s media, 1, Unconsolidated
ys., 32(9), 1688–1699, 1961.
numerical solution of elliptic
ial differential equations, in
ical Analysis, edited by J.
, McGraw-Hill, New York,

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Relationship between Horizontal Strain near a Well and Reverse Water Level Fluctuation

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Abstract. Measurements of radial surface strains near a pumping well showed compression near the well that changed to tension farther away from the well. A theoretical model is presented. In general, this model reproduces the measured strains. Observations of anomalous increase in pore pressure in a clay adjacent to the pumped aquifer are related to the development of these strains. These increases in pressure are attributed to distortion of the pore space in the clay resulting from the transference of horizontal strain from the aquifer via shear. Applying a conceptual model that permits the transference of strain across confining layers to unpumped artesian aquifers accounts for reverse water level fluctuations.

INTRODUCTION

The water level response in observation wells in the vicinity of a pumping well is occasionally observed to be the opposite of that predicted. Some 'reverse' fluctuations can be explained by pump column loading [*Barksdale et al.*, 1936, p. 87], compaction of the aquifer material due to vibration of the pumping engine (H. H. Cooper, Jr., oral communication, 1969), dilatancy [*Andreason and Brookhart,* 1963], or the Noordbergum effect [*Verruijt,* 1969]. However, these explanations do not suffice for all observed reverse fluctuations.

In an earlier report [*Wolff,* 1970, p. 197] mention was made of an observed increase in pore pressure in a clay bed. This increase accompanied the starting of the pump in a well screened in the overlying aquifer. Further investigation of this phenomenon included surface strain measurements. The purpose of this report is to: (1) present the results of the strain measurements; (2) present a possible explanation of the cause of the strain and the apparent relationship of the strain to the observed pore pressure rise at this site; and (3) extend these results to some of the reported reverse water fluctuations.

OBSERVATIONS

Pore pressure. Observations of strain around a producing well were prompted by the results observed in an earlier study. The original investigation compared values of the hydraulic diffusivity for a clay bed determined from drawdowns during a pumping test with values measured in the laboratory [*Wolff,* 1970].

In the field, observations of pressure were made by means of piezometers terminating at different depths in a clay bed underlying the pumped aquifer and located at points radially equidistant from the pumping well. A piezometer was also installed in the aquifer on the circumference of the same circle. All piezometers were equipped with electrical pressure transducers to eliminate the time lag that can result from the use of open standpipes. Simultaneous observations of the head change in the pumped aquifer and in the clay permitted an evaluation of the diffusivity of the clay. Large diameter cores were brought to the laboratory for testing.

During this earlier study it was observed that pore pressures in the clay increased very soon after the pump was started. The pressure rise was greatest near the interface between the pumped aquifer and the clay and decreased with depth in the clay (Figure 1). Shutting the pump off resulted in the opposite effect (Figure 2).

Because the measurements of pore pressure were made electrically, it was originally thought that the observed results were attributable, at least in part, to line voltage fluctuations caused by starting and stopping the electrically powered pump. To eliminate this possibility the entire measuring system was totally isolated

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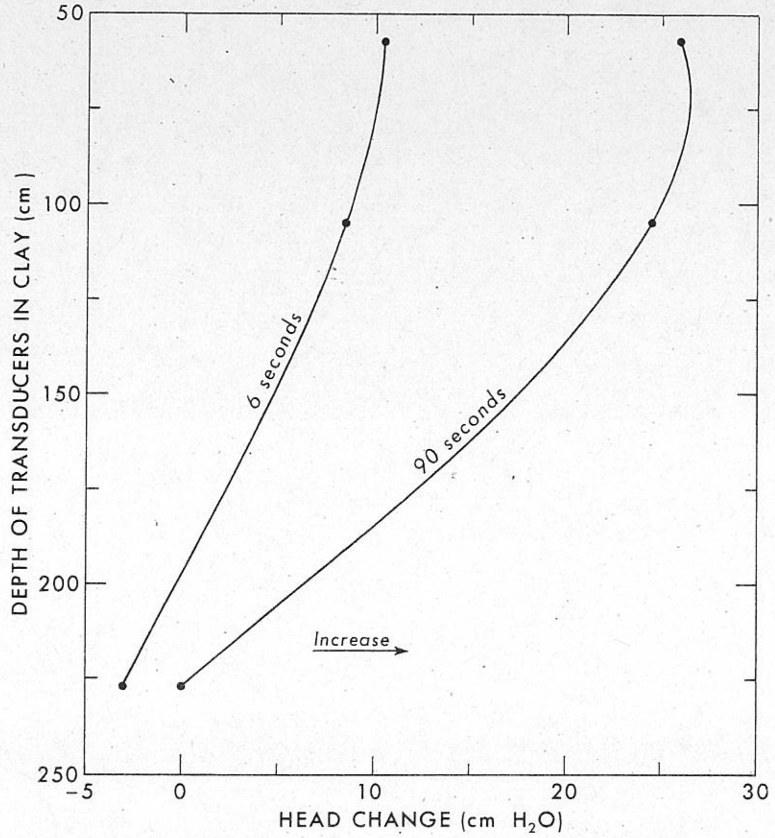

Fig. 1. Observed pore pressure changes at three different depths in a clay bed underlying the pumped aquifer. The time represents the elapsed time since pumping started.

from the pump circuit. The observations were not affected by the isolation, indicating that the phenomena were real.

HORIZONTAL STRAIN MEASUREMENTS

The reversibility of the pore pressure observations suggested elastic deformations in the aquifer similar to those described by *Peterson and Davis* [1966, p. 161] and *Davis et al.* [1969, p. 129]. To investigate the possibility of such a relationship, surface strain measurements were made in the vicinity of the pumping well. The method of measurement was similar to that described by Davis et al. Figure 3 is a diagrammatic sketch of the measuring device used.

Using a special jig, two aluminum posts were cemented into the ground. The jig permitted accurate spacing of the posts and assured that the posts were vertical and parallel; the posts were spaced 1.52 meters (5 feet) apart. One end of a Vycor glass tube was fixed on one post. The other end of the glass tubing was equipped with a ferrite core that passed through a hollow electrical extensometer transducer mounted on the opposite post. Any relative movement of the posts was sensed by the transducer and recorded on a continuous strip chart recorder. Calibration of the transducer was conducted using a micrometer head that could be read to the nearest micron.

Measurements were conducted primarily along four lines radiating from the well, as shown by the dots in Figure 5. Each set of measurements consisted of at least three pumping cycles. Each cycle consisted of turning the pump on and observing the movement until it ceased, and shutting the pump off and observing until detectable movement ceased. Typically, one cycle as described took approximately 45 min-

utes. Calibration was performed before and after each set of measurements.

All measurements were made after sundown to eliminate the thermal expansion that occurred during the day. Figure 4 presents typical results during a cycle of pumping.

Several attempts were made to detect any vertical rotation of the posts at various distances from the pumping well. *Davis et al.* [1969] observed such rotations near some wells. Within the capability of our technique, significant, repeatable rotational movements were not detected.

Strain measurement results. Figure 5 presents a contour map of the radial strain observed in the vicinity of the pumping well. Of interest is the reversal of the strain from compression to tension at a distance of 25 meters, more or less, from the pumping well and the absence of any observable strain on the opposite side of the stream. Tangential strain measurements were made at only two sites, one near the well and one in the zone of radial tension between the pumping well and well 15. Both showed compression.

Analysis. The problem of stresses and strains developed in a porous medium around a borehole as a result of fluid flow has been treated by *Paslay and Cheatham* [1963], *Seth and Gray* [1968*a*, *b*], and *Raats* [1968]. The following theoretical development differs from the studies cited above in that it is modified to apply to physical situations encountered in groundwater studies and in particular to the physical situation used in this investigation.

In the following development all discussion of stresses and strains refers only to those changes of stress and strain that are the result of pumping.

Assume an aquifer with symmetrical properties about a point $r = 0$ (that is, a single pumping well will cause symmetrical stresses and strains) in which the shear stresses are small enough to be neglected. Under these assumptions, the stress equation of equilibrium in radial coordinates is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + b = 0 \qquad (1)$$

where σ_r and σ_θ are the stresses in the radial and tangential direction, respectively, and b is the body force due to fluid friction in the r direction.

We observe that the body force on the medium is

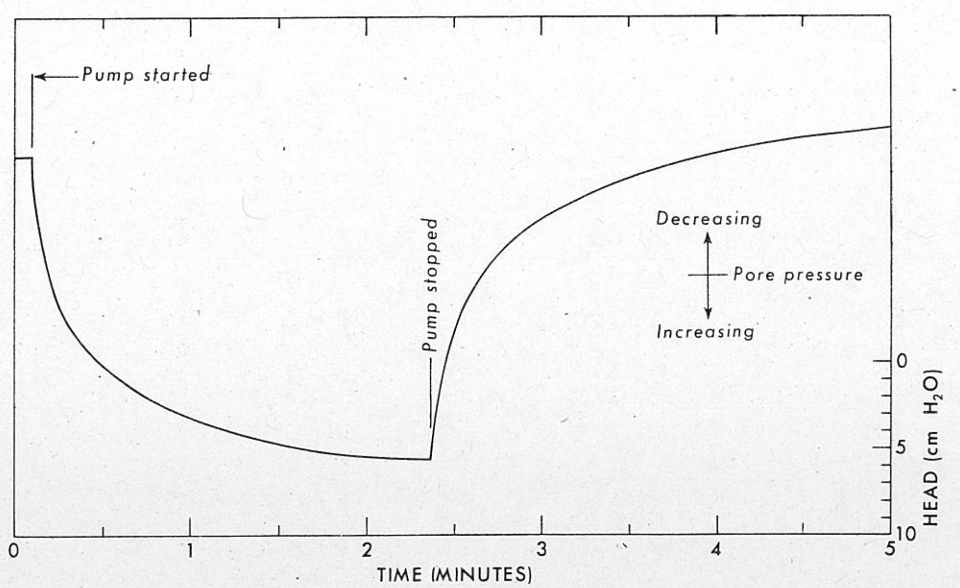

Fig. 2. Tracing of a strip chart recording for a piezometer located 0.64 meter below the pumped aquifer–clay interface and at a radial distance of 3.05 meters from the pumped well. The tracing shows the instantaneous effect of starting and stopping the pump.

$$b = -\frac{\partial p}{\partial r} \tag{2}$$

where p is the change in fluid pressure due to pumping. Substituting equation 2 into equation 1 yields (since we are assuming that shear stresses are negligible)

$$\frac{\partial(\sigma_r - p)}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{3}$$

From the general stress–strain relations it is possible to write the stress–strain relations for the physical model used.

$$E\epsilon_r = (1 + \nu)\sigma_r - \nu(\sigma_r + \sigma_\theta + \sigma_z) \tag{4a}$$

$$E\epsilon_\theta = (1 + \nu)\sigma_\theta - \nu(\sigma_r + \sigma_\theta + \sigma_z) \tag{4b}$$

$$E\epsilon_z = (1 + \nu)\sigma_z - \nu(\sigma_r + \sigma_\theta + \sigma_z) \tag{4c}$$

where E is Young's modulus, ϵ is strain, and ν is Poisson's ratio. The total load on the aquifer is assumed to be constant. Assuming that the shear stresses are negligible we can write

$$\sigma_z = p \tag{5}$$

Substituting equation 5 into equations $4a$, $4b$, and $4c$, we get

$$E\epsilon_{rr} = E\frac{\partial u_r}{\partial r} = \sigma_r - \nu\sigma_\theta - \nu p \tag{6a}$$

$$E\epsilon_{\theta\theta} = E\frac{u_r}{r} = \sigma_\theta - \nu\sigma_r - \nu p \tag{6b}$$

$$E\epsilon_{zz} = E\frac{\partial u_z}{\partial z} = p - \nu\sigma_r - \nu\sigma_\theta \tag{6c}$$

where u_r and u_z are the radial and vertical displacement, respectively, of the permeable materials.

From $6a$ and $6b$ the compatibility restrictions on u_r are found to be

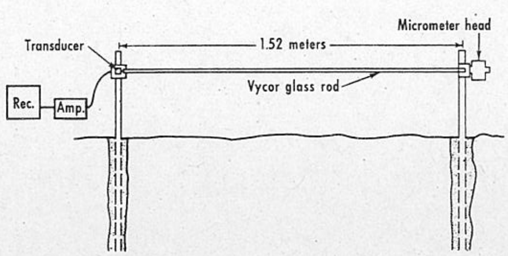

Fig. 3. Diagrammatic sketch of setup used for strain measurements.

$$\frac{\partial}{\partial r}\left(\frac{u_r}{r}\right) = \frac{1}{r}\left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r}\right) \tag{7}$$

or

$$\frac{\partial \epsilon_{\theta\theta}}{\partial r} - \frac{(\epsilon_{rr} - \epsilon_{\theta\theta})}{r} = 0 \tag{8}$$

Substituting from equations $6a$ and $6b$ into equation 8 results in an equation that can be transformed into the following form by substituting for σ_θ from equation 1:

$$3\frac{\partial(\sigma_r - p)}{\partial r} + r\partial^2\frac{(\sigma_r - p)}{\partial r^2} - 2\nu\frac{\partial p}{\partial r} = -\frac{\partial p}{\partial r} \tag{9}$$

Within distances from the pumped well such that $r^2S/4Tt \leq 0.02$, where S is the storage coefficient and T is the transmissivity, the pressure can be assumed to vary linearly with $\ln r$ [*Cooper and Jacob,* 1946, p. 527] as shown in Figure 6.

The extension of the straight-line portion of the p versus $\ln r$ line intersects $p = 0$, (p_e) at a value of r labeled r_e, the effective radius. The equation for this straight line is

$$p = p_w + \left(\frac{p_e - p_w}{\ln r_e/r_w}\right)\ln\frac{r}{r_w} \tag{10}$$

where p_w is the pressure change at the well screen, and r_w is the radius of the well screen. Differentiating equation 10 with respect to r, we get

$$\frac{\partial p}{\partial r} = \frac{p_e - p_w}{(\ln r_e/r_w)r} \tag{11}$$

Letting

$$\phi = \sigma_r - p \tag{12}$$

and substituting equations 11 and 12 into 9 we obtain

$$\frac{r^2}{\nu}\frac{\partial^2\phi}{\partial r^2} + \frac{3r}{\nu}\frac{\partial\phi}{\partial r} = -(1 - 2\nu)\frac{p_e - p_w}{\ln r_e/r_w} \tag{13}$$

If we further assume that a free surface exists at $r = r_w$ (that is, in the absence of a rigid screen), then $\sigma_r = 0$ at $r = r_w$. Thus if we use these assumptions and if we wish to compute strains within distances r such that $r^2S/4Tt \leq 0.02$, the boundary conditions

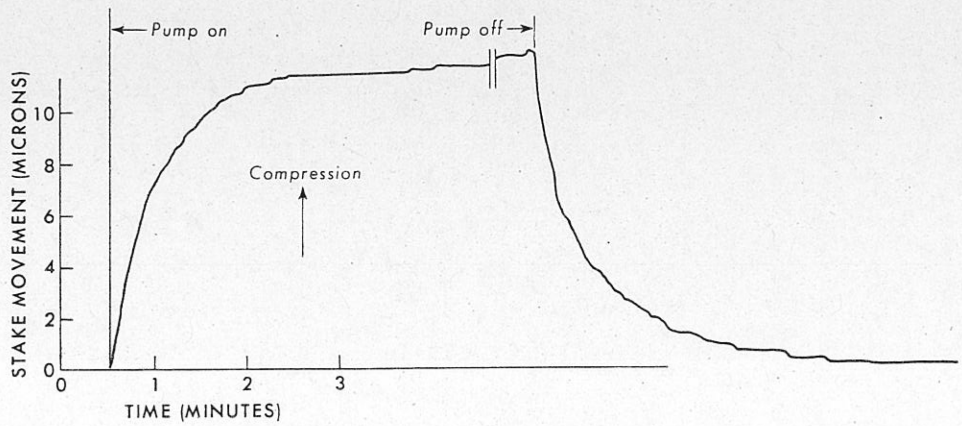

Fig. 4. Tracing of a strip chart recording showing typical relative movement of the stakes to one cycle of starting and stopping the pump.

$\sigma_r(r_w) = 0$ and $\sigma_r(r_e) = 0$ apply. With these boundary conditions the solution to equation 13 is

$$\phi = \sigma_r - p$$

$$= \left[\frac{(2\nu - 1)(p_e - p_w)}{2(\ln r_e/r_w)} \ln \frac{r_e}{r_w} - p_w\right] \cdot \left[\frac{r_w^2(r_e^2 - r^2)}{r^2(r_e^2 - r_w^2)}\right] + \frac{(2\nu - 1)(p_e - \sigma_w)}{2(\ln r_e/r_w)} \ln \frac{r}{r_e} \tag{14}$$

Substituting for p from equation 10 yields for the radial stress

$$\sigma_r = -p_w\left[\left(\frac{2\nu + 1}{2}\right)\left(\frac{r_w^2(r_e^2 - r^2)}{r^2(r_e^2 - r_w^2)}\right) + \frac{(2\nu - 1)\ \ln r/r_e}{2(\ln r_e/r_w)} + \frac{\ln r/r_w}{\ln r_e/e_w} - 1\right] \tag{15}$$

Substituting equation 14 and its derivative with respect to r into equation 3 and substituting for p in the result yields for the tangential stress

$$\sigma_\theta = -p_w\left[\left(\frac{-r_w^2(r_e^2 + r^2)}{r^2[r_e^2 - r_w^2]}\right)\left(\frac{2\nu + 1}{2}\right) + \left(\frac{2\nu - 1}{2}\right)\left(\frac{1 - \ln r_e/r}{\ln r_e/r_w}\right) + \frac{\ln r/r_w}{\ln r_e/r_w} - 1\right] \tag{16}$$

If values of Young's modulus E, and Poisson's ratio ν are known or assumed, it is now possible to obtain the strains by substituting from equations 15 and 16 into equations 6*a*, 6*b*, and 6*c*.

Interpretation of observations. Using this theory, the expected radial and tangential strains that result from pumping a well are compressive for all values of r greater than r_w and within the straight-line portion of the distance–drawdown curve (Figure 6), and for rational values of ν (< 0.5). This theoretical situation does not conform with the measured field results. However, as mentioned previously, strains were not observed on the far side of the stream away from the pumping well. Using this information as an indication that the stream serves as a line source, a recharging image well was included to analyze mathematically the effect of the line source. Because the stream does not fully penetrate the aquifer and hence does not constitute a true line source, these calculations were made only to attempt to understand the factors affecting the observations, but not to attempt to duplicate actual observed magnitudes.

Figure 7 presents a comparison of the observed and calculated results for points along the straight line between the pumping well and well 15 (Figure 5). As shown, inclusion of a line source in the calculations results in a change from compression to tension with increasing r as observed in the field. The agreement is thought to be quite good, considering the simplifying assumptions of our mathematical analysis.

Figure 7 also presents the theoretical curve resulting from equation 6*c* for vertical strain for the field study site. For the reasons cited above,

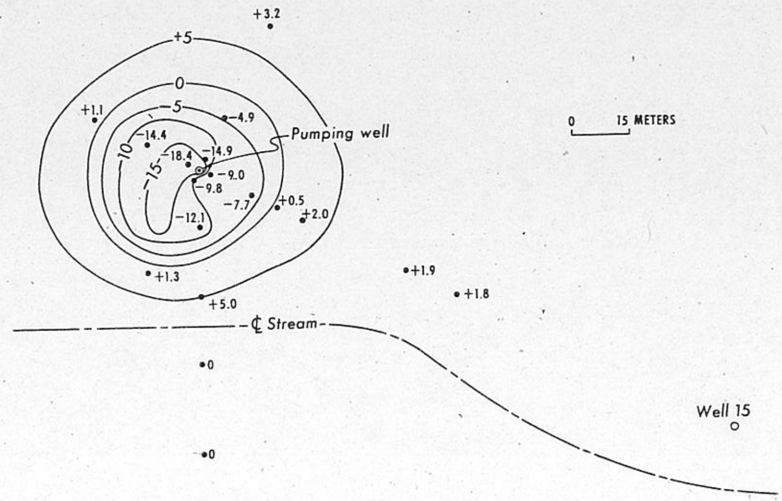

Fig. 5. Contour map of radial strains, measured at the surface, as a result of starting pumping. —, posts moving together; +, posts moving apart; contours, strain $\times 10^{-6}$.

magnitudes were not determined. The theory indicates that the magnitudes would be too small for observation with the equipment we had available. Our results do not exhibit the inflection point discussed by *Davis et al.* [1969, p. 137]. This difference may be attributable to the assumptions used in our idealized model.

Relationship of field observed negative pore pressure changes and surface strain measurements. Although the mathematical development assumed an absence of shear stress, it is apparent that some shear stress must occur along the clay–aquifer interface. Strain measurements were not made in the clay; however, it appears reasonable that the strain observed at the surface of the aquifer occurred throughout the aquifer, and that strain, transferred by shear stress at the clay–aquifer boundary, occurs in the clay. This assumption would suggest that the magnitude of strain in the clay is greatest near this boundary. Such a conclusion appears to fit our observed field data. We observe a decreasing magnitude of pore pressure rise with depth in the clay as shown in Figure 1.

Verruijt [1969, p. 362] presented a model for a reverse water level fluctuation called the 'Noordbergum effect,' which at least for short periods of time would predict an increasing magnitude of pore pressure rise with depth at the same distance from the pumped well. Although this effect may have been present in the observed field situation, it appears that other factors as described above were dominant.

Application to reverse water level fluctuations. As evidenced by the observations described earlier and those presented by *Davis et al.* [1969], and as indicated by the theoretical considerations, horizontal strain occurs in the vicinity of a pumping well. The magnitude of the strain is dependent, at least in part, on both the hydrologic and mechanical properties of the aquifer as well as its geometry and on the magnitude and method of stress application.

Most reverse water level fluctuations reported to date have been observed in strata having

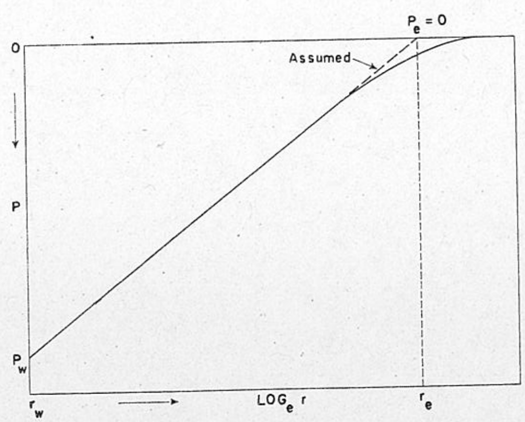

Fig. 6. Representation of assumed straight-line variation of p versus $\log_e r$.

considerably lower values of hydraulic conductivity than the pumped aquifer, or in artesian aquifers that are separated from the pumped aquifer by a confining bed. An example of the latter is presented in Figure 8.

If appreciable strain occurs in the pumped aquifer, it is not difficult to visualize the transference of this deformation across confining beds and/or adjacent aquifers with the proper physical characteristics to permit such transfer. This transference would account for some of the previously observed reverse water level fluctuations.

CONCLUSIONS

The apparent correlation of strain directions using the theoretical developments and strain directions observed in the field suggests that the latter are primarily the result of viscous drag occurring in the aquifer. Assuming shear stress occurs at aquifer boundaries where adjacent layers have a significantly lower hydraulic conductivity, the magnitude of strain and consequent pore pressure rise should decrease with distance away from the stressed aquifer. Observations substantiate this concept. In situations where ideal packing exists, strain would result in dilatation and hence pore pressure decrease.

Carrying this concept further, the transference of strain across confining beds to an artesian aquifer accounts for some of the observations of reverse water level fluctuation.

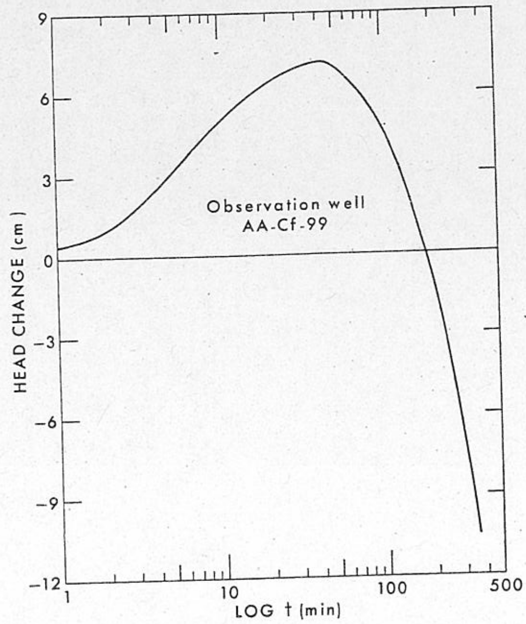

Fig. 8. Trace of water level change versus time for an observation well screened in the upper Magothy artesian aquifer. The pumping well was located 75 meters away and was screened in an underlying artesian aquifer separated from the observed zone by a clay bed that may be lenticular. The pumping test was conducted near Annapolis, Maryland, by Fred Mack, U.S. Geological Survey.

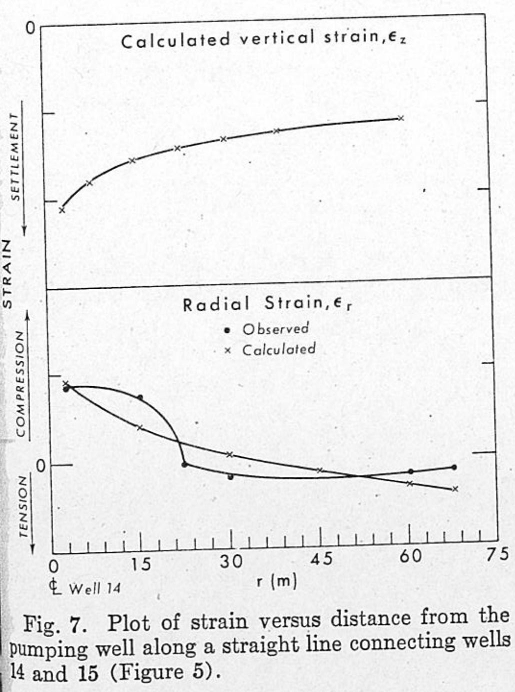

Fig. 7. Plot of strain versus distance from the pumping well along a straight line connecting wells 14 and 15 (Figure 5).

NOTATION

- b, body force per unit volume, M/L^2T^2;
- E, Young's modulus, M/LT^2;
- P, fluid pressure, M/LT^2;
- p_w, fluid pressure change at well screen, M/LT^2;
- r, radius, L;
- r_w, radius of well, L;
- S, storage coefficient, dimensionless;
- T, transmissivity, L^2/T;
- t, time, T;
- u_r, u_z, radial and vertical displacement, L
- ϵ, strain, dimensionless;
- ν, Poisson's ratio, dimensionless;
- σ_r, σ_θ, σ_z, stress in radial, tangential, and vertical direction, M/LT^2.

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REFERENCES

Andreason, G. E., and J. W. Brookhart, Reverse water-level fluctuations, in Methods of collecting and interpreting ground-water data, compiled by R. Bentall, *U.S. Geol. Surv. Water Supply Pap. 1544H*, 30–35, 1963.

Barksdale, H. C., R. W. Sundstrom, and M. S. Brunstein, Supplementary report on the ground-water supplies of the Atlantic City region, 139 pp., *New Jersey State Water Policy Comm. Spec. Rep. 6*, 1936.

Cooper, H. H., Jr., and C. E. Jacob, A generalized method for evaluating formation constants and summarizing well-field history, *Trans. Amer. Geophys. Union, 27*(4), 526–534, 1946.

Davis, S. N., F. L. Peterson, and A. D. Halderman, Measurement of small surface displacements induced by fluid flow, *Water Resour. Res., 5*(1), 129–138, 1969.

Paslay, P. R., and J. B. Cheatham, Jr., Rock stresses induced by flow of fluids into boreholes, *Soc. Petrol. Eng. J., 3*, 85–94, 1963.

Peterson, F. L., and S. N. Davis, Strain in the vicinity of pumping wells (abstract), p. 161, *Geol. Soc. Amer. Program Annu. Meet.*, 1966.

Raats, P. A. C., Forces acting upon the solid phase of a porous medium, *J. Appl. Math. Phys., 19*, 606–613, 1968.

Seth, M. S., and K. E. Gray, Transient stresses and displacement around a wellbore due to fluid flow in transversely isotropic, porous media, 1, Infinite reservoir, *Soc. Petrol. Eng. J., 8*, 63–78, 1968*a*.

Seth, M. S., and K. E. Gray, Transient stresses and displacement around a wellbore due to fluid flow in transversely isotropic, porous media, 2, Finite reservoir, *Soc. Petrol. Eng. J., 8*, 79–86, 1968*b*.

Verruijt, A., Elastic storage of aquifers, in *Flow through Porous Media,* edited by R. J. M. De Wiest, pp. 331-376, Academic, New York, 1969.

Wolff, R. G., Field and laboratory determination of the hydraulic diffusivity of a confining bed, *Water Resour. Res., 6*(1), 194–203, 1970.

(Manuscript received June 10, 1970.)