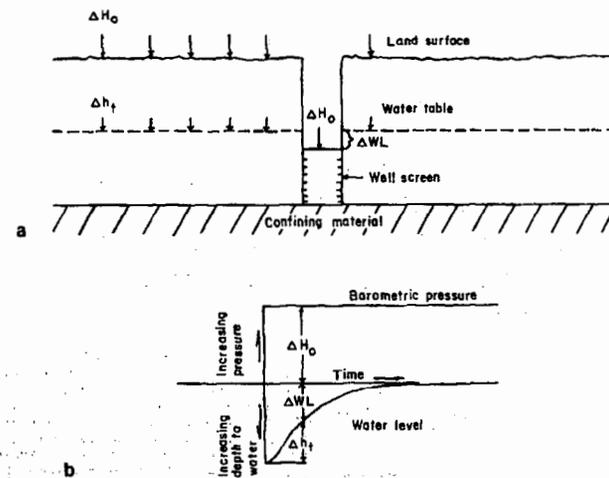


Fig. 1. Effects of a change in barometric pressure on the water level in a well tapping an unconfined aquifer. (a) Idealized section of an unconfined aquifer. (b) Idealized barograph and hydrograph showing water level response with time.

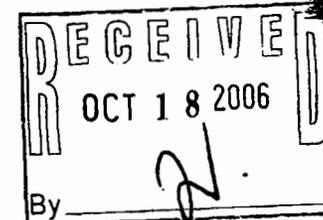
pressure at the water table lags that at land surface. However, barometric changes are transmitted essentially instantaneously in a well. This results in a pressure imbalance between water in the well and water in the adjacent aquifer. The pressure difference produces a water level fluctuation in the well.

This phenomenon may be illustrated by considering the effects of a step change in barometric pressure on the water level in a well screened only below the water table, as shown in Figure 1. Shortly after the atmospheric pressure changes at land surface by an amount ΔH_0 , only a fraction (ΔA_0) of the change has been transmitted through the unsaturated zone to the water table. However, the atmospheric pressure change has been transmitted unattenuated to the water surface in the well. Hence a temporary pressure imbalance is created between the water in the well and that in the aquifer, resulting in a water level decline equal to a pressure head difference of $(\Delta H_0 - \Delta A_0)$. As time passes, however, the entire pressure change is transmitted through the unsaturated zone, and the water level recovers to its initial position.

This phenomenon is substantially different from that causing barometric fluctuations in a well tapping a confined aquifer, always for comparison in Figure 2. On the basis of Jacob's (1940) model for barometric effects on confined aquifers, the pressure change is transmitted instantaneously without attenuation through the confining bed to the interface between the confining bed and the aquifer. At the interface a portion of the load change is borne by the confined water over the area of its contact with the interface (ΔA_0 in Figure 2), and an-

other portion (ΔA_0) is borne by the aquifer skeleton. However, within the well bore the pressure change is borne entirely by the water. Hence a pressure imbalance equal to $\Delta H_0 - \Delta A_0$ is created between the water in the well and the pore water. This pressure imbalance, expressed as a pressure head, results in an equivalent water level change in the well. Thus under confined conditions, barometrically induced water level fluctuations are in phase with the barometric change and are a constant fraction of the barometric fluctuations.

As described above, the water level in a well tapping an unconfined aquifer responds to the soil gas pressure change at the water table. Moreover, the relationship between atmospheric pressure and soil gas pressure at any depth within the unsaturated zone is governed by the differential equation describing the flow of soil gas in the unsaturated zone, subject to the appropriate boundary conditions. Equations involving the flow of gas through porous media [Katz et al., 1959, p. 408; Bear, 1972, p. 200] generally are written in terms of pressure, it being assumed that the gravity term relating pressure to fluid potential can be ignored. This assumption generally is justified in evaluating flow in laboratory gas permeameters and in natural gas reservoirs. However, for the problem of soil gas flow in the unsaturated zone, atmospheric pressure changes are of approximately the same magnitude as the gravity term. The gravity term for gas at atmospheric pressure amounts to about 1 mbar/10 m of depth, and diurnal atmospheric pressure changes commonly are 3-4 mbar. On the other hand, effects of pressure variation on gas density, usually accounted for in differential equations describing gas



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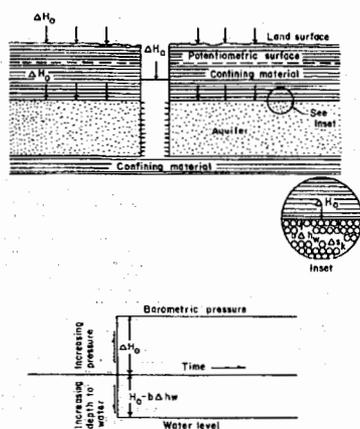


Fig. 2. Effect of a barometric pressure change on the water level in a well tapping a confined aquifer. Modified from Ferris et al. (1962, Figure 23). (a) Idealized cross section of a confined aquifer. (b) Idealized barograph and hydrograph showing water level response with time.

flow, can be ignored when the pressure changes are as slight as those occurring in the unsaturated zone (Weale, 1972, p. 4). Hence flow of gas in the unsaturated zone is adequately described by the equation for flow of a slightly compressible fluid if fluid potential, or head, rather than pressure, is used. If it is further assumed that the ideal gas laws apply, the compressibility of the soil gas is $1/P$, where P is the mean pressure during the pressure change of interest, $M/L^2 T^2$, and the specific storage coefficient of the medium to soil gas is n_a/P , where n_a is the air-filled porosity or specific yield, dimensionless.

In addition, the ability of the unsaturated materials to transmit soil gas under a head gradient is governed by the permeability of the unsaturated materials at their prevailing moisture content divided by the viscosity of the soil gas, or K_{rel}/μ_a , where K is the intrinsic permeability of the medium, L^2/T , k_{rel} is the relative permeability of the medium to air at its prevailing moisture content, dimensionless; and μ_a is the dynamic viscosity of the soil gas at its prevailing temperature, $M/L T$. Thus if one assumes that (1) soil gas flow is significant only in the vertical direction, (2) change in depth and pressure has a negligible effect on soil gas density, (3) the transmissive and storage properties of the unsaturated zone to soil gas are constant in space and over the time periods used for analysis, (4) the permeability of the medium to air is large enough that the Klinkenberg (1941) effect can be ignored, and (5) the flow of soil gas in the unsaturated zone is isothermal, the appropriate equation describing soil gas flow in the unsaturated zone is

$$\frac{K_{rel}}{n_a} \frac{\partial^2 h}{\partial z^2} = \frac{\rho_a}{\beta} \frac{\partial h}{\partial t} \quad (1)$$

where
 h pneumatic potential, L ;
 n_a air-filled porosity, dimensionless;
 μ_a dynamic viscosity of air, $M/L T$;
 K intrinsic permeability of the porous medium comprising the unsaturated zone, L^2/T ;
 k_{rel} relative permeability of the medium to air at its prevailing moisture content, dimensionless;
 ρ_a mean pressure during pressure change events, $M/L T^2$.

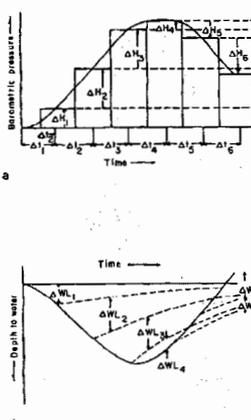


Fig. 3. Diagrammatic representation of the step change method to determine barometric effects on water levels. The time scales shown in Figures 3a and 3b are identical. (a) Hypothetical barometric pressure change with time and the step changes used to represent it. (b) Hypothetical hydrograph showing effects of barometric pressure change shown in Figure 3a upon the water level in a well tapping an unsaturated aquifer. Dashed lines represent the trend that the water level would have followed had the barometric pressure established at the end of the time step.

The Klinkenberg effect occurs during gas flow through a capillary or porous medium when, at low pressures, the length of the mean free path of the gas molecules approaches the diameter of the capillary tube or pore. Under these conditions the Hagen-Poiseuille velocity distribution no longer holds, since some gas molecules tend to slip along the capillary wall. Thus for very fine grained materials the intrinsic permeability as measured by gas flow exceeds that measured by liquid flow.

The quantity in (1) is permeance has the units T/L^2 , and its reciprocal, with dimensions L^2/T , is termed (Weale, 1972) the effective pneumatic diffusivity. The effective pneumatic diffusivity may be treated as a lumped parameter that includes both the properties of the unsaturated materials and of the soil gas. Pneumatic diffusivity will hereafter be symbolized as D . Thus (1) may be written

$$\frac{\partial^2 h}{\partial z^2} = \frac{\partial h}{\partial t} \quad (2)$$

Equation (2) is the same as that presented by Buckingham (1904) and is used to represent soil gas flow in the unsaturated zone throughout the remainder of this paper.

For most situations, soil gas flow in the unsaturated zone would be subject to the boundary conditions

$$h = f(t) \quad \text{at } z = 0 \quad (3)$$

$$\partial h / \partial z = 0 \quad \text{at } z = l \quad (4)$$

where z is taken as zero at land surface and is assumed equal to l at the water table. Equation (3) specifies that pneumatic potential at land surface varies as some arbitrary function with time. Equation (4) specifies that no soil gas flows into or from the water table. (Strictly speaking, the lower boundary for soil gas flow is the top of the capillary fringe, rather than the water table itself. This subtlety is ignored for the rest of the paper for ease of exposition. However, the thickness of the capillary fringe could be important for relatively shallow aquifers.)

In the field the unsaturated zone generally is layered rather than uniform. Effects of such layering are not explicitly considered in the methods described below, although the pneumatic diffusivity value determined by matching the observed water level response to barometric fluctuations includes the effects of layering implicitly.

DETERMINATION OF EFFECTIVE PNEUMATIC DIFFUSIVITY AND PREDICTION OF WATER LEVEL CHANGES

The effective pneumatic diffusivity D of the materials comprising the unsaturated zone must be determined if (2) is to be used to predict barometric effects on water levels. If simultaneous microbarograph and water level records are available, D may be determined using some idealization of the barometric pressure variation with time. Two of several possible methods are described below.

The step change method. A useful and general method for estimating effective pneumatic diffusivity of the unsaturated zone is to represent the actual barometric pressure variation with time as a series of discrete step changes, as shown in Figure 3. These step changes may be used with various trial values of pneumatic diffusivity to compute water level fluctuations using (2). The appropriate effective diffusivity value is that which provides the best match between computed and measured water levels.

The basic procedure involved in the step change method is to determine the change in soil gas head change at the water table produced by each step barometric change. Using the principle of superposition, the total head change at the water table at a given time is the sum of the changes caused by preceding step barometric changes. For a single step change, such as that shown in Figure 1, the pneumatic head change at the water table at any time t , assuming an initial condition of $h = 0$, $0 \leq t \leq \Delta t$, is given by Carslaw and Jaeger (1959, p. 97) as

$$h_s = H_s \left[1 - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \exp(-n^2 \pi^2 D t / L^2) \right] \quad (5)$$

where h_s is the pneumatic head at the water table, L ; L_s is the thickness of unsaturated zone, L ; t is the elapsed time since step change in barometric pressure, T ; and other symbols are as defined in the notation section. If h is expressed in terms of water head, the barometrically induced water level decline is equal to the barometric change minus the soil gas head

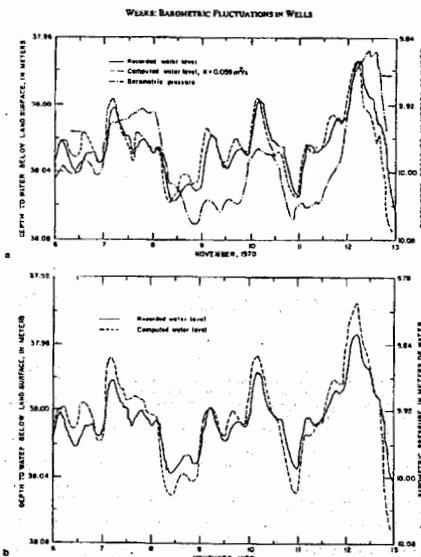


Fig. 4. Comparison of computed to measured water levels at the Lubbock, Texas, Regional Airport during the period November 6-13, 1970, for two trial values of pneumatic diffusivity: (a) Best fit match, obtained by using a pneumatic diffusivity value of 0.059 m²/s; (b) Match for a diffusivity value (0.032 m²/s) substantially different from that giving the best fit.

change, or $H_i - H_{i-1}$. Thus the change in water level is given by the equation

$$\Delta W_L = \frac{4H_i}{\pi} \sum_{n=1}^{\infty} \left[(-1)^{n-1} \frac{1}{n^2} \exp(-n^2 \pi^2 a t / 4D^2) \right] \quad (6)$$

$$\Delta W_L = \sum_{n=1}^{\infty} \frac{4H_i}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m-1} 1}{m^2} \exp[-m^2 \pi^2 a t (j-1) / 4D^2] \quad (7)$$

where ΔW_L is the barometrically induced water level change at time t , L .

The barometric effect on the water level in the well at the end of a given time step is computed as the sum of the effects of each of the preceding step changes in barometric pressure, each evaluated for the elapsed time since it was assumed to have occurred. This summation may be expressed in equation form, assuming equal time steps, as

where ΔW_L is the barometrically induced change in water level at the end of the j th time step, L , H_i is the magnitude of step change in barometric pressure head during i th time step, L ; Δt is the time step increment, T ; and other symbols are as defined above.

The general form of this equation is attributed to Dukeman, and the method has been widely used for analogous problems of groundwater flow [Rodinger and Reed, 1964; Pinder et al.,

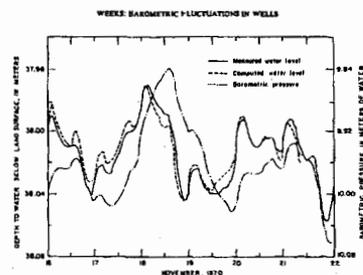


Fig. 5. Comparison of predicted and measured water levels at Lubbock, Texas, Regional Airport during the period November 16-23, 1970, based upon the best fit pneumatic diffusivity chosen from the analysis of data for the period November 6-13, 1970.

1969; Meech et al., 1974). Although (7) appears computationally formidable, it may be conveniently and efficiently programmed for a digital computer. Generally, only a few terms of the infinite series need be computed. In the program written for the analyses described below, series evaluation is halted arbitrarily when the last computed term is less than 10⁻⁶.

Application of the step change method to estimate effective pneumatic diffusivity is demonstrated using water level and barometric data recorded at the Lubbock, Texas, Regional Airport. Water level data from an observation well screened at a depth of 32-40 m in the Ogallala Formation of late Tertiary age and barometric data from the National Weather Service station at the airport, located about 0.5 km from the well, were analyzed for the period from 1200 hours, November 6, until 1200 hours, November 13, 1970. For the analysis the record was divided into 84 2-hour increments, and computations were made using eight different values for pneumatic diffusivity, ranging from 0.032 to 0.064 m²/s.

The best overall match between simulated and measured data, as determined by subjective fitting by eye, was obtained using a pneumatic diffusivity value of 0.059 m²/s and is shown in Figure 4a. The match is quite good, although there are significant departures in parts of the record. Those occurring during the 2 first days may be partially attributed to the effect of assuming equilibrium conditions at the start of simulation with (2). Also the measured water level is generally lower than the computed water level at the start of the record and higher at the end. This may be attributed to a slow upward trend in the water table during the period caused by recovery of water levels following cessation of pumping of nearby irrigation wells earlier in the fall. Finally, the computed water level is advanced in phase from the measured water level. Part of the phase advance may be due to failure to account for the effects of delayed-observation well response in the analysis.

The results of using a substantially different pneumatic diffusivity value (0.032 m²/s) are shown in Figure 4b. Note that the use of the lower diffusivity value results in a larger ampli-

tude difference between computed water levels and measured water levels. However, the phase advance between computed water levels and measured water levels appears to be smaller than it is for a diffusivity value of 0.059 m²/s.

The pneumatic diffusivity value (0.059 m²/s) giving the best match between computed and measured water levels for the November 6-13 period was also used to compute water level fluctuations for the period 1200 hours, November 16, to 1200 hours, November 23, 1970. A good match was obtained, as shown in Figure 5. However, the deviations between measured and computed values during the first day (November 16-17) are less easily explained than those on November 6-7, because the simulation was started on November 13. (Water level records were not obtained from November 13 to November 16, 1970, so no comparison of computed to measured water levels for that period is possible.)

In addition to the two November periods a 1-week period in September was also simulated using a diffusivity value of 0.059 m²/s with good results, as shown in Figure 6. Results and conclusions concerning this simulation are discussed under the section on the amplitude ratio method.

In summary, simulation with (2) of 3 weeks of water level record using step function representation of barometric records as input resulted in a good match with measured water level records. This goodness of fit supports the validity of this explanation of the mechanism producing barometric effects in unconsolidated aquifers and demonstrates the utility of the step change method.

The amplitude ratio method. An attempt was made to determine the effective pneumatic diffusivity from the ratio of the amplitude of diurnal water level fluctuations to the amplitude of the corresponding barometric fluctuations. The theory of water level fluctuations induced by sinusoidal barometric fluctuations provides useful insights concerning observed water level fluctuations. Moreover, use of the amplitude ratio to determine pneumatic diffusivity seems to be an obvious approach. However, attempts to apply the method to data ob-

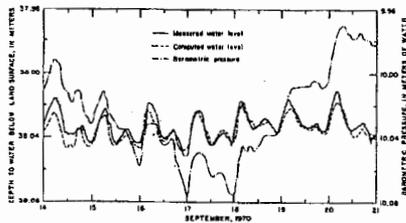


Fig. 6. Microbarograph and well hydrograph records obtained at the Lubbock, Texas, Regional Airport during the period September 14-21, 1970, showing the approximately cyclic, sinusoidal nature of the amplitude ratio analysis. Also shown is the match obtained using a pneumatic diffusivity of 0.059 m²/s to the step change computation. Note that the scale for barometric fluctuations is twice that for Figures 4 and 5.

measured at the Lubbock airport spreading site resulted in a wide range in pneumatic diffusivity values, probably because even the most cyclic barometric records available were only poorly approximated by a sine curve. The theory used attempts to use it as described below.

For this development it is assumed that the barometric pressure at land surface is described by the equation

$$H_s = A_s \sin(2\pi t/T) \quad (8)$$

where H_s is the atmospheric pressure fluctuation about its mean at land surface, expressed as water head, A_s is the amplitude or half range of the cyclic barometric pressure fluctuation, expressed as water head, T is the period of the barometric fluctuation, and t is the elapsed time measured from the start of the sinusoidal cycle, T .

Equation (8) represents the assumed boundary condition at land surface for the amplitude ratio method. It is further assumed that the atmospheric pressure has fluctuated according to (8) for enough cycles that steady cyclic conditions exist. In general, this should be true after one or two cycles (Cooper et al., 1965, p. 3920). Finally, if it is assumed that no flow of soil gas occurs across the water table, (2) may be solved to determine analytical expressions for the amplitude and phase lag of soil gas pressure changes at any depth in the unsaturated zone (Carlaw and Jaeger, 1959, p. 105). The amplitude A_d of the soil gas pressure at the water table, as derived from the general equation, is

$$A_d = A_s \frac{1}{(\cosh 2kL + \cos 2kL)^{1/2}} \quad (9)$$

where A_d is the amplitude of soil gas pressure fluctuation at the water table, in terms of water head, L ; $k = (e/\nu)^{1/2}$; L is the distance from the water table to land surface, L ; and other symbols are as described in the notation section.

Likewise, it can be shown, after considerable algebraic manipulation of an expression given by Carlaw and Jaeger, that the phase lag ϕ of the soil gas pressure at the water table behind that at land surface is given by the equation

$$\phi = \arctan(\tanh kL \tan 2kL) \quad (10)$$

where ϕ is the phase lag of soil gas pressure fluctuation at the water table behind the atmospheric pressure variation, in radians.

The barometric fluctuation ΔW_L is again equal to the difference between the barometric pressure at land surface and that at the water table, or

$$\Delta W_L = A_s \sin \frac{2\pi t}{T} - A_d \sin \left(\frac{2\pi t}{T} + \phi \right) \quad (11)$$

On the basis of wave form analysis (Monley, 1945, pp. 14-20) the amplitude A_w of the resultant water level fluctuation would be given by the expression $(A_s^2 + A_d^2 - 2A_s A_d \cos \phi)^{1/2}$. Dividing through by A_s and letting $A' = A_d/A_s$, this expression becomes

$$A_w/A_s = (1 + A'^2 - 2A' \cos \phi)^{1/2} \quad (12)$$

The ratio A_w/A_s was computed for a range of values of kL by use of (9) and (10) to compute values for A_d and ϕ , which were then substituted into (12). The results of these computations are shown graphically by the upper curve in Figure 7.

As shown in the figure, the amplitude of the water level fluctuation may exceed that of the barometric fluctuation, which at first glance appears paradoxical. The phenomenon occurs when the phase lag of the soil gas pressure change at the water table is nearly 180° behind the atmospheric pressure change. Under this condition the soil gas head is declining in response to the previous trough in barometric pressure at the same time that the barometric pressure is rising. Because the water level change is equal to the algebraic difference between the soil gas head at the water table and the barometric pressure at land surface, it may exceed the barometric pressure change alone, as shown in Figure 6.

The phase lag of the water level wave behind the barometric fluctuation wave is given by the equation (Monley, 1945, p. 14)

$$\Psi = \arctan \left(\frac{-A' \sin \phi}{1 - A' \cos \phi} \right) \quad (13)$$

where Ψ is the phase lag of water level fluctuations behind the barometric fluctuation, in radians. Values of Ψ were also

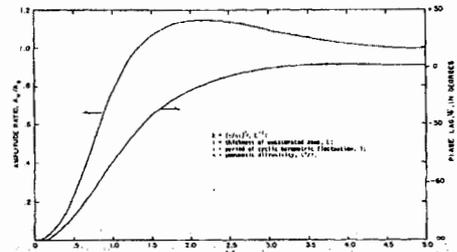


Fig. 7. Graph showing the relationship between the amplitude ratio A_w/A_s and phase lag Ψ of cyclic water level fluctuations in an unconfined aquifer in barometric fluctuations as a function of measured time (hours), pneumatic diffusivity, and cyclic period.

computed for various values of kL . Over a large range of values for kL , Ψ is negative, indicating that the water level fluctuations are advanced in phase in relation to the barometric fluctuations. This advance in phase occurs because the wave form describing the water level fluctuations represents the difference between the atmospheric pressure wave and the soil gas pressure wave at the water table. This interference wave can be advanced in phase in relation to the atmospheric wave, even though the soil gas pressure wave always lags the atmospheric wave. Values of Ψ , the phase lag of the water level fluctuations behind the barometric fluctuations, are shown by the lower curve on Figure 7.

An estimate of effective pneumatic diffusivity was made using the ratio of the amplitude of the water level fluctuations to the barometric fluctuations for the Lubbock, Texas, airport site. Two periods were chosen for this analysis, including one from September 2 to 8 and the other (shown in Figure 6) from September 14 to 21, 1970. These periods were chosen because the water level data appeared quite periodic and simultaneous water level and barometric records were available. A first attempt was made to use the amplitude ratio for individual daily records, but the results were inconclusive owing to much scatter, hence the daily ranges of water level and barometric fluctuations were summed for each period, and an amplitude ratio was computed from these sums. For the period September 2-8, 1970, the sum of the daily ranges of the water level was 17.1 cm and of the barometer was 21.5 cm of water, resulting in an amplitude ratio of 0.826. From Figure 7, $kL/(\pi)^{1/2} = 1.06$ at that ratio. Thus $\alpha = L^2/\nu/(1.06)^2$. The depth to water/liquid 38 ft at this site, and ν , the period, equals 1 day. Consequently, $\alpha = 4100 \text{ m}^2/\text{s}$, or 0.048 m²/s. A similar analysis of the record for the period September 14-21, 1970, resulted in an amplitude ratio of 0.794 and a pneumatic diffusivity of 0.051 m²/s. These values are lower than the 0.059 m²/s determined by the step change method.

As can be seen from Figure 6, the barometric and water

level fluctuations are only poorly represented by a sine curve of daily period, and the records include much noncyclic variation. Two cyclic components are present, however, including one of diurnal period that is primarily due to the effects of heating of the air during the daylight hours, and one of approximately semidiurnal period, that is due to the gravitational pull of the moon and the sun. Consequently, a harmonic analysis was made of both the barometric and water level fluctuations for the two September periods, based on readings made at 2-hour intervals, to separate the diurnal and semidiurnal components within these fluctuations. The analyses were made using the Fort program from the IBM Scientific Subroutine Package and included finding the diurnal and semidiurnal harmonics for each set of records, both for the full period and for each day. These analyses did not substantially improve the results of the amplitude ratio method. For the period September 14-21, 1970, the weekly, diurnal, and semidiurnal harmonics all yielded values of pneumatic diffusivity approximately equal to 0.030 m²/s, but the diffusivity values determined from diurnal harmonics based on individual daily records ranged from 0.018 to 0.12 m²/s and averaged 0.073 m²/s. Diffusivities computed from the amplitude ratio of the semidiurnal fluctuations based on daily records showed a similar range but averaged 0.054 m²/s.

At least part of the wide range of values determined by the amplitude ratio method results from measurement error. During times when barometric and water level fluctuations are nearly periodic, the fluctuations in typically relatively small (say, 4-5 cm of water head). Moreover, the accuracy of the data from the microbarograph and water level charts used in the accuracy of the data from the microbarograph and water level charts used in the above analyses is of the order of ±0.3 cm of water. Trial computations assuming errors of this magnitude indicate an error in computed pneumatic diffusivity of about 30%. Additional error may result from assuming that the unsaturated-zone properties are uni-

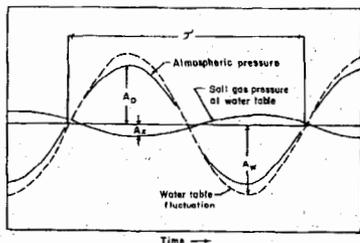


Fig. 8. Diagrammatic representation of cyclic barometric changes and soil gas pressure changes of the water table for conditions producing large-amplitude water level fluctuations.

form with depth and that water level in the well shows instantaneous pressure response.

In summary, the theoretical analysis of the amplitude ratio method provides useful insights on barometrically induced water level fluctuations. In particular, the magnitude of water level fluctuations may exceed that of the barometric fluctuation, and the water level fluctuations generally are advanced in phase in relation to the barometric fluctuations. For the data shown, attempts to identify the pneumatic properties of the unsaturated zone based on analysis of the amplitude ratio of the simultaneous barometric and water level fluctuations were disappointing. The step change method is thus recommended for this application.

Physical significance of pneumatic diffusivity index. Theoretically, pneumatic diffusivity values determined by the analyses described above could be used in conjunction with an independent specific yield estimate to determine the permeability to air of the materials composing the unsaturated zone. However, because the effects of layering of the materials and of slow observation-well response are implicitly included in the diffusivity value, it should be considered to be primarily a curve-fitting parameter. Determination of the permeability to air of the unsaturated materials should instead be made using the technology described by Weeks (1978).

SUMMARY AND CONCLUSIONS

Water levels in wells tapping unconfined aquifers are affected by variations in atmospheric pressure through a mechanism substantially different from that causing such fluctuations in confined aquifers. Moreover, the water level fluctuations in wells tapping an unconfined aquifer may exceed the magnitude of the atmospheric pressure changes under certain circumstances. In confined aquifers, on the other hand, fluctuations of such magnitude are theoretically impossible.

Barometrically induced water level fluctuations result from the resistance to soil gas flow imposed by the materials composing the unsaturated zone and to the compressibility of the soil gas within the air-filled pores. These factors may be quan-

tified as a diffusivity term, defined by Weeks [1978, p. 3] as the effective pneumatic diffusivity. Because of the pneumatic diffusivity of the unsaturated zone, pressure response in the soil gas at the water table lags that at land surface, whereas atmospheric pressure changes are transmitted instantaneously down the well bore. The difference between the atmospheric pressure exerted on the water in the well bore and the soil gas pressure exerted on water in the aquifer results in a water level change.

Any scheme to predict barometrically induced water level fluctuations in wells screened below the water table in deep unconfined aquifers requires an estimate of the pneumatic diffusivity. This parameter may be determined by matching computed to measured water levels for various trial diffusivity values or by analysis of the ratio of the amplitude of the water level fluctuations to that of the concurrent barometric fluctuations. Applications of the two procedures to data obtained at the Lubbock, Texas, Regional Airport indicates that the trial-and-error procedure is more accurate and that the diffusivity determined in this manner can provide reasonably good predictions of barometrically induced water level fluctuations. Moreover, the fluctuations are such that they could not possibly be explained by a simple barometric efficiency correction as applied to water levels in confined aquifers. Consequently, the theory described in this paper should be useful to correct water levels measured in wells tapping unconfined aquifers for barometric effects. Such a theory has not been explicitly described before.

NOTATION

- A_0 amplitude or half range of sinusoidal soil gas pressure fluctuation at the water table, expressed as water head, L .
- A_s amplitude of a cyclic barometric fluctuation, expressed as water head, L .
- A_w amplitude of barometrically induced sinusoidal water level fluctuation, L .
- A/A_0 dimensionless.
- b fraction of the interface between an artesian aquif-

er and the confining layer that is in contact with the water in the aquifer, dimensionless.

H magnitude of step change in atmospheric pressure, expressed as water head, during n th time step, L .

H_n magnitude of step change in atmospheric pressure, expressed as water head, for a single event, L .

h atmospheric pressure fluctuation about its mean, at land surface at time t , expressed as water head, L .

h_s pneumatic head at the water table, expressed as water head, L .

K intrinsic permeability of the porous medium composing the unsaturated zone, L^2 .

k $(\rho/\rho_0)^{1/2}$, $L^{-1/2}$.

α_w relative permeability of the medium to air at its prevailing moisture content, dimensionless.

ρ_0 thickness of unsaturated zone, L .

ρ air-filled porosity, dimensionless.

P mean pressure during pressure change event, M/L^2 .

τ elapsed time since step change in barometric pressure of step start of a sinusoidal cycle in barometric pressure variation, T .

z distance above the water table, L .

α effective pneumatic diffusivity, determined at the prevailing moisture content of the medium, L^2/T .

ΔP instantaneous atmospheric pressure change, expressed as water head, L .

ΔP_A portion of atmospheric pressure change, expressed as water head, transmitted through the unsaturated zone to the water table at time t following an instantaneous pressure change at land surface, L .

ΔP_s portion of the atmospheric pressure change, expressed as water head, borne by the water in a confined aquifer, L .

ΔP_w portion of the atmospheric pressure change, expressed as water head, borne by the aquifer skeleton in a confined aquifer, L .

Δt time step increment, T .

ΔWZ barometrically induced change in water level at time t , L .

ΔWZ_n barometrically induced change in water level at end of n th time step, L .

ν dynamic viscosity of air, $M/L \cdot T$.

τ period of cyclic barometric fluctuation, T .

ϕ phase lag of the soil gas head at the water table behind that at land surface, rad.

ψ phase lag of barometrically induced sinusoidal water level fluctuations behind the barometric fluctuations, rad.

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