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Universal Scaling of Hydraulic Conductivities and Dispersivities in Geologic Media

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An interpretation is offered for the observation that dispersivities increase with scale. Apparent longitudinal dispersivity data from a variety of hydrogeologic settings are assumed to represent a continuous hierarchy of log hydraulic conductivity fields with mutually uncorrelated increments, each field having its own exponential autocovariance, associated integral scale, and variance that increases as a power of scale. Such a hierarchy is shown theoretically to form a self-similar random field with homogeneous increments. Regardless of whether or not the underlying assumption is valid, one can justify interpreting the apparent dispersivities in a manner consistent with a recent quasi-linear theory of non-Fickian and Fickian dispersion in homogeneous media which supports the notion of a self-similar hierarchy a posteriori. The hierarchy is revealed to possess a semivariogram $\gamma(s) \cong cs^{1/2}$, where c is a constant, and a fractal dimension $D \cong E + 0.75$, where E is the topological dimension of interest. This can be viewed as a universal scaling rule about which large deviations occur due to local influences including the existence of discrete natural scales at which log hydraulic conductivity is statistically homogeneous. As such homogeneity is at best a local phenomenon occurring intermittently over narrow bands of the scale spectrum, one must question the utility of associating medium properties with representative elementary volumes and relying on Fickian models of dispersion over more than relatively narrow scale intervals. Porous and fractured media appear to follow the same idealized scaling rule for both flow and transport, raising a question about the validity of many distinctions commonly drawn between such media. Finally, the data suggest that conditioning transport models through calibration against hydraulic measurements has the effect of filtering out large-scale modes from the hierarchy.

INTRODUCTION

Chemical transport in geologic media is known to be strongly influenced by spatial variations in hydraulic conductivity. Such variations produce fluctuations in the groundwater velocity which in turn cause dissolved chemicals to spread at rates considerably greater than those normally observed in laboratory column experiments. Field tracer tests [e.g., *Peaudecerf and Sauty*, 1978; *Freyberg*, 1986] suggest that in relatively uniform materials the rate of longitudinal spread increases in a non-Fickian fashion with time or mean travel distance toward a constant Fickian limit. In an experiment with inorganic tracers at Borden, Ontario, Canada [*Freyberg*, 1986], the plume reached a Fickian rate of longitudinal spread in less than 3 years while the rate of transverse spread continued to vary, resulting in a quasi-Fickian regime. This behavior has been reproduced with increasing degrees of success by means of two-dimensional models based originally on a linear non-Fickian theory due to *Dagan* [1984, 1987, 1988] and more recently a quasi-linear theory developed by *Neuman and Zhang* [1990] and *Zhang and Neuman* [1990]. The latter authors have demonstrated that three-dimensional models are in fundamental conflict with observed behavior at Borden and in other stratified formations but that this conflict is easily resolved (at least in principle) by considering hydraulic anisotropy on the local scale. Both the linear and nonlinear theories deal exclusively with ensemble mean concentrations and therefore provide at best an estimate of how actual concentrations evolve with time. The fact that ensemble plume moments obtained from these theories agree quite well with observed spatial moments of the Borden plume suggests that estimation errors associated with nonergodic phenomena are small compared to sampling errors during both the non-Fickian and quasi-Fickian stages of the experiment.

Existing linear [*Dagan*, 1984, 1987, 1988] and quasi-linear [*Neuman and Zhang*, 1990; *Zhang and Neuman*, 1990] theories of non-Fickian dispersion generally deal with statistically homogeneous media in which the log hydraulic conductivity variance, σ_Y^2, and tensor of correlation or integral scales, **L**, are fixed and finite. Both theories state that if the mean seepage velocity in such a medium is uniform and independent of time, a plume attains a constant (Fickian) rate of longitudinal spread after traversing a mean distance of only a few directional correlation scales, L_μ (defined precisely in (3) below), from its source. This Fickian rate is characterized by a longitudinal macrodispersivity α_L that depends only on the constants σ_Y^2 and L_μ. Yet when one examines over 130 longitudinal dispersivities deduced by means of continuum Fickian theories from laboratory and field tracer studies in a variety of porous and fractured media under varied flow and transport regimes, one finds that they increase without limit with the scale of the study [*Lallemand-Barres and Peaudecerf*, 1978; *Anderson*, 1979; *Pickens and Grisak*, 1981; *Gelhar*, 1986; *Arya et al.*, 1988; *Lake*, 1988, 1989]. Even though a plot of α_L versus experimental scale on logarithmic paper shows a very wide scatter, there is no mistaking a systematic increase in longitudinal dispersivity with scale, known in the literature as "the scale effect."

It is common in the stochastic groundwater literature to distinguish between four major scales: pore, laboratory, local field at the formation level, and regional [*Dagan*, 1986, 1989]. However, *Sposito et al.* [1986, p. 84] have recognized that "a precise mathematical formulation and explanation of

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the scale effect" must be built on the concept of "evolving spatial heterogeneities" which "evidently . . . would require simultaneous consideration of several scales in a continuum manner." This is so because "on physical grounds, it is conceivable that in natural aquifers, more than two time scales come into play in solute transport phenomena as new spatial heterogeneities appear at larger displacement distances. These heterogeneities may have the effect that solute movement is always in a state of transition without ever reaching asymptotic behavior, as was discussed by *Gillham et al.* [1984]." Though "preliminary ideas along this direction are discussed by *Bhattacharya and Gupta* [1983], *Cushman* [1984], and *Gupta and Bhattacharya* [1986] . . . a proper understanding of this very basic and important issue awaits much more research on both the theoretical and experimental sides." *Sposito et al.* [1986, p. 86] have concluded that "the stochastic convection-dispersion model has not yet proved useful in shedding fundamental light on the scale effect."

The stochastic model referred to by *Sposito et al.* [1986] is based on the classical assumption that hydraulic heterogeneities are statistically homogeneous on a scale much smaller than the region being studied. That this assumption may not always be adequate was pointed out nearly 15 years ago by *Winograd and Pearson* [1976] in connection with a regional carbon 14 anomaly at Ash Meadows in south central Nevada. Based on their conclusion that the anomaly is related to transport through the regional carbonate aquifer of the south central Great Basin, the authors developed a doubt that [*Winograd and Pearson*, 1976, p. 1125] "all heterogeneities in fractured, solution-riddled, or lava tube–permeated aquifers are small-scale features or that they may not in some instances reinforce rather than cancel each other with regard to their effects on the distribution of velocity and concomitant dispersion of mass in a given aquifer." They pointed out that "major heterogeneities, such as shoestring sands, can also cause large local differences in velocity (and hence dispersion) even in unfractured, unconsolidated, and highly porous sediments, as was demonstrated by the classic laboratory study of *Skibitzke and Robinson* [1963]." Their own study in Nevada has suggested to the authors that "greatly differing rates of groundwater flow and concomitant distribution of mass may also occur on a regional scale, owing to channeling." Where detailed information about such channels and other flow paths is lacking (as is usually the case), the alternative is to treat them as statistical heterogeneities on a multiplicity of scales.

Past interpretations of the scale effect by *Arya et al.* [1988] (see also *Lake* [1988, 1989]) and *Wheatcraft and Tyler* [1988] have relied on various fractal representations of solute travel distance. Although the slope of log longitudinal dispersivity versus log scale appears to diminish as the latter increases, these interpretations have considered only a single straight-line fit to all or a portion of such data. A line of this kind represents a power relationship between α_L and scale which, as pointed out by *Philip* [1986], is consistent with the notion of a fractal Lagrangian velocity field that has neither a finite variance nor finite temporal correlation scales. Although Philip has not stated so directly, his analysis implies that the Eulerian velocity field, and the log hydraulic conductivity field which gives rise to spatial velocity fluctuations, might also be fractal. That log hydraulic conductivities may indeed exhibit fractal behavior is evident from the analysis of Mount Simon aquifer data by *Ababou et al.* [1988].

The present paper shows that a fractal model of log hydraulic conductivities is indeed consistent with the scale effect exhibited by field and laboratory dispersivities discussed by *Lallemand-Barres and Peaudecerf* [1978], *Pickens and Grisak* [1981], *Gelhar* [1986], and *Arya et al.* [1988] over the entire range of observed scales. These apparent dispersivities derive a priori not from a fractal model of dispersion but from Fickian models which assume (explicitly or tacitly) that log hydraulic conductivity is statistically homogeneous. This and the fact that scale dependence is a non-Fickian phenomenon suggest that the data should be consistent with the recent quasi-linear theory of *Neuman and Zhang* [1990] and *Zhang and Neuman* [1990], which accounts for both Fickian and non-Fickian dispersion in homogeneous media. Such consistency is indeed achieved by stipulating a posteriori that log hydraulic conductivities are not statistically homogeneous but form a random fractal. The result is a scaling rule which in a mean sense applies universally over the broad class of geologic media and wide range of scales represented by the available data.

THEORETICAL BACKGROUND

The quasi-linear theory of *Neuman and Zhang* [1990] and *Zhang and Neuman* [1990] predicts the manner in which ensemble mean concentrations evolve during the preasymptotic or non-Fickian stage toward an asymptotic or Fickian regime in three-dimensional geologic media that are statistically homogeneous. More precisely, the theory predicts how the dispersivity tensor $\boldsymbol{\alpha}$ and spatial covariance tensor $\mathbf{X}$ of ensemble mean concentrations in a plume having zero initial spread vary with dimensionless time (mean travel distance relative to correlation scale) t_μ, effective Peclet number P, and degree of anisotropy in a homogeneous log hydraulic conductivity field characterized by the anisotropic exponential covariance

$$\rho_Y(\tilde{\mathbf{x}}) = \sigma_Y^2 \exp\left[-\left(\frac{\tilde{x}_1^2}{L_1^2} + \frac{\tilde{x}_2^2}{L_2^2} + \frac{\tilde{x}_3^2}{L_3^2}\right)^{1/2}\right] \tag{1}$$

where σ_Y^2 is the variance of $Y = \ln K$, K being hydraulic conductivity, and L_1, L_2, and L_3 are its integral scales in the principal directions of anisotropy $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$. The dimensionless quantities t_μ and P are defined as

$$t_\mu = \mu t / L_\mu \qquad P = \mu L_\mu / d_1 \tag{2}$$

where μ is the magnitude of the constant seepage velocity vector $\boldsymbol{\mu}$, t is time,

$$L_\mu^{-2} = \boldsymbol{\beta}_1^T \mathbf{L}^{-2} \boldsymbol{\beta}_1 \tag{3}$$

$\boldsymbol{\beta}_1$ is a unit vector representing the direction cosines of $\boldsymbol{\mu}$ with respect to principal coordinates $\tilde{\mathbf{x}}$, and T indicates transpose. The scalar L_μ is the length of a radius vector parallel to $\boldsymbol{\mu}$ of an ellipsoid having semiaxes L_1, L_2, and L_3 oriented along $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$, respectively. It represents the "directional integral scale" of the covariance function $\rho_Y(\tilde{\mathbf{x}})$ parallel to the mean seepage velocity vector. The scalar d_1 is the longitudinal component of the effective macrodispersion tensor $\mathbf{d}$, where

$$\mathbf{d} = \mathbf{d}_l + \tfrac{1}{2} \lim_{t \to \infty} d\mathbf{X}/dt \tag{4}$$

and $\mathbf{d}_l$ is a tensor accounting for Fickian dispersion on the local scale (most appropriately molecular diffusion). Since $\mathbf{X}$ cannot be computed without knowing $\mathbf{d}$ and vice versa, the relationship between them is nonlinear (or rather quasi-linear because $\mathbf{d}$ is constant and hence the integrodifferential equation (7) of *Neuman and Zhang* [1990] can be solved in closed form for $\mathbf{X}$). As shown by *Zhang and Neuman* [1990], the eigenvectors of $\mathbf{d}$ are virtually parallel and normal to the velocity vector $\boldsymbol{\mu}$. This means that in the $\tilde{\mathbf{x}}$ coordinates, $\mathbf{d}$ is diagonal with principal components (eigenvalues) d_1 parallel to $\tilde{x}_1$ and $\boldsymbol{\mu}$, d_2 parallel to $\tilde{x}_2$, and d_3 parallel to $\tilde{x}_3$. The ensemble mean is of interest as the most easily obtained estimate of the actual concentration, which of course cannot be predicted without knowing the groundwater velocity at each point in space.

In the absence of local dispersion, P in (2) takes on the simpler form

$$P = L_\mu/\alpha_{\infty L} \tag{5}$$

where $\alpha_{\infty L}$ is the (constant) Fickian asymptote of the longitudinal macrodispersivity α_L as $t \to \infty$. The quasi-linear theory implies that under this condition, P is related to σ_Y^2 and the asymptotic dimensionless longitudinal dispersivity $\alpha_{\infty DL}$ through [*Neuman and Zhang*, 1990, equation (66)]

$$\alpha_{\infty DL} P = 1/\sigma_Y^2 \tag{6}$$

where $\alpha_{\infty DL} \leq 1$. It follows that in mildly nonuniform media in which $\sigma_Y^2 < 1$ the effective Peclet number satisfies $P \geq 1$. Hence the following expression for $\alpha_{\infty L}$, which according to the quasi-linear theory is valid for three-dimensional dispersion in such media, can be stated as

$$\alpha_{\infty L} = \alpha_{\infty DL} L_\mu \sigma_Y^2 \qquad \alpha_{\infty L} \leq L_\mu \tag{7}$$

In some strongly anisotropic media [*Zhang and Neuman*, 1990, Figures 1, 6, and 7], $\alpha_{\infty DL} \ll 1$, and therefore (7) is valid only when $\alpha_{\infty L} \ll L_\mu$. In both cases, $\alpha_{\infty DL}$ is independent of P.

All existing stochastic theories are nominally restricted to very mildly heterogeneous media in which $\sigma_Y^2 \ll 1$. However, the quasi-linear theory is believed by its developers to be less prone to error than linear theories when extended to strongly heterogeneous media in which $\sigma_Y \geq 1$. This is so because the quasi-linear theory deals with nonlinearity due to the deviation of plume "particles" from their mean trajectory without formally limiting σ_Y^2. The expression for $\alpha_{\infty L}$ corresponding to $\sigma_Y \gg 1$ follows from (5), (6), and the relationship [*Neuman and Zhang*, 1990, equation (47)]

$$\alpha_{\infty DL} = c^2 P \qquad P \ll 1 \tag{8}$$

where c is a constant slightly larger than 2 in isotropic media [*Neuman and Zhang*, 1990, Figure 6] but 1 or smaller in anisotropic media [*Zhang and Neuman*, 1990, Figure 1]. It takes the form

$$\alpha_{\infty L} = c L_\mu \sigma_Y \qquad \alpha_{\infty L} \gg L_\mu \tag{9}$$

and, together with (8), may be valid in anisotropic media for $\sigma_Y \ll 1$ or, equivalently, $\alpha_{\infty L} \ll L_\mu$ (i.e., $P \gg 1$ [*Zhang and Neuman*, 1990, conclusion 1]).

Two-dimensional dispersion occurs when mean flow takes place in a plane normal to a principal integral scale that is much larger than the other two principal integral scales, and one-dimensional dispersion occurs when mean flow takes place parallel to a principal integral scale that is much shorter than the other two. Hence two-dimensional dispersion of the kind generally observed in stratified formations (for reasons discussed by *Zhang and Neuman* [1990]), and one-dimensional dispersion of the kind encountered in laboratory column experiments, can be viewed as special cases of three-dimensional dispersion in anisotropic media. We thus expect (7) and (9) to be valid regardless of whether dispersion takes place in one, two, or three dimensions within a uniform mean velocity field.

Let $s(t) = \mu t$ designate the mean travel distance of a plume from its source. Then the following expression for longitudinal dispersivity during the early stages of non-Fickian transport, which the quasi-linear theory states is valid when $t_\mu \ll 1$ and $t_\mu \ll P$, can be written as [*Neuman and Zhang*, 1990, equation (42)]

$$\alpha_L(s) = c_0 s(t) \sigma_Y^2 \qquad s(t) \ll L_\mu \qquad \frac{s(t)}{L_\mu} \ll \frac{L_\mu}{\alpha_{\infty L}} \tag{10}$$

where $c_0 \cong 0.5$ in statistically isotropic media [*Neuman and Zhang*, 1990, Figure 6] and can be 1 or larger in statistically anisotropic media [*Zhang and Neuman*, 1990, Figure 1]. This shows that when L_μ is larger than both $s(t)$ and $\alpha_{\infty L}$, the preasymptotic longitudinal dispersivity varies linearly with the mean travel distance. On the other hand, when $P \ll 1$ and $P \ll t_\mu$, according to the quasi-linear theory, the non-Fickian and Fickian macrodispersivities are identical [*Neuman and Zhang*, 1990, equation (47)]. Hence we have in analogy to (9)

$$\alpha_L = c L_\mu \sigma_Y \qquad \alpha_{\infty L} \gg L_\mu \qquad \frac{L_\mu}{\alpha_{\infty L}} \ll \frac{s(t)}{L_\mu} \tag{11}$$

which, like (9), may be valid in anisotropic media when $\alpha_{\infty L} \ll L_\mu$ and $s(t) \leq L_\mu$.

The above results apply to a statistically homogeneous (in a wide sense) medium characterized by a given log hydraulic conductivity variance σ_Y^2 and tensor of integral scales, $\mathbf{L}$. However, geologic media tend to form discrete homogeneous units on not one but a hierarchy of scales. A geologic unit may exhibit homogeneity with respect to log hydraulic conductivity on a given scale and to have a mean that is drastically different from that of all neighboring units. As long as a plume evolves primarily within the confines of such a homogeneous unit, its spread is controlled almost entirely by the properties of this unit. If the plume travels a mean distance of several integral scales while remaining entirely within the confines of the same unit, it may start showing Fickian or quasi-Fickian behavior [*Neuman and Zhang*, 1990, Figures 6–8; *Zhang and Neuman*, 1990, Figures 1–3, 6–11, 14–16]. As the plume starts spreading into neighboring units, larger-scale fluctuations in log hydraulic conductivity come into play. Their effect can be included in the analysis through the notion of nested structures, as has been done for various soil properties by *Burrough* [1983*a*, *b*] (see also *Journel and Huijbregts* [1978, p. 150]) and for groundwater velocities by *Philip* [1986].

Let $Y^i(\mathbf{x})$ be the log hydraulic conductivity fluctuation

associated with the ith homogeneous unit in the hierarchy. We represent $Y^i(\mathbf{x})$ as the superposition of separate contributions $Y_j^i(\mathbf{x})$ from this and all lower levels of the hierarchy, $0 \le j \le i$,

$$Y^i(\mathbf{x}) = \sum_{j=0}^{i} Y_j^i(\mathbf{x}) \tag{12}$$

Each of the contributing fluctuations $Y_j^i(\mathbf{x})$ has a distinct variance σ_{Yj}^2 and tensor of integral scales $\mathbf{L}_j$ such that $\bar{L}_j < \bar{L}_{j+1}$ where $\bar{L}_j = \|\mathbf{L}_j\|$, the latter being a consistent norm (measure of the "magnitude") of $\mathbf{L}_j$. Spatial increments of the contributions are mutually uncorrelated so that the semivariogram of $Y^i(\mathbf{x})$, $\gamma_Y^i(\mathbf{s})$ where $\mathbf{s}$ is a displacement vector, becomes simply the sum of semivariograms contributed by $Y_j^i(\mathbf{x})$,

$$\gamma_Y^i(\mathbf{s}) = \sum_{j=0}^{i} \gamma_{Yj}(\mathbf{s}) \tag{13}$$

Although geologic media are often divisible into discrete hierarchical units, we work below with a continuous hierarchy and an infinite range of scales. In particular, we replace the summation in (13) by an infinite integral as was done for the Lagrangian correlation function of velocities in the time domain by *Philip* [1986]. We concentrate on semivariograms along a given direction defined by a unit vector $\boldsymbol{\nu}$ parallel to the displacement vector $\mathbf{s}$. Let s be the scalar magnitude of $\mathbf{s}$ and L_ν the integral scale in the direction of the displacement, defined as

$$L_\nu^{-2} = \boldsymbol{\nu}^T \mathbf{L}^{-2} \boldsymbol{\nu} \tag{14}$$

Then the semivariogram that corresponds to the exponential covariance function (1), with directional integral scale L_ν and associated variance $\sigma_Y^2(L_\nu)$, takes the form

$$\gamma_Y(s; L_\nu) = \sigma_Y^2(L_\nu)[1 - \exp(-s/L_\nu)] \tag{15}$$

Integration over all possible integral scales L_ν yields

$$\gamma_Y(s) = \int_0^\infty \sigma_Y^2(n)(1 - e^{-ns})\, dn \tag{16}$$

where n, defined as

$$n = L_\nu^{-1} \tag{17}$$

is a wave (or mode) number representing the spatial periodicity (or frequency) of log hydraulic conductivity fluctuations in the direction $\boldsymbol{\nu}$. Clearly, the semivariogram $\gamma_Y(s)$ in (16) is not associated with any preferred variance σ_Y^2 or integral scale L_ν.

Consider the special case where

$$\sigma_Y^2(n) = C/n^{1+2\omega} \qquad 0 < 2\omega < 1 \tag{18}$$

and C is a positive constant so that the variance decreases as a power of the mode. Then the integral in (16) becomes [*Gradshteyn and Ryzhik*, 1980, equation 3.551(1), p. 360]

$$\gamma_Y(s) = C_0 s^{2\omega} \tag{19}$$

where C_0 is a positive constant equal to $-C\Gamma(-2\omega)$ and Γ is the gamma function. In other words, the semivariogram of log hydraulic conductivity grows as a power of the distance s. Since the exponent 2ω of this power structure satisfies $0 < 2\omega < 1$, it follows [*Yaglom*, 1987, pp. 406–411] that $Y(\mathbf{x})$ is a self-similar random field with homogeneous increments (some prefer to call such fields self-affine [cf. *Voss*, 1985]). This means that $\gamma_Y(s)$ is invariant under a group of similarity (or affinity) transformations $s \to hs$, $Y \to f(h)Y$, where $f(h)$ is some function of h; when $f(h) = h^{-\omega}$ and $h > 0$ one has $\gamma_Y(s) = h^{-2\omega}\gamma_Y(hs)$.

Although (19) was derived above for $0 < 2\omega < 1$ under special assumptions, it is in fact known to be a valid (and the only valid) semivariogram for self-similar stochastic processes over the broader range $0 < \omega < 1$; as such, it was considered originally by Kolmogorov in 1940 [*Yaglom*, 1987]. The associated processes represent fractional Brownian motion (fBm) [*Mandelbrot and Wallis*, 1968; *Mandelbrot*, 1982] which constitutes a mathematical model for random fractals. As explained in an admirably lucid fashion by *Voss* [1985], the latter are associated with a fractal dimension

$$D = E + 1 - \omega \qquad 0 < \omega < 1 \tag{20}$$

where E is the topological dimension of interest. When $\omega = 0.5$ and $D = E + 0.5$, increments of the property are uncorrelated in E-dimensional space and thus resemble Brownian motion (which has statistically independent increments). When $\omega > 0.5$ and $D < E + 0.5$, the increments are positively correlated so that the property shows relatively smooth variations characterized by long-range persistence of positive and negative values. The tendency of many geophysical time series such as tree rings, varves, precipitation, and streamflow to exhibit such persistence is known as Hurst's phenomenon, and ω is therefore sometimes called the Hurst coefficient [cf. *Bras and Rodriguez-Iturbe*, 1985, pp. 220–221]; similar persistence is exhibited in space by landforms and the thickness as well as lithology of Paleozoic sediments [cf. *Burrough*, 1983*a*, Table 3]. When $\omega < 0.5$ and $D > E + 0.5$, the increments are negatively correlated, and the property appears to be relatively noisy. Such antipersistence seems typical of many soil parameters including percent fine or coarse material, bulk density, layer thickness, moisture capacity, and pH [*Burrough*, 1983*a*, Tables 1 and 2] as well as log hydraulic conductivities [*Ababou et al.*, 1988], all of which exhibit a dominance of short-range effects in space. Regardless of whether the correlation is positive or negative, it extends over arbitrarily large spatial scales.

INTERPRETATION OF SCALE EFFECT

We now turn our attention to over 130 longitudinal dispersivities from laboratory and field tracer studies in porous and fractured media throughout the world recorded by *Lallemand-Barres and Peaudecerf* [1978], *Pickens and Grisak* [1981], *Gelhar et al.* [1985], *Arya* [1986], and *Arya et al.* [1988]. These dispersivities vary from less than 1 mm to more than 1 km and correspond to studies conducted on scales ranging from less than 10 cm to more than 100 km. As pointed out in the introduction, these apparent dispersivities derive a priori not from a fractal model of dispersion but from Fickian models which assume (explicitly or tacitly) that log hydraulic conductivity is statistically homogeneous. This and the fact that scale dependence is a non-Fickian phenomenon suggest that the data should be consistent with the

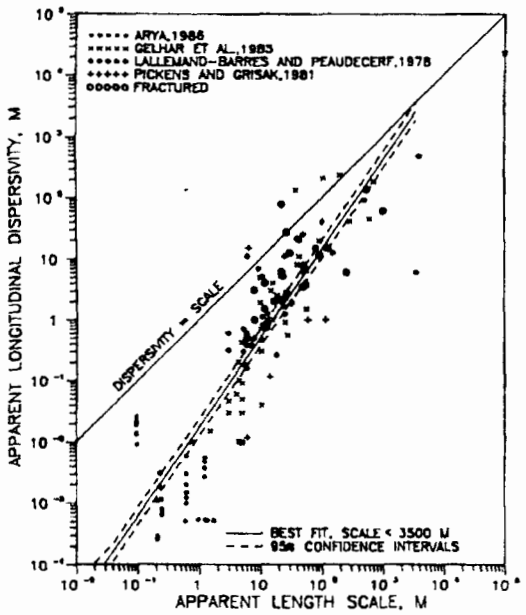

Fig. 1. Apparent longitudinal dispersivities versus scale of study excluding numerical model calibration results; first interpretation.

recent quasi-linear theory of *Neuman and Zhang* [1990] and *Zhang and Neuman* [1990] which accounts for both Fickian and non-Fickian dispersion in homogeneous media. It is shown below that such consistency is indeed achieved by stipulating, a posteriori, that log hydraulic conductivities are not statistically homogeneous but form a self-similar random field with homogeneous increments as defined in (19).

Figure 1 is a logarithmic plot of apparent longitudinal dispersivities α_{aL} versus the scale of the study L_s as reported by the aforementioned authors. For reasons to be discussed later the figure does not include dispersivities obtained by calibrating numerical grid models against solute concentrations of large-scale plumes. The remaining 134 dispersivities can be interpreted in at least two ways, both of which lead to the same scaling rule. The first interpretation, illustrated in Figure 1, is based on the assumption that (7) is valid for $\alpha_{\infty L} \leq L_\mu$ (the second assumes that (7) is not valid unless $\alpha_{\infty L} \ll L_\mu$). For reasons to become clear later, the first interpretation excludes the three points in Figure 1 which correspond to $L_s \geq 3500$ m. It consists of fitting by regression a line of the form

$$\log_{10} \alpha_{aL} = b \log_{10} L_s + c \tag{21}$$

to the remaining 131 points as shown, together with the corresponding 95% confidence limits, in the figure. This line of best fit can be expressed as

$$\alpha_{aL} = 0.0175 L_s^{1.46} \tag{22}$$

with a regression coefficient $R^2 = 0.74$ and 95% confidence intervals [0.0113, 0.0272] about the coefficient 0.0175, and [1.30, 1.61] about the exponent 1.46.

An examination of the methods that had led to the data in Figure 1 caused *Gelhar* [1986] and others to raise valid questions about the reliability of many of them. Indeed, part of the large scatter shown by these data can undoubtedly be attributed to experimental and interpretive errors; other reasons of equal or greater importance for this scatter will be suggested later. Yet the data cannot be dismissed as being devoid of quantitative significance because, despite their questionable reliability and considerable scatter, they give rise to a regression line which is confined between remarkably narrow 95% confidence limits and explains nearly three quarters (74%) of their variation about the mean. In other words, experimental and interpretive errors account for not more (or, as implied below, much less) than one quarter (26%) of the observed random scatter about the mean, and one is therefore justified to ask what gives rise to the remaining 74% of systematic variation exhibited by the data at $L_s \leq 3500$ m in Figure 1.

To answer this question in a manner which is consistent with the quasi-linear theory, we consider a plume or concentration front having initially zero spread whose center of gravity has traveled a distance $s = L_s$. This means that we treat the reported experimental scales L_s as mean travel distances and thereby introduce an error into our analysis. However, this error is expected to be small in comparison to the large scatter of data in Figure 1 because tracer studies are commonly interpreted on the basis of breakthrough curves registered at a distance L_s from the source and the time required for such a breakthrough is often much shorter than the mean travel time associated with L_s.

The data in Figure 1 were obtained from laboratory and field tracer studies interpreted on the basis of traditional Fickian models which correspond to (7). Yet such a Fickian interpretation is analogous to a non-Fickian interpretation on the basis of (10) if one replaces $\alpha_{\infty DL} L_\mu$ by $c_0 s(t)$. Since we equate s with L_s, the same is also equivalent to replacing $\alpha_{\infty DL} L_\mu$ by $c_0 L_s$. Hence the Fickian interpretation is analogous to a non-Fickian interpretation in which the apparent longitudinal dispersivity is calculated according to

$$\alpha_{aL} = c_0 L_s \sigma_Y^2 \tag{23}$$

As c_0 and $\alpha_{\infty DL}$ are of similar order and our first interpretation assumes (7) to be valid when $\alpha_{\infty L} \leq L_\mu$, (23) should provide a reasonable fit to the data in Figure 1 as long as $\alpha_{aL} \leq L_s$ (the fact that this equation derives from a theory which deals with ensemble mean rather than real concentrations can be viewed merely as another interpretive error which contributes to the aforementioned scatter). The latter condition is indeed satisfied by all data corresponding to $L_s \leq 3500$ m which form the basis for regression formula (22). For this formula to be consistent with (23) it is necessary that σ_Y^2 vary with L_s according to the power law

$$\sigma_Y^2 = C_0 L_s^{2\omega} \tag{24}$$

where $C_0 = 0.0175/c_0$ with $\omega = 0.23$ with a 95% confidence interval [0.15, 0.30]. As we equate L_s with s, (24) is analogous to (19) provided only that σ_Y^2 is given by the semivariogram

$$\gamma_Y(L_s) = (0.0175/c_0) L_s^{2\omega} \tag{25}$$

Hence the regression line in Figure 1 represents a self-similar log hydraulic conductivity field with homogeneous increments characterized by the fractal dimension $D \cong E + 0.77$ with 95% confidence interval [1.70, 1.85].

Our second interpretation considers that many of the reported tracer studies have been conducted in anisotropic

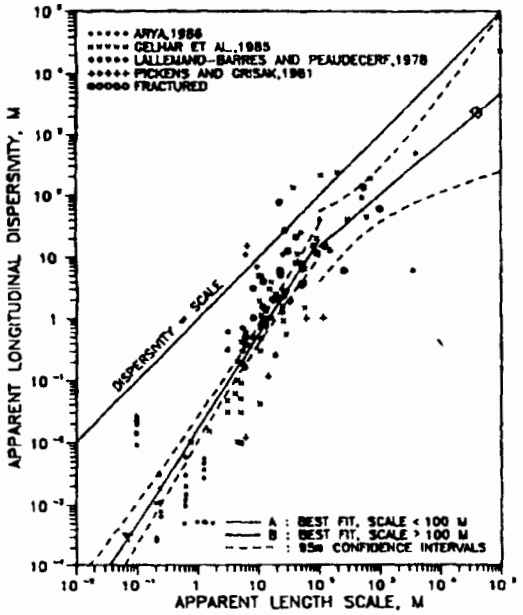

Fig. 2. Apparent longitudinal dispersivities versus scale of study excluding numerical model calibration results; second interpretation.

media and hence (7) may not be valid unless $\alpha_{\infty L} \ll L_\mu$. To account for this possibility, we fit in Figure 2 two separate regression lines of the form (21) to 119 data points associated with $L_s \leq 100$ m (line A) and 16 data points associated with $L_s \geq 100$ m (line B); one point lies at $L_s = 100$ m and belongs to both lines. Line A can be expressed as

$$\alpha_{aL} = 0.0169 L_s^{1.53} \tag{26}$$

with a regression coefficient $R^2 = 0.71$ and 95% confidence intervals [0.0108, 0.0264] about the coefficient 0.0169, and [1.35, 1.70] about the exponent 1.53. This is very similar to (22) and, in analogy to (25), leads to

$$\gamma_Y(L_s) = (0.0169/c_0) L_s^{2\omega} \tag{27}$$

where now $\omega = 0.26$ with a 95% confidence interval [0.17, 0.35]. Since this is very similar to (25), we combined the two into a single approximate scaling rule by setting $\omega \cong 0.25$ and writing

$$\gamma_Y(L_s) \cong (0.017/c_0) L_s^{1/2} \tag{28}$$

In other words, we conclude on the basis of our two interpretations that log hydraulic conductivities of the geologic media represented in Figures 1 and 2 constitute a self-similar random field with homogeneous increments characterized by the semivariogram (28) and a fractal dimension $D \cong E + 0.75$. We will show below that even though this conclusion was reached by excluding data associated with $L_s \geq 3500$ m in Figure 1 and $L_s > 100$ m in Figure 2, it is nevertheless consistent with these data.

Since the materials represented in Figures 1 and 2 include unconsolidated sediments as well as consolidated porous and fractured rocks of diverse origin from a variety of sites, the self-similarity and associated semivariogram in (28) appear to be a universal property of many geologic media. The marked scatter of the data about the regression line in Figure 1 and line A in Figure 2, attributed earlier in part to experimental and interpretive errors (including those stemming from differences between the spatial and temporal moments of ensemble mean and real concentrations), can now be attributed in large measure to local deviations from this universal self-similarity pattern. One major reason for such deviations appears to be variations from one medium to another in the coefficient C which determines the log hydraulic conductivity variance in (18) and affects the semivariogram coefficient C_0 in (19). Lesser deviations are expected to arise from fluctuations in the coefficient c_0 that derives from the quasi-linear theory and affects (10) as well as (23)–(25) and (27)–(28), due to the diversity of flow and anisotropy conditions encountered in the various tracer studies. Most importantly, the semivariogram in (28) is an idealization based on the notion of a continuous hierarchy of scales which does not strictly apply to geologic media possessing discrete natural scales at which the log hydraulic conductivity field is homogeneous. The existence of such natural scales was postulated over 30 years ago by *Hubbert* [1957], and more recent discussions of the concept can be found, for example, in publications by *Cushman* [1987, 1990]. The legality of associating medium properties with representative elementary volumes (REVs) is limited at best to natural scales, as is the possibility for the development of Fickian or quasi-Fickian dispersion regimes. Since tracer studies are often conducted within discrete geologic units exhibiting some degree of homogeneity, many of the apparent dispersivities in Figures 1 and 2 are expected to represent natural scales and therefore to deviate from the continuous model manifested by the regression lines in these figures. Hence the universal scaling rule embodied in (28) must not be viewed as a representation of actual conditions at any given locale but rather as self-similarity of log hydraulic conductivities in a mean sense over a large range of length scales in a broad variety of geologic media under diverse conditions of flow and transport.

From (22) and (26) it is clear that the universal semivariogram (28) corresponds to

$$\alpha_{aL} \cong 0.017 L_s^{1.5} \tag{29}$$

which is valid at most for $\alpha_{aL} \leq L_s$ or, equivalently, for $L_s \leq 3460$ m; this is why data points corresponding to $L_s \geq 3500$ m were excluded from the regression in Figure 1. Our second interpretation of the data according to regression line A in Figure 2 assumes that (29) is valid only for $L_s \leq 100$ m or equivalently $\alpha_{aL} \ll L_s$. Since the latter equation arises from (10), this is the same as saying that data corresponding to $L_s \gg 100$ m must be interpreted on the basis of (11) rather than (10). We stated earlier that the data were interpreted on the basis of traditional Fickian models which become analogous to (11) provided L_μ is replaced by $c_1 L_s$, where $c_1 = c_0/\alpha_{\infty DL}$ and $\alpha_{\infty DL}$ is constant. Consistent with this replacement, (11) yields the following expression for apparent longitudinal dispersivity,

$$\alpha_{aL} = c_2 L_s \sigma_Y \tag{30}$$

where $c_2 = cc_1$. Upon further replacement of σ_Y by $\gamma_Y^{1/2}$ according to the universal scaling rule (28) this becomes

$$\alpha_{aL} \cong c_3 L_s^{1.25} \tag{31}$$

where $c_3 = (0.017/c_0)^{1/2}c_2$. We see that whereas the fitted slope of α_{aL} versus L_s in Figure 2 is approximately 1.5:1 for $L_s \leq 100$ m, the quasi-linear theory predicts that the slope should drop to 1.25:1 for $L_s \gg 100$ m. In other words, the quasi-linear theory predicts a reduction in the slope of $\log_{10} \alpha_{aL}$ versus $\log_{10} L_s$ as α_{aL} approaches L_s, a phenomenon qualitatively consistent with our interpretations of the data in Figures 1 and 2. The same phenomenon was noted earlier by others but could not be interpreted theoretically. In fact, all previous attempts at a fractal analysis of the scale effect (none of which dealt directly with hydraulic conductivities) were limited to a single straight line on logarithmic paper [*Philip*, 1986] fitted either to all the available laboratory and/or field data [*Arya et al.*, 1988; *Lake*, 1988, 1989] or to a portion of these data corresponding to relatively small L_s [*Wheatcraft and Tyler*, 1988]. It is therefore important that we ask whether the reduction in slope indicated by line B in Figure 2 is quantitatively consistent with the quasi-linear prediction embodied in (31).

To answer this question, we note that data corresponding to $L_s \geq 100$ m are represented in Figure 2 by regression line B, which can be expressed as

$$\alpha_{aL} \cong 0.32 L_s^{0.83} \tag{32}$$

with a regression coefficient $R^2 = 0.44$ and 95% confidence intervals [0.026, 3.95] about the coefficient 0.32, and [0.30, 1.37] about the exponent 0.83. Upon testing the null hypothesis that the above slope is admissible under the regression leading to line B we find that it must be rejected at the 80% confidence level. In other words, our answer to the above question is negative and indicates that line B is inconsistent with the quasi-linear theory. We show below that this inconsistency is not real but an artifact of the method used to derive the data.

Equation (31) derives in part from (30), which follows from (11), the latter being based on (9) and (8). The traditional Fickian interpretation was shown to imply that $\alpha_{\infty DL}$ is a constant and L_μ can be replaced by $c_0 L_s$. Hence on the basis of (5) and (8) it implies further that

$$\alpha_{\infty L} = c_4 L_s \tag{33}$$

where c_4 is a constant. In analogy to the manner in which (9) gave rise to (11) and (30), (33) gives rise to

$$\alpha_{aL} = c_4 L_s \tag{34}$$

Comparing (34) with (31) shows that in contrast to the quasi-linear theory which states that α_{aL} should grow as $L_s^{1.25}$, the traditional (linear) model leads to apparent dispersivities that are linearly proportional to L_s. Indeed, the null hypothesis that the exponent in (32) is not 0.83 but 1.00, i.e., that (32) can be replaced by

$$\alpha_{aL} = bL_s \tag{35}$$

where b is a coefficient, cannot be rejected at the 50% confidence level. This means that (35) constitutes an acceptable alternative to line B in Figure 2 and the corresponding data are consistent with our theoretical relation (34). The latter relation, however, is an artifact arising from the interpretation of tracer studies by means of a linear theory where the quasi-linear theory is considered (in our second interpretation) to be more appropriate. The large scatter of the data about line B is due to fluctuations in the coefficient c_4 and other reasons of the type discussed earlier.

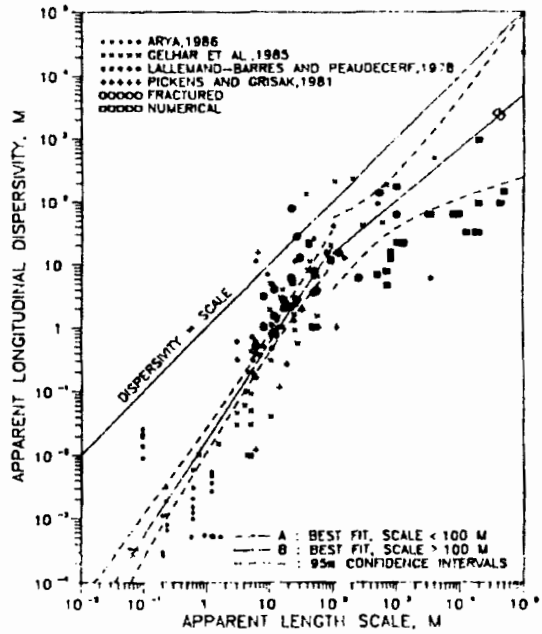

Fig. 3. Apparent longitudinal dispersivities versus scale of study including numerical model calibration results compared to second interpretation.

The dispersivities in Figures 1 and 2 were obtained by considering the hydraulic conductivity (or transmissivity) to be uniform in each tracer study. Figure 3 is similar to Figure 2 but includes dispersivities obtained from the calibration of numerical models against hydraulic and concentration data corresponding to large-scale plumes. Most of the calibrated dispersivities are seen to be associated with scales in excess of 100 m and to lie below the lower confidence limit of line B. Applying regression to the calibrated dispersivities yields a straight line with a slope of 0.54 (not shown in the figure). When the regression includes all dispersivities corresponding to $L_s \geq 100$ m, the slope reduces to 0.48 (not shown). In both cases the null hypothesis that the slope is 1 or larger must be rejected at the 99% confidence level. Hence the calibrated dispersivities are consistent neither with our first theoretical interpretation of the data in Figure 1 nor with the second in Figure 2.

The calibrated dispersivities in Figure 3 grow approximately in proportion to $L_s^{1/2}$, as does the discrepancy between them and theoretical values predicted by (34). Hence they can be made consistent with our theory if one associates them with a reduced scale L_r proportional to $L_s^{1/2}$. This reduction in scale appears to be caused by the fact that numerical model calibration often provides information about the spatial variation of hydraulic conductivities (or transmissivities) on scales exceeding the dimensions of model subregions (called "zones") within which they are kept constant or allowed to vary at a relatively slow rate (J. Carrera, personal communication, 1989). Calibration is thus tantamount to filtering out low-frequency modes from the random log hydraulic conductivity field and thereby reducing the range of length scales that affect dispersivities to below the characteristic length of the zones. In other words,

the dispersivities are associated with a length scale L_r which depends on the dimensions of the zones rather than on the mean travel distance L_s of the plume. The above regression analysis suggests that in past calibration exercises, L_r increased with the mean travel distance at an average rate proportional to $L_s^{1/2}$. This in turn implies that modelers tended to design the zones of their computational grids in a manner roughly proportional to $L_s^{1/2}$.

CONCLUSIONS

The following major conclusions can be drawn from this study:

1. Consider a hierarchy of homogeneous log hydraulic conductivity fields with mutually uncorrelated increments, each field having an exponential covariance with a distinct integral scale and a variance which increases as a power of this scale. Then the superposition of a continuous hierarchy of such fields over all scales ranging from zero to infinity yields a self-similar (or self-affine) random field with (wide sense) homogeneous increments.

2. Apparent longitudinal dispersivities α_{aL} from laboratory and field tracer studies in a variety of porous and fractured media in diverse hydrogeologic settings show a systematic increase with the scale of the study, L_s. It is possible and appropriate to interpret this scale effect by requiring that the data be consistent a priori with the recent quasi-linear theory of non-Fickian and Fickian dispersion in homogeneous media due to *Neuman and Zhang* [1990] and *Zhang and Neuman* [1990], but that they represent a posteriori a self-similar log hydraulic conductivity field with homogeneous increments. Regression against the available data implies, at a high level of confidence, that this field is associated with a semivariogram $\gamma(s) \cong cs^{1/2}$, where c is a constant and s is a measure of distance. Such a semivariogram is in turn characteristic of a random fractal with dimension $D \cong E + 0.75$, where E is the topological dimension of interest ($E = 1$ for a linear transect, $E = 2$ for a two-dimensional plane or cross section, $E = 3$ for a three-dimensional medium).

3. The above semivariogram and fractal dimension can be viewed as a universal scaling rule which does not necessarily describe conditions at any given locale but accounts for the self-similarity of log hydraulic conductivities in a mean sense over a large range of length scales in a broad variety of geologic media under diverse conditions of flow and transport. Actual α_{aL} data show sizable random fluctuations about such mean behavior due to a variety of causes including experimental and interpretive errors (caused in part by the fact that existing dispersion theories deal with ensemble mean rather than real concentrations), variations in material properties from one locale to another, and differences between flow and transport regimes during different tracer studies. A major cause for local deviations from universal behavior appears to be the presence in many geologic media of discrete natural scales at which the log hydraulic conductivity field is homogeneous. This work implies that such discrete scales occur at best intermittently over relatively narrow bands of the scale spectrum.

4. Based on the aforementioned universal scaling rule, the quasi-linear theory of *Neuman and Zhang* [1990] and *Zhang and Neuman* [1990] predicts a reduction in the slope of $\log_{10} \alpha_{aL}$ versus $\log_{10} L_s$ as α_{aL} approaches L_s. A similar reduction has been noted earlier by others but could not be interpreted theoretically. Previous attempts at a fractal analysis of the scale effect (none of which dealt directly with hydraulic conductivities) were limited to a single straight line on logarithmic paper fitted either to all the available laboratory and/or field data [*Arya et al.*, 1988] (see also *Lake* [1988, 1989]) or to a portion of these data corresponding to relatively small L_s values [*Wheatcraft and Tyler*, 1988] while saying little about dispersivities on larger scales.

5. Equations (20), (23), and (24) imply jointly that longitudinal dispersivity varies with scale as a positive power of the Hurst coefficient ω and a negative power of the fractal dimension D. This makes physical sense considering that an increase in ω (and, since $D = E + 1 - \omega$, a decrease in D) is associated with an enhancement of long-range phenomena which cause log hydraulic conductivities to persist over increased distances: the effect on dispersivity is qualitatively similar to that of lengthening the spatial correlation scale in a statistically homogeneous medium which causes it to increase. Our finding that $\omega \cong 0.25$ and $D \cong E + 0.75$ suggests that log hydraulic conductivities are generally dominated by short-range phenomena which render them relatively noisy though their increments are correlated over arbitrarily large spatial scales. This is qualitatively consistent with the conclusion of *Ababou et al.* [1988] that log hydraulic conductivities of the Mount Simon aquifer are associated with $\omega \cong 0$. Our finding that longitudinal dispersivity varies with scale as a positive power of ω and a negative power of D is consistent with related analyses by *Philip* [1986, equation (46)] and *Arya et al.* [1988, equation (24)] but inconsistent with (16) of *Wheatcraft and Tyler* [1988], which appears to state the opposite. The consistency is encouraging, and the inconsistency puzzling despite the different meanings ascribed to ω and D by the various authors.

6. Since homogeneity is at best a local phenomenon limited to random and relatively narrow intervals of scale, one must question the utility of associating medium properties with representative elementary volumes (REVs) as has been the custom in subsurface hydrology for several decades. Alternatives to the traditional REV concept as applied to porous media have been discussed in a recent review article by *Cushman* [1987]; arguments in favor of treating fractured crystalline rocks as multiscale stochastic continua to which similar alternatives apply have been put forth by *Neuman* [1987, 1988] and *Neuman and Depner* [1988].

7. The latter arguments in favor of treating crystalline fractured rocks as multiscale continua can be extended on the basis of this work to a variety of other fractured media. This follows from our finding that many fractured and porous materials follow the same universal scaling rule. Since the latter rule applies to log hydraulic conductivities which affect flow, and extends to dispersivities which affect transport, one must question the prevailing tendency among many hydrologists to draw fundamental distinctions between the manners in which fractured and porous media conduct single-phase fluids or affect the spread of chemicals dissolved in such fluids when detailed information about the geometry and transport properties of individual fractures is not available. As the lack of such information seems to be the rule rather than the exception [*Neuman*, 1987, 1988], treating fractured rocks as multiscale continua appears to be a viable alternative in many cases.

8. Given that homogeneity is at best an intermittent local occurrence on relatively narrow scale intervals, one must question the continued application of Fickian transport models to subsurface plumes that spread over more than a limited range of scales. A plume spreading through a homogeneous medium of infinite extent may attain a Fickian mode of dispersion asymptotically. However, geologic media are finite and possess a hierarchy of scales. Hence Fickian or quasi-Fickian behavior constitutes at best intermittent episodes during what is otherwise an inherently non-Fickian mode of transport.

9. Longitudinal dispersivities obtained from the calibration of numerical models against hydraulic and concentration data are found to vary more slowly with the scale of the study than do dispersivities determined by other means. This appears to be caused by the fact that calibration often provides information about the spatial variation of hydraulic conductivities (or transmissivities) on scales exceeding the dimensions of model subregions (called "zones") within which they are kept constant or allowed to vary at a relatively slow rate. Calibration is thus tantamount to filtering out low-frequency modes from the random log hydraulic conductivity field and thereby reducing the range of length scales that affect dispersivities to below the characteristic length of the zones. Stated otherwise, conditioning transport models on hydraulic data has the effect of filtering out large-scale modes from the multiscale hierarchy of log hydraulic conductivity fields. This leads to the important operational conclusion that the scale parameter which controls dispersivities in a given transport model diminishes as the density of information about hydraulic heterogeneity goes up.

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