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Generalized Scaling of Permeabilities: Validation and Effect of Support Scale

Shlomo P. Neuman

Department of Hydrology and Water Resources, The University of Arizona, Tucson, Arizona 85721

Abstract. The permeabilities and dispersivities of geologic media are known to vary with the scale of observation. Particularly well documented is the consistent increase in apparent longitudinal dispersivity with the mean travel distance of a tracer. This has been previously interpreted by the author to imply that the permeabilities of many geologic media scale, on the average, according to the power-law semivariogram $\gamma(s) = c\sqrt{s}$ where c is a constant and s is distance. Tracer test data support this conclusion indirectly at least over scales from 10 cm to 3,500 m. The present paper cites evidence for such behavior over scales from 10 cm to 45 km based directly on permeability and transmissivity data. The paper then investigates theoretically the implications of such power-law behavior on the equivalent permeability of a block of rock having a characteristic length (support scale) L. It predicts that the equivalent isotropic permeability should generally decrease with L in one-dimensional media, increase with L in three-dimensional media, and show no systematic variation with L in two-dimensional media. This prediction appears to be consistent with observations.

Introduction and Background

Longitudinal dispersivities α deduced by means of deterministic Fickian theories from laboratory and field tracer studies in a variety of porous and fractured media, under varied flow and transport regimes, appear to increase without limit as the scale of the study increases. To explain this scale effect, *Neuman* [1990, Fig. 1] considered 131 dispersivity data obtained from deterministic Fickian models which do not account explicitly for spatial variations in permeability. Such deterministic models are consistent with stochastic models which consider log permeability to be a random, statistically homogeneous field. Upon interpreting the scale of the tracer studies as the mean travel distance s(t), Neuman found by linear regression that, for a range of scales from 10 cm to 3.5 km, $\alpha \simeq 0.017\ s^{1.5}$ with a coefficient of determination $R^2 \simeq 0.75$ and remarkably narrow confidence intervals. He then noted that consistency with stochastic transport theories implies that the regression model must be associated with a log permeability field which possesses neither a finite correlation scale nor a finite variance. Instead, the corresponding log permeability field must possess a power-law semivariogram $\gamma(s) = c\ s^{2\omega}$ where c is a constant and ω is a Hurst coefficient. This in turn implies that the random log permeability field is self-affine with statistically homogeneous spatial increments and a fractal dimension $D = E + 1 - \omega$ where E is the topological dimension of relevance.

In log permeability fields which do not possess a finite correlation scale, dispersion is inherently anomalous (non-Fickian) and the traditional notion of local dispersivity does not apply. Instead, dispersion in such fields is described by integro-differential equations that involve nonlocal parameters [*Neuman*, 1993a]. This explains why α is an apparent dispersivity; rather than representing local material properties, it is merely an artifact resulting from the common practice of applying Fickian models to media in which transport is generally non-Fickian. Any spatial variations in k which are not accounted for explicitly by the Fickian model are absorbed into α [*ibid*]. Hence if the apparent correlation scale and variance of log permeabilities increase without limit, the apparent dispersivity likewise increases without bounds.

The regression model yields a Hurst coefficient $\omega \simeq 0.25$ which leads to the generalized scaling relation

$$\gamma(s) = c\sqrt{s} \qquad (1)$$

This simple relation explains 75% of the observed variation of α about its mean. As $\omega < 0.5$, the semivariogram (1) represents a log permeability field with negatively correlated (antipersistent) spatial increments and a fractal dimension $D \simeq E + 0.75$. The fact that it is supported by apparent dispersivities from both porous and fractured media was taken by *Neuman* [1990] to imply that one can often analyze flow and transport in fractured rocks by treating them as multiscale porous continua.

Neuman [1990, 1991, 1993b] recognized that the permeabilities of real geologic media scale locally in a much more complex manner than is implied by the above model. At a given locale, such media may be viewed more realistically as a nested sequence of more or less distinct hydrogeologic units associated with a discrete hierarchy of scales. When the semivariograms of such discrete units are superimposed, one obtains a function that increases with the distance s in a stepwise rather than a gradual fashion. Each step in such an echelon represents a "natural correlation scale" at which log permeability is statistically homogeneous or nearly so; other scales are locally either inactive or suppressed. As hydrogeologic conditions vary from one setting to another, so do the dominant natural scales, giving rise to an infinite variety of possible echelon-type semivariograms. It is only when one juxtaposes data

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from a sufficiently large number of different hydrogeologic settings that a sufficiently broad spectrum of scales come into play to reveal the underlying commonality of these semivariograms, in the form of a generalized power law.

Though permeabilities at each site are unique, their spatial correlation is viewed by *Neuman* [1990, 1991, 1993b] as a fluctuation about a mean semivariogram $\gamma(s) = c\sqrt{s}$. He found evidence in the data that accounting explicitly for site-specific medium heterogeneity by conditioning is tantamount to filtering out large-scale, low-frequency modes from the multiscale hierarchy of log permeabilities. This filtering effect is the reason why Neuman did not consider dispersivities obtained from calibrated (conditional) models in deriving his scaling relation $\gamma(s) = c\sqrt{s}$. A rigorous theoretical explanation of this filtering effect can be found in *Neuman* [1993a]. It forms the basis for our discussion in this paper of the manner in which block permeabilities vary with sample size (support scale).

Neuman's [1990] derivation of (1) was indirect, resting entirely on the interpretation of tracer studies. One purpose of this brief communication is to examine the extent to which the same scaling rule is supported directly by permeabilities and transmissivities obtained from hydraulic tests. Another purpose is to discuss the implications of this scaling rule on the manner in which the permeability of a block of rock changes with the dimensions and scale of the sample (or support).

Validation of Generalized Scaling Rule

To validate directly that log permeabilities do not in fact possess a finite correlation scale, we consider hydraulic conductivity and transmissivity data from a number of different sites recently summarized by *Gelhar* [1993, Table 6.1]. Fig. 1 is a plot of reported correlation scales λ versus characteristic lengths of the corresponding field sites. It is clear that the reported correlation scale increases consistently with field length, being approximately equal to one tenth of the latter. This relationship holds true over a range of λ values from 10 cm to 45 km.

Fig. 2 is based on the same data and shows how the product $\sigma^2\lambda$ of reported log hydraulic conductivity or transmissivity variance σ^2, and reported horizontal correlation scale λ, varies with this scale. Since transmissivities are proportional to hydraulic conductivities averaged over the thickness of an aquifer, one expects them to have a correspondingly smaller variance. Hence the transmissivities (circles) in Fig. 2 are expected to be offset downward relative to the hydraulic conductivities (squares and triangles). In other words, the two sets of data represent different populations and must not be analyzed jointly. The number of data in each set (14 and 10, respectively) is too small to justify fitting a theoretical model, such as $\gamma(s) = c\,s^{2\omega}$, to any one of them by regression. Instead, we prefer to ask how well does the model $\gamma(s) = c\sqrt{s}$, derived on the basis of more than 130 dispersivity data, predict the scaling behavior indicated by the two sets of hydraulic data in Fig. 2?

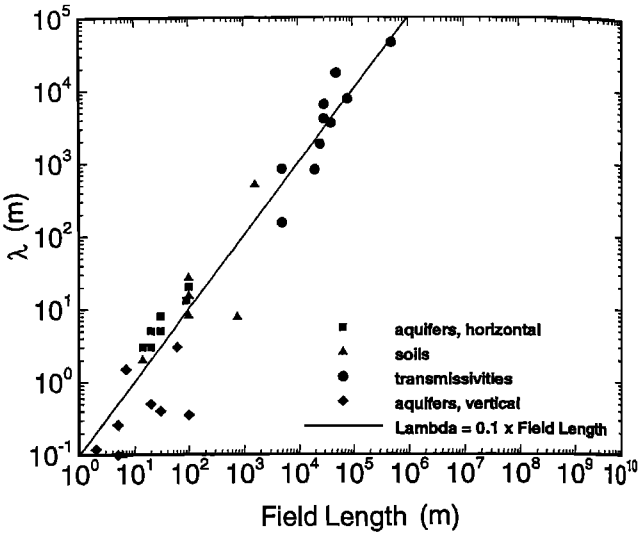

Fig. 1: Correlation scale λ of natural log hydraulic conductivities and transmissivities at various sites versus field length (data from *Gelhar* [1993, Table 6.1]).

To answer this, we first note that adjusting c so as to obtain an optimum visual fit between $\gamma(s) = c\sqrt{s}$ and the hydraulic conductivity data (squares and triangles) yields the broken line in Fig. 2. It predicts the observed variation of $\sigma^2\lambda$ with λ to within one order of magnitude over a range of scales from 10 cm to 500 m. Making no adjustment but setting c equal to 8/5 x 0.017 as implied by *Neuman's* [1990] analysis yields the solid line in Fig. 2. This line predicts the observed scaling behavior of all the data (hydraulic conductivities and transmissivities) to within two orders of magnitude over a range of scales from 10 cm to 45 km. It is clearly possible to adjust c so as to predict the observed scaling of transmissivities to within one order of magnitude over a range of scales from 150 m to 45 km. These levels of predictability are consistent with the scatter of apparent dispersivity data in Fig. 1 of *Neuman* [*ibid*] about the regression line $\alpha \simeq 0.017\, s^{1.5}$ from which the model $\gamma(s) \simeq c\sqrt{s}$ was derived. It follows that this model of permeability scaling, derived indirectly from tracer studies, is consistent with (and thus supported by) hydraulic conductivity and transmissivity data obtained directly from hydraulic tests.

Additional support can be found in a recent analysis of transmissivities from the fractured Culebra dolomite at the Waste Isolation Pilot Plant site in New Mexico by *Grindrod and Impey* [1992]. These authors showed that, at least over scales ranging from 1.7 to 18.6 km, the natural log transmissivities at the site scale according to $\gamma(s) = c\,s^{2\omega}$ where c lies between 0.014 and 0.022, and $\omega = 0.35 \pm 0.05$. This compares quite favorably with *Neuman's* [1990] predicted (see previous paragraph) c = 8/5 x 0.017 = 0.027 and $\omega = 0.25$.

Implications Concerning Block Permeabilities

The semivariogram $\gamma(s) = c\,s^{2\omega}$ is associated with a log permeability field having homogeneous spatial increments but devoid of natural scale(s) at which this

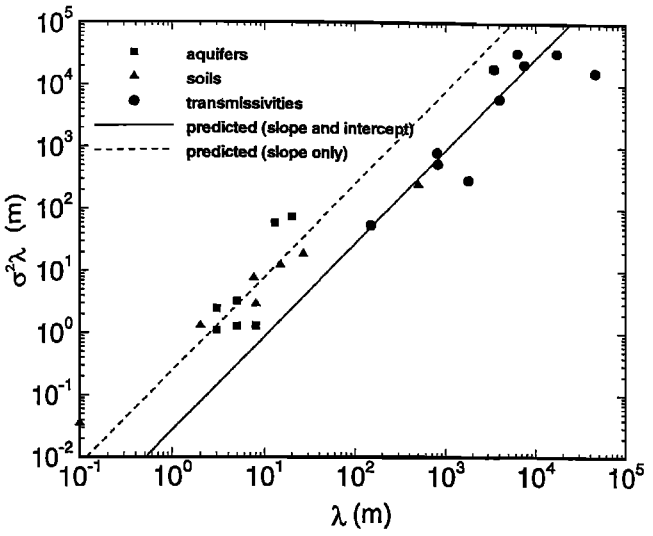

Fig. 2: Product of variance σ^2 and horizontal correlation scale λ of natural log hydraulic conductivities and transmissivities at various sites versus λ (data from *Gelhar* [1993, Table 6.1]). Predictions correspond to $\gamma(s) = \sqrt{s}$.

field would be statistically homogeneous. It implies [*Neuman*, 1990] that statistical homogeneity of log permeabilities is at best a local phenomenon occurring intermittently over narrow bands of the scale spectrum. Hence one must question the utility of routinely associating geologic medium properties with representative elementary volumes (REV's) as has been the custom for several decades. Indeed, it was found by *Brace* [1980, 1984] that the permeabilities of sedimentary and crystalline rocks tend to grow with the characteristic scale of their measurement. This is reinforced by a more recent compilation of crystalline rock permeabilities by *Clauser* [1992]. According to him, a plot of these data (Fig. 3) "confirms the increase in average permeability by about 3 decades from the laboratory to the borehole scale ... However, this increase apparently does not continue from the borehole to a regional scale, such as above 100 m." Previously, *Neuzil et al.* [1984] showed by numerical simulation that the recorded volume and rate of water withdrawal from the Dacota Aquifer could not be sustained without leakage through the overlying Pierre Shale. They found that the vertical permeability of this shale must be 10 to 1,000 times larger on the regional than on the local scales. An increase in permeability of up to four orders of magnitude was observed by *Hanor* [1993] when comparing laboratory and bulk values for clay beds at a hazardous waste site in Louisiana.

We interpret this scale effect on the premise that the log permeabilities of both porous and fractured rocks can be treated as multiscale random functions defined over a continuum. It is common to treat such log permeability as a statistically homogeneous Gaussian field. For an unbounded flow domain subject to a uniform mean hydraulic gradient, the effective permeability corresponding to a statistically isotropic field has been conjectured by *Matheron* [1967] to be

$$k_e = k_g \exp\left[\sigma^2\left[\frac{1}{2} - \frac{1}{E}\right]\right] \qquad (2)$$

where k_g is the geometric mean permeability. This is rigorously valid for the one-dimensional case (E = 1) where k_e is the harmonic mean and for the two-dimensional case (E = 2) where $k_e = k_g$. Its validity in the three-dimensional case (E = 3) has been demonstrated numerically for σ^2 as large as 7 [*Neuman and Orr*, 1993]. Equivalent expressions have also been developed for statistically orthotropic media. *Neuman and Depner* [1988] verified the three-dimensional anisotropic result at a fractured crystalline rock site for $\sigma^2 > 7$. Expressions for bounded rock volumes can be found in *Neuman et al.* [1993].

Extreme anisotropy can alter the flow regime from three-dimensional to predominantly two- or one-dimensional, and from two-dimensional to predominantly one-dimensional. The actual degree of anisotropy encountered in any given permeability test, and the prevailing boundary conditions, are often unknown or uncertain. However, regardless of what the prevailing test conditions may be, k_e is always given by

$$k_e = k_g \exp\left[\sigma^2\left[\frac{1}{2} - \frac{\beta}{E}\right]\right] \qquad (3)$$

where $0 \le \beta \le E$. In particular, $\beta = 1$ for infinite isotropic media, $0 \le \beta \le 1$ for bounded isotropic media, and $1 \le \beta \le E$ for infinite anisotropic media. This applies to (Gaussian) statistically homogeneous log permeability fields. A log permeability field having a power-law semivariogram $\gamma(s) = c\, s^{2\omega}$ with $\omega \le 0.5$ can be viewed [*Neuman*, 1990] as the superposition of an infinite hierarchy of similar (isotropic) fields having mutually uncorrelated increments and (exponential covariance functions with) variances that increase as a power of the correlation scale. Let L be the characteristic length of a rock volume which acts as the support for a given permeability test. Then low-frequency spatial variations

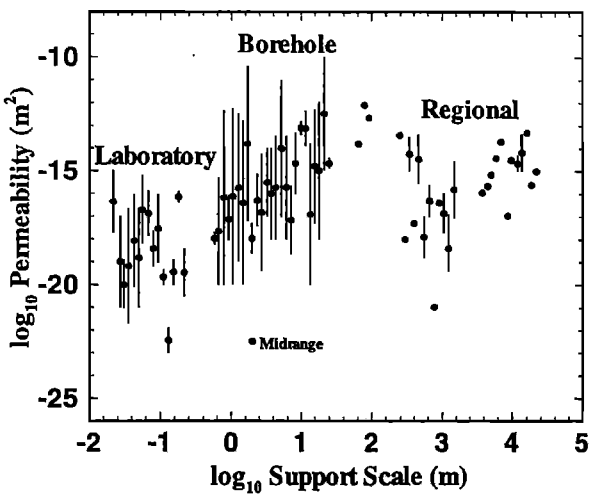

Fig. 3: Range (after *Clauser* [1992]) and midrange of permeabilities in crystalline rocks at various sites versus experimental scale.

in permeability on scales λ of order larger than L do not contribute to internal heterogeneity within the tested rock volume. Hence log permeabilities within the test volume are correlated over a distance of order L and exhibit a variance of order $\sigma^2 = \gamma(L) = c\,L^{2\omega}$. Substituting this variance into (3) yields the following relationship between effective permeability and the support scale L,

$$k_e(L) = k_g \exp\left[c\,L^{2\omega}\left[\frac{1}{2} - \frac{\beta}{E}\right]\right] \qquad (4)$$

According to (4), $k_e(0) = k_g$ is the effective permeability of a rock volume having zero support, *i.e.*, k_g represents fictitious point measurements (in reality there must always be a cutoff value of L below which the classical notion of permeability, and a power-law variogram, do not apply). Equation (4) implies that the logarithm of the effective permeability $y_e(L) = ln k_e(L)$ generally varies as a power 2ω of L. $y_e(L)$ increases as a power of L when $\beta < 0.5$ E, decreases when $\beta > 0.5$ E, and remains unchanged when $\beta = 0.5$ E. For large isotropic rock supports with $L/\lambda >> 1$, $\beta \simeq 1$. Then $y_e(L)$ increases as a power of L in three-dimensions (E = 3), decreases in one dimension (E = 1), and shows no systematic variation with L in two dimensions (E = 2).

Figure 3 shows only ranges of k which are insufficient to either validate or invalidate these theoretical predictions. It is not clear whether the laboratory permeabilities increase (as we might expect on the basis of (4) for more or less equidimensional samples) or decrease (as we might expect for long and narrow core samples) systematically with scale. The borehole log permeabilities are, as expected from (4), generally larger than those determined in the laboratory. The former show a systematic increase with the length of the test interval which however reflects only one dimension of what is likely to be a more or less cylindrical data support; it is additionally difficult to tell whether or not this increase follows a power law. The regional data in Fig. 3 do not show a consistent variation with scale. This could be due to a combination of factors: The available sample is too small to indicate a trend; some of the data correspond to near two-dimensional flow regimes; and some of the data are derived from calibrated numerical models which account explicitly for medium heterogeneity and thus filter out large-scale, low-frequency variations from the hierarchy.

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Shlomo P. Neuman, Department of Hydrology and Water Resources, The University of Arizona, Tucson, Arizona 85721.

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