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STATISTICAL ANALYSIS OF GROUNDWATER MONITORING DATA AT RCRA FACILITIES

UNIFIED GUIDANCE

OFFICE OF RESOURCE CONSERVATION AND RECOVERY

PROGRAM IMPLEMENTATION AND INFORMATION DIVISION

U.S. ENVIRONMENTAL PROTECTION AGENCY

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DISCLAIMER

This Unified Guidance has been prepared to assist EPA's Regions, the States and the regulated community in testing and evaluating groundwater monitoring data under 40 CFR Parts 264 and 265 and 40 CFR Part 258. This guidance is not a rule, is not legally enforceable, and does not confer legal rights or impose legal obligations on any member of the public, EPA, the States or any other agency. While EPA has made every effort to ensure the accuracy of the discussion in this guidance, the obligations of the regulated community are determined by the relevant statutes, regulations, or other legally binding requirements. The use of the term "should" when used in this guidance does not connote a requirement. This guidance may not apply in a particular situation based on the circumstances. Regional and State personnel retain the discretion to adopt approaches on a case-by-case basis that differ from this guidance where appropriate.

It should be stressed that this guidance is a work in progress. Given the complicated nature of groundwater and geochemical behavior, statistical applications describing and evaluating data patterns have evolved over time. While many new approaches and a conceptual framework have been provided here based on our understanding at the time of publication, outstanding issues remain. The Unified Guidance sets out mostly classical statistical methods using reasonable interpretations of existing regulatory objectives and constraints. But even these highly developed mathematical models deal primarily with sorting out chance effects from potentially real differences or trends. They do not exhaust the possibilities of groundwater definition using other technical or scientific techniques (*e.g.*, contaminant modeling or geostatistical evaluations). While providing a workable decision framework, the models and approaches offered within the Unified Guidance are only approximations of a complex underlying reality.

While providing a basic understanding of underlying statistical principles, the guidance doesn't attempt to provide the reader with more thorough explanations and derivations found in standard texts and papers. It also doesn't comprehensively cover all potential statistical approaches, and confines itself to reasonable and current methods, which will work in the present RCRA groundwater context. While it is highly likely that methods promoted in this guidance will be applied using commercial or proprietary statistical software, a detailed discussion of software applications is beyond the scope of this document.

This document has been reviewed by the Office of Resource Conservation and Recovery (former Office of Solid Waste), U.S. Environmental Protection Agency, Washington, D.C., and approved for publication. Mention of trade names, commercial products, or publications does not constitute endorsement or recommendation for use.

"It is far better to have an approximate answer to the right question than a precise answer to the wrong question…" — John Hauser

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EXECUTIVE SUMMARY

The Unified Guidance provides a suggested framework and recommendations for the statistical analysis of groundwater monitoring data at RCRA facility units subject to 40 CFR Parts 264 and 265 and 40 CFR Part 258, to determine whether groundwater has been impacted by a hazardous constituent release. Specific statistical methods are identified in the RCRA regulations, but their application is not described in any detail. The Unified Guidance provides examples and background information that will aid in successfully conducting the required statistical analyses. The Unified Guidance draws upon the experience gained in the last decade in implementing the RCRA Subtitle C and D groundwater monitoring programs and new research that has emerged since earlier Agency guidance.

The guidance is primarily oriented towards the groundwater monitoring statistical analysis provisions of 40 CFR Parts 264.90 to .100. Similar requirements for groundwater monitoring at solid waste landfill facilities under 40 CFR Part 258 are also addressed. These regulations govern the detection, characterization and response to releases from regulated units into the uppermost aquifer. Some of the methods and strategies set out in this guidance may also be appropriate for analysis of groundwater monitoring data from solid waste management units subject to 40 CFR 264.101. Although the focus of this guidance is to address the RCRA regulations, it can be used by the CERCLA program and for improving remedial actions at other groundwater monitoring programs.

Part I of the Unified Guidance introduces the context for statistical testing at RCRA facilities. It provides an ***overview of the regulatory requirements***, summarizing the current RCRA Subtitle C and D regulations and outlining the statistical methods in the final rules, as well as key regulatory sections affecting statistical decisions. It explains the basic groundwater monitoring framework, philosophy and intent of each stage of monitoring — detection, compliance (or assessment), and corrective action — and certain features common to the groundwater monitoring environment. Underlying statistical ideas common to all statistical test procedures are identified, particularly issues involving false positives arising from multiple statistical comparisons and statistical power to detect contamination.

A new component of the Unified Guidance addresses issues of ***statistical design***: what factors are important in constructing a reasonable and effective statistical monitoring program. These include the establishment and updating of background data, designing an acceptable detection monitoring plan, and statistical strategies for compliance/assessment monitoring and corrective action. This part also includes a short summary of statistical methods recommended in the Unified Guidance, detailing conditions for their appropriate use.

Part II of the Unified Guidance covers diagnostic evaluations of historical facility data for the purpose of ***checking key assumptions*** implicit in the recommended statistical tests and ***for making appropriate adjustments to the data*** (*e.g.*, consideration of outliers, seasonal autocorrelation, or non-detects). Also included is a discussion of groundwater sampling and how hydrologic factors such as flow and gradient can impact the sampling program. Concepts of statistical and physical independence are compared, with caveats provided regarding the impact of dependent data on statistical test results. Statistical methods are suggested for identifying special kinds of dependence known as spatial and temporal variation, including reasonable approaches when these dependencies are observed. Tests for trends are also included in this part.

Part III of the Unified Guidance presents a range of ***detection monitoring*** statistical procedures. First, there is a discussion of the Student's *t*-test and its non-parametric counterpart, the Wilcoxon rank-sum test, when comparing two groups of data (*e.g.*, background versus one downgradient well). This part defines both parametric and non-parametric prediction limits, and their application to groundwater analysis when multiple comparisons are involved. A variety of prediction limit possibilities are presented to cover likely interpretations of sampling and testing requirements under the RCRA regulations.

Substantial detailed guidance is offered for using prediction limits with retesting procedures, and how various retesting algorithms might be constructed. The final chapter of this Part considers another statistical method especially useful for ***intrawell*** comparisons, namely the Shewhart-CUSUM control chart. A brief discussion of analysis of variance [ANOVA] and tolerance limit tests identified in the RCRA regulations is also provided.

Part IV of the Unified Guidance is devoted to statistical methods recommended for ***compliance*** or ***assessment monitoring*** and ***corrective action***. Compliance monitoring typically involves a comparison of downgradient well data to a groundwater protection standard [GWPS], which may be a limit derived from background or a fixed concentration limit (such as in 40 CFR 264.94 Table 1, an MCL, a risk-based limit, an alternate concentration limit, or a defined clean-up standard under corrective action). The key statistical procedure is the confidence interval, and several confidence interval tests (mean, median, or upper percentile) may be appropriate for compliance evaluation depending on the circumstances. The choice depends on the distribution of the data, frequency of non-detects, the type of standard being compared, and whether or not the data exhibit a significant trend. Discussions in this part consider fixed compliance standards used in a variety of EPA programs and what they might represent in statistical terms. Strategies for corrective action differ from those appropriate for compliance monitoring primarily because statistical hypotheses are changed, although the same basic statistical methods may be employed.

Since some programs will also utilize background as standards for compliance and corrective action monitoring, those tests and discussions under **Part III** detection monitoring (including statistical design in **Part I**) may pertain in identifying the appropriate standards and tests.

A ***glossary*** of important statistical terms, ***references*** and a subject ***index*** are provided at the end of the main text. The ***Appendices*** contain additional notes on a number of topics including previous guidance, a special study for the guidance, more detailed statistical power discussions, and an extensive set of ***statistical tables*** for implementing the methods outlined in the Unified Guidance. Some tables, especially those for prediction limit retesting procedures, have been extended within the Unified Guidance beyond published sources in order to cover a wider variety of plausible scenarios.

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PART I. STATISTICAL DESIGN AND PHILOSOPHY

Chapter 1 provides introductory information, including the purposes and goals of the guidance, as well as its potential applicability to other environmental programs. **Chapter 2** presents a brief discussion of the existing regulations and identifies key portions of these rules which need to be addressed from a statistical standpoint, as well as some recommendations. In **Chapter 3**, fundamental statistical principles are highlighted which play a prominent role in the Unified Guidance including the notions of individual test false positive and negative decision errors and the accumulation of such errors across multiple tests or comparisons. **Chapter 4** sets the groundwater monitoring program context, the nature of formal statistical tests for groundwater and some caveats in identifying statistically significant increases. Typical groundwater monitoring scenarios also are described in this chapter. **Chapter 5** describes how to establish background and how to periodically update it. **Chapters 6** and **7** outline various factors to be considered when designing a reasonable statistical strategy for use in detection monitoring, compliance/assessment monitoring, or corrective action. Finally, **Chapter 8** summarizes the recommended statistical tests and methods, along with a concise review of assumptions, conditions of use, and limitations.

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CHAPTER 1. OBJECTIVES AND POTENTIAL USE OF THIS GUIDANCE

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1.1 OBJECTIVES

The fundamental goals of the RCRA groundwater monitoring regulations are fairly straightforward. Regulated parties are to accurately characterize existing groundwater quality at their facility, assess whether a hazardous constituent release has occurred and, if so, determine whether measured levels meet the compliance standards. Using accepted statistical testing, evaluation of groundwater quality should have a high probability of leading to correct decisions about a facility's regulatory status.

To implement these goals, EPA first promulgated regulations in 1980 (for interim status facilities) and 1982 (permitted facilities) for detecting contamination of groundwater at hazardous waste Subtitle C land disposal facilities. In 1988, EPA revised portions of those regulations found at 40 CFR Part 264, Subpart F. A similar set of regulations applying to Subtitle D municipal and industrial waste facilities was adopted in 1991 under 40 CFR Part 258. In April 2006, certain modifications were made to the 40 CFR Part 264 groundwater monitoring regulations affecting statistical testing and decision-making.

EPA released the ***Interim Final Guidance*** [IFG] in 1989 for implementing the statistical methods and sampling procedures identified in the 1988 rule. A second guidance document followed in July 1992 called ***Addendum to Interim Final Guidance*** [Addendum], which expanded certain techniques and also served as guidance for the newer Subpart D regulations.

As the RCRA groundwater monitoring program has matured, it became apparent that the existing guidance needed to be updated to adequately cover statistical methods and issues important to detecting changes in groundwater.[1] Research conducted in the area of groundwater statistics since 1992 has provided a number of improved statistical techniques. At the same time, experience gained in applying the regulatory statistical tests in groundwater monitoring contexts has identified certain constraints. Both needed to be factored into the guidance. This Unified Guidance document addresses these concerns and supercedes both the earlier IFG and Addendum.

The Unified Guidance offers guidance to owners and operators, EPA Regional and State personnel, and other interested parties in selecting, using, and interpreting appropriate statistical methods for evaluating data under the RCRA groundwater monitoring regulations. The guidance

[1] Some recommendations in EPA's **Statistical Training Course on Groundwater Monitoring** were developed to better reflect the reality of groundwater conditions at many sites, but were not generally available in published form. See RCRA Docket # EPA\530-R-93-003, 1993

identifies recent approaches and recommends a consistent framework for applying these methods. One key aspect of the Unified Guidance is providing a systematic application of the basic statistical principle of balancing false positives and negative errors in designing good testing procedures (*i.e.*, minimizing both the risk of falsely declaring a site to be out-of-compliance and of missing real evidence of an adverse change in the groundwater). Topics addressed in the guidance include basic statistical concepts, sampling design and sample sizes, selection of appropriate statistical approaches, how to check data and run statistical tests, and the interpretation of results. References for the suggested procedures and to more general statistical texts are provided. The guidance notes when expert statistical consultation may be advisable. Such guidance may also have applicability to other remedial activities as well.

Enough commonality exists in sampling, analysis, and evaluation under the RCRA regulatory requirements that the Unified Guidance often suggests relatively general strategies. At the same time, there may be situations where site-specific considerations for sampling and statistical analysis are appropriate or needed. EPA policy has been to promulgate regulations that are specific enough to implement, yet flexible in accommodating a wide variety of site-specific environmental factors. Usually this is accomplished by specifying criteria appropriate for the majority of monitoring situations, while at the same time allowing alternatives that are also protective of human health and the environment.

40 CFR Parts 264 and 258 allow the use of other sampling procedures and test methods[2] beyond those explicitly identified in the regulations,[3] subject to approval by the Regional Administrator or state Director. Alternative test methods must be able to meet the performance standards at §264.97(i) or §258.53(h). While these performance standards are occasionally specific, they are much less so in other instances. Accordingly, further guidance is provided concerning the types of procedures that should generally satisfy such performance standards.

Although the Part 264 and 258 regulations explicitly identify five basic formal statistical procedures for testing two- or multiple-sample comparisons characteristic of detection monitoring, the rules are silent on specific tests under compliance or corrective action monitoring when a groundwater protection standard is fixed (a one-sample comparison). The rules also require consideration of data patterns (normality, independence, outliers, non-detects, spatial and temporal dependence), but do not identify specific tests. This document expands the potential statistical procedures to cover these situations identified in earlier guidance, thus providing a comprehensive single EPA reference on statistical methods generally recommended for RCRA groundwater monitoring programs. Not every technique will be appropriate in a given situation, and in many cases more than one statistical approach can be used. The Unified Guidance is meant to be broad enough in scope to cover a high percentage of the potential situations a user might encounter.

The Unified Guidance is not designed as a treatise for statisticians; rather it is aimed at the informed groundwater professional with a limited background in statistics. Most methods discussed are well-known to statisticians, but not necessarily to regulators, groundwater engineers or scientists. A key thrust of the Unified Guidance has been to tailor the standard statistical techniques to the RCRA groundwater arena and its unique constraints. Because of this emphasis, not every variation of each test

[2] For example, §264.97(g)(2), §264.97(h)(5) and §258.53(g)(5)

[3] §264.97(g)(1), §264.97(h)(1-4), and §258.53(g)(1-4) respectively

is discussed in detail. For example, groundwater monitoring in a detection monitoring program is generally concerned with *increases* rather than *decreases* in concentration levels of monitored parameters. Thus, most detection monitoring tests in the Unified Guidance are presented as one-sided upper-tailed tests. In the sections covering compliance and corrective action monitoring (**Chapters 21** and **22** in **Part IV**), either one-sided lower-tail or upper-tail tests are recommended depending on the monitoring program. Users requiring two-tailed tests or additional information may need to consult other guidance or the statistical references listed at the end of the Unified Guidance.

The Unified Guidance is not intended to cover all statistical methods that might be applicable to groundwater. The technical literature is even more extensive, including other published frameworks for developing statistical programs at RCRA facilities. Certain statistical methods and general strategies described in the Unified Guidance are outlined in American Society for Testing and Materials [ASTM] documents entitled ***Standard Guide for Developing Appropriate Statistical Approaches for Groundwater Detection Monitoring Programs*** **(D6312-98[2005])** (ASTM, 2005) and ***Standard Guide for Applying Statistical Methods for Assessment and Corrective Action Environmental Monitoring Programs*** **(D7048-04)** (ASTM, 2004).

The first of these ASTM guidelines primarily covers strategies for detection monitoring, emphasizing the use of prediction limits and control charts. It also contains a series of flow diagrams aimed at guiding the user to an appropriate statistical approach. The second guideline covers statistical strategies useful in compliance/assessment monitoring and corrective action. While not identical to those described in the Unified Guidance, the ASTM guidelines do provide an alternative framework for developing statistical programs at RCRA facilities and are worthy of careful consideration.

EPA's primary consideration in developing the Unified Guidance was to select methods both consistent with the RCRA regulations, as well as straightforward to implement. We believe the methods in the guidance are not only effective, but also understandable and easy to use.

1.2 APPLICABILITY TO OTHER ENVIRONMENTAL PROGRAMS

The Unified Guidance is tailored to the context of the RCRA groundwater monitoring regulations. Some of the techniques described are unique to this guidance. Certain regulatory constraints and the nature of groundwater monitoring limit how statistical procedures are likely to be applied. These include typically small sample sizes during a given evaluation period, a minimum of annual monitoring and evaluation and typically at least semi-annual, often a large number of potential monitoring constituents, background-to-downgradient well comparisons, and a limited set of identified statistical methods. There are also unique regulatory performance constraints such as §264.97(i)(2), which requires a minimum single test false positive α level of 0.01 and a minimum 0.05 level for multiple comparison procedures such as analysis of variance [ANOVA].

There are enough commonalities with other regulatory groundwater monitoring programs (*e.g.*, certain distributional features of routinely monitored background groundwater constituents) to allow for more general use of the tests and methods in the Unified Guidance. Many of these test methods and the consideration of false positive and negative errors in site design are directly applicable to corrective action evaluations of solid waste management units under 40 CFR 264.101 and Comprehensive

Environmental Response, Compensation, and Liability Act [CERCLA] groundwater monitoring programs.

There are also comparable situations involving other environmental media to which the Unified Guidance statistical methods might be applied. Groundwater detection monitoring involves either a comparison between different monitoring stations (*i.e.*, downgradient compliance wells *vs.* upgradient wells) or a contrast between past and present data within a given station (*i.e.*, intrawell comparisons). To the extent that an environmental monitoring station is essentially fixed in location (*e.g.*, air quality monitors, surface water stations) and measurements are made over time, the same statistical methods may be applicable.

The Unified Guidance also details methods to compare background data against measurements from regulatory compliance points. These procedures (*e.g.*, Welch's *t*-test, prediction limits with retesting, *etc.*) are designed to contrast multiple groups of data. Many environmental problems involve similar comparisons, even if the groups of data are not collected at fixed monitoring stations (*e.g.*, as in soil sampling). Furthermore, the guidance describes diagnostic techniques for checking the assumptions underlying many statistical procedures. Testing of normality is ubiquitous in environmental statistical analysis. Also common are checks of statistical independence in time series data, the assumption of equal variances across different populations, and the need to identify outliers. The Unified Guidance addresses each of these topics, providing useful guidance and worked out examples.

Finally, the Unified Guidance discusses techniques for comparing datasets against fixed numerical standards (as in compliance monitoring or corrective action). Comparison of data against a fixed standard is encountered in many regulatory programs. The methods described in **Part IV** of the Unified Guidance could therefore have wider applicability, despite being tailored to the groundwater monitoring data context.

EPA recognizes that many guidance users will make use of either commercially available or proprietary statistical software in applying these statistical methods. Because of their wide range of diversity and coverage, the Unified Guidance does not evaluate software usage or applicability. Certain software is provided with the guidance. The guidance limits itself to describing the basic statistical principles underlying the application of the recommended tests.

CHAPTER 2. REGULATORY OVERVIEW

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This chapter generally summarizes the RCRA groundwater monitoring regulations under 40 CFR Parts 264, 265 and 258 applicable to this guidance. A second section identifies the most critical regulatory statistical issues and how they are addressed by this guidance. Finally, recommendations regarding interim status facilities and certain statistical methods in the regulations are presented at the end of the chapter.

2.1 REGULATORY SUMMARY

Section 3004 of RCRA directs EPA to establish regulations applicable to owners and operators of facilities that treat, store, or dispose of hazardous waste as may be necessary to protect human health and the environment. Section 3005 provides for the implementation of these standards under permits issued to owners and operators by EPA or authorized States. These regulations are codified in 40 CFR Part 264. Section 3005 also provides that owners and operators of facilities in existence at the time of the regulatory or statutory requirement for a permit, who apply for and comply with applicable requirements, may operate until a permit determination is made. These facilities are commonly known as interim status facilities, which must comply with the standards promulgated in 40 CFR Part 265.

EPA first promulgated the groundwater monitoring regulations under Part 265 for interim status surface impoundments, landfills and land treatment units ("regulated units") in 1980.[1] Intended as a temporary system for units awaiting full permit requirements, the rules set out a minimal detection and assessment monitoring system consisting of at least a single upgradient and three downgradient wells. Following collection of the minimum number of samples prescribed in the rule for four indicator parameters — pH, specific conductance, total organic carbon (TOC) and total organic halides (TOX) — and certain constituents defining overall groundwater quality, the owner/operator of a land disposal facility is required to implement a ***detection monitoring*** program. Detection monitoring consists of upgradient-to-downgradient comparisons using the Student's *t*-test of the four indicator parameters at no less than a .01 level of significance (α). The regulations refer to the use of "replicate" samples for contaminant indicator comparisons. Upon failure of a single detection-level test, as well as a repeated

[1] [45 FR 33232ff, May 19, 1980] Interim status regulations; later amended in 1983 and 1985

follow-up test, the facility is required to conduct an *assessment* program identifying concentrations of hazardous waste constituents from the unit in groundwater. A facility can return to detection monitoring if none of the latter constituents are detected. These regulations are still in effect today.

Building on the interim status rules, Subtitle C regulations for Part 264 permitted hazardous waste facilities followed in 1982,[2] where the basic elements of the present RCRA groundwater monitoring program are defined. In §264.91, three monitoring programs — *detection monitoring*, *compliance monitoring*, and *corrective action* — serve to protect groundwater from releases of hazardous waste constituents at certain regulated land disposal units (surface impoundments, waste piles, landfills, and land treatment). In developing permits, the Regional Administrator/State Director establishes groundwater protection standards [GWPS] under §264.92 using concentration limits [§264.94] for certain monitoring constituents [§264.93]. Compliance well monitoring locations are specified in the permit following the rules in §264.95 for the required compliance period [§264.96]. General monitoring requirements were established in §264.97, along with specific detection [§264.98], compliance [§264.99], and corrective action [§264.100] monitoring requirements. Facility owners and operators are required to sample groundwater at specified intervals and to use a statistical procedure to determine whether or not hazardous wastes or constituents from the facility are contaminating the groundwater.

As found in §264.91, detection monitoring is the first stage of monitoring when no or minimal releases have been identified, designed to allow identification of significant changes in the groundwater when compared to background or established baseline levels. Downgradient well observations are tested against established background data, including measurements from upgradient wells. These are known as two- or multiple-sample tests.

If there is statistically significant evidence of a release of hazardous constituents [§264.91(a)(1) and (2)], the regulated unit must initiate compliance monitoring, with groundwater quality measurements compared to the groundwater protection standards [GWPS]. The owner/operator is required to conduct a more extensive Part 261 Appendix VIII (later Part 264 Appendix IX)[3] evaluation to determine if additional hazardous constituents must be added to the compliance monitoring list.

Compliance/assessment as well as corrective action monitoring differ from detection monitoring in that groundwater well data are tested against the groundwater protection standards [GWPS] as established in the permit. These may be fixed health-based standards such as Safe Drinking Water Act [SDWA] maximum concentration limits [MCLs], §264.94 Table 1 values, a value defined from background, or alternate-concentration limits as provided in §264.94(a). Statistically, these are considered single-sample tests against a fixed limit (a background limit can either be a single- or two-sample test depending on how the limit is defined). An exceedance occurs when a constituent level is shown to be significantly greater than the GWPS or compliance standard.

If a hazardous monitoring constituent under compliance monitoring statistically exceeds the GWPS at any compliance well, the facility is subject to corrective action and monitoring under §264.100. Following remedial action, a return to compliance consists of a statistical demonstration that

[2] [47 FR 32274ff, July 26, 1982] Permitting Requirements for Land Disposal Facilities

[3] [52 FR 25942, July 9, 1987] List (Phase I) of Hazardous Constituents for Groundwater Monitoring; Final Rule

the concentrations of all relevant hazardous constituents lie below their respective standards. Although the rules define a three-tiered approach, the Regional Administrator or State Director can assess available information at the time of permit development to identify which monitoring program is appropriate [§264.91(b)].

Noteworthy features of the 1982 rule included retaining use of the four Part 265 indicator parameters, but allowing for additional constituents in detection monitoring. The number of upgradient and downgradient wells was not specified; rather the requirement is to have a sufficient number of wells to characterize upgradient and downgradient water quality passing beneath a regulated unit. Formalizing the "replicate" approach in the 1980 rules and the use of Student's *t*-test, rules under §264.97 required the use of aliquot replicate samples, which involved analysis of at least four physical splits of a single volume of water. In addition, Cochran's Approximation to the Behrens-Fisher [CABF] Student's *t*-test was specified for detection monitoring at no less than a .01 level of significance (α). Background sampling was specified for a one-year period consisting of four quarterly samples (also using the aliquot approach). The rules allowed use of a repeated, follow-up test subsequent to failure of a detection monitoring test. A minimum of semi-annual sampling was required.

In response to a number of concerns with these regulations, EPA amended portions of the 40 CFR Part 264 Subpart F regulations including statistical methods and sampling procedures on October 11, 1988.[4] Modifications to the regulations included requiring (if necessary) that owners and/or operators more accurately characterize the hydrogeology and potential contaminants at the facility. The rule also identifies specific performance standards in the regulations that all the statistical methods and sampling procedures must meet (discussed in a following section). That is, it is intended that the statistical methods and sampling procedures meeting these performance standards defined in §264.97 have a low probability both of indicating contamination when it is not present (Type I error), and of failing to detect contamination that actually is present (Type II error). A facility owner and/or operator must demonstrate that a procedure is appropriate for the site-specific conditions at the facility, and ensure that it meets the performance standards. This demonstration applies to any of the statistical methods and sampling procedures outlined in the regulation as well as any alternate methods or procedures proposed by facility owners and/or operators.

In addition, the amendments removed the required use of the CABF Student's t-test, in favor of five different statistical methods deemed to be more appropriate for analyzing groundwater monitoring data (discussed in a following section). The CABF procedure is still retained in Part 264, Appendix IV, as an option, but there are no longer specific citations in the regulations for this test. These newer procedures offer greater flexibility in designing a groundwater statistical program appropriate to site-specific conditions. A sixth option allows the use of alternative statistical methods, subject to approval by the Regional Administrator. EPA also instituted new groundwater monitoring sampling requirements, primarily aimed at ensuring adequate statistical sample sizes for use in analysis of variance [ANOVA] procedures, but also allowing alternative sampling plans to be approved by the Regional Administrator. The requirements identify the need for statistically independent samples to be used during evaluation. The Agency further recognizes that the selection of appropriate hazardous

[4] [53 FR 39720, October 11, 1988] 40 CFR Part 264: Statistical Methods for Evaluating Groundwater Monitoring From Hazardous Waste Facilities; Final Rule

constituent monitoring parameters is an essential part of a reliable statistical evaluation. EPA addressed this issue in a 1987 Federal Register notice.[5]

§264.101 requirements for corrective action at non-regulated units were added in 1985 and later.[6] The Agency determined that since corrective action at non-regulated units would work under a different program, these units are not required to follow the detailed steps of Subpart F monitoring.

In 1991, EPA promulgated Subtitle D groundwater monitoring regulations for municipal solid waste landfills in 40 CFR Part 258.[7] These rules also incorporate a three-tiered groundwater monitoring strategy (detection monitoring, assessment monitoring, and corrective action), and describe statistical methods for determining whether background concentrations or the groundwater protection standards [GWPS] have been exceeded.

The statistical methods and related performance standards in 40 CFR Part 258 essentially mirror the requirements found as of 1988 at 40 CFR Part 264 Subpart F, with certain differences. Minimum sampling frequencies are different than in the Subtitle C regulations. The rules also specifically provide for the GWPS using either current MCLs or standardized risk-based limits as well as background concentrations. In addition, a specific list of hazardous constituent analytes is identified in 40 CFR Part 258, Appendix I for detection-level monitoring, including the use of unfiltered (total) trace elements.

The 1988 and 1991 rule amendments identify certain statistical methods and sampling procedures believed appropriate for evaluating groundwater monitoring data under a variety of situations. Initial guidance to implement these methods was released in 1989 as: ***Statistical Analysis of Groundwater Monitoring Data at RCRA Facilities: Interim Final Guidance*** [IFG]. The IFG covered basic topics such as checking distributional assumptions, selecting one of the methods and sampling frequencies. Examples were provided for applying the recommended statistical procedures and interpreting the results. Two types of compliance tests were provided for comparison to the GWPS — mean/median confidence intervals and upper limit tolerance intervals.

Given additional interest from users of the comparable regulations adopted for Subtitle D solid waste facilities in 1991, and with experience gained in implementing various tests, EPA actively sought to improve existing groundwater statistical guidance. This culminated in a July 1992 publication of: ***Statistical Analysis of Groundwater Monitoring Data at RCRA Facilities: Addendum to Interim Final Guidance*** [Addendum].

The 1992 Addendum included a chapter devoted to retesting strategies, as well as new guidance on several non-parametric techniques not covered within the IFG. These included the Wilcoxon rank-sum test, non-parametric tolerance intervals, and non-parametric prediction intervals. The Addendum also included a reference approach for evaluating statistical power to ensure that contamination could be adequately detected. The Addendum did not replace the IFG — the two documents contained overlapping material but were mostly intended to complement one another based on newer information

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[5] [52 FR 25942, July 9, 1987] op. cit.

[6] [50 FR 28747, July 15, 1985] Amended in 1987, 1993, and 1998

[7] [56 FR 50978, October 9, 1991] 40 CFR Parts 257 & 258: Solid Waste Disposal Facility Criteria: Final Rule, especially Part 258 Subpart E Groundwater Monitoring and Corrective Action

and comments from statisticians and users of the guidance. However, the Addendum changed several recommendations within the IFG and replaced certain test methods first published in the IFG. The two documents provided contradictory guidance on several points, a concern addressed by this guidance.

More recently in April 2006, EPA promulgated further changes to certain 40 CFR Part 264 groundwater monitoring provisions as part of the Burden Reduction Initiative Rule.[8] A brief summary of the regulatory changes and the potential effects on existing RCRA groundwater monitoring programs is provided. Four items of specific interest are:

- Elimination of the requirements to sample four successive times per statistical evaluation under §264.98(d) and §264.99(f) in favor of more flexible, site-specific options as identified in §264.97(g)(1)&(2);
- Removal of the requirements in §264.98(g) and §264.99(g) to annually sample *all* monitoring wells for Part 264 Appendix IX constituents in favor of a specific subset of wells;
- Modifications of these provisions to allow for a specific subset of Part 264 Appendix IX constituents tailored to site needs; and
- A change in the resampling requirement in §264.98(g)(3) from "within a month" to a site-specific schedule.

These changes to the groundwater monitoring provisions require coordination between the regulatory agency and owner/operator with final approval by the agency. Since the regulatory changes are not issued under the 1984 Hazardous and Solid Waste Amendments [HSWA] to RCRA, authorized State RCRA program adoption of these rules is discretionary. States may choose to maintain more stringent requirements, particularly if already codified in existing regulations. Where EPA has direct implementation authority, the provisions would go into effect following promulgation.

The first provision reaffirms the flexible approach in the Unified Guidance for detection monitoring sampling frequencies and testing options. State RCRA programs using the four-successive sampling requirements can still continue to do so under §264.97(g)(1), but the rule now allows for alternate sampling frequencies under §264.97(g)(2) in both detection and compliance monitoring. The second and third provisions provide more site- and waste-specific options for Part 264 Appendix IX compliance monitoring. The final provision provides more flexibility when resampling these Appendix IX constituents.

Since portions of the earlier and the most recent rules are still operative, all are considered in the present Unified Guidance. The effort to create this guidance began in 1996, with a draft release in December 2004, a peer review in 2005, and a final version completed in 2009.

[8] [71 FR 16862-16915] April 4, 2006

2.2 SPECIFIC REGULATORY FEATURES AND STATISTICAL ISSUES

This section describes critical portions of the RCRA groundwater monitoring regulations which the present guidance addresses. The regulatory language is provided below in bold and italics.[9] A brief discussion of each issue is provided in statistical terms and how the Unified Guidance deals with it.

2.2.1 STATISTICAL METHODS IDENTIFIED UNDER §264.97(h) AND §258.53(g)

The owner or operator will specify one of the following statistical methods to be used in evaluating groundwater monitoring data for each hazardous constituent which, upon approval by the Regional Administrator, will be specified in the unit permit. The statistical test chosen shall be conducted separately for each hazardous constituent in each well…

1. ***A parametric analysis of variance (ANOVA) followed by multiple comparison procedures to identify statistically significant evidence of contamination. The method must include estimation and testing of the contrasts between each compliance well's mean and the background mean levels for each constituent.***

2. ***An analysis of variance (ANOVA) based on ranks followed by multiple comparison procedures to identify statistically significant evidence of contamination. The method must include estimation and testing of the contrasts between each compliance well's median and the background median levels for each constituent.***

3. ***A tolerance interval or prediction interval procedure in which an interval for each constituent is established from the distribution of the background data, and the level of each constituent in each compliance well is compared to the upper tolerance or prediction limit.***

4. ***A control chart approach that gives control limits for each constituent.***

5. ***Another statistical method submitted by the owner or operator and approved by the Regional Administrator.***

Part III of the Unified Guidance addresses these specific tests, as applied to a detection monitoring program. It is assumed that statistical testing will be conducted separately for each hazardous constituent in each monitoring well. The recommended non-parametric ANOVA method based on ranks is identified in this guidance as the Kruskal-Wallis test. ANOVA tests are discussed in **Chapter 17**. Tolerance interval and prediction limit tests are discussed separately in **Chapters 17** and **18**, with particular attention given to implementing prediction limits with retesting when conducting multiple comparisons in **Chapter 19**. The recommended type of control chart is the combined Shewhart-CUSUM control chart test, discussed in **Chapter 20**. Where a groundwater protection standard is based on background levels, application of these tests is discussed in **Part I**, **Chapter 7** and **Part IV**, **Chapter 22.**

[9] The following discussions somewhat condense the regulatory language for ease of presentation and understanding. Exact citations for regulatory text should be obtained from the most recent Title 40 Code of Federal Regulations.

If a groundwater protection standard involves a fixed limit, none of the listed statistical methods in these regulations directly apply. Consequently, a number of other single-sample tests for comparison with a fixed limit are recommended in **Part IV**. Certain statistical limitations encountered when using ANOVA and tolerance level tests in detection and compliance monitoring are also discussed in these chapters. Additional use of ANOVA tests for diagnostic identification of spatial variation or temporal effects is discussed in **Part II**, **Chapters 13** and **14**.

2.2.2 PERFORMANCE STANDARDS UNDER §264.97(i) AND §258.53(h)

Any statistical method chosen under §264.97(h) [or §258.53(g)] for specification in the unit permit shall comply with the following performance standards, as appropriate:

1. ***The statistical method used to evaluate ground-water monitoring data shall be appropriate for the distribution of chemical parameters or hazardous constituents. If the distribution of the chemical parameters or hazardous constituents is shown by the owner or operator to be inappropriate for a normal theory test, then the data should be transformed or a distribution-free test should be used. If the distributions for the constituents differ, more than one statistical method may be needed.***

2. ***If an individual well comparison procedure is used to compare an individual compliance well constituent concentration with background constituent concentrations or a groundwater protection standard, the test shall be done at a Type I error level no less than 0.01 for each testing period. If a multiple comparisons procedure is used, the Type I experiment-wise error rate for each testing period shall be no less than 0.05; however, the Type I error of no less than 0.01 for individual well comparisons must be maintained. This performance standard does not apply to control charts, tolerance intervals, or prediction intervals.***

3. ***If a control chart approach is used to evaluate groundwater monitoring data, the specific type of control chart and its associated parameter values shall be proposed by the owner or operator and approved by the Regional Administrator if he or she finds it to be protective of human health and the environment.***

4. ***If a tolerance interval or a prediction interval is used to evaluate groundwater monitoring data, the levels of confidence, and for tolerance intervals, the percentage of the population that the interval must contain, shall be proposed by the owner or operator and approved by the Regional Administrator if he or she finds it protective of human health and the environment. These parameters will be determined after considering the number of samples in the background data base, the data distribution, and the range of the concentration values for each constituent of concern.***

5. ***The statistical method shall account for data below the limit of detection with one or more procedures that are protective of human health and the environment. Any practical quantification limit (pql) approved by the Regional Administrator under §264.97(h) [or §258.53(g)] that is used in the statistical method shall be the lowest concentration level that can be reliably achieved within specified limits of precision and accuracy during routine laboratory operating conditions available to the facility.***

6. ***If necessary, the statistical method shall include procedures to control or correct for seasonal and spatial variability as well as temporal correlation in the data.***

These performance standards pertain to both the listed tests as well as others (such as those recommended in **Part IV** of the guidance for comparison to fixed standards). Each of the performance standards is addressed in **Part I** of the guidance for designing statistical monitoring programs and in **Part II** of the guidance covering diagnostic testing.

The ***first*** performance standard considers distributional properties of sample data; procedures for evaluating normality, transformations to normality, or use of non-parametric (distribution-free) methods are found in **Chapter 10**. Since some statistical tests also require an assumption of equal variances across groups, **Chapter 11** provides the relevant diagnostic tests. Defining an appropriate distribution also requires consideration of possible outliers. **Chapter 12** discusses techniques useful in outlier identification.

The ***second*** performance standard identifies minimum false positive error rates required when conducting certain tests. "Individual well comparison procedures" cited in the regulations include various ANOVA-type tests, Student's *t*-tests, as well as one-sample compliance monitoring/corrective action tests against a fixed standard. Per the regulations, these significance level (α) constraints do not apply to the other listed statistical methods — control charts, tolerance intervals, or prediction intervals.

When comparing an individual compliance well against background, the probability of the test resulting in a false positive or Type I error should be no *less* than 1 in 100 (1%). EPA required a minimum Type I error level for a given test and fixed sample size because false positive and negative rates are inversely related. By limiting Type I error rates to 1%, EPA felt that the risk of incurring false positives would be sufficiently low, while providing sufficient statistical power (i.e., the test's ability to control the false negative rate, that is, the rate of missing or not detecting true changes in groundwater quality).

Though a procedure to test an individual well like the Student's *t*-test may be appropriate for the smallest of facilities, more extensive networks of groundwater monitoring wells and monitoring parameters will generally require a ***multiple comparisons*** procedure. The 1988 regulations recognized this need in specifying a one-way analysis of variance [ANOVA] procedure as the method of choice for replacing the CABF Student's *t*-test. The *F*-statistic in an analysis of variance [ANOVA] does indeed control the site-wide or ***experiment-wise*** error rate when evaluating multiple upgradient and downgradient wells, at least for a single constituent. Using this technique allowed the Type I experiment-wise error rate *for each constituent* to be controlled to about 5% for each testing period.

To maintain adequate statistical power, the regulations also mandate that the ANOVA procedure be run at a *minimum* 5% false positive rate per constituent. But when a full set of well-constituent combinations are considered (particularly large suites of detection monitoring analytes at numerous compliance wells), the site-wide false positive rate can be much greater than 5%. The one-way ANOVA is inherently an interwell technique, designed to simultaneously compare datasets from different well locations. Constituents with significant natural spatial variation are likely to trigger the ANOVA *F*-statistic even in the absence of real contamination, an issue discussed in **Chapter 13**.

Control charts, tolerance intervals, and prediction intervals provide alternate testing strategies for simultaneously controlling false positive rates while maintaining adequate power to detect contamination during detection monitoring. Although the rules do not require a minimum nominal false positive rate as specified in the **second** performance standard, use of tolerance or prediction intervals combined with a retesting strategy can result in sufficiently low experiment-wise Type I error rates and the ability to detect real contamination. **Chapters 17**, **18** and **20** consider how tolerance limits, control charts, and prediction limits can be designed to meet the **third** and **fourth** performance standards specific to these tests considering the number of samples in background, the data distribution, and the range of concentration values for each constituent of concern [COC]. **Chapters 19** and **20** on multiple comparison procedures using prediction limits or control charts identify how retesting can be used to enhance power and meet the specified false positive objectives.

The ***fifth*** performance standard requires statistical tests to account for non-detect data. **Chapter 15** provides some alternative approaches for either adjusting or modeling sample data in the presence of reported non-detects. Other chapters include modifications of standard tests to properly account for the non-detect portion of data sets.

The ***sixth*** performance standard requires consideration of spatial or temporal (including seasonal) variation in the data. Such patterns can have major statistical consequences and need to be carefully addressed. Most classical statistical tests in this guidance require assumptions of data <u>independence</u> and <u>stationarity</u>. Independence roughly means that observing a given sample measurement does not allow a precise prediction of other sample measurements. Drawing colored balls from an urn at random illustrates and fits this requirement; in groundwater, sample volumes are assumed to be drawn more or less at random from the population of possible same-sized volumes comprising the underlying aquifer. Stationarity assumes that the population being sampled has a constant mean and variance across time and space. Spatial or temporal variation in the well means and/or variances can negate these test assumptions. **Chapter 13** considers the use of ANOVA techniques to establish evidence of spatial variation. Modification of the statistical approach may be necessary in this case; in particular, background levels will need to be established at each compliance well for future comparisons (termed *intrawell* tests). Control chart, tolerance limit, and prediction limit tests can be designed for intrawell comparisons; these topics are considered in **Part III** of this guidance.

Temporal variation can occur for a number of reasons — seasonal fluctuations, autocorrelation, trends over time, *etc*. **Chapter 14** addresses these forms of temporal variation, along with recommended statistical procedures. In order to achieve stationarity and independence, sample data may need to be adjusted to remove trends or other forms of temporal dependence. In these cases, the ***residuals*** remaining after trend removal or other adjustments are used for formal testing purposes. Correlation among monitoring constituents within and between compliance wells can occur, a subject also treated in this chapter.

When evaluating statistical methods by these performance standards, it is important to recognize that the ability of a particular procedure to operate correctly in minimizing unnecessary false positives while detecting possible contamination depends on several factors. These include not only the choice of significance level and test hypotheses, but also the statistical test itself, data distributions, presence or absence of outliers and non-detects, the presence or absence of spatial and temporal variation, sampling requirements, number of samples and comparisons to be made, and frequency of sampling. Since all of these statistical factors interact to determine the procedure's effectiveness, ***any proposed statistical***

***procedure needs to be evaluated in its entirety, not by individual components*.** **Part I, Chapter 5** discusses evaluation of potential background databases considering all of the performance criteria.

2.2.3 HYPOTHESIS TESTS IN DETECTION, COMPLIANCE/ASSESSMENT, AND CORRECTIVE ACTION MONITORING

The Part 264 Subpart F groundwater monitoring regulations do not specifically identify the test hypotheses to be used in detection monitoring (§264.98), compliance monitoring (§264.99), and corrective action (§264.100). The same is true for the parallel Part 258 regulations for detection monitoring (§258.54), assessment monitoring (§258.55), and assessment of corrective measures (§258.56), as well as for evaluating interim status indicator parameters (§265.93) or Appendix III constituents. However, the language of these regulations as well as accepted statistical principles allow for clear definitions of the appropriate test hypotheses. Two- or multiple-sample comparisons (background *vs*. downgradient well data) are usually involved in detection monitoring (the comparison could also be made against an ACL limit based on background data). Units under detection monitoring are initially presumed not to be contributing a release to the groundwater unless demonstrated otherwise. From a statistical testing standpoint, the population of downgradient well measurements is assumed to be equivalent to or no worse than those of the background population; typically this translates into an initial or null hypothesis that the downgradient population mean is equal to or less than the background population mean. Demonstration of a release is triggered when one or more well constituents indicate statistically significant levels above background.

Compliance and corrective action tests generally compare single sets of sample data to a fixed limit or a background standard. The language of §264.99 indicates that a significant increase above a GWPS will demonstrate the need for corrective action. Consequently, the null hypothesis is that the compliance population mean (or perhaps an upper percentile) is at or below a given standard. The statistical hypothesis is thus quite similar to that of detection monitoring. In contrast, once an exceedance has been established and §264.100 is triggered, the null hypothesis is that a site is contaminated unless demonstrated to be significantly below the GWPS. The same principles apply to Part 258 monitoring programs. In Part 265, the detection monitoring hypotheses apply to an evaluation of the contaminant indicator parameters. The general subject of hypothesis testing is discussed in **Chapter 3**, and specific statistical hypothesis formulations are found in **Parts III** and **IV** of this guidance.

2.2.4 SAMPLING FREQUENCY REQUIREMENTS

Each of the RCRA groundwater monitoring regulations defines somewhat different minimum sampling requirements. §264.97(g)(1) & (2) provides two main options:

1. ***Obtaining a sequence of at least four samples taken at an interval that ensures, to the greatest extent technically feasible, that a statistically independent sample is obtained, by reference to the uppermost aquifer effective porosity, hydraulic conductivity, and hydraulic gradient, and the fate and transport characteristics of potential contaminants; or***

2. ***An alternate sampling procedure proposed by the owner or operator and approved by the Regional Administrator if protective of human health and the environment.***

Additional regulatory language in detection [§264.98(d)] and compliance [§264.99(f)] monitoring reaffirms the first approach:

> ***[A] a sequence of at least four samples from each well (background and compliance wells) must be collected at least semi-annually during detection/compliance monitoring…***

Interim status sampling requirements under §265.92[c] read as follows:

> ***(1) For all monitoring wells, the owner or operator must establish initial background concentrations or values of all parameters specified in paragraph (b) of this section. He must do this quarterly for one year;***
>
> ***(2) For each of the indicator parameters specified in paragraph (b)(3) of this section, at least four replicate measurements must be obtained for each sample and the initial background arithmetic mean and variance must be determined by pooling the replicate measurements for the respective parameter concentrations or values in samples obtained from upgradient wells during the first year.***

The requirements under Subtitle D §258.54(b) are somewhat different:

> ***The monitoring frequency for all constituents listed in Appendix I to this part,... shall be at least semi-annual during the active life of the facility.... A minimum of four independent samples from each well (background and downgradient) must be collected and analyzed for the Appendix I constituents... during the first semi-annual event. At least one sample from each well (background and downgradient) must be collected and analyzed during subsequent semi-annual events...***

The 1980 and 1982 regulations required four analyses of essentially a single physical sample for certain constituents, i.e., the four contaminant indicator parameters. The need for statistically independent data was recognized in the 1988 revisions to Part 264 and in the Part 258 solid waste requirements. In the latter rules, only a minimum single sample is required in successive semi-annual sampling events. Individual Subtitle C programs have also made use of the provision in §264.97(g)(2) to allow for fewer than four samples collected during a given semi-annual period, while other State programs require the four successive sample measurements. As noted, by the recent changes in the April 2006 Burden Reduction Rule, the explicit requirements to obtain at least four samples during the next evaluation period under 40 CFR §264.98(d) and §264.99(f) have been removed, allowing more general flexibility under the §264.97(g) sampling options. Individual State RCRA programs should be consulted as to whether these recent rule changes may be applicable.

The requirements of Parts 264 and 258 were generally intended to provide sufficient data for ANOVA-type tests in detection monitoring. However, control chart, tolerance limit, and prediction limit tests can be applied with as few as one new sample per evaluation, once background data are established. The guidance provides maximum flexibility in offering a range of prediction limit options in **Chapter 18** in order to address these various sample size requirements. Although not discussed in detail, the same conclusions pertain to the use of control charts or tolerance limits.

The use of the term "replicate" in the Part 265 interim status regulations can be a significant problem, if interpreted to mean repeat analyses of splits (or aliquots) of a single physical sample. The

regulations indicate the need for statistical independence among sample data for testing purposes. This guidance discusses the technical statistical problems that arise if replicate (aliquot) sample data are used with the required Student's *t*-test in Part 265. Thus, the guidance recommends, if possible, that interim status statistical evaluations be based on independent sample data as discussed in **Chapters 13** and **14** and at the end of this chapter. A more standardized Welch's version of the Student-t test for unequal variances is provided as an alternative to the CABF Student's *t*-test.

2.2.5 GROUNDWATER PROTECTION STANDARDS

Part 265 does not use the term groundwater protection standards. A first-year requirement under §265.92(c)(1) is:

> ***For all monitoring wells, the owner or operator must establish background concentrations or values of all parameters specified in paragraph (b) of this section. He must do this quarterly for one year.***

Paragraph (b) includes water supply parameters listed in Part 265 Appendix III, which also provides a Maximum Level for each constituent. If a facility owner or operator does *not* develop and implement an assessment plan under §265.93(d)(4), there is a requirement in §265.94(a)(2) to report the following information to the Regional Administrator:

> ***(i) During the first year when initial background concentrations are being established for the facility: concentrations or values of the parameters listed in §265.92(b)(1) for each groundwater monitoring well within 15 days after completing each quarterly analysis. The owner or operator must separately identify for each monitoring well any parameters whose concentrations or value has been found to exceed the maximum contaminant levels in Appendix III.***

Since the Part 265 regulations are explicit in requiring a one-to-one comparison, no statistical evaluation is needed or possible.

§264.94(a) identifies the permissible concentration limits as a GWPS under §264.92:

> ***The Regional Administrator will specify in the facility permit concentrations limits in the groundwater for hazardous constituents established under §264.93. The concentration of a constituent:***
>
> ***(1) must not exceed the background level of that constituent in the groundwater at the time the limit is specified in the permit; or***
>
> ***(2) for any of the constituents listed in Table 1, must not exceed the respective value given in that table if the background level is below the value given in Table 1; or***
>
> ***(3) must not exceed an alternate limit established by the Regional Administrator under paragraph (b) of this section.***

The RCRA Subtitle D regulations establish the following standards under §258.55(h) and (i):

> ***(h) The owner or operator must establish a groundwater protection standard for each Appendix II constituent detected in groundwater. The groundwater protection standard shall be:***
>
> > ***(1) For constituents for which a maximum contaminant level (MCL) has been promulgated under Section 1412 of the Safe Drinking Water Act (codified) under 40 CFR Part 141, the MCL for that constituent;***
> >
> > ***(2) for constituents for which MCLs have not been promulgated, the background concentration for the constituent established from wells in accordance with §258.51(a)(1); or***
> >
> > ***(3) for constituents for which the background level is higher than the MCL identified under paragraph (h)(1) of this section or health based levels identified under §258(i)(1), the background concentration.***
>
> ***(i) The Director of an approved State program may establish an alternative groundwater protection standard for constituents for which MCLs have not been established. These groundwater protection standards shall be appropriate health based levels that satisfy the following criteria:***
>
> > ***(1) the level is derived in a manner consistent with Agency guidelines for assessing health risks or environmental pollutants [51 FR 33992, 34006, 34014, 34028, Sept. 24, 1986]***
> >
> > ***(2) to (4)... [other detailed requirements for health risk assessment procedures]***

The two principal alternatives for defining a groundwater protection standard [GWPS] are either a limit based on background data or a fixed health-based value (*e.g.*, MCLs, §264.94 Table 1 values, or a calculated risk limit). The Unified Guidance discusses these two basic kinds of standards in **Chapters 7 and 21**. If a background limit is applied, some definition of how the limit is constructed from prior sample data is required at the time of development. For fixed health-based limits, the regulatory program needs to consider the statistical characteristic of the data (*e.g.*, mean, median, upper percentile) that best represents the standard in order to conduct appropriate statistical comparisons. This subject is also discussed in **Chapter 21**; the guidance provides a number of testing options in this regard.

2.3 UNIFIED GUIDANCE RECOMMENDATIONS

2.3.1 INTERIM STATUS MONITORING

As discussed in **Chapter 14**, replicates required for the four contaminant indicator parameters are not statistically independent when analyzed as aliquots or splits from a single physical sample. This results in incorrect estimates of variance and the degrees of freedom when used in a Student's *t*-test. One of the most important revisions in the 1988 regulations was to require that successive samples be independent. Therefore, at a minimum, the Unified Guidance recommends that only **independent** water quality sample data be applied to the detection monitoring Student's *t*-tests in **Chapter 16**.

There are other considerations limiting the application of these tests as well. As noted in **Chapter 5**, at least two of the indicator parameters (pH and specific conductance) are likely to exhibit natural spatial differences among monitoring wells. Depending on site groundwater characteristics, TOC and TOX may also vary spatially. TOX analytical limitations described in SW-846[10] also note that levels of TOX are affected by inorganic chloride levels, which themselves can vary spatially by well. In short, all four indicator parameters may need to be evaluated on an intrawell basis, *i.e.*, using historical data from compliance monitoring wells.

Since this option is not identified in existing Part 265 regulations for indicator detection monitoring, a more appropriate strategy is to develop an alternative ***groundwater quality assessment monitoring plan*** under §265.90(d)(3) and (4) and §265.93(d)(3) and (4). These sections of the regulations require evaluation of hazardous waste constituents reasonably derived from the regulated unit (either those which served as a basis for listing in Part 265 Appendix VII or which are found in §261.24 Table 1). Interim status units subject to a permit are also subject to the groundwater contaminant information collection provisions under §270.14[c], which potentially include all hazardous constituents (a wider range of contaminants, *e.g.*, Part 264 Appendix IX) reasonably expected from the unit. While an interim status facility can return to indicator detection monitoring if no hazardous constituent releases have been identified, such a return is itself optional.

EPA recommends that interim status facilities develop the §265.90(d)(3) & (4) alternative groundwater quality assessment monitoring plan, if possible, using principles and procedures found in this guidance for monitoring design and statistical evaluation. Unlike Part 264 monitoring, there are no formal compliance/corrective action steps associated with statistical testing. A regulatory agency may take appropriate enforcement action if data indicate a release or significant adverse effect. The monitoring plan can be applied for an indefinite period until permit development. Multi-year collection of semi-annual or quarterly hazardous constituent data is more determinative of potential releases. The facility or the regulatory agency may also wish to continue evaluation of some or all of the Part 265 water quality indicators. Eventually these groundwater data can be used to establish which monitoring program(s) may be appropriate at the time of permit development under §264.91(b).

2.3.2 PARTS 264 AND 258 DETECTION MONITORING METHODS

As described in **Chapter 13**, many of the commonly monitored inorganic analytes exhibit natural spatial variation among wells. Since the two ANOVA techniques in §264.97(h) and §258.53(g) depend on an assumption of a single common background population, these tests may not be appropriate in many situations. Additionally, at least 50% of the data should be detectable in order to compare either well means or medians. For many hazardous trace elements, detectable percentages are considerably lower. Interwell ANOVA techniques would also not be generally useful in these cases. ANOVA may find limited applicability in detection monitoring with trace organic constituents, especially where downgradient levels are considerably higher than background and there is a high percentage of detects. Based on ranks alone, it may be possible to determine that compliance well(s) containing one or more hazardous constituents exceed background. However, the Unified Guidance recommends avoiding ANOVA techniques in the limiting situations just described.

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[10] Test Methods for Evaluating Solid Waste (SW-846), EPA OSWER, 3rd Edition and subsequent revisions, Method 9020B, September 1994

Another detection monitoring method receiving less emphasis in this guidance is the tolerance limit. In previous guidance, an upper tolerance limit based on background was suggested to identify significant increases in downgradient well concentration levels. While still acceptable by regulation (*e.g.*, under existing RCRA permits), use of prediction limits are preferable to tolerance limits in detection monitoring for the following reasons. The construction of a tolerance limit is nearly identical to that of a prediction limit. In parametric normal distribution applications, both methods use the general formula: $\bar{x} + \kappa s$. The kappa (κ) multiplier varies depending on the coverage and confidence levels desired, but in both cases some multiple of the standard deviation (*s*) is added or subtracted from the sample mean ($\bar{x}$). For non-parametric limits, the similarity is even more apparent. Often the identical statistic (*e.g.*, the maximum observed value in background) can either be used as an upper prediction limit or an upper tolerance limit, with only a difference in statistical interpretation.

More fundamentally, given the wide variety of circumstances in which retesting strategies are now encouraged and even necessary, the mathematical underpinnings of retesting with *prediction limits* are well established while those for retesting with *tolerance limits* are not. Monte Carlo simulations were originally conducted for the 1992 Addendum to develop appropriate retesting strategies involving tolerance limits. Such simulations were found insufficient for the Unified Guidance.[11]

While the simultaneous prediction limits presented in the Unified Guidance consider the actual number of comparisons in defining exact false positive error rates, some tolerance limit approaches (including past guidance) utilized an approximate and less precise pre-selected low level of probability. On balance, there is little practical need for recommending two highly similar (but not identical) methods in the Unified Guidance, both for the reasons just provided and to avoid confusion of which method to use. The final regulation-specified detection monitoring method — *control charts* — is comparable to prediction limits, but possesses some unique benefits and so is also recommended in this guidance.

2.3.3 PARTS 264 AND 258 COMPLIANCE/ASSESSMENT MONITORING

A second use of tolerance limits recommended in earlier guidance was for comparing downgradient monitoring well data to a fixed limit during compliance/assessment monitoring. In this case, an upper tolerance limit constructed on each compliance well data set could be used to identify non-compliance with a fixed GWPS limit. Past guidance also used upper confidence limits around an upper proportion in defining these tolerance limits. A number of problems were identified using this approach.

A tolerance limit makes statistical sense if the limit represents an upper percentile, *i.e.*, when a limit is not to be exceeded by more than, for instance, 1% or 5% or 10% of future individual concentration values. However, GWPS limits can also be interpreted as long-term averages, *e.g.*, chronic risk-based values, which are better approximated by a statistic like the mean or median. **Chapters 7 &**

[11] 1) there were minor errors in the algorithms employed; 2) Davis and McNichols (1987) demonstrated how to compute exact kappa multipliers for prediction limits using a numerical algorithm instead of employing an inefficient simulation strategy; and 3) further research (as noted in **Chapter 19**) done in preparation of the guidance has shown that repeated prediction limits are more statistically powerful than retesting strategies using tolerance limits for detecting changes in groundwater quality.

22 discuss important considerations when identifying the appropriate statistical parameter to be compared against a fixed GWPS limit.

More importantly, since the upper confidence level of tolerance limit overestimates the true population proportion by design, demonstrating an exceedance of a GWPS by this limit does not necessarily indicate that the corresponding population proportion also exceeds the standard, leading to a high false positive rate. Therefore, the Unified Guidance recommends that the compliance/assessment monitoring null hypothesis be structured so that the compliance population characteristic (***e.g.***, mean, median, upper percentile) is assumed to be less than or equal to the fixed standard unless demonstrated otherwise. The correct test statistic in this situation is then the lower confidence limit. The upper confidence limit is used in corrective action to identify whether a constituent has returned to compliance.

To ensure consistency with the underlying statistical presumptions of compliance/assessment monitoring (see **Chapter 4**) and to maintain control of false positive rates, the Unified Guidance recommends that this tolerance interval approach be replaced with a more coherent and comprehensive strategy based on the use of confidence intervals (see **Chapters 21** and **22**). Confidence intervals can be applied in a consistent fashion to GWPS concentration limits representing either long-term averages or upper percentiles.

CHAPTER 3. KEY STATISTICAL CONCEPTS

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The success of any discipline rests on its ability to accurately model and explain real problems. Spectacular successes have been registered during the past four centuries by the field of mathematics in modeling fundamental processes in mechanics and physics. The last century, in turn, saw the rise of statistics and its fundamental theory of *estimation* and *hypothesis testing*. All of the tests described in the Unified Guidance are based upon this theory and involve the same key concepts. The purpose of this chapter is to summarize the statistical concepts underlying the methods presented in the Unified Guidance, and to consider each in the practical context of groundwater monitoring. These include:

- Statistical inference: the difference between samples and populations; the concept of sampling.
- Common statistical assumptions used in groundwater monitoring: statistical independence, stationarity, lack of outliers, and normality.
- Frequently-used statistical measures: mean, standard deviation, percentiles, correlation coefficient, coefficient of variation, *etc.*
- Hypothesis testing: How probability distributions are used to model the behavior of groundwater concentrations and how the statistical evidence is used to "prove" or "disprove" the validity of competing models.
- Errors in hypothesis testing: What false positives (Type I errors) and false negatives (Type II errors) really represent.
- Sampling distributions and the Central Limit Theorem: How the statistical behavior of test statistics differs from that of individual population measurements.
- Statistical power and power curves: How the ability to detect real contamination depends on the size or degree of the concentration increase.
- Type I *vs.* Type II errors: The tradeoff between false positives and false negatives; why it is generally impossible to minimize both kinds of error simultaneously.

3.1 INTRODUCTION TO GROUNDWATER STATISTICS

This section briefly covers some basic statistical terms and principles used in this guidance. All of these topics are more thoroughly discussed in standard textbooks. It is presumed that the user already has some familiarity with the following terms and discussions.

Statistics is a branch of applied mathematics, dealing with the description, understanding, and modeling of data. An integral part of statistical analysis is the testing of competing mathematical models and the management of data uncertainty. Uncertainty is present because measurement data exhibit *variability*, with limited knowledge of the medium being sampled. The fundamental aim of almost every statistical analysis is to draw *inferences*. The data analyst must *infer* from the observed data something about the physical world without knowing or seeing all the possible facts or evidence. So the question becomes: how closely do the measured data mimic reality, or put another way, to what extent do the data correctly identify a physical truth (*e.g.*, the compliance well is contaminated with arsenic above regulatory limits)?

One way to ascertain whether an aquifer is contaminated with certain chemicals would be to exhaustively sample and measure every physical volume of groundwater underlying the site of interest. Such a collection of measurements would be impossible to procure in practice and would be infinite in size, since sampling would have to be continuously conducted over time at a huge number of wells and sampling depths. However, one would possess the entire *population* of possible measurements at that site and the exact statistical *distribution* of the measured concentration values.

A statistical *distribution* is an organized summary of a set of data values, sorted into the relative frequencies of occurrence of different measurement levels (*e.g.*, concentrations of 5 ppb or less occur among 30 percent of the values, or levels of 20 ppb or more only occur 1 percent of the time). More generally, a distribution may refer to a mathematical model (known as a *probability distribution*) used to represent the shape and statistical characteristics of a given population and chosen according to one's experience with the type of data involved.

By contrast to the population, a statistical *sample* is a finite subset of the population, typically called a *data set*. Note that the statistical definition of sample is usually different from a geological or hydrological definition of the same term. Instead of a physical volume or mass, a statistical sample is a collection of measurements, *i.e.*, a set of numbers. This collection might contain only a single value, but more generally has a number of measurements denoted as the *sample size*, *n*.

Because a sample is only a partial representation of the population, an inference is usually desired in order to conclude something from the observed data about the underlying population. One or more numerical characteristics of the population might be of interest, such as the true *average* contaminant level or the *upper 95th percentile* of the concentration distribution. Quantities computed from the sample data are known as *statistics*, and can be used to reasonably *estimate* the desired but unknown population characteristics. An example is when testing sample data against a regulatory standard such as a maximum concentration limit [MCL] or background level. *A mean sample estimate* of the average concentration can be used to judge whether the corresponding population characteristic — the true mean concentration (denoted by the Greek letter μ) — exceeds the MCL or background limit.

The accuracy of these estimates depends on how *representative* the sample measurements of the underlying population are. In a representative sample, the distribution of sample values have the best

chance of closely matching the population distribution. Unfortunately, the degree of representativeness of a given sample is almost never known. So it quite important to understand precisely how the sample values were obtained from the population and to explore whether or not they *appear* representative. Though there is no guarantee that a sample will be adequate, the best protection against an unrepresentative sample is to select measurements from the population *at random*. A *random sample* implies that each potential population value has an equivalent chance of being selected depending only on its likelihood of occurrence. Not only does random sampling guard against selection of an unrepresentative portion of the population distribution, it also enables a mathematical estimate to be drawn of the statistical uncertainty associated with the ability of a given sample to represent the desired characteristic of the population. It can be very difficult to gauge the uncertainty surrounding a sample collected haphazardly or by means of professional judgment.

As a simple example, consider an urn filled with red and green balls. By thoroughly mixing the urn and blindly sampling (*i.e.*, retrieving) 10 percent of the balls, a very nearly random sample of the population of balls will be obtained, allowing a fair estimate of the true overall proportion of one color or the other. On the other hand, if one looked into the urn while sampling and only picked red balls or tried to alternate between red and green, the sample would be far from random and likely unrepresentative of the true proportions.

At first glance, groundwater measurements obtained during routine monitoring would not seem to qualify as random samples. The well points are generally not placed in random locations or at random depths, and the physical samples are usually collected at regular, pre-specified intervals. Consequently, further distinctions and assumptions are necessary when performing statistical evaluations of groundwater data. First, the distribution of a given contaminant may not be *spatially uniform* or *homogeneous*. That is, the local distribution of measured values at one well may not be the same as at other wells. Because this is often true for naturally-occurring groundwater constituents, the statistical population(s) of interest may be well-specific. A statistical sample gathered from a particular well must then be treated as potentially representative only of that well's local population. On the other hand, samples drawn from a number of reference background wells for which no significant differences are indicated, may permit the pooled data to serve as an estimate of the overall well field behavior for that particular monitoring constituent.

The distribution of a contaminant may also not be *temporally uniform* or *stationary over time*. If concentration values indicates a trend, perhaps because a plume intensifies or dissipates or natural in-situ levels rise or fall due to drought conditions, *etc.*, the distribution is said to be *non-stationary*. In this situation, some of the measurements collected over time may not be representative of current conditions within the aquifer. Statistical adjustments might be needed or the data partitioned into usable and unusable values.

A similar difficulty is posed by *cyclical* or *seasonal* trends. A long-term constituent concentration average at a well location or the entire site may essentially be constant over time, yet temporarily fluctuate up and down on a seasonal basis. Given a fixed interval between sampling events, some of this fluctuation may go unobserved due to the non-random nature of the sampling times. This could result in a sample that is unrepresentative of the population *variance* and possibly of the population *mean* as well. In such settings, a shorter (*i.e.*, higher frequency) or staggered sampling interval may be needed to better capture key characteristics of the population as a part of the distribution of sample measurements.

The difficulties in identifying a valid statistical framework for groundwater monitoring highlight a fundamental assumption governing almost every statistical procedure and test. It is the presumption that sample data from a given population should be *independent* and *identically distributed*, commonly abbreviated as *i.i.d.* All of the mathematics and statistical formulas contained in this guidance are built on this basic assumption. If it is not satisfied, statistical conclusions and test results may be invalid or in error. The associated statistical uncertainty may be different than expected from a given test procedure.

Random sampling of a single, fixed, stationary population will guarantee independent, identically-distributed sample data. Routine groundwater sampling typically does not. Consequently, the Unified Guidance discusses both below and in later chapters what assumptions about the sample data must be routinely or periodically checked. Many but not all of these assumptions are a simple consequence of the *i.i.d.* presumption. The guidance also discusses how sampling ought to be conducted and designed to get as close as possible to the *i.i.d.* goal.

3.2 COMMON STATISTICAL ASSUMPTIONS

Every statistical test or procedure makes certain assumptions about the data used to compute the method. As noted above, many of these assumptions flow as a natural consequence of the presumption of *independent, identically-distributed* data (*i.i.d.*). The most common assumptions are briefly described below:

3.2.1 STATISTICAL INDEPENDENCE

A major advantage of truly random sampling of a population is that the measurements will be *statistically independent.* This means that observing or knowing the value of one measurement does not alter or influence the probability of observing any other measurement in the population. After one value is selected, the next value is sampled again at random without regard to the previous measurement, and so on. By contrast, groundwater samples are not chosen at random times or at random locations. The locations are fixed and typically few in number. The intervals between sampling events are fixed and fairly regular. While samples of independent data exhibit no *pairwise correlation* (*i.e.*, no statistical association of similarity or dissimilarity between pairs of sampled measurements), *non-independent* or *dependent* data *do* exhibit pairwise correlation and often other, more complex forms of correlation. Aliquot split sample pairs are generally not independent because of the *positive correlation* induced by the splitting of the same physical groundwater sample. Split measurements tend to be highly similar, much more so than the random pairings of data from distinct sampling events.

In a similar vein, measurements collected close together in time from the same well tend to be more highly correlated than pairs collected at longer intervals. This is especially true when the groundwater is so slow-moving that the same general volume of groundwater is being sampled on closely-spaced consecutive sampling events. Dependence may also be exhibited spatially across a well field. Wells located more closely in space and screened in the same hydrostratigraphic zone may show greater similarity in concentration patterns than wells that are farther apart. For both of these temporal or time-related and spatial dependencies, the observed correlations are a result not only of the non-random nature of the sampling but also the fact that many groundwater populations are not uniform throughout the subsurface. The aquifer may instead exhibit pockets or sub-zones of higher or lower concentration, perhaps due to location-specific differences in natural geochemistry or the dynamics of contaminant plume behavior over time.

As a mathematical construct, statistical independence is essentially impossible to check directly in a set of sample data — other than by ensuring ahead of time that the measurements were collected at random. However, *non-zero pairwise correlation*, a clear sign of dependent data, can be checked and estimated in a variety of ways. The Unified Guidance describes two methods for identifying temporal correlation in **Chapter 14**: the *rank von Neumann ratio* test and the *sample autocorrelation function*. Measurable correlation among consecutive sample pairs may dictate the need for decreasing the sampling frequency or for a more complicated data adjustment.

Defining and modeling wellfield spatial correlation is beyond the scope of this guidance, but is very much the purview of the field of *geostatistics*. The Unified Guidance instead looks for evidence of well-to-well *spatial variation*, *i.e.*, statistically identifiable differences in mean and/or variance levels across the well field. If evident, the statistical approach would need to be modified so that distinct wells are treated as individual populations with statistical testing being conducted separately at each one (*i.e.*, intrawell comparisons).

3.2.2 STATIONARITY

A *stationary* statistical distribution is one whose population characteristics do not change over time and/or space. In a groundwater context, this means that the true population distribution of a given contaminant is the same no matter *where or when* it is sampled. In the strictest form of *stationarity*, the full distribution must be exactly the same at every time and location. However, in practice, a weaker form is usually assumed: that the population mean (μ) and variance (denoted by the Greek symbol σ^2) are the same over time and/or space.

Stationarity is important to groundwater statistical analysis because of the way that monitoring samples must be collected. If a sample set somehow represented the entire population of possible aquifer values, stationarity would not be an issue in theory. A limited number of physical groundwater samples, however, must be individually collected from each sampled location. To generate a statistical sample, the individual measurements must be pooled together over time from multiple sampling events within a well, or pooled together across space by aggregating data from multiple wells, or both.

As long as the contaminant distribution is stationary, such pooling poses no statistical problem. But with a non-stationary distribution, either the mean and/or variance is changing over time in any given well, or the means and variances differ at distinct locations. In either case, the pooled measurements are *not identically-distributed* even if they may be statistically independent.

The effects of non-stationarity are commonly seen in four basic ways in the groundwater context: 1) as spatial variability, 2) in the existence of trends and/or seasonal variation, 3) via other forms of temporal variation, and 4) in the lack of homogeneity of variance. *Spatial variability* (discussed more extensively in **Chapter 13**) refers to statistically identifiable differences in mean and/or variance levels (but usually means) across the well field (*i.e.*, spatial non-stationarity). The existence of such variation often precludes the pooling of data across multiple background wells or the proper upgradient-to-downgradient comparison of background wells against distinct compliance wells. Instead, the usual approach is to perform intrawell comparisons, where well-specific background data is culled from the early sampling history at each well. Checks for spatial variability are conducted graphically with the aid of side-by-side box plots (**Chapter 9**) and through the use of analysis of variance [ANOVA, **Chapter 13**].

A *trend over time* at a given well location indicates that the mean level is not stationary but is instead rising or falling. A *seasonal trend* is similar in that there are periodic increases and decreases. Pooling several sampling events together thus mixes measurements with differing statistical characteristics. This can violate the identically-distributed presumption of almost all statistical tests and usually leads to an inflated estimate of the current population variance. Trends or seasonal variations identified in (upgradient) background wells or in intrawell background data from compliance wells can severely impact the accuracy and effectiveness of statistical procedures described in this guidance if data are pooled over time to establish background limits. The approach that should be taken will vary with the circumstance. Sometimes the trend component might need to be estimated and removed from the original data, so that what gets tested are the *data residuals* (*i.e.*, values that result from subtracting the estimated trend from the original data) instead of the raw measurements. In other cases, an alternate statistical approach might be needed such as a test for (positive) trend or construction of a confidence band around an estimated trend. More discussion of these options is presented in **Chapters 6**, **7**, **14**, and **21**.

To identify a linear trend, the Unified Guidance describes simple linear regression and the Mann-Kendall test in **Chapter 17**. For seasonal patterns or a combination of linear and seasonal trend effects, the guidance discusses the seasonal Mann-Kendall test and the use of ANOVA tests to identify seasonal effects. These diagnostic procedures are also presented in **Chapter 14**.

Temporal variations are distinguished in this guidance from trends or seasonal effects by the lack of a regular or identifiable pattern. Often a temporal effect will be observed as a temporary shift in concentration levels that is similar in magnitude and direction at multiple wells. This can occur at some sites, for instance, due to rainfall or recharge events. Because the mean level changes at least temporarily, pooling data over time again violates the assumption of identically-distributed data. In this case, the temporal effect can be identified by looking for parallel traces on a time series plot of multiple wells and then more formally by performing a *one-way ANOVA for temporal effects*. These procedures are described in **Chapter 14**. Once identified, the residuals from the ANOVA can be used for compliance testing, since the common temporal effect has been removed.

Lastly, *homogeneity of variance* is important in ANOVA tests, which simultaneously evaluates multiple groups of data each representing a sample from a distinct statistical population. In the latter test, well means need not be the same; the reason for performing the test in the first place is to find out whether the means do indeed differ. But the procedure assumes that all the group variances are equal or *homogeneous*. Lack of homogeneity or stationarity in the variances causes the test to be much less effective at discovering differences in the well means. In extreme cases, the concentration levels would have to differ by large amounts before the ANOVA would correctly register a statistical difference. Lack of homogeneity of variance can be identified graphically via the use of side-by-side box plots and then more formally with the use of Levene's test. Both these methods are discussed further in **Chapter 11**. Evidence of unequal variances may necessitate the use of a transformation to stabilize the variance prior to running the ANOVA. It might also preclude use of the ANOVA altogether for compliance testing, but require intrawell approaches to be considered instead.

ANOVA is not the only statistical procedure which assumes homogeneity of variance. Prediction limits and control charts require a similar assumption between background and compliance well data. But if only one new sample measurement is collected per well per evaluation period (*e.g.*, semi-annually) it can be difficult to formally test this assumption with the diagnostic methods cited above. As

an alternative, homogeneity of variance can be periodically tested when a sufficient sample size has been collected for each compliance well (see **Chapter 6**).

3.2.3 LACK OF STATISTICAL OUTLIERS

Many authors have noted that *outliers* — extreme, unusual-looking measurements — are a regular occurrence among groundwater data (Helsel and Hirsch, 2002; Gibbons and Coleman, 2001). Sometimes an outlier results from nothing more than a typographical error on a laboratory data sheet or file. In others, the fault is an incorrectly calibrated measuring device or a piece of equipment that was not properly decontaminated. An unusual measurement might also reflect the sampling of a temporary, local 'hot spot' of higher concentration. In each of these situations, outliers in a statistical context represent values that are inconsistent with the distribution of the remaining measurements. Tests for outliers thus attempt to infer whether the suspected outlier could have reasonably been drawn from the same population as the other measurements, based on the sample data observed up to that point. Statistical methods to help identify potential outliers are discussed in **Chapter 12**, including both *Dixon's* and *Rosner's* tests, as well as references to other methods.

The basic problem with including statistical outliers in analyzing groundwater data is that they do not come from the same distribution as the other measurements in the sample and so fail the identically-distributed presumption of most tests. The consequences can be dramatic, as can be seen for instance when considering *non-parametric prediction limits*. In this testing method, one of the largest values observed in the background data such as the maximum, is often the statistic selected as the prediction limit. If a large outlier is present among the background measurements, the prediction limit may be set to this value despite being unrepresentative of the background population. In effect, it arises from another population, *e.g.*, the 'population' of typographical errors. The prediction limit could then be much higher than warranted based on the observed background data and may provide little if any probability that truly contaminated compliance wells will be identified. The test will then have lower than expected *statistical power*.

Overall, it pays to try to identify possible outliers and to either correct the value(s) if possible, or exclude known outliers from subsequent statistical analysis. It is also possible to select a statistical method that is *resistant* to the presence of outliers, so that the test results are still likely to be accurate even if one or more outliers is unidentified. Examples of this last strategy include setting non-parametric prediction limits to values other than the background maximum using repeat testing (see **Chapter 18**) or using Sen's slope procedure to estimate the rate of change in a linear trend (**Chapter 17**).

3.2.4 NORMALITY

Probability distributions introduced in **Section 3.1** are mathematical models used to approximate or represent the statistical characteristics of populations. Knowing the exact form and defining equation of a probability distribution allows one to assess how likely or unlikely it will be to observe particular measurement values (or ranges of values) when selecting or drawing independent, identically distributed [*i.i.d.*] samples from the associated population. This can be done as follows. In the case of a *continuous distributional model*, a curve can be drawn to represent the probability distribution by plotting probability values along the *y*-axis and measurement or concentration values along the *x*-axis. Since the continuum of *x*-values along this curve is infinite, the probability of occurrence of any single possible value is negligible (*i.e.*, zero), and does not equal the height of the curve. Instead, positive probabilities can be computed for *ranges* of possible values by *summing the area under the distributional curve*

associated with the desired range. Since by definition the total area under any probability distribution curve sums to unity, all probabilities are then numbers between 0 and 1.

Probability distributions form the basic building blocks of all statistical testing procedures. Every test relies on comparing one or more statistics computed from the sample data against a *reference distribution*. The reference distribution is in turn a probability distribution summarizing the expected mathematical behavior of the statistic(s) of interest. A formal statistical test utilizes this reference distribution to make inferences about the sample statistic in terms of two contrasting conditions or hypotheses.

In any event, probability distributions used in statistical testing make differing assumptions about how the underlying population of measurements is distributed. A case in point is simultaneous *prediction limits using retesting* (**Chapter 19**). The first and most common version of this test (Davis and McNichols, 1987) is based on an assumption that the sample data are drawn from a *normal probability distribution*. The normal distribution is the well-known bell-shaped curve, perhaps the single most important and frequently-used distribution in statistical analysis. However, it is not the only one. Bhaumik and Gibbons (2006) proposed similar prediction limits for data drawn from a *gamma distribution* and Cameron (2008) did the same for *Weibull-distributed* measurements. This more recent research demonstrates that prediction limits with similar statistical decision error rates can vary greatly in magnitude, depending on the type of data distribution assumed.

Because many tests make an explicit assumption concerning the distribution represented by the sample data, the form and exact type of distribution often has to be checked using a *goodness-of-fit* test. A goodness-of-fit test assesses how closely the observed sample data resemble a proposed distributional model. Despite the wide variety of probability distributions identified in the statistical literature, only a very few goodness-of-fit tests generally are needed in practice. This is because most tests are based on an assumption of *normally-distributed* or *normal* data. Even when an underlying distribution is not normal, it is often possible to use a mathematical transformation of the raw measurements (*e.g.*, taking the *natural logarithm* or *log* of each value) to *normalize* the data set. The original values can be transformed into a set of numbers that behaves as if drawn from a normal distribution. The transformed values can then be utilized in and analyzed with a *normal-theory test* (*i.e.*, a procedure that assumes the input data are normal).

Specific goodness-of-fit tests for checking and identifying data distributions are found in **Chapter 10** of this guidance. These methods all are designed to check the fit to normality of the sample data. Besides the normal, the *lognormal distribution* is also commonly used as a model for groundwater data. This distribution is not symmetric in shape like the bell-shaped normal curve, nor does it have similar statistical properties. However, a simple *log transformation* of lognormal measurements works to normalize such a data set. The *transformed values* can be tested using one of the standard goodness-of-fit tests of *normality* to confirm that the original data were indeed *lognormal*.

More generally, if a sample shows evidence of *non-normality* using the techniques in **Chapter 10**, the initial remedy is to try and find a suitable *normalizing transformation*. A set of useful possible transformations in this regard has been termed the *ladder of powers* (Helsel and Hirsch, 2002). It includes not only the natural logarithm, but also other mathematical power transformations such as the square root, the cube root, the square, *etc*. If none of these transformations creates an adequately normalized data set, a second approach is to consider what are known as *non-parametric* tests. Normal-

theory and other similar *parametric* statistical procedures assume that the form of the underlying probability distribution is known. They are called parametric because the assumed probability distribution is generally characterized by a small set of *mathematical parameters*. In the case of the normal distribution, the general formula describing its shape and properties is completely specified by two parameters: the *population mean* (μ) and the *population variance* (σ^2). Once values for these quantities are known, the exact distribution representing a particular normal population can be computed or analyzed.

Most parametric tests do not require knowledge of the exact distribution represented by the sample data, but rather just the type of distribution (*e.g.*, normal, lognormal, gamma, Weibull, *etc.*). In more formal terms, the test assumes knowledge of the *family of distributions* indexed by the characterizing parameters. Every different combination of population mean and variance defines a different normal distribution, yet all belong to the normal family. Nonetheless, there are many data sets for which a known distributional family cannot be identified. Non-parametric methods may then be appropriate, since a known distributional form is not assumed. Non-parametric tests are discussed in various chapters of the Unified Guidance. These tests are typically based on either a *ranking* or an *ordering* of the sample magnitudes in order to assess their statistical performance and accuracy. But even non-parametric tests may make use of a normal approximation to define how expected rankings are distributed.

One other common difficulty in checking for normality among groundwater measurements is the frequent presence of *non-detect* values, known in statistical terms as *left-censored* measurements. The magnitude of these sample concentrations is known only to lie somewhere between zero and the *detection* or *reporting limit*; hence the true concentration is partially 'hidden' or censored on the left-hand side of the numerical concentration scale. Because the most effective normality tests assume that all the sample measurements are known and quantified and not censored, the Unified Guidance suggests two possible approaches in this circumstance. First, it is usually possible to simply assume that the true distributional form of the underlying population cannot be identified, and to instead apply a non-parametric test alternative. This solution is not always ideal, especially when using prediction limits and the background sample size is small, or when using control charts (for which there is no current non-parametric alternative to the Unified Guidance recommended test).

As a second alternative, **Chapter 10** discusses methods for assessing *approximate* normality in the presence of non-detects. If normality can be established, perhaps through a normalizing transformation, **Chapter 15** describes methods for estimating the mean and variance parameters of the specific normal distribution needed for constructing tests (such as prediction limits or control charts), even though the exact value of each non-detect is unknown.

3.3 COMMON STATISTICAL MEASURES

Due to the variety of statistical tests and other methods presented in the Unified Guidance, there are a large number of equations and formulas of relevance to specific situations. The most common statistical measures used in many settings are briefly described below.

Sample mean and standard deviation — the mean of a set of measurements of sample size n is simply the arithmetic average of each of the numbers in the sample (denoted by x_i), described by formula [3.1] below. The sample mean is a common estimate of the center or middle of a statistical distribution. That is, $\bar{x}$ is an estimate of μ, the population mean. The basic formula for the sample standard deviation

is given in equation [3.2]. The sample standard deviation is an estimate of the degree of variability within a distribution, indicating how much the values typically vary from the average value or mean. Thus, the standard deviation s is an estimate of the population standard deviation σ. Note that another measure of variability, the sample variance, is simply the square of the standard deviation (denoted by s^2) and serves as an estimate of the population variance σ^2.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad [3.1]$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \qquad [3.2]$$

Coefficient of Variation — for positively-valued measurements, the sample coefficient of variation provides a quick and useful indication of the relative degree of variability within a data set. It is computed as $s/\bar{x}$ and so indicates whether the amount of 'spread' in the sample is small or large relative to the average observed magnitude. Sample coefficients of variation can also be calculated for other distributions such as the logarithmic (see discussion on logarithmic statistics below and **Chapter 10**, **Section 10.4**).

Sample percentile — the pth percentile of a sample (denoted as $\tilde{x}_p$) is the value such that $p \times 100\,\%$ of the measurements are no greater than $\tilde{x}_p$, while $\left(1-p\right)\times 100\,\%$ of the values are no less than $\tilde{x}_p$. Sample percentiles are computed by making an ordered list of the measurements (termed the *order statistics* of the sample) and either selecting an observed value from the sample that comes closest to satisfying the above definition or interpolating between the pair of sample values closest to the definition if no single value meets it.

Slightly different estimates of the sample percentile are used to perform the interpolation depending on the software package or statistics textbook. The Unified Guidance follows Tukey's (1977) method for computing the lower and upper quartiles (*i.e.*, the 25th and 75th sample percentiles, termed *hinges* by Tukey) when constructing box plots (**Chapter 9**). In that setting, the pair of sample values closest to the desired percentile is simply averaged. Another popular method for more generally computing sample percentiles is to set the rank of the desired order statistic as $k = (n+1) \times p$. If k is not an integer, perform linear interpolation between the pair of ordered sample values with ranks just below and just above k.

Median and interquartile range — the sample median is the 50th percentile of a set of measurements, representing the midpoint of an ordered list of the values. It is usually denoted as $\tilde{x}$ or $\tilde{x}_{.5}$, and represents an alternative estimate of the center of a distribution. The interquartile range [IQR] is the difference between the 75th and 25th sample percentiles, thus equal to $\left(\tilde{x}_{.75} - \tilde{x}_{.25}\right)$. The IQR offers an alternative estimate of variability in a population, since it represents the measurement range of the middle 50% of the ordered sample values. Both the median and the interquartile range are key statistics used to construct box plots (**Chapter 9**).

The median and interquartile range can be very useful as alternative estimates of data centrality and dispersion to the mean and standard deviation, especially when samples are drawn from a highly skewed (i.e., non-symmetric) distribution or when one or more outliers is present. The table below depicts two data sets, one with an obvious outlier, and demonstrates how these statistical measures compare.

The median and interquartile ranges are not affected by the inclusion of an outlier (perhaps an inadvertent reporting of units in terms of ppb rather than ppm). Large differences between the mean and median, as well as between the standard deviation and interquartile range in the second data set can indicate that an anomalous data point may be present.

Data Set #1	**Data Set #2**
5	5
10	10
15	15
15	15
15	15
20	20
25	25,000
$\bar{x}$ = 15	$\bar{x}$ > 3,500
$\tilde{x}$ = 15	$\tilde{x}$ = 15
s = 6.5	s > 9,000
IQR = 10	IQR = 10

Log-mean, log-standard deviation and Coefficient of Variation — The lognormal distribution is a frequently-used model in groundwater statistics. When lognormally distributed data are transformed, the normally-distributed measurements can then be input into normal-theory tests. The Unified Guidance frequently makes use of quantities computed on log-transformed values. Two of these quantities, the log-mean and the log-standard deviation, represent the sample mean and standard deviation computed using log-transformed values instead of the raw measurements. Formulas for these quantities — denoted $\bar{y}$ and s_y to distinguish them from the measurement-scale mean ($\bar{x}$) and standard deviation (s) — are given below. Prior to calculating the logarithmic mean and standard deviation, the measurement scale data must first be log-transformed. Taking logarithms of the sample mean ($\bar{x}$) and the sample standard deviation (s) based on the original measurement-scale data, will not give the correct result.

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n}\log\left(x_i\right) \qquad [3.3]$$

$$s_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\log\left(x_i\right)-\bar{y}\right)^2} \qquad [3.4]$$

A population logarithmic coefficient of variation can be estimated from the logarithmically transformed data as: $CV_{\log} = \sqrt{e^{s_y^2}-1}$. It is based solely on the logarithmic standard deviation, s_y, and represents the intrinsic variability of the untransformed data. .

Sample correlation coefficient — correlation is a common numerical measure of the degree of similarity or linear association between two random variables, say x and y. A variety of statistics are used to estimate the correlation depending on the setting and how much is known about the underlying distributions of x and y. Each measure is typically designed to take on values in the range of –1 to +1, where –1 denotes perfect inverse correlation (*i.e.*, as x increases, y decreases, and vice-versa), while +1 denotes perfect correlation (*i.e.*, x and y increase or decrease together), and 0 denotes no correlation (*i.e.*, x and y behave independently of one another). The most popular measure of linear correlation is Pearson's correlation coefficient (r), which can be computed for a set of n sample pairs (x_i, y_i) as:

$$r = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}} \qquad [3.5]$$

3.4 HYPOTHESIS TESTING FRAMEWORK

An important component of statistical analysis involves the testing of competing mathematical models, an activity known as *hypothesis testing*. In hypothesis testing, a formal comparison is made between two mutually exclusive possible statements about reality. Usually these statements concern the type or form of underlying statistical population from which the sample data originated, *i.e.*, either the observed data came from one statistical population or from another, but not both. The sample data are used to judge which statistical model identified by the two hypotheses is most consistent with the collected observations.

Hypothesis testing is similar in nature to what takes place in a criminal trial. Just as one of the two statements in an hypothesis test is judged true and the other false, so the defendant is declared either innocent or guilty. The opposing lawyers each develop their theory or model of the crime and what really happened. The jury must then decide whether the available evidence better supports the prosecution's theory or the defense's explanation. Just as a strong presumption of innocence is given to a criminal defendant, one of the statements in a statistical hypothesis is initially favored over the other. This statement, known as the *null hypothesis* [H_0], is only rejected as false if the sample evidence strongly favors the other side of the hypothesis, known as the *alternative hypothesis* [H_A].

Another important parallel is that the same mistakes which can occur in statistical hypothesis testing are made in criminal trials. In a criminal proceeding, the innocent can falsely be declared guilty or the guilty can wrongly be judged innocent. In the same way, if the null hypothesis [H_0] is a true statement about reality but is rejected in favor of the alternative hypothesis [H_A], a mistake akin to convicting the innocent has occurred. Such a mistake is known in statistical terms as a *false positive* or *Type I error*. If the alternative hypothesis [H_A] is true but is rejected in favor of H_0, the mistake is akin to acquitting the guilty. This mistake is known as a *false negative* or *Type II error*.

In a criminal investigation, the test hypotheses can be reversed. A detective investigating a crime might consider a list of probable suspects as potentially guilty (the null hypothesis [H_0]), until substantial evidence is found to exclude one or more suspects [H_A]. The burden of proof for accepting the alternative hypothesis and the kinds of errors which can result are the opposite from a legal trial.

Certain steps are involved in conducting any statistical hypothesis test. First, the null hypothesis H_0 must be specified and is given presumptive weight in the hypothesis testing framework. The observed sample (or a statistic derived from these data) is assumed to follow a known statistical distribution, consistent with the distributional model used to describe reality under H_0. In groundwater monitoring, a null hypothesis might posit that concentration measurements of benzene, for instance, follow a normal distribution with zero mean. This statement is contrasted against the alternative hypothesis, which is constructed as a competing model of reality. Under H_A, the observed data or statistic follows a different distribution, corresponding to a different distributional model. In the simple example above, H_A might posit that benzene concentrations follow a normal distribution, but this time with a mean no less than 20 ppb, representing a downgradient well that has been contaminated.

Complete descriptions of statistical hypotheses are usually not made. Typically, a shorthand formula is used for the two competing statements. Denoting the true population mean as the Greek letter μ and a possible value of this mean as μ_0, a common specification is:

$$H_0 : \mu \leq \mu_0 \text{ vs. } H_A : \mu > \mu_0 \qquad [3.6]$$

This formulation clearly distinguishes between the location (*i.e.*, magnitude) of the population mean μ under the two competing models, but it does not specify the *form* of the underlying population itself. In most parametric tests, as explained in **Section 3.2**, the underlying model is assumed to be the normal distribution, but this is not a necessary condition or the basic assumption in all tests. Note also that a *family* of distributions is specified by the hypothesis, not two individual, specific distributions. Any distribution with a true mean no greater than μ_0 satisfies the null hypothesis, while any distribution from the same family with true mean larger than μ_0 satisfies the alternative hypothesis.

Once the statistical hypothesis has been specified, the next step is to actually collect the data and compute whatever test statistic is required based on the observed measurements and the kind of test. The pattern of the observed measurements or the computed test statistic is then compared with the population model predicted or described under H_0. Because this model is specified as a statistical distribution, it can be used to assign probabilities to different results. If the observed result or pattern occurs with very low probability under the null hypothesis model (*e.g.*, with at most a 5% or 1% chance), one of two outcomes is assumed to have occurred. Either the result is a "chance" fluctuation in the data representing a real but unlikely outcome under H_0, or the null hypothesis was an incorrect model to begin with.

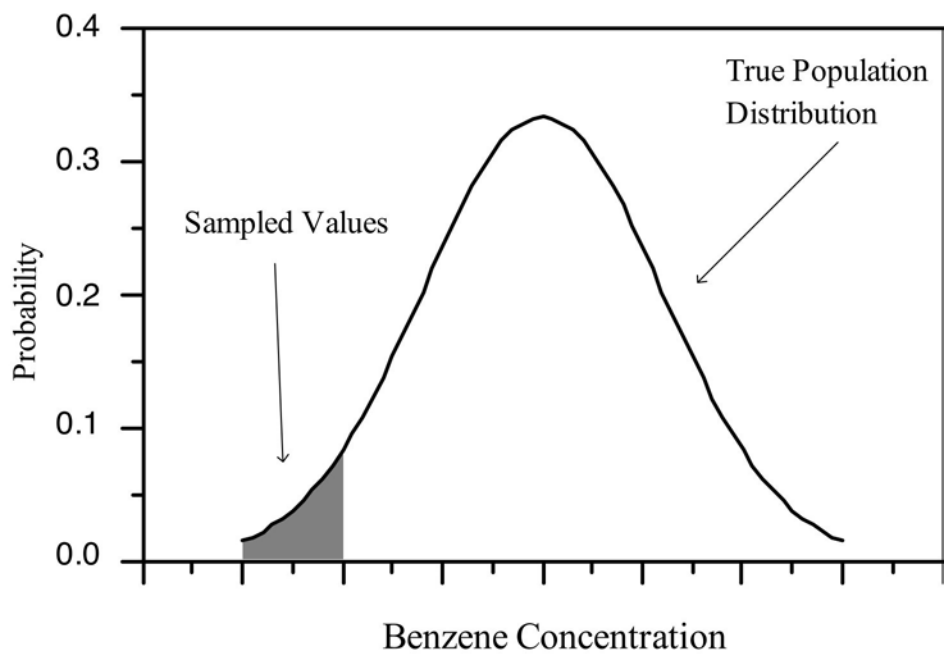
A low probability of occurrence under H_0 is cause for rejecting the null hypothesis in favor of H_A, as long as the probability of occurrence under the latter alternative is also not too small. Still, one should be careful to understand that statistics involves the art of *managing uncertainty*. The null hypothesis may indeed be true, even if the measured results seem unlikely to have arisen under the H_0 model. A *small probability* of occurrence is not the same as *no possibility* of occurrence. The judgment in favor of H_A should be made with full recognition that a *false positive mistake* is always possible even if not very likely.

Consider the measurement of benzene in groundwater in the example above. Given natural fluctuations in groundwater composition from week-to-week or month-to-month and the variability introduced in the lab during the measurement process, the fact that one or two samples show either non-detect or very low levels of benzene does not guarantee that the true mean benzene concentration at the

well is essentially zero. Perhaps the true mean is higher, but the specific sample values collected were gotten from the "lower tail" of the benzene distribution just by chance or were measured incorrectly in the lab. **Figure 3-1** illustrates this possibility, where the full benzene distribution is divided into a lower tail portion that has been sampled and a remaining portion that has not so far been observed. The sampled values are not representative of the entire population distribution, but only of a small part of it.

Along a similar vein, if the observed result or pattern can occur with moderate to high probability under the null hypothesis, the model represented by H_0 is accepted as consistent with the sample measurements. Again, this does not mean the null hypothesis is necessarily true. The alternative hypothesis could be true instead, in which case the judgment to accept H_0 would be considered a ***false negative***. Nevertheless the sample data do not provide sufficient evidence or justification to reject the initial presumption.

Figure 3-1. Actual, But Unrepresentative Benzene Measurements



3.5 ERRORS IN HYPOTHESIS TESTING

In order to properly interpret the results of any statistical test, it is important to understand the risks of making a wrong decision. The risks of the two possible errors or mistakes mentioned above are not fixed quantities; rather, false positive and false negative risks are best thought of as statistical parameters that can be adjusted when performing a particular test. This flexibility allows one, in general, to "calibrate" any test to meet specific risk or error criteria. However, it is important to recognize what the different risks represent. RCRA groundwater regulations stipulate that any test procedure maintain a "reasonable balance" between the risks of false positives and false negatives. But how does one decide on a reasonable balance? The answer lies in a proper understanding of the real-life implications attached to wrong judgments.

3.5.1 FALSE POSITIVES AND TYPE I ERRORS

A *false positive* or *Type I error* occurs whenever the null hypothesis [H_0] is falsely rejected in favor of the alternative hypothesis [H_A]. What this means in terms of the underlying statistical models is somewhat different for every test. Many of the tests in the Unified Guidance are designed to address the basic groundwater detection monitoring framework, namely, whether the concentrations at downgradient wells are significantly greater than background. In this case, the null hypothesis is that the background and downgradient wells share the same underlying distribution and that downgradient concentrations should be consistent with background in the absence of any contamination. The alternative hypothesis presumes that downgradient well concentrations are significantly greater than background and come from a distribution with an elevated concentration.

Given this formulation of H_0 and H_A, a Type I error occurs whenever one decides that the groundwater at downgradient locations is significantly higher than background when in reality it is the *same* in distribution. A judgment of this sort concerns the underlying statistical populations and not the observed sample data. The measurements at a downgradient well may indeed be higher than those collected in background. But the disparity must be great enough to decide with confidence that the underlying *populations* also differ. A proper statistical test must account for not just the difference in *observed* mean levels but also variability in the data likely to be present in the underlying statistical populations.

False positive mistakes can cause regulated facilities to incur substantial unnecessary costs and oversight agencies to become unnecessarily involved. Consequently, there is usually a desire by regulators and the regulated community alike to minimize the false positive rate (typically denoted by the Greek letter α). For reasons that will become clear below, the false positive rate is inversely related to the false negative rate for a fixed sample size n. It is impossible to completely eliminate the risk of either Type I or Type II errors, hence the regulatory mandate to minimize the inherent tradeoff by maintaining a "reasonable balance" between false positives and false negatives.

Type I errors are strictly defined in terms of the hypothesis structure of the test. While the conceptual groundwater detection monitoring framework assumes that false positive errors are incorrect judgments of a release when there is none, Type I errors in other statistical tests may have a very different meaning. For instance, in tests of normality (**Chapter 10**) the null hypothesis is that the underlying population is normally-distributed, while the alternative is that the population follows some other, non-normal pattern. In this setting, a false positive represents the mistake of falsely deciding the population to be non-normal, when in fact it *is* normal in distribution. The implication of such an error is quite different, perhaps leading one to select an alternate test method or to needlessly attempt a normalizing transformation of the data.

As a matter of terminology, the false positive rate α is also known as the *significance level* of the test. A test conducted at the α = .01 level of significance means there is at most a 1% chance or probability that a Type I error will occur in the results. The test is likely to lead to a false rejection of the null hypothesis at most about 1 out of every 100 times the same test is performed. Note that this last statement says nothing about how well the test will work if H_A is true, when H_0 *should* be rejected. The

false positive rate strictly concerns those cases where H_0 is an accurate reflection of the physical reality, but the test rejects H_0 anyway.

3.5.2 SAMPLING DISTRIBUTIONS, CENTRAL LIMIT THEOREM

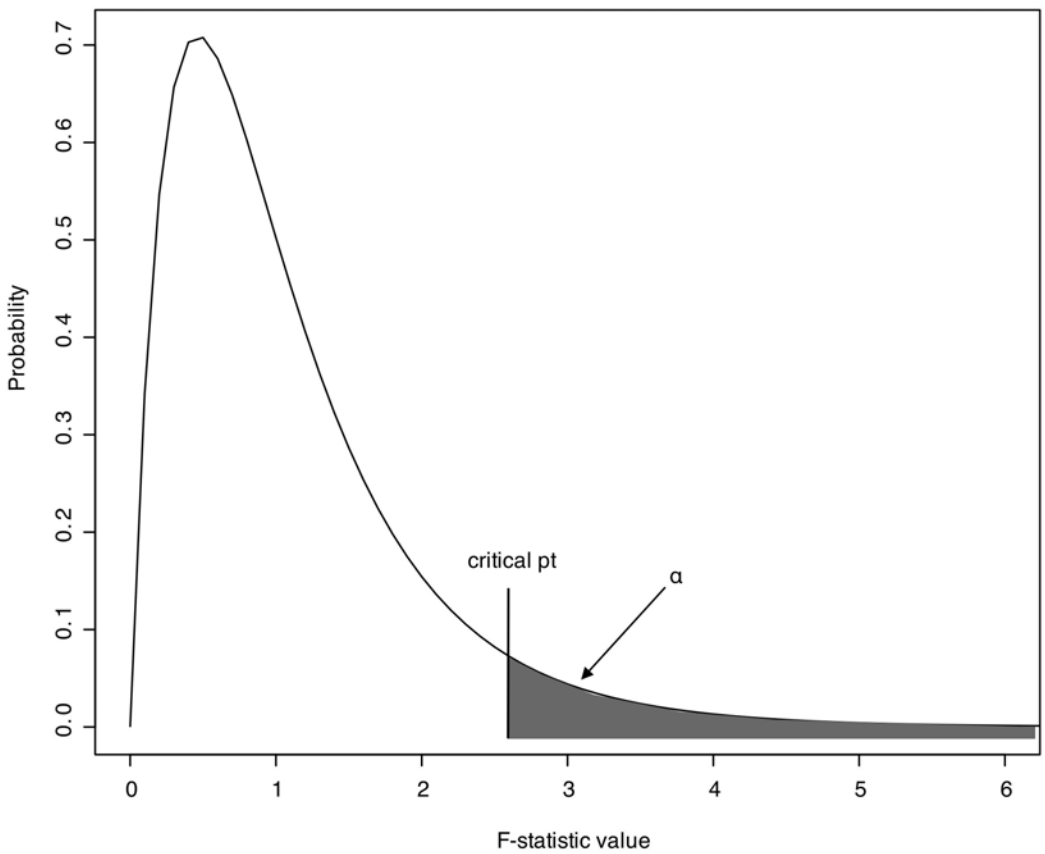
The false positive rate of any statistical test can be calibrated to meet a given risk criterion. To see how this is done, it helps to understand the concept of *sampling distribution*. Most statistical test decisions are based on the magnitude of a particular test statistic computed from the sample data. Sometimes the test statistic is relatively simple, such as the sample mean ($\bar{x}$), while in other instances the statistic is more complex and non-intuitive. In every case, however, the test statistic is formulated as it is for a specific purpose: *to enable the analyst to identify the distributional behavior of the test statistic under the null hypothesis*. Unless one knows the expected behavior of a test statistic, probabilities cannot be assigned to specific outcomes for deciding when the probability is too low to be a chance fluctuation of the data.

The distribution of the test statistic is known as its *sampling distribution*. It is given a special name, in part, to distinguish the behavior of the *test statistic* from the potentially different distribution of the *individual observations or measurements* used to calculate the test. Once identified, the sampling distribution can be used to establish *critical points* of the test associated with specific maximal false positive rates for any given α level of significance. For most tests, a single level of significance is generally chosen.

An example of this idea can be illustrated via the *F*-test. It is used for instance in parametric analysis of variance [ANOVA] to identify differences in the population means at three or more monitoring wells. Although ANOVA assumes that the individual measurements input to the test are normally-distributed, the test statistic under a null hypothesis [H_0] of no differences between the true means follows an *F*-distribution. More specifically, it applies to one member of the *F*-distribution family (an example using 5 wells and 6 measurements per well is pictured in **Figure 3-2**). As seen in the right-hand tail of this distribution by summing the area under the distributional curve, large values of the *F*-statistic become less and less probable as they increase in magnitude. For a given significance level (α), there is a corresponding *F*-statistic value such that the probability of exceeding this cutoff value is α or less. In such situations, there is at most an $\alpha \times 100\%$ chance of observing an F-statistic *under* H_0 that is as large or larger than the cutoff (shaded area in **Figure 3-2**). If α is quite small (*e.g.*, 5% or 1%), one may then judge the null hypothesis to be an untenable model and accept H_A. As a consequence, the cutoff value can be defined as an α-level *critical point* for the *F*-test.

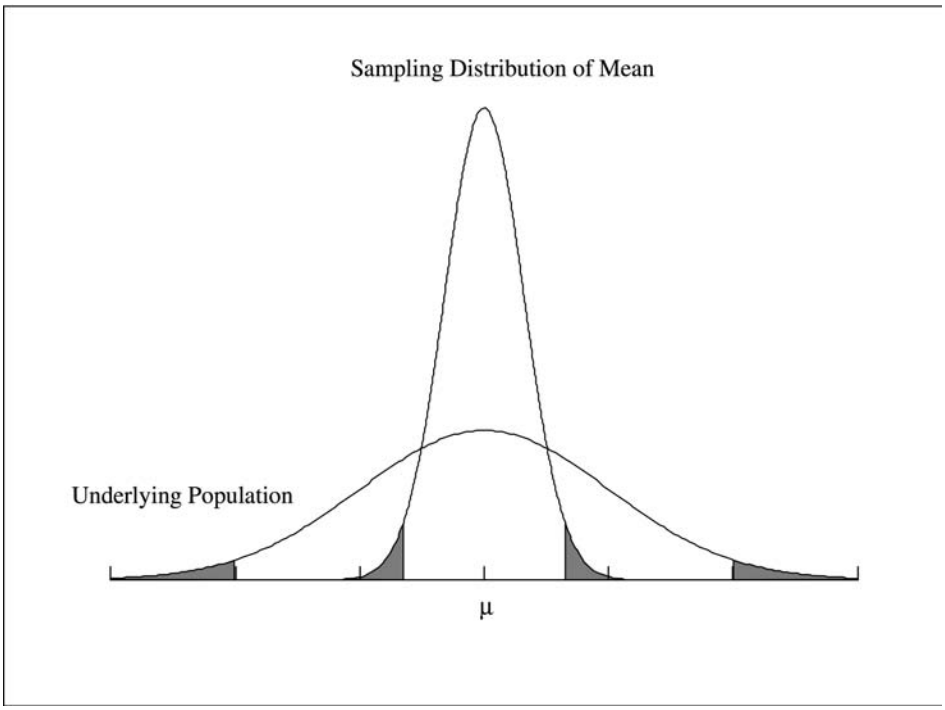
Because test statistics can be quite complicated, there is no easy rule for determining the sampling distribution of a particular test. However, the sampling behavior of some statistics is a consequence of a fundamental result known as the *Central Limit Theorem*. This theorem roughly states that averages or sums of identically-distributed random variables will follow an approximate normal distribution, regardless of the distributional behavior of the individual measurements. This averaged distribution will have the *same* mean μ as the population of individual measurements and whose variance, compared to the underlying population variance σ^2, is scaled by a factor of the sample size *n* on which the average or sum is based. Specifically, the variance is greater by a factor of *n* in the case of a sum $\left(n \cdot \sigma^2\right)$ and smaller by a factor of *n* in the case of an average $\left(\sigma^2/n\right)$. The approximation of the averages or sums to the normal distribution improves as sample size increases (also see the power discussion on page 3-21).

Figure 3-2. F-Distribution with 4 and 25 Degrees of Freedom



Because of the Central Limit Theorem, a number of test statistics at least approximately follow the normal distribution. This allows critical points for these tests to be determined from a table of the standard normal distribution. The Central Limit Theorem also explains why sample means provide a better estimate of the true population mean than individual measurements drawn from the same population (**Figure 3-3**). Since the sampling distribution of the mean is centered on the true average (μ) of the underlying population and the variance is lower by a factor of n, the sample average $\bar{x}$ will tend to be much closer to μ than a typical individual measurement.

Figure 3-3. Effect of Central Limit Theorem



3.5.3 FALSE NEGATIVES, TYPE II ERRORS, AND STATISTICAL POWER

False negatives or Type II errors are the logical opposites of false positive errors. An error of this type occurs whenever the null hypothesis [H_0] is accepted, but instead the alternative hypothesis [H_A] is true. The false negative rate is denoted by the Greek letter β. In terms of the groundwater detection monitoring framework, a Type II error represents a mistake of judging the compliance point concentrations to be consistent with background, when in reality the compliance point distribution is ***higher*** on average. False negatives in this context describe the risk of ***missing*** or ***not identifying*** contaminated groundwater when it really exists. EPA has traditionally been more concerned with such false negative errors, given its mandate to protect human health and the environment.

Statistical power is an alternate way of describing false negative errors. Power is merely the complement of the false negative rate. If β is the probability of a false negative, (1–β) is the statistical power of a particular test. In terms of the hypothesis structure, statistical power represents the probability of correctly rejecting the null hypothesis. That is, it is the minimum chance that one will decide to accept H_A, given that H_A is true. High power translates into a greater probability of identifying contaminated groundwater when it really exists.

A convenient way to keep track of the differences between false positives, false negatives, and power is via a Truth Table (**Figure 3-4**). A truth table distinguishes between the ***underlying*** truth of each hypothesis H_0 or H_A and the ***decisions*** made on the basis of statistical testing. If H_0 is true, then a decision to accept the alternative hypothesis (H_A) is a false positive error which will occur with a

probability of at most α. Because only one of two decisions is possible, H_0 will also be accepted with a probability of at least (1–α). This is also known as the confidence probability or confidence level of the test, associated with making a 'true negative' *decision*. Similarly if H_A is actually true, making a false negative decision error by accepting the null hypothesis (H_0) has at most a probability of β. Correctly accepting H_A when true then has a probability of at least (1– β) and is labeled a 'true positive' decision. This probability is also known as the statistical power of the test.

For any application of a test to a particular sample, only one of the two types of decision errors can occur. This is because only one of the two mutually exclusive hypotheses will be a true statement. In the detection monitoring context, this means that if a well is *uncontaminated* (*i.e.*, H_0 is true), it may be possible to commit a Type I false positive mistake, but it is *not* possible to make a Type II false negative error. Similarly, if a *contaminated* well is tested (*i.e.*, H_A is true), Type I false positive errors *cannot* occur, but a Type II false negative error might occur.

Figure 3-4. Truth Table in Hypothesis Testing

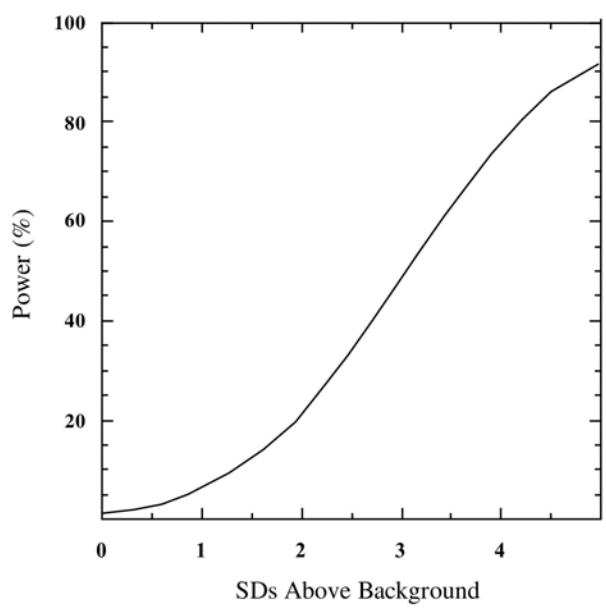
TRUTH	DECISION: Accept H_0	DECISION: Accept H_A
H_0	**OK** (True Negative) **(1-α)**	**TYPE I ERROR** (False Positive) **(α)**
H_A	**TYPE II ERROR** (False Negative) **(β)**	**OK** (True Positive) **(1-β)**

Since the false positive rate can be fixed in advance of running most statistical tests by selecting α, one might think the same could be done with statistical power. Unfortunately, neither statistical power nor the false negative rate can be fixed in advance for a number of reasons. One is that power and the false negative rate depends on the degree to which the true mean concentration level is elevated with respect to the background null condition. Large concentration increases are easier to detect than small increments. In fact, power can be graphed as an increasing function of the true concentration level in what is termed a *power curve* (**Figure 3-5**). A power curve indicates the probability of rejecting H_0 in favor of the alternative H_A for any given alternative to the null hypothesis (*i.e.*, for a range of possible mean-level increases above background).

In interpreting the power curve below, note that the *x*-axis is labeled in terms of relative background standard deviation units (σ) above the true background population mean (μ). The zero point along the *x*-axis is associated with the background mean itself, while the *k*th positive unit along the axis represents a 'true' mean concentration in the compliance well being tested equal to $\mu + k\sigma$. This mode of scaling the graph allows the same power curve to be potentially applied to any constituent of interest subject to the same test conditions. This is true no matter what the typical background concentration levels of a chemical typically found in groundwater may be. But it also means that the same point along the power curve will represent different absolute concentrations for different constituents. Even if the background means are the same, a two standard deviation increase in a chemical with highly variable background concentrations will correspond to a larger population mean increase at a compliance well than the same relative increase in a less variable constituent.

As a simple example, if the background population averages for arsenic and manganese both happen to be 10 ppb, but the arsenic standard deviation is 5 ppb while that for manganese is only 2 ppb, then a compliance well with a mean equivalent to a three standard deviation increase over background would have an average arsenic level of 25 ppb, but an average manganese level of only 16 ppb. For both constituents, however, there would be approximately a 50% probability of detecting a difference between the compliance well and background.

Figure 3-5. Example Power Curve



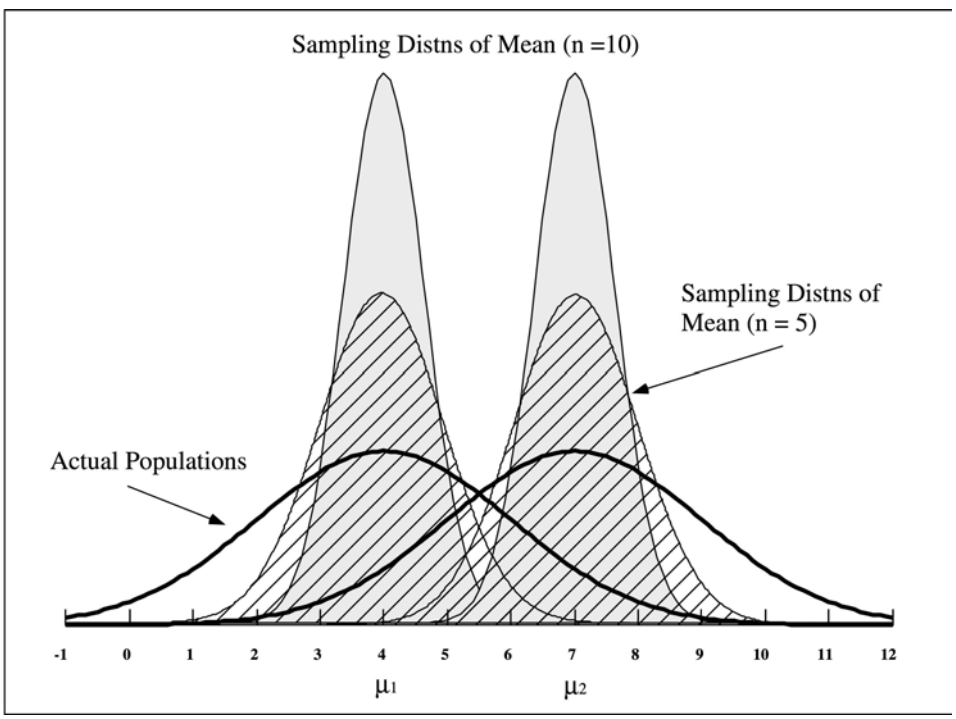
Because the power probability depends on the relative difference between the actual downgradient concentration level and background, power cannot typically be fixed ahead of time like the critical false positive rate for a test. The true concentration level (and associated power) in a compliance well is unknown. If it were known, no hypothesis test would be needed. Additionally, it is often not clear what specific magnitude of increase over background is environmentally significant. A two standard deviation increase over the background average might not be protective of human health and/or the

environment for some monitoring situations. For others, a four standard deviation increase or more may be tolerable before any threat is posed.

Since the exact ramifications of a particular concentration increase are uncertain, it points to the difficulty in setting a minimum power requirement (or a maximum false negative rate) for a given statistical test. Some State statutes contain water quality non-degradation provisions, for which *any* measurable increase might be of concern. By emphasizing relative power as in **Figure 3-5**, all detection monitoring constituents can be evaluated for significant concentration increases on a common footing, subject only to differences in measurement variability.

Another key factor affecting statistical power is sample size. All other test conditions being equal, larger sample sizes provide higher statistical power and the lower the false negative rate (β). Statistical tests perform more accurately with larger data sets, leading to greater power and fewer errors in the process. The Central Limit Theorem illustrates why this is true. Even if a downgradient well mean level is only slightly greater than background, upgradient and downgradient well sample means will have so little variance in their sampling distributions with enough measurements that they will tend to hover very close to their respective population means. True mean differences in the underlying populations can be distinguished with higher probability as sample sizes increase. In **Figure 3-6**, the sampling distributions of means of size 5 and 10 between two different normal populations are provided for illustration. The narrower width of the distribution for the n = 10 sample means are more clearly distinguished from each other than for means of sample size n = 5. This implies higher probability and power to distinguish between the two population means.

Figure 3-6. Why Statistical Power Increases with Sample Size



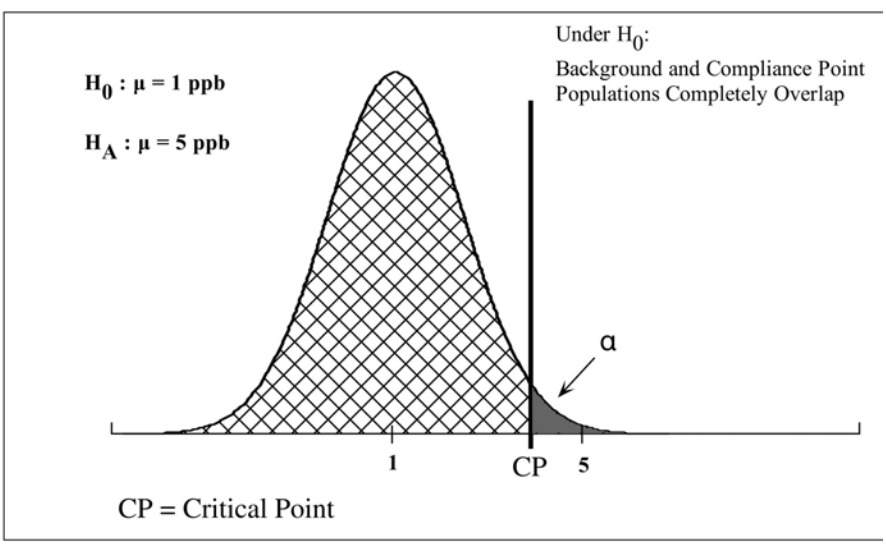
3.5.4 BALANCING TYPE I AND TYPE II ERRORS

In maintaining an appropriate balance between false positive and false negative error rates, one would ideally like to simultaneously minimize both kinds of errors. However, both risks are inherent to any statistical test procedure, and the risk of committing a Type I error is indirectly but inversely related to the risk of a Type II error unless the sample size can be increased. It is necessary to find a *balance* between the two error rates. But given that the false negative rate depends largely on the true compliance point concentrations, it is first necessary to designate what specific mean difference (known as an *effect size*) between the background and compliance point populations should be considered environmentally important. A minimum power requirement can be based on this difference (see **Chapter 6**).

►EXAMPLE 3-1

Consider a simple example of using the downgradient sample mean to test the proposition that the downgradient population mean is 4 ppb larger than background. Assume that extensive sampling has demonstrated that the background population mean is equal to 1 ppb. If the true downgradient mean were the same as the background level, curves of the two sampling distributions would coincide (as depicted in **Figure 3-7**). Then a critical point (e.g., CP = 4.5 ppb) can be selected so that the risk of a false positive mistake is α. The critical point establishes the decision criteria for the test. If the observed sample mean based on randomly selected data from the downgradient sampling distribution exceeds the critical point, the downgradient population will be declared higher in concentration than the background, even though this is not the case. The frequency that such a wrong decision will be made is just the area under the sampling distribution to the right of the critical point equal to α.

Figure 3-7. Relationship Between Type I and Type II Errors, Part A

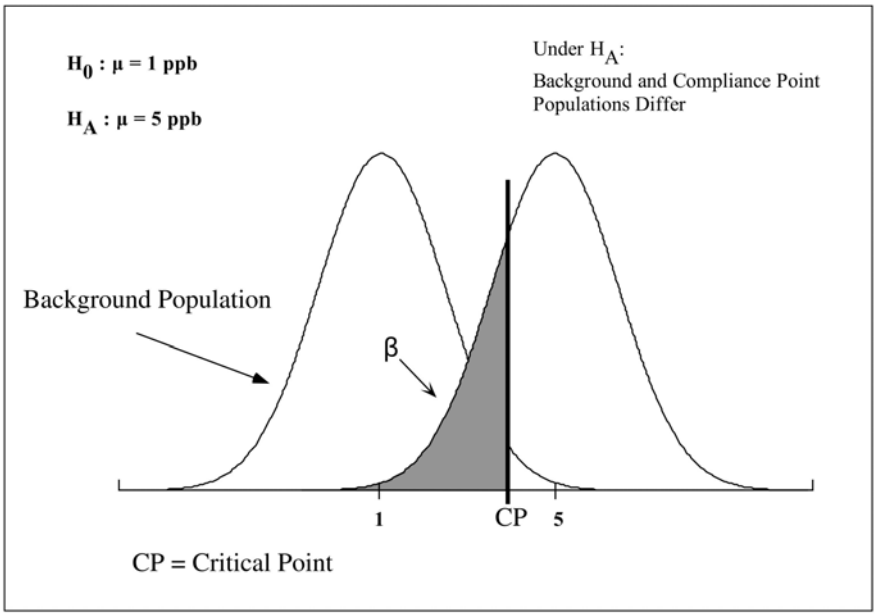


If the true downgradient mean is actually 5 ppb, the sampling distribution of the mean will instead be centered over 5 ppb as in the right-hand curve (*i.e.*, the downgradient population) in **Figure 3-8**. Since there really is a difference between the two populations, the alternative hypothesis and *not* the null

hypothesis is true. Thus, any observed sample mean drawn from the downgradient population then falling below the critical point is a false negative mistake. Consequently, the area under the right-hand sampling distribution in **Figure 3-8** to the *left* of the critical point represents the frequency of Type II errors (β).

The false negative rate (β) in **Figure 3-8** is obviously larger than the false positive rate (α) of **Figure 3-7**. This need not be the case in general, but the key point is to understand that for a fixed sample size, the Type I and Type II error rates cannot be simultaneously minimized. If α is increased, by selecting a lower critical point in **Figure 3-7**, the false negative rate will also be lowered in **Figure 3-8**. Likewise, if α is decreased by selecting a higher critical point, β will be enlarged. If the false positive rate is indiscriminately lowered, the false negative rate (or reduced power) will likely reach unacceptable levels even for mean concentration levels of environmental importance. Such reasoning lay behind EPA's decision to mandate *minimum* false positive rates for *t*-tests and ANOVA procedures in both the revised 1988 and 1991 RCRA rules.

Figure 3-8. Relationship Between Type I and Type II Errors, Part B



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CHAPTER 4. GROUNDWATER MONITORING PROGRAMS AND STATISTICAL ANALYSIS

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This chapter provides an overview of the basic groundwater monitoring framework, explaining the intent of the federal groundwater statistical regulations and offering insight into the key identification mechanism of groundwater monitoring, the ***statistically significant increase*** [SSI]:

- What are statistically significant increases and how should they be interpreted?
- What factors, both statistical and non-statistical can cause SSIs?
- What factors should be considered when demonstrating that an SSI does not represent evidence of actual contamination?

4.1 THE GROUNDWATER MONITORING CONTEXT

The RCRA regulations frame a consistent approach to groundwater monitoring, defining the conditions under which statistical testing takes place. Upgradient and downgradient wells must be installed to monitor the uppermost aquifer in order to identify releases or changes in existing conditions as expeditiously as possible. Geological and hydrological expertise is needed to properly locate the monitoring wells in the aquifer passing beneath the monitored unit(s). The regulations identify a variety of design and sampling requirements for groundwater monitoring (such as measuring well piezometric surfaces and identifying flow directions) to assure that this basic goal is achieved. Indicator or hazardous constituents are measured in these wells at regular time intervals; these sample data serve as the basis for statistical comparisons. For identifying releases under detection monitoring, the regulations generally presume comparisons of observations from downgradient wells against those from upgradient wells (designated as background). The rules also recognize certain situations (*e.g.*, mounding effects) when other means to define background may be necessary.

The Unified Guidance may apply to facility groundwater monitoring programs straddling a wide range of conditions. In addition to units regulated under Parts 264 and 265 Subpart F and Part 258 solid waste landfills, other non-regulated units at Subtitle C facilities or CERCLA sites may utilize similar programs. Monitoring can vary from a regulatory minimum of one upgradient and three downgradient wells, to very large facilities with multiple units, and perhaps 50-200 upgradient and downgradient wells. Although the rules presume that monitoring will occur in the single uppermost aquifer likely to be affected by a release, complex geologic conditions may require sampling and evaluating a number of aquifers or strata.

Detection monitoring constituents may include indicators like common ions and other general measures of water quality, pH, specific conductance, total organic carbon [TOC] and total organic halides [TOX]. Quite often, well monitoring data sets are obtained for filtered or unfiltered trace elements (or both) and sizeable suites of hazardous trace organic constituents, including volatiles, semi-volatiles, and pesticide/herbicides. Measurement and analysis of hazardous constituents using standard methods (in SW-846 or elsewhere) have become fairly routine over time. A large number of analytes may be potentially available as monitoring constituents for statistical testing, perhaps 50-100 or more. Identification of the most appropriate constituents for testing depends to a great extent on the composition of the managed wastes (or their decomposition products) as measured in leachate analyses, soil gas sampling, or from prior knowledge.

Nationally, enough groundwater monitoring experience has been gained in using routine constituent lists and analytical techniques to suggest some common underlying patterns. This is particularly true when defining background conditions in groundwater. Sampling frequencies have also been standardized enough (*e.g.*, semi-annual or quarterly sampling) to enable reasonable computation of the sorts of sample sizes that can be used for statistical testing. Nevertheless, complications can and do occur over time — in the form of changes in laboratories, analytical methods, sampled wells, and sampling frequencies — which can affect the quality and availability of sample data.

Facility status can also affect what data are potentially available for evaluation and testing — from lengthy regulated unit monitoring records under the Part 265 interim status requirements at sites awaiting either operational or post-closure 264 permits or permit re-issuance, to a new solid waste facility located in a zone of uncontaminated groundwater with little prior data. Some combined RCRA/CERCLA facilities may have collected groundwater information under differing program requirements. Contamination from offsite or non-regulated units (or solid waste management units) may complicate assessment of likely contaminant sources or contributions.

Quite often, regulators and regulated parties find themselves with considerable amounts of historical constituent-well monitoring data that must be assessed for appropriate action, such as a permit, closure, remedial action or enforcement decision. Users will need to closely consider the diagnostic procedures in **Part II** of the Unified Guidance, with an eye towards selection of one or more appropriate statistical tests in **Parts III** and **IV**. Selection will depend on key factors such as the number of wells and constituents, statistical characteristics of the observed data, and historical patterns of contamination (if present), and may also reflect preferences for certain types of tests. While the Unified Guidance purposely identifies a range of tests which might fit a situation, it is generally recommended that *one* set of tests be selected for final implementation, in order to avoid "test-shopping" (i.e., selecting tests during permit implementation based on the most favorable outcomes). EPA recognizes that the final permit requirements are approved by the regulatory agency.

All of the above situations share some features in common. A certain number of facility wells will be designated as compliance points, *i.e.*, those locations considered as significant from a regulatory standpoint for assessing potential releases. Similarly, the most appropriate and critical indicator and/or hazardous constituents for monitoring will be identified. If detection monitoring (*i.e.*, comparative evaluations of compliance wells against background) is deemed appropriate for some or all wells and constituents, definitions of background or reference comparison levels will need to be established. Background data can be obtained either from the upgradient wells or from the historical sampling database as described in **Chapter 5**. Choice of background will depend on how statistically comparable

the compliance point data are with respect to background and whether individual constituents exhibit spatial or temporal variability at the facility.

Compliance/assessment or corrective action monitoring may be appropriate choices when there is a prior or historical indication of hazardous constituent releases from a regulated unit. In those situations, the regulatory agency will establish GWPS limits. Typically, these limits are found in established tables, in SDWA drinking water MCLs, through risk-based calculations or determined from background data. For remedial actions, site-specific levels may be developed which account not only for risk, but achievability and implementation costs as well. Nationally, considerable experience has been gathered in identifying cleanup targets which might be applicable at a given facility, as well as how practical those targets are likely to be.

Use of the Unified Guidance should thus be viewed in an overall context. While the guidance offers important considerations and suggestions in selecting and designing a statistically-based approach to monitoring, it is important to realize that it is only a part of the overall decision process at a facility. Geologic and hydrologic expertise, risk-based decisions, and legal and practical considerations by the regulated entity and regulatory agency are fundamental in developing the final design and implementation. The guidance does not attempt to address the many other relevant decisions which impact the full design of a monitoring system.

4.2 RCRA GROUNDWATER MONITORING PROGRAMS

Under the RCRA regulations, some form of statistical testing of sample data will generally be needed to determine whether there has been a release, and if so, whether concentration levels lie below or above a protection standard. The regulations frame the testing programs as detection, compliance/assessment, and corrective action monitoring.

Under RCRA permit development and during routine evaluations, all three monitoring program options may need to be simultaneously considered. Where sufficient hazardous constituent data from site monitoring or other evidence of a release exists, the regulatory agency can evaluate which monitoring program(s) are appropriate under §264.91. Statistical principles and testing provided in the Unified Guidance can be used to develop presumptive evidence for one program over another.

In some applications, more than one monitoring program may be appropriate. Both the number of wells and constituents to be tested can vary among the three monitoring programs at a given site. The types of non-hazardous indicator constituents used for detection monitoring might not be applied in compliance or corrective action monitoring. The latter focus is on hazardous constituents. Only a few compliance well constituents may exceed their respective GWPSs. The focus in a corrective action monitoring program might then be placed on the latter, with the remaining well constituents evaluated under the other monitoring schemes. But following the general regulatory structure, the three monitoring systems are presented below and elsewhere in the guidance as an ordered sequence:

Detection monitoring is appropriate either when there is no evidence of a release from a regulated unit, or when the unit situated in a historically contaminated area is not impacted by current RCRA waste management practices. Care must be taken to avoid a situation where the constituents might reasonably have originated offsite or from units not subject to testing, since any adverse change in groundwater quality would be attributed to on-site causes. Whether an observed change in groundwater

quality is in fact due to a release from on-site waste activities at the facility may be open to dispute and/or further demonstration. However, this basic framework underlies each of the statistical methods used in detection monitoring.

A crucial step in setting up a detection monitoring program is to establish a set of *background* measurements, a baseline or reference level for statistical comparisons (see **Chapter 5**). Groundwater samples from compliance wells are then compared against this baseline to measure changes in groundwater quality. If at least one chemical parameter on the monitoring indicates a *statistically significant increase* above the baseline [SSI, see **Section 4.3**], the facility or regulated unit moves into the next phase: compliance or assessment monitoring.

Compliance or assessment monitoring[1] is appropriate when there is reliable statistical evidence that a concentration increase over the baseline has occurred. The purpose of compliance/assessment monitoring is two-fold: 1) to assess the extent of contamination (*i.e.*, the size of the increase, the chemical parameters involved, and the locations on-site where contamination is evident); and 2) to measure compliance with pre-established numerical concentration limits generally referred to as GWPSs. Only the second purpose is fully addressed using formal statistical tests. While important information can be gleaned from compliance well data, more complex analyses (*e.g.*, contaminant modeling) may be needed to address the first goal.

GWPSs can be fixed health- or risk-based limits, against which single-sample tests are made. At some sites, no specific fixed concentration limit may be assigned or readily available for one or more monitoring parameters. Instead, the comparison is made against a limit developed from background data. In this case, an appropriate statistical approach might be to use the background measurements to compute a statistical limit and set it as the GWPS. See **Chapter 7** for further details. Many of the detection monitoring design principles (**Chapter 6**) and statistical tests (**Part III**) can also be applied to a set of constituents defined by a background-type GWPS.

The RCRA Parts 264 and 258 regulations require an expanded analysis of potential hazardous constituents (Part 258 Appendix II for municipal landfills or Part 264 Appendix IX for hazardous waste units) when detection monitoring indicates a release and compliance monitoring is potentially triggered. The purpose is to better gauge which hazardous constituents have actually impacted groundwater. Some detection monitoring programs may require only limited testing of indicator parameters. This additional sampling can be used to determine which wells have been impacted and provide some understanding of the on-site distribution of hazardous constituent concentrations in groundwater. . The course of action decided by the Regional Administrator or State Director will depend on the number of such chemicals that are present in quantifiable levels and the actual concentration levels.

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[1] The terms compliance monitoring (§264.99 & 100) and assessment monitoring (§258.55 & 56) are used interchangeably in this document to refer to RCRA monitoring programs. Compliance monitoring is generally used for permitted hazardous waste facilities under RCRA Subtitle C, while assessment monitoring is applied to municipal solid waste landfills regulated under RCRA Subtitle D. The term "assessment" is also used in 40 CFR 265 Subpart F for a second phase of additional analyte testing. Occasional use is also made of the term "compliance wells," which refers to downgradient monitoring wells located at the point(s) of compliance under §264.95 (any of the three monitoring programs may apply when evaluating these wells).

Following the occurrence of a valid statistically significant increase [SSI] over baseline during detection monitoring, the statistical presumption in compliance/assessment monitoring is quite similar to the detection stage. Given *G* as a fixed compliance or background-derived GWPS, the null hypothesis is that true concentrations (of the underlying compliance point population) are no greater than *G*. This compares to the detection monitoring presumption that concentration levels do not exceed background. One reason for the similarity is that compliance limits may be higher than background levels in some situations. An increase over background in these situations does not necessarily imply an increase over the compliance limit, and the latter must be formally tested. On the other hand, if a health- or risk-based limit is below a background level, the RCRA regulations provide that the GWPS should be based on background.

The Subtitle D regulations for municipal solid waste landfills [MSWLF] stipulate[2] that if "the concentrations of all Appendix II constituents are shown to be at or below background values, using the statistical procedures in §258.53(g), for two consecutive sampling events, the owner or operator... may return to detection monitoring." In other words, assessment monitoring may be exited in favor of detection monitoring when concentrations at the compliance wells are statistically indistinguishable from background for two consecutive sampling periods. While a demonstration that concentration levels are below background would generally not be realistic, it may be possible to show that compliance point levels of contaminants do not exceed an upper limit computed from the background data. Conformance to the limit would then indicate an inability to statistically distinguish between background and compliance point concentration levels.

If a hazardous constituent under compliance or assessment monitoring statistically exceeds a GWPS, the facility is subject to **corrective action.** Remedial activities must be undertaken to remove and/or prevent the further spread of contamination into groundwater. **Monitoring** under corrective action is used to track the progress of remedial activities and to determine if the facility has returned to compliance. Corrective action is usually preceded or accompanied by a formal Remedial Investigation [RI] or RCRA Facility Investigation [RFI] to further delineate the nature and extent of the contaminated plume. Corrective action may be confined to a single regulated unit if only that unit exhibits SSIs above a standard during the detection and compliance/assessment monitoring phases.

Often, clean-up levels are established by the Regional Administrator or State Director during corrective action. Remediation must continue until these clean-up levels are met. The focus of remedial action and monitoring would be on those hazardous constituents and well locations exceeding the GWPSs. If specific clean-up levels have not been met, corrective action must continue until there is evidence of a *statistically significant decrease* [SSD] below the compliance limit for three consecutive years. At this point, corrective action may be exited and compliance monitoring re-started. (As described above and in **Chapter 7**, the protocol for assessing corrective action compliance with a background-type standard can differ). If subsequent concentrations are statistically indistinguishable from background or no detectable concentrations can be demonstrated for three consecutive years in any of the contaminants that triggered corrective measures in the first place, corrective action may be exited in favor of detection monitoring.

[2] [56 FR 51016] October 9, 1991

4.3 STATISTICAL SIGNIFICANCE IN GROUNDWATER TESTING

The outcome of any statistical test is judged either to be statistically significant or non-significant. In groundwater monitoring, a valid statistically significant result can force a change in the monitoring program, perhaps even leading to remedial activity. Consequently, it is important to understand what statistically significant results represent and what they do not. In the language of groundwater hypothesis testing (**Chapter 3**), a statistically significant test result is a decision to reject the null hypothesis (H_0) and to accept the alternative hypothesis (H_A), based on the observed pattern of the sample data. At the most elementary level, a *statistically significant increase* [SSI] (the kind of result typically of interest under RCRA detection and compliance monitoring) represents an observed increase in concentration at one or more compliance wells. In order to be declared an SSI, the change in concentration must be large enough after accounting for variability in the sample data, that the result is unlikely to have occurred merely by chance. What constitutes a statistically significant result depends on the phase of monitoring and the type of statistical test being employed.

If the detection monitoring statistical test being used is a *t*-test or Wilcoxon rank-sum test (**Chapter 16**), an SSI occurs whenever the *t*-statistic or *W*-statistic is larger than an α-level critical point for the test. If a retesting procedure is chosen using a prediction limit (**Chapter 19**), an SSI occurs only when both the initial compliance sample or initial mean/median and one or more resamples all exceed the upper prediction limit. For control charts (**Chapter 20**), an SSI occurs whenever either the CUSUM or Shewhart portions of the chart exceed their respective control limits. In another variation, an SSI only occurs if one or another of the CUSUM or Shewhart statistics exceeds the control limits when recomputed using one or more resamples. For tests of trend (**Chapter 17**), an SSI is declared whenever the slope is significantly greater than zero at some significance level α.

In compliance/assessment monitoring, tests are often made against a fixed compliance limit or GWPS. In this setting, one can utilize a *confidence interval* around a mean, median, upper percentile or a trend line (**Chapter 21**). A confidence interval is an estimated concentration or measurement range intended to contain a given statistical characteristic of the population from which the sample is drawn. A most common formulation is a two-way confidence interval around a normally-distributed mean μ, as shown below:

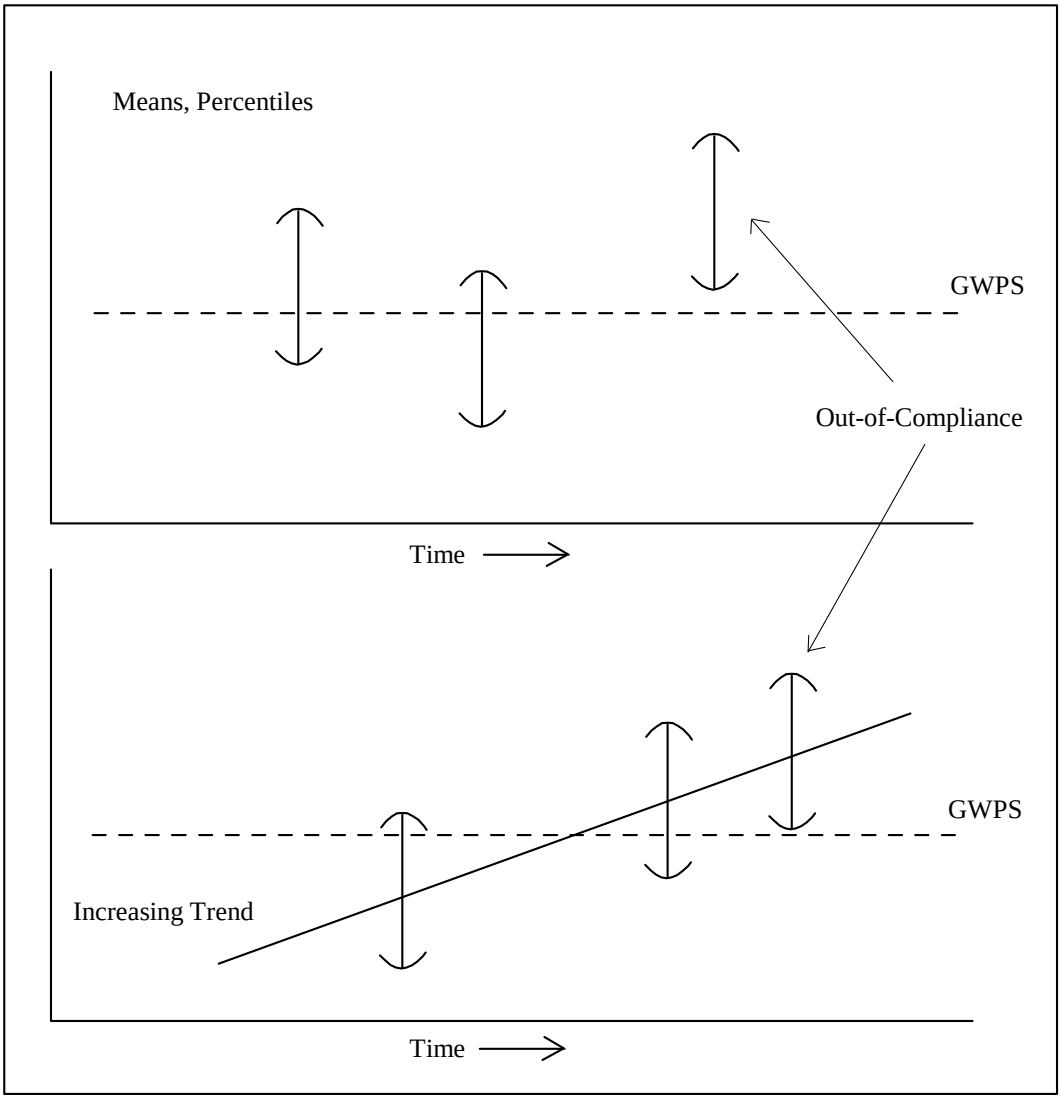
$$\left(\bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} \quad \leq \quad \mu \quad \leq \quad \bar{x} + t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} \right) \qquad [4.1]$$

where $\bar{x}$ is the mean of a sample of size *n*, *s* is the sample standard deviation, and $t_{1-\alpha,\, n-1}$ is an upper percentile selected from a Student's *t*-distribution. By constructing a range around the sample mean ($\bar{x}$), this confidence interval is designed to locate the true population mean (μ) with a high degree of *statistical confidence(1-2α)* or conversely, with a low probability of error (2α). If a one-way lower confidence interval is used, the right-hand term in equation [4.1] would be replaced by +∞ at confidence level 1-α. In a similar fashion, the upper 1-α confidence interval would be defined in the range from -∞ for the left-hand term to the right hand term in equation [4.1].

When using a *lower confidence interval* on the mean, median, or upper percentile, an SSI occurs whenever the lower edge of the confidence interval range exceeds the GWPS. For a confidence interval around a trend line, an SSI is declared whenever the lower confidence limit around the estimated trend

line *first* exceeds the GWPS at some point in time. By requiring that a lower confidence limit be used as the basis of comparison, the statistical test will account for data variability and ensure that the apparent violation is unlikely to have occurred by chance. **Figure 4-1** below visually depicts a comparison to a fixed GWPS for both lower confidence intervals for a stationary test like a mean, and around an increasing trend. Where the confidence interval straddles the limit, the test results are inconclusive. In similar fashion, an SSD can be identified by using upper confidence intervals.

Figure 4-1. Confidence Intervals Around Means, Percentiles, or Trend Lines



SSIs offer the primary *statistical* justification for moving from detection monitoring to compliance monitoring, or from compliance/assessment monitoring to corrective action. However, it is important that an SSI be interpreted correctly. Any SSI at a compliance well represents a probable increase in concentration level, but it *does not automatically imply or prove* that contaminated groundwater from the facility is the *cause* of the increase. Due to the complexities of the groundwater medium and the nature of statistical testing, there are numerous reasons why a test may exhibit a statistically significant result. These may or may not be indications of an actual release from a regulated unit.

It is always reasonable to allow for a separate demonstration once an SSI occurs, to determine whether or not the increase is actually due to a contaminant release. Such a demonstration will rely heavily on hydrological and geochemical evidence from the site, but could include additional statistical factors. Key questions and factors to consider are listed in the following sections.

4.3.1 STATISTICAL FACTORS

- Is the result a false positive? That is, were the data tested simply an unusual sample of the underlying population triggering an SSI? Generally, this can be evaluated with repeat sampling.
- Did the test correctly identify an actual release of an indicator or hazardous constituent?
- Are there corresponding SSIs in upgradient or background wells? If so, there may be evidence of a natural in-situ concentration increase, or perhaps migration from an off-site source.
- Is there evidence of significant concentration differences between separate upgradient or background wells, particularly for inorganic constituents? If so, there may be natural spatial variations between distinct well locations that have not been accounted for. These spatial differences could be local or systematic (*e.g.*, upgradient wells in one formation or zone; downgradient wells in another).
- Could observed SSIs for naturally occurring analytes be due to longer-term (*i.e.*, seasonal or multi-year) variation? Seasonal or other cyclical patterns should be observable in upgradient wells. Is this change occurring in both upgradient and downgradient wells? Depending on the statistical test and frequency of sampling involved, an observed SSI may be entirely due to temporal variation not accounted for in the sampling scheme.
- Do time series plots of the sampling data show parallel "spikes" in concentration levels from both background and compliance well samples that were analyzed at about the same time? Perhaps there was an analytical problem or change in lab methodology.
- Are there substantial correlations among within-well constituents (in both upgradient and downgradient wells)? Highly correlated analytes treated as independent monitoring constituents, may generate incorrect significance levels for individual tests.
- Were trends properly accounted for, particularly in the background data?
- Was a correct assumption made concerning the underlying distribution from which the observations were drawn (*e.g.*, was a normal assumption applied to lognormal data)?
- Was the test computed correctly?
- Were the data input to the test of poor quality? (see various factors below)

4.3.2 WELL SYSTEM DESIGN AND SAMPLING FACTORS

- Were early sample data following well installation utilized in statistical testing? Initial well measurements are sometimes highly variable during a 'break in' sampling and analysis period and potentially less trustworthy.
- Was there an effect attributable to recent well development, perhaps due to the use of hazardous constituent chemicals during development or present in drilling muds?
- Are there multiple geological formations at the site, leading to incorrect well placements?

- Has there been degradation of the well casings and screens (*e.g.*, PVC pipe)? Deteriorating PVC materials can release organic constituents under certain conditions. Occasionally, even stainless steel can corrode and release a number of metallic trace elements.
- Have there been changes in well performance over time?
- Were there excessive holding times or incorrect use of preservatives, cooling, *etc.*
- Was there incorrect calibration or drift in the field instrumentation? This effect should be observable in both upgradient and downgradient data and possibly over a number of sample events. The data itself may be compromised or useless.
- Have there been 'mid-stream' changes in sampling procedures, *e.g.*, increased or decreased well purging? Have sampling or purging techniques been consistently applied from well to well or from sampling event to sampling event?

4.3.3 HYDROLOGICAL FACTORS

- Does the site have a history of previous waste management activity (perhaps prior to RCRA), and is there any evidence of historical groundwater contamination? Previous contamination or waste management contaminant levels can limit the ability to distinguish releases from the regulated unit, particularly for those analytes found in historical contamination.
- Is there evidence of groundwater mounding or other anomalies that could lead to the lack of a reliable, definable gradient? Interwell statistical tests assume that changes in downgradient groundwater quality only affect compliance wells and not upgradient (background) wells. Changes that impact background wells also, perhaps in a complex manner involving seasonal fluctuations, are often best resolved by running intrawell tests instead.
- Is there hydrologic evidence of any migration of contaminants (including DNAPL) from off-site sources or from other non-regulated units? Are any of these contaminants observed upgradient of the regulated units?
- Have there been other prior human or site-related waste management activities which could result in the observed SSI changes for certain well locations (*e.g.*, buried waste materials, pipeline leaks, spills, *etc.*)?
- Have there been unusual changes in groundwater directions and depths? Is there confidence that the SSI did indeed correspond to a potential unit release based on observed groundwater directions, distance of the well from the unit, other well information, *etc.*?
- Is there evidence of migration of landfill gas affecting one or more wells?
- Have there been increases in well turbidity and sedimentation, which could affect observed contaminant levels?
- Are there preferential flow paths in the aquifer that could affect where contaminants are likely to be observed or not observed?
- Are the detected contaminants consistent with those found in the waste or leachate of the regulated unit?
- Are there other nearby well pumping or extraction activities?

4.3.4 GEOCHEMICAL FACTORS

- Were the measurements that triggered the SSI developed from unfiltered or filtered trace element sample data? If unfiltered, is there any information regarding associated turbidity or total suspended solid measurements? Unusual increases in well turbidity can introduce excess naturally occurring trace elements into the samples. This can be a particularly difficult problem in compliance monitoring when comparing data to a fixed standard, but can also affect detection monitoring well-to-well comparisons if turbidity levels vary.
- Were there changes in associated analytes at the "triggered" well consistent with local geochemistry? For example, given an SSI for total dissolved solids [TDS], did measured cations/anions and pH also show a consistent change? As another example, slight natural geochemical changes can result in large specific conductance changes. Did other constituents demonstrate a consistent change?
- Is there evidence of a simultaneous release of more than one analyte, consistent with the composition of the waste or leachate? In particular, is there corollary evidence of degradation or daughter products for constituents like halogenated organics? For groundwater constituents with identified SSIs, is there a probable relationship to measured concentrations in waste or waste leachate? Are leachate concentrations high enough to be detectable in groundwater?
- If an SSI is observed in one or more naturally occurring species, were organic hazardous constituents not normally present in background and found in the waste or leachate also detected? This could be an important factor in assessing the source of the possible release.
- Have aquifer mobility factors been considered? Certain soluble constituents like sodium, chloride, or conservative volatile organics might be expected to move through the aquifer much more quickly than easily adsorbed heavy metals or 4-5 ring polynuclear aromatic [PNA] compounds.
- Do the observed data patterns (particularly for naturally occurring constituents in upgradient wells or other background conditions) make sense in an overall site geochemical context, especially as compared with other available local or regional site data and published studies? If not, suspect background data may need to be further evaluated for potential errors prior to formal statistical comparisons.
- Do constituents exhibit correlated behavior among both upgradient and downgradient wells due to overall changes in the aquifer?
- Have there been natural changes in groundwater constituents over time and space due to multi-year, seasonal, or cyclical variation?
- Are there different geochemical regimes in upgradient *vs*. downgradient wells?
- Has there been a release of soil trace elements due to changes in pH?

4.3.5 ANALYTICAL FACTORS

- Have there been changes in laboratories, analytical methods, instrumentation, or procedures including specified detection limits that could cause apparent jumps in concentration levels? In some circumstances, using different values for non-detects with different reporting limits has triggered SSIs. Were inexperienced technicians involved in any of the analyses?

- Was more than one analytical method used (at different points in time) to generate the measurements?
- Were there changes in detection/quantification limits for the same constituents?
- Were there calibration problems, *e.g.*, drift in instrumentation?
- Was solvent or other laboratory contamination (*e.g.*, phthalates, methylene chloride extractant, acetone wash) introduced into any of the physical samples?
- Were there known or probable interferences among the analytes being measured?
- Were there "spikes" or unusually high values on certain sampling events (either for one constituent among many wells or related analytical constituents) that would suggest laboratory error?

4.3.6 DATA OR ANALYTIC ERRORS

- Were there data transcription errors (incorrect decimal places, analyte units, or data column entries)? These data can often be identified as being highly improbable.
- Were there calculation errors in either the analytical (*e.g.*, incorrect trace element valence assumptions or dilution factors) or in the statistical portions (mathematical mistakes, incorrect equation terms) of the analysis?

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CHAPTER 5. ESTABLISHING AND UPDATING BACKGROUND

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This chapter discusses the importance and use of background data in groundwater monitoring. Guidance is provided for the proper identification, review, and periodic updating of background. Key questions to be addressed include:

- How should background be established and defined?
- When should existing background data sets be reviewed?
- How and when should background be updated?
- What impact does retesting have on background updating?

5.1 IMPORTANCE OF BACKGROUND

High quality background data is the single most important key to a successful statistical groundwater monitoring program, especially for detection monitoring. All of the statistical tests listed in the RCRA regulations are predicated on having *appropriate* and *representative* background measurements. As indicated in **Chapter 3**, a statistical sample is representative if the distribution of the sample measurements best follows the distribution of the population from which the sample is drawn. Representative background data has a similar but slightly different connotation. The most important quality of background is that it reflects the historical conditions unaffected by the activities it is designed to be compared to. These conditions could range from an uncontaminated aquifer to an historically contaminated site baseline unaffected by recent RCRA-actionable contaminant releases. Representative background data will therefore have numerical characteristics closely matching those arising from the site-specific aquifer being evaluated.

Background must also be *appropriate* to the statistical test. All RCRA detection monitoring tests involve comparisons of compliance point data against background. If natural groundwater conditions

have changed over time — perhaps due to cycles of drought and recharge — background measurements from five or ten years ago may not reflect current uncontaminated conditions. Similarly, recent background data obtained using improved analytical methods may not be comparable to older data. In each case, older background data may have to be discarded in favor of more recent measurements in order to construct an *appropriate* comparison. If intrawell tests are utilized due to strong evidence of spatial variability, traditional upgradient well background data will not provide an appropriate comparison. Even if the upgradient measurements are reflective of uncontaminated groundwater, appropriate background data must be obtained from each compliance point well. The main point is that compliance samples should be tested against data which best can represent background conditions now and those likely to occur in the future.

5.1.1 TRACKING NATURAL GROUNDWATER CONDITIONS

Background measurements, especially from upgradient wells, can provide essential information for other than formal statistical testing. For one, background data can be used to gauge mean levels and develop estimates of variability in naturally occurring groundwater constituents. They can also be used to confirm the presence or absence of anthropogenic or non-naturally occurring constituents in the site aquifer. Ongoing sampling of upgradient background wells provides a means of tracking natural groundwater conditions. Changes that occur in parallel between the compliance point and background wells may signal site-wide aquifer changes in groundwater quality not specifically attributable to onsite waste management. Such observed changes may also be indicative of analytical problems due to common artifacts of laboratory analysis (*e.g.*, re-calibration of lab equipment, errors in batch sample handling, *etc*.), as well as indications of groundwater mounding, changes in groundwater gradients and direction, migration of contaminants from other locations or offsite, etc.

Fixed GWPS like maximum contaminant levels [MCLs] may be contemplated for compliance/assessment monitoring or corrective action. Background data analysis is important if it is suspected that naturally occurring levels of the constituent(s) in question are higher than the standards or if a given hazardous constituent does not have a health- or risk-based standard. In the first case, concentrations in downgradient wells may indeed exceed the standard, but may not be attributable to onsite waste management if natural background levels *also* exceed the standard. The Parts 264 and 258 regulations recognize these possibilities, and allow for GWPS to be based on background levels.

5.2 ESTABLISHING AND REVIEWING BACKGROUND

Establishing appropriate background depends on the statistical approach contemplated (*e.g.*, interwell *vs*. intrawell). This section outlines the major considerations concerning how to select and develop background data including monitoring constituents and sample sizes, statistical assumptions, and the presence of data outliers, spatial variation or trends. Expanding and reviewing background data are also discussed.

5.2.1 SELECTING MONITORING CONSTITUENTS AND ADEQUATE SAMPLE SIZES

Due to the cost of management, mobilization, field labor, and especially laboratory analysis, groundwater monitoring can be an expensive endeavor. The most efficient way to limit costs and still meet environmental performance requirements is to minimize the total number of samples which must be sampled and analyzed. This will require tradeoffs between the number of monitoring constituents

chosen, and the frequency of background versus compliance well testing. The number of compliance wells and annual frequency of testing also affect overall costs, but are generally site-specific considerations. By limiting the number of constituents and ensuring adequate background sample sizes, it is possible to select certain statistical tests which help minimize future compliance (and total) sample requirements.

Selection of an appropriate number of detection monitoring constituents should be dictated by the knowledge of waste or waste leachate composition and the corresponding groundwater concentrations. When historical background data are available, constituent choices may be influenced by their statistical characteristics. A few representative constituents or analytes may serve to accurately assess the potential for a release. These constituents should stem from the regulated wastes, be sufficiently mobile, stable and occur at high enough concentrations to be readily detected in the groundwater. Depending on the waste composition, some non-hazardous organic or inorganic indicator analytes may serve the same purpose. The guidance suggests that between 10-15 formal detection monitoring constituents should be adequate for most site conditions. Other constituents can still be reported but not directly incorporated into formal detection monitoring, especially when large simultaneously analyzed suites like ICP-trace elements, volatile or semi-volatile organics data are run. The focus of adequate background and future compliance test sample sizes can then be limited to the selected monitoring constituents.

The RCRA regulations do not consistently specify how many observations must be collected in background. Under the Part 265 Interim Status regulations, four quarterly background measurements are required during the first year of monitoring. Recent modifications to Part 264 for Subtitle C facilities require a sequence of at least four observations to be collected in background during an interval approved by the Regional Administrator. On the other hand, at least four measurements must be collected from each background well during the first semi-annual period along with at least one additional observation during each subsequent period, for Subtitle D facilities under Part 258. Although these are minimum requirements in the regulations, are they adequate sample sizes for background definition and use?

Four observations from a population are rarely enough to adequately characterize its statistical features; statisticians generally consider sample sizes of $n \leq 4$ to be insufficient for good statistical analysis. A decent population survey, for example, requires several hundred and often a few to several thousand participants to generate accurate results. Clinical trials of medical treatments are usually conducted on dozens to hundreds of patients. In groundwater tests, such large sample sizes are a rare luxury. However, it is feasible to obtain small sample sets of up to $n = 20$ for individual background wells, and potentially larger sample sizes if the data characteristics allow for pooling of multiple well data.

The Unified Guidance recommends that a minimum of at least 8 to 10 independent background observations be collected before running most statistical tests. Although still a small sample size by statistical standards, these levels allow for minimally acceptable estimates of variability and evaluation of trend and goodness-of fit. However, this recommendation should be considered a temporary minimum until additional background sampling can be conducted and the background sample size enlarged (see further discussions below).

Small sample sizes in background can be particularly troublesome, especially in controlling statistical test false positive and negative rates. False negative rates in detection monitoring, *i.e.*, the

statistical error of failing to identify a real concentration increase above background, are in part a function of sample size. For a fixed false positive test rate, a smaller sample size results in a higher false negative rate. This means a decreased probability (*i.e.*, *statistical power*) that real increases above background will be detected. With certain parametric tests, control of the false positive rate using very small sample sets comes at the price of extremely low power. Power may be adequate using a non-parametric test, but control of the false positive can be lost. In both cases, increased background sample sizes result in better achievable false positive and false negative errors.

The overall recommendation of the guidance is to establish background sample sizes as large as feasible. The final tradeoff comes in the selection of the type of detection tests to be used. Prediction limit, control chart, and tolerance limit tests can utilize very small future sample sizes per compliance well (in some cases a single initial sample), but require larger background sample sizes to have sufficient power. Since background samples generally are obtained from historical data sets (plus future increments as needed), total annual sample sizes (and costs) can be somewhat minimized in the future.

5.2.2 BASIC ASSUMPTIONS ABOUT BACKGROUND

Any background sample should satisfy the key statistical assumptions described in **Chapter 3**. These include statistical independence of the background measurements, temporal and spatial stationarity, lack of statistical outliers, and correct distribution assumptions of the background sample when a parametric statistical approach is selected. How independence and autocorrelation impact the establishment of background is presented below, with additional discussions on outliers, spatial variability and trends in the following sections. Stationarity assumptions are considered both in the context of temporal and spatial variation.

Both the Part 264 and 258 groundwater regulations require statistically independent measurements (**Chapter 2**). Statistical *independence* is indicated by random data sets. But randomness is only demonstrated by the presence of mean and variance *stationarity* and the lack of evidence for effects such as *autocorrelation*, *trends*, *spatial and temporal variation*. These tests (described in **Part II** of this guidance) generally require at least 8 to 10 separate background measurements.

Depending on site groundwater velocity, too-frequent sampling at any given background well can result in highly *autocorrelated*, non-independent data. Current or proposed sampling frequencies can be tested for autocorrelation or other statistical dependence using the diagnostic procedures in **Chapter 14**. Practically speaking, the best way to ensure some degree of statistical independence is to allow as much time as possible to elapse between sampling events. But a balance must be drawn between collecting as many measurements as possible from a given well over a specified time period, and ensuring that the sample measurements are statistically independent. If significant dependence is identified in already collected background, the interval between sampling events may need to be lengthened to minimize further autocorrelation. With fewer sampling events per evaluation period, it is also possible that a change in statistical method may be needed, say from analysis of variance [ANOVA], which requires at least 4 new background measurements per evaluation, to prediction limits or control charts, which may require new background only periodically (*e.g.*, during a biennial update).

5.2.3 OUTLIERS IN BACKGROUND

Outliers or observations not derived from the same population as the rest of the sample violate the basic statistical assumption of identically-distributed measurements. The Unified Guidance recommends that testing of outliers be performed on background data, but they generally not be removed unless some basis for a likely error or discrepancy can be identified. Such possible errors or discrepancies could include data recording errors, unusual sampling and laboratory procedures or conditions, inconsistent sample turbidity, and values significantly outside the historical ranges of background data. Management of potential outliers carries both positive and negative risks, which should be carefully understood.

If an outlier value with much higher concentration than other background observations is not removed from background prior to statistical testing, it will tend to increase both the background sample mean and standard deviation. In turn, this may substantially raise the magnitude of a parametric prediction limit or control limit calculated from that sample. A subsequent compliance well test against this background limit will be much less likely to identify an exceedance. The same is true with non-parametric prediction limits, especially when the maximum background value is taken as the prediction limit. If the maximum is an outlier not representative of the background population, few truly contaminated compliance wells are likely to be identified by such a test, lowering the statistical power of the method and the overall quality of the statistical monitoring program.

Because of these concerns, it may be advisable at times to remove high-magnitude outliers in background even if the reasons for these apparently extreme observations are not known. The overall impact of removal will tend to improve the power of prediction limits and control charts, and thus result in a more environmentally protective program.

But strategies that involve automated evaluation and removal of outliers may unwittingly eliminate the evidence of real and important changes to background conditions. An example of this phenomenon may have occurred during the 1970s in some early ozone depletion measurements over Antarctica (http://www.nas.nasa.gov/About/Education/Ozone/history.html). Automated computer routines for outlier detection apparently removed several measurements indicating a sharp reduction in ozone concentrations, and thus prevented identification of an enlarging ozone hole by many years. Later review of the raw observations revealed that these automated routines had statistically classified measurements as outliers, which were more extreme than most of the data from that time period. Thus, there is some merit in saving and revisiting apparent 'outliers' in future investigations, even if removed from present databases.

In groundwater data collection and testing, background conditions may not be static over time. Caution should be observed in removing observations which may signal a change in natural groundwater quality. Even when conditions have not changed, an apparently extreme measurement may represent nothing more than a portion of the background distribution that has yet to be observed. This is particularly true if the background data set contains fewer than 20 samples.

In balancing these contrasting risks in retaining or removing one or more outliers, analyses of historical data patterns can sometimes provide more definitive information depending on the types of analytes and methods. For example, if a potential order-of magnitude higher outlier is identified in a sodium data set used as a monitoring constituent, cation-anion balances can help determine if this change is geochemically probable. In this case, changes to other intrawell ions or TDS should be

observed. Similarly, if a trace element outlier is identified in a single well sampling event and occurred simultaneously with other trace element maxima measured using the same analytical method (e.g., ICP-AES) either in the same well or groups of wells, an analytical error should be strongly suspected. On the other hand, an isolated increase without any other evidence could be a real but extreme background measurement. Ideally, removal of one or more statistically identified outliers should be based on other technical information or knowledge which can support that decision.

5.2.4 IMPACT OF SPATIAL VARIABILITY

In the absence of contamination, comparisons made between upgradient-to-downgradient wells assume that the concentration distribution is *spatially stationary* across the well field (**Chapter 3**). This implies that every well should have the same population mean and variance, unless a release occurs to increase the concentration levels at one or more compliance wells. At many sites, this is not the case for many naturally occurring constituents. Natural or man-made differences in mean levels — referred to as *spatial variability* or *spatial variation* — impact how background must be established.

Evidence of spatial variation should drive the selection of an *intrawell* statistical approach if observed among wells known to be uncontaminated (*e.g.*, among a group of upgradient background locations). Lack of spatial mean differences and a common variance allow for *interwell* comparisons. Appropriate background differs between the two approaches.

With interwell tests, background is derived from distinct, initially upgradient background wells, which may be enhanced by data from historical compliance wells also shown not to exhibit significant mean and variance differences. Future data from each of these compliance wells are then tested against this common background. On the other hand, intrawell background is derived from and represents historical groundwater conditions in each individual compliance well. When the population mean levels vary across a well field, there is little likelihood that the upgradient background will provide an appropriate comparison by which to judge any given compliance well.

Although spatial variability impacts the choice of background, it does so *only* for those constituents which evidence spatial differences across the well field. Each monitoring constituent should be evaluated on its own statistical merits. Spatial variation in some constituents (*e.g.*, common ions and inorganic parameters) does not preclude the use of interwell background for other infrequently detected or non-naturally occurring analytes. At many sites, a mixture of statistical approaches may be appropriate: interwell tests for part of the monitoring list and intrawell tests for another portion. Distinct background observation sets will need to be developed under such circumstances.

Intrawell background measurements should be selected from the available historical samples at each compliance well and should include only those observations thought to be uncontaminated. Initially, this might result in very few measurements (*e.g.*, 4 to 6). With such a small background sample, it can be very difficult to develop an adequately powerful intrawell prediction limit or control chart, even when *retesting* is employed (**Chapter 19**). Thus, additional background data will be needed to augment the testing power. One option is to periodically augment the existing background data base with recent compliance well samples (discussed in a further section below). Another possible remedy is to *statistically augment* the available sample data by running an analysis of variance [ANOVA] simultaneously on all the sets of intrawell background from the various upgradient and compliance wells (see **Chapter 13**). The *root mean squared error* [RMSE] from this procedure can be used in place of the

background standard deviation in parametric prediction and control limits to substantially increase the *effective background sample size* of such tests, despite the limited number of observations available per well.

This strategy will only work if the key assumptions of ANOVA can be satisfied (**Chapter 17**), particularly the requirement of equal variances across wells. Since natural differences in mean levels often correspond to similar differences in variability, a transformation of the data will often be necessary to homogenize the variances prior to running the ANOVA. For some constituents, no transformation may work well enough to allow the RMSE to be used as a replacement estimate for the intrawell background standard deviation. In that case, it may not be possible to construct reasonably powerful intrawell background limits until background has been updated once or twice (see **Section 5.3**).

5.2.5 TRENDS IN BACKGROUND

A key implication of the independent and identically distributed assumption [*i.i.d.*] is that a series of sample measurements should be *stationary over time* (*i.e.*, stable in mean level and variance). Data that are trending upward or downward violate this assumption since the mean level is changing. Seasonal fluctuations also violate this assumption since both the mean and variance will likely oscillate. The proper handling of trends in background depends on the statistical approach and the cause of the trend. With interwell tests and a common (upgradient) background, a trend can signify several possibilities:

- Contaminated background;
- A 'break-in' period following new well installation;
- Site-wide changes in the aquifer;
- Seasonal fluctuations, perhaps on the order of several months to a few years.

If upgradient well background becomes contaminated, intrawell testing may be needed to avoid inappropriate comparisons. Groundwater flow patterns should also be re-examined to determine if gradients are properly defined or if groundwater mounding might be occurring. With newly-installed background wells, it may be necessary to discard initially collected observations and to wait several months for aquifer disturbances due to well construction to stabilize. Site-wide changes in the underlying aquifer should be identifiable as similar trends in both upgradient and compliance wells. In this case, it might be possible to remove a common trend from both the background and compliance point wells and to perform interwell testing on the *trend residuals*. However, professional statistical assistance may be needed to do this correctly. Another option would be to switch to intrawell *trend tests* (**Chapter 17**).

Seasonal fluctuations in interwell background which are also observed in compliance wells, can be accommodated by modeling the seasonal trend and removing it from all background and compliance well data. Data seasonally-adjusted in this way (see **Chapter 14** for details) will generally be less variable than the unadjusted measurements and lead to more powerful tests than if the seasonal patterns had been ignored. For this adjustment to work properly, the same seasonal trend should be observed across the well field and not be substantially different from well to well.

Roughly linear trends in *intrawell* background usually signify the need to switch from an intrawell prediction limit or control chart to an explicit trend test, such as *linear regression* or the *Mann-Kendall* (**Chapter 17**). Otherwise the background variance will be overestimated and biased on the high side, leading to higher than expected and ultimately less powerful prediction and control limits. Seasonal fluctuations in intrawell background can be treated in one of two ways. A *seasonal Mann-Kendall* trend test built to accommodate such fluctuations can be employed (**Section 14.3.4**). Otherwise, the seasonal pattern can be estimated and removed from the background data, leaving a set of seasonally-adjusted data to be analyzed with either a prediction limit or control chart. In this latter approach, the same seasonal pattern needs to be extrapolated *beyond* the current background to more recent measurements from the compliance well being tested. These later observations also need to be seasonally-adjusted prior to comparison against the adjusted background, even if there is not enough compliance data yet collected to observe the same seasonal cycles.

When trends are apparent in background, another option is to modify the groundwater monitoring list to include only those constituents that appear to be temporally stable. Only certain analytes may indicate evidence of trends or seasonal fluctuations. More powerful statistical tests might be constructed on constituents that appear to be stationary. All such changes to the monitoring list and method of testing may require approval of the Regional Administrator or State Director.

5.2.6 EXPANDING INITIAL BACKGROUND SAMPLE SIZES

In the initial development of a detection monitoring statistical program under a permit or other legal mechanism, a period of review will identify the appropriate monitoring constituents. For new sites with no prior data, plans for initial background definition need to be developed as part of permit conditions. A more typical situation occurs for interim status or older facilities which have already collected substantial historical data in site monitoring wells. For the most part, the suggestions below cover ways of expanding background data sets from existing information.

Under the RCRA interim status regulations, only a single upgradient well is required as a minimum. Generally speaking, a single background well will not generate observations that are adequately representative of the underlying aquifer. A single background well draws groundwater from only one possible background location. It is accordingly not possible to determine if spatial variation is occurring in the upgradient aquifer. In addition, a single background well can only be sampled so often since measurements that are collected too frequently run the risk of being autocorrelated. Background observations collected from a single well are typically neither representative nor constitute a large enough sample to construct powerful, accurate statistical tests. One way to expand background is to install at least 3-4 upgradient wells and collect additional data under permit.

The early RCRA regulations also allowed for aliquot replicate sampling as a means of expanding background and other well sample sizes. This approach consisted of analyzing splits or aliquots of single water quality samples. As indicated in **Chapter 2**, this approach is not recommended in the guidance. Generally limited analytical variability does not adequately capture the overall variation based on independent water quality sample data, and results in incorrect estimates of variability and degrees of freedom (a function of sample size).

Existing historical groundwater well data under consideration will need to meet the assumptions discussed earlier in this chapter– independence, stationarity, etc., including using statistical methods

which can deal with outliers, spatial and temporal variation including trends. Presuming these conditions are met, it is statistically desirable to develop as large a background sample size as practical. But no matter how many measurements are utilized, a larger sample size is advantageous only if the background samples are both appropriate to the tests selected and representative of baseline conditions.

In limited situations, upgradient-to-downgradient, interwell comparisons may be determined to be appropriate using ANOVA testing of well mean differences. To ensure appropriate and representative background, other conditions may also need to be satisfied when data from separate wells are pooled. First, each background well should be screened at the same hydrostratigraphic position as other background wells. Second, the groundwater chemistry at each of these wells should be similar. This can be checked via the use of standard geochemical bar charts, pie charts, and tri-linear diagrams of the major constituent groundwater ions and cations (Hem, 1989). Third, the ***statistical*** characteristics of the background wells should be similar — that is, they should be ***spatially stationary***, with approximately the same means and variances. These conditions are particularly important for major water quality indicators, which generally reflect aquifer-specific characteristics. For infrequently detected analytes (***e.g.***, filtered trace elements like chromium, silver, and zinc), even data collected from wells from ***different*** aquifers and/or geologic strata may be statistically indistinguishable and also eligible for pooling on an interwell basis.

If a one-way ANOVA (**Chapter 13**) on the set of background wells finds significant differences in the mean levels for some constituents, and hence, evidence of spatial variability, the guidance recommends using intrawell tests. The data gathered from the background wells will generally not be used in formal statistical testing, but are still invaluable in ensuring that appropriate background is selected.[1] As indicated in the discussions above and **Chapter 13**, it may be possible to pool constituent data from a number of upgradient and/or compliance wells having a common variance when parametric assumptions allow, even if mean differences exist.

When larger historical databases are available, the data can be reviewed and diagnostically tested to determine which observations best represent natural groundwater conditions suitable for future comparisons. During this review, ***all historical well data*** collected from both upgradient and compliance wells can be evaluated for potential inclusion into background. Wells suspected of prior contamination would need to be excluded, but otherwise each uncontaminated data point adds to the overall statistical picture of background conditions at the site and can be used to enlarge the background database. Measurements can be preferentially selected to establish background samples, so long as a consistent rationale is used (e.g., newer analytical methods, substantial outliers in a portion of a data set, etc.) Changes to an aquifer over time may require selecting newer data representing current groundwater quality over earlier results even if valid.

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[1] If the spatial variation is ignored and data are pooled across wells with differing mean levels (and perhaps variances) to run an interwell parametric prediction limit or control chart test, the pooled standard deviation will tend to be substantially larger than expected. This will result in a higher critical limit for the test. Using pooled data with spatial variation will also tends to increase observed maximum values in background, leading to higher and less powerful non-parametric prediction limit tests. In either application, there will be a loss of statistical power for detecting concentration changes at individual compliance wells. Compliance wells with naturally higher mean levels will also be more frequently determined to exceed the limit than expected, while real increases at compliance wells with naturally lower means will go undetected more often.

5.2.7 REVIEW OF BACKGROUND

As mentioned above, if a large historical database is available, a critical review of the data can be undertaken to help establish initially appropriate and representative background samples. We recommend that other reviews of background also take place periodically. These include the following situations:

- When periodically updating background, say every 1-2 years (see **Section 5.3**)
- When performing a 5-10 year permit review

During these reviews, all observations designated as background should be evaluated to ensure that they still adequately reflect current natural or baseline groundwater conditions. In particular, the background samples should be investigated for apparent trends or outliers. Statistical outliers may need to be removed, especially if an error or discrepancy can be identified, so that subsequent compliance tests can be improved. If trends are indicated, a change in the statistical method or approach may be warranted (see earlier section on "Trends in Background").

If background has been updated or enlarged since the last review, and is being utilized in parametric tests, the assumption of normality (or other distributional fit) should be re-checked to ensure that the augmented background data are still consistent with a parametric approach. The presence of non-detects and multiple reporting limits (especially with changes in analytical methods over time) can prove particularly troublesome in checking distribution assumptions. The methods of **Chapters 10** "Fitting Distributions" and **Chapter 15** "Handling Non-Detects" can be consulted for guidance.

Other periodic checks of the revised background should also be conducted, especially in relation to accumulated knowledge from other sites regarding analyte concentration patterns in groundwater. The following are potential sources for comparison and evaluation:

- reliable regional groundwater data studies or investigations from nearby sites;
- published literature; EPA or other agency groundwater databases like STORET;
- knowledge of typical patterns for background inorganic constituents and trace elements. An example is found in **Table 5-1** at the end of this chapter. Typical surface and groundwater levels for filtered trace elements can also be found in the published literature (*e.g.*, Hem, 1989).

Certain common features of routine groundwater monitoring analytes summarized in **Table 5-1** have been observed in Region 8 and other background data sets, which can have implications for statistical applications. Common water quality indicators like cations and anions, pH, TDS, specific conductance are almost always measurable (detectable) and generally have limited within-well variability. These would be more amenable to parametric applications; however, these measurable analytes are also most likely to exhibit well-to-well spatial variation and various kinds of within- and between-well temporal variation including seasonal and annual trends. Many of these within-well analytes are highly correlated, and would not meet the criterion for independent data if simultaneously used as monitoring constituents.

A second level of common indicator analytes– nitrate/nitrite species, fluoride, TOC and TOX– are less frequently detected and subject to more analytical detection instability (higher and lower

detection/quantitation limits). As such, these analyte data are somewhat less reliable. There is less likelihood of temporal variation, although they can exhibit spatial well differences.

Among routinely monitored .45μ-filtered trace elements, different groups stand out. Barium is routinely detected with limited variation within most wells, but does exhibit spatial variation. Arsenic and selenium commonly occur in groundwater as oxyanions, and data can range from virtually non-detectable to always detected in different site wells. The largest group of trace elements can be considered colloidal metals (Sb, Al, Be, Cd, Cr, Co, Fe, Hg, Mn, Pb, Ni, Sn, Tl, V and Zn). While Al, Mn and Fe are more commonly detected, variability is often quite high; well-to-well spatial variability can occur at times. The remaining colloidal metals are solubility-limited in most background groundwater, generally <1 to < 10 μg/l. But even with filtration, some natural colloidal geologic solid materials can often be detected in individual samples. Since naturally occurring Al, Mn and Fe soil solid levels are much higher, the effects on measured groundwater levels are more pronounced and variable. For most of the analytically and solubility-limited colloidal metals, there may not be any discernible well spatial differences. Often these data can be characterized by a site-wide lognormal distribution, and may be possible to pool individual well data to form larger background sizes.

With unfiltered trace element data, it is more difficult to generalize even regarding background data. The method of well sample extraction and the aquifer characteristics will determine how much solids material may be present in the samples. Excessive amounts of sample solids can result in higher levels of detection but also elevated average values and variability even for solubility-limited trace elements. The effect is most clearly seen when TSS is simultaneously collected with unfiltered data. Increases are proportional to the amount of TSS and the natural background levels for trace elements in soil/solid materials. It is recommended that TSS always be simultaneously monitored with unfiltered trace elements.

Most trace organic monitoring constituents are absent or non-detectable under clean background conditions. However, with existing up-gradient sources, it is more difficult to generalize. More soluble constituents like benzene or chlorinated hydrocarbons may be amenable to parametric distributions, but changes in groundwater levels or direction can drastically affect observed levels. For sparingly soluble compounds like polynuclear aromatics (e.g., naphthalene), aquifer effects can result in highly variable data less amenable to statistical applications.

Table 5-1 was based on the use of analytical methods common in the 1990's to the present. Detectable filtered trace element data for the most part were limited by the available analytic techniques, generally SW-846 Method 6010 ICP-AES and select AA (atomic absorption) methods with lower detection limits in the 1-10 ppb range. As newer methods are incorporated (particularly Method 6020 ICP-MS capable of parts-per-trillion detection limits for trace elements), higher quantification frequencies may result in data demonstrating more complex spatial and temporal characteristics. Table 5-1 merely provides a rough guide to where various data patterns might occur. Any extension of these patterns to other facility data sets should be determined by the formal guidance tests in **Part II**.

The background database can also be specially organized and summarized to examine common behavior among related analytes (***e.g.***, filtered trace elements using ICP-AES) either over time or across wells during common sampling events. Parallel time series plots (**Chapter 9**) are very useful in this regard. Groups of related analytes can be graphed on the same set of axes, or groups of nearby wells for the same analyte. With either plot, highly suspect sampling events can be identified if a similar spike in

concentration or other unusual pattern occurs simultaneously at all the wells or in all the analytes. Analytical measurements that appear to be in error might be removed from the background database.

Cation-anion balances and other more sophisticated geochemical analysis programs can also be used to evaluate the reliability of existing water quality background data. A suite of tests like linear or non-parametric correlations, simple or non-parametric ANOVA described in later chapters offer overall methods for evaluating historical data for background suitability.

5.3 UPDATING BACKGROUND

Due both to the complex behavior of groundwater and the need for sufficiently large sample sizes, background once obtained should not be regarded as a single fixed quantity. Background should be sampled regularly throughout the life of the facility, periodically reviewed and revised as necessary. If a site uses traditional, upgradient-to-downgradient comparisons, it might seem that updating of background is conceptually simple: collect new measurements from each background well at each sampling event and add these to the overall background sample. However, significant trends or changes in one or more upgradient wells might indicate problems with individual wells, or be part of a larger site-wide groundwater change. It is worthwhile to consider the following principles for updating, whether interwell or intrawell testing is used.

5.3.1 WHEN TO UPDATE

There are no firm rules on how often to update background data. The Unified Guidance adopts the general principle that updating should occur when enough new measurements have been collected to allow a two-sample statistical comparison between the existing background data and a potential set of newer data. As mentioned in the following section, trend testing might also be used. With quarterly sampling, at least 4 to 8 new measurements should be gathered to enable such a test; this implies that updating would take place every 1-2 years. With semi-annual sampling, the same principle would call for updating every 2-3 years.

Updating should generally not occur more frequently, since adding a new observation to background every one or two sampling rounds does not allow a statistical evaluation of whether the background mean is stationary over time. Enough new data needs to be collected to ensure that a test of means (or medians in the case of non-normal data) can be conducted. Adding individual observations to background can introduce subtle trends that might go undetected and ultimately reduce the statistical power of formal monitoring tests.

Another practical aspect is that when background is updated, all statistical background limits (*e.g.*, prediction and control limits) needs to be recomputed to account for the revised background sample. At complex sites, updating the limits at each well and constituent on the monitoring list may require substantial effort. This includes resetting the cumulative sum [CUSUM] portions of control charts to zero after re-calculating the control limits and prior to additional testing against those limits. Too-frequent updating could thereby reduce the efficacy of control chart tests.

5.3.2 HOW TO UPDATE

Updating background is primarily a concern for intrawell tests, although some of the guidelines apply to interwell data. The common (generally upgradient) interwell background pool can be tested for

trends and/or changes at intervals depending on the sampling frequencies identified above. Those recently collected measurements from the background well(s) can be added to the existing pool if a Student's *t*-test or Wilcoxon rank-sum test (**Chapter 16**) finds no significant difference between the two groups at the $\alpha = 0.01$ level of significance. Individual background wells should also be evaluated in the same manner for their respective newer data. Two-sample tests of the interwell background data are conducted to gauge whether or not background groundwater conditions have changed substantially since the last update, and are *not* tests for indicating a potential release under detection monitoring. A significant *t*-test or Wilcoxon rank-sum result should spur a closer investigation and review of the background sample, in order to determine which observations are most representative of the current groundwater conditions.

With intrawell tests using prediction limits or control charts, updating is performed both to enlarge initially small well-specific background samples and to ensure that more recent compliance measurements are not already impacted by a potential release (even if not triggered by the formal detection monitoring tests). A finding of significance using the above two-sample tests means that the most recent data *should not* be added to intrawell background. However, the same caveat as above applies: these are not formal tests for determining a potential release and the existing tests and background should continue to be used.

Updating intrawell background should also not occur until at least 4 to 8 new compliance observations have been collected. Further, a potential update is predicated on there being no *statistically significant increase* [SSI] recorded for that well constituent, including since the last update. Then a *t*-test or Wilcoxon rank-sum comparison can be conducted at each compliance well between existing intrawell background and the potential set of newer background. A non-significant result implies that the newer compliance data can be re-classified as background measurements and added to the existing intrawell background sample. On the other hand, a determination of significance suggests that the compliance observations should be reviewed to determine whether a gradual trend or other change has occurred that was missed by the intervening prediction limit or control chart tests. If intrawell tests make use of a common pooled variance, the assumption of equal variance in the pooled wells should also be checked with the newer data.

Some users may wish to evaluate historical and future background data for potential trends. If plots of data versus time suggest either an overall trend in the combined data sets or distinct differences in the respective sets, linear or non-parametric trend tests covered in **Chapter 17** might be used. A determination of a significant trend might occur even if the two-sample tests are inconclusive, but individual group sample sizes should be large enough to avoid identifying a significant trend based on too few samples and perhaps randomly occurring. A trend in the newer data may reflect or depart from the historical data conditions. Some form of statistical adjustments may be necessary, but see **Section 5.3.4** below.

5.3.3 IMPACT OF RETESTING

A key question when updating intrawell background is how to handle the results of retesting.[2] If a retest confirms an SSI, background should not be updated. Rather, some regulatory action at the site should be taken. But what if an initial exceedance of a prediction or control limit is *disconfirmed* by retesting? According to the logic of retesting (**Chapter 19**), the well passes the compliance test for that evaluation and monitoring should continue as usual. But what should be done with the initial exceedance when it comes time to update background at the well?

The initial exceedance may be due to a laboratory error or other anomaly that has caused the observation to be an outlier. If so, the error should be documented and not included in the updated background sample. But if the exceedance is not explainable as an outlier or error, it may represent a portion of the background population that has heretofore not been sampled. In that case, the data value could be included in the updated background sample (along with the repeat sample) as evidence of the expanded but true range of background variation. Ultimately, it is important to characterize the background conditions at the site as completely and accurately as possible, so as to minimize both false positive and false negative decision errors in compliance testing.

The severity and classification of the initial exceedance will depend on the specific retesting strategy that has been implemented (**Chapter 19**). Using the same background data in a parametric prediction limit or control chart test, background limits are proportionately lower as the 1-of-*m* order increases (higher *m*). Thus, a 1-of-4 prediction limit will be lower than a 1-of-3 limit, and similarly the 1-of-3 limit lower than for a 1-of-2 test. An initial exceedance triggered by a 1-of-4 test limit and disconfirmed by a repeat sample, might not trigger a lower order prediction limit test. The initial sample value may represent an upper tail value from the true distribution. Retesting schemes derive much of their statistical power by allowing more frequent initial exceedances, even if some of these represent possible measurements from background. The initial and subsequent resamples *taken together* are designed to identify which initial exceedances truly represent SSIs and which do not. These tests presume that occasional excursions beyond the background limit will occur. Unless the exceedance can be documented as an outlier or other anomaly, it should probably be included in the updated intrawell background sample.

5.3.4 UPDATING WHEN TRENDS ARE APPARENT

An increasing or decreasing trend may be apparent between the existing background and the newer set of candidate background values, either using a time series plot or applying **Chapter 17** trend analyses. Should such trend data be added to the existing background sample? Most detection monitoring tests assume that background is stationary over time, with no discernible trends or seasonal variation. A mild trend will probably make very little difference, especially if a Student-*t* or Wilcoxon rank-sum test between the existing and candidate background data sets is non-significant. More severe or continuing trends are likely to be flagged as SSIs by formal intrawell prediction limit or control chart tests.

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[2] With interwell tests, the common (upgradient) background is rarely affected by retests at compliance point wells (unless the latter were included in the common pool). Should retesting fail to confirm an initial exceedance , the initial value can be reported alongside the disconfirming resamples in statistical reports for that facility.

With interwell tests, a stronger trend in the common upgradient background may signify a change in natural groundwater quality across the aquifer or an incomplete characterization of the full range of background variation. If a change is evident, it may be necessary to delete some of the earlier background values from the updated background sample, so as to ensure that compliance testing is based on current groundwater conditions and not on outdated measures of groundwater quality.

Table 5-1. Typical Background Data Patterns for Routine Groundwater Monitoring Analytes

Analyte Groups	Detection Rates		Between Well Mean Differences	Within Well Variability (CVs)	Between Well Equal Variances	Outlier Problems	Temporal Variation					Typical Distribution within well	Data Grouping
	Frequency of Detection by Well	Multiple Reporting Limiits					Between Well by Analyte	Within Well Among Group	Within Well Auto-correl.	Within Well Seasonal Variation	Within Well Time Correl.		
Inorganic Constituents and Indicators													
Major ions, pH, TDS, Specific Conductance	High to 100%		✓✓✓	Generally low (.1-.5)	✓✓	✓	✓✓	✓✓✓	✓✓	✓✓	✓✓	Normal	Intrawell
CO3, F, NO2,NO3	Some to most detectable	✓✓	✓✓	Moderate (.2-1.5)	Variable	✓✓	✓			✓	✓	Norm, Log or NPM	Intrawell/ Interwell
.45µ Filtered Trace Elements													
Ba	High to 100%	✓✓	✓✓✓	Low (.1-.5)	✓	✓	✓				✓	Normal	Intrawell
As, Se	Some wells high, others low to zero	✓✓	✓✓ (some wells)	Moderate (.2-1.5)	Variable	✓✓	✓				✓	Normal, Log or NPM	Intrawell/ Interwell
Al, Mn, Fe	Low to Moderate	✓✓	✓	Moderate to high (.3->2.0)	✓	✓✓✓	✓				✓	Log or NPM	Intrawell/ Interwell
Sb, Be, Cd, Cr, Cu, Hg, Pb, Ni, Ag, Tl, V, Zn	Zero to low	✓✓✓		Moderate to high (.5->2.0)	✓✓	✓✓✓	✓	✓✓			✓	Log or NPM	Interwell or NDC
Trace Organic and Indicator Analytes (patterns at sites with prior contamination; generally absent in clean sites)													
VOA's-BETX and Cl-Hydrocarbons	Variable, can be high	✓	Variable by site and wells			✓	Variable by site and specific wells					Normal, Log or NPM	Intrawell, Interwell or NDC
BNAs, Other Trace Organics	Generally low-mod	✓✓	" " " " "			✓	" " " "					" "	" "
Indicators: TOX, TPH, TOC, sulfide	Variable	✓✓	" " " " "			✓✓✓	" " " "					" "	" "

NPM– non-parametric methods; **NDC-** never-detected constituents

Checks: **None–** unknown, absent or infrequently occurring; ✓ – Occasionally; ✓✓ – Frequently; ✓✓✓ – Very Frequently

CHAPTER 6. DETECTION MONITORING PROGRAM DESIGN

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6.1 INTRODUCTION

This chapter addresses the *initial statistical design* of a detection monitoring program, prior to routine implementation. It considers what important elements should be specified in site permits, monitoring development plans or during periodic reviews. A good statistical design can be critically important for ensuring that the routine process of detection monitoring meets the broad objective of the RCRA regulations: using statistical testing to accurately evaluate whether or not there is a release to groundwater at one or more compliance wells.

This guidance recommends a *comprehensive* detection monitoring program design, based on two key performance characteristics: adequate *statistical power* and a low predetermined *site-wide false positive rate* [SWFPR]. The design approach presented in **Section 6.2** was developed in response to the *multiple comparisons problem* affecting RCRA and other groundwater detection programs, discussed in **Section 6.2.1**. Greater detail in applying design cumulative false positives and assessing power follows in the next three sub-sections. In **Section 6.3**, consideration is given to data features that impact proper implementation of statistical testing, such as outliers and non-detects, using interwell versus intrawell tests, as well as the presence of spatial variability or trends. **Section 6.4** provides a general discussion of specific detection testing methods listed in the regulations and their appropriate use. Finally, **Section 6.5** applies the design concepts to three hypothetical site examples.

The principles and statistical tests which this chapter covers for a detection monitoring program can also apply to compliance/corrective action monitoring when a background standard is used. Designing a background standards compliance program is discussed in **Chapter 7** (**Section 7.5**).

6.2 ELEMENTS OF THE STATISTICAL PROGRAM DESIGN

6.2.1 THE MULTIPLE COMPARISONS PROBLEM

The foremost goal in detection monitoring is to identify a real release to groundwater when it occurs. Tests must have adequate *statistical power* to identify concentration increases above background. A second critical goal is to avoid *false positive decision errors*, evaluations where one or more wells are falsely declared to be contaminated when in fact their concentration distribution is similar to background. Unfortunately, there is a trade-off (discussed in **Chapter 3**) between maximizing power and minimizing the false positive rate in designing a statistical testing protocol. The statistical power of a given test procedure using a fixed background sample size (*n*) cannot be improved without increasing the risk of false positive error (and vice-versa).

In RCRA and other groundwater detection monitoring programs, most facilities must monitor and test for multiple constituents at all compliance wells one or more times per year. A separate statistical test[1] for each monitoring constituent-compliance well pair is generally conducted semi-annually. Each additional background comparison test increases the accumulative risk of making a false positive mistake, known statistically as the ***multiple comparisons problem***.[2]

The false positive rate α (or *Type I error*) for an individual test is the probability that the test will falsely indicate an exceedance of background. Often, a single fixed low false positive error rate typically found in textbooks or regulation, e.g., $\alpha = .01$ or .05, is applied to each statistical test performed for every well-constituent pair at a facility. Applying such a common false positive rate (α) to each of several tests can result in an acceptable cumulative false positive error if the number of tests is quite small.

But as the number of tests increases, the false positive rate associated with the testing network as a whole (*i.e.*, across all well-constituent pairs) can be surprisingly high. If enough tests are run, at least one test is likely to indicate potential contamination even if a release has not occurred. As an example, if the testing network consists of 20 separate well-constituent pairs and a 99% confidence upper prediction limit is used for each test ($\alpha = .01$), the expected overall *network-wide* false positive rate is about 18%. There is nearly a 1 in 5 chance that one or more tests will *falsely* identify a release to groundwater at uncontaminated wells. For 100 tests and the same statistical procedure, the overall network-wide false positive rate increases to more than 63%, creating additional steps to verify the lack of contamination at falsely triggered wells. This cumulative false positive error is also indicative of <u>at least</u> one well constituent false positive error, but there could be more. Controlling this cumulative false positive error rate is essential in addressing the *multiple comparisons* problem.

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[1] The number of samples collected may not be the same as the number of statistical tests (e.g., a mean test based on 2 individual samples). It is the number of <u>tests</u> which affect the multiple comparisons problem.

[2] To minimize later confusion, note that the Unified Guidance applies the term "comparison" somewhat differently than most statistical literature. In statistical theory, multiple *tests* are synonymous with multiple *comparisons*, regardless of the kind of statistical test employed. But because of its emphasis on retesting and resampling techniques, the Unified Guidance uses "comparison" in referring to the evaluation of a single sample value or sample statistic against a prediction or control chart limit. In many of the procedures described in **Chapters 19** and **20**, a single statistical test will involve two or more such individual comparisons, yet all the comparisons are part of the same (individual) test.

Three main strategies (or their combination) can be used to counter the excessive cumulative false positive error rate-- 1) the number of tests can be reduced; 2) the individual test false positive rate can be lowered, or 3) the type of statistical test can be changed. A fourth strategy to increase background sample sizes may also be appropriate. Under an initial monitoring design, one usually works with fixed historical sample sizes. However, background data can later be updated in periodic program reviews.

To make use of these strategies, a sufficiently low *target* cumulative SWFPR needs to be initially identified for design purposes. The target cumulative error applies to a certain regular time period. The guidance recommends and uses a value of 10% over a year period of testing. Reasons for this particular choice are discussed in **Section 6.2.2**. These strategies have consequences for the overall test power of a well monitoring network, which are considered following control of the false positive error.

The *number of tests* depends on the number of monitoring constituents, compliance wells and periodic evaluations. Statistical testing on a regular basis can be limited to constituents shown to be *reliable* indicators of a contaminant release (discussed further in **Section 6.2.2)**. Depending on site conditions, some constituents may need to be tested only at wells for a single regulated waste unit, rather than across the entire facility well network. The frequency of evaluation is a program decision, but might be modified in certain circumstances.

Monitoring data for other parameters should still be routinely collected and reported to trace the potential arrival of new chemicals into the groundwater, whether from changes in waste management practices or degradation over time into hazardous daughter products. By limiting *statistically evaluated* constituents to the most useful indicators, the overall number of statistical tests can be reduced to help meet the SWFPR objective. Fewer tests also imply a somewhat higher single test false positive error rate, and therefore an improvement in power.

As a second strategy, the Type I error rate (α_{test}) applied to each individual test can be lowered to meet the SWFPR. Using the *Bonferroni adjustment* (Miller, 1981), the individual test error is designed to limit the overall (or *experiment-wise*) false positive rate α associated with n individual tests by conducting each individual test at an adjusted significance level of $\alpha_{test} = \alpha/n$. Computational details for this approach are provided in a later section.

A full Bonferroni adjustment strategy was neither implemented in previous guidance[3] nor allowed by regulation. However, the principle of partitioning individual test error rates to meet an overall cumulative false positive error target is highly recommended as a design element in this guidance. Because of RCRA regulatory limitations, its application is restricted to certain detection monitoring

[3] A Bonferroni adjustment was recommended in the 1989 **Interim Final Guidance** [IFG] as a *post-hoc* (*i.e.*, 'after the fact') testing strategy for individual background-to-downgradient well comparisons following an analysis of variance [ANOVA]. However, the adjustment does not always effectively limit the risks to the intended 5% false positive error for any ANOVA test. If more than 5 compliance wells are tested, RCRA regulations restrict the single test error rate to a minimum of α = 1% for each of the individual post-hoc tests following the *F*-test. This in effect raises the cumulative ANOVA test risk above 5% and considerably higher with a larger number of tested wells. At least one contaminated well would typically be needed to trigger the initial *F*-test prior to *post-hoc* testing. This fact was also noted in the 1989 IFG. Additionally, RCRA regulations mandate a minimum α error rate of 5% *per constituent* tested with this strategy. For sites with extensive monitoring parameter lists, this means a substantial risk of at least one false positive test result during any statistical evaluation.

tests-- prediction and tolerance limits along with control charts. Where not restricted by regulation, the Bonferroni approach could be used to design workable single-test or post-hoc testing for ANOVAs to meet the overall SWFPR criterion.

Using this strategy of defining individual false positive test rates to meet a cumulative error target, the effect on statistical power is direct. Given a statistical test and fixed sample size, *a lower false positive rate coincides with lower power of the test to detect contamination at the well.* Some improvement in single test power can be gained by increasing background sample sizes at a fixed test error rate. However, the third strategy of utilizing a different or modified statistical test is generally necessary.

This strategy involves choices among certain detection monitoring tests-- prediction limits, control charts and tolerance intervals-- to enhance both power and false positive error control. Except for small sites with a very limited number of tests, any of the three detection monitoring options should incorporate some manner of *retesting*. Through proper design, retesting can simultaneously achieve sufficiently high statistical power while maintaining control of the SWFPR.

RECOMMENDED GUIDANCE CRITERIA

The design of all testing strategies should specifically address the multiple comparisons problem in light of these two fundamental concerns-- an acceptably low false positive site-wide error rate and adequate power. The Unified Guidance accordingly recommends two statistical performance criteria fundamental to good design of a detection monitoring program:

1. **Application of an annual cumulative SWFPR design target, suggested at *10% per year*.**

2. **Use of *EPA reference power curves* [ERPC] to gauge the *cumulative, annual* ability of any individual test to detect contaminated groundwater when it exists. Over the course of a single year assuming normally-distributed background data, any single test performed at the site should have the ability to detect 3 and 4 standard deviation increases above background at specific power levels at least as high as the reference curves**.

False positive rates (or errors) apply both to individual tests and cumulatively to all tests conducted in some time period. Applying the SWFPR annual 10% rate places different sites and state regulatory programs on an equal footing, so that no facility is unfairly burdened by false positive test results. Use of a single overall target allows a proper comparison to be made between alternative test methods in designing a statistical program. Additional details in applying the SWFPR include the following:

- The SWFPR false positive rate should be measured on a *site-wide* basis, partitioned among the total number of annual statistical tests.

- The SWFPR applies to all statistical tests conducted in an *annual* or calendar year period.

- The total number of *annual statistical tests* used in SWFPR calculations depends on the number of valid monitoring constituents, compliance wells and evaluation periods per year. The number of tests may or may not coincide with the number of annual sampling events, for example, if data for a future mean test are collected quarterly and tested semi-annually.

- The Unified Guidance recommends a uniform approach for dealing with monitoring constituents not historically detected in background (e.g., trace organic compounds routinely analyzed in large analytical suites). It is recommended that such constituents *not* be included in SWFPR computations, and an alternate evaluation protocol be used (referred to as the Double Quantification rule) discussed in **Section 6.2.2.**

Statistical power refers to the ability of a test to identify real increases in concentration levels above background (true SSIs). The power of a test is evaluated on population characteristics and represents average behavior defined by repeated or an infinitely large number of samples. Power is reported as a fraction between 0 and 1, representing the probability that the test will identify a *specific level or degree of increase* above background. Statistical power varies with the size of the average population concentration above background-- generally fairly low power to detect small incremental concentrations and substantially increasing power at higher concentrations.

The ERPC describe the cumulative, annual statistical power to detect increasing levels of contamination above a true background mean. These curves are based on specific normal detection monitoring prediction limit tests of single future samples against background conducted once, twice, or four times in a year. Reference curve power is linked to *relative*, not absolute, concentration levels. Actual statistical test power is closely tied to the underlying variability of the concentration measurements. Since individual data set variability will differ by site, constituent, and often by well, the EPA reference power curves provide a generalized ability to estimate power by standardizing variability. By convention, all background concentration data are assumed to follow a standard normal distribution (occasionally referred to in this document as a *Z*-normal distribution) with a true mean $\mu = 0$ and standard deviation $\sigma = 1.0$. Then, increases above background are measured in increasing the k standard deviation units corresponding to $k\sigma$ mean units above baseline. When the background population can be normalized via a transformation, the same normal-based ERPC can be used without loss of generality.

Ideally, actual test power should be assessed using the original concentration data and associated variability, referred to as *effect size* power analysis. The power of any statistical test can be readily computed and compared to the appropriate reference curve, if not analytically, then by Monte Carlo simulation. But the reference power curves laid out in the Unified Guidance offer an important standard by which to judge the adequacy of groundwater statistical programs and tests. They can be universally applied to all RCRA sites and offer a uniform way to assess the environmental and health protection afforded by a particular statistical detection monitoring program.[4]

Consequently, it is recommended that design of any detection monitoring statistical program include an assessment of its ability to meet the power standards set out in the Unified Guidance. The reference power curve approach does not place an undue statistical burden on facility owners or operators, and is believed to be generally protective of human health and the environment.

[4] The ERPCs are specifically intended for comparing background to compliance data in detection monitoring. Power issues in compliance/assessment monitoring and corrective action are considered in **Chapters 7** and **22**.

Principal features of the ERPC approach include the following:

- Reference curves are based on upper 99% prediction limit tests of single future samples against background. The background sample consists of n = 10 measurements, a minimally adequate background sample size typical of RCRA applications. It is assumed that the background sample and compliance well data are normally distributed and from the same population.

- The three reference curves described below are matched to the *annual* frequency of statistical evaluations: one each for quarterly, semi-annual, and annual evaluations. The annual cumulative false positive testing error is maintained at 1%, testing 1, 2, or 4 single future samples annually against the same background. This represents the ability to identify a release to groundwater in at least one of the 1, 2 or 4 tests over the course of a year. Reporting power on an annual basis was chosen to correspond with the application of a cumulative annual SWFPR.

- In the absence of an acceptable *effect size* increase (**Section 6.2.4**), the Unified Guidance recommends that any statistical test provide at least *55-60% annual power to detecting a* 3σ (*i.e.*, 3 standard deviation) increase above the true background mean and at least *80-85% annual power for detecting increases of* 4σ. The percent power criteria change slightly for the respective reference power curves, depending on the annual frequency of statistical evaluations. For normal populations, a 3σ increase above the background average approximately corresponds to the upper 99th percentile of the background distribution, implying better than a 50% chance of detecting such an increase. Likewise, a 4σ increase corresponds to a true mean greater than the upper 99.99th percentile of the background distribution, with better than a 4-in-5 chance of detecting it.

- A single statistical test is not adequately powerful unless its power matches or betters the appropriate reference curve, at least for mean-level increases of 3 to 4 standard deviation units. The same concept can be applied to the overall detection monitoring test design. It is assumed for statistical design purposes that each individual monitoring well and constituent is of equal importance, and assigned a common test false positive error. *Effective power* then measures the overall ability of the statistical program to identify any single constituent release in any well, assuming all remaining constituents and wells are at background levels. If a number of different statistical methods are employed in a single design, effective power can be defined with respect to the *least powerful* of the methods being employed. Applying effective power in this manner would ensure that every well and constituent is evaluated with adequate statistical power to identify potential contamination, not just those where more powerful tests are applied.

- While the Unified Guidance recommends *effective power* as a general approach, other considerations may outweigh statistical thoroughness. Not all wells and constituents are necessarily of equal practical importance. Specific site circumstances may also result in some anomalous weak test power (e.g., a number of missing samples in a background data set for one or more constituents), which might be remedied by eventually increasing background size. The user needs to consider all factors including effective statistical power criteria in assessing the overall strength of a detection monitoring program.

6.2.2 SITE-WIDE FALSE POSITIVE RATES [SWFPR]

In this section, a number of considerations in developing and applying the SWFPR are provided. Following a brief discussion of SWFPR computations, the next section explains the rationale for the 10% design target SWFPR. Additional detail regarding the selection of monitoring constituents follows, and a final discussion of the Double Quantification rule for never-detected constituents is included in the last section.

For cumulative false positive error and SWFPR computations, the following approach is used. A cumulative false positive error rate α_{cum} is calculated as the probability of at least one statistically significant outcome for a total number of tests n_T in a calendar year at a single false positive error rate α_{test} using the properties of the Binomial distribution:

$$\alpha_{cum} = 1 - (1 - \alpha_{test})^{n_T}$$

By rearranging to solve for α_{test}, the 10% design SWFPR (.1) can be substituted for α_{cum} and the needed per-test false positive error rate calculated as:

$$\alpha_{test} = 1 - (.9)^{1/n_T}$$

Although these calculations are relatively straightforward and were used to develop certain κ-factor tables in the Unified Guidance (discussed in **Section 6.5** and in later chapters), a further simplification is possible using the Bonferroni approximation. This assumes that cumulative, annual SWFPR is roughly the additive sum of all the individual test errors. For low false positive rates typical of guidance application, the Bonferroni results are satisfactorily close to the Binomial formula for most design considerations.

Using this principle, the design 10% SWFPR can be partitioned among the potential annual statistical tests at a facility in a number of ways. For facilities with different annual monitoring frequencies, the SWFPR can be divided among quarterly or semi-annual period tests. Given $\alpha_{SWFPR} = .1$ and n_E evaluation periods, the quarterly cumulative false positive target rate α_E at a facility conducting quarterly testing would be $\alpha_E = \alpha_{SWFPR}/n_E = .1/4 = .025$ or 2.5% (and similarly for semi-annual testing). The total or sub-divided SWFPR can likewise be partitioned among dedicated monitoring well groupings at a multi-unit facility or among individual monitoring constituents as needed.

DEVELOPMENT AND RATIONALE FOR THE SWFPR

The existing RCRA Part 264 regulations for parametric or non-parametric analysis of variance [ANOVA] procedures mandate a Type I error of at least 1% for any individual test, and at least 5% overall. Similarly, the RCRA Part 265 regulations require a minimum 1% error for indicator parameter tests. The rationale for minimum false positive requirements is motivated by statistical power. If the Type I error is set too low, the power of the test will be unacceptably low for any given test. EPA was historically not able to specify a minimum level of acceptable power within the RCRA regulations. To do so would require specification of a minimum difference of environmental concern between the null and alternative test hypotheses. Limits on current knowledge about the health and/or environmental effects associated with incremental changes in concentration levels of Part 264 Appendix IX or Part 258 Appendix II constituents greatly complicate this task. Tests of non-hazardous or low-hazard indicators

might have different power requirements than for hazardous constituents. Therefore, minimum false positive rates were adopted for ANOVA-type procedures until more specific guidance could be recommended. EPA's main concern was adequate statistical power to detect real contamination of groundwater, and not enforcing commonly-used false positive test rates.

This emphasis is evident in §264.98(g)(6) and §258.54(c)(3) for detection monitoring and §264.99(i) and §258.55(g)(2) for compliance monitoring. Both pairs of provisions allow the owner or operator to demonstrate that any statistically significant difference between background and compliance point wells or between compliance point wells and the GWPS is an artifact caused by an error in sampling, analysis, statistical evaluation, or natural variation in groundwater chemistry. The rules clearly expect that there will be occasional false positive errors, but existing rules are silent regarding the cumulative frequency of false positives at regulated facilities.

As previously noted, it is essentially impossible to maintain a low cumulative SWFPR for moderate to large monitoring networks if the Type I errors for individual tests must be kept at or above 1%. However, the RCRA regulations do not impose similar false positive error requirements on the remaining control chart, prediction limit and tolerance interval tests. Strategies that incorporate prediction limit or control chart *retesting* can achieve very low individual test false positive rates while maintaining adequate power to detect contamination. Based on prediction limit research in the 1990's and after, it became clear that these alternative methods with suitable retesting could also control the overall cumulative false positive error rate to manageable levels.

This guidance suggests the use of an annual SWFPR of .1 or 10% as a fundamental element of overall detection monitoring design. The choice of a 10% annual SWFPR was made in light of the tradeoffs between false positive control and testing power. An <u>annual</u> period was chosen to put different sized facilities on a common footing regardless of variations in scheduled testing. It is recognized that even with such a limited error rate, the probability of false positive outcomes over a number of years (such as in the lifetime of a 5-10 year permit) will be higher. However, such relatively limited eventualities can be identified and adjusted for, since the RCRA regulations do allow for demonstration of a false positive error. State programs may choose to use a different annual rate such as 5% depending on the circumstances. But **some** predefined SWFPR in a given evaluation period is essential for designing a detection monitoring program, which can then be translated into target individual test rates for any alternative statistical testing strategy.

To implement this recommendation, a given facility should identify its yearly evaluation schedule as quarterly, semi-annual, or annual. This designation is used both to select an appropriate EPA reference power curve by which to gauge acceptable power, and to select prediction limit and control chart multipliers useful in constructing detection monitoring tests. Some of the strategies described in the Unified Guidance in later chapters require that more than one observation per compliance well be collected prior to statistical testing. ***The cumulative, annual false positive rate is linked not to the frequency of sampling but rather to the frequency of statistical evaluation***. When resamples (or verification resamples) are incorporated into a statistical procedure (**Chapter 19**), the individual resample comparisons comprise part of a single test. When a single future mean of ***m*** individual observations is evaluated against a prediction limit, this constitutes a test based on one ***mean comparison.***

NUMBER OF TESTS AND CONSTITUENTS

In designing a detection monitoring program to achieve the target SWFPR, the number of annual statistical tests to be conducted needs to be identified. This number is calculated as the number of distinct monitoring constituents × the number of compliance wells in the network × the number of annual evaluations. Five constituents and 10 well locations statistically evaluated semi-annually constitute 100 annual tests ($5 \times 10 \times 2$), since each distinct well-constituent pair represents a different statistical test that must be evaluated against their respective backgrounds. Even smaller facilities are likely to have a substantial number of such tests, each incrementally adding to the SWFPR.

While the retesting strategies outlined in **Chapters 19** and **20** can aid tremendously in limiting the SWFPR and ensure adequate statistical power, there are practical limits to meeting these goals due to the limited number of groundwater observations that can be collected and/or the number of retests which can feasibly be run. To help balance the risks of false positive and false negative errors, the number of ***statistically-tested*** monitoring parameters should be limited to constituents thought to be reliable indicators of a contaminant release.

The guidance assumes that data from large suites of trace elements and organics along with a set of inorganic water quality indicators (pH, TDS, common ions, etc.) are routinely collected as part of historical site groundwater monitoring. The number of constituents potentially available for testing can be quite large, perhaps as many as 100 different analytes. At some sites, the full monitoring lists are too large to feasibly limit the SWFPR while maintaining sufficiently high power.

Non-naturally occurring chemicals such as volatile organic compounds [VOC] and semi-volatile organic compounds [SVOC] are often viewed as excellent indicators of groundwater contamination, and are thereby included in the monitoring programs of many facilities. There is a common misperception that the greater the number of VOCs and SVOCs on the monitoring list, the greater the statistical power of the monitoring program. The reasoning is that if none of these chemicals should normally be detected in groundwater — barring a release — testing for more of them ought to improve the chances of identifying contamination.

But including a large suite of VOCs and/or SVOCs among the mix of monitoring parameters ***can be counterproductive*** to the goal of maintaining adequate ***effective power*** for the site as a whole. Because of the trade-off between statistical power and false positive rates (**Chapter 3)**, the power to detect groundwater contamination in one of these wells even with a retesting strategy in place may be fairly low unless background sample sizes are quite large. This is especially true if the regulatory authority only allows for a single retest.

Suppose 40 VOCs and certain inorganic parameters are to be tested semi-annually at 20 compliance wells totaling 1600 annual statistical tests. To maintain a 10% cumulative annual SWFPR, the per-test false positive rate would then need to be set at approximately $\alpha_{test} = .0000625$. If only 10 constituents were selected for formal testing, the per-test rate would be increased to $\alpha_{test} = .00025$. For prediction limits and other detection tests, higher false positive test rates translate to lower κ-factors and improved power.

Some means of reducing the number of tested constituents is generally necessary to design an effective detection monitoring system. Earlier discussions have already suggested one obvious first step,

by eliminating historically non-detected constituents in background from the formal list of detction monitoring constituents (discussed further in the following section). These constituents are still analyzed and informally tested, but do not count against the SWFPR.

Results of waste and leachate testing and possibly soil gas analysis should serve as the initial basis for designating constituents that are reliable leak detection indicators. Such specific constituents actually present in, or derivable from, waste or soil gas samples, should be further evaluated to determine which can be analytically detected a reasonable proportion of the time. This evaluation should include considerations of how soluble and mobile a constituent may be in the underlying aquifer. Additionally, waste or leachate concentrations should be high enough relative to the groundwater levels to allow for adequate detection. By limiting monitoring and statistical tests to fewer parameters with reasonable detection frequencies and that are significant components of the facility's waste, unnecessary statistical tests can be avoided while focusing on the reliable identification of truly contaminated groundwater.

Initial leachate testing should not serve as the sole basis for designating monitoring parameters. At many active hazardous waste facilities and solid waste landfills, the composition of the waste may change over time. Contaminants that initially were all non-detect may not remain so. Because of this possibility, the Unified Guidance recommends that the list of monitoring parameters subject to formal statistical evaluation be periodically reviewed, for example, every three to five years. Additional leachate compositional analysis and testing may be necessary, along with the measurement of constituents not on the monitoring list but of potential health or environmental concern. If previously undetected parameters are discovered in this evaluation, the permit authority should consider revising the monitoring list to reflect those analytes that will best identify potentially contaminated groundwater in the future.

Further reductions are possible in the number of constituents used for formal detection monitoring tests, even among constituents periodically or always detected. EPA's experience at hazardous waste sites and landfills across the country has shown that VOCs and SVOCs detected in a release generally occur in clusters; it is less common to detect only a single constituent at a given location. Statistically, this implies that groups of detected VOCs or SVOCs are likely to be correlated. In effect, the correlated constituents are measuring a release in similar fashion and not providing fully independent measures. At petroleum refinery sites, benzene, toluene, ethylbenzene and xylenes measured in a VOC scan are likely to be detected together Similarly at sites having releases of 1,1,1-trichloroethane, perhaps 10-12 intermediate chlorinated hydrocarbon degradation compounds can form in the aquifer over time. Finally, among water quality indicators like common ions and TDS, there is a great deal of geochemical inter-relatedness. Again, two or three indicators from each of these analyte groups may suffice as detection monitoring constituents.

The overall goal should be to select only the most reliable monitoring constituents for detection monitoring test purposes. Perhaps 10-15 constituents may be a reasonable target, depending on site-specific needs. Those analytes not selected should still continue to be collected and evaluated. In addition to using the informal test to identify previously undetected constituents described in the next section, information on the remaining constituents (e.g., VOCs, SVOCs and trace elements) can still be important in assessing groundwater conditions, including additional confirmation of a detected release.

DOUBLE QUANTIFICATION RULE

From the previous discussion, a full set of site historical monitoring parameters can be split into three distinct groups: a) those reliable indicators and hazardous constituents selected for formal detection monitoring testing and contributing to the SWFPR; b) other analytes which may be occasionally or even frequently detected and will be monitored for general groundwater quality information but not tested; and c) those meeting the "never-detected" criteria. The last group may still be of considerable interest for eventual formal testing, should site or waste management conditions change and new compounds be detected. All background measurements in the "never-detected" group should be non-detects, whether the full historical set or a subgroup considered most representative (e.g., recently collected background measurements using an improved analytical method.[5]). The following rule is suggested to provide a means of evaluating "never-detected" constituents.

The Double Quantification rule implies that statistical tests should be designed for each of the constituents in the first group. Calculations involving the SWFPR should cover these constituents, but *not* include constituents in second and the third '100% non-detect' categories. Any constituent in this third group should be evaluated by the following simple, quasi-statistical rule[6]:

> ***A confirmed exceedance is registered if any well-constituent pair in the '100% non-detect' group exhibits quantified measurements (i.e., at or above the reporting limit [RL]) in two consecutive sample and resample events.***

It is assumed when estimating an SWFPR using the Bonferroni-type adjustment, that each well-constituent test is at *equal risk* for a *specific, definable* false positive error. As a justification for this Double Quantification rule, analytical procedures involved in identifying a reported non-detect value suggest that the error risk is probably much *lower* for most chemicals analyzed as "never-detected." Reporting limits are set high enough so that if a chemical is *not present at all* in the sample, a detected amount will rarely be recorded on the lab sheet. This is particularly the case since method detection limits [MDLs] are often intended as 99% upper prediction limits on the measured signal of an uncontaminated laboratory sample. These limits are then commonly multiplied by a factor of 3 to 10 to determine the RL.

Consequently, a series of measurements for VOCs or SVOCs on samples of uncontaminated groundwater will tend to be listed as a string of non-detects with possibly a very occasional low-level detection. Because the observed measurement levels (*i.e.*, instrument signal levels) are usually known only to the chemist, an approximate prediction limit for the chemical basically has to be set at the RL. However, the true measurement distribution is likely to be clustered much more closely around zero than the RL (**Figure 6-1**), meaning that the false positive rate associated with setting the RL as the prediction

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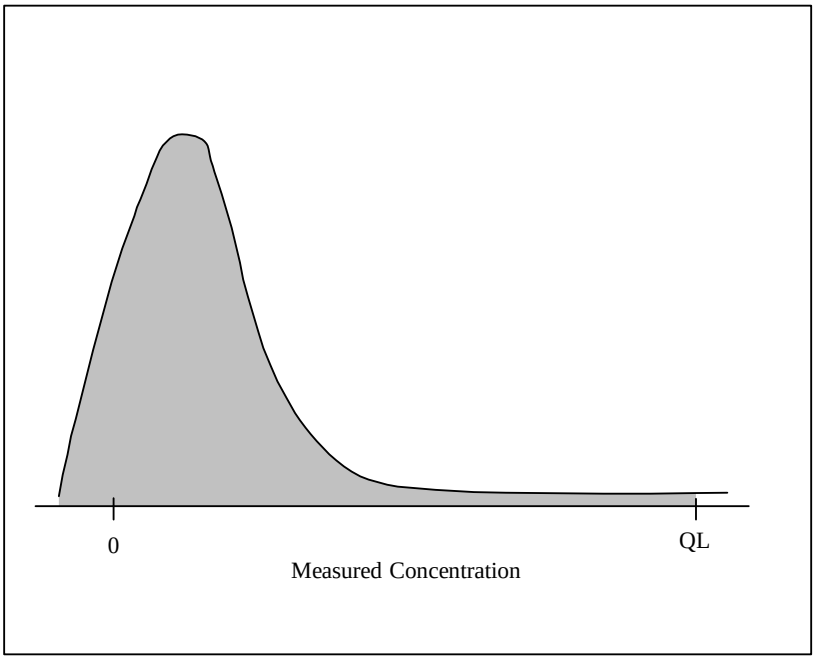
[5] Note: Early historical data for some constituents (e.g., certain filtered trace elements) may have indicated occasional and perhaps unusual detected values using older analytical techniques or elevated reporting limits. If more recent sampling exhibits no detections at lower reporting limits for a number of events, the background review discussed in **Chapter 5** may have determined that the newer, more reliable recent data should be used as background. These analytes could also be included in the '100% non-detect' group.

[6] The term "quasi-statistical" indicates that although the form is a statistical prediction limit test, only an approximate false positive error rate is implied for the reporting limit critical value. The test form follows 1-of-2 or 1-of-3 non-parametric prediction limit tests using the maximum value from a background data set (**Chapter 19**).

limit is likely already *much lower* than the Bonferroni-adjusted error rate calculated above. A similar chain of reasoning would apply to site-specific chemicals that may be on the monitoring list but have *never* been detected at the facility. Such constituents would also need a prediction limit set at the RL.

Figure 6-1. Hypothetical Distribution of Instrument Signals in Uncontaminated Groundwater



In general, there should be some minimally sufficient sample numbers to justify placing constituents in the "never-detected" category. Even such a recommendation needs to consider individual background well versus pooled well data. Depending on the number of background wells (including historical compliance well data used as background which reflect the same non-detect patterns), certain risks may have to be taken to implement this strategy. With the same total number of non-detects (e.g., 4 each in 5 wells versus 20 from a single well), the relative risk can change. Certain non-statistical judgements may be needed, such as the likelihood of particular constituents arising from the waste or waste management unit. At a minimum, we recommend that at least 6 consecutive non-detect values initially be present in each well of a pooled group, and additional background well sampling should occur to raise this number to 10-15.

Having 10-15 non-detects as a basis, a maximum worst-case probability of a future false positive exceedance under Double Quantification rule testing could be estimated. But it should be kept in mind that the true individual comparison false positive rates based on analytical considerations are likely to be considerably lower. The number of non-detect constituents evaluated under the rule will also play a role. There will be some cumulative false positive error based on the number of comparisons at some true false positive single test error or errors. Since the true false positive test rates cannot be known (and may vary considerably among analytes), it is somewhat problematic to make this cumulative false positive error estimate. Yet there is some likelihood that occasional false positive exceedances will occur under this rule.

Some flexibility will be required in evaluating such outcomes, particularly if there is doubt that a confirmed exceedance is actually due to a release from the regulated unit. In this circumstance, it might be appropriate to allow for a second resample as more definitive confirmation.

In implementing the Double Quantification rule, consideration should be given to how soon a repeat sample should be taken. Unlike detectable parameters, the question of autocorrelation is immaterial since the compound should not be present in the background aquifer. A sufficiently long interval should occur between the initial and repeat samples to minimize the possibility of a systematic analytical error. But the time interval should be short enough to avoid missing a subsequent real detection due to seasonal changes in the aquifer depth or flow direction. It is suggested that 1-2 months could be appropriate, but will depend on site-specific hydrological conditions.

Using this rule, it should be possible to construct adequately powerful prediction and control limits for naturally-occurring and detectable inorganic and organic chemicals in almost every setting. This is especially helpful at larger sites, since the total number of tests on which the per-test false positive rates (α_{test}) are based will be significantly reduced. Requiring a verified quantification for previously non-detected constituents should ensure that spurious lab results do not falsely trigger a facility into compliance/assessment monitoring, and will more reliably indicate the presence of chemicals that have heretofore not been found in background.

6.2.3 RECOMMENDATIONS FOR STATISTICAL POWER

The second but more important regulatory goal of a testing strategy is to ensure sufficient *statistical power* for detecting contaminated groundwater. Technically, in the context of groundwater monitoring, power refers to the probability that a statistical test will correctly identify a significant increase in concentration above background. Note that power is typically defined with respect to a single test, not a network of tests. In this guidance, cumulative power is assessed for a single test over an *annual* period, depending on the frequency of the evaluation. Since some testing procedures may identify contamination more readily when several wells in the network are contaminated as opposed to just one or two, the Unified Guidance recommends that all testing strategies be compared on the following more stringent common basis.

The *effective power* of a testing protocol across a network of well-constituent pairs is defined as the probability of detecting contamination in the monitoring network when *one and only one* well-constituent pair is contaminated. Effective power is a conservative measure of how a testing regimen will perform across the network, because the set of statistical tests must uncover one contaminated well among many clean ones (*i.e.*, like 'finding a needle in a haystack'). As mentioned above, this initial judgment may need to be qualified with *effect size* and other site-specific considerations.

INTRODUCTION TO POWER CURVES

Perhaps the best way to describe the power function associated with a particular testing procedure is via a graph, such as the example below of the power of a standard normal-based upper prediction limit with 99% confidence (**Figure 6-2**). The power in percent is plotted along the *y*-axis against the standardized mean level of contamination along the *x*-axis. The standardized contamination levels are presented in units of standard deviations above the *baseline* (defined as the true background mean). This

allows different power curves to be compared across constituents, wells, or well-constituent pairs. These standardized units Δ in the case of normally-distributed data may be computed as:

$$\Delta = \frac{(\text{Mean Contamination Level}) - (\text{Mean Background Level})}{(\text{SD of Background Population})} \qquad [6.1]$$

In some situations, the probability that contamination will be detected by a particular testing procedure may be difficult if not impossible to derive analytically and will have to be simulated using Monte Carlo analysis on a computer. In these cases, power is typically estimated by generating normally-distributed random values at different mean contamination levels and repeatedly simulating the test procedure. With enough repetitions a reliable *power curve* can be plotted.

In the case of the normal power curve in **Figure 6-2**, the power values were computed analytically, using properties of the *non-central t-distribution*. In particular, the statistical power of a normal 99% prediction limit for the next single future value can be calculated as

$$1-\beta = \Pr\left\{ T_{n-1}\left(\delta = \Delta \Big/ \sqrt{1+\frac{1}{n}} \right) > t_{n-1,1-\alpha} \right\} \qquad [6.2]$$

where Δ is the number of standardized (*i.e.*, standard deviation) units above the background population mean, (1–β) is the fractional power, δ is a non-centrality parameter, and:

$$T_{n-1}\left(\delta = \Delta \Big/ \sqrt{1+\frac{1}{n}} \right) \qquad [6.3]$$

represents a non-central *t*-variate with (*n*–1) degrees of freedom and non-centrality parameter δ. Equation [6.2] was used with $n = 10$ to generate **Figure 6-2**.[7]

On a general power curve, the power at Δ = 0 represents the false positive rate or *size* of the statistical test, because at that point no contamination is actually present (i.e., the background condition), even though the curve indicates how often a significant concentration increase will be detected. One should be careful to distinguish between the SWFPR across many statistical tests and the false positive rate represented on a curve measuring effective power. Since the effective power is defined as the testing procedure's ability to identify a *single* contaminated well-constituent pair, the effective power curve represents an *individual* test, *not* a network of tests. Therefore, the value of the curve at Δ = 0 will only indicate the false positive rate associated with an individual test (α_{test}), not across the network as a whole. For many of the retesting strategies discussed in **Chapters 19** and **20**, the individual per-test false positive rate will be quite small and may appear to be nearly zero on the effective power curve.

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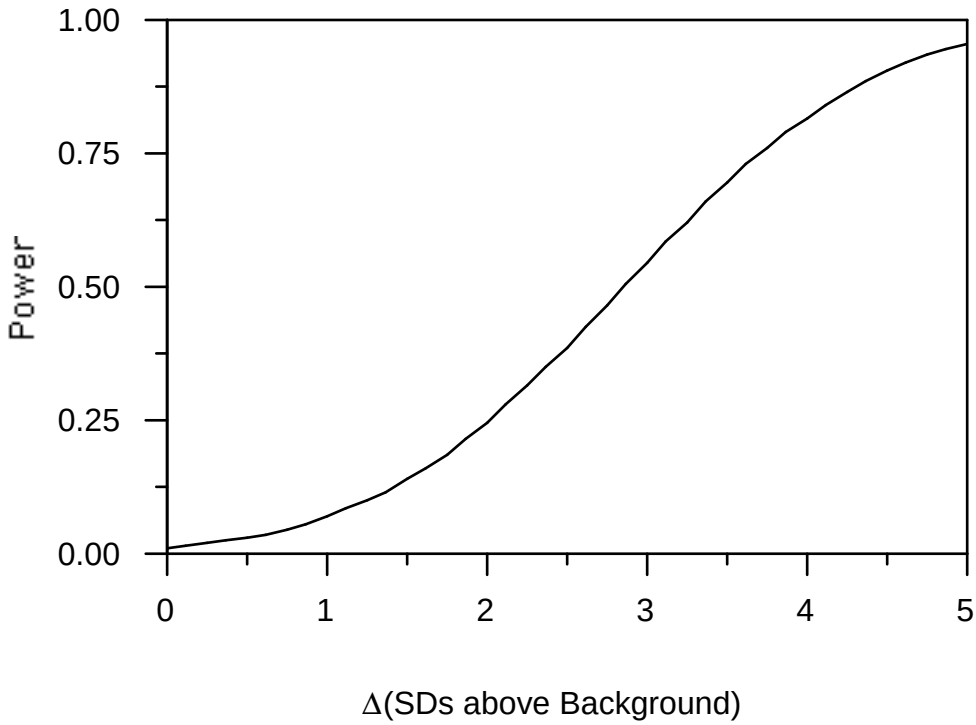
[7] For users with access to statistical software containing the non-central T-distribution, this power curve can be duplicated. For example, the Δ = 3σ fractional power can be obtained using the following inputs: a central t-value of $t_{.99,\,9} = 2.821$, 9 df, and $\delta = 3\Big/\sqrt{1+(1/10)} = 2.8604$. The fractional power is .5414. It should be noted that the software may report the

Figure 6-2. Normal Power Curve (n = 10) for 99% Prediction Limit Test



To properly interpret a power curve, note that not only is the probability greater of identifying a concentration increase above background (shown as a decimal value between 0 and 1 along the vertical axis) as the magnitude of the increase gets bigger (as measured along the horizontal axis), but one can determine the probability of identifying certain kinds of increases. For instance, with effective power equivalent to that in **Figure 6-2**, any mean concentration increase of at least 2 background standard deviations will be detected about 25% percent of the time, while an increase of 3 standard deviations will be detected with approximately 55% probability or better than 50-50 odds. A mean increase of at least 4 standard deviations will be detected with about 80% probability.

An increase of 3 or 4 standard deviations above the baseline may or may not have practical implications for human health or the environment. That will ultimately depend on site-specific factors such as the constituents being monitored, the local hydrogeologic environment, proximity to environmentally sensitive populations, and the observed variability in background concentrations. In some circumstances, more sensitive testing procedures might be warranted. As a general guide especially in the absence of direct site-specific information, the Unified Guidance recommends that when background is approximately normal in distribution,[8] any statistical test should be able to detect a 3

probability as (β) rather than (1-β). For more complex power curves involving multiple repeat samples or multiple tests, integration is necessary to generate the power estimates.

[8] If a non-parametric test is performed, power (or more technically, efficiency) is often measured by Monte Carlo simulation using normally distributed data. So these recommendations also apply to that case.

standard deviation increase at least 55-60% of the time and a 4 standard deviation increase with at least 80-85% probability.

EPA REFERENCE POWER CURVES

Since effect sizes discussed in the next section often cannot or have not been quantified, the Unified Guidance recommends using the ERPC as a suitable basis of comparison for proposed testing procedures. Each reference power curve corresponds to one of three typical yearly statistical evaluation schedules — quarterly, semi-annual, or annual — and represents the cumulative power achievable during a single year at one well-constituent pair by a 99% upper (normal) prediction limit based on n = 10 background measurements and one new measurement from the compliance well (see **Chapter 18** for discussion of normal prediction limits). The ERPC are pictured in **Figure 6-3** below.

Any proposed statistical test procedure with effective power at least as high as the appropriate ERPC, especially in the range of three or more standard deviations above the background mean, should be considered to have reasonable power.[9] In particular, if the effective power first exceeds the ERPC at a mean concentration increase no greater than 3 background standard deviations (*i.e.*, $\Delta \leq 3$), the power is labeled 'good;' if the effective power first exceeds the ERPC at a mean increase between 3 and 4 standard deviations (*i.e.*, $3 < \Delta \leq 4$), the power is considered 'acceptable;' and if the first exceedance of the ERPC does not occur until an increase greater than 4 standard deviations (*i.e.*, $\Delta > 4$), the power is considered 'low.'

With respect to the ERPCs, one should keep the following considerations in mind:

1. The effective power of any testing method applied to a groundwater monitoring network can be increased merely by relaxing the SWFPR guideline, letting the SWFPR become larger than 10%. This is why a maximum annual SWFPR of 10% is suggested as standard guidance, to ensure fair power comparisons among competing tests and to limit the overall network-wide false positive rate.

2. The ERPCs are based on *cumulative* power over a one-year period. That is, if a single well-constituent pair is contaminated at standardized level Δ during each of the yearly evaluations, the ERPC indicates the probability that a 99% upper prediction limit test will identify the groundwater as impacted during at least one of those evaluations. Because the number of evaluations not only varies by facility, but also impacts the cumulative one-year power, different reference power curves should be employed depending on a facility's evaluation schedule. Quarterly evaluators should utilize the quarterly reference power curve (Q); semi-annual evaluators the semi-annual curve (S); and annual evaluators the annual curve (A).

3. If Monte Carlo simulations are used to evaluate the power of a proposed testing method, it should incorporate every aspect of the procedure, from initial screens of the data to final

[9] When using a retesting strategy in a larger network, the false positive rate associated with a single contaminated well (used to determine the effective power) will tend to be much smaller than the targeted SWFPR. Since the point at which the effective power curve intersects $\Delta = 0$ on the standardized horizontal axis represents the false positive rate for that individual test, the effective power curve by construction will almost always be *less* than the EPA reference power curve for small concentration increases above background. Of more concern is the relative behavior of the effective power curve at larger concentration increases, say two or more standard deviations above background.

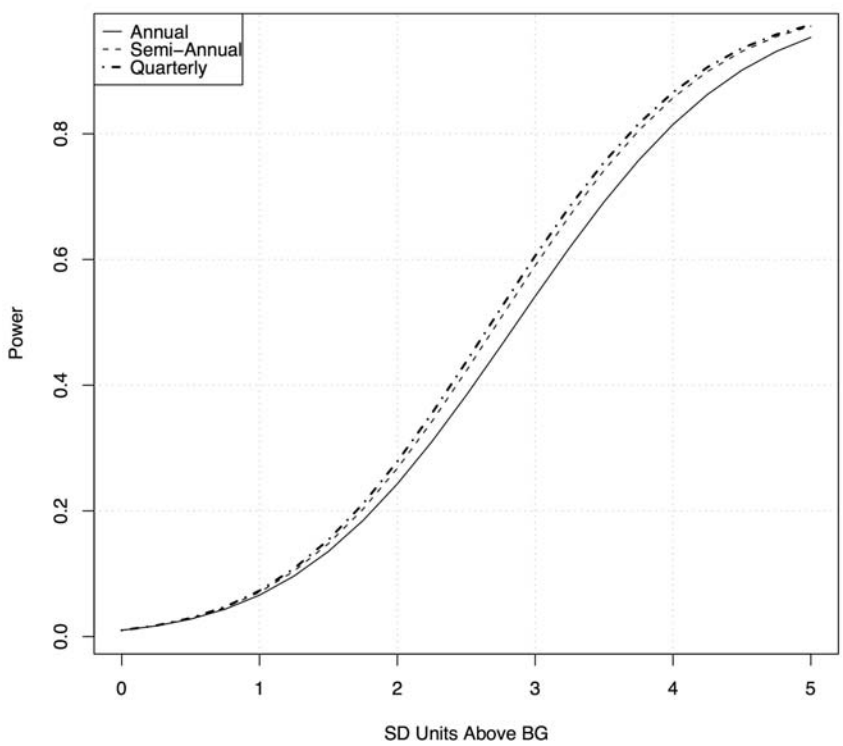
decisions concerning the presence of contamination. This is especially applicable to strategies that involve some form of retesting at potentially contaminated wells.

4. Although monitoring networks incorporate multiple well-constituent pairs, effective power can be gauged by simulating contamination in one and only one constituent at a single well.

5. The ERPCs should be considered a minimal power standard. The prediction limit test used to construct these reference curves does not incorporate retesting of any sort, and is based on evaluating a single new measurement from the contaminated well-constituent pair. In general, both retesting and/or the evaluation of multiple compliance point measurements tend to improve statistical power, so proposed tests that include such elements should be able to match the ERPC.

6. At sites employing multiple types of test procedures (*e.g.*, non-parametric prediction limits for some constituents, control charts for other constituents), effective power should be computed for each type of procedure to determine which type exhibits the least statistical power. Ensuring adequate power across the site implies that the ***least powerful*** procedure should match or exceed the appropriate ERPC, not just the most powerful procedure.

Figure 6-3. EPA Reference Power Curves



6.2.4 EFFECT SIZES AND DATA-BASED POWER CURVES

EFFECT SIZES

If site-specific or chemical-specific risk/health information is available particularly for naturally-occurring constituents, it can be used in some circumstances to develop an *effect size* of importance. An effect size (φ) is simply the smallest concentration increase above the mean background level that is presumed or known to have a measurable, deleterious impact on human health and/or the environment, or that would clearly signal the presence of contamination.

When an effect size can be quantified for a given constituent and is approved by the regulating authority, the acceptable power of the statistical test can be tailored to that amount. For instance, if an effect size for lead in groundwater at a particular site is φ = 10 ppb, one might require that the statistical procedure have an 80% or 95% chance of detecting such an increase. This would be true regardless of whether the power curve for lead at that site matches the ERPC. In some cases, an agreed-upon effect size will result in a more stringent power requirement compared to the ERPCs. In other cases, the power standard might be less stringent.

Effect sizes are not known or have not been determined for many groundwater constituents, including many inorganic parameters that have detection frequencies high enough to be amenable to effect size calculations. Because of this, many users will routinely utilize the relative power guidelines embodied in the ERPC. Even if a specific effect size cannot be determined, it is helpful to consider the site-specific and test-specific implications of a three or four standard deviation concentration increase above background. Taking the background sample mean ($\bar{x}$) as the estimated baseline, and estimating the underlying population variability by using the sample background standard deviation (*s*), one can compute the approximate actual concentrations associated with a three, four, five, *etc.* standard deviation increase above the baseline (as would be done in computing a *data-based power curve*; **Section 6.2.4**). These concentration values will only be approximate, since the true background mean (μ) and standard deviation (σ) are unknown. However, conducting this analysis can be useful in at least two ways. Each is illustrated by a simple example.

By associating the standardized units on a reference power curve with specific but approximate concentration levels, it is possible to evaluate whether the anticipated power characteristics of the chosen statistical method are adequate for the site in question. If not, another method with better power might be needed. ***Generally, it is useful to discuss and report statistical power in terms of concentration levels rather than theoretical units***.

►EXAMPLE 6-1

A potential permit GWPS for lead is 15 ppb, while natural background lead levels are normally distributed with an average of 6 ppb and a standard deviation of 2 ppb. The regulatory agency determines that a statistical test should be able to identify an exceedance of this GWPS with high power. Further assume that the power curve for a particular statistical test indicated 40% power at 3 standard deviations and 78% power at 4σ above background (a low power rating).

By comparing the actual standard deviation estimate to the required target increase φ = (15-6)/2 = 4.5 standard units, the power at the critical effect size would be 80% or higher using **Figure 6-2** as a

rough guide. This might be sufficient for monitoring needs even though the test did not meet the EPA reference criteria. Of course, the results apply only to this specific well-constituent test. ◄

For a given background sample, one can consider the regulatory and environmental impact of using *that particular background* as the basis of comparison in detection monitoring. Especially when deciding between *interwell* and *intrawell* tests at the same site, it is not unusual for the intrawell background from an individual well to exhibit much less variability than a larger set of observations pooled from multiple upgradient wells. This difference can be important since an intrawell test and an interwell test applied to the same site — *using identical relative power criteria* — might be associated with different risks to human health and the environment. A similar type of comparison might also aid in deciding whether the degrees of freedom of an intrawell test ought to be enlarged via a pooled estimate of the intrawell standard deviation (**Chapter 13**), whether a non-adjusted intrawell test is adequate, or whether more background sampling ought to be conducted prior to running intrawell tests.

►EXAMPLE 6-2

The standard deviation of an intrawell background population is σ_{intra} = 5 ppb, but that of upgradient, interwell background is σ_{inter} = 10 ppb. With the increased precision of an intrawell method, it may be possible to detect a 20 ppb increase with high probability (representing a $\Delta = 4\sigma_{intra}$ increase), while the corresponding probability for an interwell test is much lower (*i.e.*, 20 ppb = $2\sigma_{inter} = \Delta$). Of course, even if the intrawell test meets the ERPC target at four standardized units above background, consideration should be given as to whether or not 20 ppb is a meaningful increase.◄

One caveat is that calculation of either effect sizes or data-based power curves (see below) requires a reasonable estimate of the background standard deviation (σ). Such calculations may often be possible only for naturally-occurring inorganics or other constituents with fairly high detection frequencies in groundwater. Otherwise, power computations based on an effect size or the estimated standard deviation (s) are likely to be unreliable due to the presence of left-censored measurements (*i.e.*, non-detects).

A type of effect size calculation is presented in **Chapter 22** regarding methods for compliance/assessment and corrective action monitoring. A comparable effect size is computed by considering changes in mean concentration levels equal to a multiple of a fixed GWPS or clean-up/action level. While the mean level changes are multiples of the concentration limit and in that sense still relative, because they are tied to a fixed concentration standard, the power of the test can be linked to specific concentration levels.

DATA-BASED POWER CURVES

Even if basing power on a specific effect size is impractical for a given facility or constituent, it is still possible to relate power to absolute concentration levels rather than to the standardized units of the ERPC. While exact statistical power depends on the unknown population standard deviation (σ), an approximate power curve can be constructed based on the estimated background standard deviation (s). Instead of an estimate of power at a single effect size (depicted in **Example 6-1**), the actual power over a range of effect sizes can be evaluated. Such a graph is denoted in the Unified Guidance as a *data-based power curve*, a term first coined by Davis (1998).

Since the sample standard deviation (s) is calculated from actual groundwater measurements, this in turn changes an abstract power curve based on relative concentrations (*i.e.*, $k\sigma$ units above the baseline mean) into one displaying approximate, but absolute, concentrations (*i.e.*, ks units above baseline). The advantages of this approach include the following:

- Approximate data-based power curves allow the user to determine statistical power at any desired effect size (ϕ).

- Even if the effect size (ϕ) is unspecified, data-based power curves tie the performance of the statistical test back to actual concentration levels of the population being tested.

- Once the *theoretical* power curve of a particular statistical test is known, a data-based power curve is extremely easy to construct. One merely substitutes the observed background standard deviation (s) for σ and multiply by k to determine concentration values along the horizontal axis of the power curve. Even if the theoretical power curve is unknown, the same calculations can be made on the reference curve to derive an approximate site-specific, data-based power curve for tests roughly matching the performance of the ERPCs.

- If the choice between an interwell test and an intrawell approach is a difficult one (**Section 6.3.2**), helpful power comparisons can be made between intrawell and interwell tests at the same site using data-based power curves. Even if both tests meet the ERPC criteria, they may be based on different sets of background measurements, implying that the *interwell* standard deviation (s_{inter}) might differ from the *intrawell* standard deviation (s_{intra}). By plotting both data-based power curves on the same set of axes, the comparative performance of the tests can be gauged.

►EXAMPLE 6-3

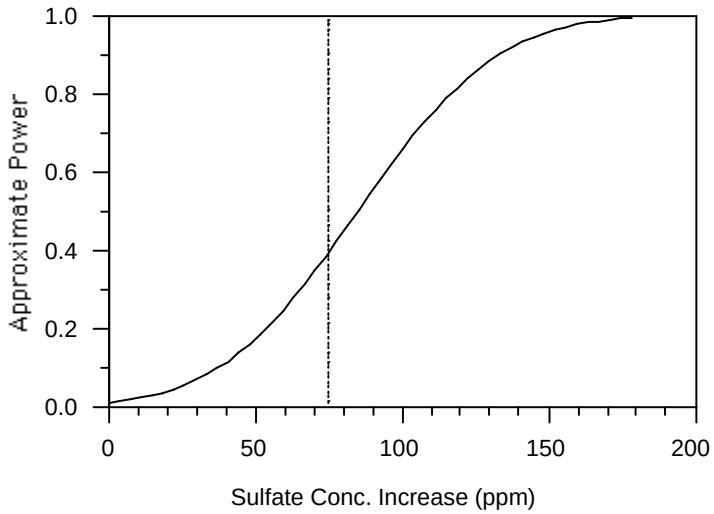
The following background sample is used to construct a test with theoretical statistical power similar to the ERPC for annual evaluations (see **Figure 6-2**). What will an approximate data-based power curve look like, and what is the approximate power for detecting a concentration increase of 75 ppm?

	Sulfate Concentrations (ppm)	
Quarter	BW-1	BW-2
1/95	560	550
4/95	530	570
7/95	568	540
10/95	490	542
1/96	510	590
Mean	545.0 ppm	
SD	29.7 ppm	

SOLUTION

The sample standard deviation of the pooled background sulfate concentrations is 29.7 ppm. Multiplying this amount by the number of standard deviations above background along the *x*-axis in **Figure 6-2** and re-plotting, the approximate data-based power curve of **Figure 6-3** can be generated. Then the statistical power for detecting an increase of 75 ppm is almost 40%.

Figure 6-3. Approximate *s*-Based Power Curve for Sulfate



Had the pooled sample size been n = 16 using the same test and sample statistics, a different and somewhat more powerful theoretical power curve would result. This theoretical curve can be generated (for a 1-of-1 prediction limit test) using the non-central T-distribution described earlier, if a user has the appropriate statistical software package. The power for a 75 ppm increase can be calculated using $\delta = 75 \big/ \sqrt{1+(1/16)} = 2.45$ and $t_{.99,\ 15}$ = 2.602, as closer to 46%. The larger background sample size makes for a more powerful test. ◄

6.2.5 SITES USING MORE THAN ONE STATISTICAL METHOD

There is no requirement that a facility apply one and only one statistical method to its groundwater monitoring program. The RCRA regulations explicitly allow for the use of multiple techniques, depending on the distributional properties of the constituents being monitored and the characteristics of the site. If some constituent data contain a high percentage of non-detect values, but others can be normalized, the statistical approach should vary by constituent.

With interwell testing, parametric prediction limits might be used with certain constituents and non-parametric prediction limits for other highly non-detect parameters. If intrawell testing is used, the most appropriate statistical technique for one constituent might differ at certain groups of wells than for others. Depending on the monitoring constituent, available individual well background, and other site-specific factors, some combination of intrawell prediction limits, control charts, and Wilcoxon rank-sum tests might come into play. At other sites, a mixture of intrawell and interwell tests might be conducted.

The Unified Guidance offers a range of possible methods which can be matched to the statistical characteristics of the observed data. The primary goal is that the statistical program should maximize the

odds of making correct judgments about groundwater quality. The guidance SWFPR and ERPC minimum power criteria serve as comprehensive guides for assessing any of the statistical methods.

One major concern is how statistical power should be compared when multiple methods are involved. Even if each method is so designed as not to exceed the recommended SWFPR, the effective power for identifying contaminated groundwater may vary considerably by technique and specific type of test. Depending on the well network and statistical characteristics of available data, a certain control chart test may or may not be as powerful as normal prediction limits. In turn, a specific non-parametric prediction limit test may be more powerful than some parametric versions. It is important that effective power be defined consistently, even at sites where more than one statistical method is employed.

The guidance encourages employing the *effective power* concept in assessing the ability of the statistical program to correctly identify and flag real concentration increases above background. As already defined, effective power is the probability that such an increase will be identified even if *only* one well-constituent pair is contaminated. Each well-constituent pair being tested should be considered equally at risk of containing a true increase above background. This also implies that the effective power of each statistical test in use should meet the criteria of the EPA reference curves. That is, the test with the *least* power should still have adequate power for identifying mean concentration increases.

The Unified Guidance does not recommend that *a single composite measure of effective power* be used to gauge a program's ability to identify potential contamination. To understand this last recommendation, consider the following hypothetical example. Two constituents exhibiting different subsurface travel times and diffusive potentials in the underlying aquifer are monitored with different statistical techniques. The constituent with the faster travel time might be measured using a test with very low effective power (compared to the ERPC), while the slower moving parameter is measured with a test having very high effective power. Averaging the separate power results into a single composite measure might result in an effective power roughly equivalent to the ERPC. Then the chances of identifying a release in a timely manner would be diminished unless rather large concentrations of the faster constituent began appearing in compliance wells. Smaller mean increases — even if 3 or 4 standard deviation units above background levels — would have little chance of being detected, while the time it took for more readily-identified levels of the slower constituent to arrive at compliance wells might be too long to be environmentally protective. Statistical power results should be reported separately, so that the effectiveness of each distinct test can be adequately judged. Further data-specific power evaluations could still be necessary to identify the appropriate test(s).

The following basic steps are recommended for assessing effective power at sites using multiple statistical methods:

1. Determine the number and assortment of distinct statistical tests. Different power characteristics may be exhibited by different statistical techniques. Specific control charts, *t*-tests, non-parametric prediction limits, *etc.* all tend to vary in their performance. The performance of a given technique is also strongly affected by the data characteristics. Background sample sizes, interwell versus intrawell choices, the number of retests and type of retesting plan, *etc.*, all affect statistical power. Each distinct data configuration and retesting plan will delineate a slightly different statistical test method.

2. Once the various methods have been identified, gauge the effective power of each.[10] Often the easiest way to measure power is via Monte Carlo simulation. Effective power involves a single well-constituent pair, so the simulation needs to incorporate only one population of background measurements representing the baseline condition and one population of compliance point measurements.

3. To run a Monte Carlo simulation, repeat the following algorithm a large number of times (*e.g.*, N = 10,000). Randomly generate a set of measurements from the background population in order to compute either a comparison limit for a control chart or some type of prediction limit test, or the background portion for a *t*-test or Wilcoxon rank-sum calculation, *etc*. Then generate compliance point samples at successively higher mean concentration levels, representing increases in standard deviation units above the baseline average. Perform each distinct test on the simulated data, recording the result of each iteration. By determining how frequently the concentration increase is identified at each successive mean level (including retests if necessary), the effective power for each distinct method can be estimated and compared.

►EXAMPLE 6-4

As a simple example of measuring effective power, consider a site using two different statistical methods. Assume that most of the constituents will be tested interwell with a 1-of-3 parametric normal prediction limit retesting plan for individual observations (**Chapter 19**). The remaining constituents having low detection rates and small well sample sizes will be tested intrawell with a Wilcoxon rank-sum test.

To measure the effective power of the normal prediction limits, note that the same number of background measurements (n = 30) is likely to be available for each of the relevant constituents. Since the per-constituent false positive rate (α_c) and the number of monitored wells (w) will also be identical for these chemicals, the same κ multiplier can be used for each prediction limit, despite the fact that the background mean and standard deviation will almost certainly vary by constituent.

Because of these identical data and well configurations, the effective power of each normal prediction limit will also be the same,[11] so that only one prediction limit test need be simulated. It is sufficient to assume the background population has a standard normal distribution. The compliance point population at the single contaminated well also has a normal distribution with the same standard deviation but a mean (μ) shifted upward to reflect successive relative concentration increases of 1 standard deviation, 2 standard deviations, 3 standard deviations, *etc*.

Simulate the power by conducting a large number of iterations (*e.g.*, N = 10,000-20,000) of the following algorithm: Generate 30 random observations from background and compute the sample mean

[10] Since power is a property of the statistical method and not linked to a specific data set, power curves are not needed for all well-constituent pairs, but only for each distinct statistical method. For instance, if intrawell prediction limits are employed to monitor barium at 10 compliance wells and the intrawell background sample size is the same for each well, only one power curve needs to be created for this group of tests.

[11] Statistical power measures the likely performance of the *technique* used to analyze the data, and is not a statement about the *data* themselves.

and standard deviation. Calculate the prediction limit by adding the background mean to κ times the background standard deviation. For a 1-of-3 retesting plan, generate 3 values from the compliance point distribution (*i.e.*, a normal distribution with unit standard deviation but mean equal to μ). If the first of these measurements does not exceed the prediction limit, record a score of zero and move on to the next iteration. If, however, the first value is an exceedance, test the second value and possibly the third. If either resample does not exceed the prediction limit, record a score of zero and move to the next iteration. But if both resamples are also exceedances, record a score of one. The fraction of iterations (N) with scores equal to one is an estimate of the effective power at a concentration level of μ standard deviations above the baseline.

In the case of the intrawell Wilcoxon rank-sum test, the power will depend on the number of intrawell background samples available at each well and for each constituent.[12] Assume for purposes of the example that all the intrawell background sizes are the same with $n = 6$ and that two new measurements will be collected at each well during the evaluation period. The power will also depend on the frequency of non-detects in the underlying groundwater population. To simulate this aspect of the distribution for each separate constituent, estimate the proportion (p) of observed non-detects across a series of wells. Then set a RL for purposes of the simulation equal to z_β, the pth quantile of the standard normal distribution.

Finally, simulate the effective power by repeating a large number of iterations of the following algorithm: Generate $n = 6$ samples from a standard normal distribution to represent intrawell background. Also generate two samples from a normal distribution with unit standard deviation and mean equal to μ to represent new compliance point measurements from a distribution with mean level equal to μ standard deviations above background. Classify any values as non-detects that fall below z_β. Then jointly rank the background and compliance values and compute the Wilcoxon rank-sum test statistic, making any necessary adjustments for ties (*e.g.*, the non-detects). If this test statistic exceeds its critical value, record a score of one for the iteration. If not, record a score of zero. Again estimate the effective power at mean concentration level μ as the proportion of iterations (N) with scores of one.

As a last step, examine the effective power for each of the two techniques. As long as the power curves of the normal prediction limit and the Wilcoxon rank-sum test *both* meet the criteria of the ERPCs, the statistical program taken as a whole should provide acceptable power. ◄

[12] Technically, since the Wilcoxon rank-sum test will often be applied to non-normal data, power will also depend fundamentally on the true underlying distribution at the compliance well. Since there may be no way to determine this distribution, approximate power is measured by assuming the underlying distribution is instead normal.

6.3 HOW KEY ASSUMPTIONS IMPACT STATISTICAL DESIGN

6.3.1 STATISTICAL INDEPENDENCE

IMPORTANCE OF INDEPENDENT, RANDOM MEASUREMENTS

Whether a facility is in detection monitoring, compliance/assessment, or corrective action, having an appropriate and valid sampling program is critical. All statistical procedures *infer* information about the underlying population from the observed sample measurements. Since these populations are only sampled a few times a year, observations should be carefully chosen to provide accurate information about the underlying population.

As discussed in **Chapter 3**, the mathematical theory behind standard statistical tests assumes that samples were *randomly* obtained from the underlying population. This is necessary to insure that the measurements are *independent* and *identically distributed* [i.i.d.]). Random sampling means that each possible concentration value in the population has an equal or known chance of being selected any time a measurement is taken. Only random sampling guarantees with sufficiently high probability that a set of measurements is adequately representative of the underlying population. It also ensures that human judgment will not bias the sample results, whether by intention or accident.

A number of factors make classical random sampling of groundwater virtually impossible. A typical small number of wells represent only a very small portion of an entire well-field. Wells are screened at specific depths and combine potentially different horizontal and vertical flow regimes. Only a minute portion of flow that passes a well is actually sampled. Sampling normally occurs at fixed schedules, not randomly.

Since a typical aquifer cannot be sampled at random, certain assumptions are made concerning the data from the available wells. It is first assumed that the selected well locations will generate concentration data similar to a randomly distributed set of wells. Secondly, it is assumed that groundwater flowing through the well screen(s) has a concentration distribution identical to the aquifer as a whole. This second assumption is unlikely to be valid unless groundwater is flowing through the aquifer at a pace fast enough and in such a way as to allow adequate mixing of the distinct water volumes over a relatively short (*e.g.*, every few months or so) period of time, so that groundwater concentrations seen at an existing well could also have been observed at other possible well locations.

Adequate sampling of aquifer concentration distributions cannot be accomplished unless enough time elapses between sampling events to allow different portions of the aquifer to pass through the well screen. Most closely-spaced sampling events will tend to exhibit a statistical dependence (*autocorrelation)*. This means that pairs of consecutive measurements taken in a series will be positively correlated, exhibiting a stronger similarity in concentration levels than expected from pairs collected at random times. This would be particularly true for overall water quality indicators which are continuous throughout an aquifer and only vary slowly with time.

Another form of statistical dependence is *spatial correlation*. Groundwater concentrations of certain constituents exhibit natural spatial variability, *i.e.*, a distribution that varies depending on the location of the sampling coordinates. Spatially variable constituents exhibit mean and occasionally

variance differences from one well to another. Pairs of spatially variable measurements collected from the same or nearby locations exhibit greater similarity than those collected from distinct, widely-spaced, or distant wells.

Natural spatial variability can result from a number of geologic and hydrological processes, including varying soil composition across an aquifer. Various geochemical, diffusion, and adsorption processes may dominate depending on the specific locations being measured. Differential flow paths can also impact the spatial distribution of contaminants in groundwater, especially if there is limited mixing of distinct groundwater volumes over the period of sampling.

An adequate groundwater monitoring sampling program needs to account for not only site-specific factors such as hydrologic characteristics, projected flow rates, and directional patterns, but also meeting data assumptions such as independence. Statistical adjustments are necessary, such as selecting intrawell comparisons for spatially distinct wells or removing autocorrelation effects in the case of time dependence.

DARCY'S EQUATION AND AUTOCORRELATION

Past EPA guidance recommended the use of Darcy's equation as a means of establishing a minimum time interval between samples. When validly applied as a basic estimate of groundwater travel time in a given aquifer, the Darcy equation ensures that separate volumes of groundwater are being sampled (***i.e.***, physical independence). This increases the probability that the samples will also be statistically independent.

The Unified Guidance in **Chapter 14** also includes a discussion on applying Darcy's equation. Caution is advised in its use, however, since Darcy's equation *cannot guarantee* temporal independence. Groundwater travel time is only one factor that can influence the temporal pattern of aquifer constituents. The measurement process itself can affect time related dependency. An imprecise analytical method might impart enough additional variability to make the measurements essentially uncorrelated even in a short sampling interval. Changes in analytical methods or laboratories and even periodic re-calibration of analytical instrumentation can impart time-related dependencies in a data set regardless of the time intervals between samples.

The overriding interest is in the behavior of chemical contaminants in groundwater, not the groundwater itself. Many chemical compounds do not travel at the same velocity as groundwater. Chemical characteristics such as adsorptive potential, specific gravity, and molecular size can influence the way chemicals move in the subsurface. Large molecules, for example, will tend to travel slower than the average linear velocity of groundwater because of matrix interactions. Compounds that exhibit a strong adsorptive potential will undergo a similar fate, dramatically changing time of travel predictions using the Darcy equation. In some cases, chemical interaction with the matrix material will alter the matrix structure and its associated hydraulic conductivity and may result in an increase in contaminant mobility. This last effect has been observed, for instance, with certain organic solvents in clay units (see Brown and Andersen, 1981).

The Darcy equation is also not valid in turbulent and non-linear laminar flow regimes. Examples of these particular hydrological environments include karst and 'pseudo-karst' (***e.g.***, cavernous basalt and extensively fractured rock) formations. Specialized methods have been investigated by Quinlan (1989)

for developing alternative monitoring procedures. Dye tracing as described by Quinlan (1989) and Mull, *et al*. (1988) can be useful for identifying flow paths and travel times in these two particular environments; conventional groundwater monitoring wells are often of little value in designing an effective monitoring system in these type of environments.

Thus, we suggest that Darcy's equation not be exclusively relied upon to gauge statistical sampling frequency. At many sites, quarterly or semi-annual sampling often provides a reasonable balance between maintaining statistical independence among observations yet enabling early detection of groundwater problems. The Unified Guidance recommends three tools to explore or test for time-related dependence among groundwater measurements. Time series plots (**Chapter 9**) can be constructed on multiple wells to examine whether there is a time-related dependence in the pattern of concentrations. Parallel traces on such a plot may indicate correlation across wells as part of a natural temporal, seasonal or induced laboratory effect. For longer data series, direct estimates of the autocorrelation in a series of measurements from a single well can be made using either the *sample autocorrelation function* or the *rank von Neumann ratio* (**Section 14.2**).

DATA MIXTURES INCLUDING ALIQUOT REPLICATE SAMPLES

Some facility data sets may contain both single and aliquot replicate groundwater measurements such as duplicate splits. An entire data set may also consist of aliquot replicates from a number of independent water quality samples. The guidance recommends against using aliquot data directly in detection monitoring tests, since they are almost *never* statistically independent. Significant positive correlation almost always exists between such duplicate samples or among aliquot sets. However, it is still possible to utilize some of the aliquot information within a larger water quality data set.

Lab duplicates and field splits can provide valuable information about the level of measurement variability attributable to sampling and/or analytical techniques. However, to use them as separate observations in a prediction limit, control chart, analysis of variance [ANOVA] or other procedure, the test must be specially structured to account for multiple data values per sampling event.

Barring the use of these more complicated methods, one suggested strategy has been to simply average each set of field splits and lab duplicates and treat the resulting mean as a single observation in the overall data set. Despite eliminating the dependence between field splits and/or lab duplicates, *such averaging is not an ideal solution*. The variability in means of two correlated measurements is approximately 30% less than the variability associated with two single independent measurements. If a data set consists of a mixture of single measurements and lab duplicates and/or field splits, the variability of the averaged values will be less than the variability of the single measurements. This would imply that the final data set is not *identically* distributed.

When data are not identically distributed, the actual false positive and false negative rates of statistical tests may be higher or lower than expected. The effect of mixing single measurements and averaged aliquot replicates might be balanced out in a two-sample t-test if sample sizes are roughly equal. However, the impact of non-identically distributed data can be substantial for an upper prediction limit test of a future single sample where the background sample includes a mixture of aliquot replicates and single measurements. Background variability will be underestimated, resulting in a lowered prediction limit and a higher false positive rate.

One statistically defensible but expensive approach is to perform the same number of aliquot replicate measurements on all physical samples collected from background and compliance wells. Aliquot replicates can be averaged, and the same variance reduction will occur in all the final observations. The statistical test degrees of freedom, however, are based on the number of independent, averaged samples.

Mixing single and averaged aliquot data is a serious problem if the component of variability due to field sampling methods and laboratory measurement error is a substantial fraction of the overall sample variance. When natural variability in groundwater concentrations is the largest component, averaging aliquot replicate measurements will do little to weaken the assumption of identically-distributed data. Even when variability due to sampling and analytical methods is a large component of the total variance, if the percentage of samples with aliquot replicate measurements is fairly small (say, 10% or less), the impact of aliquot replicate averaging should usually be negligible. However, consultation with a professional statistician is recommended.

The simplest alternative is to randomly select one value from each aliquot replicate set along with all non-replicate individual measurements, for use in statistical testing. Either this approach or the averaged replicate method described above will result in smaller degrees of freedom than the strategy of using all the aliquots, and will more accurately reflect the statistical properties of the data.

CORRECTING FOR TEMPORAL CORRELATION

The Unified Guidance recommends two general methods to correct for observable temporal correlation. Darcy's equation is mentioned above as a rough guide to physical independence of consecutive groundwater observations. A more generally applicable strategy for yet-to-be-collected measurements involves adjusting the sampling frequency to avoid autocorrelation in consecutive sampling events. Where autocorrelation is a serious concern, the Unified Guidance recommends running a *pilot study* at two or three wells and analyzing the study data by using the sample autocorrelation function (**Section 14.3.1**). The autocorrelation function plots the strength of correlation between consecutive measurements against the time lag between sampling events. When the autocorrelation becomes insignificantly different from zero at a particular sampling interval, the corresponding sampling frequency is the maximum that will ensure uncorrelated sampling events.

Two other strategies are recommended for adjusting already collected data. First, a longer data series at a single well can be corrected for seasonality by estimating and removing the seasonal trend (**Section 14.3.3**). If both a linear trend *and* seasonal fluctuations are evident, the seasonal Mann-Kendall trend test can be run to identify the trend despite the seasonal effects (**Section 14.3.4**). A second strategy is for sites where a temporal effect (*e.g.*, temporal dependence, seasonality) is apparent across multiple wells. This involves estimating a temporal effect via a *one-way* ANOVA and then creating adjusted measurements using the ANOVA residuals (**Section 14.3.3**). The adjusted data can then be utilized in subsequent statistical procedures.

6.3.2 SPATIAL VARIATION: INTERWELL VS. INTRAWELL TESTING

ASSUMPTIONS IN BACKGROUND-TO-DOWNGRADIENT COMPARISONS

The RCRA groundwater monitoring regulations initially presume that detection monitoring background can be defined on the basis of a definable groundwater gradient. In a considerable number of situations, this approach is problematic. No groundwater gradient may be measurable for identifying upgradient and downgradient well locations around a regulated unit. The hydraulic gradient may change in direction, depth or magnitude due to seasonal fluctuations. Groundwater mounding or other flow anomalies can occur. At most locations, significant spatial variability among wells exists for certain constituents. Where spatial variation is a natural artifact of the site-specific geochemistry, differences between upgradient and downgradient wells are unrelated to on-site waste management practices.

Both the Subtitle C and Subtitle D RCRA regulations allow for a determination that background quality may include sampling of wells not hydraulically upgradient of the waste management area. The rules recognize that this can occur either when hydrological information is unable to indicate which wells are hydraulically upgradient or when sampling other wells will be "representative or more representative than that provided by the upgradient wells."

For upgradient-to-downgradient well comparisons, a crucial detection monitoring assumption is that downgradient well changes in groundwater quality are only caused by on-site waste management activity. Up- and down-gradient well measurements are also assumed to be comparable and equal on average unless some waste-related change occurs. If other factors trigger significant increases in downgradient well locations, it may be very difficult to pinpoint the monitored unit as the source or cause of the contaminated groundwater.

Several other critical assumptions apply to the interwell approach. It is assumed that the upgradient and downgradient well samples are drawn from the same aquifer and that wells are screened at essentially the same hydrostratigraphic position. At some sites, more than one aquifer underlies the waste site or landfill, separated by confining layers of clay or other less permeable material. The fate and transport characteristics of groundwater contaminants likely will differ in each aquifer, resulting in unique concentration patterns. Consequently, upgradient and downgradient observations may not be comparable (*i.e.*, drawn from the same statistical population).

Another assumption is that groundwater flows in a definable pathway from upgradient to downgradient wells beneath the regulated unit. If flow paths are incorrectly determined or this does not occur, statistical comparisons can be invalidated. For example, a real release may be occurring at a site known to have groundwater mounding beneath the monitored unit. Since the groundwater may move towards both the downgradient and upgradient wells, it may not be possible to detect the release if both sets of wells become equally or similarly contaminated. One exception to this might occur if certain analytes are shown to exhibit uniform behavior in both historical upgradient and downgradient wells (e.g., certain infrequently detected trace elements). As long as the flow pathway from the unit to the

downgradient wells is assured, then an interwell test based on this combined background could still reflect a real exceedance in the downgradient wells.[13]

Groundwater flow should also move at a sufficient velocity beneath the site, so that the same groundwater observed at upgradient well locations is subsequently monitored at downgradient wells in the course of an evaluation period (*e.g.*, six months or a year). If groundwater flow is much slower, measurements from upgradient and downgradient wells may be more akin to samples from two separate aquifers. Extraneous factors may separately influence the downgradient and background populations, confusing the determination of whether or not a release has occurred.

While statistical testing can determine whether there are significant differences between upgradient and downgradient well measurements, it cannot determine ***why*** such differences exist. That is primarily the concern of a hydrologist who has carefully reviewed site-specific factors. Downgradient concentrations may be greater than background because contamination of the underlying aquifer has occurred. The increase may be due to other factors, including spatially variable concentration levels attributable to changing soil composition and geochemistry from one well location to another. It could also be due to the migration of contaminants from off-site sources reaching downgradient wells. These and other factors (including those summarized in **Chapter 4** on SSI Increases) should be considered before deciding that statistically significant background-to-downgradient differences represent site-related contamination.

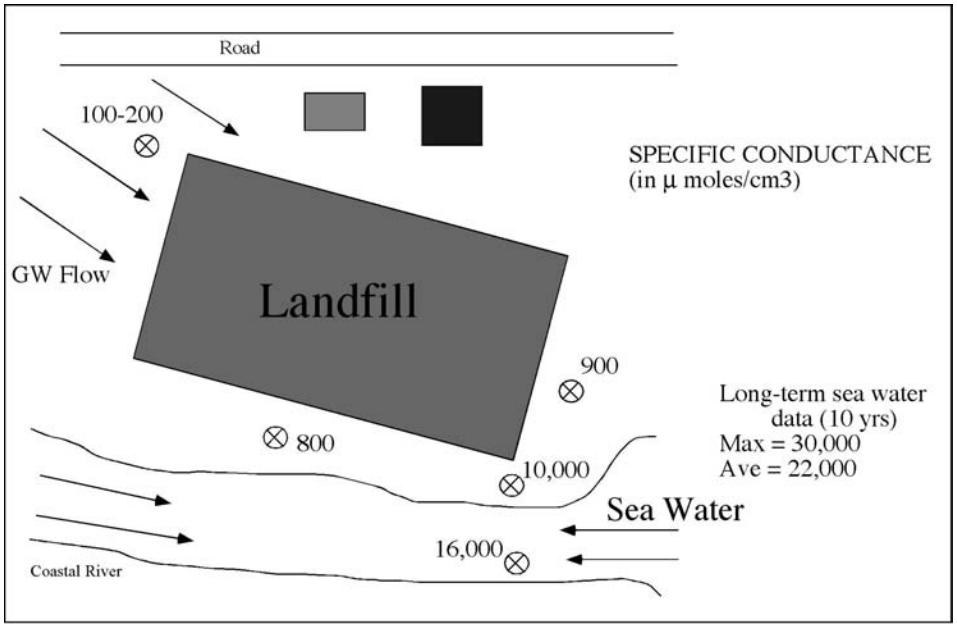
An example of how background-to-downgradient well differences can be misleading is illustrated in **Figure 6-4** below. At this Eastern coastal site, a Subtitle D landfill was located just off a coastal river emptying into the Atlantic Ocean a short distance downstream. Tests of specific conductance measurements comparing the single upgradient well to downgradient well data indicated significant increases at all downgradient wells, with one well indicating levels more than an order of magnitude higher than background concentrations.

Based on this analysis, it was initially concluded that waste management activities at the landfill had impacted groundwater. However, further hydrologic investigation showed that nearby river water also exhibited elevated levels of specific conductance, even higher than measurements at the downgradient wells. Tidal fluctuations and changes in river discharge caused sea water to periodically mix with the coastal river water at a location near the downgradient wells. Mixed river and sea water apparently seeped into the aquifer, impacting downgradient wells but not at the upgradient location. An off-site source as opposed to the landfill itself was likely responsible for the observed elevations in specific conductance. Without this additional hydrological information, the naive statistical comparison between upgradient and downgradient wells would have reached an incorrect conclusion.

[13] The same would be true of the "never-detected" constituent comparison, which does not depend on the overall flow pathway from upgradient to downgradient wells.

i

Figure 6-4. Landfill Site Configuration



TRADEOFFS IN INTERWELL AND INTRAWELL APPROACHES

The choice between interwell and intrawell testing primarily depends on the statistical characteristics of individual constituent data behavior in background wells. It is presumed that a thorough background study described in **Chapter 5** has been completed. This involves selecting the constituents deemed appropriate for detection monitoring, identifying distributional characteristics, and evaluating the constituent data for trends, stationarity, and mean spatial variability among wells. ANOVA tests can be used to assess both well mean spatial variability and the potential for pooled-variance estimates if an intrawell approach is needed.

As discussed in **Chapter 5**, certain classes of potential monitoring constituents are more likely to exhibit *spatial variation*. Water quality indicator parameters are quite frequently spatially variable. Some authors, notably Davis and McNichols (1994) and Gibbons (1994a), have suggested that significant spatial variation is a nearly ubiquitous feature at RCRA-regulated landfills and hazardous waste sites, thus invalidating the use of interwell test methods. The Unified Guidance accepts that interwell tests still have an important role in groundwater monitoring, particularly for certain classes of constituents like non-naturally occurring VOCs and some trace elements. Many sites may best be served by a statistical program which combines interwell and intrawell procedures.

Intrawell testing is an appropriate and recommended alternative strategy for many constituents. Well-specific backgrounds afford intrawell tests certain advantages over the interwell approach. One key advantage is *confounding results due to spatial variability* are eliminated, since all data used in an intrawell test are obtained from a single location. If natural background levels change substantially from

one well to the next, intrawell background provides the most accurate baseline for use in statistical comparisons.

At times, the *variability* in a set of upgradient background measurements pooled from multiple wells can be larger than the variation in individual intrawell background wells. Particularly if not checked with ANOVA well mean testing, interwell variability could substantially increase if changes in mean levels from one location to the next are also incorporated. While pooling should not occur among well means determined to be significantly different using ANOVA, a more likely situation is that pooled well true means and variance may be slightly different at each well. The ANOVA test might still conclude that the mean differences were insignificant and satisfy the equal variance assumption. The net result (as explained below) is that intrawell tests can be more statistically powerful than comparable interwell tests using upgradient background, despite employing a smaller background sample size.

Another advantage using intrawell background is that a *reasonable baseline* for tests of future observations can be established at historically contaminated wells. In this case, the intrawell background can be used to track the onset of even more extensive contamination in the future. Some compliance monitoring wells exhibit chronic elevated contaminant levels (e.g., arsenic) considerably above other site wells which may not be clearly attributed to a regulated unit release. The regulatory agency has the option of continuing detection monitoring or changing to compliance/corrective action monitoring. Unless the agency has already determined that the pre-existing contamination is subject to compliance monitoring or remedial action under RCRA, the detection monitoring option would be to test for recent or future concentration increases above the historical contamination levels by using intrawell background as a well-specific baseline.

Intrawell tests are not preferable for all groundwater monitoring scenarios. It may be unclear whether a given compliance well was historically contaminated prior to being regulated or more recently contaminated. Using intrawell background to set a baseline of comparison may ignore recent contamination subject to compliance testing and/or remedial action. Even more contamination in the future would then be required to trigger a statistically significant increase [SSI] using the intrawell test. The Unified Guidance recommends the use of intrawell testing only when it is clear that spatial variability is not the result of recent contamination attributable to the regulated unit.

A second concern is that intrawell tests typically utilize a smaller set of background data than interwell methods. Since statistical power depends significantly on background *sample size*, it may be more difficult to achieve comparable statistical power with intrawell tests than with interwell methods. For the latter, background data can be collected from multiple wells when appropriate, forming a larger pool of measurements than would be available at a single well. However, it may also be possible to enhance intrawell sample sizes for parametric tests using the pooled- variance approach.

Traditional interwell tests can be appropriate for certain constituents if the hydraulic assumptions discussed earlier are verified and there is no evidence of significant spatial variability. Background data from other historical compliance wells not significantly different from upgradient wells using ANOVA may also be used in some cases. When these conditions are met, interwell tests can be preferable as generally more powerful tests. Upgradient groundwater quality can then be more easily monitored in

parallel to downgradient locations. Such upgradient monitoring can signal changes in natural in-situ concentrations or possible migration from off-site sources. [14]

For most situations, the background constituent data patterns will determine which option is most feasible. Clear indications of spatially distinct well means through ANOVA testing will necessitate some form of intrawell methods. Further choices are then which type of statistical testing will provide the best power.

It may be possible to increase the *effective sample size* associated with a series of intrawell tests. As explained in **Chapters 13 & 19**, the κ-multipliers for intrawell prediction limits primarily depend on the number of background measurements used to estimate the standard deviation. It is first necessary to determine that the intrawell background in a series of compliance wells is both uncontaminated and exhibits similar levels of variability from well to well. Background data from these wells can then be combined to form a *pooled* intrawell standard deviation estimate with larger degrees of freedom, even though individual well means vary. A transformation may be needed to stabilize the well-to-well variances. If one or more of the compliance wells is already contaminated, these should not be mixed with uncontaminated well data in obtaining the pooled standard deviation estimate.

A site-wide constituent pattern of no significant spatial variation will generally favor the interwell testing approach. But given the potential for hydrological and other issues discussed above, further evaluation of intrawell methods may be appropriate. **Example 6-2** provided an illustration of a specific intrawell constituent having a lower absolute standard deviation than an interwell pooled data set, and hence greater relative and absolute power. In making such an interwell-intrawell comparison, the specific test and all necessary design inputs must be considered. Even if a given intrawell data set has a low background standard deviation compared to an interwell counterpart, the advantage in absolute terms over the relative power approach will change with differing design inputs. The simplest way to determine if the intrawell approach might be advantageous is to calculate the actual background limits of a potential test using existing intra- and inter-well data sets. In a given prediction limit test, for example, the actual lower limit will determine the more powerful test.

If desired, approximate data-based power curves (**Section 6.2.4**) can be constructed to evaluate absolute power over a range of concentration level increases. In practice, the method for comparing interwell versus intrawell testing strategies with the same well-constituent pair involves the following basic steps:

1. Given the interwell background sample size (n_{inter}), the statistical test method (including any retesting), and the individual per-test α for that well-constituent pair, compute or simulate the relative power of the test at multiples of ks_{inter} above the baseline mean level. Let k range from 0 to 5 in increments of 0.5, where the interwell population standard deviation (σ_{inter}) has been replaced by the sample background standard deviation (s_{inter}).

[14] The same can be accomplished via intrawell methods if upgradient wells continue to be sampled along with required compliance well locations. Continued tracking of upgradient background groundwater quality is recommended regardless of the testing strategy.

2. Repeat Step 1 for the intrawell test. Use the intrawell background sample size (n_{intra}), statistical test method, background sample standard deviation (s_{intra}), and the same individual per-test α to generate a relative power curve.

3. On the same graph, plot overlays of the estimated data-based interwell and intrawell power curves (as discussed in **Section 6.2.4**). Use the same range of (absolute, not relative) concentration increases over baseline along the horizontal axis.

4. Visually inspect the data-based power curves to determine which method offers better power over a wider range of possible concentration increases.

The Unified Guidance recommends that users apply the most powerful statistical methods available in detecting and identifying contaminant releases for each well-constituent pair. The ERPC identifies a minimum acceptable standard for judging the relative power of particular tests. However, more powerful methods based on absolute power may be considered preferable in certain circumstances.

As a final concern, very small individual well samples in the early stages of a monitoring program may make it difficult to utilize an intrawell method having both sufficient statistical power and meeting false positive design criteria. One option would be to temporarily defer tests on those well-constituent pairs until additional background observations can be collected. A second option is to use the intrawell approach despite its inadequate power, until the intrawell background is sufficiently large via periodic updates (**Chapter 5**). A third option might be to use a more powerful intrawell test (e.g., a higher order 1-of-*m* parametric or non-parametric prediction limit test). Once background is increased, a lower order test might suffice. Depending on the type of tests, some control of power may be lost (parametric) or the false positive (non-parametric tests). These tradeoffs are considered more fully in **Chapter 19**. For the first two options and the parametric test under the third option, there is some added risk that a release occurring during the period of additional data collection might be missed. For the non-parametric test under the third option, there is an increased risk of a true false positive error. Any of these options might be included as special permit conditions.

6.3.3 OUTLIERS

Evaluation of outliers should begin with historical upgradient and possibly compliance well data considered for defining initial background, as described in **Chapter 5, Section 5.2.3**. The key goal is to select the data most representative of near-term and likely future background. Potentially discrepant or unusual values can occur for many reasons including 1) a contaminant release that significantly impacts measurements at compliance wells; 2) true but extreme background groundwater measurements, 3) inconsistent sampling or analytical chemistry methodology resulting in laboratory contamination or other anomalies; and 4) errors in the transcription of data values or decimal points. While the first two conditions may appear to be discrepant values, they would not be considered outliers.

When appraising extensive background data sets with long periods of acquisition and somewhat uncertain quality, it is recommended that a formal statistical evaluation of outliers not be conducted until a thorough review of data quality (errors, etc.) has been performed. Changes in analytical methodologies, the presence of sample interferences or dilutions can affect the historical data record. Past and current treatment of non-detects should also be investigated, including whether there are multiple reporting limits in the data base. Left-censored values can impact whether or not the sample

appears normal (**Chapter 15**), especially if the data need to be normalized via a transformation. Techniques for evaluating censored data should be considered, especially those which can properly account for multiple RLs. Censored probability plots (**Chapter 15**) or quasi-nonparametric box plots (**Chapter 12**) adapted by John Tukey (1977) can be used as methods to screen for outliers.

The guidance also recommends that statistical testing of potential outliers also be performed on initial background data, including historical compliance well data potentially considered as additional background data. Recognizing the potential risks as discussed in **Chapter 5**, removal of significant outliers may be appropriate even if no probable error or discrepancy can be firmly identified. The risk is that high values registering as statistical outliers may reflect an extreme, but real value from the background population rather than a true outlier, thereby increasing the likelihood of a false positive error. But the effect of removing outliers from the background data will usually be to improve the odds of detecting upward changes in concentration levels at compliance wells, and thus providing further protection of human health and the environment. *Automated screening and removal* of background data for statistical outliers is not recommended without some consideration of the likelihood of an outlier error.

A *statistical outlier* is defined as a value originating from a different statistical population than the rest of the sample. Outliers or observations not derived from the same population as the rest of the sample violate the basic statistical assumption of identically-distributed measurements. If an outlier is suspected, an initial helpful step is to construct a probability plot of the ordered sample data versus the standardized normal distribution (**Chapter 12**). A probability plot is designed to judge whether the sample data are consistent with a normal population model. If the data can be normalized, a probability plot of the transformed observations should also be constructed. Neither is a formal test, but can still provide important visual evidence as to whether the suspected outlier(s) should be further evaluated.

Formal testing for outliers should be done only if an observation seems particularly high compared to the rest of the sample. The data can be evaluated with either Dixon's or Rosner's tests (**Chapter 12**). These outlier tests assume that the rest of the data except for the suspect observation(s), are normally-distributed (Barnett and Lewis, 1994). It is recommended that tests also be conducted on transformed data, if the original data indicates one or more potential outliers. Lognormal and other skewed distributions can exhibit apparently elevated values in the original concentration domain, but still be statistically indistinguishable when normalized via a transformation. If the latter is the case, the outlier should be retained and the data set treated as fitting the transformed distribution.

Future background and compliance well data may also be periodically tested for outliers. However, removal of outliers should only take place under certain conditions, since a true elevated value may fit the pattern of a release or a change in historical background conditions. If either Dixon's or Rosner's test identifies an observation as a statistical outlier, the measurement should not be treated as such *until* a specific physical reason for the abnormal value can be determined. Valid reasons might include contaminated sampling equipment, laboratory contamination of the sample, errors in transcription of the data values, etc. Records documenting the sampling and analysis of the measurement (*i.e.*, the "chain of custody") should be thoroughly investigated. Based on this review, one of several actions might be taken as a general rule:

- If an error in transcription, dilution, analytical procedure, *etc.* can be identified and the correct value recovered, the observation should be replaced by its corrected value and further statistical analysis done with the corrected value.

- If it can shown that the observation is in error but the correct value cannot be determined, the observation should be removed from the data set and further statistical analysis performed on the reduced data set. The fact that the observation was removed and the reason for its removal should be documented when reporting results of the analysis.

- If no error in the value can be documented, it should be assumed that the observation is a true but extreme value. In this case, it should not be altered or removed. However, it may helpful to obtain another observation in order to verify or confirm the initial measurement.

6.3.4 NON-DETECTS

Statistically, non-detects are considered 'left-censored' measurements because the concentration of any non-detect is known or assumed only to fall within a certain range of concentration values (*e.g.*, between 0 and the RL). The direct estimate has been censored by limitations of the measurement process or analytical technique.

As noted, non-detect values can affect evaluations of potential outliers. Non-detects and detection frequency also impact what detection monitoring tests are appropriate for a given constituent. A low detection frequency makes it difficult to implement parametric statistical tests, since it may not be possible to determine if the underlying population is normal or can be normalized. Higher detection frequencies offer more options, including *simple substitution* or estimating the mean and standard deviation of samples containing non-detects by means of a *censored estimation technique* (**Chapter 15**).

Estimates of the background mean and standard deviation are needed to construct parametric prediction and control chart limits, as well as confidence intervals. If simple substitution is appropriate, imputed values for individual non-detects can be used as an alternate way to construct mean and standard deviation estimates. These estimates are also needed to update the *cumulative sum* [CUSUM] portion of control charts or to compute means of order *p* compared against prediction limits.

Simple substitution is not recommended in the Unified Guidance unless no more than 10-15% of the sample observations are non-detect. In those circumstances, substituting half the RL for each non-detect is not likely to substantially impact the results of statistical testing. Censored estimation techniques like *Kaplan-Meier* or *robust regression on order statistics* [ROS] are recommended any time the detection frequency is no less than 50% (see **Chapter 15)**.

For lower detection frequencies, non-parametric tests are recommended. Non-parametric prediction limits (**Chapter 18**) can be constructed as an alternative to parametric prediction limits or control charts. The Tarone-Ware two-sample test (**Chapter 16**) is specifically designed to accommodate non-detects and serves as an alternative to the *t*-test. By the same token, the Kruskal-Wallis test (**Chapter 17**) is a non-parametric, rank-based alternative to the parametric ANOVA. These latter tests can be used when the non-detects and detects can be jointly sorted and partially ordered (except for tied values).

When *all* data are non-detect, the Double Quantification rule (**Section 6.2.2**) can be used to define an approximate non-parametric prediction limit, with the RL as an upper bound. Before doing this, it should be determined whether chemicals never or not recently detected in groundwater should even be formally tested. This will depend on whether the monitored constituent from a large analytical suite is likely to originate in the waste or leachate.

Even if a data set contains only a small proportion of non-detects, care should be taken when choosing between the *method detection limit* [MDL], the quantification limit [QL], and the RL in characterizing 'non-detect' concentrations. Many non-detects are reported with one of three data qualifier flags: "U," "J," or "E." Samples with a U data qualifier represent 'undetected' measurements, meaning that the signal characteristic of that analyte could not be observed or distinguished from 'background noise' during lab analysis. Inorganic samples with an E flag and organic samples with a J flag may or may not be reported with an estimated concentration. If no concentration estimate is reported, these samples represent 'detected, but not quantified' measurements. In this case, the actual concentration is assumed to be positive, falling somewhere between zero and the QL or possibly the RL.

Since the actual concentration is unknown, the suggested imputation when using simple substitution is to replace each non-detect having a qualifier of E or J by one-half the RL. Note, however, that E and J samples reported *with* estimated concentrations should be treated as valid measurements for statistical purposes. Substitution of one-half the RL is *not recommended* for these measurements, even though the degree of uncertainty associated with the estimated concentration is probably greater than that associated with measurements above the RL.

As a general rule, non-detect concentrations should *not* be assumed to be bounded above by the MDL. The MDL is usually estimated on the basis of ideal laboratory conditions with physical analyte samples that may or may not account for matrix or other interferences encountered when analyzing specific field samples. For certain trace element analytical methods, individual laboratories may report detectable limits closer to an MDL than a nominal QL. So long as the laboratory has confidence in the ability to quantify at its lab- or occasionally event-specific detection level, this RL may also be satisfactory. The RL should typically be taken as a more reasonable upper bound for non-detects when imputing estimated concentration values to these measurements.

RLs are sometimes but not always equivalent to a particular laboratory's QLs. While analytical techniques may change and improve over time leading to a lowering of the achievable QL, a contractually negotiated RL might be much higher. Often a multiplicative factor is built into the RL to protect a contract lab against particular liabilities. A good practice is to periodically review a given laboratory's capabilities and to encourage reporting non-detects with actual QLs whenever possible, and providing standard qualifiers with all data measurements as well as *estimated* concentrations for E- and J-flagged samples.

Even when no estimate of concentration can be made, a lab should regularly report the distinction between 'undetected' and 'detected, but not quantified' non-detect measurements. Data sets with such delineations can be used to advantage in rank-based non-parametric procedures. Rather than assigning the same tied rank to all non-detects (**Chapter 16**), 'detected but not quantified' measurements should be given larger ranks than those assigned to 'undetected' samples. These two types of non-detects should be treated as two *distinct* groups of tied observations for use in the non-parametric *Wilcoxon rank-sum* procedure.

6.4 DESIGNING DETECTION MONITORING TESTS

In the following sections, the main formal detection monitoring tests covered in this guidance are described in the context of site design choices. Advantages as well as limitations are presented, including the use of certain methods as diagnostic tools in determining the appropriate formal test(s).

6.4.1 T-TESTS

A statistical comparison between two sets of data is known as a two-sample test. When normality of the sample data can be presumed, the parametric Student *t*-test is commonly used (**Section 16.1**). This test compares two distinct populations, represented by two *samples*. These samples can either be individual well data sets, or a common pooled background versus individual compliance well data. The basic goal of the *t*-test is to determine whether there is any statistically significant difference between the two population means. Regulatory requirements for formal use of two-sample t-tests are limited to the Part 265 indicator parameters, and have generally been superseded in the Parts 264 and 258 rules by tests which can account for multiple comparisons.

When the sample data are non-normal and may contain non-detects, the Unified Guidance provides alternative two-sample tests to the parametric t-test. The Wilcoxon rank-sum test (**Section 16.2**) requires that the combined samples be sorted and ranked. This test evaluates potential differences in population *medians* rather than the *means*. The Tarone-Ware test (**Section 16.3**) is specially adapted to handle left-censored measurements, and also tests for differences in population medians.

The *t*-test or a non-parametric variant is recommended as a validation tool when updating intrawell or other background data sets (**Chapter 5**). More recently collected data considered for background addition are compared to the historical data set. A non-significant test result implies no mean differences, and the newer data may be added to the original set. These tests are generally useful for any two-sample diagnostic comparisons.

6.4.2 ANALYSIS OF VARIANCE [ANOVA]

The parametric one-way ANOVA is an extension of the t-test to multiple sample groups. Like its two-sample counterpart, ANOVA tests for significant differences in one or more group (e.g., well) means. If an overall significant difference is found as measured by the F-statistic, *post-hoc* statistical contrasts may be used to determine where the differences lie among individual group means. In the groundwater detection monitoring context, only differences of mean well increases relative to background are considered of importance. The ANOVA test also has wide applicability as a diagnostic tool.

USE OF ANOVA IN FORMAL DETECTION MONITORING TESTS

RCRA regulations under Parts 264 and 258 identify parametric and non-parametric ANOVA as potential detection monitoring tests. Because of its flexibility and power, ANOVA can sometimes be an appropriate method of statistical analysis when groundwater monitoring is based on an *interwell* comparison of background and compliance well data. Two types of ANOVA are presented in the Unified Guidance: parametric and non-parametric one-way ANOVA (**Section 17.1**). Both methods

attempt to assess whether distinct monitoring wells differ in average concentration during a given evaluation period.[15]

Despite the potential attractiveness of ANOVA tests, use in formal detection monitoring is limited by these important factors:

- Many monitoring constituents exhibit significant spatial variability and cannot make use of interwell comparisons;
- The test can be confounded by a large number of well network comparisons;
- A minimum well sample size must be available for testing; and
- Regulatory false positive error rate restrictions limit the ability to effectively control the overall false positive rate.

As discussed in **Section 6.2.3**, many if not most inorganic monitoring constituents exhibit spatial variability, precluding an interwell form of testing. Since ANOVA is inherently an interwell procedure, the guidance recommends against its use for these constituents and conditions. Spatial variability implies that the average groundwater concentration levels vary from well to well because of existing on-site conditions. Mean differences of this sort can be identified by ANOVA, but the cause of the differences cannot. Therefore, results of a statistically significant ANOVA might be falsely attributed as a regulated unit release to groundwater.

ANOVA testing might be applied to synthetic organic and trace element constituent data. However, spatial variation across a site is also likely to occur from offsite or prior site-related organic releases. An existing contamination plume generally exhibits varying average concentrations longitudinally, as well as in cross-section and depth. For other organic constituents never detected at a site, ANOVA testing would be unnecessary. Certain trace elements like barium, arsenic and selenium do often exhibit some spatial variability. Other trace element data generally have low overall detection rates, which may also preclude ANOVA applications. Overall, very few routine monitoring constituents are measurable (i.e., mostly detectable) yet not spatially distinct to warrant using ANOVA as a formal detection monitoring test. Other guidance tests better serve this purpose.

ANOVA has good power for detecting real contamination provided the network is small to moderate in size. But for large monitoring networks, it may be difficult to identify single well contamination. One explanation is that the ANOVA F-statistic simultaneously combines all compliance well effects into a single number, so that many other uncontaminated wells with their own variability can mask the test effectiveness to detect the contaminated well. This might occur at larger sites with multiple waste units, or if only the edge of a plume happens to intersect one or two boundary wells.

The statistical power of ANOVA depends significantly on having at least 4 observations per well available for testing. Since the measurements must be statistically independent, collection of four well observations may necessitate a wait of several months to a few years if the natural groundwater velocity

[15] Parametric ANOVA assesses differences in means; the non-parametric ANOVA compares *median* concentration levels. Both statistical measures are a kind of average.

is low. In this case, other strategies (e.g., prediction limits) might be considered that allow each new groundwater measurement to be tested as it is collected and analyzed.

The one-way ANOVA test in the RCRA regulations is not designed to control the false positive error rate for multiple constituents. The rules mandate a minimum false positive error rate (α) of 5% per test application. With an overall false positive rate of approximately 5% per constituent, a potentially very high SWFPR can result as the number of constituents tested by ANOVA increases and if tests are conducted more than once per year.

For these reasons, the Unified Guidance does not generally recommend ANOVA for formal detection monitoring. ANOVA might be applicable to a small number of constituents, depending on the site. Prediction limit and control chart strategies using retesting are usually more flexible and offer the ability to accommodate even very large monitoring networks, while meeting the false positive and statistical power targets recommended by the guidance.

USE OF ANOVA IN DIAGNOSTIC TESTING

In contrast, ANOVA is a versatile tool for diagnostic testing, and is frequently used in the guidance for that purpose. Parametric or non-parametric one-way versions are the principal means of identifying prior spatial variability among background monitoring wells (**Chapter 13**). Improving sample sizes using intrawell pooled variances also makes use of ANOVA (**Chapter 13**). Equality of variances among wells is evaluated with ANOVA (**Chapter 11**). ANOVA is also applied when determining certain temporal trends in parallel well sample constituent data (**Chapter 14**).

Tests of natural spatial variability can be made by running ANOVA prior to any waste disposal at a new facility located above an undisturbed aquifer (Gibbons, 1994a). If ANOVA identifies significant upgradient and downgradient well differences when wastes have not yet been managed on-site, natural spatial variability is the likely cause. Prior on-site contamination might also be revealed in the form of significant ANOVA differences.

Sites with multiple upgradient background wells can initially conduct an ANOVA on historical data from just these locations. Where upgradient wells are not significantly different for a given constituent, ANOVA testing can be extended to existing historical compliance well data for evaluating potential additions to the upgradient background data base.

If intrawell tests are chosen because of natural spatial variation, the results of a one-way ANOVA on background data from multiple wells can sometimes be used to improve intrawell background limits (**Section 13.3**). Though the amount of intrawell background at any given well may be small, the ANOVA provides an estimate of the *root mean squared error* [RMSE], which is very close to an estimate of the *average per-well standard deviation*. By substituting the RMSE for the usual well-specific standard deviation (s), a more powerful and accurate intrawell limit can be constructed, at least at those sites where intrawell background across the group of wells can be normalized and the variances approximately equalized using a common transformation.

Although the Unified Guidance primarily makes use of one-way ANOVA, many kinds of ANOVA exist. The one-way ANOVA applications so far discussed— in formal detection monitoring or to assess well mean differences— utilize data from spatial locations as the factor of interest. In some situations,

correlated behavior may exist for a constituent among well samples evaluated in different temporal events. A constituent measured in a group of wells may simultaneously rise or fall in different time periods. Under these conditions, the data are no longer random and independent. ANOVA can be used to assess the significance of such systematic changes, making *time* the factor of interest. Time can also play a role if the sample data exhibit cyclical seasonal patterns or if parallel upward or downward trends are observed both in background and compliance point wells.

If time is an important second factor, a *two-way* ANOVA is probably appropriate. This procedure is discussed in Davis (1994). Such a method can be used to test for and adjust data either for seasonality, parallel trends, or changes in lab performance that cause temporal (*i.e.*, time-related) effects. It is somewhat more complicated to apply than a one-way test. The main advantage of a two-way ANOVA is to separate components of overall data variation into three sources: well-to-well mean-level differences, temporal effects, and random variation or statistical error. Distinguishing the sources of variation provides a more powerful test of whether significant well-to-well differences actually exist compared to using only a one-way procedure.

A significant temporal factor does not necessarily mean that the one-way ANOVA will *not* identify actual well-to-well spatial differences. It merely does not have as strong a chance of doing so. Rarely will the one-way ANOVA identify non-existent well-to-well differences. One situation where this can occur is when there is a strong *statistical interaction* between the well-to-well factor and the time factor in the two-way ANOVA. This would imply that changes in lab performance or seasonal cycles affect certain wells (*e.g.*, compliance point) to a different degree or in a different manner than other wells (*e.g.*, background). If this is the case, professional consultation is recommended before conducting more definitive statistical analyses.

6.4.3 TREND TESTS

Most formal detection monitoring tests in the guidance compare background and compliance point populations under the key assumption that the populations *are stationary over time*. The distributions in each group or well are assumed to be stable during the period of monitoring, with only random fluctuations around a constant mean level. If a significant trend occurs in the background data, these tests cannot be directly used. Trends can occur for several reasons including natural cycles, gradual changes in aquifer parameters or the effects of contaminant migration from off-site sources.

Although not specifically provided for in the RCRA regulations, the guidance necessarily includes a number of tests for evaluating potential trends. **Chapter 17, Section 17.3** covers three basic trend tests. (1) *Linear regression* is a parametric method requiring normal and independent trend residuals, and can be used both to identify a linear trend and estimate its magnitude; (2) For non-normal data (including sample data with left-censored measurements), the *Mann-Kendall* test offers a non-parametric method for identifying trends; and (3) To gauge trend magnitude with non-normal data, the *Theil-Sen* trend line can be used.

Trend analyses are primarily diagnostic tests, which should be applied to background data prior to implementing formal detection monitoring tests. If a significant trend is uncovered, two options may apply. The particular monitoring constituent may be dropped in favor of alternate constituents not exhibiting non-stationary behavior. Alternatively, prediction limit or control chart testing can make use of stationary *trend residuals* for testing purposes. One limitation of the latter approach requires making

an assumption that the historical trend will continue into future monitoring periods. In addition, future data needs to be de-trended prior to testing. If a trend happened to be of limited duration, this assumption may not be reasonable and could result in identifying a background exceedance when it does not exist. If a trend occurs in future data at a compliance well and prior background data was stationary, other detection monitoring tests are likely to eventually identify it. Trend testing may also be applied to once-future data considered for a periodic background update, although the guidance primarily relies on t-testing of historical and future groups to assess data suitability.

At historically contaminated compliance wells, establishing a proper baseline for a prediction limit or control chart is problematic, since uncontaminated concentration data cannot be collected. Depending on the pattern of contamination, an intrawell background may either have a stable mean concentration level or exhibit an increasing or decreasing trend. Particularly when intrawell background concentrations are rising, the assumption of a static baseline population required by prediction limits and control charts will be violated.

As an alternative, the Unified Guidance recommends a test for trend to measure the extent and nature of the apparent increase. Trend testing can determine if there is a statistically significant positive trend over the period of monitoring and can also determine the magnitude (*i.e.*, slope) of the trend. In identifying a positive trend, it might be possible to demonstrate that the level of contamination has increased relative to historical behavior and indicate how rapidly levels are increasing.

Trend analyses can be used directly as an alternative test against a GWPS in compliance and corrective action monitoring. For typical compliance monitoring, data collected at each compliance well are used to generate a lower confidence limit compared to the fixed standard (**Chapters 7, 21 and 22**). A similar situation occurs when corrective action is triggered, but making use of an upper confidence interval for comparison. For compliance well data containing a trend, the appropriate confidence interval is constructed around a linear regression trend line (or its non-parametric alternative) in order to better estimate the most current concentration levels. Instead of a single confidence limit for stationary tests, the confidence limit (or band) estimate changes with time.

6.4.4 STATISTICAL INTERVALS

Prediction limits, tolerance limits, control chart limits and confidence limits belong to the class of methods known as statistical intervals. The first three are used to define their respective detection monitoring test limits, while the last is used in fixed standard compliance and corrective action tests. When using a background GWPS, either approach is possible (see **Section 7.5**). Intervals are generated as a statistic from reference sample data, and represent a probable range of occurrence either for a future sample statistic or some parameter of the population (in the case of confidence intervals) from which the sample was drawn. A future sample statistic might be one or more single values, as well as a future mean or median of specific size, drawn from one or more sample sets to be compared with the interval (generally an upper limit). Both the reference and comparison sample populations are themselves unknown, with the latter initially presumed to be identical to the reference set population. In the groundwater monitoring context, the initial reference sample is the background data set.

The key difference in confidence limits[16] is that a statistical interval based on a single sample is used to estimate the probable range of a population parameter like the true mean, median or variance. The three detection monitoring tests use intervals to identify ranges of future sample statistics likely to arise from the background population based on the initial sample, and are hence two- or multiple-sample tests.

Statistical intervals are inherently two-sided, since they represent a finite range in which the desired statistic or population parameter is expected to occur. Formally, an interval is associated with a level of confidence (1-α); by construction, the error rate α represents the remaining likelihood that the interval *does not contain* the appropriate statistic or parameter. In a two-sided interval, the α-probability is associated with ranges both above and below the statistical interval. A one-sided upper interval is designed to contain the desired statistic or parameter at the same (1-α) level of confidence, but the remaining error represents only the range above the limit. As a general rule, detection monitoring options discussed below use one-sided upper limits because of the nature of the test hypotheses.

PREDICTION LIMITS

Upper prediction limits (or intervals) are constructed to contain with (1-α) probability, the next few sample value(s) or sample statistic(s) such as a mean from a background population. Prediction limits are exceptionally versatile, since they can be designed to accommodate a wide variety of potential site monitoring conditions. They have been extensively researched, and provide a straightforward interpretation of the test results. Since this guidance strongly encourages use of a comprehensive design strategy to account for both the cumulative SWFPR and effective power to identify real exceedances, prediction limit options offer a most effective means of accounting for both criteria. The guidance provides test options in the form of parametric normal and non-parametric prediction limit methods. Since a retesting strategy of some form is usually necessary to meet both criteria, prediction limit options are constructed to formally include resampling as part of the overall tests.

Chapters 18 and 19 provide nine parametric normal prediction limit test options: four tests of future values (1-of-2, 1-of-3, 1-of-4 or a modified California plan) and five future mean options (1-of-1, 1-of-2, or 1-of-3 tests of mean size 2, and 1-of-1 or 1-of-2 tests of mean size 3). Non-parametric prediction limit options cover the same future value test options as the parametric versions, as well as two median tests of size 3 (1-of-1 or 1-of-2 tests). **Appendix D** tables provide the relevant κ-factors for each parametric normal test option, the achievable false positive rates for non-parametric tests, and a categorical rating of relative test power for each set of input conditions. Prediction limits can be used both for interwell and intrawell testing. Selecting from among these options should allow the two site design criteria to be addressed for most groundwater site conditions.

The options provided in the guidance are based on a wider class known in the statistical literature as *p-of-m* prediction limit tests. Except for the two modified California plan options, those selected are 1-of-*m* test varieties. The number of future measurements to be predicted (i.e., contained) by the interval is also denoted in the Unified Guidance by m and can be as small as $m = 1$. To test for a release to groundwater, compliance well measurements are designated as future observations. Then a limit is constructed on the background sample, with the prediction limit formula based on the number of m

[16] Confidence limits are further discussed **in Chapters 7, 21 and 22** for use in compliance and corrective action testing.

future values or statistics to be tested against the limit. As long as the compliance point measurements are similar to background, the prediction limit should contain all *m* of the future values or statistics with high probability (the level of confidence). For a 1-of-*m* test, all *m* values must be larger than the prediction limit to be declared an exceedance, as initial evidence that compliance point concentrations are higher than background.

Prediction limits with retesting are presented in **Chapter 19**. When retesting is part of the procedure, there are significant and instructive differences in statistical performance between parametric and non-parametric prediction limits.

Parametric prediction limits are constructed using the general formula: $PL = \bar{x} + \kappa \cdot s$, where $\bar{x}$ and *s* are the background sample mean and standard deviation, and κ is the specific multiplicative factor for the type of test, background sample size, and the number of annual tests. The number of tests made against a common background is also an input factor for interwell comparison. The **Appendix D** κ-factors are specifically designed to meet the SWFPR objective, but power will vary. Larger background sample sizes and higher order (*m*) tests afford greater power.

When background data cannot be normalized, a non-parametric prediction limit can be used instead. A non-parametric prediction limit test makes use of one or another of the largest sample values from the background data set as the limit. For a given background sample size and test type, the level of confidence of that maximal value is fixed.

Using the absolute maximum of a background data set affords the highest confidence and lowest single-test false positive error. However, even this confidence level may not be adequate to meet the SWFPR objective, especially for lower order 1-of-*m* tests. A higher order future values test using the same maximum and background sample size will provide greater false positive confidence and hence a lower false positive error rate. For a fixed background sample size, a 1-of-4 retesting scheme will have a lower achievable significance level (α) than a 1-of-3 or 1-of-2 plan for any specific maximal value. A larger background sample size using a fixed maximal value for any test also has a higher confidence level (lower α) than a smaller sample.

But for a fixed non-parametric limit of a given background sample size, the power decreases as the test order increases. If the non-parametric prediction limit is set at the maximum, a 1-of-2 plan will be more powerful than a 1-of-4 plan. It is relatively easy to understand why this is the case. A verified exceedance in a 1-of-2 test occurs only if two values exceed the limit, but would require four to exceed for the 1-of-4 plan. As a rule, even the highest order non-parametric test using some maximal background value will be powerful enough to meet the ERPC power criteria, but achieving a sufficiently low single-test error rate to meet the SWFPR is more problematic.

If the SWFPR objective can be attained at a maximum value for higher order 1-of-*m* tests, it may be possible to utilize lower maxima from a large background data base. Lower maxima will have greater power and a somewhat higher false positive rate. Limited comparisons of this type can be made when choosing between the largest or second-largest order statistics in the Unified Guidance **Appendix D Tables 19-19** to **19-24**. A more useful and flexible comparison for 1-of-*m* future value plans can be obtained using the EPA Region 8 *Optimal Rank Values Calculator* discussed in **Chapter 19**. The calculator identifies the lowest ranked maximal value of a background data set for 1-of-1 to 1-of-4 future

value non-parametric tests which can meet the SWFPR objective, while providing ERPC ratings and fractional power estimates at 2, 3, and 4 standard deviations above background.

TOLERANCE INTERVALS

Tolerance intervals are presented in **Section 17.2**. A tolerance interval is generated from background sample data to contain a pre-specified *proportion* of the underlying population (*e.g.*, 99% of all possible population measurements) at a certain level of confidence. Measurements falling outside the tolerance interval can be judged to be statistically different from background.

While tolerance intervals are an acceptable statistical technique under RCRA as discussed in **Section 2.3**, the Unified Guidance generally recommends prediction limits instead. Both methods can be used to compare compliance point measurements to background in detection monitoring. The same general formula is used in both tests for constructing a parametric upper limit of comparison: $\bar{x} + \kappa s$. For non-parametric upper limit tests, both prediction limits and tolerance intervals use an observed order statistic in background (often the background maximum). But prediction limits are ultimately more flexible and easier to interpret than tolerance intervals.

Consider a parametric upper prediction limit test for the next two compliance point measurements with 95% confidence. If either measurement exceeds the limit, one of two conditions is true: either the compliance point distribution is significantly different and higher than background, or a false positive has been observed and the two distributions are similar. False positives in this case are expected to occur 5% of the time. Using an upper tolerance interval is not so straightforward. The tolerance interval has an extra statistical parameter that must be specified — the coverage (γ) — representing the fraction of background to be contained beneath the upper limit. Since the confidence level ($1-\alpha$) governs how often a statistical interval contains its target population parameter (**Section 7.4**), the complement α does not necessarily represent the false positive rate in this case.

In fact, a tolerance interval constructed with 95% confidence to cover 80% of background is designed so that as many as 20% of all background measurements will exceed the limit with 95% probability. Here, $\alpha = 5\%$ represents the probability that the true coverage will be less than 80%. But less clear is the false positive rate of a tolerance interval test in which as many as 1 in 5 background measurements are expected to exceed the upper background limit. Are compliance point values above the tolerance interval indicative of contaminated groundwater or merely representative of the upper ranges of background?

Besides a more confusing interpretation, there is an added concern. Mathematically valid retesting strategies can be computed for prediction limits, but not yet for tolerance intervals, further limiting their usefulness in groundwater testing. It is also difficult to construct powerful *intrawell* tolerance intervals, especially when the intrawell background sample size is small. Overall, there is little practical need for two similar (but not identical) methods in the Unified Guidance, at least in detection monitoring.

If tolerance intervals *are* employed as an alternative to *t*-tests or ANOVA when performing interwell tests, the RCRA regulations allow substantial flexibility in the choice of α. This means that a somewhat arbitrarily high confidence level ($1-\alpha$) can be specified when constructing a tolerance interval. However, unless the coverage coefficient (γ) is also set to a high value (*e.g.*, $\geq 95\%$), the test is likely to incur a large risk of false positives despite a small α.

One setting in which an upper tolerance interval is very appropriate is discussed in **Section 7.5**. Some constituents that must be evaluated under compliance/assessment or corrective action may not have a fixed GWPS. Existing background levels may also exceed a fixed GWPS. In these cases, a background standard can be constructed using an upper tolerance interval on background with 95% confidence and 95% coverage. The standard will then represent a reasonable upper bound on background and an achievable target for compliance and remediation testing.

6.4.5 CONTROL CHARTS

Control charts (**Chapter 20**) are a viable alternative to prediction limits in detection monitoring. One advantage of a control chart over a prediction limit is that control charts allow compliance point data to be viewed and assessed graphically over time. Trends and changes in concentration levels can be easily seen, because the compliance measurements are consecutively plotted on the chart as they are collected, giving the data analyst an historical overview of the concentration pattern. Standard prediction limits allow only *point-in-time comparisons* between the most recent data and background, making long-term trends more difficult to identify.

The guidance recommends use of the combined *Shewhart-CUSUM control chart.* The advantage is that *two* statistical quantities are assessed at every sampling event, both the new individual measurement and the cumulative sum [CUSUM] of past and current measurements. Prediction limits do not incorporate a CUSUM, and this can give control charts comparatively greater sensitivity to gradual (upward) trends and shifts in concentration levels. To enhance false positive error rate control and power, retesting can also be incorporated into the Shewhart-CUSUM control chart. Following the same restrictions as for prediction limits, they may be applied either to interwell or intrawell testing.

A disadvantage in applying control charts to groundwater monitoring data is that less is understood about their statistical performance, *i.e.*, false positive rates and power. The control limit used to identify potential releases to groundwater is not based on a formula incorporating a desired false positive rate (α). Unlike prediction limits, the control limit cannot be precisely set to meet a pre-specified SWFPR, unless the behavior of the control chart is modeled via Monte Carlo simulation. The same is true for assessing statistical power. Control charts usually provide less flexibility than prediction limits in designing a statistical monitoring program for a network.

In addition, Shewhart-CUSUM control charts are a parametric procedure with no existing non-parametric counterpart. Non-parametric prediction limit tests are still generally needed when the background data on which the control chart is constructed cannot be normalized. Control charts are mostly appropriate for analytes with a reasonably high detection frequency in monitoring wells. These include inorganic constituents (*e.g.*, detectable trace elements and geochemical monitoring parameters) occurring naturally in groundwater, and other persistently-found, site-specific chemicals.

6.5 SITE DESIGN EXAMPLES

Three hypothetical design examples consider a small, medium and large facility, illustrating the principles discussed in this chapter. In each example, the goal is to determine what statistical method or methods should be chosen and how those methods can be implemented in light of the two fundamental design criteria. Further design details are covered in respective **Part III** detection monitoring test

chapters, although very detailed site design is beyond the scope of the guidance. More detailed evaluations and examples of diagnostic tests are found in **Part II** of the guidance.

►EXAMPLE 6-5 SMALL FACILITY

A municipal landfill has 3 upgradient wells and 8 downgradient wells. Semi-annual statistical evaluations are required for five inorganic constituents. So far, six observations have been collected at each well. Exploratory analysis has shown that the concentration measurements appear to be approximately normal in distribution. However, each of the five monitored parameters exhibits significant levels of natural spatial variation from well to well. What statistical approach should be recommended at this landfill?

SOLUTION

Since the inorganic monitoring parameters are measurable and have significant spatial variability, it is recommended that parametric intrawell rather than interwell tests should be considered. Assuming that none of the downgradient wells is recently contaminated, each well has n = 6 observations available for its respective intrawell background. Six background measurements may or may not be enough for a sufficiently powerful test.

To address the potential problem of inadequate power, a one-way ANOVA should be run on the combined set of wells (including background locations). If the well-to-well variances are significantly different, individual standard deviation estimates should be made from the six observations at the eight downgradient wells. If the variances are approximately equal, a pooled standard deviation estimate can instead be computed from the ANOVA table. With 11 total wells and 6 measurements per well, the pooled standard deviation has $df = 11 \times 5 = 55$ degrees of freedom, instead of df = 5 for each individual well.

Regardless of ANOVA results, the per-test false positive rate is approximately the design SWFPR divided by the annual number of tests. For w = 8 compliance wells, c = 5 parameters monitored, and n_E = 2 statistical evaluations per year, the per-test false positive rate is approximately α_{test} = SWFPR/($w \times c \times n_E$) = 0.00125. Given normal distribution data, several different parametric prediction limit retesting plans can be examined,[17] using either the combined sample size of df + 1= 56 or the per-well sample size of n = 6.

Explained in greater detail in **Chapter 19**, κ-multiples and power ratings for each test type (using the inputs w = 8 and n = 6 or 56 are obtained from the nine parametric **Appendix D** Intrawell tables labeled *'5 COC, Semi-Annual'*. The following κ-factors were obtained for tests of future values at n = 6: κ = 3.46 (1-of-2 test); κ = **2.41** (1-of-3); κ = **1.81** (1-of-4); and κ = 2.97 (modified California) plans. For future means, the corresponding κ-factors were: κ = 4.46 (1-of-1 mean size 2); κ = 2.78 (1-of-2 mean size 2); κ = 2.06 (1-of-3 mean size 2); κ = 3.85 (1-of-1 mean size 3); and κ = 2.51 (1-of-2 mean size 3). In these tables, κ-factors reported in **Bold** have good power, those ***Italicized*** have acceptable power and Plain Text indicates low power. For single well intrawell tests, only 1-of-3 or 1-of-4 plans for future values, 1-of-2 or 1-of-3 mean size 2 or 1-of-2 mean size 3 plans meet the ERPC criteria.

[17] Intrawell control charts with retesting are also an option, though the control limits associated with each retesting scheme need to be simulated.

Although each of these retesting plans is adequately powerful, a final choice would be made by balancing 1) the cost of sampling and chemical analysis at the site; 2) the ability to collect statistically independent samples should the sampling frequency be increased; and 3) a comparison of the actual power curves of the three plans. The last can be used to assess how differences in power might impact the rapid identification of a groundwater release. Since a 1-of-3 test for future observations has good power, it is unnecessary to make use of a 1-of-4 test. Similarly, the 1-of-3 test for mean size 2 and a 1-of-2 test for mean size 3 might also be eliminated, since a 1-of-2 test of a mean size 2 is more than adequate. This leaves the 1-of-3 future values and 1-of-2 mean 2 tests as the final prediction limit options to consider.

Though prediction limits around future means are more powerful than plans for observations, only 3 independent measurements might be required for a 1-of-3 test, while 4 might be necessary for the 1-of-2 test for mean size 2. For most tests at background, a single sample might suffice for the 1-of-3 test and 2 independent samples for the test using a 1-of-2 mean size 2.

Much greater flexibility is afforded if the pooled intrawell standard deviation estimate can be used. For this example, any of the nine parametric intrawell retesting plans is sufficiently powerful, including a 1-of-2 prediction limit test on observations and a 1-of-1 test of mean size 2. In order to make this assessment using the pooled-variance approach, a careful reading of **Chapter 13, Section 13.3**. is necessary to generate comparative κ-factors.

Less overall sampling is needed with the 1-of-2 plan on observations, since only a single sample may be needed for most background conditions. Two observations are always required for the 1-of-1 mean size 2 test. More prediction limit testing options are generally available for a small facility. ◄

►EXAMPLE 6-6 MEDIUM FACILITY

A medium-sized hazardous waste facility has 4 upgradient background wells and 20 downgradient compliance wells. Ten initial measurements have been collected at each upgradient well and 8 at downgradient wells. The permitted monitoring list includes 10 inorganic parameters and 30 VOCs. No VOCs have yet been detected in groundwater. The remaining 10 inorganic constituents are normal or can be normalized, and five show evidence of significant spatial variation across the site. Assume that pooled-variances cannot be obtained from the historical upgradient or downgradient well data. If one statistical evaluation must be conducted each year, what statistical method and approach are recommended?

SOLUTION

At this site, there are potentially 800 distinct well-constituent pairs that might be tested. But since none of the VOCs has been detected in groundwater in background wells, all 30 of the VOCs should be handled using the *double quantification rule* (**Section 6.2.2).** A second confirmatory resample should be analyzed at those compliance wells for any of the 30 VOC constituents initially detected. Two successive quantified detections above the RL are considered significant evidence of groundwater contamination at that well and VOC constituent. To properly limit the SWFPR, the 30 VOC constituents are excluded from further SWFPR calculations, which is now based on $w \times c \times n_E = 20 \times 10 \times 1 = 200$ annual tests.

The five inorganic constituent background data sets indicate insignificant spatial variation and can be normalized. The observations from the four upgradient wells can be pooled to form background data sets with an n = 40 for each of these five constituents. Future samples from the 20 compliance wells are then compared against the respective *interwell* background data. With one annual evaluation, c = 10 constituents, w = 20 wells and n = 40 background samples, the Interwell *'10 COC, Annual'* tables for parametric prediction limits with retesting can be searched in **Appendix D**. Alternatively, control chart limits can be fit to this configuration via Monte Carlo simulations. Even though only five constituents will be tested this way, all of the legitimate constituents (c) affecting the SWFPR calculation, are used in applying the tables.

Most of the interwell prediction limit retesting plans, whether for observations or means, offer good power relative to the annual evaluation ERPC. The final choice of a plan may be resolved by a consideration of sampling effort and cost, as well as perhaps a more detailed power comparison using simulated curves. For prediction limits, a 1-of-2 test for observations (κ = **2.18**) and the 1-of-1 prediction limit for a mean of order 2 (κ = **2.56**) both offer good power. These two plans also require the least amount of sampling to identify a potential release (as discussed in Example 6-6). Beyond this rationale, the more powerful 1-of-1 test of a future mean size 2 might be selected. Full power curves could be constructed and overlaid for several competing plans.

The remaining 5 inorganic constituents must be managed using intrawell methods based on individual compliance well sizes of n = 8. For the same c, w, and n_E inputs as above, the Appendix D Intrawell '10 COC, Annual' tables should be used. Only four of the higher order prediction limit tests have acceptable or good power: 1-of-4 future values (κ = **1.84**); 1-of-2 mean size 2 (κ = *2.68*); 1-of-3 mean size 2 (κ = **2.00**); and 1-of-2 mean size 3 (κ = **2.39**) tests. The 1-of-2 mean size 2 has only acceptable power. The first two tests require the fewest samples under most background conditions and in total, with the 1-of-4 test having superior power. ◄

►EXAMPLE 6-7 LARGE FACILITY

A larger solid waste facility must conduct two statistical evaluations per year at two background wells and 30 compliance wells. Parameters on the monitoring list include five trace metals with a high percentage of non-detect measurements, and five other inorganic constituents. While the inorganic parameters are either normal or can be normalized, a significant degree of spatial variation is present from one well to the next. If 12 observations were collected from each background well, but only 4 quarterly measurements from each compliance well, what statistical approach is recommended?

SOLUTION

Because the two groups of constituents evidence distinctly different statistical characteristics, each needs to be separately considered. Since the trace metals have occasional detections or 'hits,' they cannot be excluded from the SWFPR computation. Because of their high non-detect rates, parametric prediction limits or control charts may not be appropriate or valid unless a non-detect adjustment such as Kaplan-Meier or robust regression on order statistics is used (**Chapter 15**). Assuming for this example that parametric tests cannot be applied, the trace metals should be analyzed using non-parametric prediction limits. The presence of frequent non-detects may substantially limit the potential degree of spatial variation, making an *interwell* non-parametric test potentially feasible. The Kruskal-Wallis non-parametric ANOVA (**Chapter 17**) could be used to test this assumption.

In this case, the number of background measurements is n = 24, and this value along with w = 30 compliance wells would be used to examine possible non-parametric retesting plans in the **Appendix D** tables for non-parametric prediction limits. As these tables offer achievable per-evaluation, per-constituent false positive rates for each configuration of compliance wells and background levels, the target α level must be determined. Given semi-annual evaluations, the per-evaluation false positive rate is approximately $\alpha_E = 0.10/n_E = 0.05$. Then, with 10 constituents altogether, the approximate per-constituent false positive rate for each trace metal becomes $\alpha_{const} = 0.05/10 = 0.005$.

Only one retesting plan meets the target false positive rate, a 1-of-4 non-parametric prediction limit using the maximum value in background as the comparison limit. This plan has 'acceptable' power relative to the ERPC. Other more powerful plans all have higher-than-targeted false positive rates.

For the remaining 5 inorganic constituents, the presence of significant spatial variation and the fact that the observations can be normalized, suggests the use of parametric intrawell prediction or control limits. As in the previous **Example 6-6**, interwell prediction limit tables in **Appendix D** are used by identifying κ multipliers and power ratings based on *all* 10 constituents subject to the SWFPR calculations. This is true even though these parametric options only pertain to 5 constituents. The total number of well-constituent pair tests per year is equal to $w \times c \times n_E = 30 \times 10 \times 2 = 600$ annual tests.

Assuming none of the observed spatial variation is due to already contaminated compliance wells, the number of measurements that can be used as intrawell background per well is small (n = 4). A quick scan of the intrawell prediction limit retesting plans in **Appendix D** *'10COC, Semi-Annual'* tables indicates that none of the plans offer even acceptable power for identifying a potential release. A one-way ANOVA should be run on the combined set of w = 30 compliance wells to determine if a pooled intrawell standard deviation estimate can be used.

If levels of variance across these wells are roughly the same, the pooled standard deviation will have $df = w(n-1) = 30 \times 3 = 90$ degrees of freedom, making each intrawell prediction or control limit much more powerful. Using the **R** script provided in **Appendix C** for intrawell prediction limits with a pooled standard deviation estimate (see **Section 13.3**), based on n = 4 and df = 90, all of the relevant intrawell prediction limits are sufficiently powerful compared to the semi-annual ERPC. With the exception of the 1-of-2 future values test at acceptable power, the other tests have good power. The final choice of retesting plan can be made by weighing the costs of required sampling versus perhaps a more detailed comparison of the full power curves. Plans with lower sampling requirements may be the most attractive. ◄

CHAPTER 7. STRATEGIES FOR COMPLIANCE/ASSESSMENT AND CORRECTIVE ACTION

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This chapter covers the fundamental design principles for compliance/assessment and corrective action statistical monitoring programs. One important difference between these programs and detection monitoring is that a fixed external GWPS is often used in evaluating compliance. These GWPS can be an MCL, risk-based or background limit as well as a remedial action goal. Comparisons to a GWPS in compliance/assessment and corrective action are generally *one-sample* tests as opposed to the two- or multi-sample tests in detection monitoring. Depending on the program design, *two- or multiple-sample* detection monitoring strategies can be used with well constituents subject to background compliance/corrective action testing. While a general framework is presented in this chapter, specific test applications and strategies are presented in **Chapters 21 and 22** for fixed GWPS comparisons. **Sections 7.1** through **7.4** discuss comparisons to fixed GWPSs, while **Section 7.5** covers background GWPS testing (either as a fixed limit or based on a background statistic). Discussions of regulatory issues are generally limited to 40 CFR Part 264, although they also apply to corresponding sections of the 40 CFR Part 258 solid waste rules.

7.1 INTRODUCTION

The RCRA regulatory structure for compliance/assessment and corrective action monitoring is outlined in **Chapter 2**. In detection and compliance/assessment monitoring phases, a facility is presumed not to be 'out of compliance' until significant evidence of an impact or groundwater release can be identified. In corrective action monitoring, the presumption is reversed since contamination of the groundwater has already been identified and confirmed. The null hypothesis of onsite contamination is rejected only when there is significant evidence that the clean-up or remediation strategy has been successful.

Compliance/assessment monitoring is generally begun when statistically significant concentration exceedances above background have been confirmed for one or more detection monitoring constituents. *Corrective action* is undertaken when at least one exceedance of a *hazardous* constituent GWPS has been identified in compliance/assessment monitoring. The suite of constituents subject to compliance/assessment monitoring is determined from Part 264 Appendix IX or Part 258 Appendix II testing, along with prior hazardous constituent data evaluated under the detection monitoring program. Following a compliance monitoring statistical exceedance, only a few of these constituents may require

the change in hypothesis structure to corrective action monitoring. This formal corrective action testing will need to await completion of remedial activities, while continued monitoring can track progress in meeting standards.

The same general statistical method of *confidence interval testing against a fixed GWPS* is recommended in both compliance/assessment and corrective action programs. As discussed more fully below and in **Chapter 21**, confidence intervals provide a flexible and statistically accurate method to test how a parameter estimated from a single sample compares to a fixed numerical limit. Confidence intervals explicitly account for variation and uncertainty in the sample data used to construct them.

Most decisions about a statistical program under §264.98 detection monitoring are tailored to facility conditions, other than selecting a target site-wide cumulative false positive rate and a scheme for evaluating power. Statistical design details are likely to be site-specific, depending on the available data, observed distributions and the scope of the monitoring network. For compliance/assessment and corrective action testing under §264.99 and §264.100 or similar tests against fixed health-based or risk-based standards, the testing regimen is instead likely to be determined in advance by the regulatory agency. The Regional Administrator or State Director is charged with defining the nature of the tests, constituents to be tested, and the wells or compliance points to be evaluated. Specific decisions concerning false positive rates and power may also need to be defined at a regulatory program level.

The advantage of a consistent approach for compliance/assessment and corrective action monitoring tests is that it can be applied across all Regional or State facilities. Facility-specific input is still needed, including the observed distributions of key constituents and the selection of statistical power and false positive criteria for permits. Because of the asymmetric nature of the risks involved, regulatory agency and facility perspectives may differ on which statistical risks are most critical. Therefore, we recommend that the following issues be addressed for compliance/assessment and corrective action monitoring (both §264.99 and §264.100), as well as for other programs involving comparisons to fixed standards:

- What are the appropriate hypothesis testing structures for making comparisons to a fixed standard?
- What do fixed GWPS represent in statistical terms and which population parameter(s) should be tested against them?
- What is a desirable frequency of sampling and testing, which test(s), and for what constituents?
- What statistical power requirements should be included to ensure protection of health and the environment?
- What confidence level(s) should be selected to control false positive error rates, especially considering sites with multiple wells and/or constituents?

Decisions regarding these five questions are complex and interrelated, and have not been fully addressed by previous RCRA guidance or existing regulations. This chapter addresses each of these points for both §264.99 and §264.100 testing. By developing answers at a regulatory program level, the necessity of re-evaluating the same questions at each specific site may be avoided.

7.2 HYPOTHESIS TESTING STRUCTURES

Compliance testing under §264.99 specifically requires a determination that one or more well constituents exceeds a permit-specific GWPS. The correct statistical hypothesis during compliance/assessment monitoring is that groundwater concentrations are presumed *not* to exceed the fixed standard unless sampling data from one or more well constituents indicates otherwise. The null hypothesis, H_0, assumes that downgradient well concentration levels are less than or equal to a standard, while the alternative hypothesis, H_A, is accepted only if the standard is significantly exceeded. Formally, for some parameter (Θ) estimated from sample data and representing a standard G, the relevant hypotheses under §264.99 compliance monitoring are stated as:

$$H_0 : \Theta \leq G \ vs. \ H_A : \Theta > G \qquad [7.1]$$

Once a positive determination has been made that at least one compliance well constituent exceeds the fixed standard (*i.e.*, GWPS), the facility is subject to corrective action requirements under §264.100. At this point, the regulations imply and statistical principles dictate that the hypothesis structure should be *reversed* (for those compliance wells and constituents indicating exceedances). Other compliance constituents (*i.e.*, those not exceeding their respective GWPSs) may continue to be tested using equation 7.1 hypotheses. It is then assumed that contamination equal to or in excess of the GWPS exists and is presumed to be the case unless demonstrated otherwise. A positive determination that groundwater concentrations are below the standard is necessary to demonstrate regulatory compliance for any wells and constituents under remediation. In statistical terms, the relevant hypotheses for §264.100 are:

$$H_0 : \Theta \geq G \ vs. \ H_A : \Theta < G \qquad [7.2]$$

The reasoning behind this approach is as follows. Background exceedances by one or more well constituents under §264.98 detection monitoring do not predetermine any particular relationship of these increased concentration levels to fixed limits used as GWPS. Standards for different constituents vary over orders of magnitude. The actual concentration level triggering a statistically significant increase above background can vary considerably and bear little or no relationship to risk-based standards. Use of the initial compliance monitoring hypothesis framework in [7.1] ensures positive evidence that at least one hazardous constituent is truly above a GWPS. Since corrective action can be expensive and difficult, this provides important assurance that site program monitoring decisions are made correctly.

This guidance recognizes that not all regulatory programs are constructed alike. Objectives and regulatory interpretations may differ as to the basic goals of compliance/assessment or corrective action monitoring. When large numbers of sites with available hazardous constituent data are being screened to determine their need for remediation (perhaps outside the formal RCRA regulatory framework), the assessment may be conducted with the *explicit presumption* that contamination exists onsite. Presumably, elevated hazardous constituent concentrations have already been detected at these facilities. For these assessments, the compliance/assessment statistical hypothesis framework follows that presented in Equation 7.2. Instead of a *lower* confidence limit as recommended below, the appropriate statistical approach involves an *upper* confidence limit, as is appropriate for corrective action.

Non-RCRA programs seeking to use methods presented in the Unified Guidance may also presume a different statistical hypothesis structure from that presented here. The primary goal is to ensure that the statistical approach matches the appropriate hypothesis framework. It is also allowable under RCRA regulations to define GWPS based on background data, discussed further in **Section 7.5**.

Whatever the population parameter (Θ) selected as representative of the GWPS, testing consists of a confidence interval derived from the compliance point data at some choice of significance level (α), and then compared to the standard *G*. The confidence intervals describe the probable distribution of the sample statistic, θ, employed to estimate the true parameter Θ. For testing under compliance/assessment monitoring, a lower confidence limit around the true parameter — LCL(Θ) — is utilized. If LCL(Θ) exceeds the standard, there is statistically significant evidence in favor of the alternative hypothesis, H_A: $\Theta > G$, that the compliance standard has been violated. If not, the confidence limit test is inconclusive and the null hypothesis accepted.

When the corrective action hypothesis of [7.2] is employed, an *upper* confidence limit UCL(Θ) is generated from the compliance point data and compared to the standard *G*. In this case, the UCL(Θ) should lie *below* the standard to accept the alternative hypothesis that concentration levels are in compliance, H_A: $\Theta < G$. If the UCL(Θ) is larger than the standard, the test is inconclusive. It should be recognized that once corrective action or remediation activities are initiated, there will be a considerable time during which the GWPS may still be exceeded. As provided in the RCRA regulations, it is at the conclusion of remediation activities that formal corrective action monitoring evaluation is appropriate. However, in the intervening period of remedial activity, well constituents can still be monitored and the relative efficacy of remediation measures tracked. The same corrective action statistical hypotheses can be assumed for the targeted constituents; techniques such as trend testing may be appropriate interim applications.

If the entire confidence interval (considering both the lower and upper confidence limits) lies below the fixed standard *G* in either a compliance/assessment or corrective action setting, there is statistically significant evidence that the true parameter or characteristic (*e.g.*, the mean) is less than the standard. The constituent concentrations at the well are considered to be in compliance. Conversely, if the confidence interval lies entirely above *G*, the evidence suggests that the true parameter or characteristic exceeds the standard, and that concentrations at the well are out of compliance.

When the confidence interval straddles the standard *G* (as with the example confidence interval around the upper 95th percentile in **Figure 7-1** below), the correct decision is uncertain. When the population mean is being tested, and a confidence interval around the mean has accurately estimated its location, the true mean lies somewhere between the lower and upper confidence limits. But the *precise* value of the population mean within that range is unknown. The mean might be less than *G* or it might be greater than *G*. No clear decision with high statistical confidence is possible. **W*hen testing the compliance/assessment monitoring hypothesis of* [7.1]*, we recommend that the null hypothesis should not be rejected unless the entire confidence interval defined by and including the lower confidence limit exceeds the GWPS*. *By the same token, when testing the corrective action hypothesis of equation* [7.2]*, we recommend that the null hypothesis not be rejected unless the entire upper confidence interval and limit lies below the GWPS*.

These ideas can be illustrated with a normal confidence interval around the arithmetic mean. In this case, the population parameter Θ equals μ, the true population mean of a given compliance well

constituent. The statistic used to estimate μ is the sample mean ($\bar{x}$). With this statistic and normally-distributed data, the lower and upper confidence limits are symmetric:

$$LCL(\mu) = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} \qquad [7.3]$$

$$UCL(\mu) = \bar{x} + t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} \qquad [7.4]$$

for a selected significance level (α) and sample size *n*. Note in these formulas that *s* is the sample standard deviation, and $t_{1-\alpha,n-1}$ is a central Student's *t*-value with *n*–1 degrees of freedom.

The two hypothesis structures and tests are defined as follows:

Case A. Test of non-compliance (§264.99) *vs*. a fixed standard (compliance/assessment monitoring):

Test Hypothesis: $H_0 : \mu \le G \ vs. \ H_A : \mu > G$

Test Statistic: $LCL_{1-\alpha} = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}}$

Rejection Region: Reject null hypothesis (H_0) if $LCL_{1-\alpha} > G$; otherwise, accept null hypothesis

Case B. Test of compliance (§264.100) *vs*. a fixed standard (corrective action):

Test Hypothesis: $H_0 : \mu \ge G \ vs. \ H_A : \mu < G$

Test Statistic: $UCL_{1-\alpha} = \bar{x} + t_{1-\alpha,n-1}\frac{s}{\sqrt{n}}$

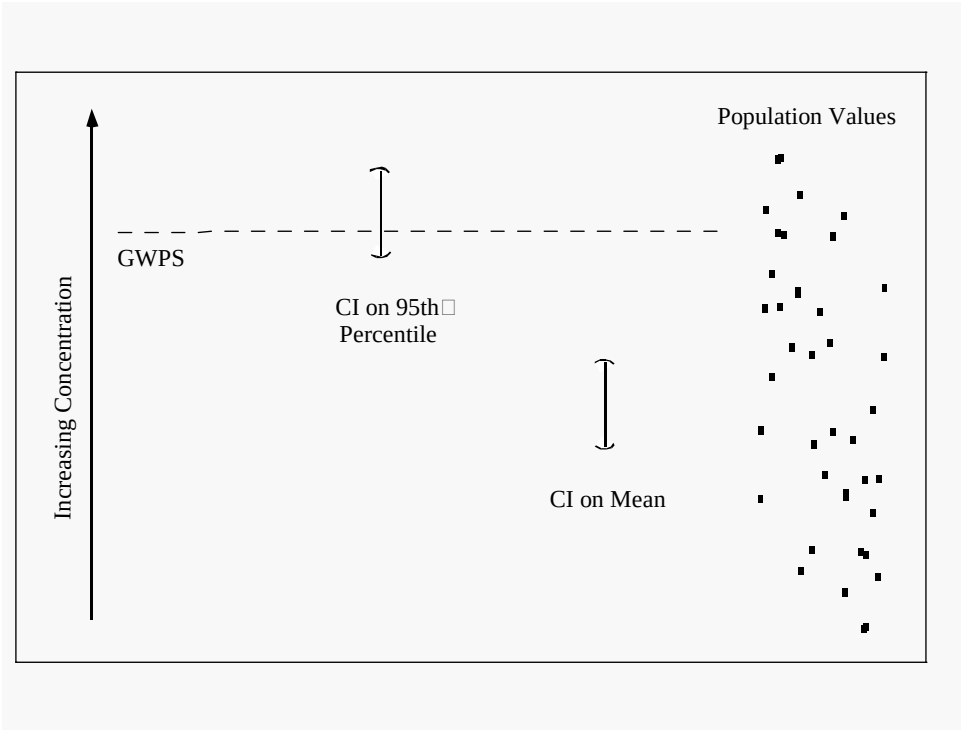
Rejection Region: Reject null hypothesis (H_0) if $UCL_{1-\alpha} < G$; otherwise, accept null hypothesis

For all confidence intervals and tests presented in **Chapters 21** and **22**, the test structures are similar to those above. But not every pair of lower and upper confidence limits (*i.e.*, LCL and UCL) will be symmetric, particularly for skewed distributions and in non-parametric tests on upper percentiles. For a non-parametric technique such as a confidence interval around the median, exact confidence levels will depend on the available sample size and which *order statistics* are used to estimate the desired population parameter. In these cases, an exact target confidence level may or may not be attainable.

When calculating confidence intervals, assignment of the false positive error (α) differs between a one-sided and two-sided confidence interval test. The symmetric upper and lower confidence intervals are shown in **Figure 7-1** largely for illustration purposes. If the *lower* confidence interval for some tested parameter Θ is the critical limit, all of the α error is assigned to the region below the LCL(Θ). Hence, a 1-α confidence level covers the range from the lower limit to positive infinity. Similarly, all of the α error for an upper confidence limit UCL(Θ) is assigned to the region above this value. For a two-

sided interval, the error rate is equally partitioned on both sides of the respective confidence interval limits. A 95% lower confidence limit implies that a 5% chance of an error exists for values lying below the limit. In contrast, a two-sided 95% confidence interval implies a 2.5% chance above and a 2.5% chance of an error below the confidence level. Depending on how confidence intervals are defined, the appropriate statistical adjustment (e.g., the *t*-value in **Equations 7-3** and **7-4**) needs to take this into account.

Figure 7-1. Confidence Interval on Mean vs. Fixed Upper Percentile Limit



7.3 GROUNDWATER PROTECTION STANDARDS

A second essential design step is to identify the appropriate population parameter and its associated statistical estimate. This is primarily a determination of what a given fixed GWPS represents in statistical terms. Not all fixed concentration standards are meant to represent the same statistical quantities. A distinction is drawn between 1) those central tendency standards designed to represent a mean or average concentration level and 2) those which represent either an upper percentile or the maximum of the concentration distribution. If the fixed standard represents an average concentration, it is assumed in the Unified Guidance that the *mean* concentration (or possibly the *median* concentration) in groundwater should not exceed the limit. When a fixed standard represents an *upper percentile* or *maximum*, no more than a small, specified fraction of the individual concentration measurements should exceed the limit.

The choice of confidence interval should be based on the type of fixed standard to which the groundwater data will be compared. A fixed limit best representing an upper percentile concentration (*e.g.*, the upper 95th percentile) should not be compared to a confidence interval constructed around the arithmetic mean. Such an interval only estimates the location of the population mean, but says nothing about the specific upper percentile of the concentration distribution. The *average* concentration level

could be substantially less than the standard even though a significant fraction of the individual measurements exceeds the standard (see **Figure 7-1**).

There are a variety of fixed standards to which different statistical measures apply. Alternative GWPSs based on Agency risk-assessment protocols are cited as an option in the solid waste regulations at §258.55(i)(1). Many of the risk-assessment procedures identified in the CERCLA program make use of chronic, long-term exposure models for ingestion or inhalation. These procedures are identified in the (EPA, 1989b) Risk Assessment Guidance for Superfund (RAGS) and the Supplemental Guidance for Calculating the Concentration Term (EPA, 1992c), and serve as guidance for other EPA programs. In the latter document, the *arithmetic mean* is identified as the appropriate parameter for identifying environmental exposure levels. The levels are intended to identify chronic, time-weighted *concentration averages* based on lifetime exposure scenarios.

The primary *maximum contaminant levels* [MCL] promulgated under the Safe Drinking Water Act (SDWA) follow the same exposure evaluation principles. An MCL is typically based on 70-year risk-exposure scenarios (for carcinogenic compounds), assuming an ingestion rate of 2 liters of water per day at the average concentration over time. Similarly, long-term risk periods (*e.g.*, 6-years) are used for non-carcinogenic constituents, assuming average exposure concentrations. The promulgated levels also contain a safety multiplicative factor and are applied at the end-user tap. Calculations for ingestion exposure risk to soil contaminants by an individual randomly traversing a contaminated site are based on the average estimated soil concentration. It is expected that an exposed individual drinking the water or ingesting the soil is not afforded any protection in the form of prior treatment.

Other standards which may represent a population mean include some RCRA site permits that include comparisons against an *alternate concentration limit* [ACL] based on the average value of background data. In addition, some standards represent time-weighted averages used for carcinogenic risk assessments such as the *lifetime average daily dose* [LADD].

Fixed limits based explicitly on the *median concentration* include *fish ingestion exposure factors*, used in testing fish tissue for certain contaminants. The exposure factors represent the allowable concentration level below which at least half of the fish sample concentrations should lie, the 50th percentile of the observed concentration distribution. If this distribution is symmetric, the mean and median will be identical. For positively skewed populations, the mean concentration could exceed the exposure factor even though the median (and hence, a majority of the individual concentrations) is below the limit. It would therefore not be appropriate to compare such exposure factors against a confidence interval around the mean contaminant level, unless one could be certain the distribution was symmetric.

Fixed standards are sometimes based on *upper percentiles*. Scenarios of this type include risk-based standards designed to limit acute effects that result from short-term exposures to certain chemicals (*e.g.*, chlorine gas leaking from a rail car or tanker). There is greater interest in possible acute effects or transient exposures having a significant short-term risk. Such exposure events may not happen often, but can be important to track for monitoring and/or compliance purposes.

When even short exposures can result in deleterious health or environmental effects, the fixed limit can be specified as a maximum allowable concentration. From a statistical standpoint, the standard identifies a level which can only be exceeded a small fraction of the time (*e.g.*, the upper 90th percentile). If a larger than allowable fraction of the individual exposures exceeds the standard, action is

likely warranted, even if the average concentration level is below the standard. Certain MCLs are interpreted in this same manner; the term 'maximum' in maximum contaminant level would be treated statistically as an upper percentile limit. Examples include criteria for bacterial counts and nitrate/nitrite concentrations, best regarded as upper percentile limits.

As an example, exposure of infants to nitrate concentrations in excess of 10 mg/L (NO_3^- as N) in drinking water is a case where greater concern surrounds acute effects resulting from short-term exposure. The flora in the intestinal tract of infant humans and animals does not fully develop until the age of about six months. This results in a lower acidity in the intestinal tract, which permits the growth of nitrate reducing bacteria. These bacteria convert nitrate to nitrite. When absorbed into the bloodstream, nitrite interferes with the absorption of oxygen. Suffocation by oxygen starvation in this manner produces a bluish skin discoloration — a condition known as "blue baby" syndrome (or methemoglobinemia) — which can result in serious health problems, even death. In such a scenario, suppose that acute effects resulting from short-term exposure above some critical level should normally occur in no more than 10 percent of all exposure events. Then the critical level so identified would be equivalent to the upper 90th percentile of all exposure events.

Another example is the so-called 20-year flood recurrence interval for structural design. Flood walls and drainage culverts are designed to handle not just the average flood level, but also flood levels that have a 1 in 20 chance of being equaled or exceeded in any single year. A 20-year flood recurrence level is essentially equivalent to estimating the upper 95th percentile of the distribution of flood levels (*e.g.*, a flood of this magnitude is expected to occur only 5 times every 100 years).

The various limits identified as potential GWPS in **Chapter 2** pose some interpretation problems. §264.94 Table 1 values are identified as "Maximum Concentration[s] of Constituents for Groundwater Protection" for 14 hazardous constituents, originating from earlier Federal Water Pollution Control Administration efforts. While not a definitive protocol for comparison, it was indicated that the limits were intended to represent a concentration level that should not be exceeded most of the time. In an early Water Quality Criteria report (USDI, 1968), the language is as follows:

> "It is clearly not possible to apply these (drinking water) criteria solely as maximum single sample values. The criteria should ***not be exceeded over substantial portions*** of time."

Similarly, the more current MCLs promulgated under the SDWA are identified as "maximum contaminant limits". Even if the limits were derived from chronic, risk-based assessments, the same implication is that these limits should not be exceeded.

Individual EPA programs make sample data comparisons to MCLs using different approaches. For small-facility systems monitored under the SDWA, only one or two samples a year might be collected for comparison. Anything other than direct comparisons isn't possible. Some Clean Water Act programs use arithmetic comparisons (means or medians) rather than a fully statistical approach. CERCLA typically utilizes these standards in mean statistical comparisons, consistent with other chronic health-based levels derived from their program risk assessment equations. In short, EPA nationwide does not have a single operational definition or measure for assessing MCLs with sample data.

The Unified Guidance cannot directly resolve these issues. Since the regulations promulgated under RCRA presume the use of fully statistical measures for groundwater monitoring program

evaluations, the guidance provides a number of options for both centrality-based and upper limit tests. It falls upon State or Regional programs to determine which is the most appropriate parameter for comparison to a GWPS. As indicated above, the guidance does recommend that any operational definition of the appropriate parameter of comparison to GWPS's be applied uniformly across a program.

If a mean- or median-based centrality parameter is chosen, the guidance offers fairly straightforward confidence interval testing options. For a parameter representing some infrequent level of exceedance to address the "maximum" or "most" criteria, the program would need to identify a specific upper proportion and confidence level that the GWPS represents. Perhaps a proportion of 80 to 95% would be appropriate, at 90-95% confidence. It is presumed that the same standard would apply to both compliance and corrective action testing under §264.99 and §264.100. If non-parametric upper proportion tests must be used for certain data, very high proportions make for especially difficult tests to determine a return to compliance (**Chapter 22**) because of the number of samples required.

7.4 DESIGNING A STATISTICAL PROGRAM

7.4.1 FALSE POSITIVES AND STATISTICAL POWER IN COMPLIANCE/ASSESSMENT

As discussed in **Chapters 3 and 6**, the twin criteria in designing an acceptable detection monitoring statistical program are the site-wide false positive rate [SWFPR] and the effective power of the testing regimen. Both statistical measures are crucial to good statistical design, although from a regulatory perspective, ensuring adequate power to detect contaminated groundwater is of primary importance.

In compliance/assessment monitoring, statistical power is also of prime concern to EPA. There should be a high probability that the statistical test will positively identify concentrations that have exceeded a fixed, regulatory standard. In typical applications where a confidence interval is compared against a fixed standard, a low false positive error rate (α) is chosen without respect to the power of the test. Partly this is due to a natural desire to have high *statistical confidence* in the test, where ($1-\alpha$) designates the confidence level of the interval. But statistical confidence is *not* the same as power. The confidence level merely indicates how often — in repeated applications — the interval will contain the true population parameter (Θ); not how often the test will indicate an exceedance of a fixed standard. It has historically been much easier to select a single value for the false positive rate (α) than to measure power, especially since power is not a single number but a *function* of the level of contamination (as discussed in **Section 3.5**).

The power to detect increases above a fixed standard using a lower confidence limit can be negligible when contaminant variability is high, the sample size is small and especially when a high degree of confidence has been selected. To remedy this problem, the Unified Guidance recommends reversing the usual sequence: first select a desired level of power for the test ($1-\beta$), and then compute the associated (maximum) false positive rate (α). In this way, a pre-specified power can be maintained even if the sample size is too low to simultaneously minimize the risks of both Type I and Type II errors (*i.e.*, false positives and false negatives).

Specific methods for choosing power and computing false positive rates with confidence interval tests are presented in **Chapter 22**. Detailed applications of confidence interval tests are provided in **Chapter 21**. The focus here is on setting a basic framework and consistent strategies.

As noted above, selecting false positive error rates in compliance or assessment testing (§264.99) has traditionally been accomplished under RCRA by choosing a fixed, individual test α. This strategy is attractive if only for the sake of simplicity. Individual test-wise false positive rates in the range of α = .01 to α = .10 are traditional and easily understood. In addition, the Part 264 regulations in §264.97(i)(2) require a minimum individual false positive rate of α = .01 in both compliance and corrective action testing against a fixed standard, as well as in those tests not specifically exempted under detection monitoring.[1]

Given a *fixed* sample size and constant level of variation, the statistical power of a test method drops as the false positive rate decreases. A low false positive rate is often associated with low power. Since statistical power is of particular concern to EPA in compliance/assessment monitoring, somewhat higher false positive rates than the minimum α = .01 RCRA requirement may be necessary to maintain a pre-specified power over the range of sample sizes and variability likely to be encountered in RCRA testing situations. The key is sample variability. When the true population coefficient of variation [CV] is no greater than 0.5 (whether the underlying distribution is normal or lognormal), almost all lower confidence limit tests exhibit adequate power. When the variation is higher, the risk of false negative error is typically much greater (and thus the power is lower), which may necessitate setting a larger than usual individual α.

Based on the discussion regarding false positives in detection monitoring in **Chapter 6**, some might be concerned about the use of relatively high individual test-wise false positive rates (α) in order to meet a pre-specified power, especially when considering the cumulative false positive error rate across multiple wells and/or constituents (*i.e.*, SWFPR). Given that a number of compliance wells and constituents might need to be tested, the likelihood of occurrence of at least one false positive error increases dramatically. However, several factors specific to compliance/assessment monitoring need to be considered. Unlike detection monitoring where the number of tests is easily identified, the issue is less obvious for compliance/assessment or corrective action testing. The RCRA regulations do not clearly specify which wells and constituents must be compared to the GWPS in compliance/assessment monitoring other than wells at the 'compliance point.' In some situations, this has been interpreted to mean all compliance wells; in other instances, only at those wells with a documented exceedance.

While all hazardous constituents including additional ones detected in Part 264 Appendix IX monitoring are potentially subject to testing, many may still be at concentration levels insignificantly different from onsite background. Constituents without health-based limits may or may not be included in compliance testing. The latter would be tested against background levels, using perhaps an ACL computed as a *tolerance limit on background* (see **Section 7.5**). This also tends to complicate derivation of SWFPRs in compliance testing. It was also noted in **Section 7.2** that the levels at which contaminants are released bear no necessary relationship to fixed, health-based standards. In a typical release, some constituent levels from a suite of analytical parameters may lie orders of magnitude below their GWPS, while certain carcinogenic compounds may easily exceed their standards.

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[1] In some instances, a test with "reasonable confidence" (that is, having adequate statistical power) for identifying compliance violations can be designed even if $\alpha < 0.01$. This is particularly the case when the sample coefficient of variation is quite low, indicating small degrees of sample variability.

The simple example below illustrates typical low-level aquifer concentrations following a release of four common petrochemical facility hazardous organic constituents often detected together:

Analyte	Aquifer Concentration (ug/l)		MCL (ug/l)
	Mean	SD	
Benzene	20	10	5
Toluene	35	15	1,000
Ethylbenzene	40	20	700
Xylene	100	35	10,000

While benzene as a carcinogen has a very low health standard, the remaining three constituents have aquifer concentrations orders of magnitude lower than their respective MCLs. Realistically, only benzene is likely to impact the cumulative false positive rate in LCL testing. Similar relationships occur in releases measured by trace element and semi-volatile organic suites.

Even though the null hypotheses in detection and compliance/assessment monitoring are similar (and compound) in nature (see [7.1]), it is reasonable to presume in detection monitoring that the compliance wells have average concentrations *no less* than mean background levels.[2] Since it is these background levels to which the compliance point data are compared in the absence of a release, the compound null hypothesis in detection monitoring (H_0: $\mu_C \leq \mu_{BG}$) can be reformulated practically as (H_0: $\mu_C = \mu_{BG}$). In this framework, individual concentration measurements are likely to occasionally exceed the background average and at times cause false positives to be identified even when there has been no change in average groundwater quality.

In compliance/assessment monitoring, the situation is generally different. The compound null hypothesis (H_0: $\mu_C \leq$ GWPS) will include some wells and constituents where the sample mean equals or nearly equals the GWPS when testing begins. But many well-constituent pairs may have true means considerably less than the standard, making false positives much less likely for those comparisons and lowering the overall SWFPR. How much so will depend on both the variability of each individual constituent and the degree to which the true mean (or relevant statistical parameter Θ) is lower than the GWPS for that analyte.

Because of this, determining the relevant number of comparisons with non-negligible false positive error rates may be quite difficult. The SWFPR in this situation would be defined as the probability that at least one or more lower confidence limits exceeded the fixed standard G, when the true parameter Θ (usually the mean) was actually below the standard. However, the relevant number of comparisons will depend on the nature and extent of the release. For a more extensive release, there is greater likelihood that the null hypothesis is no longer true at one or more wells. Instead of computing false positive rates, the focus should shift to minimizing false negative errors (*i.e.*, the risk of missing contamination above the GWPS).

[2] Note that background might consist of early intrawell measurements from compliance wells when substantial spatial variability exists.

On balance, the Unified Guidance considers computation of cumulative SWFPRs in compliance/assessment testing to be problematic, and reliance on individual test false positive rates preferable. The above arguments also suggest that flexibility in setting individual test-wise α levels may be appropriate.

7.4.2 FALSE POSITIVES AND STATISTICAL POWER IN CORRECTIVE ACTION

When contamination above a GWPS is confirmed, corrective action is triggered. Following a period of remediation activity, formal statistical testing will usually involve an *upper* confidence limit around the mean or an upper percentile compared against a GWPS. EPA's overriding concern in corrective action is that remediation efforts not be declared successful without sufficient statistical proof. Since groundwater is now presumed to be impacted at unacceptable levels, a facility should not exit corrective action until there is sufficient evidence that contamination has been abated.

Given the reversal of test hypotheses from compliance/assessment monitoring to corrective action (*i.e.*, comparing equation [7.1] with [7.2]), there is an asymmetry in regulatory considerations of false positive and false negative rates depending on the stage of monitoring. In compliance/assessment monitoring using tests of the lower confidence limit, the principal regulatory concern is that a given test has adequate statistical power to detect exceedances above the GWPS.

Permitted RCRA monitoring is likely to involve small annual well sample sizes based on quarterly or semi-annual sampling. To meet a pre-specified level of power by controlling the false negative rate (β) necessitates varying the false positive rate (α) for individual tests. Controlling an SWFPR for these tests (using a criterion like the SWFPR) is usually not practical because of the ambiguity in identifying the relevant number of potential tests and the difficulty of properly assigning via the subdivision principle (**Chapter 19**) individual fractions of a targeted SWFPR.

By contrast under corrective action using an *upper* confidence limit for testing, the principal regulatory and environmental concern is that one or more constituents might falsely be declared below a GWPS in concentration. Under the corrective action null hypothesis [7.2] this would be a *false positive error*, implying that α should be minimized during this sort of testing, instead of β. Specific methods for accomplishing this goal are presented in **Chapter 22**.

A remaining question is whether SWFPRs should be controlled during corrective action. While potentially desirable, the number of well-constituent pairs exceeding their respective GWPS and subject to corrective action testing is likely to be small relative to compliance testing. Not all compliance wells or constituents may have been impacted, and some may not be contaminated to levels exceeding the GWPS, depending on the nature, extent, and intensity of the plume. Remediation efforts would focus on those constituents exceeding their GWPS.

As noted in **Section 7.4.1**, the tenuous relationship between ambient background levels, contaminant magnitudes, and risk-based health standards implies that most GWPS exceedances are likely to be carcinogens, usually representing a small portion of all monitored constituents. Some exceedances may also be related compounds, for instance, chlorinated hydrocarbon daughter degradation products.

Statistically, the fact that some wells are contaminated while others may not be makes it difficult to define SWFPRs in corrective action. Instead, the Unified Guidance attempts to limit the individual test-wise α at those wells where exceedances have been confirmed and that are undergoing remediation. Since the most important consideration is to ensure that the true population parameter (Θ) is actually below the clean-up standard before declaring remediation a success, this guidance recommends the use of a reasonably low, *fixed* test-wise false positive rate (*e.g.*, α = .05 or .10). Under this framework, there will be a 5% to 10% chance of incorrectly declaring any single well-constituent pair of being in compliance when its concentrations are truly above the remedial standard.

The regulatory position in corrective action concerning statistical power is one of relative indifference. Although power under [7.2] represents the probability that the confidence interval test will correctly identify concentrations to be below the regulatory standard when in fact they are, the onus of proof for demonstrating that remediation has succeeded (*e.g.*, μ_C < GWPS) falls on the regulated facility. As it is the facility's interest to demonstrate compliance, it may wish to develop statistical power criteria which would enhance this possibility (including increasing test sample sizes).

7.4.3 RECOMMENDED STRATEGIES

As noted in **Section 7.1**, the Unified Guidance recommends the use of confidence intervals in both compliance/assessment and corrective action testing. In compliance/assessment, the lower confidence limit is the appropriate statistic of interest, while in corrective action it is the upper confidence limit. In either case, the confidence limit is compared against a fixed, regulatory standard as a one-sample test. These recommendations are consistent with good statistical practice, as well as literature in the field, such as Gibbons and Coleman (2001).

The type of confidence interval test will initially be determined by the choice of parameter(s) to represent the GWPS (**Section 7.2**). While this discussion has suggested that the mean may be the most appropriate parameter for chronic, health-based limits, other choices are possible. **Chapter 21** identifies potential test statistical tests of a mean, median or upper percentile as the most appropriate parameters for comparison to a GWPS. In turn, data characteristics will determine whether parametric or non-parametric test versions can be used. Depending on whether normality can be assumed for the original data or following transformation, somewhat different approaches may be needed. Finally, the presence of data trends affects how confidence interval testing can be applied.

Some regulatory programs prefer to compare each *individual* measurement against *G*, identifying a well as out-of-compliance if any of the individual concentrations exceeds the standard. However, the false positive rate associated with such strategies tends to be quite high if the parameter choice has not been clearly specified. Using this individual comparison approach and assuming a mean as the parameter of choice, is of particular concern. If the true mean is *less than but close to* the standard, chances are very high that one or more individual measurements will be greater than the limit even though the hypothesis in [7.1] has not been violated. Corrective action could then be initiated on a false premise. To evaluate whether a limited number of sample data exceed a standard, a lower confidence interval test would need to be based on a pre-specified upper percentile assumed to be the appropriate parameter for comparison to the GWPS.

Small individual well sample sizes and data uncertainty can rarely be avoided in compliance/assessment and corrective action. Given the nature of RCRA permits, sampling frequencies

in compliance/assessment or corrective action monitoring are likely to be established in advance. Relatively small sample sizes per well-constituent pair each year are likely to be the rule; the Unified Guidance assumes that quarterly and semi-annual sampling will be very typical.

For small and highly variable sample data sets, compliance/assessment monitoring and corrective action tests will have low statistical power either to detect exceedances above fixed standards or to demonstrate compliance in corrective action. One way to both enhance statistical power and control false positive error rates is through *incremental* or *sequential pooling* of compliance point data over time. Adding more data into a test of non-compliance or compliance will generally result in narrower confidence intervals and a clearer decision with respect to a compliance standard.

The Unified Guidance recommends accumulating compliance data over time at each well, by allowing construction of confidence limits on *overlapping* as opposed to *distinct* or *mutually exclusive* data sets. If the lower confidence limit [LCL] exceeds the GWPS in compliance/assessment, a clear exceedance can be identified. If the upper confidence limit [UCL] is below the GWPS in corrective action, remediation at that well can be declared a success. If neither of these respective events occurs, further sampling should continue. A confidence interval can be recomputed after each additional 1 or 2 measurements and a determination made whether the position of the confidence limit has changed relative to the compliance standard.

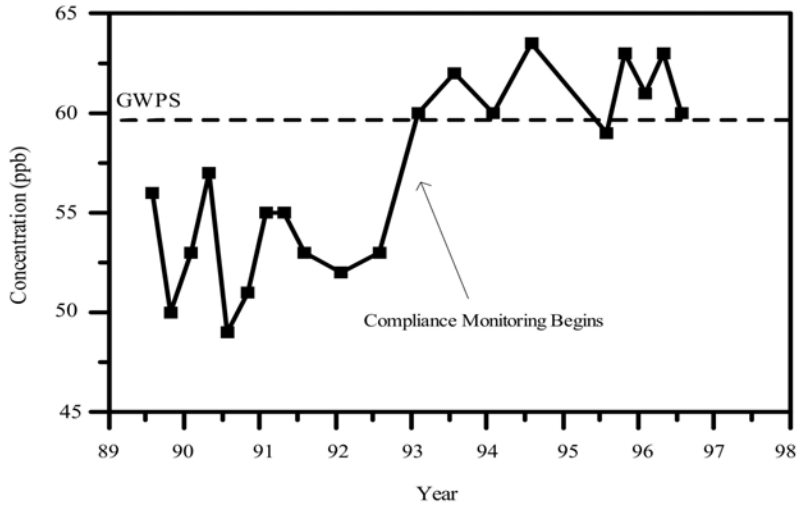
Tests constructed in this way at each successive evaluation period will not be statistically independent; instead, the proposed testing strategy falls into the realm of *sequential analysis*. But it should help to minimize the possibility that a small group of spurious values will either push a facility into needless corrective action or prevent a successful remedial effort from being identified.

One caveat with this approach is that it must be reasonable to assume that the population parameter Θ is stable over time. If a release has occurred and a contaminant plume is spreading through the aquifer, concentration shifts in the form of increasing trends over time may be more likely at contaminated wells. Likewise under active remediation, decreasing trends for a period of time may be more likely. Therefore, it is recommended that the sequential testing approach be used *after* aquifer conditions have stabilized to some degree. While concentration levels are actively changing with time, use of confidence intervals around a trend line should be pursued (see **Section 7.4.4** and **Chapter 21**).

7.4.4 ACCOUNTING FOR SHIFTS AND TRENDS

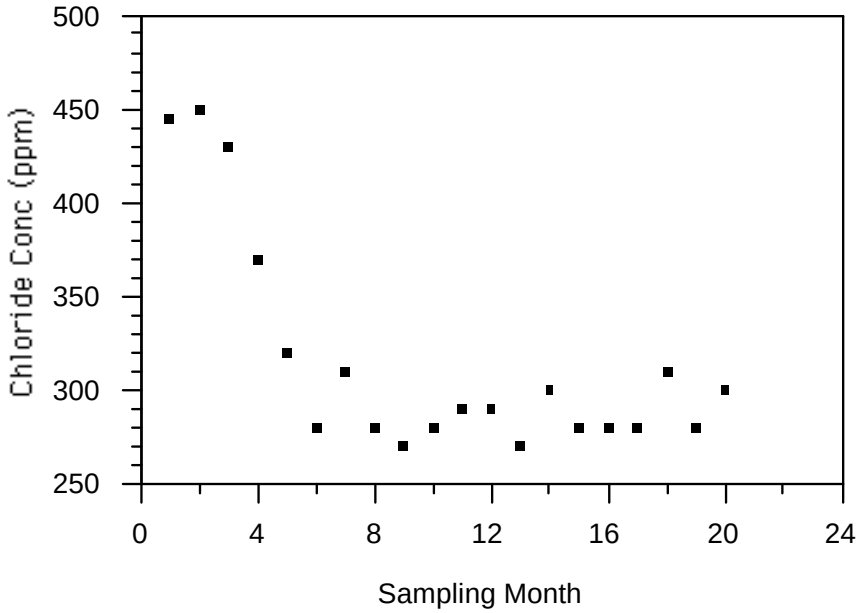
While accumulating compliance point data over time and successively re-computing confidence limits is appropriate for stable (*i.e.*, stationary) populations, it can give misleading or false results when the underlying population is changing. Should a release create an expanding contaminant plume within the aquifer, concentration levels at some or all of the compliance wells will tend to shift upward, either in discrete jumps (as illustrated in **Figure 7-2**) or an increasing trend over time. In these cases, a lower confidence limit constructed on accumulated data will be overly wide (due to high sample variability caused by combining pre- and post-shift data) and not be reflective of the more recent upward shift in the contaminant distribution.

Figure 7-2. Effect on Confidence Intervals of Stable Contamination Level



A similar problem can arise with corrective action data. Aquifer modifications as part of contaminant removals are likely to result in observable declines in constituent concentrations during the active treatment phase. At some point following cessation of remedial action, a new steady-state equilibrium may be established (**Figure 7-3**).

Figure 7-3. Decreasing Trend During Corrective Action

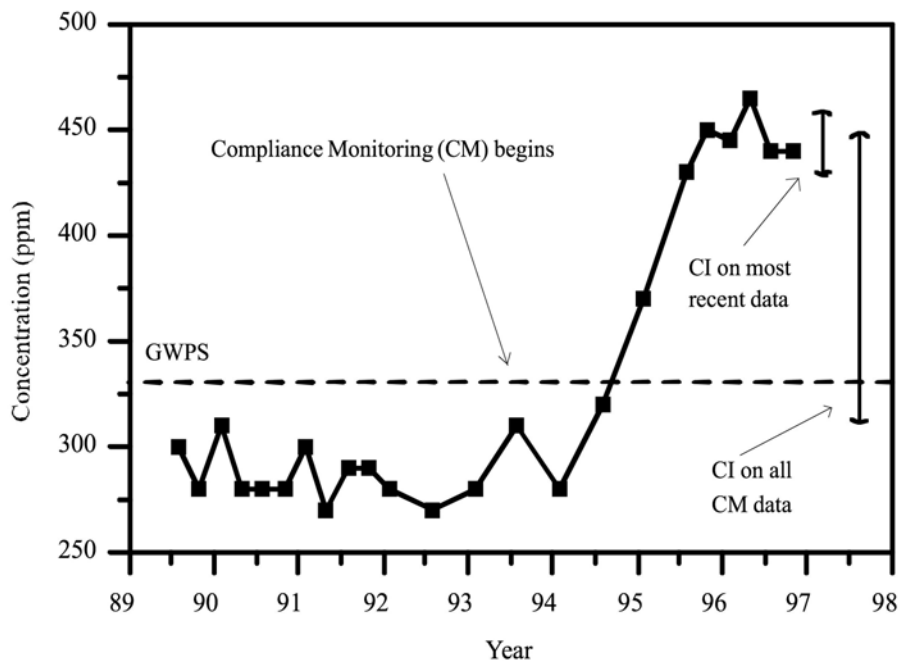


Until then, it is inappropriate to use a confidence interval test around the mean or an upper percentile to evaluate remedial success with respect to a clean-up standard. During active treatment phases and under non-steady state conditions, other forms of analysis such as confidence bands around a trend (see below), are recommended and should be pursued.

The Unified Guidance considers two basic types of non-stationary behavior: shifts and (linear) trends. A shift refers to a significant mean concentration increase or decrease departing from a roughly stable mean level. A trend refers to a series of consecutive measurements that evidence successively increasing or decreasing concentration levels. More complicated non-random data patterns are also possible, but beyond the scope of this guidance. With these two basic scenarios, the strategy for constructing an appropriate confidence interval differs.

An important preliminary step is to track the individual compliance point measurements on a time series plot (**Chapter 9**). If a discrete shift in concentration level is evident, a confidence limit should be computed on the most recent stable measurements. Limiting the observations in this fashion to a specific time period is often termed a 'moving window.' The reduction in sample size will often be more than offset by the gain in statistical power. More recent measurements may exhibit less variation around the shifted mean value, resulting in a shorter confidence interval (**Figure 7-4**). The sample size included in the moving window should be sufficient to achieve the desired statistical power (compliance/assessment) or false positive rate (corrective action). However, measurements that are clearly unrepresentative of the newly shifted distribution should not be included, even if the sample size suffers. Once a stable mean can be assumed, the strategy of sequential pooling can be used.

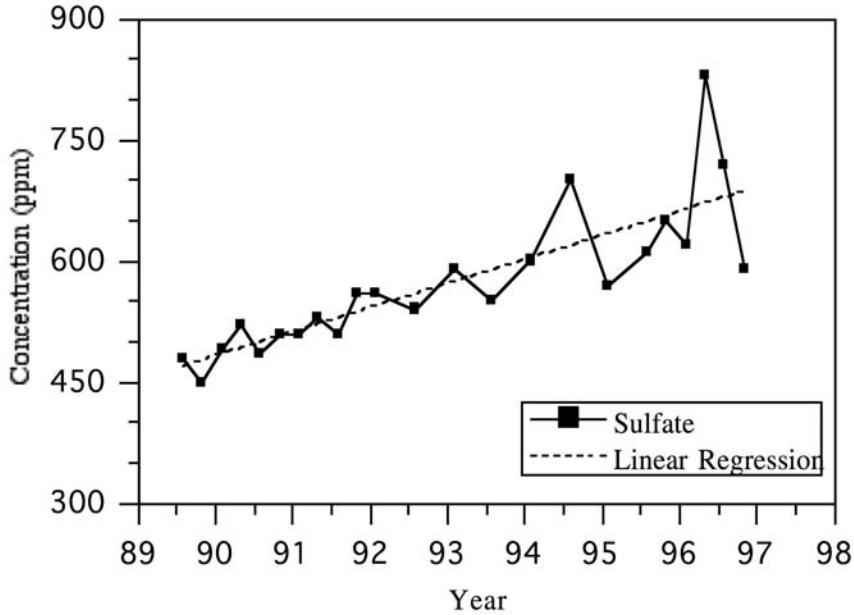
Figure 7-4. Effect on Confidence Intervals of Concentration Shift



If well concentration levels exhibit an increasing or decreasing trend over time (such as the example in **Figure 7-5**) and the pattern is reasonably linear or monotone, the trend can be identified using the methods detailed in **Chapter 17**. To measure compliance or non-compliance, a *confidence band* can be constructed around the estimated trend line, as described in **Chapter 21**. A confidence band is essentially a continuous series of confidence intervals estimated along every point of the trend. Using this technique, the appropriate upper or lower confidence limits at one or more points in the most recent

portion of the end of the sampling record can be compared against the fixed standard. The lower band is used to determine whether or not an exceedance has occurred in compliance/assessment, and an upper confidence band to determine if remedial success has been achieved in corrective action.

Figure 7-5. Rising Trend During Compliance Monitoring



By explicitly accounting for the trend, the confidence interval in **Chapter 21** will adjust upward or downward with the trend and thus more accurately estimate the current true concentration levels. Trend techniques are not just used to track progress towards exceeding or meeting a fixed standard. Confidence bands around the trend line can also provide an estimate of confidence in the average concentration as it changes over time. This subject is further covered in the Comprehensive Environmental Response, Compensation, and Liability Act [CERCLA] guidance ***Methods for Evaluating the Attainment of Cleanup Standards — Volume 2: Groundwater*** (EPA, 1992a).

A final determination of remedial success should not solely be a statistical decision. In many hydrologic settings, contaminant concentrations tend to rise after groundwater pumping wells are turned off due to changes in well drawdown patterns. Concentration levels may exhibit more complicated behavior than the two situations considered above. Thus, on balance, it is recommended that determining achievement of corrective action goals be done in consultation with the site manager, geologist, and/or remedial engineer.

7.4.5 IMPACT OF SAMPLE VARIABILITY, NON-DETECTS, AND NON-NORMAL DATA

Selection of hazardous constituents to be monitored in compliance/assessment or corrective action is largely determined by permit decisions. Regulatory requirements (*e.g.*, Part 264, Appendix IX) may also dictate the number of constituents. As a practical matter, the most reliable indicators of contamination should be favored. Occasionally, constituents subject to degradation and transformation in the aquifer (*e.g.*, chlorinated hydrocarbon suites) may result in additional, related constituents of concern.

Since health-based considerations are paramount in this type of monitoring, the most sensitive constituents from a health risk standpoint could be selected. But even with population parameters (Θ), sample sizes, and constituents determined, selecting an appropriate confidence interval test from **Chapter 21** can be problematic. For mildly variable sample data, measured at relatively stable levels, tests based on the normal distribution should be favored, whether constructed around a mean or an upper percentile. With highly variable sample data, selection of a test is less straightforward. If the observed data happen to be lognormal, Land's confidence interval around the arithmetic mean is a valid option; however, it has low power to measure compliance as the observations become more variable, and upward adjustment of the false positive rate (α) may be necessary to maintain sufficient power.

In addition, the extreme variability of an upper confidence limit using Land's technique can severely restrict its usage in tests of compliance during corrective action. Depending on the data pattern observed, degree of variability, and how closely the sample mimics the lognormal model, consultation with a professional statistician should be considered to resolve unusual cases. When the lognormal coefficient of variation is quite high, one alternative is to construct an upper confidence limit around the lognormal geometric mean (**Chapter 21**). Although such a confidence limit does not fully account for extreme concentration values in the right-hand tail of the lognormal distribution, a bound on the geometric mean will account for the bulk of possible measurements. Nonetheless, use of a geometric mean as a surrogate for the population arithmetic mean leads to distinctly different statistical test characteristics in terms of power and false positive rates.

In sum, excessive sample variability can severely limit the effectiveness of traditional compliance/assessment and corrective action testing. On the other hand, if excessive variability is primarily due to trends observable in the data, confidence bands around a linear trend can be constructed (**Section 7.4.4**).

LEFT-CENSORED SAMPLES

For compliance point data sets containing left-censored measurements (*i.e.*, non-detects), parametric confidence intervals cannot be computed directly without some adjustment. All of the parametric confidence intervals described in **Chapter 21** require estimates of the population mean μ and standard deviation σ. A number of adjustment strategies are presented in **Chapter 15.** If the percentage of non-detects is small — no more than 10-15% — *simple substitution* of half the reporting limit [RL] for each non-detect will generally work to give an approximately correct confidence interval.

For samples of at least 8-10 measurements and up to 50% non-detects, the *Kaplan-Meier* or *robust regression on order statistics* [ROS] methods can be used. Data should first be assessed via a *censored probability plot* whether the sample can be normalized. If so, these techniques can be used to compute estimates of the mean μ and standard deviation σ adjusted for the presence of left-censored values. These adjusted estimates can be used in place of the sample mean ($\bar{x}$) and standard deviation (s) listed in the confidence interval formulas of **Chapter 21** around either a mean or upper percentile.

If none of these adjustments is appropriate, non-parametric confidence intervals on either the median or an upper percentile (**Section 21.2**) can be calculated. Larger sample sizes are needed than with parametric confidence interval counterparts, especially for intervals around an upper percentile, to ensure a high level of confidence and a sufficiently narrow interval. The principal advantage of non-parametric

intervals is their flexibility. Not only can large fractions of non-detects be accommodated, but non-parametric confidence intervals can also be applied to data sets which cannot be normalized.

For heavily censored small data sets of 4-6 observations, the options are limited. One approach is to replace each non-detect by half its RL and compute the confidence interval *as if* the sample were normal. Though the resulting interval will be approximate, it can provide a preliminary indication of the well's compliance with the standard until further sampling data can be accumulated and the confidence interval recomputed.

Confidence bands around a trend can be constructed with censored data using a bootstrapped Theil-Sen non-parametric trend line (**Section 21.3.2**). In this method, the Theil-Sen trend is first computed using the sample data, accounting for the non-detects. Then a large number bootstrap resamples are drawn from the original sample, and an alternate Theil-Sen trend is conducted on each bootstrap sample. Variability in these alternate trend estimates is then used to construct a confidence band around the original trend.

LOGNORMAL AND OTHER NORMALIZED DATA

Lognormal data may require special treatment when building a confidence interval around the mean. Land's method (**Section 21.1.3**) can offer a reasonable way to accommodate the transformation bias associated with the logarithm, particularly when computing a lower confidence limit as recommended in compliance/assessment monitoring. For data normalized by transformations other than the logarithm, one option is to calculate a normal-based confidence interval around the mean using the transformed measurements, then back-transform the limits to the original concentration scale. The resulting interval will *not* represent a confidence interval around the arithmetic mean of the original data, but rather will estimate the confidence intervals of the *median* and/or *geometric mean*.

If the difference between the arithmetic mean and median is not considered important for a given GWPS, this strategy will be the easiest to implement. A wide range of results can occur with Land's method on highly skewed lognormal populations especially when computing an upper confidence limit around the arithmetic mean (Singh *et al.*, **1997**). It may be better to either construct a confidence interval around the lognormal *geometric* mean (**Section 21.1.2**) or to use the technique of *bootstrapping* (Efron, 1979; Davison and Hinkley, 1997) to create a non-parametric interval around the arithmetic mean.[3]

For confidence intervals around an upper percentile, no bias is induced by data that have been normalized via a transformation. Whatever the transformation used (*e.g.*, logarithm, square root, cube, *etc.*), a confidence interval can be constructed on the transformed data. The resulting limits can then be back-transformed to provide confidence limits around the desired upper percentile in the concentration domain.

[3] Bootstrapping is widely available in statistical software, including the open source **R** computing environment and EPA's free-of-charge **ProUCL** package. In some cases, setting up the procedure correctly may require professional statistical consultation.

7.5 COMPARISONS TO BACKGROUND DATA

Statistical tests in compliance/assessment and corrective action monitoring will often involve a comparison between compliance point measurements and a promulgated fixed health-based limit or a risk-based remedial action goal as the GWPS, described earlier. But a number of situations arise where a GWPS must be based on a background limit. The Part 264 regulations presume such a standard as one of the options under §264.94(a); an ACL may also be determined from background under §264.94(b). More recent Part 258 rules specify a background GWPS where a promulgated or risk-based standard is not available or if the historical background is greater than an MCL [§258.55(h)(2) & (3)].

Health-based risk standards bear no necessary relationship to site-specific aquifer concentration levels. At many sites this poses no problem, since the observed levels of many constituents may be considerably lower than their GWPS. However, either naturally-occurring or pre-existing aquifer concentrations of certain analytes can exceed promulgated standards. Two commonly monitored trace elements in particular-- arsenic and selenium-- are occasionally found at uncontaminated background well concentrations exceeding their respective MCLs. The regulations then provide that a GWPS based on background levels is appropriate.

A number of factors should be considered in designing a background-type GWPS testing program for compliance/assessment or corrective action monitoring. The most fundamental decision is whether to base such comparisons on *two- (or multiple-) sample* versus *single-sample* tests. For the first, many of the design factors discussed for detection monitoring in **Chapter 6** will be appropriate; for single sample comparisons to a fixed background GWPS, a confidence level approach similar to that discussed earlier for testing fixed health standards in this **Chapter 7** would be applied. This basic decision then determines how the GWPS is defined, the appropriate test hypotheses, types of statistical tests, what the background GWPS represents in statistical terms, and the relevance of individual test and cumulative false positive error rates. Such decisions may also be constrained by State groundwater anti-degradation policies. Other design factors to consider are the number of wells and constituents tested, interwell versus intrawell options, background sample sizes, and power. Unlike a single fixed standard like an MCL, background GWPS's may be uniquely defined for a given monitoring well constituent by a number of these factors.

SINGLE- VERSUS TWO-SAMPLE TESTING

One of two fundamental testing approaches can be used with site-specific background GWPSs. Either 1) a GWPS is defined as the critical limit from a pre-selected detection-level statistical test (e.g., a prediction limit) based on background measurements, or 2) background data are used to generate a fixed GWPS somewhat elevated above current background levels. In both cases, the resulting GWPS will be constituent- and possibly compliance well- specific. The first represents a *two-sample test* of two distinct populations (or more if a multiple-sample test) similar to those utilized in detection monitoring. As such, the individual test false positive rate, historical background sample size, cumulative false positive considerations, number of annual tests and desired future sample size will uniquely determine the limit. Whatever the critical value for a selected background test, it becomes the GWPS under compliance/assessment or corrective action monitoring.

The only allowable hypothesis test structure for the two-sample approach follows that of detection and compliance monitoring [7.1]. Once exceeded and in corrective action, a return to compliance is through evidence that future samples lie below the GWPS using the same hypothesis structure.

The second option uses a fixed statistic from the background data as the GWPS in a *single-sample* confidence interval test. Samples from a single population are compared to the fixed limit. In other respects, the strategy follows that outlined in **Chapter 7** for fixed health- or risk-based GWPS tests. The compliance/assessment test hypothesis structure also follows [7.1], but the hypotheses are reversed as in [7.2] for corrective action testing.

The choice of the single-sample GWPS deserves careful consideration. In the past, many such standards were simply computed as multiples of the background sample average (*i.e.*, GWPS = $2\cdot\bar{x}$). However, this approach may not fully account for natural variation in background levels and lead to higher than expected false positive rates. If the GWPS were to be set at the historical background sample mean, even higher false positive rates would occur during compliance monitoring, and demonstrating corrective action compliance becomes almost impossible.

In the recommendations which follow below, an upper tolerance limit based on both background sample size and sample variability is recommended for identifying the background GWPS at a suitably high enough level above current background to allow for reversal of the test hypotheses. Although a somewhat arbitrary choice, a GWPS based on this method allows for a variety of confidence interval tests (e.g., a one-way normal mean confidence interval identified in equations [7.3] and [7.4]).

WHAT A BACKGROUND GWPS REPRESENTS

If the testing protocol involves two-sample comparisons, the background GWPS is an upper limit statistical interval derived from a given set of background data based on one or another detection monitoring tests discussed in **Chapter 6** and detailed in **Part III**. In these cases, the appropriate testing parameter is the true *mean* for the parametric tests, and the true *median* for non-parametric tests. This would include 1-of-*m* prediction limit detection tests involving future values. If a single-sample comparison against a fixed background GWPS is used, the appropriate parameter will also depend upon the type of confidence interval test to be used (**Part IV**). Except for parametric or non-parametric upper percentile comparisons, the likely statistical parameter would again be a mean (arithmetic, logarithmic, geometric) or the median. A background GWPS could be defined as an upper percentile parameter, making use of normal test confidence interval structures found in **Section 21.1.4**. Non-parametric percentile options would likely require test sample sizes too large for most applications. The Unified Guidance recommended approaches for defining single-sample GWPSs discussed later in this section presume a central tendency test parameter like the mean or median.

NUMBER OF MONITORED WELLS AND CONSTITUENTS

Compliance/assessment or corrective action monitoring tests against a fixed health- or risk-based standard (including single-sample background GWPSs) are not affected in a significant manner by the number of annual tests. But this would not be true for two- or multiple-sample background GWPS testing. In similar fashion to detection monitoring, the total number of tests is an important consideration in defining the appropriate false positive error test rate (α_{test}). The total number of annual tests is determined by how many compliance wells, constituents and evaluations occur per year.

Regulatory agency interpretations will determine the number and location of compliance monitoring wells. These can differ depending on whether the wells are unit-specific, and if a reasonable subset can be shown to be affected by a release. Perhaps only those compliance wells containing detectable levels of a compliance monitoring constituent need be included. Formal annual tests are generally required semi-annually, but other approaches may be applied.

The number of constituents subject to two-sample background GWPS testing will also depend on several factors. Only *hazardous* constituents not having a health- or risk-based standard are considered here. The basic criterion in interpreting required Part 264 Appendix IX or Part 258 Appendix II analyses is to identify those hazardous constituents found in downgradient compliance wells. Some initially detected common laboratory or sampling contaminants might be eliminated following a repeat scan. The remainder of the qualifying constituents will then require some form of background GWPS's. Along with the number of wells and annual evaluations, the total annual number of background tests will then be used in addressing an overall design cumulative design false positive rate.

In corrective action testing (for either the one- or two-sample approaches), the number of compliance wells and constituents may differ. Only those wells and constituents showing a significant compliance test exceedance might be used. However, from a standpoint of eventually demonstrating compliance under corrective action, it might be appropriate to still use the compliance/assessment GWPS for two-sample tests. With single-sample tests, the GWPS is compared individually by well and constituent as described.

BACKGROUND SAMPLE SIZES and INTERWELL vs. INTRAWELL TESTING

Some potential constituents may already have been monitored during the detection phase, and have a reasonable background size. Others identified under Part 264 Appendix IX or part 258 Appendix II testing may have no historical background data bases and require a period of background sampling.

Historical constituent well data patterns and the results of this testing may help determine if an interwell or intrawell approach should be used for a given constituent. For example, if arsenic and selenium were historical constituents in detection monitoring, they might also be identified as candidates for compliance background GWPS testing. There may already be indications that individual well spatial differences will need to be taken into account and an intrawell approach followed. In this case, individual compliance well background GWPSs need to be established and tested. On the other hand, certain hazardous trace elements and organics may only be detected and confirmed in one or more compliance wells with non-detects in background upgradient wells and possibly historical compliance well data. Under the latter conditions, the simpler Double Quantification Rule (**Section 6.2.2**.) might be used with the GWPS set at a quantification limit. However, this could pose some interpretation problems. Subsequent testing against the background GWPS at the same compliance well concentration levels causing the initial detection monitoring exceedance, might very likely result in further excursions above the background GWPS. The more realistic option would be to collect and use additional compliance well data to establish a specific minimum intrawell background, and only apply the Double Quantification Rule at other wells not exhibiting detections. Even this approach might be unnecessarily stringent if a contaminant plume were to expand in size and gradually affect other compliance wells (now subject to GWPS testing).

CUMULATIVE & INDIVIDUAL TEST FALSE POSITIVE RATES

Each of the independent two-sample tests against background standards will have a roughly equal probability of being exceeded by chance alone. Since an exceedance in the compliance monitoring mode based on background can result in a need for corrective action, it is recommended that the individual test false positive rate be set sufficiently low. Much of the discussion in **Chapter 6**, **Section 6.2.2** is relevant here. An *a priori*, cumulative error design rate must first be identified. To allow for application of the Unified Guidance detection monitoring strategies and **Appendix D** tables, it is suggested that the .1 *SWFPR* value also be applied to two-sample background GWPS testing. In similar fashion to **Chapter 6** and **Part III**, this can be translated into individual test configurations.

If the single-sample confidence interval option will be used with an elevated GWPS, the compliance level test will have a very low probability of being exceeded by truly background data. Cumulative false positive error considerations are generally negligible. For testing compliance/assessment or corrective action hypotheses, there is still a need to identify an appropriately low single test false positive rate which meets the regulatory goals. Generally, a single test false positive error rate of .1 to .05 will be suitable with the recommended approach for defining the background GWPS.

UNIFIED GUIDANCE RECOMMENDATIONS

Two-Sample GWPS Definition and Testing

As indicated above, any of the detection monitoring tests described in **Chapter 6** might be selected for two- or multiple- sample background compliance testing. One highly recommended statistical test approach is a prediction limit. Either a parametric prediction limit for a future mean (**Section 18.2.2**) or a non-parametric prediction limit for a future median (**Section 18.3.2**) can be used, depending on the constituent being tested and its statistical and distributional characteristics (*e.g.*, detection rate, normality, *etc.*). It would be equally possible to utilize one of the 1-of-*m* future value prediction limit tests, on an interwell or intrawell basis. Use of repeat samples as part of the selected test is appropriate, although the expected number of annual compliance/corrective action samples may dictate which tests can apply.

One parametric example is the 1-of-1 future mean test. If the background data can be normalized, background observations are used to construct a parametric prediction limit with (1–α) confidence around a mean of order *p*, using the equation:

$$PL = \bar{x} + t_{1-\alpha,n-1} \cdot s \cdot \sqrt{\frac{1}{p} + \frac{1}{n}} \qquad [7.5]$$

The next *p* measurements from each compliance well are averaged and the future mean compared to the background prediction limit, *PL* (considered the background GWPS). In compliance/assessment monitoring, if any of the means exceeds the limit, those well-constituent pairs are deemed to be out of compliance. In corrective action, if the future mean is no greater than *PL*, it can be concluded that the well-constituent pair is sufficiently similar to background to be within the remediation goal. In both monitoring phases, the prediction limit is constructed to represent a reasonable upper limit on the

background distribution. Compliance point means above this limit are statistically different from background; means below it are similar to background.

If the background sample cannot be normalized perhaps due to a large fraction of non-detects, two-sample non-parametric upper prediction limit detection monitoring tests (**Chapters 18 & 19**) can be used. As an example, a maximal order statistic (often the highest or second-highest value) can be selected from background as a non-parametric 1-of-1 upper prediction limit test of the median. **Table 18-2** is used to guide the choice based on background sample size (n) and the achievable confidence level (α). The median of the next 3 measurements from each compliance well is compared to the upper prediction limit. As with the parametric case in compliance/assessment, if any of the medians exceeds the limit, those well-constituent pairs would be considered out of compliance. In corrective action, well-constituent pairs with medians no greater than the background prediction limit would be considered as having met the standard.

If background measurements for a particular constituent are all non-detect, the GWPS should be set equal to the highest RL. In similar fashion to detection monitoring, 1-of-2 or 1-of-3 future value prediction limit tests can be applied (**Section 6.2.2** Double Quantification rule).

Single-Sample GWPS Definition and Testing

For single-sample testing, the Unified Guidance recommendation is to define a fixed GWPS or ACL based on a background *upper tolerance limit* with 95% confidence and 95% coverage (**Chapter 17**). For normal background, the appropriate formula for the GWPS would be the same as that given in **Section 17.2.1**, namely:

$$GWPS = \bar{x} + \tau(n,.95,.95) \cdot s \qquad [7.6]$$

where n = number of background measurements, $\bar{x}$ and s represent the background sample mean and standard deviation, and τ is a tolerance factor selected from **Table 17-3**. If the background sample is a mixture of detects and non-detects, but the non-detect fraction is no more than 50%, a censored estimation method such as Kaplan-Meier or robust regression on order statistics [ROS] (**Chapter 15**) can be attempted to compute adjusted estimates of the background mean μ and standard deviation σ in equation [7.5].

For larger fractions of non-detects, a non-parametric tolerance limit can be constructed, as explained in **Section 17.2.2**. In this case, the GWPS median will often be set to the largest or second-largest observed value in background. **Table 17-4** can be used to determine the achieved confidence level ($1-\alpha$) associated with a 95% coverage GWPS constructed in this way. Ideally, enough background measurements should be used to set the tolerance limit as close to the target of 95% coverage, 95% confidence as possible. However, this could require very large background sample sizes ($n \geq 60$).

Multiple independent measurements are used to form either a mean or median confidence interval for comparison with the background GWPS. Preferably at least 4 distinct compliance point measurements should be used to define the mean confidence interval in the parametric case, and 3-7 values should be used with a non-parametric median test. The guidance does not recommend retesting in single-sample background GWPS compliance/assessment monitoring. An implicit kind of retesting is built in to any test of a sample mean or median as explained in **Section 19.3.2.**

In essence, the background tolerance limit is used to set a somewhat higher mean target GWPS which can accommodate both compliance and corrective action testing under background conditions. The GWPS in equation [7.6] can be interpreted as an approximation to the upper 95th percentile of the background distribution. It is designed to be a reasonable maximum on the likely range of background concentrations. It is high enough that compliance wells exceeding the GWPS via a confidence interval test (*i.e.*, LCL > GWPS) are probably impacted and not mere false positives. At the same time, successful remedial efforts must show that concentrations at contaminated wells have decreased to levels similar to background. The GWPS above represents an upper bound on background but is not so low as to make proof of remediation via an upper confidence limit [GWPS] impossible.

To ensure that the GWPS in equation [7.6] sets a reasonable target, the Unified Guidance recommends that at least 8 to 10 background measurements (n) be utilized, and more if available. If the background sample is not normal, but can be normalized via a transformation, the tolerance limit should be computed on the transformed measurements and the result ***back-transformed*** to obtain a limit in the concentration scale (see **Chapter 17** for further details).

TRADEOFFS IN BACKGROUND GWPS TESTING METHODS

A two-sample GWPS approach offers a stricter test of background exceedances. There is also greater flexibility in designing tests for a variety of future comparison values (single with repeat, small sample means, etc.). The true test parameter is explicitly defined by the type of test chosen. Non-parametric upper prediction limit tests also allow for greater flexibility when data sets include significant non-detect values or are not transformable to a normal distribution assumption. The approach suggested in this section accounts for the cumulative false positive error rate.

One negative feature of two-sample GWPS testing is that the test hypotheses cannot be reversed for correction action monitoring. The trigger for compliance/assessment testing may also be quite small, resulting in important consequences (the need to move to corrective action). It may also be difficult to demonstrate longer-term compliance following remedial activities, if the actual background is somewhat elevated.

Single-sample GWPS testing, by contrast, does allow for the reversal of test hypotheses. Using a suitable definition of the somewhat elevated GWPS takes into account background sample variability and size. Cumulative false positive error rates for compliance or corrective action testing are not considered, and standardized alpha error levels (.1 or .05) can be used. Exceedances under compliance monitoring also offer clear evidence of a considerable increase above background.

But applying an arbitrary increase above background recommended for single-sample testing may conflict with State anti-degradation policy. Defining the GWPS as a specific population parameter is also somewhat arbitrary. Using the suggested guidance approach for defining the GWPS in equation [7.6] above, may result in very high values if the data are not normal (including logarithmic or non-parametric applications). There is also less flexibility in identifying testing options, especially with data sets containing significant non-detect values. Annual testing with quarterly sampling may be the only realistic choice.

A possible compromise might utilize both approaches. That is, initially apply the two-sample approach for compliance/assessment testing. Then evaluate the single-sample approach with reversed

hypotheses. Some of the initially significant increases under the two-sample approach may also meet the upper confidence level limit when tested against the higher GWPS. Those well constituents that cannot meet this limit can then be subjected to corrective action remediation and full post-treatment testing. This implies that the background GWPS would be a range based on the two testing methods rather than a single value.

►EXAMPLE 7-1

A facility has triggered a significant increase under detection monitoring. One hazardous constituent (arsenic) was identified which must be tested against a background GWPS at six different compliance wells, since background well levels were above the appropriate arsenic MCL of 10 ug/l. Two semi-annual tests are required for compliance/assessment monitoring. Assume that arsenic had been detected in both background and downgradient wells, but was significantly higher in one of the compliance wells. It must be determined whether any of the compliance wells have exceeded their background GWPS, and might require corrective action.

Design a background GWPS monitoring system for the following arsenic data from the elevated Well #1, consisting of eight hypothetical historical intrawell background samples and four future annual values for two different simulated data distribution cases shown in the table below. Sample means and standard deviations are provided in the bottom row:

Compliance Well #1 Arsenic (μg/l)

Historical Well Data		Case 1	Case2
74.1	41.5	61.5	95.0
10.8	41.0	58.7	73.4
32.8	30.8	76.8	73.3
25.0	40.0	81.3	90.0
$\bar{x}$ = 37.0		$\bar{x}$ = 69.58	$\bar{x}$ = 82.93
s = 18.16		s = 11.15	s = 11.24

Background values were randomly generated from a normal distribution with a true mean of μ = 40 and a population standard deviation of σ = 16. Case 1 future data were from a normal distribution with a mean 1.5 times higher, while Case 2 data were from a normal distribution twice as high as the background true mean. Both cases used the same background population standard deviation. The intent of these simulated values is to allow exploration of both of the Unified Guidance recommended background GWPS methods when background increases are relatively modest and sample sizes small.

The two-sample background GWPS approach is first evaluated. Assume that the background data are normal and stationary (no evidence of spatial or temporal variation and other forms of statistical dependence). Given a likely limit of future quarterly sampling and required semi-annual evaluations, two guidance prediction limit options would seem appropriate—either a 1-of-2 future values or a 1-of-1 future mean size 2 test conducted twice a year. The 1-of-2 future values option is chosen.

Since there are a total of 6 compliance wells, one background constituent and two annual evaluations, there are a total of 12 annual background tests to be conducted. Either the Unified Guidance tables in **Appendix D** or R-script can be used to identify the appropriate prediction limit κ-

factor. For the 1-of-2 future values test, κ = **1.83** (found by interpolation from the second table on page D-118), based on w = 6, COC = 1, and two tests per year. The calculated prediction limit using the background data set statistics and κ-factor is 70.2 µg/l, serving as the background GWPS.

When the future values from the table above are tested against the GWPS, the following results are obtained. A "Pass" indicates that the compliance/assessment null hypothesis was achieved, while a "Fail" indicates that the alternative hypothesis (the GWPS has been exceeded) is accepted.

Well #1 As Compliance Comparisons
1-of-2 Future Values Test (µg/l)

Case 1 (data)	Result	Case 2 (data)	Result
61.5		95.0	
58.7	Pass	73.4	Fail
76.8		73.3	
81.3	Fail	90.0	Fail

GWPS = 70.2

Both cases indicate at least one GWPS exceedance using the 1-of-2 future values tests. These may be indications of a statistically significant increase above background, but the outcome for Case 1 is somewhat troubling. While a 50% increase above background (based on the simulated population parameters) is potentially significant, more detailed power evaluations indicate that such a detected exceedance would only be expected about 24% of the time (using R-script power calculations with a Z-value of 1.25 standard deviations above background for the 1-of-2 future values test). In contrast, the 2.5 Z-value for Case 2 would be expected to be exceeded about 76% of the time. In order to further evaluate the extent of significance of these results, the single-sample GWPS method is also considered.

Following the guidance above, define the single-sample mean GWPS using equation [7.6] for the upper 95% confidence, 95% proportion tolerance limit. Then apply upper and lower normal mean confidence intervals tests of the Case 1 and 2 n = 4 sample data using equations [7.3] and [7.4].

From **Table 21-9** on page D-246, a τ-factor of 3.187 is used with the background mean and standard deviation to generate the GWPS = 94.9. One-way upper and lower mean confidence levels are evaluated at 90 or 95% confidence for the tests and compared to the fixed background GWPS.

LCL test Pass/Fail results are the same as above for the two-sample compliance test. However, a "Pass" for the UCL test implies that the alternative hypothesis (less than the standard) is accepted while a "Fail" implies greater than or equal to the GWPS under corrective action monitoring hypotheses:

As Mean Confidence Interval Tests Against Background GWPS (μg/l)

LCL Test				UCL Test			
90% LCL	Result	95% LCL	Result	90% UCL	Result	95% UCL	Result
				Case 1 Data			
60.5	Pass	56.5	Pass	78.7	Pass	82.7	Pass
				Case 2 Data			
73.7	Pass	69.7	Pass	92.1	Pass	96.2	Fail

GWPS = 94.9

For either chosen significance level, the Case 1 90% and 95% UCLs of 78.7 and 82.7 are below the GWPS and the alternative corrective action hypothesis (the mean is less than the standard) can be accepted. For Case 2, the 90% UCL of 92.1 is below the GWPS, but the 95% UCL of 96.2 is above. If a higher level of test confidence is appropriate, the Case 2 arsenic values can be considered indicative of the need for corrective action.

If only the single-sample background GWPS approach were applied to the same data as above in compliance/assessment monitoring tests, neither case mean LCLs would exceed the standard, and no corrective action monitoring would be necessary. However, it should be noted from the example that this approach does allow for a significant increase above the reference background level before any action would be indicated. ◄

The approaches provided above presume that well constituent data subject to background GWPS testing are *stationary* over time. If sampling data show evidence of a trend, the situation becomes more complicated in making compliance or corrective action test decisions. Two- and single-sample stationary scenarios for identifying standards may not be appropriate. Trend behavior can be determined by applying one of the methods provided in **Chapter 17** (e.g., linear regression or Mann-Kendall trend tests) to historical data. A significant increasing slope can be indicative of a background exceedance, although it should be clear that the increase is not due to natural conditions. A decreasing or non-significant slope can be considered evidence for compliance with historical background. The most problematic standard would be setting an eventual background target for compliance testing under corrective action. To a great extent, it will depend on site-specific conditions including the behavior of specific constituent subject to remediation. A background GWPS might be determined following the period of remediation and monitoring when aquifer conditions have hopefully stabilized.

Setting and applying background GWPSs have not received a great deal of attention in previous guidance. The discussions and example above help illustrate the somewhat difficult regulatory choices that need to be made. A regulatory agency needs to determine what levels, if any, above background can be considered acceptable. A further consideration is the degree of importance placed on background GWPS exceedances, particularly when tested along with constituents having health-based limits. Existing regulatory programs may have already developed procedures to deal with many of the issues discussed in this section.

CHAPTER 8. SUMMARY OF RECOMMENDED METHODS

8.1 SELECTING THE RIGHT STATISTICAL METHODS 8-1
8.2 TABLE 8.1 INVENTORY OF RECOMMENDED METHODS 8-4
8.3 METHOD SUMMARIES 8-9

This chapter provides a quick guide to the statistical procedures discussed within the Unified Guidance. The first section is a basic road map designed to encourage the user to ask a series of key questions. The other sections offer thumbnail sketches of each method and a matrix of options to help in selecting the right procedure, depending on site-specific characteristics and constraints.

8.1 SELECTING THE RIGHT STATISTICAL METHODS

Choosing appropriate statistical methods is important in developing a sound groundwater monitoring statistical program. The statistical test(s) should be selected to match basic site-specific characteristics such as number and configuration of wells, the water quality constituents being measured, and general hydrology. Statistical methods should also be selected with reference to the statistical characteristics of the monitored parameters — proportion of non-detects, type of concentration distribution (*e.g.*, normal, lognormal), presence or absence of spatial variability, *etc.*

Because site conditions and permit requirements vary considerably, no single "cookbook" approach is readily available to select the right statistical method. The best strategy is to consider site-specific conditions and ask a series of questions. A table of recommended options (**Table 8-1**) and summary descriptions is presented in **Section 8.2** to help select an appropriate basic approach.

The first question is: what stage of monitoring is required? Detection monitoring is the first stage of any groundwater monitoring program and typically involves comparisons between measurements of background and compliance point groundwater. Most of the methods described in this document (*e.g.*, prediction limits, control charts, tests for trend, *etc.*) are designed for facilities engaged in detection monitoring. However, it must be determined whether an interwell (*e.g.*, upgradient-to-downgradient) or an intrawell test is warranted. This entails consideration of the site hydrology, constituent detection rates, and deciding whether separate (upgradient) wells or past intrawell data serves as the most appropriate and representative background.

Compliance/assessment monitoring is required for facilities that no longer meet the requirements of a detection monitoring program by exhibiting statistically significant indications of a release to groundwater. Once in compliance/assessment, compliance point measurements are typically tested against a fixed GWPS. Examples of fixed standards include Maximum Concentration Limits [MCL], risk-derived limits or a single limit derived from background data. The most appropriate statistical method for tests against GWPS is a lower confidence limit. The type of confidence limit will depend on whether the regulatory standard represents an average concentration; an absolute maximum, ceiling, or upper percentile; or whether the compliance data exhibit a trend over time.

In cases where no fixed GWPS is specified for a particular constituent, compliance point data may be directly compared against background data. In this situation, the most appropriate statistical method is

one or another detection monitoring two- or multiple-sample tests using the critical design limit as the GWPS (discussed in **Section 7.5**).

Corrective action is reserved for facilities where evidence of a groundwater release is confirmed above a GWPS. In these situations, the facility is required to submit an appropriate remediation plan to the Regional Administrator and to institute steps to insure adequate containment and/or clean-up of the release. Remediation of groundwater can be very costly and also difficult to measure. EPA has not adopted a uniform approach in the setting of clean-up standards or how one should determine whether those clean-up standards have been attained. Some guidance on this issue is given in the EPA document, ***Methods for Evaluating the Attainment of Cleanup Standards, Volume II: Groundwater*** (EPA, 1992).

The null hypothesis in corrective action testing is reversed from that of detection and compliance/assessment monitoring. Not only is it assumed that contamination is above the compliance or clean-up standard, but corrective action should continue until the average concentration level is below the clean-up limit for periods specified in the regulations. For any fixed-value standard (e.g., the GWPS or a remediation goal) a reasonable and consistent statistical test for corrective action is an *upper* confidence limit. The type of confidence limit will depend on whether the data have a stable mean concentration or exhibit a trend over time. For those well constituents requiring remediation, there will be a period of activity before formal testing can take place. A number of statistical techniques (e.g. trend testing) can be applied to the data collected in this interim period to gauge prospects for eventual GWPS compliance. **Section 7.5** describes corrective action testing limitations involving a two-sample GWPS.

Another major question involves the statistical distribution most appropriate to the observed measurements. Parametric tests are those which assume the underlying population follows a known and identifiable distribution, the most common examples in groundwater monitoring being the normal and the lognormal. If a specific distribution cannot be determined, non-parametric test methods can be used. Non-parametric tests do not require a known statistical distribution and can be helpful when the data contain a substantial proportion of non-detects. All of the parametric tests described in the Unified Guidance, except for control charts, have non-parametric counterparts that can be used when the underlying distribution is uncertain or difficult to test.

A special consideration in fitting distributions is the presence of non-detects, also known as left-censored measurements. As long as a sample contains a small fraction of non-detects (*i.e.*, no more than 10-15%), simple substitution of half the reporting limit [RL] is generally adequate. If the proportion of non-detects is substantial, it may be difficult or impossible to determine whether a specific parametric distributional model provides a good fit to the data. For some tests, such as the *t*-test, one can switch to a non-parametric test with little loss of power or accuracy. Non-parametric interval tests, however, such as prediction and tolerance limits, require substantially more data before providing statistical power equivalent to *parametric* intervals. Partly because of this drawback, the Unified Guidance discusses methods to adjust datasets with significant fractions of non-detects so that parametric distributional models may still be used (**Chapter 15**).

The Unified Guidance now recommends a single, consistent D*ouble Quantification rule* approach for handling constituents that have either never been detected or have not been recently detected. Such constituents are *not* included in cumulative annual *site-wide false positive error rate* [SWFPR] computations; and no special adjustment for non-detects is necessary. Any confirmed quantification (*i.e.*,

two consecutive detections above the RL) at a compliance point provides sufficient evidence of groundwater contamination by that parameter.

A key question when picking a test for detection monitoring is whether traditional background-to-downgradient interwell or single-well intrawell tests are appropriate. If intrawell testing is appropriate, historical measurements form the individual compliance well's own background while future values are tested against these data. Intrawell tests eliminate any natural spatial differences among monitoring wells. They can also be used when the groundwater flow gradient is uncertain or unstable, since all samples being tested are collected from the same well.

Possible disadvantages to intrawell tests also need to be considered. First, if the compliance well has already been impacted, intrawell background will also be impacted. Such contaminated background may provide a skewed comparison to later data from the same well, making it difficult to identify contaminated groundwater in the future. Secondly, if intrawell background is constructed from only a few early measurements, considerable time may be needed to accumulate a sufficient number of background observations (via periodic updating) to run a statistically powerful test.

If a compliance well has already been impacted by previous contamination, trend testing can still indicate whether conditions have deteriorated since intrawell background was collected. For sites historically contaminated above background, the only way to effectively monitor compliance wells may be to establish an historical intrawell baseline and measure increases above this baseline.

Besides trend tests, techniques recommended for intrawell comparisons include intrawell prediction limits, control charts, and sometimes the Wilcoxon rank-sum test. The best choice between these methods is not always clear. Since there is no non-parametric counterpart to control charts, the choice will depend on whether the data is normal or can be normalized via a transformation. New guidance for control charts shows they also can be designed to incorporate retesting. For sites with a large number of well-constituent pairs, intrawell prediction limits can incorporate retesting to meet specific site-wide false positive rate and statistical power characteristics. Parametric intrawell prediction limits can be used with background that is normal or transformable to normality; non-parametric versions can also be applied for many other data sets.

If interwell, upgradient-to-downgradient tests are appropriate, the choice of statistical method depends primarily on the number of compliance wells and constituents being monitored, the number of observations available from each of these wells, and the detection rates and distributional properties of these parameters. If a very small number of comparisons must be tested (***i.e.***, two or three compliance wells versus background, for one or two constituents), a *t*-test or Wilcoxon rank-sum test may be appropriate if there are a sufficient number of compliance measurements (***i.e.***, at least two per well).

For other cases, the Unified Guidance recommends a prediction limit or control chart constructed from background. Whenever more than a few statistical tests must be run, retesting should be incorporated into the procedure. If multiple observations per compliance well can be collected during a given evaluation period, either a prediction limit for 'future' observations, a prediction limit for means or medians, or a control chart can be considered, depending on which option best achieves statistical power and SWFPR targets, while balancing the site-specific costs and feasibility of sampling. If only one observation per compliance well can be collected per evaluation, the only practical choices are a prediction limit for individual observations or a control chart.

8.2 TABLE 8-1 INVENTORY OF RECOMMENDED METHODS

Chapter 9. Exploratory Tools		
Statistical Method	Chapter	Use
Time Series Plot	**§9.1**	Plot of measurement levels over time; Useful for assessing trends, data inconsistencies, etc.
Box Plot	**§9.2**	Graphical summary of sample distribution; Useful for comparing key statistical characteristics in multiple wells
Histogram	**§9.3**	Graphical summary of sample distribution; Useful for assessing probability density of single data set
Scatter Plot	**§9.4**	Diagnostic tool; Plot of one variable vs. another; Useful for exploring statistical associations
Probability Plot	**§9.5**	Graphical fit to normality; Useful for raw or transformed data
Chapter 10. Fitting Distributions		
Statistical Method	Chapter	Use
Skewness Coefficient	**§10.4**	Measures symmetry/asymmetry in distribution; Screening level test for plausibility of normal fit
Coefficient of Variation	**§10.4**	Measures symmetry/asymmetry in distribution; Screening tool for plausibility of normal fit; Only for non-negative data
Shapiro-Wilk Test	**§10.5.1**	Numerical normality test of a single sample; for $n \leq 50$
Shapiro-Francía Test	**§10.5.2**	Numerical test of normality for a single sample; Supplement to Shapiro-Wilk; Use with $n > 50$
Filliben's Probability Plot Correlation Coefficient	**§10.6**	Numerical test of normality for a single sample; Interchangeable with Shapiro-Wilk; Use with $n \leq 100$; Good supplement to probability plot
Shapiro-Wilk Multiple Group Test	**§10.7**	Extension of Shapiro-Wilk test for multiple samples with possibly different means and/or variances; Good check to use with Welch's *t*-test
Chapter 11. Equality of Variance		
Statistical Method	Chapter	Use
Box Plots (side-by-side)	**§11.1**	Graphical test of differences in population variances; Good screening tool for equal variance assumption in ANOVA
Levene's Test	**§11.2**	Numerical, robust ANOVA-type test of equality of variance for ≥ 2 populations; Useful for testing assumptions in ANOVA
Mean-SD Scatter Plot	**§11.3**	Visual test of association between SD and mean levels across group of wells; Use to check for proportional effect or if variance-stabilizing transformation is needed
Chapter 12. Outliers		
Statistical Method	Chapter	Use
Probability Plot	**§12.1**	Graphical fit of distribution to normality; Useful for identifying extreme points not coinciding with predicted tail of distribution
Box Plot	**§12.2**	Graphical screening tool for outliers; quasi-non-parametric, only requires rough symmetry in distribution
Dixon's Test	**§12.3**	Numerical test for single low or single high outlier; Use when $n \leq 25$
Rosner's Test	**§12.4**	Numerical test for up to 5 outliers in single dataset; Recommended when $n \geq 20$; User must identify a specific number of possible outliers before running

Chapter 13. Spatial Variation		
Statistical Method	Chapter	Use
Box Plots (side-by-side)	**§13.2.1**	Quick screen for spatial variability; Look for noticeably staggered boxes
One-Way Analysis of Variance [ANOVA] for Spatial Variation	**§13.2.2**	Test to compare means of several populations; Use to identify spatial variability across a group of wells and to estimate pooled (background) standard deviation for use in intrawell tests; Data must be normal or normalized; Assumption of equal variances across populations

Chapter 14. Temporal Variability		
Statistical Method	Chapter	Use
Time Series Plot (parallel)	**§14.2.1**	Quick screen for temporal (and/or spatial) variation; Look for parallel movement in the graph traces at several wells over time
One-way ANOVA for Temporal Effects	**§14.2.2**	Test to compare means of distinct sampling events, in order to assess systematic temporal dependence across wells; Use to get better estimate of (background) variance and degrees of freedom in data with temporal patterns; Residuals from ANOVA also used to create stationary, adjusted data
Sample Autocorrelation Function	**§14.2.3**	Plot of autocorrelation by lag between sampling events; Requires approximately normal data; Use to test for temporal correlation and/or to adjust sampling frequency
Rank von Neumann Ratio	**§14.2.4**	Non-parametric numerical test of dependence in time-ordered data series; Use to test for first-order autocorrelation in data from single well or population
Darcy Equation	**§14.3.2**	Method to approximate groundwater flow velocity; Use to determine sampling interval guaranteeing physical independence of consecutive groundwater samples; Does not ensure statistical independence
Seasonal Adjustment (single well)	**§14.3.3**	Method to adjust single data series exhibiting seasonal correlations (*i.e.*, cyclical fluctuations); At least 3 seasonal cycles must be evident on time series plot
Temporally-Adjusted Data Using ANOVA	**§14.3.3**	Method to adjust multiple wells for a common temporal dependence; Use adjusted data in subsequent tests
Seasonal Mann-Kendall Test	**§14.3.4**	Extension of Mann-Kendall trend test when seasonality is present; At least 3 seasonal cycles must be evident

Chapter 15. Managing Non-Detect Data		
Statistical Method	Chapter	Use
Simple Substitution	**§15.2**	Simplest imputation scheme for non-detects; Useful when ≤ 10-15% of dataset is non-detect
Censored Probability Plot	**§15.3**	Probability plot for mixture of non-detects and detects; Use to check normality of left-censored sample
Kaplan-Meier	**§15.3**	Method to estimate mean and standard deviation of left-censored sample; Use when ≤ 50% of dataset is non-detect; Multiple detects and non-detects must originate from same distribution
Robust Regression on Order Statistics	**§15.4**	Method to estimate mean and standard deviation of left-censored sample; Use when ≤ 50% of dataset is non-detect; Multiple detects and non-detects must originate from same distribution
Cohen' Method and Parametric Regression on Order Statistics	**§15.5**	Other methods to estimate mean and standard deviation of left-censored sample; Use when ≤ 50% of dataset is non-detect; Detects and non-detects must originate from same distribution and there must be a single censoring limit

Chapter 16. Two-sample Tests		
Statistical Method	Chapter	Use
Pooled Variance *t*-Test	**§16.1.1**	Test to compare means of two populations; Data must be normal or normalized, with no significant spatial variability; Useful at very small sites in upgradient-to-downgradient comparisons; Also useful for updating background; Population variances must be equal
Welch's *t*-Test	**§16.1.2**	Test to compare means of two populations; Data must be normal or normalized, with no significant spatial variability; Useful at very small sites in interwell comparisons; Also useful for updating background; Population variances can differ
Wilcoxon Rank-Sum Test	**§16.2**	Non-parametric test to compare medians of two populations; Data need not be normal; Some non-detects OK; Should have no significant spatial variability; Useful at very small sites in interwell comparisons and for certain intrawell comparisons; Also useful for updating background
Tarone-Ware Test	**§16.3**	Extension of Wilcoxon rank-sum; non-parametric test to compare medians of two populations; Data need not be normal; Designed to accommodate left-censored data; Should have no significant spatial variability; Useful at very small sites in interwell comparisons and for certain intrawell comparisons; Also useful for updating background

Chapter 17. ANOVA, Tolerance Limits, & Trend Tests		
Statistical Method	Chapter	Use
One-Way ANOVA	**§17.1.1**	Test to compare means across multiple populations; Data must be normal or normalized; Should have no significant spatial variability if used as interwell test; Assumes equal variances; Mandated in some permits, but generally superceded by other tests; Useful for identifying spatial variation; RMSE from ANOVA can be used to improve intrawell background limits
Kruskal-Wallis Test	**§17.1.2**	Test to compare medians across multiple populations; Data need not be normal; some non-detects OK; Should have no significant spatial variability if used as interwell test; Useful alternative to ANOVA for identifying spatial variation
Tolerance Limit	**§17.2.1**	Test to compare background vs. ≥ 1 compliance well; Data must be normal or normalized; Should have no significant spatial variability if used as interwell test; Alternative to ANOVA; Mostly superceded by prediction limits; Useful for constructing alternate clean-up standard in corrective action
Non-parametric Tolerance Limit	**§17.2.2**	Test to compare background vs. ≥ 1 compliance well; Data need not be normal; Non-Detects OK; Should have no significant spatial variability if used as interwell test; Alternative to Kruskal-Wallis; Mostly superceded by prediction limits
Linear Regression	**§17.3.1**	Parametric estimate of linear trend; Trend residuals must be normal or normalized; Useful for testing trends in background or at already contaminated wells; Can be used to estimate linear association between two random variables
Mann-Kendall Trend Test	**§17.3.2**	Non-parametric test for linear trend; Non-detects OK; Useful for documenting upward trend at already contaminated wells or where trend already exists in background
Theil-Sen Trend Line	**§17.3.3**	Non-parametric estimate of linear trend; Non-detects OK; Useful for estimating magnitude of an increasing trend in conjunction with Mann-Kendall test

Chapter 18. Prediction Limit Primer		
Statistical Method	Chapter	Use
Prediction Limit for m Future Values	**§18.2.1**	Test to compare m measurements from compliance well against background; Data must be normal normalized; Useful in retesting schemes; Can be adapted to either intrawell or interwell tests; No significant spatial variability allowed if used as interwell test
Prediction Limit for Future Mean	**§18.2.2**	Test to compare mean of compliance well against background; Data must be normal or normalized; Useful alternative to traditional ANOVA; Can be useful in retesting schemes; Most useful for interwell (*e.g.*, upgradient to downgradient) comparisons; No significant spatial variability allowed if used as interwell test
Non-Parametric Prediction Limit for m Future Values	**§18.3.1**	Non-parametric test to compare m measurements from compliance well against order statistics of background; Non-normal data and/or non-detects OK; Useful in non-parametric retesting schemes; Should have no significant spatial variability if used as interwell test
Non-parametric Prediction Limit for Future Median	**§18.3.2**	Test to compare median of compliance well against order statistics of background; Non-normal data and/or non-detects OK; Useful in non-parametric retesting schemes; Most useful for interwell (*e.g.*, upgradient to downgradient) comparisons; No significant spatial variability allowed if used as interwell test

Chapter 19. Prediction Limit Strategies with Retesting		
Statistical Method	Chapter	Use
Prediction Limits for Individual Observations With Retesting	**§19.3.1**	Tests individual compliance point measurements against background; Data must be normal or normalized; Assumes common population variance across wells; No significant spatial variability allowed if used as interwell test; Replacement for traditional ANOVA, extends Dunnett's multiple comparison with control (MCC) procedure; Allows control of SWFPR across multiple well-constituent pairs; Retesting explicitly incorporated; Useful at any size site
Prediction Limits for Means With Retesting	**§19.3.2**	Tests compliance point means against background; Data must be normal or normalized; Assumes common population variance across wells; No significant spatial variability allowed if used as interwell test; Replacement for traditional ANOVA, extends Dunnett's multiple comparison with control (MCC) procedure; More flexible than a series of intrawell t-tests if used as intrawell test; Allows control of SWFPR across multiple well-constituent pairs; Must be feasible to collect ≥2 resamples per evaluation period to incorporate retesting; 1-of-1 scheme does not require explicit retesting
Non-Parametric Prediction Limits for Individual Observations With Retesting	**§19.4.1**	Non-parametric test of individual compliance point observations against background; Non-normal data and/or non-detects OK; No significant spatial variability allowed if used as interwell test; Retesting explicitly incorporated; Large background sample size helpful
Non-Parametric Prediction Limits for Medians With Retesting	**§19.4.2**	Non-parametric test of compliance point medians against background; Non-normal and/or non-detects OK; No significant spatial variability allowed if used as interwell test; Large background sample size helpful; Must be feasible to collect ≥ 3 resamples per evaluation period to incorporate retesting; 1-of-1 scheme does not require explicit retesting

Chapter 20. Control Charts		
Statistical Method	Chapter	Use
Shewhart-CUSUM Control Chart	**§20.2**	Graphical test of significant increase above background; Data must be normal or normalized; Some non-detects OK if left-censored adjustment made; At least 8 background observations recommended; Viable alternative to prediction limits; Retesting can be explicitly incorporated; Control limits can be set via published literature or Monte Carlo simulation
Chapter 21. Confidence Intervals		
Statistical Method	Chapter	Use
Confidence Interval Around Normal Mean	**§21.1.1**	Data must be normal; Some non-detects OK if left-censored adjustment made; Used in compliance/assessment or corrective action to compare compliance well against fixed, mean-based groundwater standard; Should be no significant trend; 4 or more observations recommended
Confidence Interval Around Lognormal Geometric Mean	**§21.1.2**	Data must be lognormal; Some non-detects OK if left-censored adjustment made; Used in compliance/assessment or corrective action to compare compliance well against fixed, mean-based groundwater standard; Should be no significant trend; 4 or more observations recommended; Geometric mean equivalent to lognormal median, smaller than lognormal mean
Confidence Interval Around Lognormal Arithmetic Mean	**§21.1.3**	Data must be lognormal; Some non-detects OK if left-censored adjustment made; Used in compliance/assessment or corrective action to compare compliance well against fixed, mean-based groundwater standard; Should be no significant trend; 4 or more observations recommended; Lognormal arithmetic mean larger than lognormal geometric mean
Confidence Interval Around Upper Percentile	**§21.1.4**	Data must be normal or normalized; Some non-detects OK if left-censored adjustment made; Used in compliance/assessment to compare compliance well against percentile-based or maximum groundwater standard; Should be no significant trend
Non-Parametric Confidence Interval around Median	**§21.2**	For non-normal, non-lognormal data; Non-detects OK; Used in compliance/assessment or corrective action to compare compliance well against fixed, mean-based groundwater standard; Should be no significant trend; 7 or more observations recommended
Non-Parametric Confidence Interval Around Upper Percentile	**§21.2**	For non-normal, non-lognormal data; Non-detects OK; Used in compliance/assessment or corrective action to compare compliance well against percentile-based or maximum groundwater standard; Should be no significant trend; Large background sample size helpful
Confidence Band Around Linear Regression	**§21.3.1**	Use on data with significant trend; Trend residuals must be normal or normalized; Used in compliance/assessment or corrective action to compare compliance well against fixed groundwater standard; ≥ 8 observations recommended
Non-parametric Confidence Band Around Theil-Sen Line	**§21.3.2**	Use on data with significant trend; Non-normal data and/or non-detects OK; Used in compliance/assessment or corrective action to compare compliance well against fixed groundwater standard; Bootstrapping of Theil-Sen trend line used to construct confidence band

8.3 METHOD SUMMARIES

TIME SERIES PLOT (SECTIONS 9.1 AND 14.2.1)

Basic purpose: Diagnostic and exploratory tool. It is a graphical technique to display changes in concentrations at one or more wells over a specified period of time or series of sampling events.

Hypothesis tested: Not a formal statistical test. Time series plots can be used to informally gauge the presence of temporal and/or spatial variability in a collection of distinct wells sampled during the same time frame.

Underlying assumptions: None.

When to use: Given a collection of wells with several sampling events recorded at each well, a time series plot can provide information not only on whether the mean concentration level changes from well to well (an indication of possible spatial variation), but also on whether there exists time-related or temporal dependence in the data. Such temporal dependence can be seen in parallel movement on the time series plot, that is, when several wells exhibit the same pattern of up-and-down fluctuations over time.

Steps involved: 1) For each well, make a plot of concentration against time or date of sampling for the sampling events that occurred during the specified time period; 2) Make sure each well is identified on the plot with a distinct symbol and/or connected line pattern (or trace); 3) To observe possible spatial variation, look for well traces that are substantially separated from one another in concentration level; 4) To look for temporal dependence, look for well traces that rise and fall together in roughly the same (parallel) pattern; 5) To ensure that artificial trends due to changing reporting limits are not reported, plot any non-detects with a distinct symbol, color, and/or fill.

Advantages/Disadvantages: Time series plots are an excellent tool for examining the behavior of one or more samples over time. Although, they do not offer the compact summary of distributional characteristics that, say, box plots do, time series plots display each and every data point and provide an excellent initial indication of temporal dependence. Since temporal dependence affects the underlying variability in the data, its identification is important so adjustments can be made to the estimated standard deviation.

BOX PLOT (SECTIONS 9.2, 12.2, AND 13.2.1)

Basic purpose: Diagnostic and exploratory tool. Graphical summary of data distribution; gives compact picture of central tendency and dispersion.

Hypothesis tested: Although not a formal statistical test, a side-by-side box plot of multiple datasets can be used as a rough indicator of either unequal variances or spatial variation (via unequal means/medians). Also serves as a quasi-non-parametric screening tool for outliers in a symmetric population.

Underlying assumptions: When used to screen outliers, underlying population should be approximately symmetric.

When to use: Can be used as a quick screen in testing for unequal variances across multiple populations. Box lengths indicate the range of the central 50% of sample data values. Substantially different box lengths suggest possibly different population variances. It is useful as a rough indication of spatial variability across multiple well locations. Since the median (and often the mean) are graphed on each box, significantly staggered medians and/or means on a multiple side-by-side box plot can suggest possibly different population means at distinct well locations. Can also be used to screen for outliers: values falling beyond the 'whiskers' on the box plot are labeled as potential outliers.

Steps involved: 1) Compute the median, mean, lower and upper quartiles (*i.e.*, 25th and 75th percentiles) of each dataset; 2) Graph each set of summary statistics side-by-side on the same set of axes. Connect the lower and upper quartiles as the ends of a box, cut the box in two with a line at the median, and use an 'X' or other symbol to represent the mean. 3) Compute the 'whiskers' by extending lines below and above the box by an amount equal to 1.5 times the interquartile range [IQR].

Advantages/Disadvantages: The box plot is an excellent screening tool and visual aid in diagnosing either unequal variances for testing the assumptions of ANOVA, the possible presence of spatial variability, or potential outliers. It is not a formal statistical test, however, and should generally be used in conjunction with numerical test procedures.

HISTOGRAM (SECTION 9.3)

Basic purpose: Diagnostic and exploratory tool. It is a graphical summary of an entire data distribution.

Hypothesis tested: Not a formal statistical test.

Underlying assumptions: None.

When to use: Can be used as a rough estimate of the probability density of a single sample. Shape of histogram helps determine whether the distribution is symmetric or skewed. For larger data sets, histogram can be visually compared to a normal distribution or other known model to assess whether the shapes are similar.

Steps involved: 1) Sort and bin the data set into non-overlapping concentration segments that span the range of measurement values; 2) Create a bar chart of the bins created in **Step 1**: put the height of each bar equal to the number or fraction of values falling into each bin.

Advantages/Disadvantages: The histogram is a good visual aid in exploring possible distributional models that might be appropriate. Since it is not a formal test, there is no way to judge possible models solely on the basis of the histogram; however, it provides a visual 'feel' for a data set.

SCATTER PLOT (SECTION 9.4)

Basic purpose: Diagnostic tool. It is a graphical method to explore the association between two random variables or two paired statistical samples.

Hypothesis tested: None.

Underlying Assumptions: None.

When to use: Useful as an exploratory tool for discovering or identifying statistical relationships between pairs of variables. Graphically illustrates the degree of correlation or association between two quantities.

Steps involved: Using Cartesian pairs of the variables of interest, graph each pair on the scatter plot, using one symbol per pair.

Advantages/Disadvantages: A scatter plot is not a formal test, but rather an excellent exploratory tool. Helps identify statistical relationships.

PROBABILITY PLOT (SECTIONS 9.5 AND 12.1)

Basic purpose: Diagnostic tool. A graphical method to compare a dataset against a particular statistical distribution, usually the normal. Designed to show how well the data match up to or 'fit' the hypothesized distribution. An absolutely straight line fit indicates perfect consistency with the hypothesized model.

Hypothesis tested: Although not a formal test, the probability plot can be used to graphically indicate whether a dataset is normal. The straighter the plot, the more consistent the dataset with a null hypothesis of normality; significant curves, bends, or other non-linear patterns suggest a rejection of the normal model as a poor fit.

Underlying Assumptions: All observations come from a single statistical population.

When to use: Can be used as a graphical indication of normality on a set of raw measurements or, by first making a transformation, as an indication of normality on the transformed scale. It should generally be supplemented by a formal numerical test of normality. It can be used on the residuals from a one-way ANOVA to test the joint normality of the groups being compared. The test can also be used to help identify potential outliers (*i.e.*, individual values not part of the same basic underlying population).

Steps involved: 1) Order the dataset and determine matching percentiles (or quantiles) from the hypothesized distribution (typically the standard normal); 2) Plot the ordered data values against the matching percentiles; 3) Examine the plot for a straight line fit.

Advantages/Disadvantages: Not a formal test of normality; however, the probability plot is an excellent graphical supplement to any goodness-of-fit test. Because each data value is depicted, specific departures from normality can be identified (*e.g.*, excessive skewness, possible outliers, *etc.*).

SKEWNESS COEFFICIENT (SECTION 10.4)

Basic purpose: Diagnostic tool. Sample statistic designed to measure the degree of symmetry in a sample. Because the normal distribution is perfectly symmetric, the skewness coefficient can provide a quick indication of whether a given dataset is symmetric enough to be consistent with the normal model. Skewness coefficients close to zero are consistent with normality; skewness values large in absolute value suggest the underlying population is asymmetric and non-normal.

Hypothesis tested: The skewness coefficient is used in groundwater monitoring as a screening tool rather than a formal hypothesis test. Still, it can be used to roughly test whether a given sample is normal by using the following rule of thumb: if the skewness coefficient is no greater than one in absolute value, accept a null hypothesis of normality; if not, reject the normal model as ill-fitting.

Underlying Assumptions: None

Steps involved: 1) Compute skewness coefficient; 2) Compare to cutoff of 1; 3) If skewness is greater than 1, considering running a formal test of normality.

Advantages/Disadvantages: Fairly simple calculation, good screening tool. Skewness coefficient can be positive or negative, indicating positive or negative skewness in the dataset, respectively. Measures symmetry rather than normality, per se; since other non-normal distributions can also be symmetric, might give a misleading result. Not as powerful or accurate a test of normality as either the Shapiro-Wilk or Filliben tests, but a more accurate indicator than the coefficient of variation, particularly for data on a transformed scale.

Coefficient of Variation [CV] (Section 10.4)

Basic purpose: Diagnostic tool. Sample statistic used to measure skewness in a sample of positively-valued measurements. Because the CV of positively-valued normal measurements must be close to zero, the CV provides an easy indication of whether a given sample is symmetric enough to be normal. Coefficients of variation close to zero are consistent with normality; large CVs indicate a skewed, non-normal population.

Hypothesis tested: The coefficient of variation is not a formal hypothesis test. Still, it can be used to provide a 'quick and easy' gauge of non-normality: if the CV exceeds 0.5, the population is probably not normal.

Underlying Assumptions: Sample must be positively-valued for CV to have meaningful interpretation.

Steps involved: 1) Compute sample mean and standard deviation; 2) Divide standard deviation by mean to get coefficient of variation.

Advantages/Disadvantages: Simple calculation, good screening tool. It measures skewness and variability in positively-valued data. Not an accurate a test of normality, especially if data have been transformed.

Shapiro-Wilk and Shapiro-Francía Tests (Section 10.5)

Basic purpose: Diagnostic tool and a formal numerical goodness-of-fit test of normality. Shapiro-Francía test is a close variant of the Shapiro-Wilk useful when the sample size is larger than 50.

Hypothesis tested: H_0 — the dataset being tested comes from an underlying normal population. H_A — the underlying population is non-normal (note that the form of this alternative population is not specified).

Underlying assumptions: All observations come from a single normal population.

When to use: To test normality on a set of raw measurements or following transformation of the data. It can also be used with the residuals from a one-way ANOVA to test the joint normality of the groups being compared.

Steps involved (for Shapiro-Wilk): 1) Order the dataset and compute successive differences between pairs of extreme values (*i.e.*, most extreme pair = maximum – minimum, next most extreme pair = 2nd largest – 2nd smallest, *etc.*); 2) Multiply the pair differences by the Shapiro-Wilk coefficients and compute the Shapiro-Wilk test statistic; 3) Compare the test statistic against an α-level critical point; 4) Values higher than the critical point are consistent with the null hypothesis of normality, while values lower than the critical point suggest a non-normal fit.

Advantages/Disadvantages: The Shapiro-Wilk procedure is considered one of the very best tests of normality. It is much more powerful than the skewness coefficient or chi-square goodness-of-fit test. The Shapiro-Wilk and Shapiro-Francía test statistics will tend to be large (and more indicative of normality) when a probability plot of the same data exhibits a close-to-linear pattern. Special Shapiro-Wilk coefficients are available for sample sizes up to 50. For larger sample sizes, the Shapiro-Francía test does not require a table of special coefficients, just the ability to compute inverse normal probabilities.

Filliben's Probability Plot Correlation Coefficient Test (Section 10.6)

Basic purpose: Diagnostic tool and a formal numerical goodness-of-fit procedure to test for normality.

Hypothesis tested: H_0 — the dataset being tested comes from an underlying normal population. H_A — the underlying population is non-normal (note that the form of this alternative population is not specified).

Underlying assumptions: All observations come from a single normal population.

When to use: To test normality on a set of raw measurements or following transformation of the data on the transformed scale. It can also be used on the residuals from a one-way ANOVA to test the joint normality of the groups being compared.

Steps involved: 1) Construct a normal probability plot of the dataset; 2) Calculate the correlation between the pairs on the probability plot; 3) Compare the test statistic against an α-level critical point; 4) Values higher than the critical point are consistent with the null hypothesis of normality, while values lower than the critical point suggest a non-normal fit.

Advantages/Disadvantages: Filliben's procedure is an excellent test of normality, with very similar characteristics to the Shapiro-Wilk test. As a correlation on a probability plot, the Filliben's test statistic will tend to be close to one (and more indicative of normality) when a probability plot of the same data exhibits a close-to-linear pattern. Critical points for Filliben's test are available for sample sizes up to 100. A table of special coefficients is not needed to run Filliben's test, only the ability to compute inverse normal probabilities.

Shapiro-Wilk Multiple Group Test (Section 10.7)

Basic purpose: Diagnostic tool and a formal normality goodness-of-fit test for multiple groups.

Hypothesis tested: H_0 — datasets being tested all come from underlying normal populations, possibly with different means and/or variances. H_A — at least one underlying population is non-normal (note that the form of this alternative population is not specified).

Underlying assumptions: The observations in each group all come from, possibly different, normal populations.

When to use: Can be used to test normality on multiple sets of raw measurements or, by first making a transformation, to test normality of the data groups on the transformed scale. It is particularly helpful when used in conjunction with Welch's *t*-test.

Steps involved: 1) Compute Shapiro-Wilk statistic (**Section 10.5**) on each group separately; 2) Transform the Shapiro-Wilk statistics into *z*-scores and combine into an omnibus *z*-score; 3) Compare the test statistic against an α-level critical point; 4) Values higher than the critical point are consistent with the null hypothesis of normality for all the populations, while values lower than the critical point suggest a non-normal fit of one or more groups.

Advantages/Disadvantages: As an extension of the Shapiro-Wilk test, the multiple group test shares many of its desirable properties. Users should be careful, however, not to assume that a result consistent with the hypothesis of normality implies that all groups follow the ***same*** normal distribution. The multiple group test does not assume that all groups have the same means or variances. Special coefficients are needed to convert Shapiro-Wilk statistics into *z*-scores, but once converted, no other special tables needed to run test besides a standard normal table.

LEVENE'S TEST (SECTION 11.2)

Basic purpose: Diagnostic tool. Levene's test is a formal numerical test of equality of variances across multiple populations.

Hypothesis tested: H_0 — The population variances across all the datasets being tested are equal. H_A — One or more pairs of population variances are unequal.

Underlying assumptions: The data set from each population is assumed to be roughly normal in distribution. Since Levene's test is designed to work well even with somewhat non-normal data (***i.e.***, it is fairly robust to non-normality), precise normality is not an overriding concern.

When to use: Levene's method can be used to test the equal variance assumption underlying one-way ANOVA for a group of wells. Used in this way, the test is run on the absolute values of the residuals after first subtracting the mean of each group being compared. If Levene's test is significant, the original data may need to be transformed to stabilize the variances before running an ANOVA.

Steps involved: 1) Compute the residuals of each group by subtracting the group mean; 2) conduct a one-way ANOVA on the absolute values of the residuals; and 3) if the ANOVA *F*-statistic is significant at the 5% α-level, conclude the underlying population variances are unequal. If not, conclude the data are consistent with the null hypothesis of equal variances.

Advantages/Disadvantages: As a test of equal variances, Levene's test is reasonably robust to non-normality. It is much more so than for Bartlett's test (recommended within the 1989 ***Interim Final***

Guidance [IFG]). In addition, Levene's method uses the same basic equations as those needed to run a one-way ANOVA.

MEAN-STANDARD DEVIATION SCATTER PLOT (SECTION 11.3)

Basic purpose: Diagnostic tool. It is a graphical method to examine degree of association between mean levels and standard deviations at a series of wells. Positive correlation or association between these quantities is known as a 'proportional effect' and is characteristic of skewed distributions such as the lognormal.

Hypothesis tested: Though not a formal test, the mean-standard deviation scatter plot provides a visual indication of whether variances are roughly equal from well to well, or whether the variance depends on the well mean.

Underlying Assumptions: None.

When to use: Useful as a graphical indication of 1) equal variances or 2) proportional effects between the standard deviation and mean levels. A positive correlation between well means and standard deviations may signify that a transformation is needed to stabilize the variances.

Steps involved: 1) Compute the sample mean and standard deviation for each well; 2) plot the mean-standard deviation pairs on a scatter plot; and 3) examine the plot for any association between the two quantities.

Advantages/Disadvantages: Not a formal test of homoscedasticity (*i.e.*, equal variances). It is helpful in assessing whether a transformation might be warranted to stabilize unequal variances.

DIXON'S TEST (SECTION 12.3)

Basic purpose: Diagnostic tool. It is used to identify (single) outliers within smaller datasets.

Hypothesis tested: H_0 — Outlier(s) comes from same normal distribution as rest of the dataset. H_A — Outlier(s) comes from different distribution than rest of the dataset.

Underlying assumptions: Data without the suspected outlier(s) are normally distributed. Test recommended only for sample sizes up to 25.

When to use: Try Dixon's test when one value in a dataset appears anomalously low or anomalously high when compared to the other data values. Be cautious about screening apparent high outliers in compliance point wells. Even if found to be statistical outliers, such extreme concentrations may represent contamination events. A safer application of outlier tests is with background or baseline samples. Even then, always try to establish a physical reason for the outlier if possible (*e.g.*, analytical error, transcription mistake, *etc.*).

Steps involved: 1) Remove the suspected outlier and test remaining data for normality. If non-normal, try a transformation to achieve normality; 2) Once remaining data are normal, calculate Dixon's statistic, depending on the sample size n; 3) Compare Dixon's statistic against an α-level critical point; and 4) If Dixon's statistic exceeds the critical point, conclude the suspected value is a statistical outlier. Investigate this measurement further.

Advantages/Disadvantages: Dixon's test is only recommended for sample sizes up to 25. Furthermore, if there is more than one outlier, Dixon's test may lead to masking (*i.e.*, a non-significant result) where two or more outliers close in value 'hide' one another. If more than one outlier is suspected, always test the *least* extreme value first.

ROSNER'S TEST (SECTION 12.4)

Basic purpose: Diagnostic tool. It is used to identify multiple outliers within larger datasets.

Hypothesis tested: H_0 — Outliers come from same normal distribution as the rest of the dataset. H_A — Outliers come from different distribution than the rest of the dataset.

Underlying assumptions: Data without the suspected outliers are normally distributed. Test recommended for sample sizes of at least 20.

When to use: Try Rosners's test when multiple values in a dataset appear anomalously low or anomalously high when compared to the other data values. As Dixon's test, be cautious about screening apparent high outliers in compliance point wells. Always try to establish a physical reason for an outlier if possible (*e.g.*, analytical error, transcription mistake, *etc.*).

Steps involved: 1) Identify the maximum number of possible outliers ($r_0 \leq 5$) and the number of suspected outliers ($r \leq r_0$). Remove the suspected outliers and test the remaining data for normality. If non-normal, try a transformation to achieve normality; 2) Once remaining data are normal, successively compute the mean and standard deviation, removing the next most extreme value each time until r_0 possible outliers have been removed; 3) Compute Rosner's statistic based on the number (r) of suspected outliers; and 4) If Rosner's statistic exceeds an α-level critical point, conclude there are r statistical outliers. Investigate these measurements further. If Rosner's statistic does not exceed the critical point, recompute the test for (r–1) possible outliers, successively reducing r until either the critical point is exceeded or $r = 0$.

Advantages/Disadvantages: Rosner's test is only recommended for sample sizes of 20 or more, but can be used to identify up to 5 outliers per use. It is more complicated to use than some other outlier tests, but does not require special tables other than to determine α-level critical points.

ONE-WAY ANALYSIS OF VARIANCE [ANOVA] FOR SPATIAL VARIATION (SECTION 13.2.2)

Basic purpose: Diagnostic tool. Test to compare population means at multiple wells, in order to gauge the presence of spatial variability.

Hypothesis tested: H_0 — Population means across all tested wells are equal. H_A — One or more pairs of population means are unequal.

Underlying assumptions: 1) ANOVA residuals at each well or group must be normally distributed using the original data or after transformation. Residuals should be tested for normality using a goodness-of-fit procedure; 2) population variances across all wells must be equal. This assumption can be tested with box plots and Levene's test; and 3) each tested well should have at least 3 to 4 separate observations.

When to use: The one-way ANOVA procedure can be used to identify significant spatial variation across a group of distinct well locations. The method is particularly useful for a group of multiple upgradient wells, to determine whether or not there are large average concentration differences from one location to the next due to natural groundwater fluctuations and/or differences in geochemistry. If downgradient wells are included in an ANOVA, the downgradient groundwater should not be contaminated, at least if a test of *natural* spatial variation is desired. Otherwise, a significant difference in population means could reflect the presence of either recent or historical contamination.

Steps involved: 1) Form the ANOVA residuals by subtracting from each measurement its sample well mean; 2) test the ANOVA residuals for normality and equal variance. If either of these assumptions is violated, try a transformation of the data and retest the assumptions; 3) compute the one-way ANOVA F-statistic; 4) if the F-statistic exceeds an α-level critical point, conclude the null hypothesis of equal population means has been violated and that there is some (perhaps substantial) degree of spatial variation; 5) if the F-statistic does not exceed the critical point, conclude that the well averages are close enough to treat the combined data as coming from the same statistical population.

Advantages/Disadvantages: One-way ANOVA is an excellent technique for identifying differences in separate well populations, as long as the assumptions are generally met. However, a finding of significant spatial variability does not specify the reason for the well-to-well differences. Additional information or investigation may be necessary to determine why the spatial differences exist. Be especially careful when (1) testing a combination of upgradient and downgradient wells that downgradient contamination is not the source of the difference found with ANOVA; and 2) when ANOVA identifies significant spatial variation and intrawell tests are called for. In the latter case, the ANOVA results can sometimes be used to estimate more powerful intrawell prediction and control limits. Such an adjustment comes directly from the ANOVA computations, requiring no additional calculation.

Analysis of Variance [ANOVA] for Temporal Effects (Sections 14.2.2 & 14.3.3)

Basic purpose: Diagnostic tool. It is a test to compare population means at multiple sampling events, after pooling the event data across wells. The test can also used to adjust data across multiple wells for common temporal dependence.

Hypothesis tested: H_0 — Population means across all sampling events are equal. H_A — One or more pairs of population means are unequal.

Underlying assumptions: 1) ANOVA residuals from the population at each sampling event must be normal or normalized. These should be tested for normality using a goodness-of-fit procedure; 2) the population variances across all sampling events must be equal. Test this assumption with box plots and Levene's test; and 3) each tested well should have at least 3 to 4 observations per sampling event.

When to use: 1) The ANOVA procedure for temporal effects should be used to identify significant temporal variation over a series of distinct sampling events. The method assumes that spatial variation by well location is not a significant factor (this should have already been tested). ANOVA for temporal effects should be used when a time series plot of a group of wells exhibits roughly parallel traces over time, indicating a time-related phenomenon affecting all the wells in a similar

way on any given sampling event. If a significant temporal effect is found, the results of the ANOVA can be employed to adjust the standard deviation estimate and the degrees of freedom quantities needed for further upgradient-to-downgradient comparisons; 2) compliance wells can be included in ANOVA for temporal effects, since the temporal pattern is assumed to affect all the wells on-site, regardless of gradient; and 3) residuals from ANOVA for temporal effects can be used to create adjusted, temporally-stationary measurements in order to eliminate the temporal dependence.

Steps involved: 1) Compute the mean (across wells) from data collected on each separate sampling event; 2) form the ANOVA residuals by subtracting from each measurement its sampling event mean; 3) test the ANOVA residuals for normality and equal variance. If either of these assumptions is violated, try a transformation of the data and retest the assumptions; 4) compute the one-way ANOVA F-statistic; 5) if the F-statistic exceeds an α-level critical point, conclude the null hypothesis of equal population means has been violated and that there is some (perhaps substantial) degree of temporal dependence; 6) compute the degrees of freedom adjustment factor and the adjusted standard deviation for use in *interwell* comparisons; 7) if the F-statistic does not exceed the critical point, conclude that the sampling event averages are close enough to treat the combined data as if there were no temporal dependence; and use the residuals, if necessary, to create adjusted, temporally-stationary measurements, regardless of the significance of the F-test (**Section 14.3.3**).

Advantages/Disadvantages: 1) One-way ANOVA for temporal effects is a good technique for identifying time-related effects among a group of wells. The procedure should be employed when a strong temporal dependence is indicated by parallel traces in time series plots; 2) if there is both temporal dependence and strong spatial variability, the ANOVA for temporal effects may be non-significant due to the added spatial variation. A two-way ANOVA for temporal and spatial effects might be considered instead; and 3) even if the ANOVA is non-significant, the ANOVA residuals can still be used to adjust data for apparent temporal dependence.

SAMPLE AUTOCORRELATION FUNCTION (SECTION 14.2.3)

Basic purpose: Diagnostic tool. This is a parametric estimate and test of autocorrelation (*i.e.*, time-related dependence) in a data series from a single population.

Hypothesis tested: H_0 — Measurements from the population are independent of sampling events (*i.e.*, they are not influenced by the time when the data were collected). H_A — The distribution of measurements is impacted by the time of data collection.

Underlying assumptions: Data should be approximately normal, with few non-detects. Sampling events represented in the sample should be fairly regular and evenly spaced in time.

When to use: When testing a data series from a single population (*e.g.*, a single well), the sample autocorrelation function (also known as the correlogram) can determine whether there is a significant temporal dependence in the data.

Steps involved: 1) Form overlapping ordered pairs from the data series by pairing measurements 'lagged' by a certain number of sampling events (*e.g.*, all pairs with measurements spaced by $k = 2$ sampling events); 2) for each distinct lag (k), compute the sample autocorrelation; 3) plot the

autocorrelations from **Step 2** by lag (k) on a scatter plot; and 4) count any autocorrelation as significantly different from zero if its absolute magnitude exceeds $2/\sqrt{n}$, where n is the sample size.

Advantages/Disadvantages: 1) The sample autocorrelation function provides a graphical test of temporal dependence. It can be used not only to identify autocorrelation, but also as a planning tool for adjusting the sampling interval between events. The smallest lag (k) at which the autocorrelation is insignificantly different from zero is the minimum sampling interval ensuring temporally uncorrelated data; 2) the test only applies to a single population at a time and cannot be used to identify temporal effects that span across groups of wells simultaneously. In that scenario, use a one-way ANOVA for temporal effects; and 3) tests for significant autocorrelation depend on the data being approximately normal; use the rank von Neumann ratio for non-normal samples.

RANK VON NEUMANN RATIO (SECTION 9.4)

Basic purpose: Diagnostic tool. It is a non-parametric test of first-order autocorrelation (*i.e.*, time-related dependence) in a data series from a single population.

Hypothesis tested: H_0 — Measurements from the population are independent of sampling events (*i.e.*, they are not influenced by the time when the data were collected). H_A — The distribution of measurements is impacted by the time of data collection.

Underlying assumptions: Data need not be normally distributed. However, it is assumed that the data series can be uniquely ranked according to concentration level. Ties in the data (*e.g.*, non-detects) are not technically allowed. Although a mid-rank procedure (as used in the Wilcoxon rank-sum test) to rank tied values might be considered, the available critical points for the rank von Neumann ratio statistic only directly apply to cases where a unique ranking is possible.

When to use: When testing a data series from a single population (*e.g.*, a single well) for use in, perhaps, an intrawell prediction limit, control chart, or test of trend, the rank von Neumann ratio can determine whether there is a significant temporal dependence in the data. If the dependence is seasonal, the data may be adjusted using a seasonal correction (**Section 14.3.3**). If the dependence is a linear trend, remove the estimated trend and re-run the rank von Neumann ratio on the trend residuals before concluding there are additional time-related effects. Complex dependence may require consultation with a professional statistician.

Steps involved: 1) Rank the measurements by concentration level, but then list the ranks in the order the samples were collected; 2) using the ranks, compute the von Neumann ratio; 3) if the rank von Neumann ratio exceeds an α-level critical point, conclude the data exhibit no significant temporal correlation. Otherwise, conclude that a time-related pattern does exist. Check for seasonal cycles or linear trends using time series plots. Consult a professional statistician regarding possible statistical adjustments if the pattern is more complex.

Advantages/Disadvantages: The rank von Neumann ratio, as opposed to other common time series methods for determining autocorrelation, is a non-parametric test based on using the ranks of the data instead of the actual concentration measurements. The test is simple to compute and can be used as a formal confirmation of temporal dependence, even if the autocorrelation appears fairly obvious on a time series plot. As a limiting feature, the test only applies to a single population at a time and

cannot be used to identify temporal effects that span across groups of wells simultaneously. In that scenario, a one-way ANOVA for temporal effects is a better diagnostic tool. Because critical points for the rank von Neumann ratio have not been developed for the presence of ties, the test will not be useful for datasets with substantial portions of non-detects.

DARCY EQUATION (SECTION 14.3.2)

Basic purpose: Method to determine a sampling interval ensuring that distinct physical volumes of groundwater are sampled on any pair of consecutive events.

Hypothesis tested: Not a statistical test or formal procedure.

Underlying assumptions: Flow regime is one in which Darcy's equation is approximately valid.

When to use: Use Darcy's equation to gauge the minimum travel time necessary for distinct volumes of groundwater to pass through each well screen. Physical independence of samples does not guarantee statistical independence, but it increases the likelihood of statistical independence. Use to design or plan for a site-specific sampling frequency, as well as what formal statistical tests and retesting strategies are possible given the amount of temporally-independent data that can be collected each evaluation period.

Steps involved: 1) Using knowledge of the site hydrogeology, calculate the horizontal and vertical components of average groundwater velocity with Darcy's equation; 2) Determine the minimum travel time needed between field samples to ensure physical independence; 3) Specify a sampling interval during monitoring no less than the travel time obtained via the Darcy computation.

Advantages/Disadvantages: Darcy's equation is relatively straightforward, but is not a statistical procedure. It is not applicable to certain hydrologic environments. Further, it is not a substitute for a direct estimate of autocorrelation. Statistical independence is not assured using Darcy's equation, so caution is advised.

SEASONAL CORRECTION (SECTION 14.3.3)

Basic purpose: Method to adjust a longer data series from a single population for an obvious seasonal cycle or fluctuation pattern. By removing the seasonal pattern, the remaining residuals can be used in further statistical procedures (*e.g.*, prediction limits, control charts) and treated as independent of the seasonal correlation.

Hypothesis tested: The seasonal correction is not a formal statistical test. Rather, it is a statistical adjustment to data for which a definite seasonal pattern has been identified.

Underlying assumptions: There should be enough data so that at least 3 full seasonal cycles are displayed on a time series plot. It is also assumed that the seasonal component has a stationary (*i.e.*, stable) mean and variance during the period of data collection.

When to use: Use the seasonal correction when a longer series of data must be examined, but a time series plot indicates a clearly recurring, seasonal fluctuation of concentration levels. If not removed, the seasonal dependence will tend to upwardly bias the estimated variability and could lead to inaccurate or insufficiently powerful tests.

Steps involved: 1) Using a time series plot of the data series, separate the values into common sampling events for each year (*e.g.*, all January measurements, all third quarter values, *etc.*); 2) compute the average of each subgroup and the overall mean of the dataset; and 3) adjust the data by removing the seasonal pattern.

Advantages/Disadvantages: The seasonal correction described in the Unified Guidance is relatively simple to perform and offers a more accurate standard deviation estimates compared to using unadjusted data. Removal of the seasonal component may reveal other previously unnoticed features of the data, such as a slow-moving trend. A fairly long data series is required to confirm the presence of a recurring seasonal cycle. Furthermore, many complex time-related patterns cannot be handled by this simple correction. In such cases, consultation with a professional statistician may be necessary.

SEASONAL MANN-KENDALL TEST FOR TREND (SECTION 14.3.4)

Basic purpose: Method for detection monitoring. It is used to identify the presence of a significant (upward) trend at a compliance point when data also exhibit seasonal fluctuations. It may also be used in compliance/assessment and corrective action monitoring to track upward or downward trends.

Hypothesis tested: H_0 — No discernible linear trend exists in the concentration data over time. H_A — A non-zero, (upward) linear component to the trend does exist.

Underlying assumptions: Since the seasonal Mann-Kendall trend test is a non-parametric method, the underlying data need not be normal or follow a particular distribution. No special adjustment for ties is needed.

When to use: Use when 1) upgradient-to-downgradient comparisons are inappropriate so that intrawell tests are called for; 2) a control chart or intrawell prediction limit cannot be used because of possible trends in the intrawell background, and 3) the data also exhibit seasonality. A trend test can be particularly helpful at sites with recent or historical contamination where it is uncertain if background is already contaminated. An upward trend in these cases will document the changing concentration levels more accurately than either a control chart or intrawell prediction limit, both of which assume a stationary background mean concentration.

Steps involved: 1) Divide the data into separate groups representing common sampling events from each year; 2) compute the Mann-Kendall test statistic (S) and its standard deviation ($SD[S]$) on each group; 3) sum the separate Mann-Kendall statistics into an overall test statistic; 4) compare this statistic against an α-level critical point; and 5) if the statistic exceeds the critical point, conclude that a significant upward trend exists. If not, conclude there is insufficient evidence for identifying a significant, non-zero trend.

Advantages/Disadvantages: 1) The seasonal Mann-Kendall test does not require any special treatment for non-detects, only that all non-detects be set to a common value lower than any of the detected values; and 2) the test is easy to compute and reasonably efficient for detecting (upward) trends in the presence of seasonality. Approximate critical points are derived from the standard normal distribution.

SIMPLE SUBSTITUTION (SECTION 15.2)

Basic purpose: A simple adjustment for non-detects in a dataset. One-half the reporting limit [RL] is substituted for each non-detect to provide a numerical approximation to the unknown true concentration.

Hypothesis tested: None.

Underlying assumptions: The true non-detect concentration is assumed to lie somewhere between zero and the reporting limit. Furthermore, that the probability of the true concentration being less than half the RL is about the same as the probability of it being greater than half the RL.

When to use: In general, simple substitution should be used when the dataset contains a relatively small proportion of non-detects, say no more than 10-15%. Use with larger non-detect proportions can result in biased estimates, especially if most of the detected concentrations are recorded at low levels (*e.g.*, at or near RL).

Steps involved: 1) Determine the reporting limit; and 2) replace each non-detect with one-half RL as a numerical approximation.

Advantages/Disadvantages: Simple substitution of half the RL is the easiest adjustment available for non-detect data. However, it can lead to biased estimates of the mean and particularly the variance if employed when more than 10-15% of the data are non-detects.

CENSORED PROBABILITY PLOT (SECTIONS 15.3 AND 15.4)

Basic purpose: Diagnostic tool. It is a graphical fit to normality of a mixture of detected and non-detect measurements. Adjustments are made to the plotting positions of the detected data under the assumption that all measurements come from a common distributional model.

Hypothesis tested: As a graphical tool, the censored probability plot is not a formal statistical test. However, it can provide an indication as to whether a dataset is consistent with the hypothesis that the mixture of detects and non-detects come from the same distribution and that the non-detects make up the lower tail of that distribution.

Underlying assumptions: Dataset consists of a mixture of detects and non-detects, all arising from a common distribution. Data must be normal or normalized.

When to use: Use the censored probability plot to check the viability of the Kaplan-Meier or robust regression on order statistics [ROS] adjustments for non-detect measurements. If the plot is linear, the data are consistent with a model in which the unobserved non-detect concentrations comprise the lower tail of the underlying distribution.

Steps involved: 1) Using either Kaplan-Meier or ROS, construct a partial ranking of the detected values to account for the presence of non-detects; 2) determine standard normal quantiles that match the ranking of the detects; and 3) graph the detected values against their matched normal quantiles on a probability plot and examine for a linear fit.

Advantages/Disadvantages: The censored probability plot offers a visual indication of whether a mixture of detects and non-detects come from the same (normal) distribution. There are, however, no formal critical points to aid in deciding when the fit is 'linear enough.' Correlation coefficients can be computed to informally aid the assessment. Censored probability plots can also be constructed on transformed data to help select a normalizing transformation.

KAPLAN-MEIER ADJUSTMENT (SECTION 15.3)

Basic purpose: Diagnostic tool. It is used to adjust a mixture of detected and non-detect data for the unknown concentrations of non-detect values. The Kaplan-Meier procedure leads to adjusted estimates for the mean and standard deviation of the underlying population.

Hypothesis tested: As a statistical adjustment procedure, the Kaplan-Meier method is not a formal statistical test. Rather, it allows estimation of characteristics of the population by assuming the combined group of detects and non-detects come from a common distribution.

Underlying assumptions: Dataset consists of a mixture of detects and non-detects, all arising from the same distribution. Data must be normal or normalized in the context of the Unified Guidance. Kaplan-Meier should not be used when more than 50% of the data are non-detects.

When to use: Since the Kaplan-Meier adjustment assumes all the measurements arise from the same statistical process, but that some of these measurements (*i.e.*, the non-detects) are unobservable due to limitations in analytical technology, Kaplan-Meier should be used when this model is the most realistic or reasonable choice. In particular, when constructing prediction limits, confidence limits, or control charts, the mean and standard deviation of the underlying population must be estimated. If non-detects occur in the dataset (but do not account for more than half of the observations), the Kaplan-Meier adjustment can be used to determine these estimates, which in turn can be utilized in constructing the desired statistical test.

Steps involved: 1) Sort the detected values and compute the 'risk set' associated with each detect; 2) using the risk set, compute the Kaplan-Meier cumulative distribution function [CDF] estimate associated with each detect; 3) calculate adjusted estimates of the population mean and standard deviation using the Kaplan-Meier CDF values; and 4) use these adjusted population estimates in place of the sample mean and standard deviation in prediction limits, confidence limits, and control charts.

Advantages/Disadvantages: Kaplan-Meier offers a way to adjust for significant fractions of non-detects without having to know the actual non-detect concentration values. It is more difficult to use than simple substitution, but avoids the biases inherent in that method.

ROBUST REGRESSION ON ORDER STATISTICS [ROS] (SECTION 15.4)

Basic purpose: Diagnostic tool. It is a method to adjust mixture of detects and non-detects for the unknown concentrations of non-detect values. Robust ROS leads to adjusted estimates for the mean and standard deviation of the underlying population by imputing a distinct estimated value for each non-detect.

Hypothesis tested: As a statistical adjustment procedure, robust ROS is not a formal statistical test. Rather, it allows estimation of characteristics of the population by assuming the combined group of detects and non-detects come from a common distribution.

Underlying assumptions: Dataset consists of a mixture of detects and non-detects, all arising from the same distribution. Data must be normal or normalized in the context of the Unified Guidance. Robust ROS should not be used when more than 50% of the data are non-detects.

When to use: Since robust regression on order statistics assumes all the measurements arise from the same statistical process, robust ROS should be used when this model is reasonable. In particular, when constructing prediction limits, confidence limits, or control charts, the mean and standard deviation of the underlying population must be estimated. If non-detects occur in the dataset (but do not account for more than half of the observations), robust ROS can be used to determine these estimates, which in turn can be utilized to construct the desired statistical test.

Steps involved: 1) Sort the distinct reporting limits [RL] for non-detect values and compute 'exceedance probabilities' associated with each RL; 2) using the exceedance probabilities, compute 'plotting positions' for the non-detects, essentially representing CDF estimates associated with each RL; 3) impute values for individual non-detects based on their RLs and plotting positions; 4) compute adjusted mean and standard deviation estimates via the sample mean and standard deviation of the combined set of detects and imputed non-detects; and 5) use these adjusted population estimates in place of the (unadjusted) sample mean and standard deviation in prediction limits, confidence limits, and control charts.

Advantages/Disadvantages: Robust ROS offers an alternative to Kaplan-Meier to adjust for significant fractions of non-detects without having to know the actual non-detect concentration values. It is more difficult to use than simple substitution, but avoids the biases inherent in that method.

Cohen's Method and Parametric ROS (Section 15.5)

Basic purpose: Diagnostic tools. These are other methods to adjust mixture of detects and non-detects to obtain the unknown mean and standard deviation for the entire data set

Hypothesis tested: **N**either technique is a formal statistical test. Rather, they allow estimation of characteristics of the population by assuming the combined group of detects and non-detects come from a common distribution.

Underlying assumptions: Dataset consists of a mixture of detects and non-detects, all arising from the same distribution. Data must be normal or normalized in the context of the Unified Guidance. Neither should be used when more than 50% of the data are non-detects nor when data contain multiple non-detect levels.

When to use: Since these methods assume that all the measurements arise from the same statistical process, they should be used when this model is reasonable. In particular, when constructing prediction limits, confidence limits, or control charts, the mean and standard deviation of the underlying population must be estimated. If non-detects occur in the dataset (but do not account for more than half of the observations), they can be used to determine these estimates, which in turn can be utilized to construct the desired statistical test.

Steps involved: Cohen's Method: 1) data are sorted into non-detect and detected portions; 2) detect mean and standard deviation estimates are calculated; 3) intermediate quantities of the ND% and a factor γ are calculated and used to locate the appropriate λ value from a table; and 4) full data set mean and standard deviation estimates are then obtained using formulas based on the detected mean, standard deviation, the detection limit and λ. Parametric ROS: 1) detected data are sorted in ascending order; 2) standardized normal distribution Z-values are generated from the full set of ranked values. Those corresponding to the sorted detected values are retained; 3) the detected values are then regressed against the Z-values; and 4) the resulting regression intercept and slope are the estimates of the mean and standard deviation for the full data set.

Advantages/Disadvantages: These two methods offer alternatives to Kaplan-Meier and robust ROS. The key limitation is that only data containing a single censoring limit can be used. In some situations using logarithmic data, their application can lead to biased estimates of the mean and standard deviation. Where appropriate, these methods are less computationally intensive that either Kaplan-Meier or robust ROS.

POOLED VARIANCE T-TEST (SECTION 16.1.1)

Basic purpose: Method for detection monitoring. This test compares the means of two populations.

Hypothesis tested: H_0 — Means of the two populations are equal; H_A — Means of the two populations are unequal (for the usual one-sided alternative, the hypothesis would state that the mean of the second population is greater than the mean of the first population).

Underlying assumptions: 1) The data from each population must be normal or normalized; 2) when used for interwell tests, there should be no significant spatial variability; 3) at least 4 observations per well should be available before applying the test; and 4) the two group variances are equal.

When to use: The pooled variance *t*-test can be used to test for groundwater contamination at very small sites, those consisting of maybe 3 or 4 wells and monitoring for 1 or 2 constituents. Site configurations with larger combinations of wells and constituents should employ a retesting scheme using either prediction limits or control charts. The pooled variance *t*-test can also be used to test proposed updates to intrawell background. A *non-significant t*-test in this latter case suggests the two sets of data are sufficiently similar to allow the initial background to be updated by augmenting with more recent measurements.

Steps involved: 1) Test the combined residuals from each population for normality. Make a data transformation if necessary; 2) test for equal variances, and if equal, compute a pooled variance estimate; 3) compute the pooled variance *t*-statistic and the degrees of freedom; 3) compare the *t*-statistic against a critical point based on both the α-level and the degrees of freedom; and 4) if the *t*-statistic exceeds the critical point, conclude the null hypothesis of equal means has been violated.

Advantages/Disadvantages: 1) The pooled variance *t*-test is one of the easiest to compute *t*-test procedures, but requires an assumption of equal variances across both populations; 2) because the *t*-test is a well-understood statistical procedure, the Unified Guidance recommends its use at very small groundwater monitoring facilities. For larger sites, however, repeated use of the *t*-test at a given α-level will lead to an unacceptably high risk of false positive error; and 3) if substantial spatial variability exists, the use of any *t*-test for upgradient-to-downgradient comparisons may lead

to inaccurate conclusions. A significant difference in the population averages could also indicate the presence of natural geochemical factors differentially affecting the concentration levels at different wells. In these situations, consider an intrawell test instead.

WELCH'S T-TEST (SECTION 16.1.2)

Basic purpose: Method for detection monitoring. This test compares the means of two populations.

Hypothesis tested: H_0 — Means of the two populations are equal; H_A — Means of the two populations are unequal (for the usual one-sided alternative, the hypothesis would state that the mean of the second population is greater than the mean of the first population).

Underlying assumptions: 1) The data from each population must be normal or normalized; 2) when used for interwell tests, there should be no significant spatial variability; and 3) At least 4 observations per well should be available before applying the test.

When to use: Welch's *t*-test can be used to test for groundwater contamination at very small sites, those consisting of maybe 3 or 4 wells and monitoring for 1 or 2 constituents. Site configurations with larger combinations of wells and constituents should employ a retesting scheme using either prediction limits or control charts. Welch's *t*-test can also be used to test proposed updates to intrawell background data. A *non-significant t*-test in this latter case suggests the two sets of data are sufficiently similar to allow the initial background to be updated by augmenting with the more recent measurements.

Steps involved: 1) Test the combined residuals from each population for normality. Make a data transformation if necessary; 2) compute Welch's *t*-statistic and approximate degrees of freedom; 3) compare the *t*-statistic against a critical point based on both the α-level and the estimated degrees of freedom; and 4) if the *t*-statistic exceeds the critical point, conclude the null hypothesis of equal means has been violated.

Advantages/Disadvantages: 1) Welch's *t*-test is slightly more difficult to compute than other common *t*-test procedures, but has the advantage of *not* requiring equal variances across both populations. Furthermore, it has been shown to perform statistically as well or better than other *t*-tests; 2) it can be used at very small groundwater monitoring facilities, but should be avoided at larger sites. Repeated use of the *t*-test at a given α-level will lead to an unacceptably high risk of false positive error; and 3) if there is substantial spatial variability, use of Welch's *t*-test for interwell tests may lead to inaccurate conclusions. A significant difference in the population averages may reflect the presence of natural geochemical factors differentially affecting the concentration levels at different wells. In these situations, consider an intrawell test instead.

WILCOXON RANK-SUM TEST (SECTION 16.2)

Basic purpose: Method for detection monitoring. This test compares the medians of two populations.

Hypothesis tested: H_0 — Both populations have equal medians (and, in fact, are identical in distribution). H_A — The two population medians are unequal (in the usual one-sided alternative, the hypothesis would state that the median of the second population is larger than the median of the first).

Underlying assumptions: 1) While the Wilcoxon rank-sum test does not require normal data, it does assume both populations have the same distributional form and that the variances are equal. If the data are non-normal but there at most a few non-detects, the equal variance assumption may be tested through the use of box plots and/or Levene's test. If non-detects make-up a large fraction of the observations, equal variances may have to be assumed rather than formally verified; 2) use of the Wilcoxon rank-sum procedure for interwell tests assumes there is no significant spatial variability. This is more likely to be the case in precisely those circumstances where the Wilcoxon procedure might be used: when there are high fractions of non-detects, so that most of the concentration measurements at any location are at low levels; and 3) there should be at least 4 background measurements and at least 2-4 compliance point values.

When to use: The Wilcoxon rank-sum test can be used to test for groundwater contamination at very small sites, those consisting of maybe 3 or 4 wells and monitoring for 1 or 2 constituents. Site configurations with larger combinations of wells and constituents should employ a retesting scheme using non-parametric prediction limits. Note, however, that non-parametric prediction limits often require large background sample sizes to be effective. The Wilcoxon rank-sum can be useful when a high percentage of the data is non-detect, but the amount of available background data is limited. Indeed, an *intrawell* Wilcoxon procedure may be helpful in some situations where the false positive rate would otherwise be too high to run intrawell prediction limits.

Steps involved: 1) Rank the combined set of values from the two datasets, breaking ties if necessary by using midranks; 2) compute the sum of the ranks from the compliance point well and calculate the Wilcoxon test statistic; 3) compare the Wilcoxon test statistic against an α-level critical point; and 4) if the test statistic exceeds the critical point, conclude that the null hypothesis of equal medians has been violated.

Advantages/Disadvantages: 1) The Wilcoxon rank-sum test is an excellent technique for small sites with constituent non-detect data. Compared to other possible methods such as the test of proportions or exact binomial prediction limits, the Wilcoxon rank-sum does a better job overall of correctly identifying elevated groundwater concentrations while limiting false positive error; 2) because the Wilcoxon rank-sum is easy to compute and understand, the Unified Guidance recommends its use at very small groundwater monitoring facilities. For larger sites, repeated use of the Wilcoxon rank-sum at a given α-level will lead to an unacceptably high risk of false positive error; and 3) if substantial spatial variability exists, the use of the Wilcoxon rank-sum for interwell tests may lead to inaccurate conclusions. A significant difference in the population medians may signal the presence of natural geochemical differences rather than contaminated groundwater. In these situations, consider an intrawell test instead.

TARONE-WARE TEST (SECTION 16.3)

Basic purpose: Non-parametric method for detection monitoring. This is an extension of Wilcoxon rank-sum, an alternative test to compare the medians in two populations when non-detects are prevalent.

Hypothesis tested: H_0 — Both populations have equal medians (and, in fact, are identical in distribution). H_A — The two population medians are unequal (in the usual one-sided alternative, the hypothesis would state that the median of the second population is larger than the median of the first).

Underlying assumptions: 1) The Tarone-Ware test does not require normal data, but does assume both populations have the same distributional form and that the variances are equal; and 2) use of the Tarone-Ware procedure for interwell tests assumes there is no significant spatial variability. This is more likely to be the case when there are high fractions of data non-detects, so that most of the concentration measurements at any location are at low and similar levels.

When to use: The Tarone-Ware test can be used to test for groundwater contamination at very small sites, those consisting of perhaps 3 or 4 wells and monitoring for 1 or 2 constituents. Site configurations with larger combinations of wells and constituents should employ a retesting scheme using non-parametric prediction limits. Note, however, that non-parametric prediction limits often require large background sample sizes to be effective. The Tarone-Ware test can be useful when a high percentage of the data is non-detect, but the amount of available background data is limited. The Tarone-Ware test is also an alternative to the Wilcoxon rank-sum when there are multiple reporting limits and/or it is unclear how to fully rank the data as required by the Wilcoxon.

Steps involved: 1) Sort the distinct detected values in the combined data set; 2) count the 'risk set' associated with each distinct value from **Step 1** and compute the expected number of compliance point detections within each risk set; 3) form the Tarone-Ware test statistic from the expected counts in **Step 2**; 4) compare the test statistic against a standard normal α-level critical point; and 5) if the test statistic exceeds the critical point, conclude that the null hypothesis of equal medians has been violated.

Advantages/Disadvantages: The Tarone-Ware test is an excellent technique for small sites with constituent non-detect data having multiple reporting limits. If substantial spatial variability exists, use of the Tarone-Ware test for interwell tests may lead to inaccurate conclusions. A significant difference in the population medians may signal the presence of natural geochemical differences rather than contaminated groundwater. In these situations, consider an intrawell test instead.

ONE-WAY ANALYSIS OF VARIANCE [ANOVA] (SECTION 17.1.1)

Basic purpose: Formal interwell detection monitoring test and diagnostic tool. It compares population means at multiple wells, in order to detect contaminated groundwater when tested against background.

Hypothesis tested: H_0 — Population means across all tested wells are equal. H_A — One or more pairs of population means are unequal.

Underlying assumptions: 1) ANOVA residuals at each well or population must be normally distributed or transformable to normality. These should be tested for normality using a goodness-of-fit procedure; 2) the population variances across all wells must be equal. This assumption can be tested with box plots and Levene's test; and 3) each tested well should have at least 3 to 4 separate observations.

When to use: The one-way ANOVA can sometimes be used to identify to simultaneously test for contaminated groundwater across a group of distinct well locations. As an inherently interwell test, ANOVA should be utilized only on constituents exhibiting little to no spatial variation. Most uses of ANOVA have been superseded by prediction limits and control charts, although it is commonly employed to identify spatial variability or temporal dependence across a group of wells.

Steps involved: 1) Form the ANOVA residuals by subtracting from each measurement its sample well mean; 2) tcst the ANOVA residuals for normality and equal variance. If either of these assumptions is violated, try a transformation of the data and retest the assumptions; 3) compute the one-way ANOVA *F*-statistic; 4) if the *F*-statistic exceeds an α-level critical point, conclude the null hypothesis of equal population means has been violated and that at least one pair of wells shows a significant difference in concentration levels; and 5) test each compliance well individually to determine which one or more exceeds background.

Advantages/Disadvantages: ANOVA is only likely to be infrequently used to make upgradient-to-downgradient comparisons in formal detection monitoring testing. The regulatory restrictions for per-constituent α-levels using ANOVA make it difficult to adequately control site-wide false positive rates [SWFPR]. Even if spatial variability is not a significant problem, users are advised to consider interwell prediction limits or control charts, and to incorporate some form of retesting

KRUSKAL-WALLIS TEST (SECTION 17.1.2)

Basic purpose: Formal interwell detection monitoring test and diagnostic tool. It compares population medians at multiple wells, in order to detect contaminated groundwater when tested against background. It is also useful as a non-parametric alternative to ANOVA for identifying spatial variability in constituents with non-detects or for data that cannot be normalized.

Hypothesis tested: H_0 — Population medians across all tested wells are equal. H_A — One or more pairs of population medians are unequal.

Underlying assumptions: 1) As a non-parametric alternative to ANOVA, data need not be normal; 2) the population variances across all wells must be equal. This assumption can be tested with box plots and Levene's test if the non-detect proportion is not too high; and 3) each tested well should have at least 3 to 4 separate observations.

When to use: The Kruskal-Wallis test can sometimes be used to identify to simultaneously test for contaminated groundwater across a group of distinct well locations. As an inherently interwell test, Kruskal-Wallis should be utilized for this purpose only with constituents exhibiting little to no spatial variation. Most uses of the Kruskal-Wallis (similar to ANOVA) have been superseded by prediction limits, although it can be used to identify spatial variability and/or temporal dependence across a group of wells when the sample data are non-normal or have higher proportions of non-detects.

Steps involved: 1) Sort and form the ranks of the combined measurements; 2) compute the rank-based Kruskal-Wallis test statistic (*H*); 3) if the *H*-statistic exceeds an α-level critical point, conclude the null hypothesis of equal population medians has been violated and that at least one pair of wells shows a significant difference in concentration levels; and 5) test each compliance well individually to determine which one or more exceeds background.

Advantages/Disadvantages: 1) The Kruskal-Wallis test is only likely to be infrequently used to make upgradient-to-downgradient comparisons in formal detection monitoring testing. The regulatory restrictions for per-constituent α-levels using ANOVA make it difficult to adequately control the SWFPR. Even if spatial variability is not a significant problem, users are advised to consider

interwell prediction limits, and to incorporate some form of retesting; and 2) the Kruskal-Wallis test can be used to test for spatial variability in constituents with significant fractions of non-detects.

TOLERANCE LIMIT (SECTION 17.2.1)

Basic purpose: Formal interwell detection monitoring test of background versus one or more compliance wells. Tolerance limits can be used as an alternative to one-way ANOVA. These can also be used in corrective action as an alternative clean-up limit.

Hypothesis tested: H_0 — Population means across all tested wells are equal. H_A — One or more pairs of population means are unequal.

Underlying assumptions: 1) Data should be normal or normalized; 2) the population variances across all wells are assumed to be equal. This assumption can be difficult to test when comparing a single new observation from each compliance well against a tolerance limit based on background; and 3) there should be a minimum of 4 background measurements, preferably 8-10 or more.

When to use: A tolerance limit can be used in place of ANOVA for detecting contaminated groundwater. It is more flexible than ANOVA since 1) as few as one new measurement per compliance well is needed to run a tolerance limit test, and 2) no post-hoc testing is necessary to identify which compliance wells are elevated over background. Most uses of tolerance limits (similar to ANOVA) have been superseded by prediction limits, due to difficulty of incorporating retesting into tolerance limit schemes. If a hazardous constituent requires a background-type standard in compliance/assessment or corrective action, a tolerance limit can be computed on background and used as a fixed GWPS.

Steps involved: 1) Compute background sample mean and standard deviation; 2) calculate upper tolerance limit on background with high confidence and high coverage; 3) collect one or more observations from each compliance well and test each against the tolerance limit; and 4) identify a well as contaminated if any of its observations exceed the tolerance limit.

Advantages/Disadvantages: Tolerance limits are likely to be used only infrequently to be used as either interwell or intrawell tests. Prediction limits or control charts offer better control of false positive rates, and less is known about the impact of retesting on tolerance limit performance.

NON-PARAMETRIC TOLERANCE LIMIT (SECTION 17.2.2)

Basic purpose: Formal interwell detection monitoring test of background versus one or more compliance wells. Non-parametric tolerance limits can be used as an alternative to the Kruskal-Wallis test. They may also be used in compliance/assessment or corrective action to define a background GWPS.

Hypothesis tested: H_0 — Population medians across all tested wells are equal. H_A — One or more pairs of population medians are unequal.

Underlying assumptions: 1) As a non-parametric test, non-normal data with non-detects can be used; and 2) there should be a minimum of 8-10 background measurements and preferably more.

When to use: A non-parametric tolerance limit can be used in place of the Kruskal-Wallis test for detecting contaminated groundwater. It is more flexible than Kruskal-Wallis since 1) as few as one new measurement per compliance well is needed to run a tolerance limit test, and 2) no post-hoc testing is necessary to identify which compliance wells are elevated over background. Most uses of tolerance limits have been superseded by prediction limits, due to difficulty of incorporating retesting into tolerance limit schemes. However, when a clean-up limit cannot or has not been specified in corrective action, a tolerance limit can be computed on background and used as a site-specific alternate concentration limit [ACL].

Steps involved: 1) Compute a large order statistic from background and set this value as the upper tolerance limit; 2) calculate the confidence and coverage associated with the tolerance limit; 3) collect one or more observations from each compliance well and test each against the tolerance limit; and 4) identify a well as contaminated if any of its observations exceed the tolerance limit.

Advantages/Disadvantages: 1) Tolerance limits are likely to be used only infrequently to be used as either interwell or intrawell tests. Prediction limits or control charts offer better control of false positive rates, and less is known about the impact of retesting on tolerance limit performance; and 2) non-parametric tolerance limits have the added disadvantage of generally requiring large background samples to ensure adequate confidence and/or coverage. For this reason, it is strongly recommended that a parametric tolerance limit be constructed whenever possible.

LINEAR REGRESSION (SECTION 14.4)

Basic purpose: Method for detection monitoring and diagnostic tool. It is used to identify the presence of a significantly increasing trend at a compliance point or any trend in background data sets.

Hypothesis tested: H_0 — No discernible linear trend exists in the concentration data over time. H_A — A non-zero, (upward) linear component to the trend does exist.

Underlying assumptions: Trend residuals should be normal or normalized, equal in variance, and statistically independent. If a small fraction of non-detects exists (≤10-15%), use simple substitution to replace each non-detect by half the reporting limit [RL]. Test homoscedasticity of residuals with a *scatter plot* (**Section 9.1**).

When to use: Use a test for trend when 1) upgradient-to-downgradient comparisons are inappropriate so that intrawell tests are called for, and 2) a control chart or intrawell prediction limit cannot be used because of possible trends in the intrawell background. A trend test can be particularly helpful at sites with recent or historical contamination where it is uncertain to what degree intrawell background is already contaminated. The presence of an upward trend in these cases will document the changing nature of the concentration data much more accurately than either a control chart or intrawell prediction limit, both of which assume a stable baseline concentration.

Steps involved: 1) If a linear trend is evident on a time series plot, construct the linear regression equation; 2) subtract the estimated trend line from each observation to form residuals; 3) test residuals for assumptions listed above; and 4) test regression slope to determine whether it is significantly different from zero. If so and the slope is positive, conclude there is evidence of a significant upward trend.

Advantages/Disadvantages: Linear regression is a standard statistical method for identifying trends and other linear associations between pairs of random variables. However, it requires approximate normality of the trend residuals. Confidence bands around regression trends can be used in compliance/assessment and corrective action to determine compliance with fixed standards even when concentration levels are actively changing (*i.e.*, when a trend is apparent).

MANN-KENDALL TEST FOR TREND (SECTION 17.3.2)

Basic purpose: Method for detection monitoring and diagnostic tool. It is used to identify the presence of a significant (upward) trend at a compliance point or any trend in background data.

Hypothesis tested: H_0 — No discernible linear trend exists in the concentration data over time. H_A — A non-zero, (upward) linear component to the trend does exist.

Underlying assumptions: Since the Mann-Kendall trend test is a non-parametric method, the underlying data need not be normal or follow any particular distribution. No special adjustment for ties is needed.

When to use: Use a test for trend when 1) interwell tests are inappropriate so that intrawell tests are called for, and 2) a control chart or intrawell prediction limit cannot be used because of possible trends in intrawell background. A trend test can be particularly helpful at sites with recent or historical contamination where it is uncertain if intrawell background is already contaminated. An upward trend in these cases documents changing concentration levels more accurately than either a control chart or intrawell prediction limit, both of which assume a stationary background mean concentration.

Steps involved: 1) Sort the data values by time of sampling/collection; 2) consider all possible pairs of measurements from different sampling events; 3) score each pair depending on whether the later data point is higher or lower in concentration than the earlier one, and sum the scores to get Mann-Kendall statistic; 4) compare this statistic against an α-level critical point; and 5) if the statistic exceeds the critical point, conclude that a significant upward trend exists. If not, conclude there is insufficient evidence for identifying a significant, non-zero trend.

Advantages/Disadvantages: The Mann-Kendall test does not require any special treatment for non-detects, only that all non-detects can be set to a common value lower than any of the detects. The test is easy to compute and reasonably efficient for detecting (upward) trends. Exact critical points are provided in the Unified Guidance for $n \leq 20$; a normal approximation can be used for $n > 20$. 3) A version of the Mann-Kendall test (the seasonal Mann-Kendall, **Section 14.3.4**) can be used to test for trends in data that exhibit seasonality.

THEIL-SEN TREND LINE (SECTION 17.3.3)

Basic purpose: Method for detection monitoring. This is a non-parametric alternative to linear regression for estimating a linear trend.

Hypothesis tested: As presented in the Unified Guidance, the Theil-Sen trend line is not a formal hypothesis test but rather an estimation procedure. The algorithm can be modified to formally test whether the true slope is significantly different from zero, but this question will already be answered if used in conjunction with the Mann-Kendall procedure.

Underlying assumptions: Like the Mann-Kendall trend test, the Theil-Sen trend line is non-parametric, so the underlying data need not be normal or follow a particular distribution. Furthermore, data ranks are not used, so no special adjustment for ties is needed.

When to use: It is particularly helpful when used in conjunction with the Mann-Kendall test for trend. The latter test offers information about whether a trend exists, but does not estimate the trend line itself. Once a trend is identified, the Theil-Sen procedure indicates how quickly the concentration level is changing with time.

Steps involved: 1) Sort the data set by date/time of sampling; 2) for each pair of distinct sampling events, compute the simple pairwise slope; 3) sort the list of pairwise slopes and set the overall slope estimate (Q) as the median slope in this list; 4) compute the median concentration and the median date/time of sampling; and 5) construct the Theil-Sen trend as the line passing through the median scatter point from **Step 4** with slope Q.

Advantages/Disadvantages: Although non-parametric, the Theil-Sen slope estimator does not use data ranks but rather the concentrations themselves. The method is non-parametric because the median pairwise slope is utilized, thus ignoring extreme values that might otherwise skew the slope estimate. The Theil-Sen trend line is as easy to compute as the Mann-Kendall test and does not require any special adjustment for ties (*e.g.*, non-detects).

PREDICTION LIMIT FOR *M* FUTURE VALUES (SECTION 18.2.1)

Basic purpose: Method for detection monitoring. This technique estimates numerical bound(s) on a series of m independent future values. The prediction limit(s) can be used to test whether the mean of one or more compliance well populations are equal to the mean of a background population.

Hypothesis tested: H_0 — The true mean of m future observations arises from the same population as the mean of measurements used to construct the prediction limit. H_A — The m future observations come from a distribution with a different mean than the population of measurements. Since an upper prediction limit is of interest in detection monitoring, the alternative hypothesis would state that the future observations are distributed with a larger mean than the background population.

Underlying assumptions: 1) Data used to construct the prediction limit must be normal or normalized. Adjustments for small to moderate fractions of non-detects can be made, perhaps using Kaplan-Meier or robust ROS; 2) although the variances of both populations (background and future values) are assumed to be equal, rarely will there be enough data from the future population to verify this assumption except during periodic updates to background; and 3) if used for upgradient-to-downgradient comparisons, there should be no significant spatial variability.

When to use: Prediction limits on individual observations can be used as an alternative in detection monitoring to either one-way ANOVA or Dunnett's multiple comparison with control [MCC] procedure. Assuming there is insignificant natural spatial variability, an interwell prediction limit can be constructed using upgradient or other representative background data. The number of future samples (m) should be chosen to reflect a single new observation collected from each downgradient or compliance well prior to the next statistical evaluation, plus a fixed number (m–1) of possible resamples. The initial future observation at each compliance point is then compared against the prediction limit. If it exceeds the prediction limit, one or more resamples are collected from the

'triggered' well and also tested against the prediction limit. If substantial spatial variability exists, prediction limits for individual values can be constructed on a well-specific basis using intrawell background. The larger the intrawell background size, the better. To incorporate retesting, it must be feasible to collect up to (m–1) additional, but independent, resamples from each well.

Steps involved: 1) Compute the estimated mean and standard deviation of the background data; 2) considering the type of prediction limit (*i.e.*, interwell or intrawell), the number of future samples m, the desired site-wide false positive rate, and the number of wells and monitoring parameters, determine the prediction limit multiplier (κ); 3) compute the prediction limit as the background mean plus κ times the background standard deviation; and 4) compare each initial future observation against the prediction limit. If both the initial measurement and resample(s) exceed the limit, conclude the null hypothesis of equal means has been violated.

Advantages/Disadvantages: Prediction limits for individual values offer several advantages compared to the traditional one-way ANOVA and Dunnett's multiple comparison with control [MCC] procedures. Prediction limits are not bound to a minimum 5% per-constituent false positive rate and can be constructed to meet a target site-wide false positive rate [SWFPR] while maintaining acceptable statistical power. Unlike the one-way ANOVA F-test, only the comparisons of interest (*i.e.*, each compliance point against background) are tested. This gives the prediction limit more statistical power. Prediction limits can be designed for intrawell as well as interwell comparisons.

Prediction Limit for Future Mean (Section 18.2.2)

Basic purpose: Method for detection monitoring or compliance monitoring. It is used to estimate numerical limit(s) on an independent mean constructed from p future values. The prediction limits(s) can be used to test whether the mean of one population is equal to the mean of a separate (background) population.

Hypothesis tested: H_0 — The true mean of p future observations arise from the same population as the mean of measurements used to construct the prediction limit. H_A — The p future observations come from a distribution with a different mean than the population of background measurements. Since an upper prediction limit is of interest in both detection and compliance monitoring, the alternative hypothesis would state that the future observations are distributed with a larger mean than that of the background population.

Underlying assumptions: 1) Data used to construct the prediction limit must be normal or normalized. Adjustments for small to moderate fractions of non-detects can be made, perhaps using Kaplan-Meier or robust ROS; 2) although the variances of both populations (background and future values) are assumed to be equal, rarely will there be enough data from the future population to verify this assumption; and 3) if used for upgradient-to-downgradient comparisons, there should be no significant spatial variability.

When to use: Prediction limits on means can be used as an alternative in detection monitoring to either one-way ANOVA or Dunnett's multiple comparison with control [MCC] procedure. Assuming there is insignificant natural spatial variability, an interwell prediction limit can be constructed using upgradient or other representative background data. The number of future samples p should be chosen to reflect the number of samples that will be collected at each compliance well prior to the next statistical evaluation (*e.g.*, 2, 4, *etc.*). The average of these p observations at each compliance

point is then compared against the prediction limit. If it is feasible to collect at least *p* additional, but independent, resamples from each well, retesting can be incorporated into the procedure by using independent mean(s) of *p* samples as confirmation value(s).

If substantial spatial variability exists, prediction limits for means can be constructed on a well-specific basis using intrawell background. At least two future values must be available per well. Larger intrawell background size are preferable. To incorporate retesting, it must be feasible to collect at least *p* independent resamples from each well, in addition to the initial set of *p* samples. A prediction limit can also be used in some compliance monitoring settings when a fixed compliance health based limit cannot be use and the compliance point data must be compared directly to a background GWPS. In this case, the compliance point mean concentration is tested against an upper prediction limit computed from background. No retesting would be employed for this latter kind of test.

Steps involved: 1) Compute the background sample mean and standard deviation; 2) considering the type of prediction limit (*i.e.*, interwell or intrawell), the number of future samples *p*, use of retesting, the desired site-wide false positive rate, and the number of wells and monitoring parameters, determine the prediction limit multiplier (κ); 3) compute the prediction limit as the background mean plus κ times the background standard deviation; 4) compare each future mean of order *p* (*i.e.*, a mean constructed from *p* values) against the prediction limit; and 5) if the future mean exceeds the limit and retesting is not feasible (or if used for compliance monitoring), conclude the null hypothesis of equal means has been violated. If retesting is feasible, conclude the null hypothesis has been violated only when the resampled mean(s) of order *p* also exceeds the prediction limit.

Advantages/Disadvantages: Prediction limits on means offer several advantages compared to the traditional one-way ANOVA and Dunnett's multiple comparison with control [MCC] procedure: Prediction limits are not bound to a minimum 5% per-constituent false positive rate. As such, prediction limits can be constructed to meet a target SWFPR, while maintaining acceptable statistical power. Unlike the one-way *F*-test, only the comparisons of interest (*i.e.*, each compliance point against background) are tested, giving the prediction limit more statistical power. Prediction limits can be designed for intrawell as well as interwell comparisons. One slight disadvantage is that ANOVA combines compliance point data with background to give a somewhat better per-well estimate of variability. But even this disadvantage can be overcome when using an interwell prediction limit by first running ANOVA on the combined background and compliance point data to generate a better variance estimate with a larger degree of freedom. A disadvantage compared to prediction limits on individual future values is that two or more new compliance point observations per well must be available to run the prediction limit on means. If only one new measurement per evaluation period can be collected, the user should instead construct a prediction limit on individual values.

NON-PARAMETRIC PREDICTION LIMIT FOR *M* FUTURE VALUES (SECTION 18.3.1)

Basic purpose: Method for detection monitoring. It is a non-parametric technique to estimate numerical limits(s) on a series of *m* independent future values. The prediction limit(s) can be used to test whether two samples are drawn from the same or different populations.

Hypothesis tested: H_0 — The *m* future observations come from the same distribution as the measurements used to construct the prediction limit. H_A — The *m* future observations come from a

different distribution than the population of measurements used to build the prediction limit. Since an upper prediction limit is of interest in detection monitoring, the alternative hypothesis is that the future observations are distributed with a larger median than the background population.

Underlying assumptions: 1) The data used to construct the prediction limit need not be normal; however, the forms of the both the background distribution and the future distribution are assumed to be the same. Since the non-parametric prediction limit is constructed as an order statistic of background, high fractions of non-detects are acceptable; 2) although the variances of both populations (background and future values) are assumed to be equal, rarely will there be enough data from the future population to verify this assumption; and 3) if used for upgradient-to-downgradient comparisons, there should be no significant spatial variability. Spatial variation is less likely to be significant in many cases where constituent data are primarily non-detect, allowing the use of a non-parametric interwell prediction limit test.

When to use: Prediction limits on individual values can be used as a non-parametric alternative in detection monitoring to either one-way ANOVA or Dunnett's multiple comparison with control [MCC] procedure. Assuming there is insignificant natural spatial variability, an interwell prediction limit can be constructed using upgradient or other representative background data. The number of future samples m should be chosen to reflect a single new observation collected from each compliance well prior to the next statistical evaluation, plus a fixed number (m–1) of possible resamples. The initial future observation at each compliance point is then compared against the prediction limit. If it exceeds the prediction limit, one or more resamples are collected from the 'triggered' well and also compared to the prediction limit.

Steps involved: 1) Determine the maximum, second-largest, or other highly ranked value in background and set the non-parametric prediction limit equal to this level; 2) considering the number of future samples m, and the number of wells and monitoring parameters, determine the achievable site-wide false positive rate [SWFPR]. If the error rate is not acceptable, consider possibly enlarging the pool of background data used to construct the limit or increasing the number of future samples m; 3) compare each initial future observation against the prediction limit; and 4) if both the initial measurement and resample(s) exceed the limit, conclude the null hypothesis of equal distributions has been violated.

Advantages/Disadvantages: Non-parametric prediction limits on individual values offer distinct advantages compared to the Kruskal-Wallis non-parametric ANOVA test. Prediction limits are not bound to a minimum 5% per-constituent false positive rate. As such, prediction limits can be constructed to meet a target SWFPR, while maintaining acceptable statistical power. Unlike the Kruskal-Wallis test, only the comparisons of interest (*i.e.*, each compliance point against background) are tested, giving the prediction limit more statistical power. Non-parametric prediction limits have the disadvantage of generally requiring fairly large background samples to effectively control false positive error and ensure adequate power.

PREDICTION LIMIT FOR FUTURE MEDIAN (SECTION 18.3.2)

Basic purpose: Method for detection monitoring and compliance monitoring. This is a non-parametric technique to estimate numerical limits(s) on the median of p independent future values. The prediction limit(s) is used to test whether the median of one or more compliance well populations is equal to the median of the background population.

Hypothesis tested: H_0 — The true median of p future observations arise from the same population as the median of measurements used to construct the prediction limit. H_A — The p future observations come from a distribution with a different median than the background population of measurements. Since an upper prediction limit is of interest in both detection monitoring and compliance monitoring, the alternative hypothesis is that the future observations are distributed with a larger median than the background population.

Underlying assumptions: 1) The data used to construct the prediction limit need not be normal; however, the forms of the both the background distribution and the future distribution are assumed to be the same. Since the non-parametric prediction limit is constructed as an order statistic of background, high fractions of non-detects are acceptable: 2) although the variances of both populations (background and future values) are assumed to be equal, rarely will there be enough data from the future population to verify this assumption; and 3) if used for upgradient-to-downgradient comparisons, there should be no significant spatial variability.

When to use: Prediction limits on medians can be used as a non-parametric alternative in detection monitoring to either one-way ANOVA or Dunnett's multiple comparison with control [MCC] procedure. Assuming there is insignificant natural spatial variability, an interwell prediction limit can be constructed using upgradient or other representative background data. The number of future samples p should be odd and chosen to reflect the number of samples that will be collected at each compliance well prior to the next statistical evaluation (*e.g.*, 3). The median of these p observations at each compliance point is then compared against the prediction limit. If it is feasible to collect at least p additional, but independent, resamples from each well, retesting can be incorporated into the procedure by using independent median(s) of p samples as confirmation value(s). A prediction limit for a compliance point median can also be constructed in certain compliance monitoring settings, when no fixed health-based compliance limit can be used and the compliance point data must be directly compared against a background GWPS. In this case, the compliance point median concentration is compared to an upper prediction limit computed from background. No retesting is employed for this latter kind of test.

Steps involved: 1) Determine the maximum, second-largest, or other highly ranked value in background and set the non-parametric prediction limit equal to this level; 2) considering the number of future samples p, whether or not retesting will be incorporated, and the number of wells and monitoring parameters, determine the achievable SWFPR. If the error rate is not acceptable, increase the background sample size or consider a non-parametric prediction limit on individual future values instead; 3) compare each future median of order p (*i.e.*, a median of p values) against the prediction limit; and 4) if the future median exceeds the limit and retesting is not feasible (or if the test is used for compliance monitoring), conclude the null hypothesis of equal medians has been violated. If retesting is feasible, conclude the null hypothesis has been violated only when the resampled median(s) of order p also exceeds the prediction limit.

Advantages/Disadvantages: Non-parametric prediction limits on medians offer distinct advantages compared to the Kruskal-Wallis test (a non-parametric one-way ANOVA). Prediction limits are not bound to a minimum 5% per-constituent false positive rate. As such, prediction limits can be constructed to meet a target SWFPR, while maintaining acceptable statistical power. Unlike the Kruskal-Wallis test, only the comparisons of interest (*i.e.*, each compliance point against background) are tested, giving the prediction limit more statistical power. A disadvantage in

detection monitoring compared to non-parametric prediction limits on individual future values is that at least three new compliance point observations per well must be available to run the prediction limit on medians. If only one new observation per evaluation period can be collected, construct instead a non-parametric prediction limit for individual values. All non-parametric prediction limits have the disadvantage of usually requiring fairly large background samples to effectively control false positive error and ensure adequate power.

SHEWHART-CUSUM CONTROL CHART (SECTION 20.2)

Basic purpose: Method for detection monitoring. These are used to quantitatively and visually track concentrations at a given well over time to determine whether they exceed a critical threshold (*i.e.*, control limit), thus implying a significant increase above background conditions.

Hypothesis tested: H_0 — Data plotted on the control chart follow the same distribution as the background data used to compute the baseline chart parameters. H_A — Data plotted on the chart follow a different distribution with higher mean level than the baseline data.

Underlying assumptions: Data used to construct the control chart must be approximately normal or normalized. Adjustments for small to moderate fractions of non-detects, perhaps using Kaplan-Meier or ROS, can be acceptable. There should be no discernible trend in the baseline data used to calculate the control limit.

When to use: Use control charts as an alternative to parametric prediction limits, when 1) there are enough uncontaminated baseline data to compute an accurate control limit, and 2) there are no trends in intrawell background. Retesting can be incorporated into control charts by judicious choice of control limit. This may need to be estimated using Monte Carlo simulations.

Steps involved: 1) Compute the intrawell baseline mean and standard deviation; 2) calculate an appropriate control limit from these baseline parameters, the desired retesting strategy and number of well-constituent pairs in the network; 3) construct the chart, plotting the control limit, the compliance point observations, and the cumulative sums [CUSUM]; and 4) determine that the null hypothesis is violated when either an individual concentration measurement or the cumulative sum exceeds the control limit.

Advantages/Disadvantages: Unlike prediction limits, control charts offer an explicit visual tracking of compliance point values over time and provide a method to judge whether these concentrations have exceeded a critical threshold. The Shewhart portion of the chart is especially good at detecting sudden concentration increases, while the CUSUM portion is preferred for detecting slower, steady increases over time. No non-parametric version of the combined Shewhart-CUSUM control chart exists, so non-parametric prediction limits should be considered if the data cannot be normalized.

CONFIDENCE INTERVAL AROUND NORMAL MEAN (SECTION 21.1.1)

Basic purpose: Method for compliance/assessment monitoring or corrective action. This is a technique for estimating a range of concentration values from sample data, in which the true mean of a normal population is expected to occur at a certain probability.

Hypothesis tested: In compliance monitoring, H_0 — True mean concentration at the compliance point is no greater than the predetermined groundwater protection standard [GWPS]. H_A — True mean

concentration is greater than the GWPS. In corrective action, H_0 — True mean concentration at the compliance point is greater than or equal to the fixed GWPS. H_A — True mean concentration is less than or equal to the fixed standard.

Underlying assumptions: 1) Compliance point data are approximately normal in distribution. Adjustments for small to moderate fractions of non-detects, perhaps using Kaplan-Meier or ROS, are encouraged; 2) data do not exhibit any significant trend over time; 3) there are a minimum of 4 observations for testing. Generally, at least 8 to 10 measurements are recommended; and 4) the fixed GWPS is assumed to represent a true mean average concentration, rather than a maximum or upper percentile.

When to use: A mean confidence interval can be used for normal data to determine whether there is statistically significant evidence that the average is either above a fixed GWPS (in compliance monitoring) or below the fixed standard (in corrective action). In either case, the null hypothesis is rejected only when the *entire* confidence interval lies on one or the other side of the GWPS. The key determinant in compliance monitoring is whether the *lower* confidence limit exceeds the GWPS, while in corrective action the *upper* confidence limit lies below the clean-up standard. Because of bias introduced by transformations when estimating a mean, this approach should not be used for highly-skewed or non-normal data. Instead consider a confidence interval around a lognormal mean or a non-parametric confidence interval. It is also not recommended for use when the data exhibit a significant trend. In that case, the estimate of variability will likely be too high, leading to an unnecessarily wide interval and possibly little chance of deciding the hypothesis. When a trend is present, consider instead a confidence interval around a trend line.

Steps involved: 1) Compute the sample mean and standard deviation; 2) based on the sample size and choice of a confidence level ($1-\alpha$), calculate either the lower confidence limit (for use in compliance monitoring) or the upper confidence limit (for use in corrective action); 3) compare the confidence limit against the GWPS or clean-up standard; and 4) if the lower confidence limit exceeds the GWPS in compliance monitoring or the upper confidence limit is below the clean-up standard, conclude that the null hypothesis should be rejected.

Advantages/Disadvantages: Use of a confidence interval instead of simply the sample mean for comparison to a fixed standard accounts for both the level of statistical variation in the data and the desired or targeted confidence level. The same basic test can be used both to document contamination above the compliance standard in compliance/assessment and to show a sufficient decrease in concentration levels below the clean-up standard in corrective action.

CONFIDENCE INTERVAL ON LOGNORMAL GEOMETRIC MEAN (SECTION 21.1.2)

Basic purpose: Method for compliance/assessment monitoring or corrective action. It is a technique to estimate the range of concentration values from sample data, in which the true geometric mean of a lognormal population is expected to occur at a certain probability.

Hypothesis tested: In compliance monitoring, H_0 — True mean concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True mean concentration is greater than the GWPS. In corrective action, H_0 — True mean concentration at the compliance point is greater than the fixed compliance or clean-up standard. H_A — True mean concentration is less than or equal to the fixed standard.

Underlying assumptions: 1) Compliance point data are approximately lognormal in distribution. Adjustments for small to moderate fractions of non-detects, perhaps using Kaplan-Meier or ROS, are encouraged; 2) data do not exhibit any significant trend over time; 3) there are a minimum of 4 observations. Generally, at least 8 to 10 measurements are recommended; and 4) the fixed GWPS is assumed to represent a true geometric mean average concentration following a lognormal distribution, rather than a maximum or upper percentile. The GWPS also represents the true median.

When to use: A confidence interval on the geometric mean can be used for lognormal data to determine whether there is statistically significant evidence that the geometric average is either above a fixed numerical standard (in compliance monitoring) or below a fixed standard (in corrective action). In either case, the null hypothesis is rejected only when the *entire* confidence interval is to one side of the compliance or clean-up standard. Because of this fact, the key question in compliance monitoring is whether the *lower* confidence limit exceeds the GWPS, while in corrective action the user must determine whether the *upper* confidence limit is below the clean-up standard. Because of bias introduced by transformations when estimating the arithmetic lognormal mean, and the often unreasonably high upper confidence limits generated by Land's method for lognormal mean confidence intervals (see below), this approach is an alternative approach for lognormal data. One could also consider a non-parametric confidence interval. It is also not recommended for use when data exhibit a significant trend. In that case, the estimate of variability will likely be too high, leading to an unnecessarily wide interval and possibly little chance of deciding the hypothesis. When a trend is present, consider instead a confidence interval around a trend line.

Steps involved: 1) Compute the sample log-mean and log-standard deviation; 2) based on the sample size and choice of confidence level ($1–\alpha$), calculate either the lower confidence limit (for use in compliance monitoring) or the upper confidence limit (for use in corrective action) using the logged measurements and exponentiate the result; 3) compare the confidence limit against the GWPS or clean-up standard; and 4) if the lower confidence limit exceeds the GWPS in compliance monitoring or the upper confidence limit is below the clean-up standard, conclude that the null hypothesis should be rejected.

Advantages/Disadvantages: Use of a confidence interval instead of simply the sample geometric mean for comparison to a fixed standard accounts for both statistical variation in the data and the targeted confidence level. The same basic test can be used both to document contamination above the compliance standard in compliance/assessment and to show a sufficient decrease in concentration levels below the clean-up standard in corrective action.

CONFIDENCE INTERVAL ON LOGNORMAL ARITHMETIC MEAN (SECTION 21.1.3)

Basic purpose: Test for compliance/assessment monitoring or corrective action. This is a method by Land (1971) used to estimate the range of concentration values from sample data, in which the true arithmetic mean of a lognormal population is expected to occur at a certain probability.

Hypothesis tested: In compliance monitoring, H_0 — True mean concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True mean concentration is greater than the GWPS. In corrective action, H_0 — True mean concentration at the compliance point is greater than the fixed compliance or clean-up standard. H_A — True mean concentration is less than or equal to the fixed standard.

Underlying assumptions: 1) Compliance point data are approximately lognormal in distribution. Adjustments for small to moderate fractions of non-detects, perhaps using Kaplan-Meier or ROS, are encouraged; 2) data do not exhibit any significant trend over time; 3) there are a minimum of 4 observations. Generally, at least 8 to 10 measurements are strongly recommended; and 4) the fixed GWPS is assumed to represent the true arithmetic mean average concentration, rather than a maximum or upper percentile.

When to use: Land's confidence interval procedure can be used for lognormally-distributed data to determine whether there is statistically significant evidence that the average is either above a fixed numerical standard (in compliance monitoring) or below a fixed standard (in corrective action). In either case, the null hypothesis is rejected only when the *entire* confidence interval is to one side of the compliance or clean-up standard. Because of this fact, the key question in compliance monitoring is whether the *lower* confidence limit exceeds the GWPS, while in corrective action the user must determine whether the *upper* confidence limit is below the clean-up standard. Because the lognormal distribution can have a highly skewed upper tail, this approach should only be used when the data fit the lognormal model rather closely, especially if used in corrective action. Consider instead a confidence interval around the lognormal geometric mean or a non-parametric confidence interval if this is not the case. It is also not recommended for data that exhibit a significant trend. In that situation, the estimate of variability will likely be too high, leading to an unnecessarily wide interval and possibly little chance of deciding the hypothesis. When a trend is present, consider instead a confidence interval around a trend line.

Steps involved: 1) Compute the sample log-mean and log-standard deviation; 2) based on the sample size, magnitude of the log-standard deviation and choice of confidence level ($1-\alpha$), determine Land's adjustment factor; 3) then calculate either the lower confidence limit (for use in compliance monitoring) or the upper confidence limit (for use in corrective action); 4) compare the confidence limit against the GWPS or clean-up standard; and 5) if the lower confidence limit exceeds the GWPS in compliance montoring or the upper confidence limit is below the clean-up standard, conclude that the null hypothesis should be rejected.

Advantages/Disadvantages: Use of a confidence interval instead of simply the sample mean for comparison to a fixed standard accounts for both statistical variation in the data and the targeted confidence level. The same basic test can be used both to document contamination above the compliance standard in compliance/assessment and to show a sufficient decrease in concentration levels below the clean-up standard in corrective action. Since the upper confidence limit on a lognormal mean can be extremely high for some populations, the user may need to consider a non-parametric upper confidence limit on the median concentration as an alternative or use a program such as **Pro-UCL** to determine an alternate upper confidence limit.

CONFIDENCE INTERVAL ON UPPER PERCENTILE (SECTION 21.1.4)

Basic purpose: Method for compliance monitoring. It is used to estimate the range of concentration values from sample data in which a pre-specified true proportion of a normal population is expected to occur at a certain probability. The test can also be used to identify the range of a true proportion or percentile (*e.g.*, the 95th) in population data which can be normalized.

Hypothesis tested: H_0 — True upper percentile concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True upper percentile concentration is greater than the fixed GWPS.

Underlying assumptions: 1) Compliance point data are either normal in distribution or can be normalized. Adjustments for small to moderate fractions of non-detects, perhaps using Kaplan-Meier or ROS, are encouraged; 2) data do not exhibit any significant trend over time; 3) there are a minimum of at least 8 to 10 measurements; and 4) the fixed GWPS is assumed to represent a maximum or upper percentile, rather than an average concentration.

When to use: A confidence interval around an upper percentile can be used to determine whether there is statistically significant evidence that the percentile is above a fixed numerical standard. The null hypothesis is rejected only when the *entire* confidence interval is greater than the compliance standard. Because of this fact, the key question in compliance monitoring is whether the *lower* confidence limit exceeds the GWPS. This approach is not recommended for use when the data exhibit a significant trend. The estimate of variability will likely be too high, leading to an unnecessarily wide interval and possibly little chance of deciding the hypothesis.

Steps involved: 1) Compute the sample mean and standard deviation; 2) based on the sample size, pre-determined true proportion and test confidence level ($1-\alpha$), calculate the lower confidence limit; 3) compare the confidence limit against the GWPS; and 4) if the lower confidence limit exceeds the GWPS, conclude that the true upper percentile is larger than the compliance standard.

Advantages/Disadvantages: If a fixed GWPS is intended to represent a 'not-to-be-exceeded' maximum or an upper percentile, statistical comparison requires the prior definition of a true or expected upper percentile against which sample data can be compared. Some standards may explicitly identify the expected percentile. The appropriate test then must estimate the confidence interval in which this true proportion is expected to lie. Either an upper or lower confidence limit can be generated, depending on whether compliance or corrective action hypothesis testing is appropriate. Whatever the interpretation of a given limit used as a GWPS, it should be determined in advance what a given standard represents before choosing which type of confidence interval to construct.

NON-PARAMETRIC CONFIDENCE INTERVAL ON MEDIAN (SECTION 21.2)

Basic purpose: Test for compliance/assessment monitoring or corrective action. It is a non-parametric method used to estimate the range of concentration values from sample data in which the true median of a population is expected to occur at a certain probability.

Hypothesis tested: In compliance monitoring, H_0 — True median concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True median concentration is greater than the GWPS. In corrective action, H_0 — True median concentration at the compliance point is greater than the fixed compliance or clean-up standard. H_A — True median concentration is less than or equal to the fixed standard.

Underlying assumptions: 1) Compliance data need not be normal in distribution; up to 50% non-detects are acceptable; 2) data do not exhibit any significant trend over time; 3) there are a *minimum* of at least 7 measurements; and 4) the fixed GWPS is assumed to represent a true median average concentration, rather than a maximum or upper percentile.

When to use: A confidence interval on the median can be used for non-normal data (*e.g.*, samples with non-detects) to determine whether there is statistically significant evidence that the average (*i.e.*, median) is either above a fixed numerical standard (in compliance monitoring) or below a fixed standard (in corrective action). In either case, the null hypothesis is rejected only when the *entire* confidence interval is to one side of the compliance or clean-up standard. Because of this fact, the key question in compliance monitoring is whether the *lower* confidence limit exceeds the GWPS, while in corrective action the user must determine whether the *upper* confidence limit is below the clean-up standard. This approach is not recommended for use when data exhibit a significant trend. In that case, the variation in the data will likely be too high, leading to an unnecessarily wide interval and possibly little chance of deciding the hypothesis. It is also possible that the apparent trend is an artifact of differing detection or reporting limits that have changed over time. The trend may disappear if all non-detects are imputed at a common value or RL. If a trend is still present after investigating this possibility, but a significant portion of the data are non-detect, consultation with a professional statistician is recommended.

Steps involved: 1) Order and rank the data values; 2) pick tentative interval endpoints close to the estimated median concentration; 3) using the selected endpoints, compute the achieved confidence level of the lower confidence limit for use in compliance monitoring or that of the upper confidence limit for corrective action; 4) iteratively expand the interval until either the selected endpoints achieve the targeted confidence level or the maximum or minimum data value is chosen as the confidence limit; and 5) compare the confidence limit against the GWPS or clean-up standard. If the lower confidence limit exceeds the GWPS in compliance monitoring or the upper confidence limit is below the clean-up standard, conclude that the null hypothesis should be rejected.

Advantages/Disadvantages: Use of a confidence interval instead of simply the sample median for comparison to a fixed limit accounts for both statistical variation in the data and the targeted confidence level. The same basic test can be used both to document contamination above the compliance standard in compliance/assessment and to show a sufficient decrease in concentration levels below the clean-up standard in corrective action. By not requiring normal or normalized data, the non-parametric confidence interval can accommodate a substantial fraction of non-detects. A minor disadvantage is that a non-parametric confidence interval estimates the location of the median, instead of the mean. For symmetric populations, these quantities will be the same, but for skewed distributions they will differ. So if the compliance or clean-up standard is designed to represent a mean concentration, the non-parametric interval around the median may not provide a completely fair and/or accurate comparison. In some cases, the non-parametric confidence limit will not achieve the desired confidence level even if set to the maximum or minimum data value, leading to a higher risk of false positive error.

NON-PARAMETRIC CONFIDENCE INTERVAL ON UPPER PERCENTILE (SECTION 21.2)

Basic purpose: Non-parametric method for compliance monitoring. It is used to estimate the range of concentration values from sample data in which a pre-specified true proportion of a population is expected to occur at a certain probability. Exact probabilities will depend upon sample data ranks.

Hypothesis tested: H_0 — True upper percentile concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True upper percentile concentration is greater than the GWPS.

Underlying assumptions: 1) Compliance point data need not be normal; large fractions of non-detects can be acceptable; 2) data do not exhibit any significant trend over time; 3) there are a minimum of at least 8 to 10 measurements; and 4) the fixed GWPS is assumed to represent a true upper percentile of the population, rather than an average concentration.

When to use: A confidence interval on an upper percentile can be used to determine whether there is statistically significant evidence that the percentile is above a fixed numerical standard. The null hypothesis is rejected only when the *entire* confidence interval is greater than the compliance standard. Because of this fact, the key determinant in compliance/assessment monitoring is whether the *lower* confidence limit exceeds the GWPS. This approach is not recommended for use when data exhibit a significant trend. In that case, the estimate of variability will likely be too high, leading to an unnecessarily wide interval and possibly little chance of deciding the hypothesis.

Steps involved: 1) Order and rank the data values; 2) select tentative interval endpoints close to the estimated upper percentile concentration; 3) using the selected endpoints, compute the achieved confidence level of the lower confidence limit; 4) iteratively expand the interval until either the selected lower endpoint achieves the targeted confidence level or the minimum data value is chosen as the confidence limit; and 5) compare the confidence limit against the GWPS. If the lower confidence limit exceeds the GWPS, conclude that the population upper percentile is larger than the compliance standard.

Advantages/Disadvantages: If a fixed GWPS is intended to represent a 'not-to-be-exceeded' maximum or an upper percentile, statistical comparison requires the prior definition of a true or expected upper percentile against which sample data can be compared. Some standards may explicitly identify the expected percentile. The appropriate test then must estimate the confidence interval in which this true proportion is expected to lie. Either an upper or lower confidence limit can be generated, depending on whether compliance or corrective action hypothesis testing is appropriate. Whatever the interpretation of a given limit used as a GWPS, it should be determined in advance what a given standard represents before choosing which type of confidence interval to construct. However, precise non-parametric estimation of upper percentiles often requires much larger sample sizes than the parametric option (**Section 21.1.4**). For this reason, a *parametric* confidence interval for upper percentile tests is recommended whenever possible, especially if a suitable transformation can be found or adjustments made for non-detect values.

Confidence Band Around Linear Regression (Section 21.3.1)

Basic purpose: Method for compliance/assessment monitoring or corrective action when stationarity cannot be assumed. It is used to estimate ranges of concentration values from sample data around each point of a predicted linear regression line at a specified probability. The prediction line (based on regression of concentration values against time) represents the best estimate of gradually changing true mean levels over the time period.

Hypothesis tested: In compliance monitoring, H_0 — True mean concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True mean concentration is greater than the GWPS. In corrective action, H_0 — True mean concentration at the compliance point is greater than the fixed compliance or clean-up standard. H_A — True mean concentration is less than or equal to the fixed standard.

Underlying assumptions: 1) Compliance point values exhibit a linear trend with time, with normally distributed residuals. Use simple substitution with small (≤10-15%) fractions of non-detects. Non-detect adjustment methods are not recommended; 2) there are a minimum of 4 observations. Generally, at least 8 to 10 measurements are recommended; and 3) the fixed GWPS is assumed to represent an average concentration, rather than a maximum or upper percentile.

When to use: A confidence interval around a trend line should be used in cases where a linear trend is apparent on a time series plot of the compliance point data. Even if observed well concentrations are either increasing under compliance monitoring or decreasing in corrective action, it does not necessarily imply that the true mean concentration at the current time is either above or below the fixed GWPS. While the trend line properly accounts for the fact that the mean is changing with time, the null hypothesis is rejected only when the *entire* confidence interval is to one side of the compliance or clean-up standard at the most recent point(s) in time. The key determinant in compliance monitoring is whether the *lower* confidence limit at a specified point in time exceeds the GWPS, while in corrective action the *upper* confidence limit at a specific time must lie below the clean-up standard to be considered in compliance.

Steps involved: 1) Check for presence of a trend on a time series plot; 2) estimate the coefficients of the best-fitting linear regression line; 3) compute the trend line residuals and check for normality; 4) if data are non-normal, try re-computing the regression and residuals after transforming the data; 5) compute the lower confidence limit band around the trend line for compliance monitoring or the upper confidence limit band around the trend line for corrective action; and 6) compare the confidence limit at each sampling event against the GWPS or clean-up standard. If the lower confidence limit exceeds the GWPS in compliance/assessment or the upper confidence limit is below the clean-up standard on one or more recent sampling events, conclude that the null hypothesis should be rejected.

Advantages/Disadvantages: Use of a confidence interval around the trend line instead of simply the regression line itself for comparison to a fixed standard accounts for both statistical variation in the data and the targeted confidence level. The same basic test can be used both to document contamination above the compliance standard in compliance/assessment and to show a sufficient decrease in concentration levels below the clean-up standard in corrective action. By estimating the trend line first and then using the residuals to construct the confidence interval, variation due to the trend itself is removed, providing a more powerful test (via a narrower interval) of whether or not the true mean is on one side of the fixed standard. This technique can only be used when the identified trend is reasonably linear and the trend residuals are approximately normal.

NON-PARAMETRIC CONFIDENCE BAND AROUND THEIL-SEN TREND (SECTION 21.3.1)

Basic purpose: Non-parametric method for compliance/assessment or corrective action when stationarity cannot be assumed. It is used to estimate ranges of concentration values from sample data around each point of a predicted Theil-Sen trend line at a specified probability. The prediction line represents the best estimate of gradually changing true median levels over the time period.

Hypothesis tested: In compliance monitoring, H_0 — True mean concentration at the compliance point is no greater than the fixed compliance or groundwater protection standard [GWPS]. H_A — True mean concentration is greater than the GWPS. In corrective action, H_0 — True mean concentration at the

compliance point is greater than the fixed compliance or clean-up standard. H_A — True mean concentration is less than or equal to the fixed standard.

Underlying assumptions: 1) Compliance point values exhibit a linear trend with time; 2) non-normal data and substantial levels of non-detects up to 50% are acceptable; 3) there are a minimum of 8-10 observations available to construct the confidence band; and 4) the fixed GWPS is assumed to represent a median average concentration, rather than a maximum or upper percentile.

When to use: A confidence interval around a trend line should be used in cases where a linear trend is apparent on a time series plot of the compliance point data. Even if observed well concentrations are either increasing under compliance monitoring or decreasing in corrective action, it does not necessarily imply that the true mean concentration at the current time is either above or below the fixed GWPS. While the trend line properly accounts for the fact that the mean is changing with time, the null hypothesis is rejected only when the *entire* confidence interval is to one side of the compliance or clean-up standard at the most recent point(s) in time. The key determinant in compliance monitoring is whether the *lower* confidence limit at a specified point in time exceeds the GWPS, while in corrective action the *upper* confidence limit at a specific time must lie below the clean-up standard to be considered in compliance.

Steps involved: 1) Check for presence of a trend on a time series plot; 2) construct a Theil-Sen trend line; 3) use bootstrapping to create a large number of simulated Theil-Sen trends on the sample data; 4) construct a confidence band by selecting lower and upper percentiles from the set of bootstrapped Theil-Sen trend estimates; and 5) compare the confidence band at each sampling event against the GWPS or clean-up standard. If the lower confidence band exceeds the GWPS in compliance/assessment or the upper confidence band is below the clean-up standard on one or more recent sampling events, conclude that the null hypothesis should be rejected.

Advantages/Disadvantages: Use of a confidence band around the trend line instead of simply the Theil-Sen trend line itself for comparison to a fixed standard accounts for both statistical variation in the data and the targeted confidence level. The same basic test can be used both in compliance/assessment and in corrective action. By estimating the trend line first and then using bootstrapping to construct the confidence band, variation due to the trend itself is removed, providing a more powerful test (via a narrower interval) of whether or not the true mean is on one side of the fixed standard. This technique can only be used when the identified trend is reasonably linear. The Theil-Sen trend estimates the change in median level rather than the mean. For roughly symmetric populations, this will make little difference; for highly skewed populations, the trend in the median may not accurately reflect changes in mean concentration levels.

PART II: DIAGNOSTIC METHODS AND TESTING

Part II covers diagnostic evaluations of historical facility data for checking key assumptions implicit in the recommended statistical tests and for making appropriate adjustments to the data (e.g., consideration of outliers, seasonal autocorrelation, or non-detects). Also included is a discussion of groundwater sampling and how hydrologic factors such as flow and gradient can impact the sampling program.

Chapter 9 provides a number of exploratory data tools and examples, which can generally be used in data evaluations. Approaches for fitting data sets to normal and other parametric distributions follows in **Chapter 10**. The importance of the normal distribution and its potential uses is also discussed. **Chapter 11** provides methods for assessing the equality of variance necessary for some formal testing. The subject of outliers and means of testing for them is covered in **Chapter 12**. **Chapter 13** addresses spatial variability, with particular emphasis on ANOVA means testing. In **Chapter 14**, a number of topics concerning temporal variation are provided. In addition to providing tests for identifying the presence of temporal variation, specific adjustments for certain types of temporal dependence are covered. The final **Chapter 15** of **Part II** discusses non-detect data and offers several methods for estimating missing data. In particular, methods are provided to deal with data containing multiple non-detection limits.

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CHAPTER 9. COMMON EXPLORATORY TOOLS

9.1 TIME SERIES PLOTS 9-1
9.2 BOX PLOTS 9-5
9.3 HISTOGRAMS 9-8
9.4 SCATTER PLOTS 9-13
9.5 PROBABILITY PLOTS 9-16

Graphs are an important tool for exploring and understanding patterns in any data set. Plotting the data visually depicts the structure and helps unmask possible relationships between variables affecting the data set. Data plots which accompany quantitative statistical tests can better demonstrate the reasons for the results of a formal test. For example, a Shapiro-Wilk test may conclude that data are not normally distributed. A probability plot or histogram of the data can confirm this conclusion graphically to show why the data are not normally distributed (e.g., heavy skewness, bimodality, a single outlier, etc.).

Several common exploratory tools are presented in **Chapter 9**. These graphical techniques are discussed in statistical texts, but are presented here in detail for easy reference for the data analyst. An example data set is used to demonstrate how each of the following plots is created.

- Time series plots (**Section 9.1**)
- Box plots (**Section 9.2**)
- Histograms (**Section 9.3**)
- Scatter plots (**Section 9.4**)
- Probability plots (**Section 9.5**)

9.1 TIME SERIES PLOTS

Data collected over specific time intervals (e.g., monthly, biweekly, or hourly) have a temporal component. For example, air monitoring measurements of a pollutant may be collected once a minute or once a day. Water quality monitoring measurements may be collected weekly or monthly. Typically, groundwater sample data are collected quarterly from the same monitoring wells, either for detection monitoring testing or demonstrating compliance to a GWPS. An analyst examining temporal data may be interested in the trends over time, correlation among time periods, or cyclical patterns. Some graphical techniques specific to temporal data are the time plot, lag plot, correlogram, and variogram. The degree to which some of these techniques can be used will depend in part on the frequency and number of data collected over time.

A data sequence collected at regular time intervals is called a time series. More sophisticated time series data analyses are beyond the scope of this guidance. If needed, the interested user should consult with a statistician or appropriate statistical texts. The graphical representations presented in this section are recommended for any data set that includes a temporal component. Techniques described below will help identify temporal patterns that need to be accounted for in any analysis of the data. The analyst examining temporal environmental data may be interested in seasonal trends, directional trends, serial correlation, or stationarity. *Seasonal trends* are patterns in the data that repeat over time, i.e., the data

rise and fall regularly over one or more time periods. Seasonal trends may occur over long periods of time (large scale), such as a yearly cycle where the data show the same pattern of rising and falling from year to year, or the trends may be over a relatively short period of time (small scale), such as a daily cycle. Examples of seasonal trends are quarterly seasons (winter, spring, summer and fall), monthly seasons, or even hourly (e.g., air temperature rising and falling over the course of a day). *Directional trends* are increasing or decreasing patterns over time in monitored constituent data, which may be of importance in assessing the levels of contaminants. *Serial correlation* is a measure of the strength in the linear relationship of successive observations. If successive observations are related, statistical quantities calculated without accounting for the serial correlation may be biased. A time series is *stationary* if there is no systematic change in the mean (i.e., no trend) and variance across time. Stationary data look the same over all time periods except for random behavior. Directional trends or a change in the variability in the data imply non-stationarity.

A time series plot of concentration data versus time makes it easy to identify lack of randomness, changes in location, change in scale, small scale trends, or large-scale trends over time. Small-scale trends are displayed as fluctuations over smaller time periods. For example, ozone levels over the course of one day typically rise until the afternoon, then decrease, and this process is repeated every day. Larger scale trends such as seasonal fluctuations appear as regular rises and drops in the graph. Ozone levels tend to be higher in the summer than in the winter, so ozone data tend to show both a daily trend and a seasonal trend. A time plot can also show directional trends or changing variability over time.

A time plot is constructed by plotting the measurements on the vertical axis versus the actual time of observation or the order of observation on the horizontal axis. The points plotted may be connected by lines, but this may create an unfounded sense of continuity. It is important to use the actual date, time or number at which the observation was made. This can create discontinuities in the plot but are needed as the data that should have been collected now appear as “missing values” but do not disturb the integrity of the plot. Plotting the data at equally spaced intervals when in reality there were different time periods between observations is not advised.

For environmental data, it is also important to use a different symbol or color to distinguish non-detects from detected data. Non-detects are often reported by the analytical laboratory with a “U” or “<” analytical qualifier associated with the reporting limit [RL]. In statistical terminology, they are left-censored data, meaning the actual concentration of the chemical is known only to be below the RL. Non-detects contrast with detected data, where the laboratory reports the result as a known concentration that is statistically higher than the analytical limit of detection. For example, the laboratory may report a trichloroethene concentration in groundwater of “5 U” or “< 5” µg/L, meaning the actual trichloroethene concentration is unknown, but is bounded between zero and 5 µg/L. This result is different than a detected concentration of 5 µg/L which is unqualified by the laboratory or data validator. Non-detects are handled differently than detected data when calculating summary statistics. A statistician should be consulted on the proper use of non-detects in statistical analysis. For radionuclides negative and zero concentrations should be plotted as reported by the laboratory, showing the detection status.

The scaling of the vertical axis of a time plot is of some importance. A wider scale tends to emphasize large-scale trends, whereas a narrower scale tends to emphasize small-scale trends. A wide scale would emphasize the seasonal component of the data, whereas a smaller scale would tend to

emphasize the daily fluctuations. The scale needs to contain the full range of the data. Directions for constructing a time plot are contained in **Example 9-1** and **Figure 9-1**.

►EXAMPLE 9-1

Construct a time series plot using trichloroethene groundwater data in **Table 9-1** for each well. Examine the time series for seasonality, directional trends and stationarity.

Table 9-1. Trichloroethene (TCE) Groundwater Concentrations

	Well 1		**Well 2**	
Date Collected	**TCE (mg/L)**	**Data Qualifier**	**TCE (mg/L)**	**Data Qualifier**
1/2/2005	0.005	U	0.10	U
4/7/2005	0.005	U	0.12	
7/13/2005	0.004	J	0.125	
10/24/2005	0.006		0.107	
1/7/2006	0.004	U	0.099	U
3/30/2006	0.009		0.11	
6/28/2006	0.017		0.13	
10/2/2006	0.045		0.109	
10/17/2006	0.05		NA	
1/15/2007	0.07		0.10	U
4/10/2007	0.12		0.115	
7/9/2007	0.10		0.14	
10/5/2007	NA		0.17	
10/29/2007	0.20		NA	
12/30/2007	0.25		0.11	

NA = Not available (missing data).
U denotes a non-detect.
J denotes an estimated detected concentration.

SOLUTION

Step 1. Import the data into data analysis software capable of producing graphics.

Step 2. Sort the data by date collected.

Step 3. Determine the range of the data by calculating the minimum and maximum concentrations for each well, shown in the table below:

	Well 1		Well 2	
	TCE (mg/L)	**Data Qualifier**	**TCE (mg/L)**	**Data Qualifier**
Min	0.004	U	0.099	U
Max	0.25		0.17	

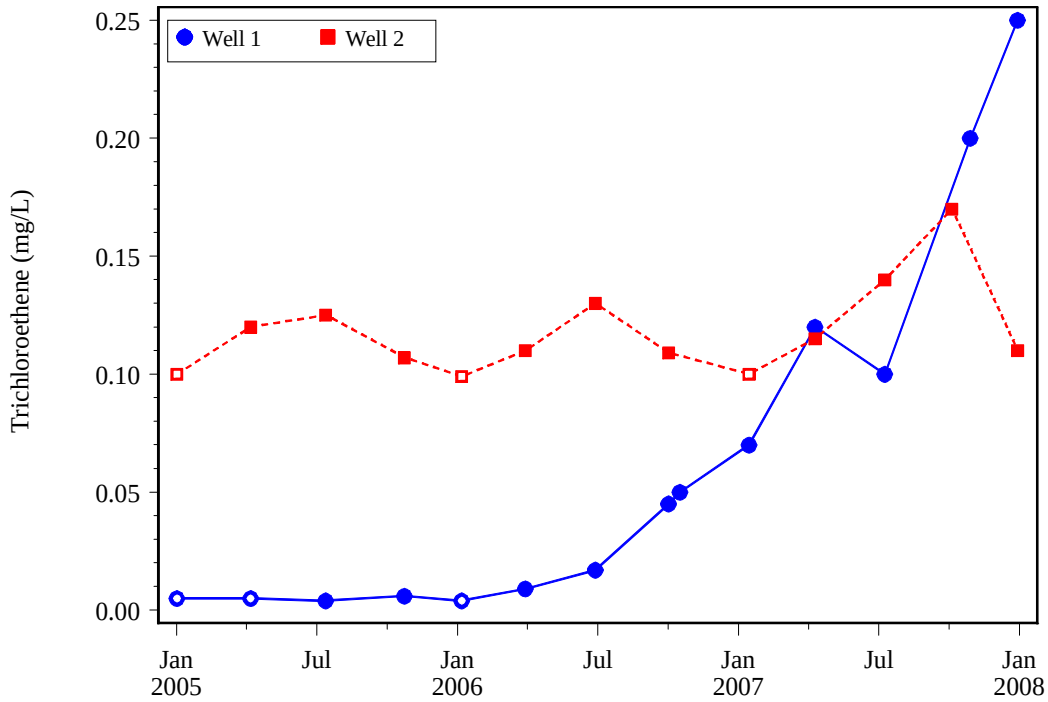
Step 4. Plot the data using a scale from 0 to 0.25 if data from both wells are plotted together on the same time series plot. Use separate symbols for non-detects and detected concentrations. One suggestion is to use "open" symbols (whose centers are white) for non-detects and "closed" symbols for detects.

Step 5. Examine each series for directional trends, seasonality and stationarity. Note that Well 1 demonstrates a positive directional trend across time, while Well 2 shows seasonality within each year. Neither well exhibits stationarity.

Step 6. Examine each series for missing values. Inquire from the project laboratory why data are missing or collected at unequal time intervals. A response from the laboratory for this data set noted that on 10/5/2007 the sample was accidentally broken in the laboratory from Well 1, so Well 1 was resampled on 10/29/2007. Well 1 was resampled on 10/17/2006 to confirm the historically high concentration collected on 10/2/2006. Well 2 was not sampled on 10/17/2006 because the data collected on 10/2/2006 from Well 2 did not merit a resample, as did Well 1.

Step 7. Examine each series for elevated detection limits. Inquire why the detection limits for Well 2 are much larger than detection limits for Well 1. A reason may be that different laboratories analyzed the samples from the two wells. The laboratory analyzing samples from Well 1 used lower detection limits than did the laboratory analyzing samples from Well 2.◀

Figure 9-1. Time Series Plot of Trichloroethene Groundwater for Wells 1 and 2 from 2005-2007.



Open symbols denote non-detects. Closed symbols denote detected concentrations.

9.2 BOX PLOTS

Box plots (also known as Box and Whisker plots) are useful in situations where a picture of the distribution is desired, but it is not necessary or feasible to portray all the details of the data. A box plot displays several percentiles of the data set. It is a simple plot, yet provides insight into the location, shape, and spread of the data and underlying distribution. A simple box plot contains only the 0th (minimum data value), 25th, 50th, 75th and 100th (maximum data value) percentiles. A box-plot divides the data into 4 sections, each containing 25% of the data. Whiskers are the lines drawn to the minimum and maximum data values from the 25th and 75th percentiles. The box shows the interquartile range (IQR) which is defined as the difference between the 75th and the 25th percentiles. The length of the central box indicates the spread of the data (the central 50%), while the length of the whiskers shows the breadth of the tails of the distribution. The 50th percentile (median) is the line within the box. In addition, the mean and the 95% confidence limits around the mean are shown. Potential outliers are categorized into two groups:

- data points between 1.5 and 3 times the IQR above the 75th percentile or between 1.5 and 3 times the IQR below the 25th percentile, and
- data points that exceed 3 times the IQR above the 75th percentile or exceed 3 times the IQR below the 25th percentile.

The mean is shown as a star, while the lower and upper 95% confidence limits around the mean are shown as bars. Individual data points between 1.5 and 3 times the IQR above the 75th percentile or below the 25th percentile are shown as circles. Individual data points at least 3 times the IQR above the 75th percentile or below the 25th percentile are shown as squares.

Information from box plots can assist in identifying potential data distributions. If the upper box and whisker are approximately the same length as the lower box and whisker, with the mean and median approximately equal, then the data are distributed symmetrically. The normal distribution is one of a number that is symmetric. If the upper box and whisker are longer than the lower box and whisker, with the mean greater than the median, then the data are right-skewed (such as lognormal or square root normal distributions in original units). Conversely, if the upper box and whisker are shorter than the lower box and whisker with the mean less than the median, then the data are left-skewed.

A box plot showing a normal distribution will have the following characteristics: the mean and median will be in the center of the box, whiskers to the minimum and maximum values are the same length, and there would be no potential outliers. A box plot showing a lognormal distribution (in original units) typical of environmental applications will have the following characteristics: the mean will be larger than the median, the whisker above the 75th percentile will be longer than the whisker below the 25th percentile, and extreme upper values may be indicated as potential outliers. Once the data have been logarithmically transformed, the pattern should follow that described for a normal distribution. Other right-skewed distributions transformable to normality would indicate similar patterns.

It is often helpful to show box plots of different sets of data side by side to show differences between monitoring stations (see **Figure 9-2**). This allows a simple method to compare the locations, spreads and shapes of several data sets or different groups within a single data set. In this situation, the width of the box can be proportional to the sample size of each data set. If the data will be compared to a standard, such as a preliminary remediation goal (PRG) or maximum contaminant level (MCL), a line on the graph can be drawn to show if any results exceed the criteria.

It is important to plot the data as reported by the laboratory for non-detects or negative radionuclide data. Proxy values for non-detects should not be plotted since we want to see the distribution of the original data. Different symbols can be used to display non-detects, such as the open symbols described in **Section 9.1**. The mean will be biased high if using the RL of non-detects in the calculation, but the purpose of the box plot is to assess the distribution of the data, not quantifying a precise estimate of an unbiased mean. Displaying the frequency of detection (number of detected values / number of total samples) under the station name is also useful. Unlike time series plots, box plots cannot use missing data, so missing data should be removed before producing a box plot.

Directions for generating a box plot are contained in **Example 9-2**, and an example is shown in **Figure 9-2.** It is important to remove lab and field duplicates from the data before calculating summary statistics such as the mean and UCL since these statistics assume independent data. The box plot assumes the data are statistically independent.

►EXAMPLE 9-2

Construct a box plot using the trichloroethene groundwater data in **Table 9-1** for each well. Examine the box plot to assess how each well is distributed (normal, lognormal, skewed, symmetric, etc.). Identify possible outliers.

SOLUTION

Step 1. Import the data into data analysis software capable of producing box plots.

Step 2. Sort the data from smallest to largest results by well.

Step 3. Compute the 0th (minimum value), 25th, 50th (median), 75th and 100th (maximum value) percentiles by well.

Step 4. Plot these points vertically. Draw a box around the 25th and 75th percentiles and add a line through the box at the 50th percentile. Optionally, make the width of the box proportional to the sample size. Narrow boxes reflect smaller sample sizes, while wider boxes reflect larger sample sizes.

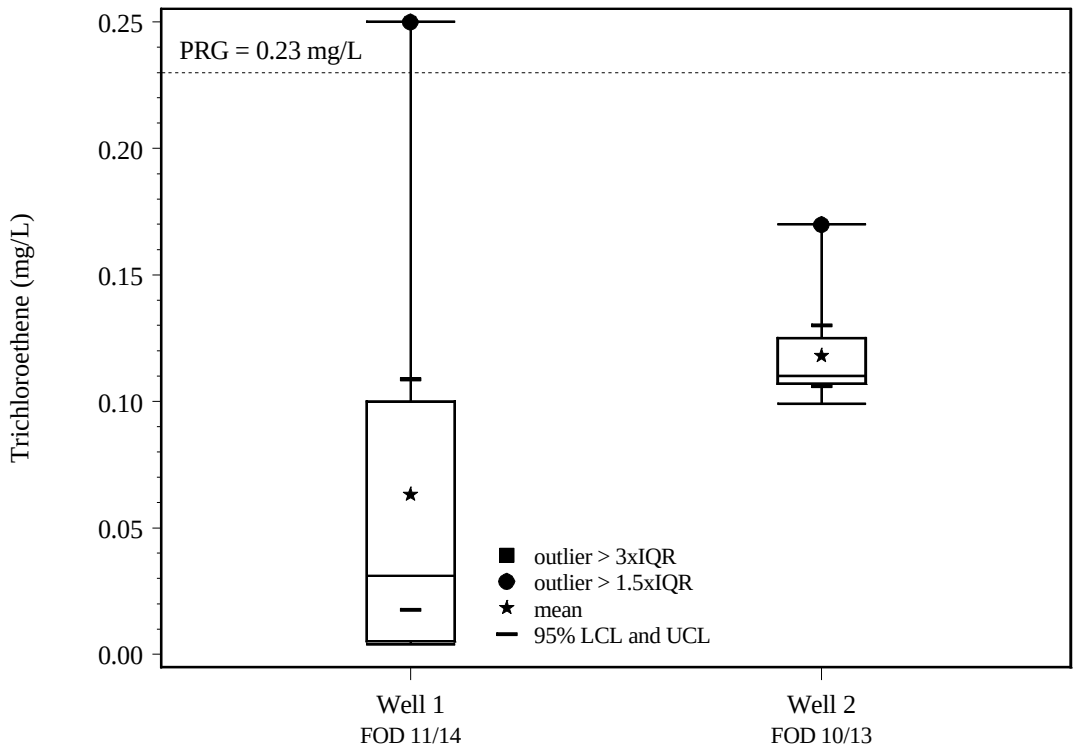
Step 5. Compute the mean and the lower and upper 95% confidence limits. Denote the mean with a star and the confidence limits as bars. Also, identify potential outliers between 1.5×IQR and 3×IQR beyond the box with a circle. Identify potential outliers exceeding 3×IQR beyond the box with a ` square.

Step 6. Draw the whiskers from each end of the box to the furthest data point to show the full range of the data.

INTERPRETATION

The box plots in **Figure 9-2** show the similarities and differences in the distributions of trichloroethene in Wells 1 and 2. The mean of trichloroethene in Well 1 is significantly lower than the mean in Well 2. The variance of the data from Well 1 is significantly larger than the variance from Well 2. A parametric t-test or nonparametric Wilcoxon Rank Sum test can quantitatively confirm these conclusions. Since the mean exceeds the median for both wells and the whiskers at the top of each box are much longer than the whiskers at the bottom of each box, we can conclude both distributions are skewed to the right, resembling a lognormal distribution. In fact, the Shapiro-Wilk test quantitatively confirms that both distributions are lognormally distributed. Both wells have their largest concentrations between 1.5 and 3 times the IQR, as denoted by a black circle. No data point lies outside 3 times the IQR. Since the data for both wells are lognormally distributed, the maximum concentrations in each well should not be removed just because they exceed 1.5 times the IQR. Long tails are expected for the lognormal distribution. The width of the 95% confidence limits confirms the large variability in Well 1 compared to the width of the confidence limits in Well 2. Well 1 has one concentration exceeding the PRG of 0.23 mg/L, while Well 2 has all concentrations below the PRG. The width of each box is similar since the sample size as shown in the frequency of detection (FOD) are nearly the same (11 detects out of 14 samples for Well 1 and 10 detects out of 13 samples for Well 2). ◄

Figure 9-2. Box Plots of Trichloroethene Data for Wells 1 & 2



9.3 HISTOGRAMS

A histogram is a visual representation of the data collected into groups. This graphical technique provides a visual method of identifying the underlying distribution of the data. The data range is divided into several bins or classes and the data is sorted into the bins. A histogram is a bar graph conveying the bins and the frequency of data points in each bin. Other forms of the histogram use a normalization of the bin frequencies for the heights of the bars. The two most common normalizations are relative frequencies (frequencies divided by sample size) and densities (relative frequency divided by the bin width). **Figure 9-3** is an example of a histogram using frequencies and **Figure 9-4** is a histogram of densities. Histograms provide a visual method of accessing location, shape and spread of the data. Also, extreme values and multiple modes can be identified. The details of the data are lost, but an overall picture of the data is obtained. A stem and leaf plot offers the same insights into the data as a histogram, but the data values are retained.

The visual impression of a histogram is sensitive to the number of bins selected. A large number of bins will increase data detail, while fewer bins will increase the smoothness of the histogram. A good starting point when choosing the number of bins is the square root of the sample size n. The minimum number of bins for any histogram should be at least 4. Another factor in choosing bins is the choice of endpoints. When feasible, using simple bin endpoints can improve the readability of the histogram. Simple bin endpoints include multiples of $5k$ units for some integer $k > 0$ (e.g., 0 to <5, 5 to <10, etc. or 1 to <1.5, 1.5 to <2, etc.). Finally, when plotting a histogram for a continuous variable (e.g.,

concentration), it is necessary to decide on an endpoint convention; that is, what to do with data points that fall on the boundary of a bin. Also, use the data as reported by the laboratory for non-detects and eliminate any missing values, since histograms cannot include missing data. With discrete variables, (e.g., family size) the intervals can be centered in between the variables. For the family size data, the intervals can span between 1.5 and 2.5, 2.5 and 3.5, and so on. Then the whole numbers that relate to the family size can be centered within the box. Directions for generating a histogram are contained in **Example 9-3**.

►EXAMPLE 9-3

Construct a histogram using the trichloroethene groundwater data in **Table 9-1** for each well. Examine the histogram to assess how each well is distributed (normal, lognormal, skewed, symmetric, etc.).

SOLUTION

Step 1. Import the data into data analysis software capable of producing histograms.

Step 2. Sort the data from smallest to largest results by well.

Step 3. With n = 14 concentrations for Well 1, a rough estimate of the number of bins is $\sqrt{14}$ = 3.74 or 4 bins. Since the data from Well 1 range from 0.004 to 0.25, the suggested bin width is calculated as (maximum concentration – minimum concentration) / number of bins = (0.25 – 0.004) / 4 = 0.0615. Therefore, the bins for Well 1 are 0.004 to <0.0655, 0.0655 to <0.127, 0.127 to <0.1885, and 0.1885 to 0.25 mg/L.

Similarly, with n = 13 concentrations for Well 2, the number of bins is $\sqrt{13}$ = 3.61 or 4 bins. Since the data from Well 2 range from 0.099 to 0.17, the suggested bin width is calculated as (maximum concentration – minimum concentration) / number of bins = (0.17 – 0.099) / 4 = 0.01775. Therefore, the bins for Well 2 are 0.099 to <0.11675, 0.11675 to <0.1345, 0.1345 to <0.15225, and 0.15225 to 0.17 mg/L.

Step 4. Construct a frequency table using the bins defined in Step 3. **Table 9-2** shows the frequency or number of observations within each bin defined in Step 3 for Wells 1 and 2. The third column shows the relative frequency which is the frequency divided by the sample size n. The final column of the table gives the densities or the relative frequencies divided by the bin widths calculated in Step 3.

Step 5. The horizontal axis for the data is from 0.004 to 0.25 mg/L for Well 1 and 0.099 to 0.17 for Well 2. The vertical axis for the histogram of frequencies is from 0 to 9 and the vertical axis for the histogram of relative frequencies is from 0% - 70%.

Step 6. The histograms of frequencies are shown in **Figure 9-3**. The histograms of relative frequencies or densities are shown in **Figure 9-4**. Note that frequency, relative frequency and density histograms all show the same shape since the scale of the vertical axis is divided by

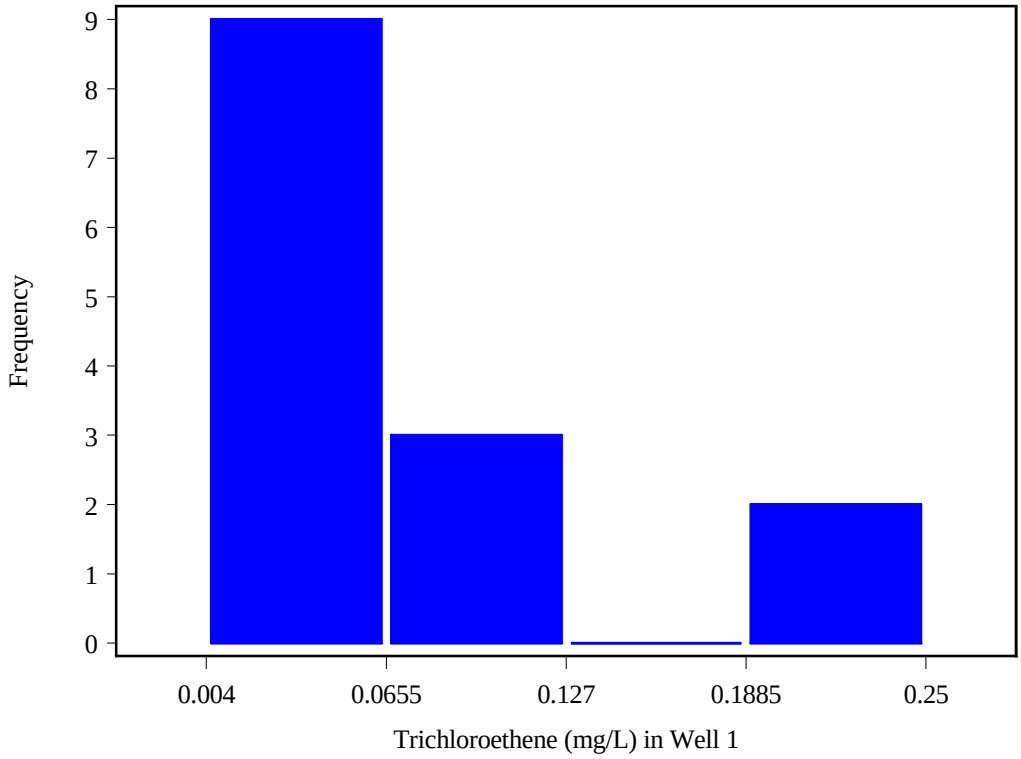
the sample size or the bin width. These histograms confirm the data are not normally distributed for either well, but are closer to lognormal.

Table 9-2. Histogram Bins for Trichloroethene Groundwater Data

Bin	Frequency	Relative Frequency (%)	Density
Well 1			
0.0040 to <0.0655 mg/L	9	64.3	10.5
0.0655 to <0.1270 mg/L	3	21.4	3.5
0.1270 to <0.1885 mg/L	0	0	0
0.1885 to 0.2500 mg/L	2	14.3	2.3
Well 2			
0.099 to <0.11675 mg/L	8	61.5	34.7
0.11675 to <0.1345 mg/L	3	23.1	13.0
0.1345 to <0.15225 mg/L	1	7.7	4.3
0.15225 to 0.17 mg/L	1	7.7	4.3

◄

Figure 9-3. Frequency Histograms of Trichloroethene by Well.



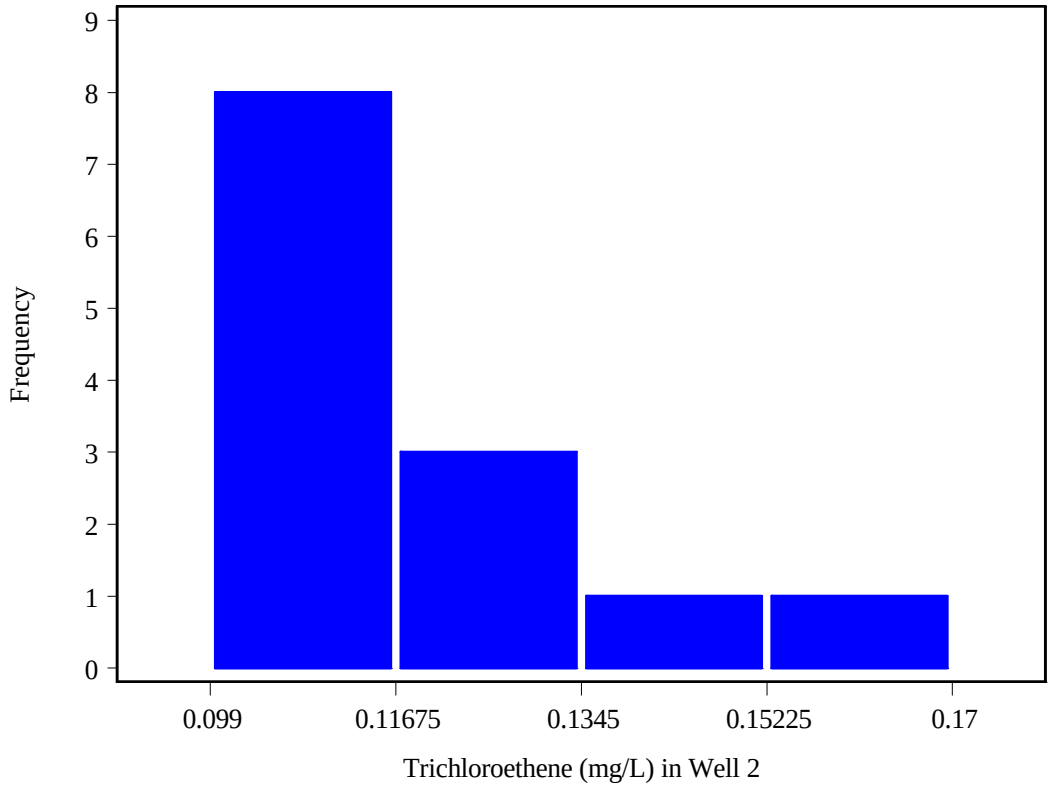
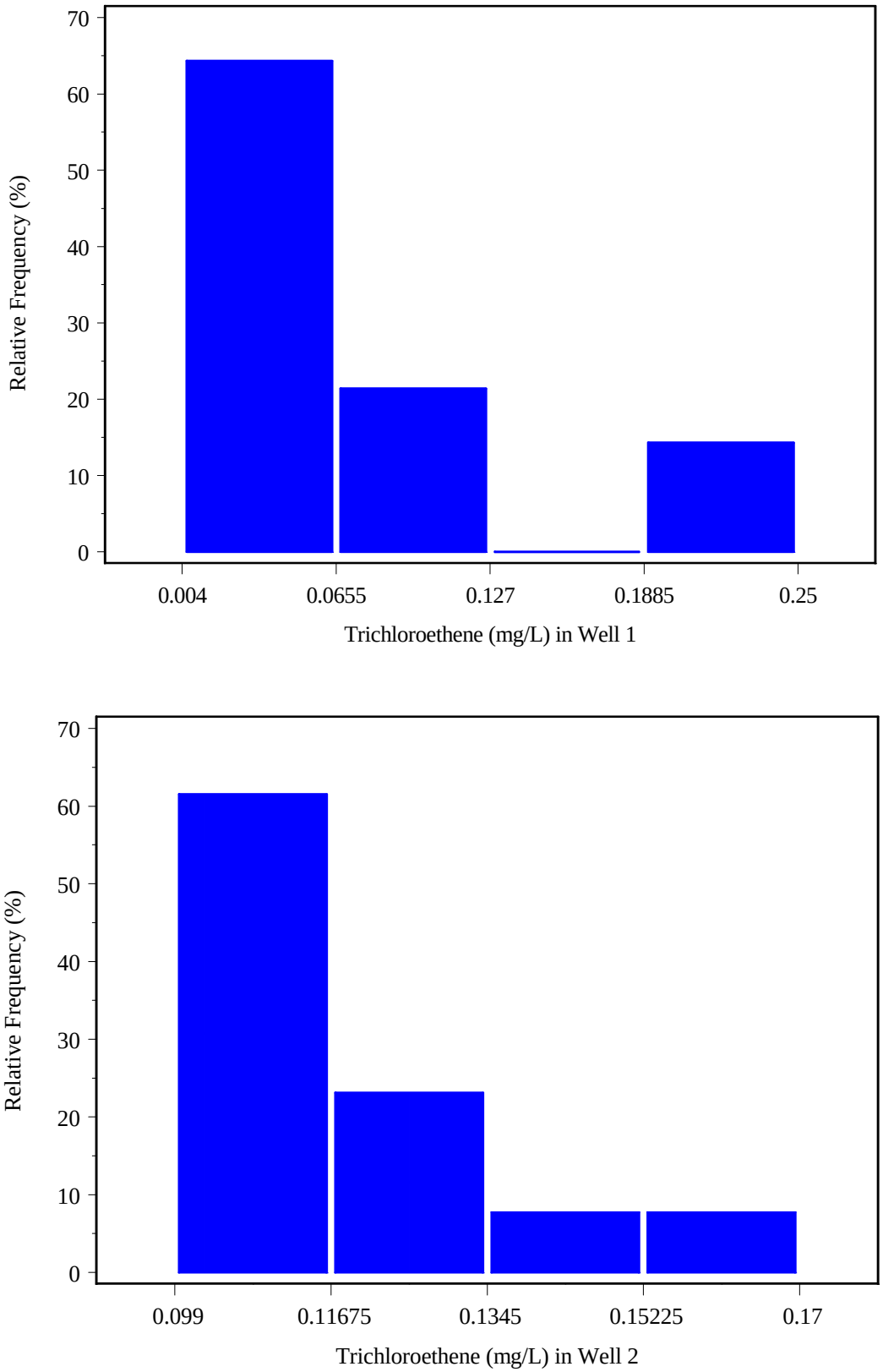


Figure 9-4. Relative Frequency Histograms of Trichloroethene by Well.



9.4 SCATTER PLOTS

For data sets consisting of multiple observations per sampling point, a scatter plot is one of the most powerful graphical tools for analyzing the relationship between two or more variables. Scatter plots are easy to construct for two variables, and many software packages can construct 3-dimensional scatter plots. A scatter plot can clearly show the relationship between two variables if the data range is sufficiently large. Truly linear relationships can always be identified in scatter plots, but truly nonlinear relationships may appear linear (or some other form) if the data range is relatively small. Scatter plots of linearly correlated variables cluster about a straight line.

As an example of a nonlinear relationship, consider two variables where one variable is approximately equal to the square of the other. With an adequate range in the data, a scatter plot of this data would display a partial parabolic curve. Other important modeling relationships that may appear are exponential or logarithmic. Two additional uses of scatter plots are the identification of potential outliers for a single variable or for the paired variables and the identification of clustering in the data. Directions for generating a scatter plot are contained in **Example 9-4**.

►EXAMPLE 9-4

Construct a scatter plot using the groundwater data in **Table 9-3** for arsenic and mercury from a single well collected approximately quarterly across time. Examine the scatter plot for linear or quadratic relationships between arsenic and mercury, correlation, and for potential outliers.

Table 9-3. Groundwater Concentrations from Well 3

	Arsenic		**Mercury**		**Strontium**	
Date Collected	**Conc. (mg/L)**	**Data Qualifier**	**Conc. (mg/L)**	**Data Qualifier**	**Conc. (mg/L)**	**Data Qualifier**
1/2/2005	0.01	U	0.02	U	0.10	
4/7/2005	0.01	U	0.03		0.02	U
7/13/2005	0.02		0.04	U	0.05	U
10/24/2005	0.04		0.06		0.11	
1/7/2006	0.01		0.02		0.05	
3/30/2006	0.05		0.07		0.07	
6/28/2006	0.09		0.10		0.03	
10/2/2006	0.07		0.08		0.04	
10/17/2006	0.10		NA		0.02	U
1/15/2007	0.02	U	0.03	U	0.15	
4/10/2007	0.15		0.11		0.03	
7/9/2007	0.12		0.08		0.10	
10/5/2007	0.10		0.07		0.09	
10/29/2007	0.30		0.29		0.05	
12/30/2007	0.25		0.23		0.22	

NA = Not available (missing data).

U denotes a non-detect.

SOLUTION

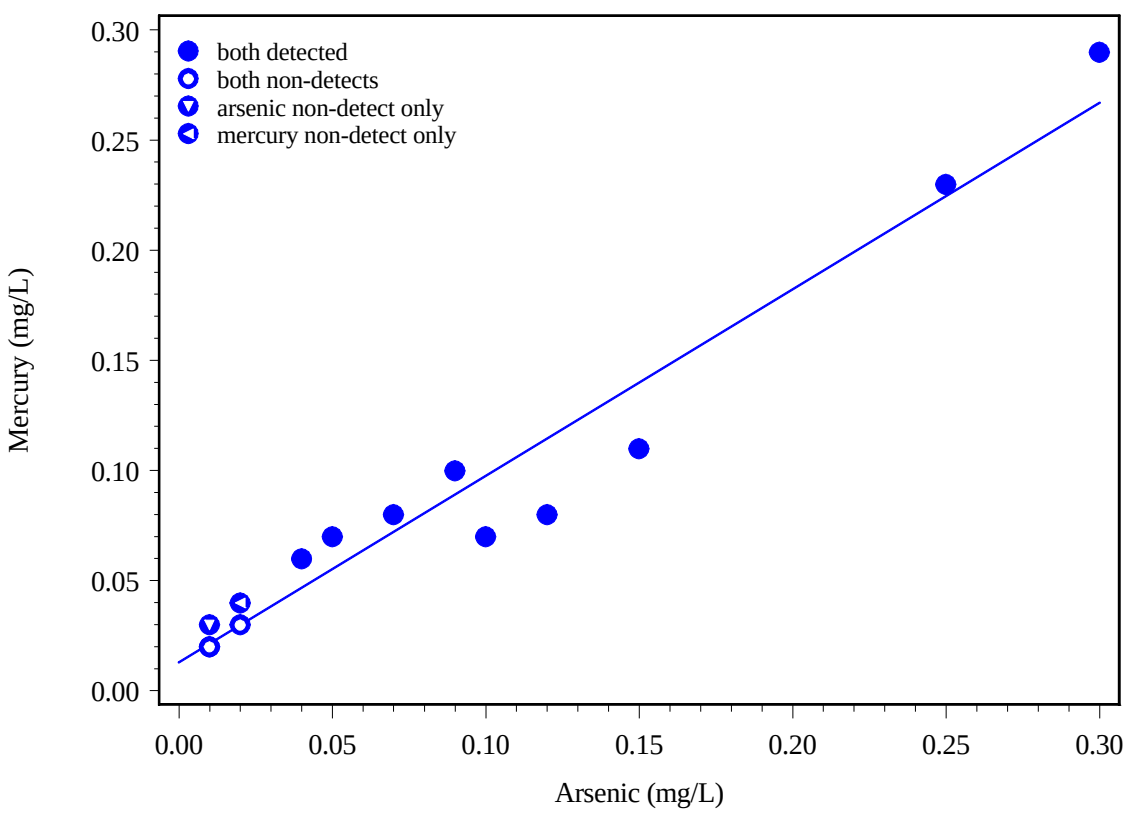
Step 1. Import the data into data analysis software capable of producing scatter plots.

Step 2. Sort the data by date collected.

Step 3. Calculate the range of concentrations for each constituent. If the range of both constituents are similar, then scale both the X and Y axes from the minimum to the maximum concentrations of both constituents. If the range of concentrations are very different (e.g., two or more orders of magnitude), then perhaps the scales for both axes should be logarithmic ($\log_{10}$). The data will be plotted as pairs from (X_1, Y_1) to (X_n, Y_n) for each sampling date, where n = number of samples.

Step 4. Use separate symbols to distinguish detected from non-detected concentrations. Note that the concentration for one constituent may be detected, while the concentration for the other constituent may not be detected for the same sampling date. If the concentration for one constituent is missing, then the pair (X_i, Y_i) cannot be plotted since both concentrations are required. **Figure 9-5** shows a linear correlation between arsenic and mercury with two possible outliers. The Pearson correlation coefficient is 0.97, indicating a significantly high correlation. The linear regression line is displayed to show the linear correlation between arsenic and mercury.◀

Figure 9-5. Scatter Plot of Arsenic with Mercury from Well 3



Many software packages can extend the 2-dimensional scatter plot by constructing a 3-dimensional scatter plot for 3 constituents. However, with more than 3 variables, it is difficult to construct and interpret a scatter plot. Therefore, several graphical representations have been developed that extend the idea of a scatter plot for data consisting of more than 2 variables. The simplest of these graphical techniques is a coded scatter plot. All possible two-way combinations are given a symbol and the pairs of data are plotted on one 2-dimensional scatter plot. The coded scatter plot does not provide information on three way or higher interactions between the variables since only two dimensions are plotted. If the data ranges for the variables are comparable, then a single set of axes may suffice. If the data ranges are too dissimilar (e.g., at least two orders of magnitude), different scales may be required.

►EXAMPLE 9-5

Construct a coded scatter plot using the groundwater data in **Table 9-3** for arsenic, mercury, and strontium from Well 3 collected approximately quarterly across time. Examine the scatter plot for linear or quadratic relationships between the three inorganics, correlation, and for potential outliers.

SOLUTION

Step 1. Import the data into data analysis software capable of producing scatter plots.

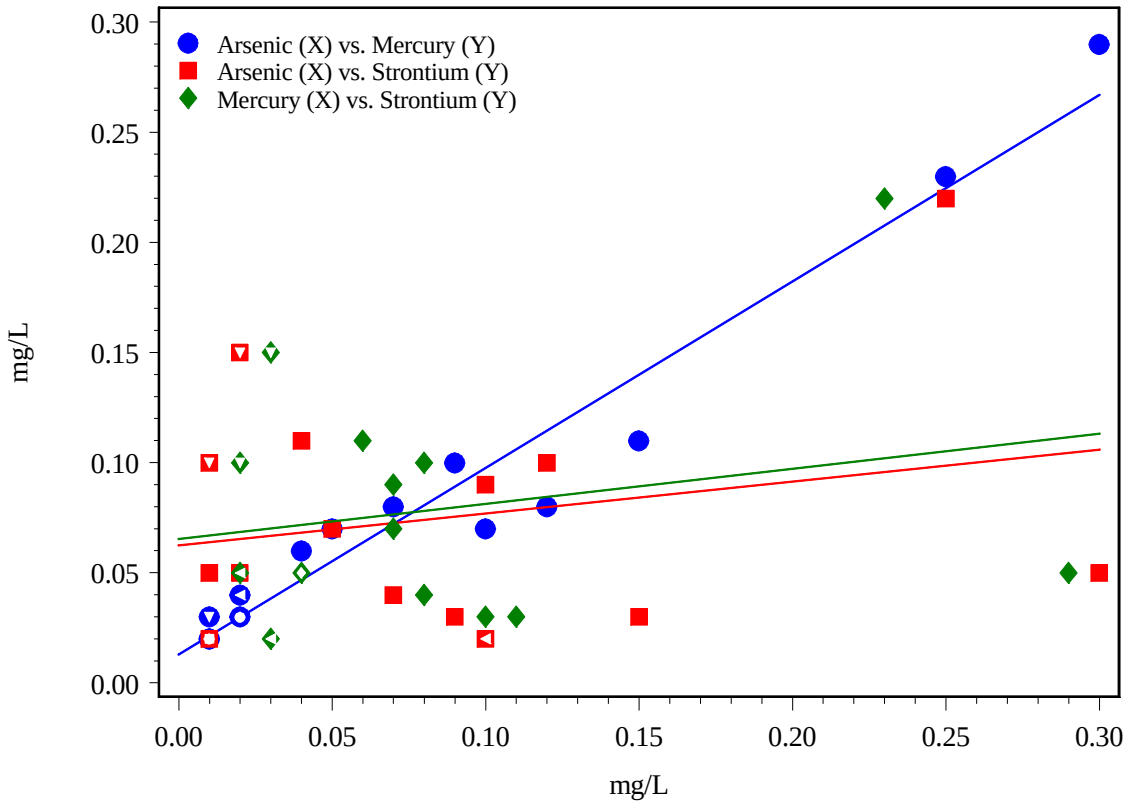
Step 2. Sort the data by date collected.

Step 3. Calculate the range of concentrations for each constituent. If the ranges of both constituents are similar, then scale both the X and Y axes from the minimum to the maximum concentrations of all three constituents. Since the ranges of concentrations are very similar, the minimum to the maximum concentrations of all three constituents will be used for both axes.

Step 4. Let each arsenic concentration be denoted by X_i, each mercury concentration be denoted by Y_i, and each strontium concentration be denoted by Z_i. The arsenic and mercury paired data will be plotted as pairs (X_i, Y_i) with solid blue circles for $1 \leq i \leq n$. The arsenic and strontium paired data will be plotted as pairs (X_i, Z_i) with solid red squares. The mercury and strontium paired data will be plotted as pairs (Y_i, Z_i) with solid green diamonds. If either concentration in each pair is a non-detect, then the non-detects will be displayed similar to **Figure 9-5**.

Step 5. Interpret the plot. **Figure 9-6** shows the linear correlation between arsenic and mercury with two possible outliers. The Pearson correlation coefficient is 0.97, indicating a significantly high correlation. The approximate 45º slope of the regression line indicates a strong correlation between arsenic and mercury. However, the nearly zero slope of the regression line between arsenic and strontium indicates little or no correlation between arsenic and strontium. There are two possible outliers for arsenic and strontium. Similarly, the nearly zero slope of the regression line between mercury and strontium indicates little or no correlation between mercury and strontium. There are also two possible outliers for mercury and strontium. The Pearson correlation coefficients for both arsenic with strontium and mercury with strontium are 0.23 which are not significantly different from zero.◄

Figure 9-6. Coded Scatter Plot of Well 3 Arsenic, Mercury, and Strontium



9.5 PROBABILITY PLOTS

A simple, but extremely useful visual assessment of normality is to graph the data as a probability plot. The *y*-axis is scaled to represent quantiles or *z*-scores from a standard normal distribution and the concentration measurements are arranged in increasing order along the *x*-axis. As each observed value is plotted on the *x*-axis, the *z*-score corresponding to the proportion of observations less than or equal to that measurement is plotted as the *y*-coordinate. Often, the *y*-coordinate is computed by the following formula:

$$y_i = \Phi^{-1}\left(\frac{i}{n+1}\right) \qquad [9.1]$$

where Φ^{-1} denotes the inverse of the cumulative standard normal distribution, n represents the sample size, and i represents the rank position of the ith ordered concentration. The plot is constructed so that, if the data are normal, the points when plotted will lie on a straight line. Visually apparent curves or bends indicate that the data do not follow a normal distribution.

Probability plots are particularly useful for spotting irregularities within the data when compared to a specific distributional model (usually, but not always, the normal). It is easy to determine whether departures from normality are occurring more or less in the middle ranges of the data or in the extreme

tails. Probability plots can also indicate the presence of possible outlier values that do not follow the basic pattern of the data and can show the presence of significant positive or negative skewness.

If a (normal) probability plot is constructed on the combined data from several wells and normality is accepted, it suggests — but does not prove — that all of the data came from the same normal distribution. Consequently, each subgroup of the data set (*e.g.*, observations from distinct wells) probably has the same mean and standard deviation. If a probability plot is constructed on the data residuals (each value minus its subgroup mean) and is not a straight line, the interpretation is more complicated. In this case, either the residuals are not normally-distributed, or there is a subgroup of the data with a normal distribution but a different mean or standard deviation than the other subgroups. The probability plot will indicate a deviation from the underlying assumption of a common normal distribution in either case. It would be prudent to examine normal probability plots by well on the same plot if the ranges of the data are similar. This would show how the data are distributed by well to determine which wells may depart from normality.

The same probability plot technique may be used to investigate whether a set of data or residuals follows a lognormal distribution. The procedure is generally the same, except that one first replaces each observation by its natural logarithm. After the data have been transformed to their natural logarithms, the probability plot is constructed as before. The only difference is that the natural logarithms of the observations are used on the *x*-axis. If the data are lognormal, the probability plot of the logged observations will approximate a straight line.

►EXAMPLE 9-6

Determine whether the dataset in **Table 9-4** is normal by using a probability plot.

SOLUTION

Step 1. After combining the data into a single group, list the measured nickel concentrations in order from lowest to highest.

Step 2. The cumulative probabilities, representing for each observation (x_i) the proportion of values less than or equal to x_i, are given in the third column of the table below. These are computed as $i / (n + 1)$ where n is the total number of samples ($n = 20$).

Step 3. Determine the quantiles or *z*-scores from the standard normal distribution corresponding to the cumulative probabilities in Step 2. These can be found by successively letting P equal each cumulative probability and then looking up the entry in **Table 10-1 (Appendix D)** corresponding to P. Since the standard normal distribution is symmetric about zero, for cumulative probabilities $P < 0.50$, look up the entry for $(1–P)$ and give this value a negative sign.

Step 4. Plot the normal quantile (*z*-score) versus the ordered concentration for each sample, as in the plot below (**Figure 9-7**). The curvature found in the probability plot indicates that there is evidence of non-normality in the data. ◄

Table 9-4. Nickel Concentrations from a Single Well

Nickel Concentration (ppb)	Order (i)	Cumulative Probability [$i/(n+1)$]	Normal Quantile (z-score)
1.0	1	0.048	–1.668
3.1	2	0.095	–1.309
8.7	3	0.143	–1.068
10.0	4	0.190	–0.876
14.0	5	0.238	–0.712
19.0	6	0.286	–0.566
21.4	7	0.333	–0.431
27.0	8	0.381	–0.303
39.0	9	0.429	–0.180
56.0	10	0.476	–0.060
58.8	11	0.524	0.060
64.4	12	0.571	0.180
81.5	13	0.619	0.303
85.6	14	0.667	0.431
151.0	15	0.714	0.566
262.0	16	0.762	0.712
331.0	17	0.810	0.876
578.0	18	0.857	1.068
637.0	19	0.905	1.309
942.0	20	0.952	1.668

PROBABILITY PLOTS FOR LOG TRANSFORMED DATA

Step 1. List the natural logarithms of the measured nickel concentrations in **Table 9-4** in order from lowest to highest. These are shown in **Table 9-5**.

Step 2. The cumulative probabilities representing the proportion of values less than or equal to x_i for each observation (x_i), are given in the third column of **Table 9-4**. These are computed as i / (n + 1) where n is the total number of samples (n = 20).

Step 3. Determine the quantiles or z-scores from the standard normal distribution corresponding to the cumulative probabilities in Step 2. These can be found by successively letting P equal each cumulative probability and then looking up the entry in **Table 10-1 Appendix D** corresponding to P. Since the standard normal distribution is symmetric about zero, for cumulative probabilities $P < 0.50$, look up the entry for (1–P) and give this value a negative sign.

Table 9-5. Nickel Log Concentrations from a Single Well

Order (i)	Log Nickel Concentration log(ppb)	Cumulative Probability [$i/(n+1)$]	Normal Quantile (z-score)
1	0.00	0.048	–1.668
2	1.13	0.095	–1.309
3	2.16	0.143	–1.068
4	2.30	0.190	–0.876
5	2.64	0.238	–0.712
6	2.94	0.286	–0.566
7	3.06	0.333	–0.431
8	3.30	0.381	–0.303
9	3.66	0.429	–0.180
10	4.03	0.476	–0.060
11	4.07	0.524	0.060
12	4.17	0.571	0.180
13	4.40	0.619	0.303
14	4.45	0.667	0.431
15	5.02	0.714	0.566
16	5.57	0.762	0.712
17	5.80	0.810	0.876
18	6.36	0.857	1.068
19	6.46	0.905	1.309
20	6.85	0.952	1.668

Step 4. Plot the normal quantile (z-score) versus the ordered logged concentration for each sample, as in the plot below (**Figure 9-8**). The reasonably linear trend found in the probability plot indicates that the log-scale data closely follow a normal pattern, further suggesting that the original data closely follow a lognormal distribution.

Figure 9-7. Nickel Normal Probability Plot

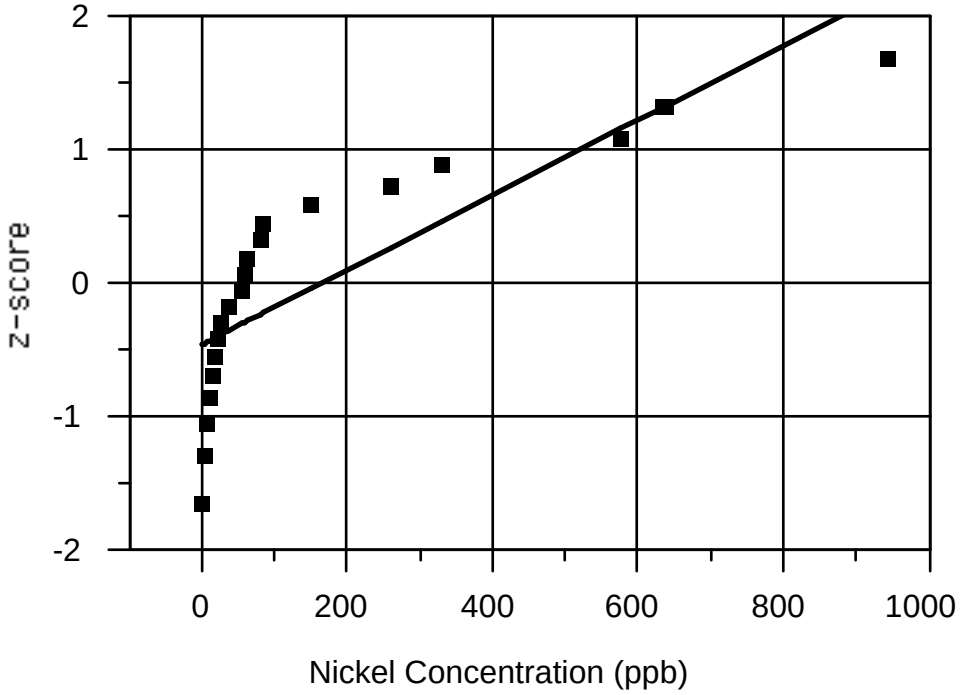
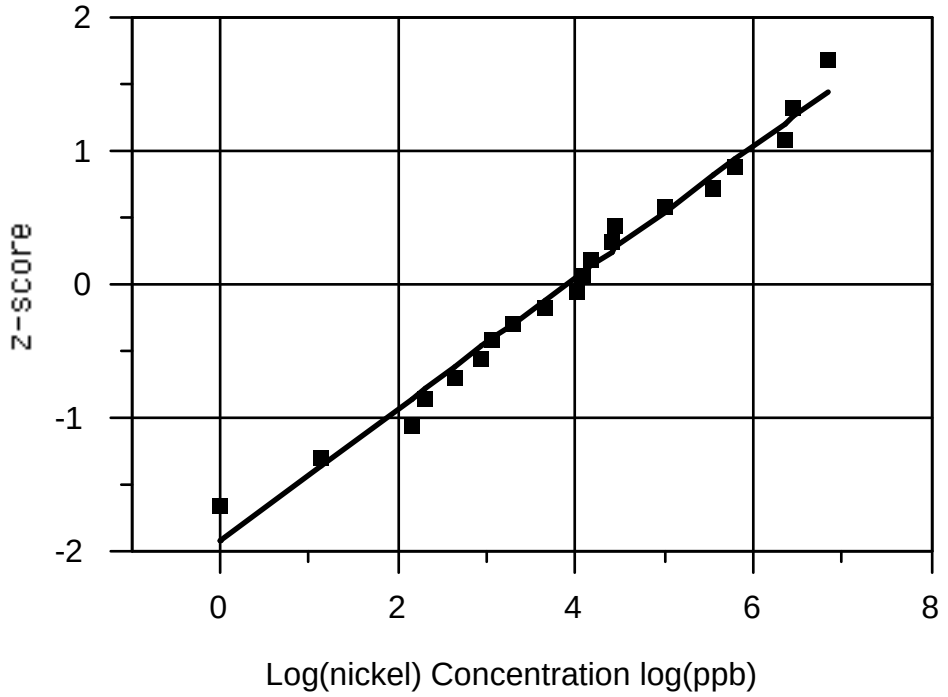


Figure 9-8. Probability Plot of Log Transformed Nickel Data



CHAPTER 10. FITTING DISTRIBUTIONS

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Because a statistical or mathematical model is at best an approximation of reality, all statistical tests and procedures require certain assumptions for the methods to be used correctly and for the results to be properly interpreted. Many tests make an assumption regarding the underlying distribution of the observed data; in particular, that the original or transformed sample measurements follow a normal distribution. Data transformations are discussed in **Section 10.2** while considerations as to whether the normal distribution should be used as a 'default' are explored in **Section 10.3**. Several techniques for assessing normality are also examined, including:

- The skewness coefficient (**Section 10.4**)
- The Shapiro-Wilk test of normality and its close variant, the Shapiro-Francía test (**Section 10.5**)
- Filliben's probability plot correlation coefficient test (**Section 10.6**)
- The Shapiro-Wilk multiple group test of normality (**Section 10.7**)

10.1 IMPORTANCE OF DISTRIBUTIONAL MODELS

As introduced in **Chapter 3**, all statistical testing relies on the critical assumption that the sample data are *representative* of the population from which they are selected. The statistical distribution of the *sample* is assumed to be similar to the distribution of the mostly unobserved *population* of possible measurements. Many *parametric* testing methods make a further assumption: that the form or type of the underlying population is at least approximately known or can be identified through diagnostic testing. Most of these parametric tests assume that the population is *normal* in distribution; the validity or accuracy of the test results may be in question if that assumption is violated.

Consequently, an important facet of choosing among appropriate test methods is determining whether a commonly-used statistical distribution such as the normal, adequately models the observed sample data. A large variety of possible distributional models exist in the statistical literature; most are not typically applied to groundwater measurements and often introduce additional statistical or mathematical complexity in working with them. So groundwater statistical models are usually confined to the gamma distribution, the Weibull distribution, or distributions that are normal or can be normalized via a transformation (e.g., the logarithmic or square root).

Although the Unified Guidance will occasionally reference procedures that assume an underlying gamma or Weibull distribution, the presentation in this guidance will focus on distributions that can be normalized and diagnostic tools for assessing normality. The principal reasons for limiting the discussion in this manner are: 1) the same tools useful for testing normality can be utilized with any distribution that can be normalized-- the only change needed is perform the normality test after first making a data transformation; 2) if no transformation works to adequately normalize the sample data, a non-parametric test can often be used as an alternative statistical approach; and 3) addressing more complicated scenarios is outside the scope of the guidance and may require professional statistical consultation.

Understanding the statistical behavior of groundwater measurements can be very challenging. The constituents of interest may occur at relatively low concentrations and frequently be left-censored because of current analytical method limitations. Sample data are often positively skewed and asymmetrical in distributional pattern, perhaps due to the presence of outliers, inhomogeneous mixing of contaminants in the subsurface, or spatially variable soils deposition affecting the local groundwater geochemistry. For some constituents, the distribution in groundwater is not stationary over time (*e.g.*, due to linear or seasonal trends) or not stationary across space (due to spatial variability in mean levels from well to well). A set of these measurements pooled over time and/or space may appear highly non-normal, even if the underlying population at any fixed point in time or space *is* normal.

Because of these complexities, fitting a distributional model to a set of sample data cannot be done in isolation from checks of other key statistical assumptions. The data must also be evaluated for outliers (**Chapter 12**), since the presence of even one extreme outlier may cause an otherwise recognizable distribution from being correctly identified. For data grouped across wells, the possible presence of spatial variability must be considered (**Chapter 13**). If identified, the Shapiro-Wilk multiple group test of normality may be needed to account for differing means and/or variances at distinct wells. Data pooled across sampling events (*i.e.*, over time) must be examined for the presence of trends or seasonal patterns (**Chapter 14**). A clearly identified pattern may need to be removed and the *data residuals* tested for normality, instead of the raw measurements.

A frequently encountered problem involves testing normality on data sets containing non-detect values. The best goodness-of-fit tests attempt to assess whether the sample data closely resemble the *tails* of the candidate distributional model. Since non-detects represent *left-censored observations* where the exact concentrations are unknown for the lower tail of the sample distribution, standard normality tests cannot be run without some estimate or *imputation* of these unknown values. For a small fraction of non-detects in a sample (10-15% or less) censored at a single reporting limit, it may be possible to apply a normality test by simply replacing each non-detect with an imputed value of half the RL. However, more complicated situations arise when there is a combination of multiple RLs (detected values intermingled with different non-detect levels), or the proportion of non-detects is larger. The Unified Guidance recommends different strategies in these circumstances.

Properly *ordering* the sample observations (*i.e.*, from least to greatest) is critical to *any* distributional goodness-of-fit test. Because the concentration of a non-detect measurement is only known to be in the range from zero to the RL, it is generally impossible to construct a full ordering of the

sample.[1] There are methods, however, to construct *partial orderings* of the data that allow the assignment of relative rankings to each of the detected measurements and which account for the presence of censored values. In turn, a partial ordering enables construction of an approximate normality test. This subject is covered in **Chapter 15**.

10.2 TRANSFORMATIONS TO NORMALITY

Guidance users will often encounter data sets indicating significant evidence of non-normality. Due to the presumption of most parametric tests that the underlying population is normal, a common statistical strategy for apparently non-normal observations is to search for a normalizing mathematical transformation. Because of the complexities associated with interpreting statistical results from data that have been transformed to another scale, some care must be taken in applying statistical procedures to transformed measurements. In questionable or disputable circumstances, it may be wise to analyze the same data with an equivalent non-parametric version of the same test (if it exists) to see if the same general conclusion is reached. If not, the data transformation and its interpretation may need further scrutiny.

Particularly with prediction limits, control charts, and some of the confidence intervals described in **Chapters 18**, **20,** and **21**, the parametric versions of these procedures are especially advantageous. Here, a transformation may be warranted to approximately normalize the statistical sample. Transformations are also often useful when combining or pooling intrawell background from several wells in order to increase the degrees of freedom available for intrawell testing (**Chapter 13**). Slight differences in the distributional pattern from well to well can skew the resulting pooled dataset, necessitating a transformation to bring about approximate normality and to equalize the variances.

The interpretation of transformed data is straightforward in the case of prediction limits for individual observations or when building a confidence interval around an upper percentile. An interval with limits constructed from the transformed data and then re-transformed (or *back-transformed*) to the original measurement domain will retain its original probabilistic interpretation. For instance, if the data are approximately normal under a square root transformation and a 95% confidence prediction limit is constructed on the square roots of the original measurements, *squaring* the resulting prediction limit allows for a 95% confidence level when applied to the original data.

The same ease of interpretation does not apply to prediction limits for a future arithmetic mean (**Chapter 18**) or to confidence intervals around an arithmetic mean compared to a fixed GWPS (**Chapter 21**). A back-transformed confidence interval constructed around the mean of log-transformed data (*i.e.*, the log-mean) corresponds to a confidence interval around the *geometric mean* of the raw (untransformed) data. For the lognormal distribution, the geometric mean is equal to the median, but it is *not* the same as the arithmetic mean. Using this back-transformation to bracket the location of the true arithmetic population mean will result in an incorrect interval.

For these particular applications, a similar problem of *scale bias* occurs with other potential normality transformations. Care is needed when applying and interpreting transformations to a data set

[1] Even when all the non-detects represent the lowest values in the sample, there is still no way to determine how this subset is internally ordered.

for which either a confidence interval around the mean or a prediction limit for a future mean is desired. The interpretation depends on which statistical parameter is being estimated or predicted. The geometric mean or median in some situations may be a satisfactory alternative as a central tendency parameter, although that decision must be weighed carefully when making comparisons against a GWPS.

Common normalizing transformations include the natural logarithm, the square root, the cube root, the square, the cube, and the reciprocal functions, as well as a few others. More generally, one might consider the "ladder of powers" (Helsel and Hirsch, 2002) technically known as the set of Box-Cox transformations (Box and Cox, 1964). The heart of these transformations is a power transformation of the original data, expressed by the equations:

$$y_\lambda = \begin{cases} \left(x^\lambda - 1\right)/\lambda & \text{for } \lambda \neq 0 \\ \log x & \text{for } \lambda = 0 \end{cases} \qquad [10.1]$$

The goal of a Box-Cox analysis is to find the value λ that best transforms the data to approximate normality, using a procedure such as maximum likelihood. Such algorithms are beyond the scope of this guidance, although an excellent discussion can be found in Helsel and Hirsch (2002). In practice, slightly different equation formulations can be used:

$$y_\lambda = \begin{cases} x^\lambda & \text{for } \lambda \neq 0 \\ \log x & \text{for } \lambda = 0 \end{cases} \qquad [10.2]$$

where the parameter λ can generally be limited to the choices 0, -1, 1/4, 1/3, 1/2, 1, 2, 3, and 4, except for unusual cases of more extreme powers.

As noted in **Section 10.1**, checking normality with transformed data does not require any additional tools. Standard normality tests can be applied using the transformed scale measurements. Only the interpretation of the test changes. A goodness-of-fit test can assess the normality of the raw measurements. Under a transformation, the same test checks for normality on the transformed scale. The data will still follow the non-normal distribution in the original concentration domain. So if a cube root transformation is attempted and the transformed data are found to be approximately normal, the original data are not normal but rather cube-root normal in distribution. If a log transformation is successfully used, the original measurements are not normal but lognormal instead. In sum, a series of non-normal distributions can be fitted to data with the goodness-of-fit tests described in this chapter without needing specific tests for other potential distributions.

Finding a reasonable transformation in practice amounts to systematically 'climbing' the "ladder of powers" described above. In other words, different choices of the power parameter λ would be attempted — beginning with λ = 0 and working upward from -1 toward more extreme power transformations — until a specific λ normalizes the data or all choices have been attempted. If no transformation seems to work, the user should instead consider a non-parametric test alternative.

10.3 USING THE NORMAL DISTRIBUTION AS A DEFAULT

Normal and lognormal distributions are frequently applied models in groundwater data because of their general utility. One or the other of these models might be chosen as a *default distribution* when designing a statistical approach, particularly when relatively little data has been collected at a site. Since the statistical behavior of these two models is very different and can lead to substantially different conclusions, the choice is not arbitrary. The type of test involved, the monitoring program, and the sample size can all affect the decision. For many data sets and situations, however, the normal distribution can be assumed as a default unless and until a better model can be pinpointed through specific *goodness-of-fit* testing provided in this chapter.

Assumptions of normality are most easily made with regard to naturally-occurring and measurable inorganic parameters, particularly under background conditions. Many ionic and other inorganic water quality analyte measurements exhibit decent symmetry and low variability within a given well data set, making these data amenable to assumptions of normality. Less frequently detected analytes (*e.g.*, certain colloidal trace elements) may be better fit either by a site-wide lognormal or another distribution that can be normalized, as well as evaluated with non-parametric methods.

Where contamination in groundwater is known to exist *a priori* (whether in background or compliance wells), default distributional assumptions become more problematic. At a given well, organic or inorganic contaminants may exhibit high or low variability, depending on local hydrogeologic conditions, the pattern of release from the source, the degree of solid phase absorption, degradability of a given constituent, and the variation in groundwater flow direction and depths. Non-steady state releases may result in a historical, occasionally non-linear pattern of trend increases or decreases. Such data might be fit by an apparent lognormal distribution, although removal of the trend may lead to normally-distributed residuals.

Sample size is also a consideration. With fewer than 8 samples in a data set, formal goodness-of-fit tests are often of limited value. Where larger sample sizes are available, goodness-of-fit tests should be conducted. The Shapiro-Wilk multiple group well test (**Section 10.7**) — even with small sample sizes — can sometimes be used to identify individual anomalous wells which might otherwise be presumed to meet the criterion of normality. Under compliance/assessment or corrective action monitoring, one might anticipate only four samples per well in the first year after instituting such monitoring. Under these conditions, a default assumption of normality for testing of the mean against a fixed standard is probably necessary. Aggregation of multi-year data when conducting compliance tests (see **Chapter 7**) may allow large enough sample sizes to warrant formal goodness-of-fit testing. With 8 (or more) samples, it may be possible to determine that a lognormal distribution is an appropriate fit for the data. Even in this latter approach, caution may be needed in applying Land's confidence interval for a lognormal mean (**Chapter 21**) if the sample variability is large and especially if the upper confidence limit is used in the comparison (*i.e.*, in corrective action monitoring).

The normal distribution may also serve as a reasonable default when it is not critical to ensure that sample data closely follow a specific distribution. For example, statistical tests on the mean are generally considered more *robust* with respect to departures from normality than procedures which involve upper or lower limits of an assumed distribution. Even if the data are not quite normal, tests on the mean such

as a Student's *t*-test will often still provide a valid result. However, one might need to consider transformations of the data for other reasons. Analysis of variance [ANOVA] can be run with small individual well samples (*e.g.*, $n = 4$), and as a comparison of means, it is fairly robust to departures from normality. A logarithmic or other transformation may be needed to stabilize or equalize the well-to-well variability (*i.e.*, achieve *homoscedasticity*), a separate and more critical assumption of the test.

Given their importance in statistical testing and the risks that sometimes occur in trying to interpret tests on other data transformation possibilities, it is useful to briefly consider the logarithmic transformation in more detail. As noted in **Section 10.1**, groundwater data can frequently be normalized using a logarithmic distribution model. Despite this, objections are sometimes raised that the log transformation is merely used to "make large numbers look smaller."

To better understand the log transformation, it should be recognized that logarithms are, in fact, exponents to some unit base. Given a concentration-scale variable x, re-expressed as $x = 10^y$ or $x = e^y$, the logarithm y is the exponent of that base (10 or the natural base e). It is the behavior of the resultant y *values* that is assessed when data are log-transformed. When data relationships are multiplicative in the original arithmetic domain ($x_1 \times x_2$), the relationships between exponents (*i.e.*, logarithms) are additive ($y_1 + y_2$). Since the logarithmic distribution by mathematical definition is normal in a log-transformed domain, working with the logarithms instead of the original concentration measurements may offer a sample distribution much closer to normal.

Similar to a unit scale transformation (ppm to ppb or Fahrenheit to Centigrade), the relative ordering of log-transformed measurements does not change. When non-parametric tests based on ranks (*e.g.*, the Wilcoxon rank-sum test) are applied to data transformed either to a different unit scale or by logarithms, the outcomes are identical. However, other relationships among the log-transformed data *do* change, so that the log-scale numerical 'spacing' between lower values is more similar to the log-scale spacing between higher values. While parametric tests like prediction limits, *t*-tests, *etc.*, are not affected by unit scale transformations, these tests may have different outcomes depending on whether raw concentrations or log-transformed measurements are used. The justification for utilizing log-transformed data is that the transformation helps to normalize the data so that these tests can be properly applied.

There is also a plausible physical explanation as to why pollutant concentrations often follow a logarithmic pattern (Ott, 1990). In Ott's model, pollutant sources are randomly dispersed through the subsurface or atmosphere in a multiplicative fashion through repeated dilutions when mixing with volumes of (uncontaminated) water or air, depending on the medium. Such random and repeated dilutions can mathematically lead to a lognormal distribution. In particular, if a final concentration (c_0) is the product of several random dilutions (c_i) as suggested by the following equation:

$$c_0 = \prod_{i=1}^{n} c_i = (c_1 \times c_2 \times \ldots \times c_n) \qquad [10.3]$$

the logarithm of this concentration is equivalent to the *sum* of the logarithms of the individual dilutions:

$$\log(c_0) = \sum_{i=1}^{n} \log(c_i) \qquad [10.4]$$

The Central Limit Theorem (**Chapter 3**) can be applied to conclude that the logged concentration in equation [10.4] should be approximately normal, implying that the original concentration (c_0) should be approximately lognormal in distribution. Contaminant fate-and-transport models more or less follow this same approach, using successive multiplicative dilutions (while accounting for absorption and degradation effects) across grids in time and space.

Despite the mathematical elegance of the Ott model, experience with groundwater monitoring data has shown that the lognormal model alone is not adequate to account for observed distribution patterns. While contaminant modeling might predict a lognormal contaminant distribution in space (and often in time at a fixed point during transient phases), individual well location points fixed in space and at rough contaminant equilibrium are more likely to be subject to a variety of local hydrologic and other factors, and the observed distributions can be almost limitless in form. Since most of the tests within the Unified Guidance presume a stationary population over time at a given well location (subject to identification and removal of trends), the resultant distributions may be other than lognormal in character. Individual constituents may also exhibit varying aquifer-related distributional characteristics.

A practical issue in selecting a default transformation is ease of use. Distributions like the lognormal usually entail more complicated statistical adjustments or calculations than the normal distribution. A confidence interval around the arithmetic mean of a lognormal distribution utilizes Land's *H*-factor, which is a function of both log sample data variability and sample size, and is only readily available for specific confidence levels. By contrast, a normal confidence interval around the sample mean based on the *t*-statistic can easily be defined for virtually any confidence level. As noted earlier, correct use of these confidence intervals depends on selecting the appropriate parameter and statistical measure (arithmetic mean versus the geometric mean).

While a transformation does not always necessitate using a different statistical formula to ensure unbiased results, use of a transformation *does* assume that the underlying population is non-normal. Since the true population will almost never be known with certainty, it may not be advantageous to simply default to a lognormal assumption for a variety of reasons. Under detection monitoring, the presumption is made that a statistically significant increase above background concentrations will trigger a monitoring exceedance. But the larger the prediction limit computed from background, the less *statistical power* the test will have for detecting true increases. An important question to answer is what the consequences are when incorrectly applying statistical techniques based on one distributional assumption (normal or lognormal), when the underlying distribution is in fact the other. More specifically, what is the impact on statistical power and accuracy of assuming the wrong underlying distribution? The general effects of violating underlying test assumptions can be measured in terms of false positive and negative error rates (and therefore power). These questions are particularly pertinent for prediction limit and control chart tests in detection monitoring. Similar questions could be raised regarding the application of confidence interval tests on the mean when compared against fixed standards.

To answer these questions, a series of Monte Carlo simulations was generated for the Unified Guidance to evaluate the impacts on prediction limit false positive error rates and statistical power of using normal and lognormal distributions (correctly and incorrectly applied to the underlying distributions). Detailed results of this study are provided in **Appendix C, Section C.1**.

The conclusions of the Monte Carlo study are summarized as follows:

- If an underlying population is truly normal, *treating the sample data as lognormal* in constructing a prediction limit can have significant consequences. With no retesting, the lognormal prediction limits were in every case considerably larger and thus less powerful than the normal prediction limits. Further, the lognormal limits consistently exhibited less than the expected (nominal) false positive rate, while the normal prediction limits tended to have slightly higher than nominal error rates.
- When retesting was added to the procedure, both types of prediction limits improved. While power uniformly improved compared to no retest, the normal limits were still on average about 13% shorter than the lognormal limits, leading again to a measurable loss of statistical power in the lognormal case.
- On balance, *misapplication* of logarithmic prediction limits to normally-distributed data consistently resulted in (often considerably) lower power and false positive rates that were lower than expected. The results argue *against* presuming the underlying data to be lognormal without specific goodness-of-fit testing.
- The highest penalties from misapplying lognormal prediction limits occurred for smaller background sizes. Since goodness-of-fit tests are least able to distinguish between normal and lognormal data with small samples, small background samples should not be presumed to be lognormal *as a default* unless other evidence from the site suggests otherwise. For larger samples, goodness-of-fit tests have much better discriminatory power, enabling a better indication of which model to use.
- If the underlying population is truly lognormal but the sample data *are treated as normal*, the penalty in overall statistical performance is substantial *only* if no retesting is conducted. With no retesting, the false positive rates of normal-based limits were often substantially higher than the expected rate. Under conditions of no retesting, *misapplying* normal prediction limits to lognormal data would result in an excessive site-wide false positive rate (SWFPR).
- If at least one retest was added, the achieved false positive rates for the misapplied normal limits tended to be *less* than the expected rates, especially for moderate to larger sample sizes. Except for highly skewed lognormal distributions, the power of the normal limits was comparable or greater than the power of the lognormal limits.

Overall, the Monte Carlo study indicated that adding a retest to the testing procedure significantly minimized the penalty of misapplying normal prediction limits to lognormal data, as long as the sample size was at least 8 and the distribution was not too skewed. Consequently, there is *less* penalty associated with making a default assumption of *normality* than in making a default assumption of *lognormality* under most situations. With highly skewed data, goodness-of-fit tests tend to better discriminate between the normal and lognormal models. The Unified Guidance therefore recommends that such diagnostic testing be done *explicitly* rather than simply assuming the data to be normal or lognormal.

The most problematic cases in the study occurred for very small background sample sizes, where a misapplication of prediction limits in either direction often resulted in poorer statistical performance, even with retesting. In some situations, compliance testing may need to be conducted on an interim basis until enough data has been collected to accurately identify a distributional model. The Unified Guidance does not recommend an automatic default assumption of lognormality.

In summary, during detection and compliance/assessment monitoring, data sets should be treated initially as normal in distribution unless a better model can be pinpointed through specific testing. The normal distribution is a fairly safe assumption for background distributions, particularly for naturally occurring, measurable constituents and when sample sizes are small. Goodness-of-fit tests provided in this chapter can be used to more closely identify the appropriate distributions for larger sample sizes. If the initial assumption of normality is not rejected, further statistical analyses should be performed on the raw observations. If the normal distribution *is* rejected by a goodness-of-fit test, one should generally test the normality of the logged data, in order to check for lognormality of the original observations. If this test also fails, one can either look for an alternate transformation to achieve approximate normality (**Section 10.2**) or use a non-parametric technique.

Since tests of normality have low power for rejecting the null hypothesis when the data are really lognormal but the sample size and degree of skewness are small, it is reassuring that a "wrong" default assumption of normality will infrequently lead to an incorrect statistical conclusion. In fact, the statistical power for detecting real concentration increases will generally be better than if the data were assumed to be lognormal. If the data *are* truly lognormal, there *is* a risk of greater-than-expected site-wide false positive error rates.

When the population is more skewed, normality tests in the Unified Guidance have much greater power for correctly rejecting the normal model in favor of the lognormal distribution. Consequently, an initial assumption of normality will not, in most cases, lead to an incorrect final conclusion, since the presumed normal model will tend to be rejected before further testing is conducted.

These recommendations do not apply to corrective action monitoring or other programs where it either known or reasonable to presume that groundwater is already impacted or has a non-normal distribution. In such settings, a default presumption of lognormality could be made, or a series of normalizing transformations could be attempted until a suitable fit is determined. Furthermore, even in detection monitoring, there are situations that often require the use of alternate transformations, for instance when pooling intrawell background across several wells to increase the degrees of freedom available for intrawell testing (**Chapter 13**).

Whatever the circumstance, the Unified Guidance recommends whenever possible that site-specific data be used to test the distributional presumption. If no data are initially available to do this, "referencing" may be employed to justify the use of, say, a normal or lognormal assumption in developing statistical tests at a particular site. Referencing involves the use of historical data or data from sites in similar hydrologic settings to justify the assumptions applied to the proposed statistical regimen. These initial assumptions should be checked when data from the site become available, using the procedures described in the Unified Guidance. Subsequent changes to the initial assumptions should be made if goodness-of-fit testing contradicts the initial hypothesis.

10.4 COEFFICIENT OF VARIATION AND COEFFICIENT OF SKEWNESS

PURPOSE AND BACKGROUND

Because the normal distribution has a symmetric 'bell-shape,' the normal mean and median coincide and random observations drawn from a normal population are just as likely to occur below the mean as above it. More generally, in any symmetric distribution the distributional pattern below the

mean is a mirror-image of the pattern above the mean. By definition, such distributions have no degree of *skewness* or asymmetry.

Since the normal distribution has zero skewness, one way to look for non-normality is to estimate the degree of skewness. Non-zero values of this measure imply that the population is asymmetric and therefore something different from normal. Two exploratory screening tools useful for this task are the *coefficient of variation* and the *coefficient of skewness*.

The coefficient of variation [CV] is extremely easy to compute, but only indirectly offers an estimate of skewness and hence normality/non-normality. A more direct estimate can be determined via the coefficient of skewness. Furthermore, better, formal tests can be used instead of either coefficient to directly assess normality. Nevertheless, the CV provides a measure of intrinsic variability in positive-valued data sets. Although approximate, CVs can indicate the relative variability of certain data, especially with small sample sizes and in the absence of other formal tests (*e.g.*, see **Chapter 22**, when comparing confidence limits on the mean to a fixed standard in compliance monitoring).

The CV is also a valid measure of the multiplicative relationship between the population mean and the standard deviation for positively-valued random variables. Using sample statistics for the mean ($\bar{x}$) and standard deviation (s), the true CV for non-negative normal populations can be reasonably estimated as:

$$CV = s / \bar{x} \qquad [10.5]$$

In lognormal populations, the CV is also used in evaluations of statistical power. In this latter case, the population CV works out to be:

$$CV = \sqrt{\exp\left(\sigma_y^2\right) - 1} \qquad [10.6]$$

where σ_y is the population log-standard deviation. Instead of a ratio between the original scale standard deviation and the mean, the lognormal CV is estimated with the equation:

$$CV = \sqrt{\exp\left(s_y^2\right) - 1} \qquad [10.7]$$

where s_y is the sample log-standard deviation. The estimate in equation [10.7] is usually more accurate than the simple CV ratio of the arithmetic standard deviation-to-mean, especially when the underlying population coefficient of variation is high. Similar to using the normal CV as a formal indicator of normality, the lognormal coefficient of variation estimator in equation [10.7] will have little relevance ***as a test of lognormality*** of the data. Using it for that purpose is not recommended in the Unified Guidance. But it can provide a sense of how variable a data set is and whether a lognormal assumption might need to be tested.

While others have reported a ratio CV on logged measurements as $CV = s_y / \bar{y}$ for the transformation $y = \log x$, the result is essentially meaningless. The actual logarithmic CV in equations [10.6] and [10.7] is solely determined by the logarithmic variability of σ_y or s_y. Negative logarithmic mean values are always possible, and the log ratio statistic is not invariant under a unit scale

transformation (*e.g.*, ppb to ppm or ppt). Similar problems in interpretation occur when CV estimators are applied to any variable which can be negatively valued, such as following a *z*-transformation to a standard normal distribution. This log ratio statistic is not recommended for any application in the guidance.

The coefficient of skewness (γ_1) directly indicates to what degree a dataset is skewed or asymmetric with respect to the mean. Sample data from a normal distribution will have a skewness coefficient near zero, while data from an asymmetric distribution will have a positive or negative skewness depending on whether the right- or left-hand tail of the distribution is longer and skinnier than the opposite tail.

Since groundwater monitoring concentrations are inherently non-negative, such data often exhibit skewness. A small degree of skewness is not likely to affect the results of statistical tests that assume normality. However, if the skewness coefficient is larger than 1 (in absolute value) and the sample size is small (*e.g.*, $n < 25$), past research has shown that standard normal theory-based tests are much less powerful than when the absolute skewness is less than 1 (Gayen, 1949).

Calculating the skewness coefficient is useful and only slightly more difficult than computing the CV. It provides a quick indication of whether the skewness is minimal enough to assume that the data are roughly symmetric and hopefully normal in distribution. If the original data exhibit a high skewness coefficient, the normal distribution will provide a poor approximation to the dataset. In that case — and unlike the CV — γ_1 can be computed on the log-transformed data to test for symmetry of the logged measurements, or similarly for other transformations.

PROCEDURE

The CV is calculated simply by taking the ratio of the sample standard deviation to the sample mean, $CV = s/\bar{x}$ or its corresponding logarithmic version $CV = \sqrt{\exp\left(s_y^2\right) - 1}$.

The skewness coefficient may be computed using the following equation:

$$\gamma_1 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^3 \Bigg/ \left[\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2\right]^{3/2} = n^{1/2}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^3 \Big/ \left(n-1\right)^{3/2} s^3 \qquad [10.8]$$

where the numerator represents the average cubed residual after subtracting the sample mean.

►EXAMPLE 10-1

Using the following data, compute the CVs and the coefficient of skewness to test for approximate symmetry.

	Nickel Concentration (ppb)			
Month	Year 1	Year 2	Year 3	Year 4
Jan	58.8	19	39	3.1
Mar	1.0	81.5	151	942
Jun	262	331	27	85.6
Aug	56	14	21.4	10
Oct	8.7	64.4	578	637

SOLUTION

Step 1. Compute the mean, standard deviation (s), and sum of the cubed residuals for the nickel concentrations:

$$\bar{x} = \frac{1}{20}(58.8 + 1 + \ldots + 637) = 169.52 \ ppb$$

$$s = \sqrt{\frac{1}{19}\left[(58.8 - 169.52)^2 + (1 - 169.52)^2 + \ldots + (637 - 169.52)^2\right]} = 259.7175 \ ppb$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^3 = \left[(58.8 - 169.52)^3 + \ldots + (637 - 169.52)^3\right] = 5.97845791 \times 10^8 \ ppb^3$$

Step 2. Compute the arithmetic normal coefficient of variation following equation [10.5]: $CV = 259.7175/169.52 = 1.53$

Step 3. Calculate the coefficient of skewness using equation [10.8]:

$$\gamma_1 = (20)^{1/2}\left(5.97845791 \times 10^8\right) / (19)^{3/2}(259.7175)^3 = 1.84$$

Both the CV and the coefficient of skewness are much larger than 1, so the data appear to be significantly positively skewed. Do not assume that the underlying population is normal.

Step 4. Since the original data evidence a high degree of skewness, one can instead compute the skewness coefficient and corresponding sample CV with equation [10.7] on the logged nickel concentrations. The logarithmic CV equals 4.97, a much more variable data set than suggested by the arithmetic CV. The skewness coefficient works out to be $|\gamma_1| = 0.24 < 1$, indicating that the logged data values are slightly skewed but not enough to clearly reject an assumption of normality in the logged data. In other words, the original nickel values may be lognormally distributed. ◄

10.5 SHAPIRO-WILK AND SHAPIRO-FRANCÍA NORMALITY TESTS

10.5.1 SHAPIRO-WILK TEST (N ≤ 50)

PURPOSE AND BACKGROUND

The Shapiro-Wilk test is based on the premise that if a data set is normally distributed, the ordered values should be highly correlated with corresponding *quantiles* (*z*-scores) taken from a normal distribution (Shapiro and Wilk, 1965). In particular, the Shapiro-Wilk test gives substantial weight to evidence of non-normality in the tails of a distribution, where the robustness of statistical tests based on the normality assumption is most severely affected. A variant of this test, the Shapiro-Francía test, is useful for sample sizes greater than 50 (see **Section 10.5.2**).

The Shapiro-Wilk test statistic (*SW*) will tend to be large when a probability plot of the data indicates a nearly straight line. Only when the plotted data show significant bends or curves will the test statistic be small. The Shapiro-Wilk test is considered one of the best tests of normality available (Miller, 1986; Madansky, 1988).

PROCEDURE

Step 1. Order and rank the dataset from least to greatest, labeling the observations as x_i for rank i = 1…n. Using the notation $x_{(i)}$, let the ith rank statistic from a data set represent the ith smallest value.

Step 2. Compute differences $\left[x_{(n-i+1)} - x_{(i)}\right]$ for each i = 1…n. Then determine k as the greatest integer less than or equal to (n/2).

Step 3. Use **Table 10-2** in **Appendix D** to determine the Shapiro-Wilk coefficients, a_{n-i+1} , for i = 1…k. Note that while these coefficients depend only on the sample size (n), the order of the coefficients must be preserved when used in Step 4. The coefficients can be determined for any sample size from n = 3 up to n = 50.

Step 4. Compute the quantity b given by the following equation:

$$b = \sum_{i=1}^{k} b_i = \sum_{i=1}^{k} a_{n-i+1}(x_{(n-i+1)} - x_{(i)}) \qquad [10.9]$$

Note that the values b_i are simply intermediate quantities represented by the terms in the sum of the right-hand expression in equation [10.9].

Step 5. Calculate the standard deviation (s) of the dataset. Then compute the Shapiro-Wilk test statistic using the equation:

$$SW = \left[\frac{b}{s\sqrt{n-1}}\right]^2 \qquad [10.10]$$

Step 6. Given the significance level (α) of the test, determine the critical point of the Shapiro-Wilk test with n observations using **Table 10-3** in **Appendix D**. To maximize the utility and power of the test, choose $\alpha = .10$ for very small data sets ($n < 10$), $\alpha = .05$ for moderately sized data sets ($10 \le n < 20$), and $\alpha = .01$ for larger sized data sets ($n \ge 20$). Compare the SW against the critical point (sw_c). If the test statistic exceeds the critical point, accept normality as a reasonable model for the underlying population. However, if $SW < sw_c$, reject the null hypothesis of normality at the α-level and decide that another distributional model might provide a better fit.

►EXAMPLE 10-2

Use the nickel data of **Example 10-1** to compute the Shapiro-Wilk test of normality.

SOLUTION

Step 1. Order the data from smallest to largest, rank in ascending order and list, as shown in columns 1 and 2 of the table below. Next list the data in reverse order in a third column.

i	$x_{(i)}$	$x_{(n-i+1)}$	$x_{(n-i+1)} - x_{(i)}$	a_{n-i+1}	b_i
1	1.0	942.0	941.0	.4734	445.47
2	3.1	637.0	633.9	.3211	203.55
3	8.7	578.0	569.3	.2565	146.03
4	10.0	331.0	321.0	.2085	66.93
5	14.0	262.0	248.0	.1686	41.81
6	19.0	151.0	132.0	.1334	17.61
7	21.4	85.6	64.2	.1013	6.50
8	27.0	81.5	54.5	.0711	3.87
9	39.0	64.4	25.4	.0422	1.07
10	56.0	58.8	2.8	.0140	0.04
11	58.8	56.0	−2.8		b = 932.88
12	64.4	39.0	−25.4		
13	81.5	27.0	−54.5		
14	85.6	21.4	−64.2		
15	151.0	19.0	−132.0		
16	262.0	14.0	−248.0		
17	331.0	10.0	−321.0		
18	578.0	8.7	−569.3		
19	637.0	3.1	−633.9		
20	942.0	1.0	−941.0		

Step 2. Compute the differences $\left[x_{(n-i+1)} - x_{(i)}\right]$ in column 4 of the table by subtracting column 2 from column 3. Since the total sample size is $n = 20$, the largest integer less than or equal to (n/2) is $k = 10$.

Step 3. Look up the coefficients a_{n-i+1} from **Table 10-2** in **Appendix D** and list in column 4.

Step 4. Multiply the differences in column 3 by the coefficients in column 4 and add the first k products (b_i) to get quantity b, using equation [10.9].

$$b = [.4734(941.0) + .3211(633.9) + \ldots + .0140(2.8)] = 932.88$$

Step 5. Compute the standard deviation of the sample, s = 259.72. Then use equation [10.10] to calculate the SW:

$$SW = \left[\frac{932.88}{259.72\sqrt{19}}\right]^2 = 0.679$$

Step 6. Use **Table 10-3** in **Appendix D** to determine the 0.01-level critical point for the Shapiro-Wilk test when n = 20. This gives sw_c = 0.868. Then compare the observed value of SW = 0.679 to the 1% critical point. Since SW < 0.868, the sample shows significant evidence of non-normality by the Shapiro-Wilk test. The data should be transformed using logarithms or another transformation on the ladder of powers and re-checked using the Shapiro-Wilk test before proceeding with further statistical analysis. ◀

10.5.2 SHAPIRO-FRANCÍA TEST (N > 50)

The Shapiro-Wilk test of normality can be used for sample sizes up to 50. When n is larger than 50, a slight modification of the procedure called the Shapiro-Francía test (Shapiro and Francia, 1972) can be used instead. Like the Shapiro-Wilk test, the Shapiro-Francía test statistic (SF) will tend to be large when a probability plot of the data indicates a nearly straight line. Only when the plotted data show significant bends or curves will the test statistic be small.

To calculate the test statistic SF, one can use the following equation:

$$SF = \left[\sum_{i=1}^{n} m_i x_{(i)}\right]^2 \Bigg/ \left[(n-1)s^2 \sum_{i=1}^{n} m_i^2\right] \qquad [10.11]$$

where $x_{(i)}$ represents the ith ranked value of the sample and where m_i denotes the approximate expected value of the ith rank normal quantile (or z-score). The values for m_i are approximately equal to

$$m_i = \Phi^{-1}\left(\frac{i}{n+1}\right) \qquad [10.12]$$

where Φ^{-1} denotes the inverse of the standard normal distribution with zero mean and unit variance. These values can be computed by hand using the normal distribution in **Table 10-1** of **Appendix D** or via simple commands found in many statistical computer packages.

Normality of the data should be rejected if the Shapiro-Francía statistic is too low when compared to the critical points provided in **Table 10-4** of **Appendix D**. Otherwise one can assume the data are approximately normal for purposes of further statistical analysis.

10.6 PROBABILITY PLOT CORRELATION COEFFICIENT

BACKGROUND AND PURPOSE

Another test for normality that is essentially equivalent to the Shapiro-Wilk and Shapiro-Francía tests is the *probability plot correlation coefficient* test described by Filliben (1975). This test meshes perfectly with the use of probability plots, because the essence of the test is to compute the usual *correlation coefficient* for points on a probability plot. Since the correlation coefficient is a measure of the linearity of the points on a scatterplot, the probability plot correlation coefficient, like the *SW* test statistic, will be high when the plotted points fall along a straight line and low when there are significant bends and curves in the probability plot. Comparison of the Shapiro-Wilk and probability plot correlation coefficient tests has indicated very similar statistical power for detecting non-normality (Ryan and Joiner, 1990).

It should be noted that although some statistical software may not compute Filliben's test directly, the usual Pearson's correlation coefficient computed on the data pairs used to construct a probability plot will provide a very close approximation to the Filliben statistic. Some users may find this latter correlation easier to compute or more accessible in their software.

PROCEDURE

Step 1. List the observations in order from smallest to largest, denoting $x_{(i)}$ as the ith smallest rank statistic in the data set. Then let n = sample size and compute the sample mean ($\bar{x}$) and the standard deviation (s).

Step 2. Consider a random sample drawn from a standard normal distribution. The ith rank statistic of this sample is fixed once the sample is drawn, but beforehand it can be considered a random variable, denoted as $X_{(i)}$. Likewise, by considering all possible datasets of size n that might be drawn from the normal distribution, one can think of the sampling distribution of the statistic $X_{(i)}$. This sampling distribution has its own mean and variance, and, of importance to the probability plot correlation coefficient, its own *median*, which can be denoted M_i.

To compute the median of the ith rank statistic, first compute intermediate probabilities m_i for $i = 1 \ldots n$ using the equation:

$$m_i = \begin{cases} 1-(.5)^{1/n} & \text{for } i=1 \\ (i-.3175)/(n+.365) & \text{for } 1<i<n \\ (.5)^{1/n} & \text{for } i=n \end{cases} \qquad [10.13]$$

Then compute the medians M_i as the standard normal quantiles or z-scores associated with the intermediate probabilities m_i. These can be determined from **Table 10-1** in **Appendix D** or computed according to the following equation, where Φ^{-1} represents the inverse of the standard normal distribution:

$$M_i = \Phi^{-1}(m_i) \qquad [10.14]$$

Step 3. With the rank statistic medians in hand, calculate the arithmetic mean of the M_i's, denoted $\bar{M}$, and the intermediate quantity C_n, given by the equation:

$$C_n = \sqrt{\sum_{i=1}^{n} M_i^2 - n\bar{M}^2} \qquad [10.15]$$

Note that when the dataset is "complete" (meaning it contains no non-detects, ties, or censored values), the mean of the order statistic medians reduces to $\bar{M} = 0$. This in turn reduces the calculation of C_n to:

$$C_n = \sqrt{\sum_{i=1}^{n} M_i^2} \qquad [10.16]$$

Step 4. Finally compute Filliben's probability plot correlation coefficient:

$$r = \frac{\sum_{i=1}^{n} x_{(i)} M_i - n\bar{x}\bar{M}}{C_n \cdot s\sqrt{n-1}} \qquad [10.17]$$

When the dataset is complete, the equation for the probability plot correlation coefficient also has a simplified form:

$$r = \sum_{i=1}^{n} x_{(i)} M_i \Big/ \left[C_n \cdot s\sqrt{n-1} \right] \qquad [10.18]$$

Step 5. Given the level of significance (α), determine the critical point (r_{cp}) for Filliben's test with sample size n from **Table 10-5** in **Appendix D**. Compare the probability plot correlation coefficient (r) against the critical point (r_{cp}). If $r \geq r_{cp}$, conclude that normality is a reasonable model for the underlying population at the α-level of significance. If, however, $r < r_{cp}$, reject the null hypothesis and conclude that another distributional model would provide a better fit.

►EXAMPLE 10-3

Use the data of **Example 10-1** to compute Filliben's probability plot correlation coefficient test at the α = .01 level of significance.

SOLUTION

Step 1. Order and rank the nickel data from smallest to largest and list, as in the table below. The sample size is n = 20, with sample mean $\bar{x}$ = 169.52 and the standard deviation s = 259.72.

Step 2. Compute the intermediate probabilities m_i from equation [10.13] for each i in column 3 and the rank statistic medians, M_i, in column 4 by applying the inverse normal transformation to column 3 using equation [10.14] and **Table 10-1** of **Appendix D**.

Step 3. Since this sample contains no non-detects or ties, the simplified equations for C_n in equation [10.16] and for r in equation [10.18] may be used. First compute C_n using the squared order statistic medians in column 5:

$$C_n = \sqrt{[3.328 + 1.926 + \ldots + 3.328]} \quad = \quad 4.138$$

Step 4. Next compute the products $x_{(i)} \times M_i$ in column 6 and sum to get the numerator of the correlation coefficient (equal to 3,836.81 in this case). Then compute the final correlation coefficient:

$$r = 3{,}836.81 \Big/ \left[4.138 \times 259.72\sqrt{19} \right] = 0.819$$

i	$x_{(i)}$	m_i	M_i	$(M_i)^2$	$x_{(i)} \times M_i$
1	1.0	.03406	−1.8242	3.328	−1.824
2	3.1	.08262	−1.3877	1.926	−4.302
3	8.7	.13172	−1.1183	1.251	−9.729
4	10.0	.18082	−0.9122	0.832	−9.122
5	14.0	.22993	−0.7391	0.546	−10.347
6	19.0	.27903	−0.5857	0.343	−11.129
7	21.4	.32814	−0.4451	0.198	−9.524
8	27.0	.37724	−0.3127	0.098	−8.444
9	39.0	.42634	−0.1857	0.034	−7.242
10	56.0	.47545	−0.0616	0.004	−3.448
11	58.8	.52455	0.0616	0.004	3.621
12	64.4	.57366	0.1857	0.034	11.959
13	81.5	.62276	0.3127	0.098	25.488
14	85.6	.67186	0.4451	0.198	38.097
15	151.0	.72097	0.5857	0.343	88.445
16	262.0	.77007	0.7391	0.546	193.638
17	331.0	.81918	0.9122	0.832	301.953
18	578.0	.86828	1.1183	1.251	646.376
19	637.0	.91738	1.3877	1.926	883.941
20	942.0	.96594	1.8242	3.328	1718.408

Step 5. Compare Filliben's test statistic of $r = 0.819$ to the 1% critical point for a sample of size 20 in **Table 10-5** of **Appendix D**, namely $r_{cp} = 925$. Since $r < 0.925$, the sample shows significant evidence of non-normality by the probability plot correlation coefficient. The data should be transformed and the correlation coefficient re-calculated before proceeding with further statistical analysis. ◀

10.7 SHAPIRO-WILK MULTIPLE GROUP TEST OF NORMALITY

BACKGROUND AND PURPOSE

The main purpose for including the multiple group test normality (Wilk and Shapiro, 1968) in the Unified Guidance is to serve as a check for normality when using a Student's *t*-test (**Chapter 16**) or

when assessing the joint normality of multiple intrawell data sets. The multiple group test is an extension of the Shapiro-Wilk procedure for assessing the joint normality of several independent samples. Each sample may have a different mean and/or variance, but as long as the underlying distribution of each group is normal, the multiple group test statistic will tend to be non-significant. Conversely, the multiple group test is designed to identify when at least one of the groups being tested is definitely non-normal.

This test extends the Shapiro-Wilk procedure for a single sample, using individual *SW* test statistics computed separately for each group or sample. Then the individual *SW* statistics are transformed and combined into an overall or "omnibus" statistic (G). Like the single sample procedure — where non-normality is indicated when the test statistic *SW* is too low — non-normality in one or more groups is indicated when G is too low. However, instead of a special table of critical points, G is constructed to follow a standard normal distribution under the null hypothesis of normality. The value of G can simply be compared to an α-level z-score or normal quantile to decide whether the null or alternative hypothesis is better supported.

Since it may be unclear which one or more of the groups is actually non-normal when the G statistic is significant, Wilk and Shapiro recommend that a probability plot (**Chapter 9**) be examined on the intermediate quantities, G_i (at least for the case where several groups are being simultaneously tested). One of these statistics is computed for each separate sample/group and is designed to follow a standard normal distribution under H_0. Because of this, the G_i statistics for non-normal groups will tend to look like outliers on a normal probability plot (see **Chapter 12**).

The multiple group test can also be used to check normality when performing Welch's *t*-test, a two-sample procedure in which the underlying data of both groups are assumed to be normal, but no assumption is made that the means or variances are the same. This is different from either the pooled variance *t*-test or the one-way analysis of variance [ANOVA], both of which assume *homoscedasticity* (*i.e.*, equal variances across groups). If the group variances can be shown to be equal, the single sample Shapiro-Wilk test can be run on the combined residuals, where the residuals of each group are formed by subtracting off the group mean from each of the individual measurements. However, if the group variances are possibly different, testing the residuals as a single group using the *SW* statistic may give an inaccurate or misleading result. Consequently, since a test of homoscedasticity is not required for Welch's *t*-test, it is suggested to first use the multiple group test to check normality.

Although the Shapiro-Wilk multiple group method is an attractive procedure for accommodating several groups of data at once, the user is cautioned against indiscriminate use. While many of the methods described in the Unified Guidance assume underlying normality, they also assume homoscedasticity. Other parametric multi-sample methods recommended for detection monitoring — prediction limits in **Chapter 18** and control charts in **Chapter 20** — all assume that each group has the same variance. Even if normality of the joint data can be demonstrated using the Shapiro-Wilk multiple group test, it says nothing about whether the assumption of equal variances is also satisfied. Generally speaking, except for Welch's *t*-test, a separate test of homoscedasticity may also be needed. Such tests are described in **Chapter 11**.

PROCEDURE

Step 1. Assuming there are K groups to be tested, let the sample size of the ith group be denoted n_i. Then compute the SW_i test statistic for each of the K groups using equation [10.10].

Step 2. Transform the SW_i statistics to the intermediate quantities (G_i). If the sample size (n_i) of the ith group is at least 7, compute G_i with the equation:

$$G_i = \gamma + \delta \ln\left(\frac{SW_i - \varepsilon}{1 - SW_i}\right) \qquad [10.19]$$

where the quantities γ, δ, and ε can be found in **Table 10-6** of **Appendix D** for $7 \le n_i \le 50$. If the sample size (n_i) is less than 7, determine G_i directly from **Table 10-7** in **Appendix D** by first computing the intermediate value

$$u_i = \ln\left(\frac{SW_i - \varepsilon}{1 - SW_i}\right) \qquad [10.20]$$

(obtaining ε from the top of **Table 10-7**), and then using linear interpolation to find the closest value G_i associated with u_i.

Step 3. Once the G_i statistics are derived, compute the Shapiro-Wilk multiple group statistic with the equation:

$$G = \frac{1}{\sqrt{K}} \sum_{i=1}^{K} G_i \qquad [10.21]$$

Step 4. Under the null hypothesis that all K groups are normally-distributed, G will follow a standard normal distribution. Given the significance level (α), determine an α-level critical point from **Table 10-1** of **Appendix D** as the *lower* $\alpha \times 100th$ normal quantile (z_α). Then compare G to z_α. If $G < z_\alpha$, there is significant evidence of non-normality at the α level. Otherwise, the hypothesis of normality cannot be rejected.

►EXAMPLE 10-4

The previous examples in this chapter pooled the data of **Example 10-1** into a single group before testing for normality. This time, treat each well separately and compute the Shapiro-Wilk multiple group test of normality at the α = .05 level.

SOLUTION

Step 1. The nickel data in **Example 10-1** come from K = 4 wells with n_i = 5 observations per well. Using equation [10.10], the SW_i individual well test statistics are calculated as:

Well 1: $SW_1 = 0.6062$

Well 2: $SW_2 = 0.5917$

Well 3: $SW_3 = 0.5652$

Well 4: $SW_4 = 0.6519$

Step 2. Since n_i = 5 for each well, use **Table 10-7** of **Appendix D** to find ε = .5521. First calculating u_1 with equation [10.20]:

$$u_1 = \ln\left(\frac{.6062 - .5521}{1 - .6062}\right) = -1.985$$

The performing this step for each well group and using linear interpolation on u in **Table 10-7**, the approximate G_i statistics are:

Well 1: $u_1 = -1.985$ $G_1 = -3.238$

Well 2: $u_2 = -2.333$ $G_2 = -3.488$

Well 3: $u_3 = -3.502$ $G_3 = -4.013$ (taking the last and closest entry)

Well 4: $u_4 = -1.249$ $G_4 = -2.755$

Step 3. Compute the multiple group test statistic using equation [10.21]:

$$G = \frac{1}{\sqrt{4}}\left[(-1.985) + (-2.333) + (-4.013) + (-2.755)\right] = -5.543$$

Step 4. Since α = 0.05, the lower α × 100*th* critical point from the standard normal distribution in **Table 10-1** of **Appendix D** is $z_{.05} = -1.645$. Clearly, $G < z_{.05}$; in fact G is smaller than just about any α-level critical point that might be selected. Thus, there is significant evidence of non-normality in at least one of these wells (and probably all of them). ◄

►EXAMPLE 10-5

The data in **Example 10-1** showed significant evidence of non-normality. In this example, use the same nickel data applying the coefficient of skewness, Shapiro-Wilk and the Probability Plot Correlation Coefficient tests to determine whether the combined well measurements better follow a lognormal distribution by first log-transforming the measurements. Computing the natural logarithms of the data gives the table below:

	Logged Nickel Concentrations log(ppb)			
Month	Well 1	Well 2	Well 3	Well 4
1	4.07	2.94	3.66	1.13
2	0.00	4.40	5.02	6.85
3	5.57	5.80	3.30	4.45
4	4.03	2.64	3.06	2.30
5	2.16	4.17	6.36	6.46

SOLUTION

METHOD 1. COEFFICIENT OF SKEWNESS

Step 1. Compute the log-mean ($\bar{y}$), log-standard deviation (s_y), and sum of the cubed residuals for the logged nickel concentrations (y_i):

$$\bar{y} = \frac{1}{20}(4.07 + 0.00 + \ldots + 6.46) \quad = \quad 3.918 \log(ppb)$$

$$s_y \quad = \quad \sqrt{\frac{1}{19}\left[(4.07 - 3.918)^2 + (0.00 - 3.918)^2 + \ldots + (6.46 - 3.918)^2\right]} \quad = \quad 1.8014 \log(ppb)$$

$$\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^3 \quad = \quad \left[(4.07 - 3.918)^3 + \ldots + (6.46 - 3.918)^3\right] = -26.528 \log^3(ppb)$$

Step 2. Calculate the coefficient of skewness using equation [10.8] with Step 1 values as:

$$\gamma_1 = (20)^{1/2}(-26.528) \Big/ (19)^{3/2}(1.8014)^3 = -0.245$$

Since the absolute value of the skewness is less than 1, the data do not show evidence of significant skewness. Applying a normal distribution to the log-transformed data may therefore be appropriate, but this model should be further checked. The logarithmic CV of 4.97 computed in Example 10-1 was also suggestive of a highly skewed distribution, but can be difficult to interpret in determining if measurements, in fact, follow a logarithmic distribution.

METHOD 2. SHAPIRO-WILK TEST

Step 1. Order and rank the data from smallest to largest and list, as in the table below. List the data in reverse order alongside the first column. Denote the *i*th logged observation by $y_i = \log(x_i)$.

Step 2. Compute differences $\left[y_{(n-i+1)} - y_{(i)}\right]$ in column 4 of the table by subtracting column 2 from column 3. Since $n = 20$, the largest integer less than or equal to $(n/2)$ is $k = 10$.

Step 3. Look up the coefficients a_{n-i+1} from **Table 10-2** of **Appendix D** and list in column 5.

Step 4. Multiply the differences in column 4 by the coefficients in column 5 and add the first k products (b_i) to get quantity b, using equation [10.9].

$$b = [.4734(6.85) + .3211(5.33) + \ldots + .0140(.04)] = 7.77$$

i	$y_{(i)}$	$y_{(n-i+1)}$	$y_{(n-i+1)} - y_{(i)}$	a_{n-i+1}	b_i
1	0.00	6.85	6.85	.4734	3.24
2	1.13	6.46	5.33	.3211	1.71
3	2.16	6.36	4.20	.2565	1.08
4	2.30	5.80	3.50	.2085	0.73
5	2.64	5.57	2.93	.1686	0.49
6	2.94	5.02	2.08	.1334	0.28
7	3.06	4.45	1.39	.1013	0.14
8	3.30	4.40	1.10	.0711	0.08
9	3.66	4.17	0.51	.0422	0.02
10	4.03	4.07	0.04	.0140	0.00
11	4.07	4.03	−0.04		b = 7.77
12	4.17	3.66	−0.51		
13	4.40	3.30	−1.10		
14	4.45	3.06	−1.39		
15	5.02	2.94	−2.08		
16	5.57	2.64	−2.93		
17	5.80	2.30	−3.50		
18	6.36	2.16	−4.20		
19	6.46	1.13	−5.33		
20	6.85	0.00	−6.85		

Step 5. Compute the log-standard deviation of the sample, $s_y = 1.8014$. Then use [10.10] to calculate the *SW* test statistic:

$$SW = \left[\frac{7.77}{1.8014\sqrt{19}}\right]^2 = 0.979$$

Step 6. Use **Table 10-3** of **Appendix D** to determine the .01-level critical point for the Shapiro-Wilk test when $n = 20$. This gives $sw_{cp} = 0.868$. Then compare the observed value of $SW = 0.979$ to the 1% critical point. Since $SW > 0.868$, the sample shows no significant evidence of non-normality by the Shapiro-Wilk test. Proceed with further statistical analysis using the log-transformed data or by assuming the underlying population is lognormal.

METHOD 3. PROBABILITY PLOT CORRELATION COEFFICIENT

Step 1. Order and rank the logged nickel data from smallest to largest and list, as in the table below. Again let the *i*th logged value be denoted by $y_i = \log(x_i)$. The sample size is $n = 20$, the log-mean is $\bar{y} = 3.918$, and the log-standard deviation is $s_y = 1.8014$.

Step 2. Compute the intermediate probabilities m_i from equation [10.13] for each *i* in column 3 and the rank statistic medians, M_i , in column 4 by applying the inverse normal transformation to column 3 using equation [10.14] and **Table 10-1** of **Appendix D**.

i	$y_{(i)}$	m_i	M_i	$(M_i)^2$	$y_{(i)} \times M_i$
1	0.00	.03406	−1.8242	3.328	0.000
2	1.13	.08262	−1.3877	1.926	−1.568
3	2.16	.13172	−1.1183	1.251	−2.416
4	2.30	.18082	−0.9122	0.832	−2.098
5	2.64	.22993	−0.7391	0.546	−1.951
6	2.94	.27903	−0.5857	0.343	−1.722
7	3.06	.32814	−0.4451	0.198	−1.362
8	3.30	.37724	−0.3127	0.098	−1.032
9	3.66	.42634	−0.1857	0.034	−0.680
10	4.03	.47545	−0.0616	0.004	−0.248
11	4.07	.52455	0.0616	0.004	0.251
12	4.17	.57366	0.1857	0.034	0.774
13	4.40	.62276	0.3127	0.098	1.376
14	4.45	.67186	0.4451	0.198	1.981
15	5.02	.72097	0.5857	0.343	2.940
16	5.57	.77007	0.7391	0.546	4.117
17	5.80	.81918	0.9122	0.832	5.291
18	6.36	.86828	1.1183	1.251	7.112
19	6.46	.91738	1.3877	1.926	8.965
20	6.85	.96594	1.8242	3.328	12.496

Step 3. Since this sample contains no non-detects or ties, the simplified equations for C_n in [10.16] and for r in [10.18] may be used. First compute C_n using the squared order statistic medians in column 5:

$$C_n = \sqrt{[3.328 + 1.926 + \ldots + 3.328]} \quad = \quad 4.138$$

Step 4. Next compute the products $y_{(i)} \times M_i$ in column 6 and sum to get the numerator of the correlation coefficient (equal to 32.226 in this case). Then compute the final correlation coefficient:

$$r = 32.226 \Big/ \left[4.138 \times 1.8014\sqrt{19} \right] = 0.992$$

Step 5. Compare the Filliben's test statistic of r = 0.992 to the 1% critical point for a sample of size 20 in **Table 10-5** in **Appendix D**, namely r_{cp} = 925. Since r > 0.925, the sample shows no significant evidence of non-normality by the probability plot correlation coefficient test. Therefore, lognormality of the original data can be assumed in subsequent statistical procedures.

Note: the Shapiro-Wilk and Filliben's Probability Plot Correlation Coefficient tests for normality on a single data set perform quite comparably. Only one of these tests need be run in routine applications. ◄

CHAPTER 11. TESTING EQUALITY OF VARIANCE

11.1 BOX PLOTS 11-2
11.2 LEVENE'S TEST 11-4
11.3 MEAN-STANDARD DEVIATION SCATTER PLOT 11-8

Many of the methods described in the Unified Guidance assume that the different groups under comparison have the same variance (*i.e.*, are *homoscedastic*). This chapter covers procedures for assessing homoscedasticity and its counterpart, *heteroscedasticity* (*i.e.*, unequal variances). Equality of variance is assumed, for instance, when using prediction limits to make either upgradient-to-downgradient or intrawell comparisons. In the former case, the method assumes that the upgradient variance is equal to the variance in each downgradient well. In the latter case, the presumption is that the well variance is stable over time (*i.e.*, stationary) when comparing intrawell background versus more recent measurements.

If a prediction limit is constructed on a single new measurement at each downgradient well, it isn't feasible to test the variance equality assumption prior to each statistical evaluation. Homoscedasticity *can* be tested after several new rounds of compliance sampling by pooling collected compliance measurements within a well. The Unified Guidance recommends periodic testing of the presumption of equal variances by comparing newer data to historical background (**Chapter 6**).

Equality of variance between different groups (*e.g.*, different wells) is also an important assumption for an analysis of variance [ANOVA]. If equality of variance does not hold, the power of the *F*-test (its ability to detect differences among the group means) is reduced. Mild differences in variance are generally acceptable. But the effect becomes noticeable when the largest and smallest group variances differ by a ratio of about 4, and becomes quite severe when the ratio is 10 or more (Milliken and Johnson, 1984).

Three procedures for assessing or testing homogeneity of variance are described in the Unified Guidance, two of which that are more robust to departures from normality (*i.e.*, less sensitive to non-normality). These include:

1. The box plot (**Chapter 9**), a graphical method useful not only for checking equality of variance but also as an exploratory tool for visualizing the basic statistical characteristics of data sets. It can also provide a rough indication of differences in mean or median concentration levels across several wells;
2. Levene's test (**Section 11.2**), a formal ANOVA-type procedure for testing variance inequality; and
3. The mean-standard deviation scatter plot (**Chapter 9** and **Section 11.3**), a visual tool for assessing whether the degree of variability in a set of data groups or wells is correlated with the mean levels for those groups. This could potentially indicate whether a *variance stabilizing transformation* might be needed.

11.1 BOX PLOTS

PURPOSE AND BACKGROUND

Box plots are described in **Chapter 9**. In the context of variance testing, one can construct a box plot for each well group and compare the boxes to see if the assumption of equal variances is reasonable. The comparison is not a formal test procedure, but is easier to perform and is often sufficient for checking the group variance assumption.

Box plots for each data group simultaneously graphed side-by-side provide a direct visual comparison of the dispersion in each group. As a rule of thumb, if the box length for each group is less than 1.5–2 times the length of the shortest box, the sample variances may be close enough to assume equal group variances. If the box length for any group is greater than 1.5–2 times the length of the box for another group, the variances may be significantly different. A formal test such as Levene's might be needed to more accurately decide. Sample data sets with unequal variances may need a *variance stabilizing transformation*, *i.e.*, one in which the transformed measurements have approximately equal variances.

Most statistical software packages will calculate the statistics needed to draw a box plot, and many will construct side-by-side box plots directly. Usually a box plot will also be shown with two "whiskers" extending from the edges of the box. These lines indicate either the positions of extreme minimum or maximum values in the data set. In Tukey's original formulation (Tukey, 1977), they indicate the most extreme lower and upper data points outside the box but falling within a distance of 1.5 times the interquartile range (that is, the length of the box) from either edge. The whiskers should generally *not* be used to approximate the overall variance under either formulation.

A convenient tactic when using box plots to screen for heteroscedasticity is to plot the *residuals* of each data group rather than the measurements themselves. This will line the boxes up at roughly a common level (close to zero), so that a visual comparison of box lengths is easier.

REQUIREMENTS AND ASSUMPTIONS

The requirements and assumptions for box plots are discussed in **Section 9.2.**

PROCEDURE

Step 1. For each of j wells or data groups, compute the sample mean of that group $\bar{x}_j$. Then compute the residuals (r_{ij}) for each group by subtracting the group mean from each individual measurement: $r_{ij} = x_{ij} - \bar{x}_j$.

Step 2. Use the procedure outlined in **Section 9.2** to create side-by-side box plots of the residuals formed in Step 1. Then compare the box lengths to check for possibly unequal variances.

►EXAMPLE 11-1

Construct box plots on the residuals for each of the following well groups to check for homoscedasticity.

	Arsenic Concentration (ppb)					
Month	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
1	22.9	2.0	2.0	7.8	24.9	0.3
2	3.1	1.2	109.4	9.3	1.3	4.8
3	35.7	7.8	4.5	25.9	0.8	2.8
4	4.2	52	2.5	2.0	27	1.2

SOLUTION

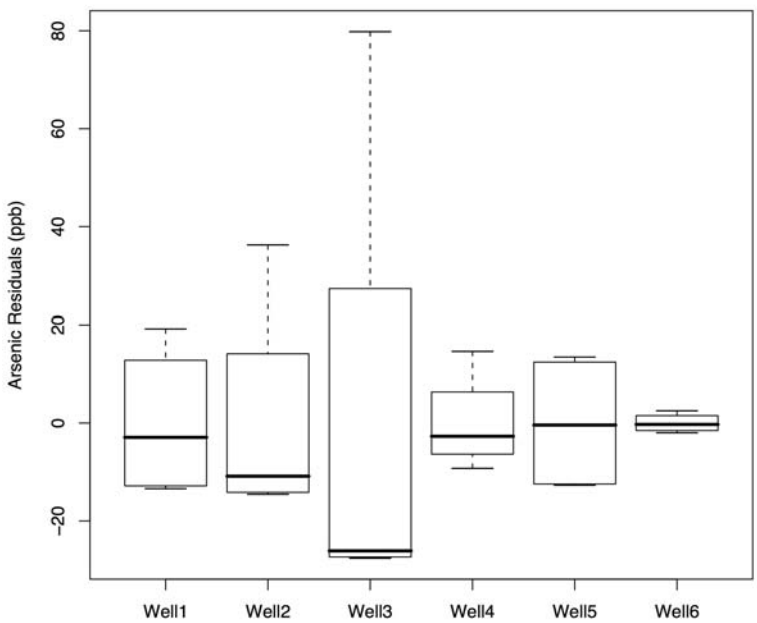
Step 1. Form the residuals for each well by subtracting the sample well mean from each observation, as shown in the table below.

	Arsenic Residuals (ppb)					
Month	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
1	6.43	−13.75	−27.6	−3.45	11.4	−1.98
2	−13.38	−14.55	79.8	−1.95	−12.2	2.52
3	19.22	−7.95	−25.1	14.65	−12.7	0.52
4	−12.28	36.25	−27.1	−9.25	13.5	−1.08
Mean	16.48	15.75	29.6	11.25	13.5	2.28

Step 2. Follow the procedure in **Section 9.2** to compute a box plot of the residuals for each well. Line these up side by side on the same graph, as in **Figure 11-1**.

Step 3. Compare the box lengths. Since the box length for Well 3 is more than three times the box lengths of Wells 4 and 6, there is informal evidence that the population group variances may be different. These data should be further checked using a formal test and perhaps a variance stabilizing transformation attempted. ◄

Figure 11-1. Side-by-Side Box Plots of Arsenic Residuals



11.2 LEVENE'S TEST

PURPOSE AND BACKGROUND

Levene's test is a formal procedure for testing homogeneity of variance that is fairly robust (i.e., not overly sensitive) to non-normality in the data. It is based on computing the new variables:

$$z_{ij} = \left| x_{ij} - \bar{x}_{i\bullet} \right| \qquad [11.1]$$

where x_{ij} represents the *j*th sample value from the *i*th group (*e.g.*, well) and $\bar{x}_{i\bullet}$ is the *i*th group sample mean. The symbol $(\bullet)$ in the notation for the group sample mean represents an averaging over subscript *j*. The values z_{ij} then represent the absolute values of the ***residuals***. Levene's test involves running a standard one-way ANOVA (**Chapter 17**) on the variables z_{ij}. If the *F*-test is significant, reject the hypothesis of equal group variances and perhaps seek a variance stabilizing transformation. Otherwise, proceed with analysis of the original x_{ij}'s.

Levene's test is based on a one-way ANOVA and contrasts the means of the groups being tested. This implies a comparison between averages of the form:

$$\bar{z}_i = \frac{1}{n_i}\sum_{j=1}^{n_i}\left|x_{ij} - \bar{x}_{i\bullet}\right| \qquad [11.2]$$

Such averages of the z_{ij}'s are very similar to the standard deviations of the original data groups, given by the formula:

$$s_i = \sqrt{\frac{1}{n_i - 1}\sum_{j=1}^{n_i}\left(x_{ij} - \bar{x}_{i\bullet}\right)^2} \qquad [11.3]$$

In both cases, the statistics are akin to an average absolute residual. Therefore, the comparison of means in Levene's test is closely related to a direct comparison of the group standard deviations, the underlying aim of any test of variance equality.

REQUIREMENTS AND ASSUMPTIONS

The requirements and assumptions for Levene's test are essentially the same as the one-way ANOVA in **Section 17.1**, but applied to the absolute residuals instead of the raw measurements.

PROCEDURE

Step 1. Suppose there are p data groups to be compared. Because there may be different numbers of observations per well, denote the sample size of the ith group by n_i and the total number of data points across all groups by N.

Denote the observations in the ith group by x_{ij} for $i = 1\ldots p$ and $j = 1\ldots n_i$. The first subscript then designates the well, while the second denotes the jth value in the ith well. After computing the sample mean ($\bar{x}_i$) for each group, calculate the absolute residuals (z_{ij}) using equation [11.1].

Step 2. Utilizing the absolute residuals — and not the original data — compute the mean of each group along with the overall (grand) mean of the combined data set using the formula:

$$\bar{z}_{\bullet\bullet} = \frac{1}{N}\sum_{i=1}^{p}\sum_{j=1}^{n_i} z_{ij} \qquad [11.4]$$

Step 3. Compute the sum of squares of differences between the group means and the grand mean, denoted SS_{grps}:

$$SS_{grps} = \sum_{i=1}^{p} n_i\left(\bar{z}_{i\bullet} - \bar{z}_{\bullet\bullet}\right)^2 = \sum_{i=1}^{p} n_i\bar{z}_{i\bullet}^2 - N\bar{z}_{\bullet\bullet}^2 \qquad [11.5]$$

The formula on the far right is usually the most convenient for calculation. This sum of squares has (p–1) degrees of freedom associated with it and is a measure of the variability *between* groups. It constitutes the numerator of the F-statistic.

Step 4. Compute the corrected total sum of squares, denoted by SS_{total}:

$$SS_{total} = \sum_{i=1}^{p}\sum_{j=1}^{n_i}\left(z_{ij} - \bar{z}_{\bullet\bullet}\right)^2 = \sum_{i=1}^{p}\sum_{j=1}^{n_i} z_{ij}^2 - N\bar{z}_{\bullet\bullet}^2 \qquad [11.6]$$

Again, the formula on the far right is usually the most computationally convenient. This sum of squares has (N–1) associated degrees of freedom.

Step 5. Compute the sum of squares of differences between the absolute residuals and the group means. This is known as the within-groups component of the total sum of squares or, equivalently, as the sum of squares due to error. It is easiest to obtain by subtracting SS_{grps} from SS_{total} and is denoted SS_{error}:

$$SS_{error} = \sum_{i=1}^{p}\sum_{j=1}^{n_i}\left(z_{ij} - \bar{z}_{i\bullet}\right)^2 = SS_{total} - SS_{grps} = \sum_{i=1}^{p}\sum_{j=1}^{n_i} z_{ij}^2 - \sum_{i=1}^{p} n_i\bar{z}_{i\bullet}^2 \qquad [11.7]$$

SS_{error} is associated with (N–p) degrees of freedom and is a measure of the variability *within* groups. This quantity goes into the denominator of the F-statistic.

Step 6. Compute the mean sum of squares for both the between-groups and within-groups components of the total sum of squares, denoted by MS_{grps} and MS_{error}. These quantities are obtained by dividing each sum of squares by its corresponding degrees of freedom:

$$MS_{grps} = SS_{grps} \big/ \left(p - 1\right) \qquad [11.8]$$

$$MS_{error} = SS_{error} \big/ \left(N - p\right) \qquad [11.9]$$

Step 7. Compute the F-statistic by forming the ratio between the mean sum of squares for wells and the mean sum of squares due to error, as in **Figure 11-2** below. This layout is known as the one-way parametric ANOVA table and illustrates each sum of squares component of the total variability, along with the corresponding degrees of freedom, the mean squares components, and the final F-statistic calculated as $F = MS_{\text{grps}}/MS_{\text{error}}$. Note that the first two rows of the one-way table sum to the last row.

Step 8. **Figure 11-2** is a generalized ANOVA table for Levene's test. To test the hypothesis of equal variances across all p well groups, compare the F-statistic in **Figure 11-2** to the α-level critical point found from the F-distribution with (p–1) and (N–p) degrees of freedom in **Appendix D Table 17-1**. When testing variance equality, only severe levels of difference typically impact test performance in a substantial way. For this reason, the Unified Guidance recommends setting α = .01 when screening multiple wells and/or constituents using Levene's test. In that case, the needed critical point equals the upper 99th percentage point of the F-distribution. If the observed F-statistic exceeds the critical point ($F_{.99,\text{p}-1,\text{N}-\text{p}}$), reject the hypothesis of equal group population variances. Otherwise, conclude that there is insufficient evidence of a significant difference between the variances.

Figure 11-2. ANOVA Table for Levene's Test

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	*F*-Statistic
Between Wells	SS_{grps}	$p-1$	$MS_{grps} = SS_{grps}/(p-1)$	$F = MS_{grps}/MS_{error}$
Error (within wells)	SS_{error}	$N-p$	$MS_{error} = SS_{error}/(N-p)$	
Total	SS_{total}	$N-1$		

►EXAMPLE 11-2

Use the data from **Example 11-1** to conduct Levene's test of equal variances at the α = 0.01 level of significance.

SOLUTION

Step 1. Calculate the group arsenic mean for each well ($\bar{x}_{i\bullet}$):

Well 1 mean = 16.47 ppm Well 4 mean = 11.26 ppm

Well 2 mean = 15.76 ppm Well 5 mean = 13.49 ppm

Well 3 mean = 29.60 ppm Well 6 mean = 2.29 ppm

Then compute the absolute residuals z_{ij} in each well using equation [11.1] as in the table below.

	Absolute Arsenic Residuals (z_{ij})					
Month	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
1	6.43	13.76	27.6	3.42	11.41	1.95
2	13.38	14.51	79.8	1.96	12.19	2.49
3	19.23	7.96	25.1	14.64	12.74	0.56
4	12.29	36.24	27.1	9.26	13.51	1.09
Well Mean ($\bar{z}_{i\bullet}$)	12.83	18.12	39.9	7.32	12.46	1.52
Overall Mean ($\bar{z}_{\bullet\bullet}$)	15.36					

Step 2. Compute the mean absolute residual ($\bar{z}_{i\bullet}$) in each well and then the overall grand mean using equation [11.4]. These results are listed above.

Step 3. Compute the between-groups sum of squares for the absolute residuals using equation [11.5]:

$$SS_{grps} = \left[4(12.83)^2 + 4(18.12)^2 + \ldots + 4(1.52)^2\right] - 24 \cdot (15.36)^2 = 3{,}522.90$$

Step 4. Compute the corrected total sum of squares using equation [11.6]:

$$SS_{total} = \left[(6.43)^2 + (13.38)^2 + \ldots + (1.09)^2\right] - 24 \cdot (15.36)^2 = 6{,}300.89$$

Step 5. Compute the within-groups or error sum of squares using equation [11.7]:

$$SS_{error} = 6{,}300.89 - 3{,}522.90 = 2{,}777.99$$

Step 6. Given that the number of groups is p = 6 and the total sample size is N = 24, calculate the mean squares for the between-groups and error components using formulas [11.8] and [11.9]:

$$MS_{grps} = 3{,}522.90 \big/ (6-1) = 704.58$$

$$MS_{error} = 2{,}777.99 \big/ (24-6) = 154.33$$

Step 7. Construct an ANOVA table following **Figure 11-2** to calculate the F-statistic. The numerator degrees of freedom [df] is computed as (p–1) = 5, while the denominator df is equal to (N–p) = 18.

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	F-Statistic
Between Well Grps	3,522.90	5	704.58	4.56
Error (within grps)	2,777.99	18	154.33	
Total	6,300.89	23		

Step 8. Determine the .01-level critical point for the F-test with 5 and 18 degrees of freedom from **Table 17-1**. This gives $F_{.99,5,18}$ = 4.25. Since the F-statistic of 4.56 exceeds the critical point, the assumption of equal variances should be rejected. Since the original concentration data are used in this example, a transformation such as the natural logarithm might be tried and the transformed data retested. ◄

11.3 MEAN-STANDARD DEVIATION SCATTER PLOT

BACKGROUND AND PURPOSE

The mean-standard deviation scatter plot is described in **Chapter 9**. It is useful as an exploratory tool for multiple groups of data (*e.g.*, wells) to aid in identifying relationships between mean levels and variability. It is also helpful in providing a visual assessment of variance homogeneity across data groups. Like side-by-side box plots, the mean-standard deviation scatter plot graphs a measure of variability for each well. In the latter, however, the standard deviation is plotted rather than the interquartile range, so a more direct assessment of variance equality can be made. Since standard

deviations (and consequently variances) are often positively correlated with sample mean levels in skewed populations, the observed pattern on the mean-standard deviation scatter plot can offer valuable clues as to what sort of variance stabilizing transformation if any might work.

REQUIREMENTS AND ASSUMPTIONS

The requirements for the mean-standard deviation scatter plot are listed in **Section 9.4.**

PROCEDURE

See **Section 9.4.**

►EXAMPLE 11-3

Use the data from **Example 11-1** to construct a mean-standard deviation scatter plot.

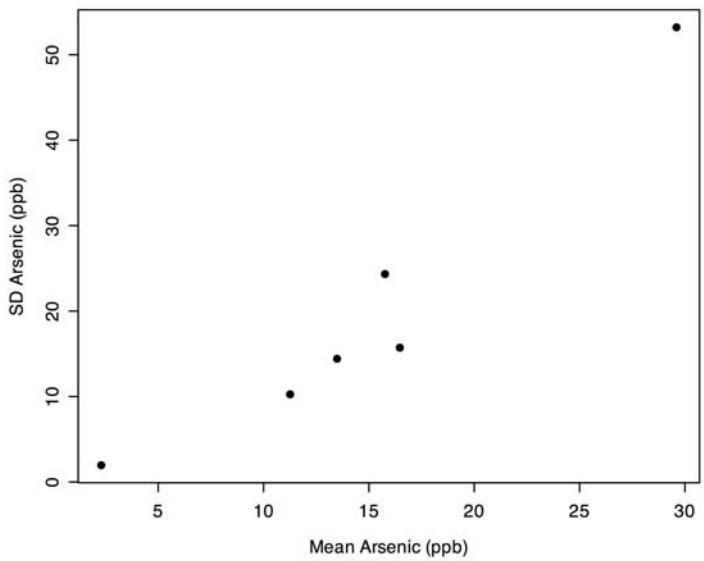
SOLUTION

Step 1. First compute the sample mean ($\bar{x}$) and standard deviation (s) of each well, as listed below.

Well	Mean	Std Dev
1	16.468	15.718
2	15.762	24.335
3	29.600	53.211
4	11.260	10.257
5	13.488	14.418
6	2.292	1.958

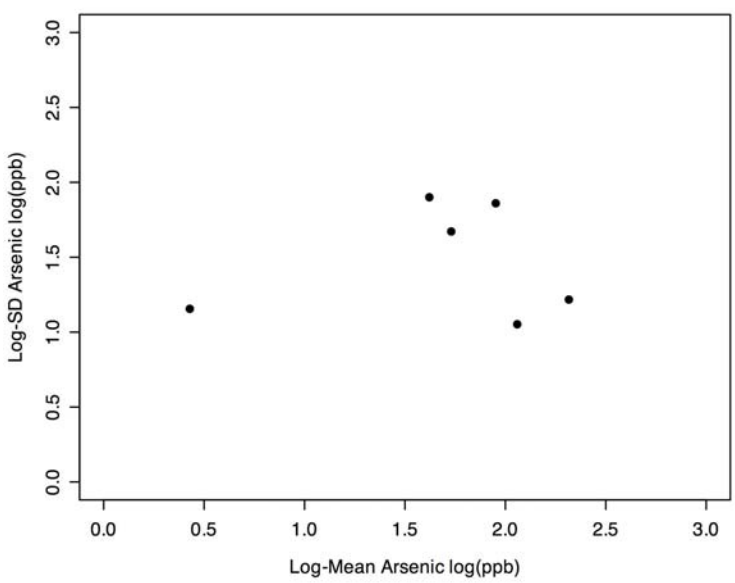
Step 2. Plot the well means versus the standard deviations as in **Figure 11-3** below. Note the roughly linear relationship between the magnitude of the standard deviations and their corresponding means. The data suggest unequal variances among the wells, as indicated by the large range in the standard deviations.

Figure 11-3. Arsenic Mean-Standard Deviation Plot



Step 3. Because lognormal data groups will tend to show a linear association between the sample means and standard deviations, apply a log transformation to the original arsenic measurements and reconstruct the mean-standard deviation scatter plot on the log scale. Computing the log-means and log-standard deviations and then re-plotting gives **Figure 11-4**. Now the apparent trend between the means and standard deviations is gone. Further, on the log scale, the standard deviations are much more similar in magnitude, all with values between 1 and 2. The log transformation thus appears to roughly stabilize the arsenic variances. ◄

Figure 11-4. Log(Arsenic) Mean-Standard Deviation Plot



CHAPTER 12. IDENTIFYING OUTLIERS

12.1 SCREENING WITH PROBABILITY PLOTS 12-1
12.2 SCREENING WITH BOX PLOTS 12-5
12.3 DIXON'S TEST 12-8
12.4 ROSNER'S TEST 12-10

This chapter discusses screening tools and formal tests for identifying statistical outliers. Two screening tools are first presented: probability plots (**Section 12.1**) and box plots (**Section 12.2**). These are followed by two formal outlier tests:

- Dixon's test (**Section 12.3**) for a single outlier in smaller data sets, and
- Rosner's test (**Section 12.4**) for up to five separate outliers in larger data sets.

A statistical determination of one or more statistical outliers does not indicate *why* the measurements are discrepant from the rest of the data set. The Unified Guidance does not recommend that outliers be removed *solely* on a statistical basis. The outlier tests can provide supportive information, but generally a reasonable rationale needs to be identified for removal of suspect outlier values (usually limited to background data**).** At the same time there must be some level of confidence that the data are representative of ground water quality. A number of factors and considerations in removing outliers from potential background data are discussed in **Section 5.2.3.**

12.1 SCREENING WITH PROBABILITY PLOTS

BACKGROUND AND PURPOSE

Probability plots (**Chapter 9**) are helpful in identifying outliers in at least two ways. First, since the straightness of the plot indicates how closely the data fit the pattern of a normal distribution, values that appear "out of line" with the remaining data can be visually identified as possible outliers. Secondly, the two formal outlier tests presented in the Unified Guidance assume that the underlying population minus the suspected outlier(s) is normal. Probability plots can provide visual evidence for this assumption. Data that appear non-normal after the suspected outliers have been removed from the probability plot may need to be transformed (e.g., via the natural logarithm) and re-examined on the transformed scale to see if potential outliers are still apparent.

As an aid to the interpretation of a given probability plot, the Unified Guidance recommends computation of the probability plot correlation coefficient, using either Filliben's procedure (**Chapter 10**) or the simple (Pearson) correlation (**Chapter 3**) between the numerical pairs plotted on the graph. The higher the correlation, the more linear the pattern is on the probability plot and therefore a better fit to normality. Note that while the Filliben correlation coefficient can be compared to critical points derived for that test of normality (**Chapter 10**), a low correlation may be related to other causes of non-normality besides the presence of outliers. The correlation coefficient is not a substitute for a formal outlier test, but can be useful as a screening tool.

REQUIREMENTS AND ASSUMPTIONS

Probability plots are primarily a tool to assess normality, and not to identify outliers *per se*. It is critical that the remaining data without potential outliers is either normal in distribution or can be normalized via a transformation. Otherwise, the probability plot may appear non-linear and non-normal for reasons unrelated to the presence of outliers. Right-skewed lognormal distributions can appear to have one or more outliers on a probability plot unless the original data are first log-transformed. As a general rule, probability plots should be constructed on the original (or raw) measurements and one or more transformed data sets (*e.g.*, log or square root), in order to avoid mistaking inherent data skewness for outliers.

If the raw and transformed-data probability plots both indicate one or more values inconsistent with the pattern of the remaining values, continue with a second level of screening by temporarily removing the suspected outlier(s) and re-constructing the probability plots. If the raw-scale plot is reasonably linear, consider running a formal outlier test on the original measurements. On the other hand, if the raw-scale plot is skewed but the transformed-scale plot is linear, consider conducting a formal outlier test on the transformed measurements.

A related difficulty occurs when sample data includes censored or non-detect values. If simple substitution is used to estimate a value for each non-detect prior to plotting, the resulting probability plot may appear non-linear simply because the censored observations were not properly handled. In this case, a censored probability plot (**Chapter 15**) should be constructed instead of an uncensored, complete sample plot (**Chapter 9**). The same caveats apply to normalizing the sample data, perhaps by attempting at least one transformation. The only difference is that each probability plot constructed must appropriately account for the observed censoring in the sample.

PROCEDURE

Step 1. After identifying one or more possible outliers (*e.g.*, values much higher in concentration than the remaining measurements), construct a probability plot on the entire sample using the procedure described in **Section 9.5**. Construct a *censored probability plot* from **Section 15.3** if the sample contains non-detects. If the data including the suspected outlier(s) follow a reasonably linear pattern, a formal outlier test is probably unnecessary. However, if one or more values are out of line compared to the pattern of the remaining data, construct a similar probability plot after applying one or more transformations. If one or more suspected outliers is still inconsistent, proceed to Step 2.

Step 2. Compute a probability plot correlation coefficient for each plot constructed in Step 1. Use these correlations as an aid to interpreting the degree of linearity in each probability plot.

Step 3. Reconstruct the probability plots from Step 1 after removing the suspected outlier(s). Recompute the correlation coefficients from Step 2 on this reduced sample.

Step 4. If the 'outlier-deleted' probability plot on the raw concentration scale indicates a linear pattern with high correlation, consider running a formal outlier test on the original measurements. When the pattern is distinctly non-linear but the corresponding probability plot on the transformed-scale is fairly linear (and higher in correlation), conduct the outlier test on the transformed values.

►EXAMPLE 12-1

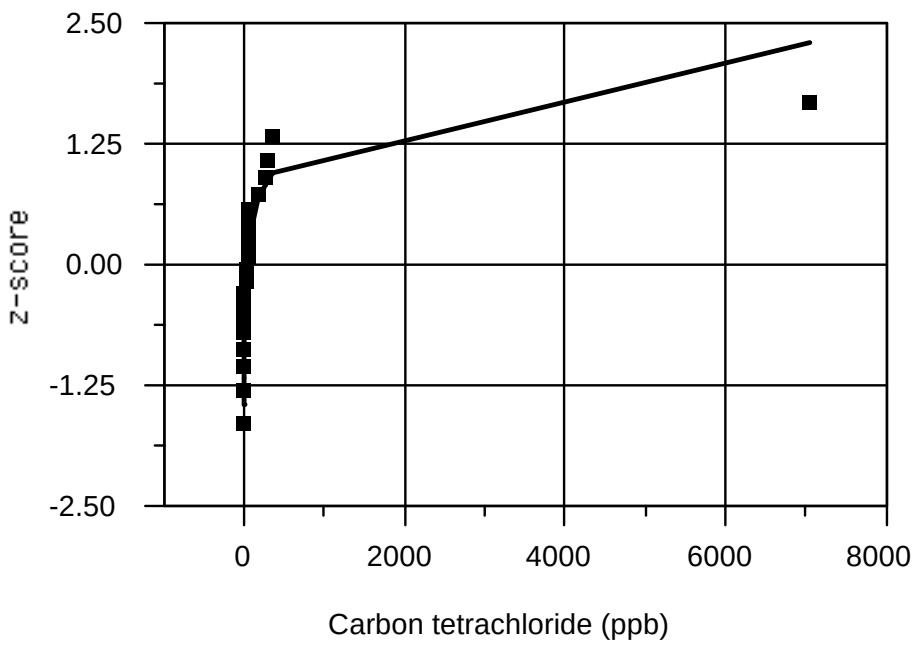
The table below contains data from five background wells measured over a four month period. The value 7,066 is found in the second month at Well 3. Use probability plots on the combined sample to determine whether or not a formal outlier test is warranted.

Carbon Tetrachloride Concentrations (ppb)				
Well 1	Well 2	Well 3	Well 4	Well 5
1.7	302	16.2	199	275
3.2	35.1	7066	41.6	6.5
7.3	15.6	350	75.4	59.7
12.1	13.7	70.1	57.9	68.4

SOLUTION

Step 1. Examine the probability plots of the entire sample first using the raw measurements and then log-transformed values (**Figures 12-1** and **12-2**). Both these plots indicate that the suspected outlier does not follow the pattern of the remaining observations, but seems 'out of line.' The Pearson correlation coefficients for these probability plots are, respectively, r = 0.513 and 0.975, indicating that the fit to normality overall is much closer using log-transformed measurements.

Figure 12-1. Probability Plot on Raw Concentrations (r = .513)



Step 2. Next remove the suspected outlier and reconstruct the probability plots on both the original and logged observations (**Figures 12-3** and **12-4**). The plot on the original scale indicates heavy positive (or right-) skewness and a non-linear pattern, while the plot on the log-scale exhibits a fairly linear pattern. The respective correlation coefficients now become r = 0.854 and 0.985, again favoring the log-transformed sample. On the basis of these plots, the

underlying data should be modeled as lognormal and the observations logged prior to running a formal outlier test. ◄

Figure 12-2. Probability Plot on Logged Observations (*r* = .975)

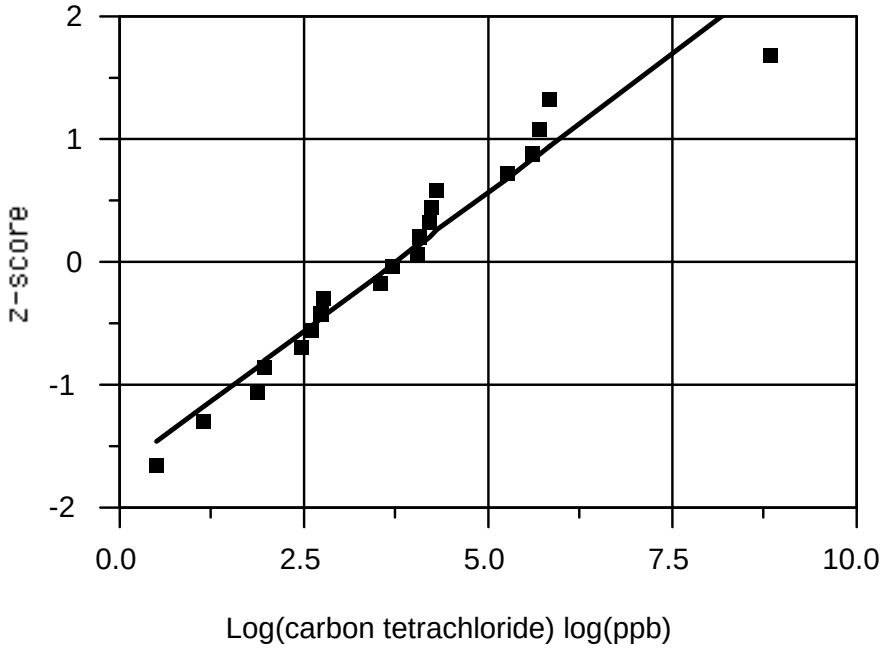


Figure 12-3. Outlier-Deleted Probability Plot on Original Scale (*r* = .854)

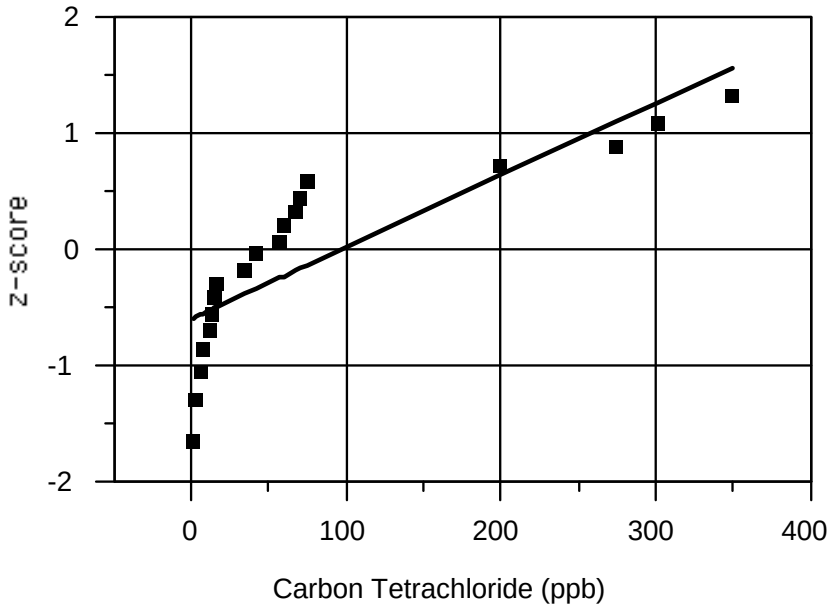
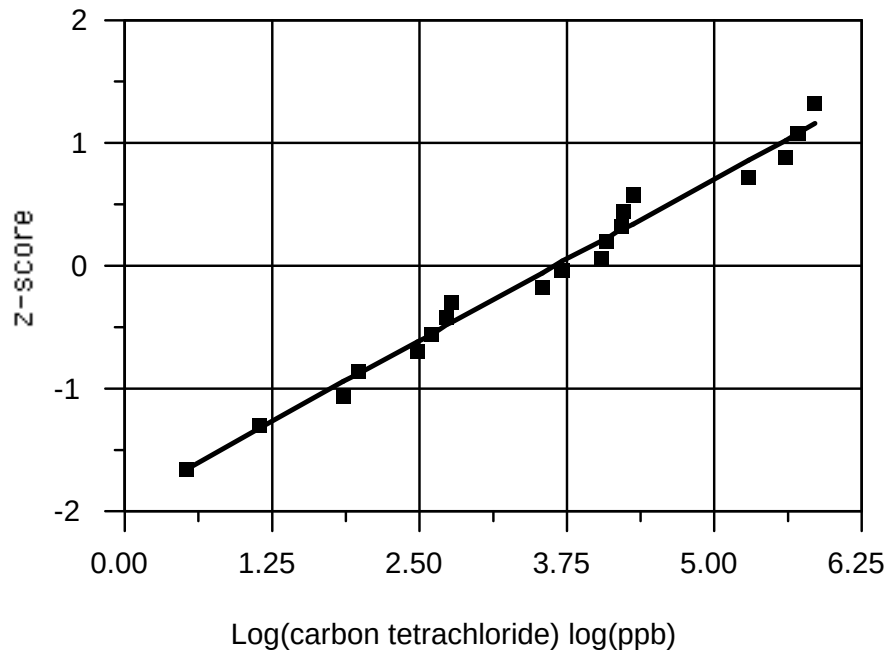


Figure 12-4. Outlier-Deleted Probability Plot on Logarithmic Scale (r = .985)



12.2 SCREENING WITH BOX PLOTS

BACKGROUND AND PURPOSE

Probability plots as described in **Section 12.1** require the remaining observations following removal of one or more suspected outliers to be either approximately normal or normalized via transformation. Box plots (**Chapter 9**) provide an alternate method to perform outlier screening, one not dependent on normality of the underlying measurement population. Instead of looking for points inconsistent with a linear pattern on a probability plot, the box plot flags as possible outliers values that are located in either or both of the ***extreme tails*** of the sample.

To define the extreme tails, Tukey (1977) proposed the concept of 'hinges' that would 'swing' off either end of a box plot, defining the range of concentrations consistent with the bulk of the data. Data points outside this concentration range could then be identified as potential outliers. Tukey defined the hinges, i.e., the lower and upper edges of the box plot, essentially as the lower and upper quartiles of the data set. Then multiples of the interquartile range [IQR] (***i.e.***, the range represented by the middle half of the sample) were added to or subtracted from these hinges as potential outlier boundaries. Any observation from 1.5 × IQR to 3 × IQR below the lower edge of the box plot was labeled a 'mild' low outlier; any value more than 3 × IQR below the lower edge of the box plot was labeled an 'extreme' low outlier. Similarly, values greater than the upper edge of the box plot in the range of 1.5 to 3 times the IQR were labeled 'mild' higher outliers, and 'extreme' high outliers if more than 3 times the IQR beyond the upper box plot edge.

REQUIREMENTS AND ASSUMPTIONS

By using hinges and multiples of the interquartile range, Tukey's box plot method utilizes statistics (***i.e.***, the lower and upper quartiles) that are generally not or minimally affected by one or a few outliers

in the sample. Consequently, it isn't necessary to first delete possible outliers before constructing the box plot.

Screening for outliers with box plots is a very simple technique. Since no assumption of normality is needed, Tukey's procedure can be considered quasi-non-parametric. But note that rough symmetry of the underlying distribution is implicitly assumed. Legitimate observations from highly skewed distributions could be flagged as potential outliers on a box plot if no transformation of the data is first attempted. It may be necessary to first conduct multiple data transformations in order to achieve approximate symmetry before applying and evaluating potential outliers with box plots.

PROCEDURE

Step 1. Construct a box plot on the sample using the method given in **Section 9.2**. Using the IQR from that calculation, along with the lower and upper quartiles ($\tilde{x}_{.25}$ and $\tilde{x}_{.75}$), compute the first pair of lower and upper boundaries as:

$$LB_1 = \tilde{x}_{.25} - 1.5 \times IQR \tag{12.1}$$

$$UB_1 = \tilde{x}_{.75} + 1.5 \times IQR \tag{12.2}$$

Step 2. Construct the second pair of lower and upper boundaries as:

$$LB_2 = \tilde{x}_{.25} - 3 \times IQR \tag{12.3}$$

$$UB_2 = \tilde{x}_{.75} + 3 \times IQR \tag{12.4}$$

Step 3. Label any sample measurement lower than the first lower boundary (LB_1) but no less than the second lower boundary (LB_2) as a mild low outlier. Label any measurement greater than the first upper boundary (UB_1) but no greater than the second upper boundary (UB_2) as a mild high outlier.

Step 4. Label any sample measurement lower than the second lower boundary (LB_2) as an extreme low outlier. Label any value higher than the second upper boundary (UB_2) as an extreme high outlier.

►EXAMPLE 12-2

Use the carbon tetrachloride data from **Example 12-1** to screen for possible outliers using Tukey's box plot.

SOLUTION

Step 1. Using the procedure described in **Section 9.2**, the upper and lower quartiles of carbon tetrachloride sample are found to be $\tilde{x}_{.25} = 12.9$ and $\tilde{x}_{.75} = 137.2$, leading to an IQR = 124.3.

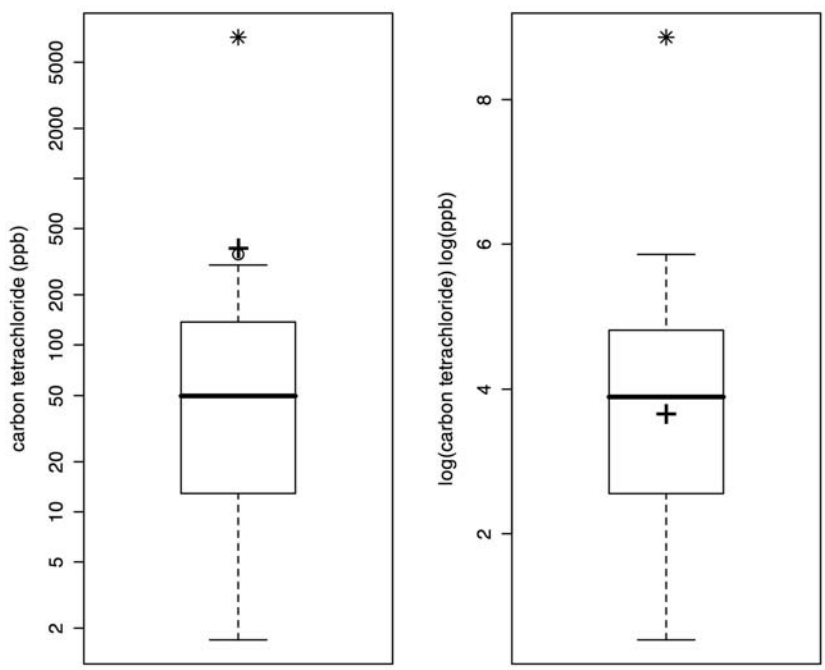
Step 2. Compute the two pairs of lower and upper boundaries using equations (12.1), (12.2), (12.3), and (12.4):

$$LB_1 = 12.9 - 1.5 \times 124.3 = -173.55$$
$$UB_1 = 137.2 + 1.5 \times 124.3 = 323.65$$
$$LB_2 = 12.9 - 3 \times 124.3 = -360$$
$$UB_2 = 137.2 + 3 \times 124.3 = 510.1$$

Step 3. Scan the list of carbon tetrachloride measurements and compare against the boundaries of **Step 2**. It can be seen that the value of 350 from Well 3 is greater than UB_1 but lower than UB_2, thus qualifying as a mild high outlier. Also, the measurement 7,066 from the same well is higher than UB_2 and so qualifies as an extreme high outlier.

Step 4. Because the box plot outlier screening method assumes roughly symmetric data, recompute the box plot on the log-transformed measurements (as shown in **Figure 12-5** alongside a similar box plot of the raw concentrations). Transforming the sample to the log-scale does result in much greater symmetry compared to the original measurement scale. This can be seen in the close similarity between the mean and median on the log-scale box plot. With a more symmetric data set, the mild high outlier from Step 3 disappears, but the extreme high value is still classified as an outlier. ◄

Figure 12-5. Comparative Carbon Tetrachloride Box Plots Indicating Outliers



12.3 DIXON'S TEST

BACKGROUND AND PURPOSE

Dixon's test is helpful for documenting statistical outliers in smaller data sets (*i.e.*, $n \leq 25$). The test is particularly designed for cases where there is only a single high or low outlier, although it can also be adapted to test for multiple outliers. The test falls in the general class of tests for *discordancy* (Barnett and Lewis, 1994). The test statistic for such procedures is generally a ratio: the numerator is the difference between the suspected outlier and some summary statistic of the data set, while the denominator is always a measure of spread within the data. In this version of Dixon's test, the summary statistic in the numerator is an order statistic nearby to the potential outlier (*e.g.*, the second or third most extreme value). The measure of spread is essentially the observed sample range.

If there is more than one outlier in the data set, Dixon's test can be vulnerable to *masking*, at least for very small samples. Masking in the statistical literature refers to the problem of an extreme outlier being missed because one or more additional extreme outliers are also present. For instance, if the data consist of the values {2, 4, 10, 12, 15, 18, 19, 22, 200, 202}, identification of the maximum value (202) as an outlier might fail since the maximum by itself is not extreme with respect to the next highest value (200). However, both of these values are clearly much higher than the rest of the data set and might jointly be considered outliers.

If more than one outlier is suspected, the user is encouraged to consider Rosner's test (**Section 12.4**) as an alternative to Dixon's test, at least if the sample size is 20 or more. If the data set is smaller, Dixon's test should be modified so that the *least extreme* of the suspected outliers is tested first. This will help avoid the risk of masking. The same equations given below can be used, but the data set and sample size should be temporarily reduced to exclude any suspected outliers that are *more* extreme than the one being tested. If a less extreme value is found to be an outlier, then that observation and any more extreme values can also be regarded as outliers. Otherwise, add back the next most extreme value and test it in the same way.

REQUIREMENTS AND ASSUMPTIONS

Dixon's test is only recommended for sample sizes $n \leq 25$. It assumes that the data set (minus the suspected outlier) is normally-distributed. This assumption should be checked prior to running Dixon's test using a goodness-of-fit technique such as the probability plots described in **Section 12.2**.

PROCEDURE

Step 1. Order the data set and label the ordered values, $x_{(i)}$.

Step 2. If a "low" outlier is suspected (*i.e.*, $x_{(1)}$), compute the test statistic C using the appropriate equation [12.5] depending on the sample size (n):

$$C = \begin{cases} \left(x_{(2)} - x_{(1)}\right) \Big/ \left(x_{(n)} - x_{(1)}\right) & \text{for } 3 \le n \le 7 \\ \left(x_{(2)} - x_{(1)}\right) \Big/ \left(x_{(n-1)} - x_{(1)}\right) & \text{for } 8 \le n \le 10 \\ \left(x_{(3)} - x_{(1)}\right) \Big/ \left(x_{(n-1)} - x_{(1)}\right) & \text{for } 11 \le n \le 13 \\ \left(x_{(3)} - x_{(1)}\right) \Big/ \left(x_{(n-2)} - x_{(1)}\right) & \text{for } 14 \le n \le 25 \end{cases} \qquad [12.5]$$

Step 3. If a "high" outlier is suspected (*i.e.*, $x_{(n)}$), and again depending on the sample size (n), compute the test statistic C using the appropriate equation [12.6] as:

$$C = \begin{cases} \left(x_{(n)} - x_{(n-1)}\right) \Big/ \left(x_{(n)} - x_{(1)}\right) & \text{for } 3 \le n \le 7 \\ \left(x_{(n)} - x_{(n-1)}\right) \Big/ \left(x_{(n)} - x_{(2)}\right) & \text{for } 8 \le n \le 10 \\ \left(x_{(n)} - x_{(n-2)}\right) \Big/ \left(x_{(n)} - x_{(2)}\right) & \text{for } 11 \le n \le 13 \\ \left(x_{(n)} - x_{(n-2)}\right) \Big/ \left(x_{(n)} - x_{(3)}\right) & \text{for } 14 \le n \le 25 \end{cases} \qquad [12.6]$$

Step 4. In either case, given the significance level (α), determine a critical point for Dixon's test with n observations from **Table 12-1** in **Appendix D**. If C exceeds this critical point, the suspected value should be declared a statistical outlier and investigated further (see discussion in **Chapter 6**).

►EXAMPLE 12-3

Use the data from **Example 12-1** in Dixon's test to determine if the anomalous high value is a statistical outlier at an $\alpha = 0.05$ level of significance.

SOLUTION

Step 1. In **Example 12-1**, probability plots of the carbon tetrachloride data indicated that the highest value might be an outlier, but that the distribution of the measurements was more nearly lognormal than normal. Since the sample size $n = 20$, Dixon's test can be used on the logged observations. Logging the values and ordering them leads to the following table:

Order	Concentration (ppb)	Logged Concentration
1	1.7	0.531
2	3.2	1.163
3	6.5	1.872
4	7.3	1.988
5	12.1	2.493
6	13.7	2.617
7	15.6	2.747
8	16.2	2.785
9	35.1	3.558
10	41.6	3.728
11	57.9	4.059
12	59.7	4.089
13	68.4	4.225
14	70.1	4.250
15	75.4	4.323
16	199.0	5.293
17	275.0	5.617
18	302.0	5.710
19	350.0	5.878
20	7066.0	8.863

Step 2. Because a high outlier is suspected and n = 20, use the last option of equation [12.6] to calculate the test statistic C:

$$C = \frac{8.863 - 5.710}{8.863 - 1.872} = 0.451$$

Step 3. With n = 20 and α = .05, the critical point from **Table 12-1** in **Appendix D** is equal to 0.450. Since the test statistic C exceeds this critical point, the extreme high value can be declared a statistical outlier. Before excluding this value from further analysis, however, a valid explanation for this unusually high value should be sought. Otherwise, the outlier may need to be treated as an extreme but valid concentration measurement. ◄

12.4 ROSNER'S TEST

BACKGROUND AND PURPOSE

Rosner's test (Rosner, 1975) is a useful method for identifying multiple outliers in moderate to large-sized data sets. The approach developed in Rosner's method is known as a *block*-style test. Instead of testing for outliers one-by-one in a consecutive manner from most extreme to least extreme (*i.e.*, most to least suspicious), the data are examined first to identify the total number of possible outliers, k. Once k is determined, the set of possible outliers is tested together as a block. If the test is significant, all k measurements are regarded as statistical outliers. If not, the set of possible outliers is reduced by one and the test repeated on the smaller block. This procedure is iterated until either a set of outliers is identified

or none of the observations are labeled an outlier. By testing outliers in blocks instead of one-by-one, Rosner's test largely avoids the problem of *masking* of one outlier by another (as discussed in **Section 12.3** regarding Dixon's test).

Although Rosner's test avoids the problem of masking when multiple outliers are present in the same data set, it is not immune to the related problem of *swamping*. A good discussion is found in Barnett and Lewis, 1994, Outliers in Statistical Data (3rd Edition), p. 236. Swamping refers to a block of measurements all being *labeled* as outliers even though only *some* of the observations are actually outliers. This can occur with Rosner's test especially if all the outliers tend to be at one end of the data set (*e.g.*, as upper extremes). The difficulty is in properly identifying the total number of possible outliers (k), which can be low outliers, high outliers, or some combination of the two extremes. If k is made too large, swamping may occur. Again, the user is reminded to always do a preliminary screening for outliers via box plots (**Section 12.2**) and probability plots (**Section 12.1**).

REQUIREMENTS AND ASSUMPTIONS

Rosner's test is recommended when the sample size (n) is 20 or larger. The critical points provided in **Table 12-2** in **Appendix D** can be used to identify from 2 to 5 outliers in a given data set. Like Dixon's test, Rosner's method assumes the underlying data set (minus any outliers) is normally distributed. If a probability plot of the data exhibits significant bends or curves, the data should first be transformed (*e.g.*, via a logarithm) and then re-plotted. The formal test for outliers should only be performed on (outlier-deleted) data sets that have been approximately normalized.

A potential drawback of Rosner's test is that the user must first identify the maximum number of potential outliers (k) prior to running the test. Therefore, this requirement makes the test ill-advised as an automatic outlier screening tool, and somewhat reliant on the user to identify candidate outliers.

PROCEDURE

Step 1. Order the data set and denote the ordered values $x_{(i)}$. Then by simple inspection, identify the maximum number of possible outliers, r_0.

Step 2. Compute the sample mean and standard deviation of all the data; denote these values by $\bar{x}^{(0)}$ and $s^{(0)}$. Then determine the measurement furthest from $\bar{x}^{(0)}$ and denote it $y^{(0)}$. Note that $y^{(0)}$ could be either a potentially low or a high outlier.

Step 3. Delete $y^{(0)}$ from the data set and compute the sample mean and standard deviation from the remaining observations. Label these new values $\bar{x}^{(1)}$ and $s^{(1)}$. Again find the value in this reduced data set furthest from $\bar{x}^{(1)}$ and label it $y^{(1)}$.

Step 4. Delete $y^{(1)}$, recompute the mean and standard deviation, and continue this process until all r_0 potential outliers have been removed. At this point, the following set of statistics will be available:

$$\left[\bar{x}^{(0)}, s^{(0)}, y^{(0)}\right], \left[\bar{x}^{(1)}, s^{(1)}, y^{(1)}\right], \ldots, \left[\bar{x}^{(r_0-1)}, s^{(r_0-1)}, y^{(r_0-1)}\right] \qquad [12.7]$$

Step 5. Now test for r outliers (where $r \le r_0$) by iteratively computing the test statistic:

$$R_{r-1} = \left| y^{(r-1)} - \bar{x}^{(r-1)} \right| \Big/ s^{(r-1)} \qquad [12.8]$$

First test for r_0 outliers. If the test statistic R_{r_0-1} in equation [12.8] exceeds the first critical point from **Table 12-2** in **Appendix D** based on sample size (n) and the Type I error (α), conclude there are r_0 outliers. If not, test for r_0–1 outliers in the same fashion using the next critical point, continuing until a certain number of outliers have either been identified or Rosner's test finds no outliers at all.

►EXAMPLE 12-4

Consider the following series of 25 background napthalene measurements (in ppb). Use Rosner's test to determine whether any of the values should be deemed statistical outliers.

	Naphthalene Concentrations (ppb)				
Qtr	BW-1	BW-2	BW-3	BW-4	BW-5
1	3.34	5.59	1.91	6.12	8.64
2	5.39	5.96	1.74	6.05	5.34
3	5.74	1.47	23.23	5.18	5.53
4	6.88	2.57	1.82	4.43	4.42
5	5.85	5.39	2.02	1.00	35.45

SOLUTION

Step 1. Screening with probability plots of the combined data indicates a less than linear fit with both the raw measurements and log-transformed data (see **Figures 12-6 and 12-7**); two points appear rather discrepant from the rest. Correlation coefficients for these plots are 0.740 on the concentration scale and 0.951 on the log-scale. Re-plotting after removing the two possible outliers gives a substantially improved correlation on the concentration scale of 0.958 but reduces the log-scale correlation to 0.929. Normality appears to be a slightly better default distribution for the outlier-deleted data set. Run Rosner's test on the original data with k = 2 possible outliers.

Step 2. Compute the mean and standard deviation of the complete data set. Then identify the observation farthest from the mean. These results are listed, along with the ordered data, in the table below. After removing the farthest value (35.45), recompute the mean and standard deviation on the remaining values and again identify the most discrepant observation (23.23). Repeat this process one more time so that both suspected outliers have been removed (see table below).

Step 3. Now test for 2 joint outliers by computing Rosner's statistic on subset $SS_{k-1} = SS_1$ using equation [12.8]:

$$R_1 = \frac{23.23 - 5.23}{4.326} = 4.16$$

Figure 12-6. Napthalene Probability Plot

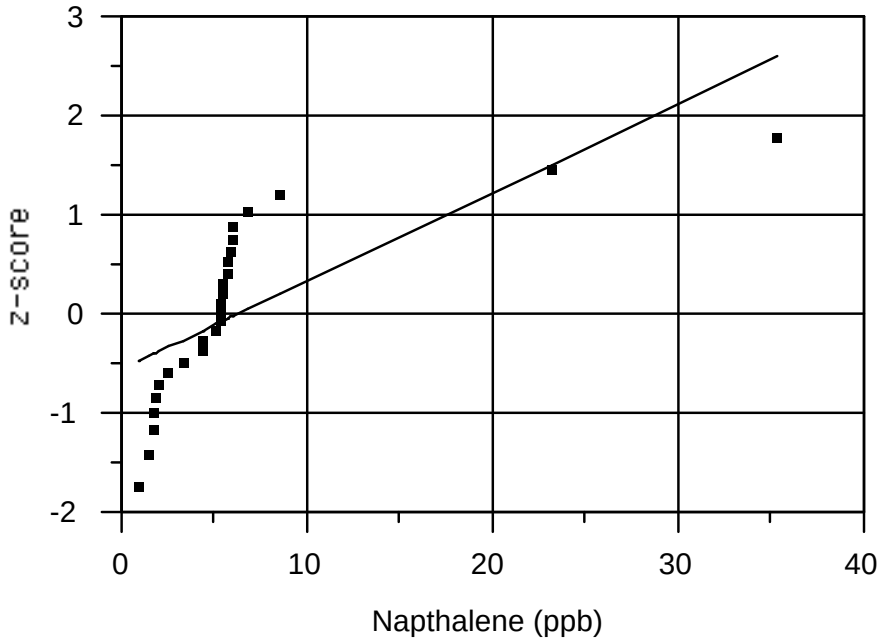
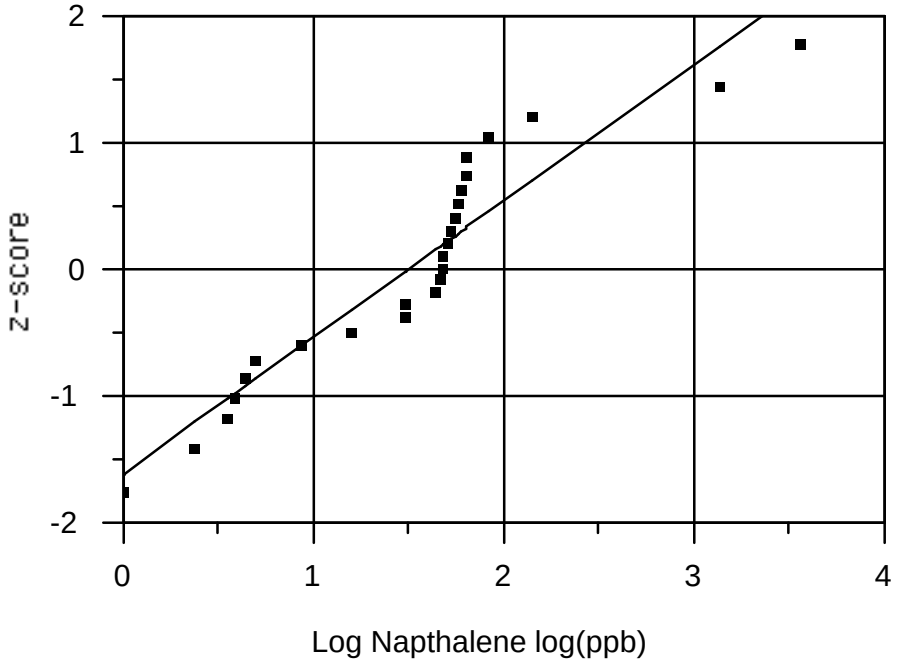


Figure 12-7. Log Napthalene Probability Plot



Successive Naphthalene Subsets (SS_i)		
SS_0	SS_1	SS_2
1.00	1.00	1.00
1.47	1.47	1.47
1.74	1.74	1.74
1.82	1.82	1.82
1.91	1.91	1.91
2.02	2.02	2.02
2.57	2.57	2.57
3.34	3.34	3.34
4.42	4.42	4.42
4.43	4.43	4.43
5.18	5.18	5.18
5.34	5.34	5.34
5.39	5.39	5.39
5.39	5.39	5.39
5.53	5.53	5.53
5.59	5.59	5.59
5.74	5.74	5.74
5.85	5.85	5.85
5.96	5.96	5.96
6.05	6.05	6.05
6.12	6.12	6.12
6.88	6.88	6.88
8.64	8.64	8.64
23.23	23.23	
35.45		
$\bar{x}_0$ = 6.44	$\bar{x}_1$ = 5.23	$\bar{x}_2$ = 4.45
s_0 = 7.379	s_1 = 4.326	s_2 = 2.050
y_0 = 35.45	y_1 = 23.23	y_2 = 8.64

Step 4. Given $\alpha = 0.05$, a sample size of $n = 25$, and $k = 2$, the first critical point in **Table 12-2** in **Appendix D** equals 2.83 for $n = 20$ and 3.05 for $n = 30$. The value R_1 in Step 3 is larger than either of these critical points, so both suspected values may be declared statistical outliers by Rosner's test at the 5% significance level. Before excluding these values from further analysis, however, a valid explanation for them should be found. Otherwise, treat the outliers as extreme but valid concentration measurements.

Note: had R_1 been *less* than these values, a test could still be run for a *single* outlier using the second critical point for each sample size (or a critical point interpolated between them). ◄

The guidance considers Dixon's and Rosner's outlier evaluation methods preferable for groundwater monitoring data situations, when assumptions of normality are reasonable and data are quantified. We did not include the older method found in the 1989 guidance based on ASTM paper E178-75, which can still be used as an alternative. Where data do not appear to be fit by a normal or transformably normal distribution, other robust outlier evaluation methods can be considered from the wider statistical literature. The literature will also need to be consulted when data contains non-detect values along with potential outliers.

CHAPTER 13. SPATIAL VARIABILITY

13.1 INTRODUCTION TO SPATIAL VARIATION 13-1
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This chapter discusses a type of ***statistical dependence*** in groundwater monitoring data known as ***spatial variability***. Spatial variability exists when the distribution or pattern of concentration measurements changes from well location to well location (most typically in the form of differing mean levels). Such variation may be natural or ***synthetic***, depending on whether it is caused by natural or anthropogenic factors. Methods for ***identifying*** spatial variation are detailed via the use of box plots (**Section 13.2.1**) and analysis of variance [ANOVA] (**Section 13.2.2**). Once identified, ANOVA can sometimes be employed to construct more powerful intrawell background limits. This topic is addressed in **Section 13.3**.

13.1 INTRODUCTION TO SPATIAL VARIATION

Spatial dependence, ***spatial variation*** or ***variability***, and ***spatial correlation*** are closely related concepts. All refer to the notion of measurement levels that vary in a structured way as a function of the ***location*** of sampling. Although spatial variation can apply to any statistical characteristic of the underlying population (including the population variance or upper percentiles), the usual sense in groundwater monitoring is that ***mean*** levels of a given constituent vary from one well to the next.

Standard geostatistical models posit that an area exhibits positive spatial correlation if any two sampling locations share a greater similarity in concentration level the closer the distance between them, and more dissimilarity the further apart they are. Such models have been applied to both groundwater and soil sampling problems, but are not applicable in all geological configurations. It may be, for instance, that mean concentration levels differ across wells but vary in a seemingly random way with no apparent connection to the distance between the sampling points. In that case, the concentrations between pairs of wells are not correlated with distance, yet the measurements within each well are strongly associated with the mean level at that particular location, whether due to a change in soil composition, hydrological characteristics or some other factor. In other words, spatial ***variation*** may exist even when spatial ***correlation*** does not.

Spatial variation is important in the guidance context since substantial differences in mean concentration levels between different wells can ***invalidate*** interwell, upgradient-to-downgradient comparisons and point instead toward ***intrawell*** tests (**Chapter 6**). Not all spatial variability is natural. Average concentration levels can vary from well to well for a variety of reasons.

In this guidance, a distinction is occasionally made between ***natural*** versus ***synthetic*** spatial variation. Natural spatial variability refers to a pattern of changing mean levels in groundwater associated with normal geochemical behavior unaffected by human activities. Natural spatial variability

is not an indication of groundwater contamination, even if concentrations at one or more compliance wells exceed (upgradient) background. In contrast, synthetic spatial variability is related to human activity. Sources can include recent releases affecting compliance wells, migration of contaminants from off-site sources, or historic contamination at certain wells due to past industrial activity or pre-RCRA waste disposal. Whether natural or synthetic, techniques and test methods for dealing with spatial variation will still be identical from a purely statistical standpoint. It is interpreting the testing outcomes which will necessitate a consideration of why the spatial variation occurs.

The goal of groundwater analysis is not simply to identify significant concentration differences among monitoring wells at compliance point locations. It is also to determine *why* those differences exist. Especially with prior groundwater contamination, regulatory decisions outside the scope of this guidance need to address the problem. In some cases, compliance/assessment monitoring or remedial action may be warranted. In other cases, chronic contamination from offsite sources may simply have to be considered as the current background condition at a given location. At least the ability to attribute certain mean differences to *natural* spatial variation allows the range of potential concerns to be somewhat narrowed. Of course, deciding that an observed pattern of spatial variation is natural and not synthetic may not be easy. Ultimately, expert judgment and knowledge concerning site hydrology, geology and geochemistry are important in providing more definitive answers.

One statistical approach to use when a site has multiple, non-impacted background wells is to conduct a one-way ANOVA for inorganic constituents on those wells. Substantial differences among the mean levels at a set of *uncontaminated* sampling locations are suggestive of natural spatial variability. At a true 'greenfield' site, ANOVA can be run on all the wells — both background and compliance — after a few preliminary sampling rounds have been collected.

The Unified Guidance offers two basic tools to explore and test for spatial correlation. The first, side-by-side box plots (**Section 13.2.1**), provides a quick screen for possible spatial variation. When multiple well data are plotted on the same concentration axis, noticeably staggered boxes are often an indication of significantly different mean levels.

A more formal test of spatial variation is the one-way ANOVA (**Section 13.3.2**). When significant spatial variation exists and an intrawell test strategy is pursued, one-way ANOVA can also be used to adjust the standard deviation estimate used in forming intrawell prediction and control chart limits, and to increase the effective sample size of the test, via the *degrees of freedom*. This is discussed in **Section 13.3**.

13.2 IDENTIFYING SPATIAL VARIABILITY

13.2.1 SIDE-BY-SIDE BOX PLOTS

BACKGROUND AND PURPOSE

Box plots for graphing side-by-side statistical summaries of multiple wells were introduced in **Chapter 9**. They are also discussed in **Chapter 11** as an initial screen for differences in population variances and as a tool to check the assumption of equal variances in ANOVA. They can further be employed to screen for possible spatial variation in mean levels. While variability in a sample from a given well is roughly indicated by the length of the box, the average concentration level is indicated by the position of the box relative to the concentration axis. Many standard box plot software routines

display both the sample median value and the sample mean on each box, so these values may be compared from well to well. A high degree of staggering in the box positions is then indicative of potentially significant spatial variation.

Since side-by-side box plots provide a picture of the variability at each well, the extent to which apparent differences in mean levels seem to be real rather than chance fluctuations can be examined. If the boxes are staggered but there is substantial overlap between them, the degree of spatial variability may not be significant. A more formal ANOVA might still be warranted as a follow-up test, but side-by-side box plots will offer a initial sense of how spatially variable the groundwater data appear.

REQUIREMENTS, ASSUMPTIONS AND PROCEDURE

Requirements, assumptions and the procedure for box plots are outlined in **Chapter 9, Section 9.2**.

►EXAMPLE 13-1

Quarterly dissolved iron concentrations measured at each of six upgradient wells are listed below. Construct side-by-side box plots to initially screen for the presence of spatial variability.

	Iron Concentrations (ppm)					
Date	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
Jan 1997	57.97	46.06	100.48	34.12	60.95	83.10
Apr 1997	54.05	76.71	170.72	93.69	72.97	183.09
Jul 1997	29.96	32.14	39.25	70.81	244.69	198.34
Oct 1997	46.06	68.03	52.98	83.10	202.35	160.77
Mean	47.01	55.71	90.86	70.43	145.24	156.32
Median	55.06	57.04	76.73	76.96	137.66	171.93

SOLUTION

Step 1. Determine the median, mean, lower and upper quartiles of each well. Then plot these against a concentration axis to form side-by-by side box plots (**Figure 13-1**) using the procedure in **Section 9.2**. .

Step 2. From this plot, the means and medians at the last two wells (Wells 5 and 6) appear elevated above the rest. This is a possible indication of spatial variation. However, the variances as represented by the box lengths also appear to differ, with the highest means associated with the largest boxes. A transformation of the data should be attempted and the data re-plotted. Spatial variability is only a significant problem if it is apparent on the scale of the data actually used for statistical analysis.

Step 3. Take the logarithm of each measurement as in the table below. Recompute the mean, median, lower and upper quartiles, and then re-construct the box plot as in **Figure 13-2**.

Date	Log Iron Concentrations log(ppm)					
	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
Jan 1997	4.06	3.83	4.61	3.53	4.11	4.42
Apr 1997	3.99	4.34	5.14	4.54	4.29	5.21
Jul 1997	3.40	3.47	3.67	4.26	5.50	5.29
Oct 1997	3.83	4.22	3.97	4.42	5.31	5.08
Mean	3.82	3.96	4.35	4.19	4.80	5.00
Median	3.70	4.02	4.29	4.34	4.80	5.14

Figure 13-1. Side-by-Side Iron Box Plots

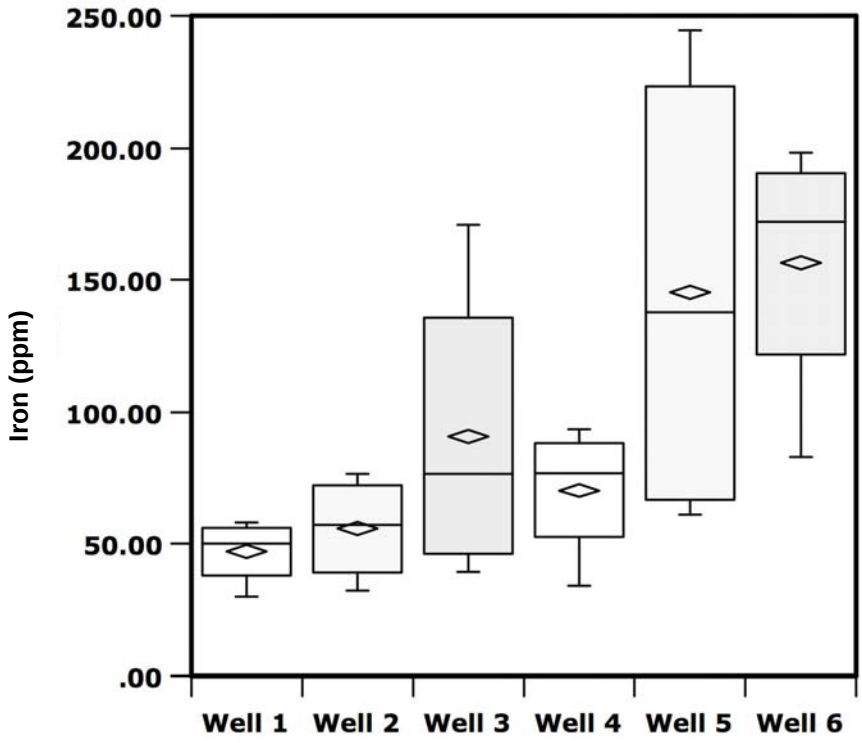
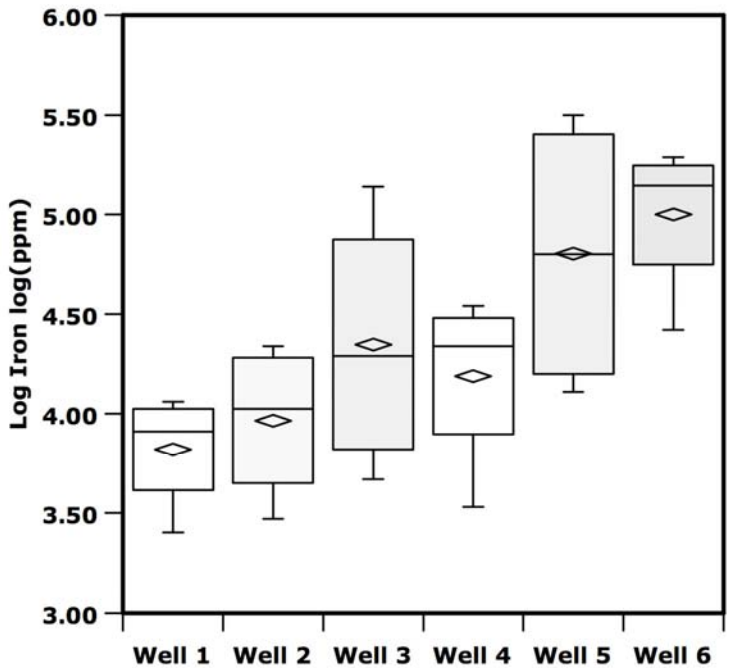


Figure 13-2. Side-by-Side Log(Iron) Box Plots



Step 4. While more nearly similar on the log-scale, the means and medians are still elevated in Wells 5 and 6. Since the differences in box lengths are much less on the log-scale, the log transformation has worked to somewhat stabilize the variances. These data should be tested formally for significant spatial variation using an ANOVA, probably on the log-scale. ◄

13.2.2 ONE-WAY ANALYSIS OF VARIANCE FOR SPATIAL VARIABILITY

PURPOSE AND BACKGROUND

Chapter 17 presents Analysis of Variance [ANOVA] in greater detail. When using ANOVA to check for spatial variability, the observations from each well are taken as a single group. Significant differences between data groups represent monitoring wells with different mean concentration levels. The lack of significant well mean differences may afford an opportunity to pool the data for larger background sizes and conduct interwell detection monitoring tests.

ANOVA used for this purpose should be performed either on a set of multiple non-impacted upgradient wells, or on historically uncontaminated compliance and upgradient background wells. If significant mean differences exist among naturally occurring constituent data at upgradient wells, natural spatial variability is the likely reason. Synthetic consitituents in upgradient wells might also exhibit spatial differences if affected by an offsite- plume. Presumably, if the flow gradient has been correctly

assessed and no migration of contaminants from off-site has occurred, differences in mean levels across upgradient wells ought to signal the influence of factors not attributable to a monitored release. A similar, but potentially weaker, argument can be made if spatial differences exist between uncontaminated historical data at compliance wells. The lack of spatial differences between uncontaminated compliance and upgradient background well data, may again allow for even larger background sample sizes.

REQUIREMENTS AND ASSUMPTIONS

The basic assumptions and data requirements for one-way ANOVA are presented in **Section 17.1**. If the assumption that the observations are statistically independent over time is not met, both identifying spatial variability using ANOVA as well as improving intrawell prediction limits and control charts can be impacted. It is usually difficult to verify that the measurements are temporally independent with only a limited number of observations per well. This potential problem can be somewhat minimized by collecting samples far enough apart in time to guard against autocorrelation. Another option is to construct a parallel time series plot (**Chapter 14**) to look for time-related effects or dependencies occurring simultaneously across the set of wells.

If a significant temporal dependence or autocorrelation exists, the one-way ANOVA can still identify well-to-well mean level differences. But the power of the test to do so is lessened. If a parallel time series plot indicates a potentially strong time-related effect, a two-way ANOVA including temporal effects can be performed to test and correct for a significant temporal factor. This slightly more complicated procedure is discussed in Davis (1994).

Another key assumption of parametric ANOVA is that the residuals are normal or can be normalized. If a normalizing transformation cannot be found, a test for spatial variability can be made using the Kruskal-Wallis non-parametric ANOVA (**Chapter 17**). As long as the measurements can be ranked, average ranks that differ significantly across wells provide evidence of spatial variation.

PROCEDURE

Step 1. Assuming there are p distinct wells to test, designate the measurements from each well as a separate group for purposes of computing the ANOVA. Then follow **Steps 1** through **7** of the procedure in **Section 17.1.1** to compute the overall F-statistic and the quantities of the ANOVA table in **Figure 13-3** below.

Figure 13-3. One-Way Parametric ANOVA Table

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	*F*-Statistic
Between Wells	SS_{wells}	$p-1$	$MS_{wells} = SS_{wells}/(p-1)$	$F = MS_{wells}/MS_{error}$
Error (within wells)	SS_{error}	$n-p$	$MS_{error} = SS_{error}/(n-p)$	
Total	SS_{total}	$n-1$		

Step 2. To test the hypothesis of equal means for all p wells, compare the F-statistic from Step 1 to the α-level critical point found from the F-distribution with (p–1) and (n–p) degrees of freedom in **Table 17-1** of **Appendix D**. Usually α is taken to be 5%, so that the needed comparison value equals the upper 95th percentage point of the F-distribution. If the observed F-statistic exceeds the critical point ($F_{.95,p-1,n-p}$), reject the hypothesis of equal well population means and conclude there is significant spatial variability. Otherwise, the evidence is insufficient to conclude there are significant differences between the means at the p wells.

►EXAMPLE 13-2

The iron concentrations in **Example 13-1** show evidence of spatial variability in side-by-side box plots. Tested for equal variances and normality, these same data are best fit by a lognormal distribution. The statistics for natural logarithms of the iron measurements are shown below; individual log data are provided in the **Example 13-1** second table. Compute a one-way parametric ANOVA to determine whether there is significant spatial variation at the α = .05 significance level.

	Log Iron Concentration Statistics log(ppm)					
Date	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
N	4	4	4	4	4	4
Mean	3.820	3.965	4.348	4.188	4.802	5.000
SD	0.296	0.395	0.658	0.453	0.704	0.396
			Grand Mean = 4.354			

SOLUTION

Step 1. With 6 wells and 4 observations per well, n_i = 4 for all the wells. The total sample size is n = 24 and p = 6. Compute the (overall) grand mean and the sample mean concentrations in each of the well groups using equations [17.1] and [17.2]. These values are listed (along with each group's standard deviation) in the above table.

Step 2. Compute the sum of squares due to well-to-well differences using equation [17.3]:

$$SS_{wells} = \left[4(3.820)^2 + 4(3.965)^2 + \ \ldots \ + 4(5.000)^2\right] - 24(4.354)^2 = 4.294$$

This quantity has (6 – 1) = 5 degrees of freedom.

Step 3. Compute the corrected total sum of squares using equation [17.4] with (n – 1) = 23 df:

$$SS_{total} = \left[(4.06)^2 + \ \ldots \ + (5.08)^2\right] - 24(4.354)^2 = 8.934$$

Step 4. Obtain the within-well or error sum of squares by subtraction using equation [17.5]:

$$SS_{error} = 8.934 - 4.294 = 4.640$$

This quantity has (n – p) = 24–6 = 18 degrees of freedom.

Step 5. Compute the well and error mean sum of squares using equations [17.6] and [17.7]:

$$MS_{wells} = 4.294/5 = 0.859$$

$$MS_{error} = 4.640/18 = 0.258$$

Step 6. Construct the *F*-statistic and the one-way ANOVA table, using **Figure 13-3** as a guide:

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	*F*-Statistic
Between Wells	4.294	5	0.859	*F* = 0.859/0.258=3.33
Error (within wells)	4.640	18	0.258	
Total	8.934	23		

Step 7. Compare the observed *F*-statistic of 3.33 against the critical point taken as the upper 95th percentage point from the *F*-distribution with 5 and 18 degrees of freedom. Using **Table 17-1** of **Appendix D**, this gives a value of $F_{.95,5,18}$ = 2.77. Since the *F*-statistic exceeds the critical point, the null hypothesis of equal well means can be rejected, suggesting the presence of significant spatial variation. ◄

13.3 USING ANOVA TO IMPROVE PARAMETRIC INTRAWELL TESTS

BACKGROUND AND PURPOSE

Constituents that exhibit significant spatial variability usually should be formally tested with intrawell procedures such as a prediction limit or control chart. Historical data from each compliance well are used as background for these tests instead of from upgradient wells. At an early stage of intrawell testing, there may only be a few measurements per well which can be designated as background. Depending on the number of statistical tests that need to be performed across the monitoring network, available intrawell background at individual compliance wells may not provide sufficient statistical power or meet the false positive rate criteria (**Chapter 19**).

One remedy first suggested by Davis (1998) can increase the *degrees of freedom* of the test by using one-way ANOVA results (**Section 13.2**) from a number of wells to provide an alternate estimate of the average intrawell variance. In constructing a parametric intrawell prediction limit for a single compliance well, the intrawell background of sample size *n* is used to compute a well-specific sample mean ($\bar{x}$). The intrawell standard deviation (*s*) is replaced by the root mean squared error [RMSE] component from an ANOVA of the intrawell background associated with a series of compliance and/or background wells.[1] This raises the degrees of freedom from (*n*–1) to (*N*–*p*), where *N* is the total sample size across the group of wells input to the ANOVA and *p* is the number of distinct wells.

[1] RMSE is another name for the square root of the mean error sum of squares (MS_{error}) in the ANOVA table of **Figure 13-3**.

As an example of the difference this adjustment can make, consider a site with 6 upgradient wells and 15 compliance wells. Assuming n = 6 observations per well that have been collected over the last year, a total of 36 potential background measurements are available to construct an interwell test. If there is significant natural spatial variation in the mean levels from well to well, an interwell test is probably not appropriate. Switching to an intrawell method is the next best solution, but with only six observations per compliance well, either the power of an intrawell test to identify contaminated groundwater is likely to be quite low (even with retesting) or the site-wide false positive rate [SWFPR] will exceed the recommended target.

If the six upgradient wells were tested for spatial variability using a one-way ANOVA (presuming that the equal variance assumption is met), the degrees of freedom [*df*] associated with the mean error sum of squares term is (6 wells × 5 *df* per well) = 30 *df* (see **Section 13.2**). Thus by substituting the RMSE in place of each compliance well's intrawell standard deviation (s), the degrees of freedom for the modified intrawell prediction or control chart limit is 30 instead of 5.

ANOVA can be usefully employed in this manner since the RMSE is very close to being a weighted average of the individual well sample standard deviations. As such, it can be considered a measure of average within-well variability across the wells input to the ANOVA. Substituting the RMSE for s at an individual well consequently provides a better estimate of the typical within-well variation, since the RMSE is based on levels of fluctuation averaged across several wells. In addition, the number of observations used to construct the RMSE is much greater than the n values used to compute the intrawell sample standard deviation (s). Since both statistical measures are estimates of within-well variation, the RMSE with its larger degrees of freedom is generally a superior estimate if certain assumptions are met.

REQUIREMENTS AND ASSUMPTIONS

Using ANOVA to bolster parametric intrawell prediction or control chart limits will not work at every site or for every constituent. Replacement of the well-specific, intrawell sample standard deviation (s) by the RMSE from ANOVA assumes that the true within-well variability is *approximately the same* at all the wells for which an intrawell background limit (*i.e.*, prediction or control chart) will be constructed, and not just those wells tested in the ANOVA procedure. This last assumption can be difficult to verify if the ANOVA includes only background or upgradient wells. But to the extent that uncontaminated intrawell background measurements from compliance point wells can be included, the ANOVA should be run on all or a substantial fraction of the site's wells (excluding those which might already be contaminated). Whatever mix of upgradient and downgradient wells are included in the ANOVA, the purpose of the procedure is *not* to identify groundwater contamination, but rather to compute a better and more powerful estimate of the average intrawell standard deviation.

For the ANOVA to be valid and the RMSE to be a reasonable estimate of average within-well variability, a formal check of the equal variance assumption should be conducted using **Chapter 11** methods. A spatially variable constituent will often exhibit well-specific standard deviations that increase with the well-specific mean concentration. Equalizing the variances in these cases will require a data transformation, with an ANOVA conducted on the transformed data. Ultimately, any transformation applied to the wells in the ANOVA *also* need to be applied to intrawell background before computing intrawell prediction or control chart limits. The *same transformation* has to be appropriate for *both* sets

of data (*i.e.*, wells included in ANOVA *and* intrawell background at wells for which background limits are desired).

Even when the ANOVA procedure described in this section is utilized, the resulting intrawell limits should also be designed to incorporate retesting. When intrawell background is employed to estimate both a well-specific background mean ($\bar{x}$) and well-specific standard deviation (s), the **Appendix D** tables associated with **Chapters 19** and **20** can be used to look up the intrawell sample size (n) and number of wells (w) in the network in order to find a prediction or control chart multiplier that meets the targeted SWFPR and has acceptable statistical power. However, these tables implicitly assume that the degrees of freedom [df] associated with the test is equal to (n–1). The ANOVA method of this section results in a much larger df, and more importantly, in a df that does not 'match' the intrawell sample size (n).

Consequently, the parametric multipliers in the **Appendix D** tables cannot be directly used when constructing prediction or control chart limits with retesting. Instead, a multiplier must be computed for the specific combination of n and df computed as a result of the ANOVA. Tabulating all such possibilities would be prohibitive. For prediction limits, the Unified Guidance recommends the free-of-charge, open source **R** statistical computing environment. A pre-scripted program is included in **Appendix C** that can be run in **R** to calculate appropriate prediction limit multipliers, once the user has supplied an intrawell sample size (n), network size (w), and type of retesting scheme.

If guidance users are unable to utilize the **R-script** approach, the following approximation for the well-specific prediction limit κ-factors is suggested based on EPA Region 8 Monte Carlo evaluations. Given a per- test confidence level of $1-\alpha$, r total tests of $w \cdot c$ well-constituents, an individual well size n_i, a pooled variance sample size of $n_{df} = df + 1$, and $\kappa_{ndf,1-\alpha}$ obtained from annual intrawell Unified Guidance tables, the individual well $\kappa_{ni,1-\alpha}$ factor can be estimated using the following equation:

$$\kappa_{n_i,1-\alpha} = \kappa_{n_{df},1-\alpha} \cdot \left[\frac{(n_i + \mu) \cdot n_{df}}{n_i \cdot (n_{df} + \mu)} \right]^{m^* = \frac{A \cdot n_i^b}{r^c}}$$

where μ =1 for future $1{:}m$ observations or μ is the size of a future mean. The value of m^* is specific to each of the nine parametric prediction limit tests and is a function of the three coefficients A, b and c, individual well sample size n_i and r tests. For a 1:1 test of future means or observations, the equation is exact; for higher order 1:m tests, the results are approximate.[2] The equation is also useful in

[2] For each of the nine prediction limit tests, the following coefficients (A, b & c) are recommended: a 1:2 future values test (1.01, .0524 & .0158); a 1:3 test (1.63, .108 & .0407); a 1:4 test (2.41, .157 & .0668); the modified California plan (1.36, .103 & .0182); a 1:1 mean size 2 test (.5, 0 & 0); a 1:2 mean size 2 test (.898, .0856 & .0172); a 1:3 mean size 2 test (1.27, .168 & .0363); a 1:1 mean size 3 test (.5, 0 & 0); and a 1:2 mean size 3 test (.817, .108 & .0158). %. The coefficients were obtained from regression analysis; approximation values were compared with R-script values for κ-factors. In 1260 comparisons of the seven tests using repeat values ($m > 1$), 86% of the approximations lay within or equal to ± 1% of the true value and 96% within or equal to ± 2%. The 1:4 test had the greatest variability, but all values lay within ± 4%. 81% of the values lay within or equal to ± .01 κ-units and 93% less than or equal to ± .02 units.

gauging **R-script** method results. Another virtue of this equation is that it can be readily applied to different individual well sample sizes based on the common $\kappa_{ndf,1-\alpha}$ for pooled variance data.

A less elegant solution is available for intrawell control charts. Currently, an appropriate multiplier needs to be simulated via Monte Carlo methods. The approach is to simulate separate normally-distributed data sets for the background mean based on n measurements, and the background standard deviation based on $df + 1$ measurements. Statistical independence of the sample mean ($\bar{x}$) and standard deviation (s) for normal populations allows this to work. With the background mean and standard deviation available, a series of possible multipliers (h) can be investigated in simulations of control chart performance. The multiplier which meets the targeted SWFPR and provides acceptable power should be selected. Further detail is presented in **Chapter 20**. **R** can also be used to conduct these simulations.

►EXAMPLE 13-3

The logged iron concentrations from **Example 13-2** showed significant evidence of spatial variability. Use the results of the one-way ANOVA to compute adjusted intrawell prediction limits (without retesting) for each of the wells in that example and compare them to the unadjusted prediction limits.

SOLUTION

Step 1. Summary statistics by well for the logged iron measurements are listed in the table below. With n = 4 measurements per well, use equation [13.1] and $t_{1-\alpha,n-1} = t_{.99,3}$ = 4.541 from **Table 16-1** in **Appendix D** to compute at each well an unadjusted 99% intrawell prediction limit for the next single measurement, based on lognormal data:

$$PL_{1-\alpha} = \exp\left[\bar{y} + s_y t_{1-\alpha,n-1}\sqrt{1+\frac{1}{n}}\right] \qquad [13.1]$$

	Unadjusted 99% Prediction Limits for Iron (ppm)					
	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
Log-mean	3.820	3.965	4.348	4.188	4.802	5.000
Log-SD	0.296	0.395	0.658	0.453	0.704	0.396
n	4	4	4	4	4	4
$t_{.99,3}$	4.541	4.541	4.541	4.541	4.541	4.541
99% PL	204.9	391.6	2183.0	657.0	4341.5	1108.1

Step 2. Use the RMSE (*i.e.*, square root of the mean error sum of squares [MS_{error}] component) of the ANOVA in **Example 13-2** as an estimate of the adjusted, pooled standard deviation, giving $\sqrt{MS_{error}} = \sqrt{.258} = .5079$. The degrees of freedom (df) associated with this pooled standard deviation is $p(n-1) = 6(3) = 18$, the same as listed in the ANOVA table of **Example 13-2**.

Step 3. Use equation [13.2], along with the adjusted pooled standard deviation and its associated *df*, to compute an adjusted 99% prediction limit for each well, as given in the table below. Note that the adjusted t-value based on the larger *df* is $t_{1-\alpha,\mathrm{df}} = t_{.99,18} = 2.552$.

$$PL_{1-\alpha} = \exp\left[\bar{y} + t_{1-\alpha,df}\sqrt{MS_{error}\left(1+\frac{1}{n}\right)}\right] \qquad [13.2]$$

	Adjusted 99% Prediction Limits for Iron (ppm)					
	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
Log-mean	3.820	3.965	4.348	4.188	4.802	5.000
RMSE	0.5079	0.5079	0.5079	0.5079	0.5079	0.5079
df	18	18	18	18	18	18
$t_{.99,18}$	2.552	2.552	2.552	2.552	2.552	2.552
99% PL	194.3	224.6	329.4	280.7	518.7	632.3

Step 4. Compare the adjusted and unadjusted lognormal prediction limits. By estimating the average intrawell standard deviation using ANOVA, the adjusted prediction limits are significantly lower and thus more powerful than the unadjusted limits, especially at Wells 3, 5, and 6.

In this example, use of the **R**-script approach was unnecessary, since the corresponding κ-multiple used in 1-of-1 prediction limit tests can be directly derived analytically. ◄

CHAPTER 14. TEMPORAL VARIABILITY

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This chapter discusses the importance of ***statistical independence*** in groundwater monitoring data with respect to ***temporal variability***. Temporal variability exists when the distribution of measurements varies with the times at which sampling or analytical measurement occurs. This variation can be caused by seasonal fluctuations in the groundwater itself, changes in the analytical method used, the re-calibration of instruments, anomalies in sampling method, ***etc***.

Methods to ***identify*** temporal variability are discussed for both groups of wells (parallel time series plots; one-way analysis of variance [ANOVA] for temporal effects) and single data series (sample autocorrelation function; rank von Neumann ratio). Procedures are also presented for correcting or accommodating temporal effects. These include guidance on adjusting the sampling frequency to avoid temporal correlation, choosing a sampling interval using the Darcy equation, removing seasonality or other temporal dependence, and finally testing for trends with seasonal data.

14.1 TEMPORAL DEPENDENCE

A key assumption underlying most statistical tests is that the sample data are independent and identically distributed [*i.i.d.*] (**Chapter 3**). In part, this means that measurements collected over a period of time should not exhibit a clear ***time dependence*** or significant ***autocorrelation***. Time dependence refers to the presence of trends or cyclical patterns when the observations are graphed on a time series plot. The closely related concept of autocorrelation is essentially the degree to which measurements collected later in a series can be predicted from previous measurements. Strongly autocorrelated data are highly predictable from one value to the next. Statistically independent values vary in a random, unpredictable fashion.

While temporal independence is a complex topic, there are several common types of temporal dependence. Some of these include: 1) correlation across wells over time in the concentration pattern of a single constituent (***i.e.***, concentrations tending to jointly rise or fall at each of the wells on common sampling events); 2) correlation across multiple constituents over time in their concentration patterns (***i.e.***, a parallel rise or fall in concentration across several parameters on common sampling events); 3) seasonal cycles; 4) trends, linear or otherwise; and 5) serial dependence or autocorrelation (***i.e.***, greater correlation between sampling events more closely spaced in time).

Any of these patterns can invalidate or weaken the results of statistical testing. In some cases, a statistical method can be chosen that specifically accounts for temporal dependence (e.g., seasonal Mann-Kendall trend test). In other instances, the sample data need to be adjusted for the dependence. Future data might also need to be collected in a manner that avoids temporal correlation. The goal of this chapter is to present straightforward tools that can be used to first identify temporal dependence and then to adjust for this correlation.

To better understand why most statistical tests depend on the assumption of statistical independence, consider a hypothetical series of groundwater measurements exhibiting an obvious pattern of seasonal fluctuation (**Figure 14-1**). These data demonstrate regular and repeated cycles of higher and lower values. Even though fluctuating predictably and highly dependent, the characteristics of the entire groundwater population will be observed over a long period of monitoring. This provides an estimate of the full range of concentrations and an accurate gauge of total variability.

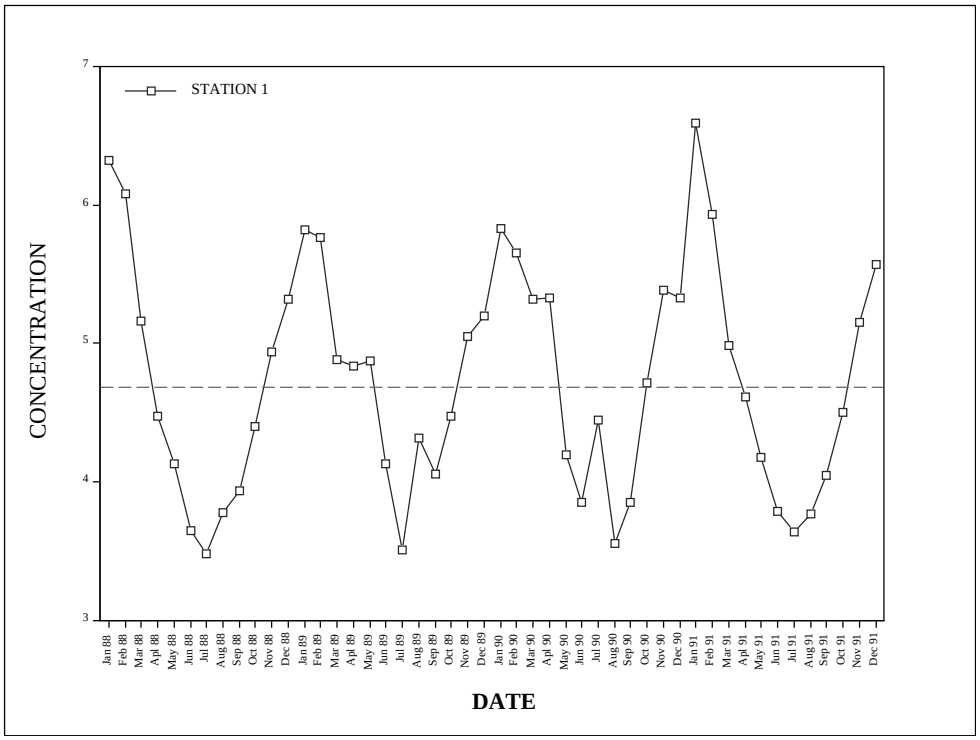
The same is not true for data collected from the same population over a much shorter span, say in five to six months. A much narrower range of sample concentrations would be observed due to the cyclical pattern. Depending on when the sampling was conducted, the average concentration level would either be much higher or much lower than the overall average; no single sampling period is likely to accurately estimate either the true population mean or its variance.

From this example, an important lesson can be drawn about temporally dependent data. Variance estimates in a sample of dependent, positively autocorrelated data are likely to be biased low. This is important because the guidance methods require and assume that an accurate and unbiased estimate of the sample standard deviation be available. A case in point was the practice of using aliquot replicates of a single physical sample for comparison with other combined replicate aliquot samples from a number of individual physical water quality samples (*e.g.*, in a Student-*t* test). Aliquot replicate values are much more similar to each other than to measurements made on physically discrete groundwater samples. Consequently, the estimate of variance was too low and the *t*-test frequently registered false positives.

Using physically discrete samples is not always sufficient. If the sampling interval ensures that discrete volumes of groundwater are being sampled on consecutive sampling events, the observations can be described as *physically independent*. However, they are not necessarily *statistically* independent. Statistical independence is based not on the physical characteristics of the sample data, but rather on the statistical pattern of measurements.

Temporally dependent and autocorrelated data generally contain both a truly random and non-random component. The relative strength of the latter effect is a measured by one or more correlation techniques. The degree of correlation among dependent sample measurements lies on a continuum. Sample pairs can be mildly correlated or strongly correlated. Only strong correlations are likely to substantially impact the results of further statistical testing.

Figure 14-1. Seasonal Fluctuations



14.2 IDENTIFYING TEMPORAL EFFECTS AND CORRELATION

14.2.1 PARALLEL TIME SERIES PLOTS

BACKGROUND AND PURPOSE

Time series plots were introduced in **Chapter 9**. A time series plot such as **Figure 14-1** is a simple graph of concentration versus time of sample collection. Such plots are useful for identifying a variety of temporal patterns. These include identifying a trend over time, one or more sampling events that may signal contaminant releases, measurement outliers resulting in anomalous 'spikes' due to field handling or analytical problems, cyclical and seasonal fluctuations, as well as the presence of other time-related dependencies.

Time series plots can be used in two basic ways to identify temporal dependence. By graphing single constituent data from multiple wells together on a time series plot, potentially significant temporal components of variability can be identified. For example, seasonal fluctuations can cause the mean concentration levels at a number of wells to vary with the time of sampling events. This dependency will show up in the time series plot as a pattern of *parallel traces*, in which the individual wells will tend to rise and fall together across the sequence of sampling dates. The parallel pattern may be the result of the measurement process such as mid-stream changes in field handling or sample collection procedures, periodic re-calibration of analytical instrumentation, and changes in laboratory or analytical methods. It could also be the result from significant autocorrelation present in the groundwater population itself. Hydrologic factors such as drought, recharge patterns or regular (*e.g.*, seasonal) water table fluctuations may be responsible. In these cases, it may be useful to test for the presence of a significant temporal

effect by first constructing a parallel time series plot and then running a formal one-way ANOVA for temporal effects (**Section 14.2.2**).

The second way time series plots can be helpful is by plotting multiple constituents over time for the same well, or averaging values for each constituent across wells on each sampling event and then plotting the averages over time. In either case, the plot can signify whether the general concentration pattern over time is simultaneously observed for different constituents. If so, it may indicate that a group of constituents is highly correlated in groundwater or that the same artifacts of sampling and/or lab analysis impacted the results of several monitoring parameters.

REQUIREMENTS AND ASSUMPTIONS

The requirements for time series plots were discussed in **Chapter 9**. Two very useful recommendations follow from that discussion. First, a different plot symbol should be used to display any non-detect measurements (*e.g.*, solid symbols for detected values, hollow symbols for non-detects). This can help prevent mistaking a change over time in reporting limits as a trend, since detected and non-detected data are clearly distinguished on the plot. It also allows one to determine whether non-detects are more prevalent during certain portions of the sample record and less prevalent at other times. Secondly, when multiple constituents are plotted on the same graph, it may be necessary to *standardize* each constituent prior to plotting to avoid trying to simultaneously visualize high-valued and low-valued traces on the same *y*-axis (*i.e.*, concentration axis). The goal of such a plot is to identify parallel concentration patterns over time. This can be done most readily by subtracting each constituent's sample mean ($\bar{x}$) from the measurements for that constituent and dividing by the standard deviation (s), so that every constituent is plotted on roughly the same scale.

PROCEDURE FOR MULTIPLE WELLS, ONE CONSTITUENT

Step 1. For each well to be plotted, form data pairs by matching each concentration value with its sampling date.

Step 2. Graph the data pairs for each well on the same set of axes, the horizontal axis representing time and the vertical axis representing concentration. Connect the points for each individual well to form a 'trace' for that well.

Step 3. Look for parallel movement in the traces across the wells. Even if all the well concentrations tend to rise on a given sampling event, but not to the same magnitude or degree, this is evidence of a possible temporal effect.

PROCEDURE FOR MULTIPLE CONSTITUENTS, ONE OR MANY WELLS

Step 1. For each constituent to be plotted, compute the constituent-specific sample mean ($\bar{x}$) and standard deviation (s). Form standardized measurements (z_i) by subtracting the mean from each concentration (x_i) and dividing by the standard deviation, using the equation:

$$z_i = \frac{x_i - \bar{x}}{s} \qquad [14.1]$$

Form data pairs by matching each standardized concentration with its sampling event.

Step 2. If correlation is suspected in a group of wells, average the standardized concentrations for each given constituent *across wells* for each specific sampling event. Otherwise, form a multi-constituent time series plot separately for each well.

Step 3. Graph the data pairs for each constituent on the same set of axes, the horizontal axis representing time and the vertical axis representing standardized concentrations. Connect the points for each constituent to form a trace for that parameter.

Step 4. Look for parallel movement in the traces across the constituents. A strong degree of parallelism indicates a high degree of correlation among the monitoring parameters.

►EXAMPLE 14-1

The following well sets of manganese measurements were collected over a two-year period. Construct a time series plot of these data to check for possible temporal effects.

	Manganese Concentrations (ppm)			
Qtr	BW-1	BW-2	BW-3	BW-4
1	28.14	31.41	27.15	30.46
2	29.33	30.27	30.24	30.60
3	30.45	32.57	29.14	30.96
4	32.42	32.77	30.59	30.70
5	34.37	33.03	34.88	32.71
6	33.25	32.18	30.53	31.76
7	31.02	28.85	30.33	31.85
8	28.50	32.88	30.42	29.58

SOLUTION

Step 1. Graph each well's concentrations versus sampling event on the same set of axes to construct the following time series plot (**Figure 14-2**).

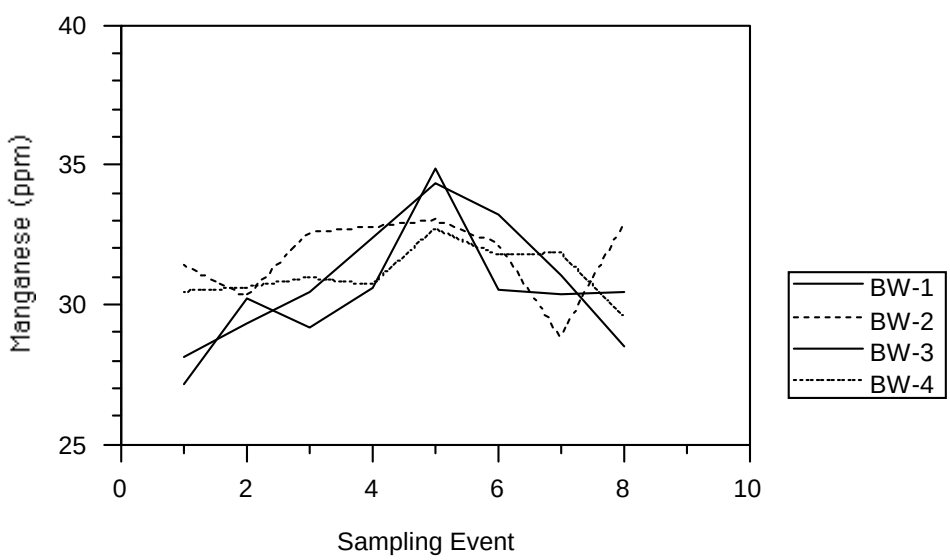


Figure 14-2. Manganese Parallel Time Series Plot

Step 2. Examining the traces on the plot, there is some degree of parallelism in the pattern over time. Particularly for the fifth quarter, there is an across-the-board increase in the manganese level, followed by a general decline the next two quarterly events. Note, however, that there is little evidence of differences in mean levels by *well location.* ◄

14.2.2 ONE-WAY ANALYSIS OF VARIANCE FOR TEMPORAL EFFECTS

PURPOSE AND BACKGROUND

Parametric ANOVA is a comparison of means among a set of populations. The one-way ANOVA for temporal effects is no exception. A one-way ANOVA for spatial variation (**Chapter 13**) uses well data sets to represent *locations* as the statistical factor of interest. In contrast, a one-way ANOVA for temporal effects considers multiple well data sets for individual *sampling events* or *seasons* as the relevant statistical factor. A significant temporal factor implies that the average concentration depends to some degree on *when* sampling takes place.

Three common examples of temporal factors include: 1) an irregular, but consistent shift of average concentrations over time perhaps due to changes in laboratories or analytical method interferences; 2) cyclical seasonal patterns; or 3) parallel upward or downward trends. These can occur in both upgradient and downgradient well data.

If event-specific analytical differences or seasonality appear to be an important temporal factor, the one-way ANOVA for temporal effects can be used to formally identify seasonality, parallel trends, or changes in lab performance that affect other temporal effects. Results of the ANOVA can also be used to create temporally *stationary residuals*, where the temporal effect has been 'subtracted from' the original measurements. These stationary residuals may be used to replace the original data in subsequent statistical testing.

The one-way ANOVA for a temporal factor described below can be used for an additional purpose when interwell testing is appropriate. For this situation, *there can be no significant spatial variability*. If a group of upgradient or other background wells indicates a significant temporal effect, an interwell prediction limit can be designed which properly accounts for this temporal dependence. A more powerful interwell test of upgradient-to-downgradient differences can be developed than otherwise would be possible. This can occur because the ANOVA separates or 'decomposes' the overall data variation into two sources: a) temporal effects and b) random variation or statistical error. It also estimates how the background mean is changing from one sampling event to the next. The final prediction limit is formed by computing the background mean, using the separate structural and random variation components of the ANOVA to better estimate the standard deviation, and then adjusting the effective sample size (via the degrees of freedom) to account for these factors.

REQUIREMENTS AND ASSUMPTIONS

Like the one-way ANOVA for spatial variation (**Chapter 13**), the one-way ANOVA for temporal effects assumes that the data groups are normally-distributed with constant variance. This requirement means that the group residuals should be tested for normality (**Chapter 10**) and also for equality of

variance (**Chapter 11**). It is also assumed that for each of a series of background wells, measurements are collected at each well on sampling events or dates *common* to all the wells.

Two variations in the basic procedure are described below. For cases of temporal effects *excluding* seasonality, each sampling event is treated as a separate population. The ANOVA residuals are grouped and tested *by sampling event* to test for equality of variance. In cases of apparent seasonality, each *season* is treated as a distinct population. The difference is that seasons contain multiple sampling events across a span of multiple years, with sampling events collected at the same time of year assigned to one of the seasons (*e.g.*, all January or first quarter measurements). Here, the ANOVA residuals are grouped by season to test for homoscedasticity.

If the assumption of equal variances or normal residuals is violated, a data transformation should be considered. This should be followed by testing of the assumptions on the transformed scale. The one-way ANOVA for a non-seasonal effect should include a minimum of four wells and at least 4 observations (*i.e.*, distinct sampling dates) per well. In the seasonal case, there should be a minimum of 3-4 sampling events per distinct season, with the events thus spanning at least three years (*i.e.*, one per year per season). Larger numbers of both wells and observations are preferable. Sampling dates should also be approximately the *same* for each well if a temporal effect is to be tested.

If the data cannot be normalized, a similar test for a temporal or seasonal effect can be performed using the Kruskal-Wallis test (**Chapter 17**). The only difference from the procedure outlined in **Section 17.1.2** is that the roles of wells/groups and sampling events have to be reversed. That is, each sampling event should be treated as a separate 'well,' while each well is treated as a separate 'sampling event.' Then the same equations can be applied to the reversed data set to test for a significant temporal dependence. If testing for a seasonal effect, the wells in the notation of **Section 17.1.2** become the groups of common sampling events from different years, while the sampling events are again the distinct wells.

Even when a temporal effect exists and is apparent on a time series plot, the variation between well locations (*i.e.*, spatial variability) may overshadow the temporal variability. This could result in a non-significant one-way ANOVA finding for the temporal factor. In these cases, a two-way ANOVA can be considered where both well location and sampling event/season are treated as statistical factors. This procedure is described in Davis (1994). Evidence for a temporal effect can be documented using this last technique, although the two-way ANOVA isn't necessary if the goal is simply to construct temporally stationary residuals. That can be accomplished with a one-way ANOVA even when significant spatial variability exists.

PROCEDURE

Step 1. Given a set of W wells and measurements from each of T sampling events at each well on each of K years, label the observations as x_{ijk}, for i = 1 to W, j = 1 to T, and k = 1 to K. Then x_{ijk} represents the measurement from the ith well on the jth sampling event during the kth year.

Step 2. When testing for a *non-seasonal* temporal effect, form the set of event means ($x_{\bullet jk}$) and the grand mean ($x_{\bullet\bullet\bullet}$) using equations [14.2] and [14.3] respectively:

$$x_{\bullet jk} = \frac{1}{W}\sum_{i=1}^{W} x_{ijk} \quad \text{for } j = 1 \text{ to } T \text{ and } k = 1 \text{ to } K \qquad [14.2]$$

$$x_{\bullet\bullet\bullet} = \sum_{i=1}^{W}\sum_{j=1}^{T}\sum_{k=1}^{K} x_{ijk} \Big/ WTK \qquad [14.3]$$

Step 2a. If testing for a seasonal effect common to all wells, form the seasonal means ($x_{\bullet j\bullet}$) instead of the event means of **Step 2**, using the equation:

$$x_{\bullet j\bullet} = \frac{1}{WK}\sum_{i=1}^{W} x_{ijk} \quad \text{for } j = 1 \text{ to } T \qquad [14.4]$$

Step 3. Compute the set of residuals for each sampling event or season using either equation [14.5] or equation [14.6] respectively:

$$r_{ijk} = x_{ijk} - x_{\bullet jk} \quad \text{for } i = 1 \text{ to } W \qquad [14.5]$$

$$r_{ijk} = x_{ijk} - x_{\bullet j\bullet} \quad \text{for } i = 1 \text{ to } W \text{ and } k = 1 \text{ to } K \qquad [14.6]$$

Step 4. Test the residuals for normality (**Chapter 10**). If significant non-normality is evident, consider transforming the data and re-doing the computations in **Steps 1** through **4** on the transformed scale.

Step 5. Test the sets of residuals grouped either by sampling event or season for equal variance (**Chapter 11**). If the variances are significantly different, consider transforming the data and re-doing the computations in **Steps 1** through **5** on the transformed data.

Step 6. If testing for a non-seasonal temporal effect, compute the mean error sum of squares term (MS_E) using equation:

$$MS_E = \sum_{i=1}^{W}\sum_{j=1}^{T}\sum_{k=1}^{K} r_{ijk}^2 \Big/ TK\left(W-1\right) \qquad [14.7]$$

This term is associated with $TK(W–1)$ degrees of freedom. Also compute the mean sum of squares for the temporal effect (MS_T) with degrees of freedom ($TK–1$), using equation:

$$MS_T = W\sum_{j=1}^{T}\sum_{k=1}^{K}\left(x_{\bullet jk} - x_{\bullet\bullet\bullet}\right)^2 \Big/ \left(TK-1\right) \qquad [14.8]$$

Step 6a. If testing for a seasonal effect, compute the mean error sum of squares (MS_E) using equation:

$$MS_E = \sum_{i=1}^{W}\sum_{j=1}^{T}\sum_{k=1}^{K} r_{ijk}^2 \Big/ T\left(WK-1\right) \qquad [14.9]$$

This term is associated with $T(WK-1)$ degrees of freedom. Also compute the mean sum of squares for the seasonal effect (MS_T) with degrees of freedom ($T-1$), using equation:

$$MS_T = WK\sum_{j=1}^{T}\left(x_{\bullet j \bullet} - x_{\bullet\bullet\bullet}\right)^2 \Big/ \left(T-1\right) \qquad [14.10]$$

Step 7. Test for a significant event-to-event or seasonal effect by computing the ratio of the mean sum of squares for time and the mean error sum of squares:

$$F_T = MS_T \big/ MS_E \qquad [14.11]$$

Step 8. If testing for a non-seasonal temporal effect, the test statistic F_T under the null hypothesis (*i.e.*, of no significant time-related variability among the sampling events) will follow an F-distribution with $(TK-1)$ and $TK(W-1)$ degrees of freedom. Therefore, using a significance level of $\alpha = 0.05$, compare F_T against the critical point $F_{.05,\ TK-1,TK(W-1)}$ taken from the F-distribution in **Table 17-1** in **Appendix D**. If the critical point is exceeded, conclude there is a significant temporal effect.

Step 8a. If testing for a seasonal effect, the test statistic F_T under the null hypothesis (*i.e.*, of no seasonal pattern) will follow an F-distribution with $(T-1)$ and $T(WK-1)$ degrees of freedom. Therefore, using a significance level of $\alpha = 0.05$, compare F_T against the critical point $F_{.05,\ T-1,T(WK-1)}$ taken from the F-distribution in **Table 17-1** of **Appendix D**. If the critical point is exceeded, conclude there is a significant seasonal pattern.

Step 9. If there is no spatial variability but a significant temporal effect exists among a set of background wells, compute an appropriate *interwell* prediction or control chart limit as follows. First replace the background sample standard deviation (s) with the following estimate built from the one-way ANOVA table:

$$\hat{\sigma} = \sqrt{\frac{1}{W}\left[MS_T + (W-1)\,MS_E\right]} \qquad [14.12]$$

Then calculate the effective sample size for the prediction limit as:

$$n^* = 1 + \left\{\left[T\cdot(T-1)\cdot(F_T + W - 1)^2\right] \Big/ \left[T\cdot F_T^2 + (T-1)\cdot(W-1)\right]\right\} \qquad [14.13]$$

►EXAMPLE 14-2

Some parallelism was found in the time series plot of **Example 14-1**. Test those same manganese data for a significant, non-seasonal temporal effect using a one-way ANOVA at the 5% significance level.

Qtr	Manganese Concentrations (ppm)				
	Event Mean	BW-1	BW-2	BW-3	BW-4
1	29.290	28.14	31.41	27.15	30.46
2	30.110	29.33	30.27	30.24	30.60
3	30.780	30.45	32.57	29.14	30.96
4	31.620	32.42	32.77	30.59	30.70
5	33.747	34.37	33.03	34.88	32.71
6	31.930	33.25	32.18	30.53	31.76
7	30.513	31.02	28.85	30.33	31.85
8	30.345	28.50	32.88	30.42	29.58
	Grand mean = 31.042				

SOLUTION

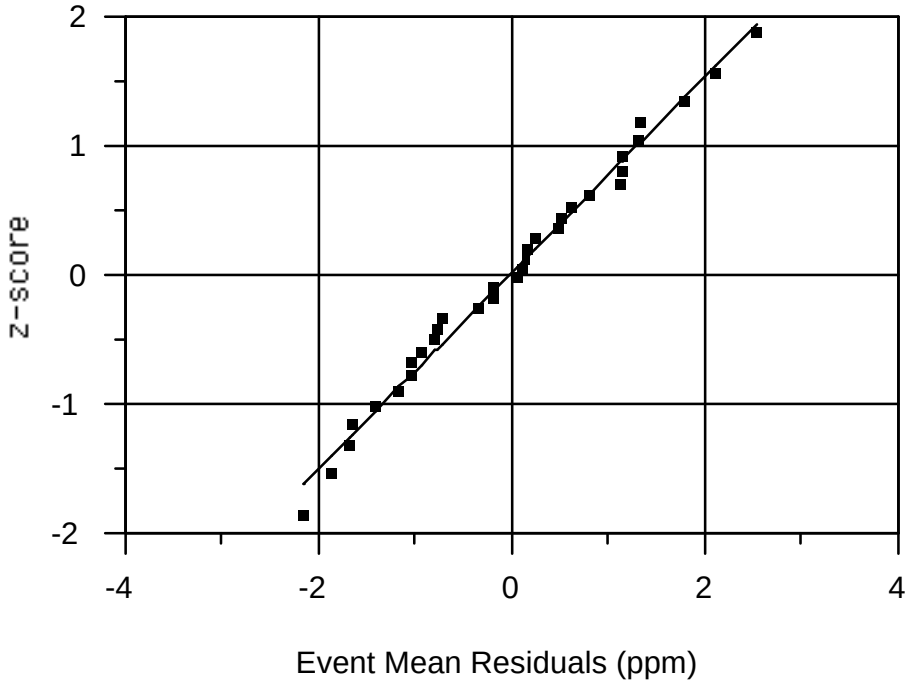
Step 1. First compute the means for each sampling event and the grand mean of all the data. These values are listed in the table above. With four wells and eight quarterly events per well, $W = 4$, $T = 4$, and $K = 2$.

Step 2. Determine the residuals for each sampling event by subtracting off the event mean. These values are listed in the table below.

Qtr	Manganese Event Residuals (ppm)			
	BW-1	BW-2	BW-3	BW-4
1	-1.150	2.120	-2.140	1.170
2	-0.780	0.160	0.130	0.490
3	-0.330	1.790	-1.640	0.180
4	0.800	1.150	-1.030	-0.920
5	0.622	-0.718	1.132	-1.038
6	1.320	0.250	-1.400	-0.170
7	0.508	-1.662	-0.182	1.338
8	-1.845	2.535	0.075	-0.765

Step 3. Test the residuals for normality. A probability plot of these residuals is given in **Figure 14-3**. An adequate fit to normality is suggested by Filliben's probability plot correlation coefficient test.

Figure 14-3. Probability Plot of Manganese Sampling Event Residuals



Step 4. Next, test the groups of residuals for equal variance across sampling events. Levene's test (**Chapter 11**) gives an *F*-statistic of 1.30, well below the 5% critical point with 7 and 24 degrees of freedom of $F_{.95,7,24} = 2.42$. Therefore, the group variances test out as adequately homogeneous.

Step 5. Compute the mean error sum of squares term using equation [14.7]:

$$MS_E = \left[(-1.150)^2 + (-.780)^2 + \ldots + (1.338)^2 + (-.765)^2\right] / (4 \cdot 2)(7) = 1.87$$

Step 6. Compute the mean sum of squares term for the time effect using equation [14.8]:

$$MS_T = 4\left[(29.290 - 31.042)^2 + (30.11 - 31.042)^2 + \ldots + (30.345 - 31.042)^2\right] / 7 = 7.55$$

Step 7. Test for a significant temporal effect, computing the *F*-statistic in equation [14.11]:

$$F_T = 7.55 / 1.87 = 4.04$$

The degrees of freedom associated with the numerator and denominator respectively are $(T–1) = 7$ and $T(W–1) = 24$. Just as with Levene's test run earlier, the 5% level critical point for the test is $F_{.95,7,24} = 2.42$. Since F_T exceeds this value, there is evidence of a significant temporal effect in the manganese background data.

Step 8. Assuming a lack of spatial variation, the presence of a temporal effect can be used to compute a standard deviation estimate and effective background sample size appropriate for an

interwell prediction limit test, using equations [14.12] and [14.13] respectively. The adjusted standard deviation becomes:

$$\hat{\sigma} = \sqrt{\frac{1}{4}\left[7.55 + 7(1.87)\right]}$$

while the effective sample size is:

$$n^* = 1 + \left\{\left[8 \cdot 7 \cdot (4.04 + 4 - 1)^2\right] \Big/ \left[8 \cdot (4.04)^2 + 7 \cdot 3\right]\right\} = 19.31 \approx 19$$

If the background data had simply been pooled together and the sample standard deviation computed, s = 1.776 ppm with a sample size of n = 32. So the adjustments based on the temporal effect alter the final prediction limit by enlarging it to account for the additional component of variation. ◀

14.2.3 SAMPLE AUTOCORRELATION FUNCTION

BACKGROUND AND PURPOSE

The sample autocorrelation function enables a test of temporal autocorrelation in a single data series (*e.g.*, from a single well over time). When a time-related dependency affects several wells simultaneously, parallel time series plots (**Section 14.2.1**) and one-way ANOVA for temporal effects (**Section 14.2.2**) should be considered. But when a longer data series is to be used for an intrawell test such as a prediction limit or control chart, the autocorrelation function does an excellent job of identifying temporal dependence.

Given a sequence of consecutively-collected measurements, x_1, x_2,…, x_n, form the set of overlapping pairs (x_i, x_{i+1}) for i = 1,…, n–1. The approximate first-order sample autocorrelation coefficient is then computed from these pairs as (Chatfield, 2004):

$$r_1 = \frac{\sum_{i=1}^{n-1}\left(x_i - \bar{x}\right)\left(x_{i+1} - \bar{x}\right)}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \qquad [14.14]$$

Equation [14.14] estimates the *first-order autocorrelation*, that is, the correlation between pairs of sample measurements collected one event apart (*i.e.*, consecutive events). The number of sampling events separating each pair is called the *lag*, representing the temporal distance between the pair measurements.

Autocorrelation can also be computed at other lags. The general approximate equation for the *k*th lag is given by:

$$r_k = \frac{\sum_{i=1}^{n-k}\left(x_i - \bar{x}\right)\left(x_{i+k} - \bar{x}\right)}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \qquad [14.15]$$

which estimates the kth-order autocorrelation for pairs of measurements separated in time by k sampling events. Note that the number of pairs used to compute r_k decreases with increasing k due to the fact that fewer and fewer sample pairs can be formed which are separated by that many lags.

By computing the first few sample autocorrelation coefficients and defining $r_0 = 1$, the sample autocorrelation function can be formed by plotting r_k against k. Since the autocorrelation coefficients are approximately normal in distribution with zero mean and variance equal to $1/n$, a test of significant autocorrelation at the 95% significance level can be made by examining the sample autocorrelation function to see if any coefficients exceed $2/\sqrt{n}$ in absolute value ($\pm 2/\sqrt{n}$ represent approximate upper and lower confidence limits).

The sample autocorrelation function is a valuable visual tool for assessing different types of autocorrelation (Chatfield, 2004). For instance, a stationary (*i.e.*, stable, non-trending) but non-random series of measurements will often exhibit a large value of r_1 followed by perhaps one or two other significantly non-zero coefficients. The remaining coefficients will be progressively smaller and smaller, tending towards zero. A series with a clear seasonal pattern will exhibit a seasonal (i.e., approximately sinusoidal) autocorrelation function. If the series tends to alternate between high and low values, the autocorrelation function will also alternate, with r_1 being negative to reflect that consecutive measurements tend to be on 'opposite sides' of the sample mean. Finally, if the series contains a trend, the sample autocorrelation function will not drop to zero as the lag k increases. Rather, there will a persistent autocorrelation even at very large lags.

REQUIREMENTS AND ASSUMPTIONS

The approximate distribution of the sample autocorrelation coefficients is predicated on the sample measurements following a normal distribution. A test for significant autocorrelation may therefore be inaccurate unless the sample measurements are roughly normal. Non-normal data series should be tested for temporal autocorrelation using the non-parametric rank Von Neumann ratio (**Section 14.2.4**).

Outliers can drastically affect the sample autocorrelation function (Chatfield, 2004). Before assessing autocorrelation, check the sample for possible outliers, removing those that are identified. A series of at least 10-12 measurements is minimally recommended to construct the autocorrelation function. Otherwise, the number of lagged data pairs will be too small to reliably estimate the correlation, especially for larger lags. Sampling events should be regularly spaced so that pairs lagged by the same number of events (k) represent the same approximate time interval.

PROCEDURE

Step 1. Given a series of n measurements, $x_1,\ldots, x_n$, form sets of lagged data pairs (x_i, x_{i+k}), $i = 1,\ldots, n–k$, for $k \leq [n/3]$, where the notation $[c]$ represents the largest integer no greater than c. For longer series, computing lags to a maximum of $k = 15$ is generally sufficient.

Step 2. For each set of lagged pairs from **Step 1**, compute the sample autocorrelation coefficient, r_k, using equation [14.15]. Also define $r_0 = 1$.

Step 3. Graph the sample autocorrelation function by plotting r_k versus k for $k = 0,\ldots,[n/3]$, generally up to a maximum lag of 15. Also plot horizontal lines at levels equal to: $\pm 2/\sqrt{n}$.

Step 4. Examine the sample autocorrelation function. If any coefficient r_k exceeds $2/\sqrt{n}$ in absolute value, conclude that the sample has significant autocorrelation.

►EXAMPLE 14-3

The following series of monthly total alkalinity measurements were collected from leachate at a solid waste landfill during a four and a half year period. Use the sample autocorrelation function to test for significant temporal dependence in this series.

Date	Total Alkalinity (mg/L)	Date	Total Alkalinity (mg/L)	Date	Total Alkalinity (mg/L)
01/26/96	1400	07/01/97	2400	01/15/99	1350
02/20/96	1700	08/15/97	3500	02/02/99	1560
03/19/96	1900	09/15/97	3100	03/02/99	1220
04/22/96	1800	10/15/97	3300	04/15/99	1390
05/22/96	1300	11/15/97	2100	05/04/99	1940
06/24/96	2000	12/15/97	2100	06/02/99	2160
07/15/96	2300	01/15/98	1500	07/07/99	1990
08/21/96	2500	02/15/98	710	08/03/99	2540
09/15/96	1700	03/15/98	1100	09/02/99	2250
10/15/96	1600	04/15/98	1900	10/07/99	1630
11/11/96	1400	05/08/98	2100	11/02/99	1710
12/10/96	1600	06/15/98	2000	12/07/99	1210
01/22/97	1800	07/15/98	2500	01/06/00	1170
02/11/97	1000	08/15/98	2700	02/02/00	1330
03/04/97	720	09/02/98	2400	03/02/00	1540
04/07/97	1400	10/06/98	3000	04/04/00	1670
05/01/97	1600	11/03/98	2700	05/02/00	1520
06/09/97	990	12/15/98	2680	06/06/00	2080

SOLUTION

Step 1. Create a time series plot of the $n = 54$ alkalinity measurements, as in **Figure 14-4**. The series indicates an apparent seasonal fluctuation.

Step 2. Form lagged data pairs from the alkalinity series for each lag k = 1,…, [n/3] = 18. The first two pairs for k = 1 (i.e., first order lag) are (1400, 1700) and (1700, 1900). For k = 2, the first two pairs are (1400, 1900) and (1700, 1800), etc.

Step 3. At each lag (k), compute the sample autocorrelation coefficient using equation [14.15]. Note that the denominator of this equation equals $(n–1)s^2$. For the alkalinity data, the sample mean and variance are $\bar{x} = 1865.93$ and $s^2 = 392349.1$ respectively. The lag-1 autocorrelation is thus:

$$r_1 = \frac{(1400-1865.93)\cdot(1700-1865.93)+\ldots+(1520-1865.93)\cdot(2080-1865.93)}{(54-1)\cdot 392349.1} = .64$$

Other lags are computed similarly.

Step 4. Plot the sample autocorrelation function as in **Figure 14-5**. Overlay the plot with 95% confidence limits (dotted lines) shown at $\pm 2/\sqrt{n} = \pm 2/\sqrt{54} = 0.27$.

Step 5. The autocorrelation function indicates coefficients at several lags that lie outside the 95% confidence limits, confirming the presence of temporal dependence. Further, the shape of autocorrelation function is sinusoidal, suggesting a strong seasonal fluctuation in the alkalinity levels. ◄

Figure 14-4. Time Series Plot of Total Alkalinity (mg/L)

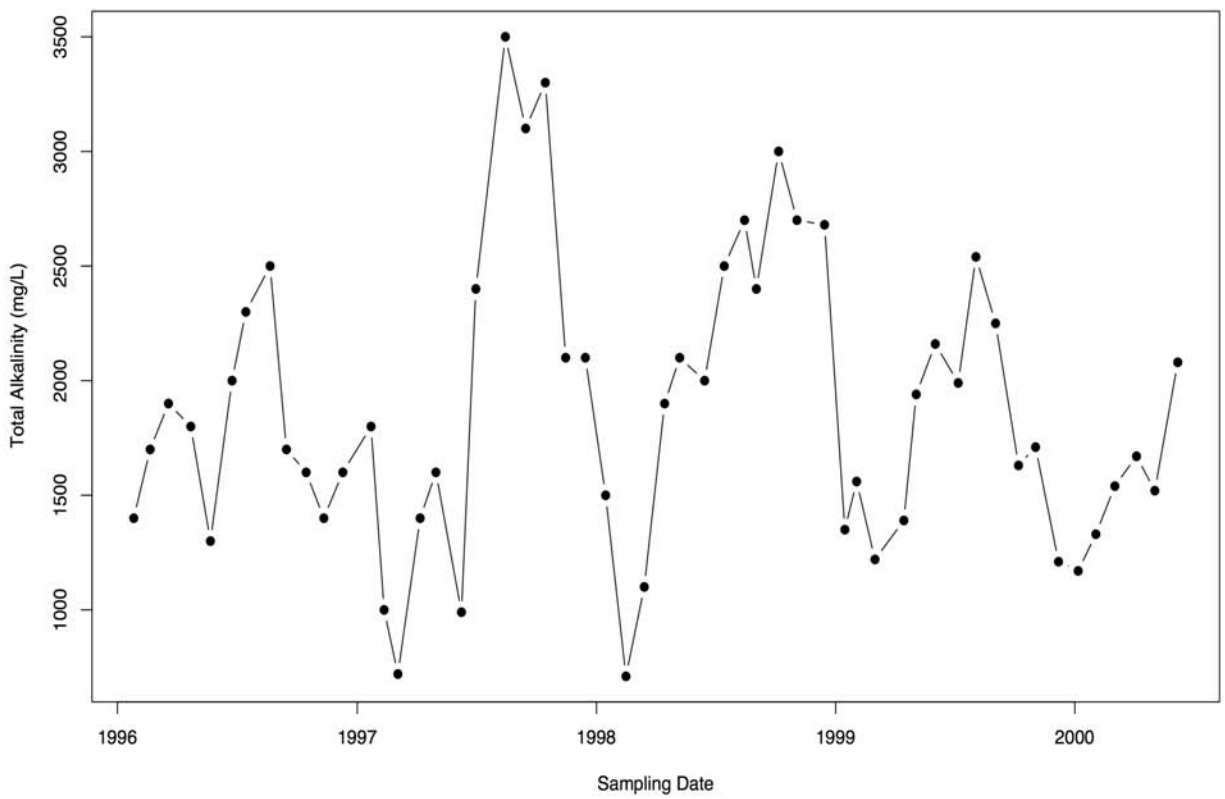
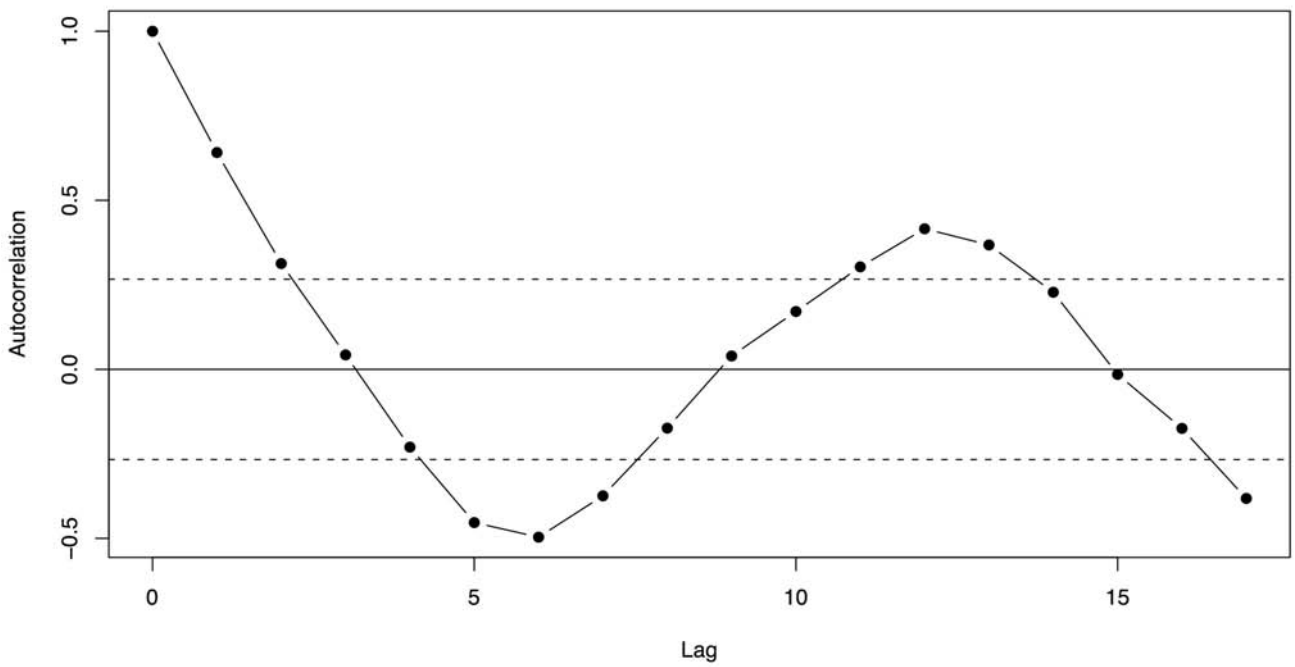


Figure 14-5. Sample Autocorrelation Function for Total Alkalinity



14.2.4 RANK VON NEUMANN RATIO TEST

BACKGROUND AND PURPOSE

The rank von Neumann ratio is a non-parametric test of first-order temporal autocorrelation in a single data series (*e.g.*, from a single well over time). It can be used as an alternative to the sample autocorrelation function (**Section 14.2.3**) for non-normal data, and is both easily computed and effective.

The rank von Neumann ratio is based on the idea that a truly independent series of data will vary in an unpredictable fashion as the list is examined sequentially. The first order or lag-1 autocorrelation will be approximately zero. By contrast, the first-order autocorrelation in *dependent* data will tend to be positive (or negative), implying that lag-1 data pairs in the series will tend to be more similar (or dissimilar) in magnitude than would expected by chance.

Not only will the concentrations of lag-1 data pairs tend to be similar (or dissimilar) when the series is autocorrelated, but the *ranks* of lag-1 data pairs will share that similarity or dissimilarity. Although the test is non-parametric and rank-based, the ranks of non-independent data still follow a discernible pattern. Therefore, the rank von Neumann ratio is constructed from the sum of differences between the *ranks* of lag-1 data pairs. When these differences are *small*, the ranks of consecutive data measurements need to be fairly similar, implying that the pattern of observations is somewhat predictable. Given the relative position and magnitude of one observation, the approximate relative position and magnitude of the next sample measurement can be predicted. Low values of the rank von Neumann ratio are therefore indicative of temporally dependent data series.

Compared to other tests of statistical independence, the rank von Neumann ratio has been shown to be more powerful than non-parametric methods such as the Runs up-and-down test (Madansky, 1988). It is also a reasonable test when the data follow a normal distribution. In that case, the efficiency of the test is always close to 90 percent when compared to the von Neumann ratio computed on concentrations instead of the ranks. Thus, very little effectiveness is lost by using the ranks in place of the original measurements. The rank von Neumann ratio will correctly detect dependent data and do so over a variety of underlying data distributions. The rank von Neumann ratio is also fairly robust to departures from normality, such as when the data derive from a skewed distribution like the lognormal.

REQUIREMENTS AND ASSUMPTIONS

An unresolved problem with the rank von Neumann ratio test is the presence of a substantial fraction of tied observations. Like the Wilcoxon rank-sum test (**Chapter 16**), Bartels (1982) recommends replacing each tied value by its mid-rank (*i.e.*, the average of all the ranks that would have been assigned to that set of ties). However, no explicit adjustment of the ratio for ties has been developed. The rank von Neumann critical points may not be appropriate (or at best very approximate) when a large portion of the data consists of non-detects or other tied values. Especially in the case of frequent non-detects, too much information is lost regarding the pattern of variability to use the rank von Neumann ratio as an accurate indication of autocorrelation. In fact, no test is likely to provide a good estimate of temporal correlation, whether non-parametric or parametric.

While the rank von Neumann ratio test is recommended in the Unified Guidance for its ease of use and robustness when applied to either normal or non-normal distributions, the literature on time series analysis and temporal correlation is extensive with respect to other potential tests. Many other tests of autocorrelation are available, especially when either the original measurements or the residuals of the data are normally distributed after a trend has been removed. Chatfield (2004) and (Madansky, 1988) are two good references for some of these alternate tests.

PROCEDURE

Step 1. Order the sample from least to greatest and assign a unique rank to each measurement. If some data values are tied, replace tied values with their mid-ranks as in the Wilcoxon rank-sum test (**Chapter 16**). Then list the observations and their corresponding ranks in the order that they were collected (*i.e.*, by sampling event or time order).

Step 2. Using the list of ranks, R_i, for the sampling events $i = 1 \ldots n$, compute the rank von Neumann ratio with the equation:

$$v = \sum_{i=2}^{n} \left(R_i - R_{i-1}\right)^2 \Big/ \left[n\left(n^2 - 1\right)/12 \right] \qquad [14.16]$$

Step 3. Given sample size (n) and desired significance level (α), find the lower critical point of the rank von Neumann ratio in **Table 14-1** of **Appendix D**. In most cases, a choice of $\alpha = .01$ should be sufficient, since only substantial non-independence is likely to affect subsequent statistical testing. If the computed ratio, v, is *smaller* than this critical point, conclude that the data series is strongly autocorrelated. If not, there is insufficient evidence to reject the

hypothesis of independence; treat the data as temporally independent in subsequent statistical testing.

►EXAMPLE 14-4

Use the rank von Neumann ratio test on the following series of 16 quarterly measurements of arsenic (ppb) to determine whether or not the data set should be treated as temporally independent in subsequent tests. Compute the test at the α = .01 level of significance.

Sample Date	Arsenic (ppb)	Rank (R_i)
Jan 1990	4.0	5
Apr 1990	7.2	15
Jul 1990	3.1	2
Oct 1990	3.5	3
Jan 1991	4.4	8
Apr 1991	5.1	9
Jul 1991	2.2	1
Oct 1991	6.3	13
Jan 1992	6.5	14
Apr 1992	7.5	16
Jul 1992	5.8	11
Oct 1992	5.9	12
Jan 1993	5.7	10
Apr 1993	4.1	6
Jul 1993	3.8	4
Oct 1993	4.3	7

SOLUTION

Step 1. Assign ranks to the data values as in the table above. Then list the data in chronological order so that each rank value occurs in the order sampled.

Step 2. Compute the von Neumann ratio using the set of ranks in column 3 using equation [14.16], being sure to take squared differences of successive, *overlapping* pairs of rank values:

$$v = \frac{\left[(15-5)^2 + (2-15)^2 + \ldots + (7-4)^2\right]}{16 \cdot \left(16^2 - 1\right)/12} = 1.67$$

Step 3. Look up the lower critical point (v_{cp}) for the rank von Neumann ratio in **Table 14-1** of **Appendix D**. For n = 16 and α = .01, the lower critical point is equal to 0.93. Since the test statistic v is larger than v_{cp}, there is insufficient evidence of autocorrelation at the α = .01 level of significance. Therefore, treat these data as statistically independent in subsequent testing. ◄

14.3 CORRECTING FOR TEMPORAL EFFECTS AND CORRELATION

14.3.1 ADJUSTING THE SAMPLING FREQUENCY AND/OR TEST METHOD

If a data series is temporally correlated, a simple remedy (if allowable under program rules) is to change the sampling frequency and/or statistical method used to analyze the data. In some cases, increasing the sampling interval will effectively eliminate the statistical dependence exhibited by the series. This may happen because the longer time between sampling events allows more groundwater to flow through the well screen, further differentiating measurements of consecutive volumes of groundwater and lessening the impact of seasonal fluctuations or other time-dependent patterns in the underlying concentration distribution.

Many authors including Gibbons (1994a) and ASTM (1994) have recommended that sampling be conducted no more often than quarterly to avoid temporal dependence. If the sampling frequency is reduced, there are obviously fewer measurements available for statistical analysis during any given evaluation period. A *t*-test or ANOVA cannot realistically be run with fewer than four measurements per well. A prediction limit for a future mean requires at least two new observations, and a prediction limit for a future median requires at least three measurements, not counting any resamples. Depending on the length of the evaluation period (i.e., quarterly, semi-annual, annual), a change of statistical method may also be necessary when groundwater measurements are autocorrelated.

When sufficient background data have been collected over a longer period of time, a prediction limit test for future values can be run with as few as one or two new measurements per compliance well. The same is true for control charts. Therefore, if a low groundwater flow velocity and/or evidence of statistical dependence suggest a reduction in sampling frequency, certain prediction limits and control charts should be strongly considered as alternate statistical procedures.

RUNNING A PILOT STUDY

An optional approach to adjusting the sampling frequency is to run a site-specific *pilot study* of autocorrelation. Such a study can be conducted in several ways, but perhaps the easiest is to pick two or three wells from the network (perhaps one background well and one or two compliance wells) and then conduct weekly sampling at these wells over a one year period. For each well in the study, construct the sample autocorrelation function (**Section 14.2.3**) for a variety of constituents, and determine from these graphs the smallest lagged interval at which the autocorrelation coefficient becomes insignificantly different from zero for most of the study constituents.

Since an autocorrelation of zero is equivalent to temporal independence for practical purposes, finding the smallest lag between sampling events with no correlation indicates the minimum sampling frequency needed to approximately ensure statistical independence. If the sample autocorrelation function does not drop down to zero with increasing lag (k), there may be a strong seasonal component or a trend involved. In these circumstances, lengthening the sampling frequency may do little to lessen the temporal dependence. A seasonal pattern may need to be estimated instead and regularly removed from the data prior to statistical testing. Likewise, any apparent trends should be investigated to determine if there is evidence of increasing concentration levels indicative of a possible release.

14.3.2 CHOOSING A SAMPLING INTERVAL VIA DARCY'S EQUATION

Another strategy for determining an appropriate sampling interval is to use Darcy's equation. The goal of this approach is to calculate groundwater flow velocity and the time needed to ensure that physically independent or distinct volumes of groundwater are collected on each sampling trip. As noted in **Chapter 6**, physical independence does not guarantee statistical independence. However, statistical independence may be more likely if the same general volume of groundwater is not re-sampled on multiple occasions.

This section discusses the important hydrological parameters to consider when choosing a sampling interval. The *Darcy equation* is used to determine the horizontal component of the average linear velocity of ground water for confined, semi-confined, and unconfined aquifers. This value provides a good estimate of travel time for most soluble constituents in groundwater, and can be used to determine a minimal sampling interval. Example calculations are provided to further assist the reader. Alternative methods should be employed to determine a sampling interval in groundwater environments where Darcy's law is invalid. Karst, cavernous basalt, fractured rocks, and other 'pseudo-karst' terranes usually require specialized monitoring approaches.

Section 264.97(g) of 40 CFR Part 264 Subpart F allows the owner or operator of a RCRA facility to choose a sampling procedure that will reflect site-specific concerns. It specifies that the owner or operator shall obtain a sequence of at least four samples from each well collected at least semi-annually. The interval is determined after evaluating the uppermost aquifer's effective porosity, hydraulic conductivity, and hydraulic gradient, and the fate and transport characteristics of potential contaminants. The intent of this provision is to set a sampling frequency that allows sufficient time between sampling events to ensure, to the greatest extent technically feasible, that independent groundwater observations are taken from each well.

The sampling frequency required in Part 264 Subpart F can be based on estimates using the average linear velocity of ground water. Two forms of the Darcy equation stated below relate groundwater velocity (V) to effective porosity (Ne), hydraulic gradient (i), and hydraulic conductivity (K):

$$V_h = \left(K_h \cdot i\right) / Ne \qquad [14.17]$$

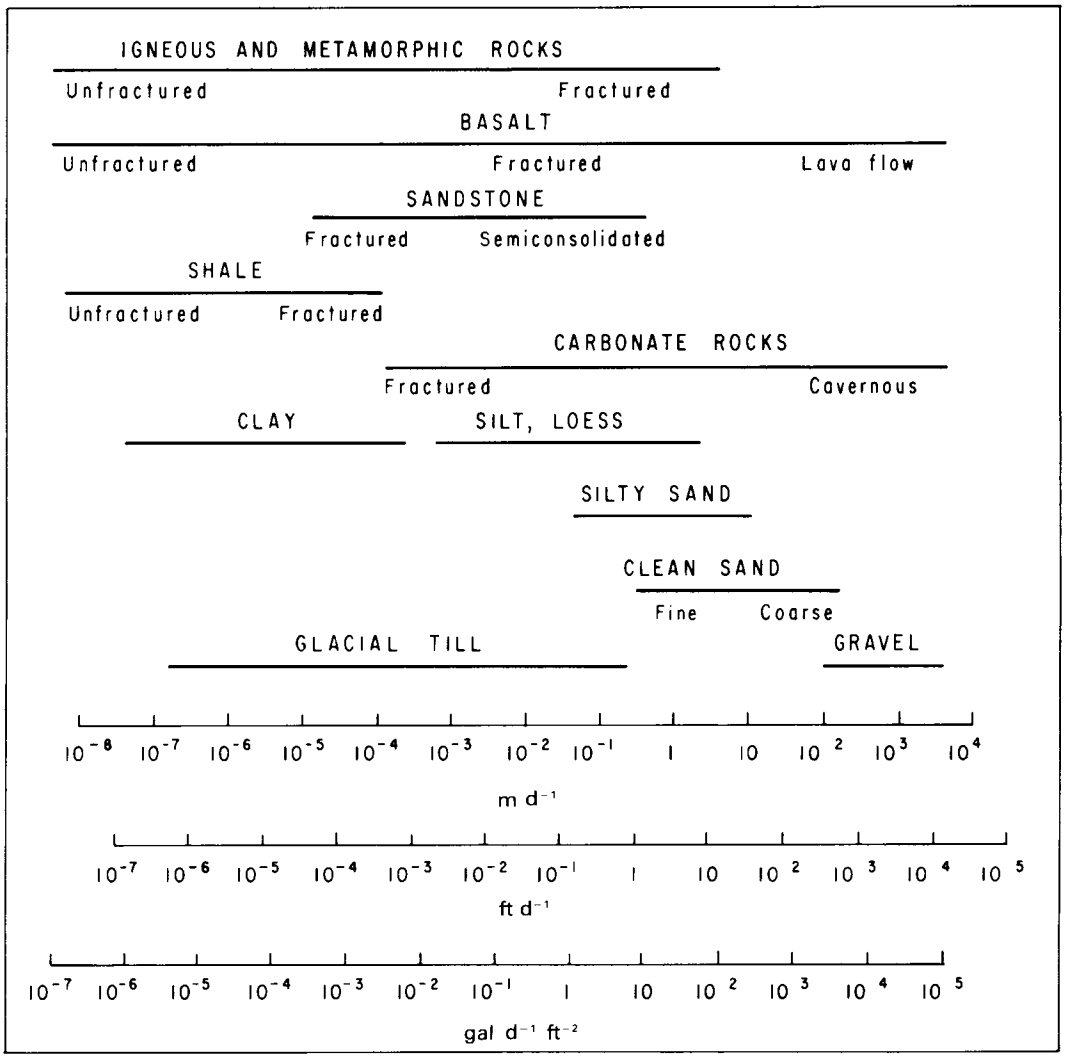
$$V_v = \left(K_v \cdot i\right) / Ne \qquad [14.18]$$

where V_h and V_v are the horizontal and vertical components of the average linear velocity of groundwater, respectively; K_h and K_v are the horizontal and vertical components of hydraulic conductivity, respectively; i is the head gradient; and Ne is the effective porosity.

In applying these equations to ground-water monitoring, the horizontal component of the average linear velocity (V_h) can be used to determine an appropriate sampling interval. Usually, field investigations will yield bulk values for hydraulic conductivity. In most cases, the bulk hydraulic conductivity determined by a pump test, tracer test, or a slug test will be sufficient for these calculations. The vertical component (V_v), however, should be considered in estimating flow velocities in areas with significant components of vertical velocity such as recharge and discharge zones.

To apply the Darcy equation to groundwater monitoring, the parameters *K*, *i*, and *Ne* need to be determined. The hydraulic conductivity, *K*, is the volume of water at the existing kinematic viscosity that will move in unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow. "[E]xisting kinematic viscosity" refers to the fact that hydraulic conductivity is not only determined by the media (aquifer), but also by fluid properties (groundwater or potential contaminants). Thus, it is possible to have several hydraulic conductivity values for different chemical substances present in the same aquifer. The lowest velocity value calculated using the Darcy equation should be used to determine sampling intervals, ensuring physical independence of consecutive sample measurements.

Figure 14-6. Hydraulic Conductivity of Selected Rocks

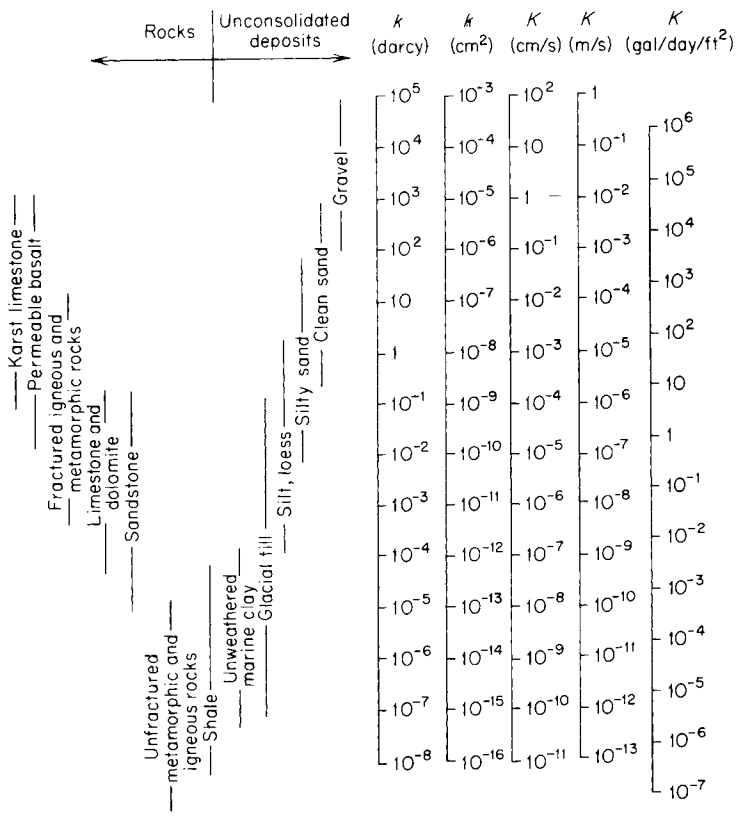


Source: Heath, R.C. 1987. Basic Ground-Water Hydrology. U.S. Geological Survey Water Supply Paper, 2220, 13 pp.

A range of hydraulic conductivities (the transmitted fluid is water) for various aquifer materials is given in **Figures 14-6** and **14-7**. The conductivities are given in several units. **Figure 14-8** lists conversion factors to change between various permeability and hydraulic conductivity units.

The hydraulic gradient, *i*, is the change in hydraulic head per unit of distance in a given direction. It can be determined by dividing the difference in head between two points on a potentiometric surface map by the orthogonal distance between those two points (see calculation in **Example 14-5**). Water level measurements are normally used to determine the natural hydraulic gradient at a facility. However, the effects of mounding in the event of a release may produce a steeper local hydraulic gradient in the vicinity of the monitoring well. These local changes in hydraulic gradient should be accounted for in the velocity calculations.

Figure 14-7. Range of Values of Hydraulic Conductivity and Permeability



Source: Freeze, R.A., and J.A. Cherry. 1979. Ground Water. Prentice Hall, Inc., Englewood Cliffs, New Jersey. p. 29.

Figure 14-8. Conversion Factors for Permeability and Hydraulic Conductivity Units

	Permeability, *k**			Hydraulic conductivity, *K*		
	cm^2	ft^2	darcy	m/s	ft/s	$gal/day/ft^2$
cm^2	1	1.08×10^{-3}	1.01×10^{8}	9.80×10^{2}	3.22×10^{3}	1.85×10^{9}
ft^2	9.29×10^{2}	1	9.42×10^{10}	9.11×10^{5}	2.99×10^{6}	1.71×10^{12}
darcy	9.87×10^{-9}	1.06×10^{-11}	1	9.66×10^{-6}	3.17×10^{-5}	1.82×10^{1}
m/s	1.02×10^{-3}	1.10×10^{-6}	1.04×10^{5}	1	3.28	2.12×10^{6}
ft/s	3.11×10^{-4}	3.35×10^{-7}	3.15×10^{4}	3.05×10^{-1}	1	6.46×10^{5}
$gal/day/ft^2$	5.42×10^{-10}	5.83×10^{-13}	5.49×10^{-2}	4.72×10^{-7}	1.55×10^{-6}	1

*To obtain *k* in ft^2, multiply *k* in cm^2 by 1.08×10^{-3}

Source: Freeze, R.A., and J.A. Cherry (1979). *Ground Water*. Prentice Hall, Inc., Englewood Cliffs, New Jersey, p. 29.

The effective porosity, *Ne*, is the ratio, usually expressed as a percentage, of the total volume of voids available for fluid transmission to the total volume of the porous medium de-watered. It can be estimated during a pump test by dividing the volume of water removed from an aquifer by the total volume of aquifer dewatered (see calculation in **Example 14-5**). **Figure 14-9** presents approximate effective porosity values for a variety of aquifer materials. In cases where the effective porosity is unknown, specific yield may be substituted into the equation. Specific yields of selected rock units are given in **Figure 14-10**. In the absence of measured values, drainable porosity is often used to approximate effective porosity. **Figure 14-11** illustrates representative values of drainable porosity and total porosity as a function of aquifer particle size. If available, field measurements of effective porosity are preferred.

Figure 14-9. Default Values of Effective Porosity (*Ne*) For Travel Time Analyses

Soil textural classes	Effective porosity of saturation[a]
Unified soil classification system	
GS, GP, GM, GC, SW, SP, SM, SC	0.20 (20%)
ML, MH	0.15 (15%)
CL, OL, CH, OH, PT	0.01 (1%)[b]
USDA soil textural classes	
Clays, silty clays, sandy clays	0.01 (1%)b
Silts, silt loams, silty clay loams	0.10 (10%)
All others	0.20 (20%)
Rock units (all)	
Porous media (non-fractured rocks such as sandstone and some carbonates)	0.15 (15%)
Fractured rocks (most carbonates, shales, granites, etc.)	0.0001 (0.01%)

Source: Barari, A., and L. S. Hedges (1985). Movement of Water in Glacial Till. *Proceedings of the 17th International Congress of the International Association of Hydrogeologists*, pp. 129-134.

[a]These values are estimates and there may be differences between similar units. For example, recent studies indicate that weathered and unweathered glacial till may have markedly different effective porosities (Barari and Hedges, 1985; Bradbury et al., 1985).

[b]Assumes *de minimus* secondary porosity. If fractures or soil structure are present, effective porosity should be 0.001 (0.1%).

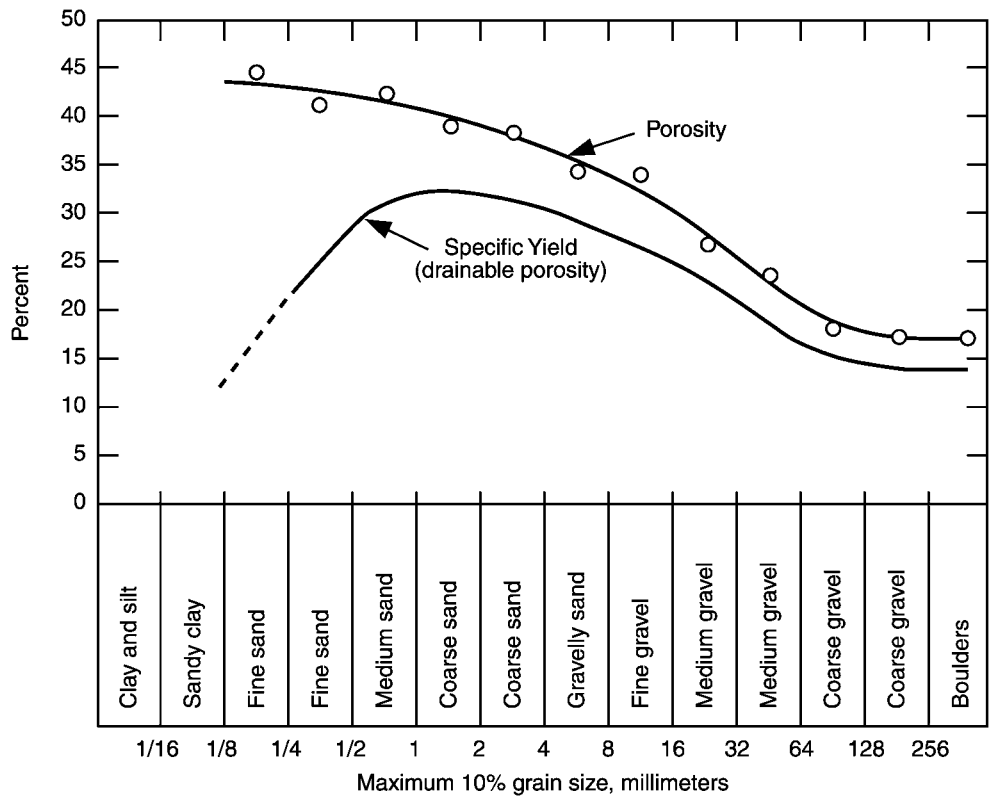
Figure 14-10. Specific Yield Values for Selected Rock Types

Rock Type	Specific Yield (%)
Clay	2
Sand	22
Gravel	19
Limestone	18
Sandstone (*semi-consolidated*)	6
Granite	0.09
Basalt (*young*)	8

Source: Heath, R.C. (1983). *Basic Ground-Water Hydrology*. U.S. Geological Survey, Water Supply Paper 2220, 84 pp.

Once the values for *K*, *i*, and *Ne* are determined, the horizontal component of average linear groundwater velocity can be calculated. Using the Darcy equation [14.17], the time required for groundwater to pass through the complete monitoring well diameter can be determined by dividing the well diameter by the horizontal component of the average linear groundwater velocity. If considerable exchange of water occurs during well purging, the diameter of the filter pack may be used rather than the well diameter. This value represents the ***minimum*** time interval required between sampling events yielding a physically independent (*i.e.*, distinct) ground-water sample. Note that three-dimensional mixing of groundwater in the vicinity of the monitoring well is likely to occur when the well is purged before sampling. Partly for that reason, this method can only provide an estimated travel time.

Figure 14-11. Total Porosity and Drainable Porosity for Typical Geologic Materials



(The grain size in which the cumulative total, beginning with the coarsest material, reaches 10% of the total sample.)

Source: Todd, D.K. 1980. Ground Water Hydrology. John Wiley and Sons, New York, 534 pp.

In determining these sampling intervals, many chemical compounds do not travel at the same velocity as groundwater. Chemical characteristics such as adsorptive potential, specific gravity, and molecular size influence the way chemicals travel in the subsurface. Large molecules, for example, tend to travel slower than the average linear groundwater velocity because of matrix interactions. Compounds that exhibit a strong adsorptive potential undergo a similar fate that dramatically changes time of travel predictions using the Darcy equation. In some cases chemical interaction with the matrix material alters the matrix structure and its associated hydraulic conductivity and may result in an increase in contaminant mobility. This effect has been observed with certain organic solvents in clay units (see Brown and Andersen, 1981). Contaminant fate and transport models may be useful in determining the influence of these effects on movement in the subsurface.

►EXAMPLE 14-5

Compute the effective porosity, *Ne*, expressed as a percent (%), using results obtained during a pump test.

SOLUTION

Step 1. Compute the effective porosity using the following equation:

$$Ne = 100\% \times \text{volume of water removed} / \text{volume of aquifer dewatered} \qquad [14.19]$$

Step 2. Based on a pumping rate of 50 gal/min and a pumping duration of 30 min, compute the volume of water removed as:

$$\text{volume of water removed} = 50 \text{ gal/min} \times 30 \text{ min} = 1{,}500 \text{ gal}$$

Step 3. To calculate the volume of aquifer de-watered, use the equation:

$$V = \frac{1}{3}\pi h r^2 \qquad [14.20]$$

where r is the radius (in ft) of the area affected by pumping and h (ft) is the drop in the water level. If, for example, h = 3 ft and r = 18 ft, then:

$$V = \frac{1}{3}\left(3.14 \times 3 \times 18^2\right) = 1{,}018 \text{ ft}^3$$

Next, converting cubic feet of water to gallons of water,

$$V = 1{,}018 \text{ ft}^3 \times 7.48 \text{ gal/ft}^3 = 7{,}615 \text{ gal}$$

Step 4. Finally, substitute the two volumes from **Step 3** into equation [14.19] to obtain the effective porosity:

$$Ne = 100\% \times \left(1{,}500 \text{ gal}/7{,}615 \text{ gal}\right) = 19.7\% \quad ◄$$

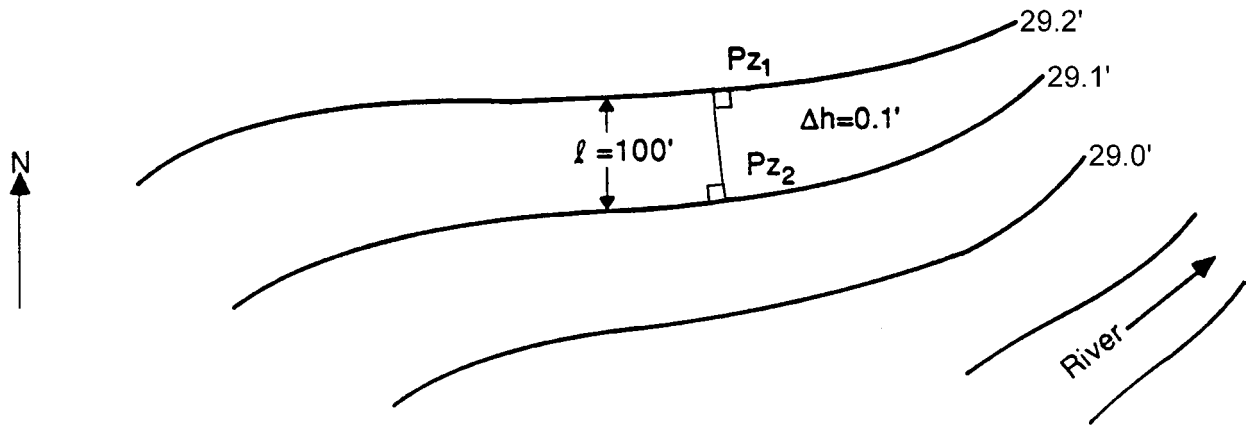
►EXAMPLE 14-6

Determine the hydraulic gradient, *i*, from a potentiometric surface map.

SOLUTION

Step 1. Consider the potentiometric surface map in **Figure 14-12**. The hydraulic gradient can be constructed as $i = \Delta h/l$, where Δh is the difference measured in the gradient at piezometers Pz_1 and Pz_2,and l is the orthogonal distance between the two piezometers.

Figure 14-12. Potentiometric Surface Map for Computation of Hydraulic Gradient



Step 2. Using the values given in **Figure 14-12**, the hydraulic gradient is computed as:

$$i = \Delta h / l = (29.2 \text{ ft} - 29.1 \text{ ft}) / 100 \text{ ft} = 0.001 \text{ ft/ft}$$

Step 3. Note that this method provides only a very general estimate of the natural hydraulic gradient existing in the vicinity of the two piezometers. Chemical gradients are known to exist and may override the effects of the hydraulic gradient. A detailed study of the effects of multiple chemical contaminants may be necessary to determine the actual average linear groundwater velocity (horizontal component) in the vicinity of the monitoring wells. ◄

►EXAMPLE 14-7

Determine the horizontal component of the average linear groundwater velocity (V_h) at a land disposal facility which has monitoring wells screened in an unconfined silty sand aquifer.

SOLUTION

Step 1. Slug tests, pump tests, and tracer tests conducted during a hydrologic site investigation have revealed that the aquifer has a horizontal hydraulic conductivity (K_h) of 15 ft/day and an effective porosity (Ne) of 15%. Using a potentiometric map (as in **Example 14-6**), the regional hydraulic gradient (i) has been determined to be 0.003 ft/ft.

Step 2. To estimate the minimum time interval between sampling events enabling the collection of physically independent samples of ground water, calculate the horizontal component of the average linear groundwater velocity (V_h) using Darcy's equation [14.17]. With K_h = 15 ft/day, Ne = 15%, and i = 0.003 ft/ft, the velocity becomes:

$$V_h = (15 \text{ ft/day} \times 0.003 \text{ ft/ft}) / 15\% = 0.3 \text{ ft/day or } 3.6 \text{ in/day}$$

Step 3. Based on these calculations, the horizontal component of the average linear groundwater velocity, V_h, is equal to 3.6 in/day. Since monitoring well diameters at this particular facility are 4 inches, the minimum time interval between sampling events enabling a physically

independent groundwater sample can be computed by dividing the horizontal component into the monitoring well diameter:

$$\text{Minimum time interval} = (4\ \text{in})/(3.6\ \text{in/day}) = 1.1\ \text{days}$$

As a result, the facility could theoretically sample every other day. However, this may be unwise because velocity can seasonally vary with recharge rates. It is also emphasized that *physical* independence does not guarantee *statistical* independence. **Figure 14-13** gives results for common situations. The overriding point is that it may not be necessary to set the minimum sampling frequency to quarterly at every site. Some hydrologic environments may allow for more frequent sampling, some less. ◀

Figure 14-13. Typical Darcy Equation Results in Determining a Sampling Interval

Unit	K_h (ft/day)	*Ne* (%)	V_h (in/mo)	Sampling Interval
Gravel	10^4	19	9.6×10^4	Daily
Sand	10^2	22	8.3×10^2	Daily
Silty Sand	10	14	1.3×10^2	Weekly
Till	10^{-3}	2	9.1×10^{-2}	Monthly
Silty Sand (semi-consolidated)	1	6	30	Weekly
Basalt	10^{-1}	8	2.28	Monthly

14.3.3 CREATING ADJUSTED, STATIONARY MEASUREMENTS

When an existing data set exhibits temporal correlation or other variability, it is sometimes possible to *remove* the temporal pattern and thereby create a set of adjusted data which are uncorrelated and stationary over time in mean level. As long as the same temporal pattern seems to affect both background and the compliance point data to be tested, the effect (*e.g.*, regular seasonal fluctuation) can be estimated and removed from both data sets prior to statistical testing. Testing the adjusted data instead of the raw measurements in this way results in a more powerful and accurate test. An extraneous source of variation *not related* to identifying a contaminant release has been removed from the sample data.

The general topic of stationary, adjusted data is complex, contained within the extensive literature on time series. The Unified Guidance discusses two simple cases below: removing a seasonal pattern from a single well and creating adjusted data from a one-way ANOVA for temporal effects across several wells. More complicated situations may need professional consultation.

14.3.3.1 CORRECTING FOR SEASONAL PATTERN IN A SINGLE WELL

BACKGROUND AND PURPOSE

Sometimes an obvious cyclical seasonal pattern can be seen in a time series plot. Such data are not statistically independent. They do not fluctuate randomly but rather in a predictable way from one sampling event to the next. Data from such patterns can be adjusted to correct for or remove the seasonal fluctuation, but only if a longer series of data is available. This is also known as *deseasonalizing* the data. Seasonal correction should be done both to minimize the chance of mistaking a seasonal effect for

evidence of contaminated groundwater, and also to build more powerful background-to-compliance point tests.

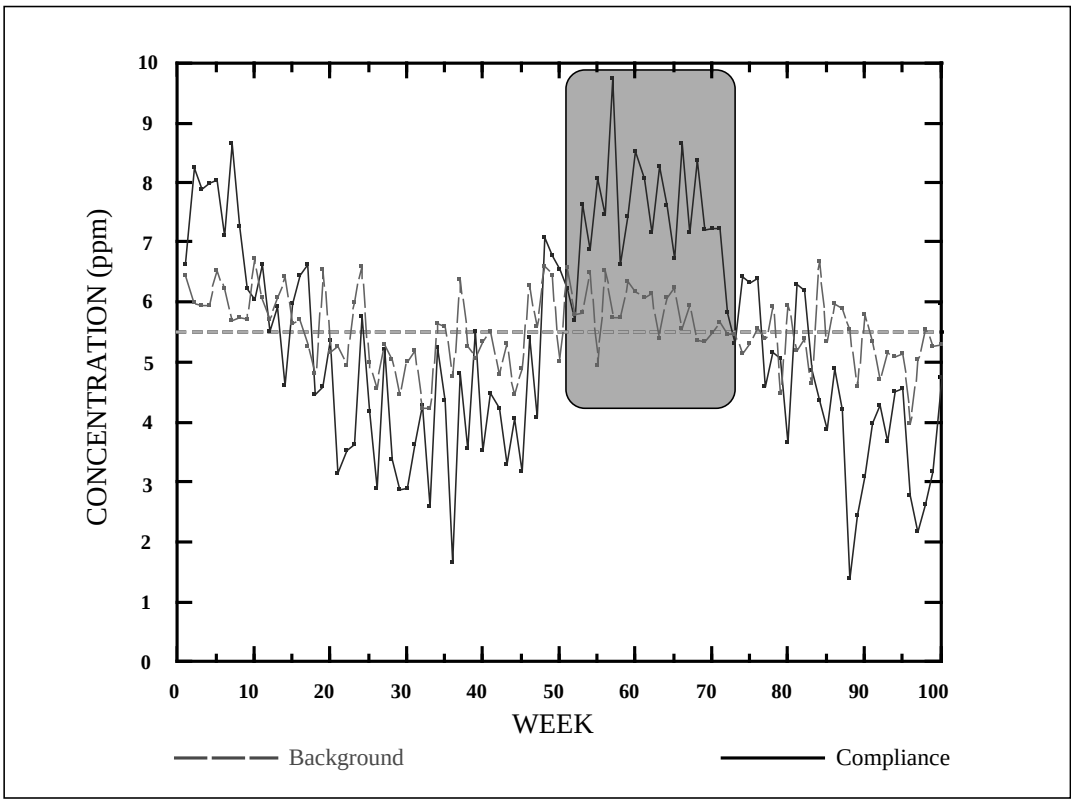
Problems can arise, for instance, from measurement variations associated with changing recharge rates during different seasons. Compliance point concentrations can exceed a groundwater protection standard [GWPS] for a portion of the year, but on average lie below. If the long-term average is of regulatory concern, the data should first be de-seasonalized before comparing it against a GWPS.

If *point-in-time, interwell comparisons* are being made between simultaneously collected background and downgradient data, a correction may not be necessary even when seasonal fluctuations exist. A temporal cycle may cover a period of several years so that both the background and downgradient values are observed on essentially the same parts of the overall cycle. In this case, the short-term averages in both data sets will be fairly stable and the seasonal or cyclical effect may equivalently impact both sets of data.

For intrawell tests, the data need to be collected sequentially at each well, with background formed from the earliest measurements in the series. The point-in-time argument would not apply and the presence of seasonality should be checked and accounted for.

Even with interwell testing, it is sometimes difficult to verify whether or not a seasonal pattern is impacting upgradient and compliance point wells similarly. If the groundwater velocity is low, the lag between the time groundwater passes through a background well screen and then travels downgradient may create a noticeable shift as to when corresponding portions of the seasonal cycle are observed in compliance point locations. It also may be the case that differences in geochemistry from well to well may cause the same seasonal pattern to differentially impact concentration levels at distinct wells (**Figure 14-14**).

Figure 14-14. Differential Seasonal Effects: Background vs. Compliance Wells



If the timing of the cycle and the relative *magnitude* of the concentration swings are essentially the same in upgradient and downgradient wells, both data sets should be deseasonalized prior to statistical analysis. If the seasonal effects are ignored, real differences in mean levels between upgradient and downgradient well data may not be observed, simply because the short-term seasonal fluctuations add variability that can mask the difference being tested. In this case, the non-independent nature of the seasonal pattern adds unwanted noise to the observations, obscuring statistical evidence of groundwater contamination.

REQUIREMENTS AND ASSUMPTIONS

Seasonal correction is only appropriate for wells where a cyclical pattern is clearly present and very regular over time. Many approaches to deseasonalizing data exist. If the seasonal pattern is highly regular, it may be modeled with a sine or cosine function. Often, moving averages and/or lag-based differences (of order 12 for monthly data, for example) are used. General time series models may include these and other more complicated methods for deseasonalizing the data.

The simple method described in the Unified Guidance has the advantage of being easy to understand and apply, and of providing natural estimates of the monthly or quarterly seasonal effects via the monthly or quarterly means. The method can be applied to any seasonal or recurring cycle-- perhaps an annual cycle for monthly or quarterly data or a longer cycle for certain kinds of geologic environments. In some cases, recharge rates are linked to drought cycles that may be on the order of

several years long. For these situations, the 'seasonal' cycle may not correspond to typical fluctuations over the course of a single year.

Corrections for seasonality should be used cautiously, as they represent extrapolation into the future. There should be a good physical explanation for the seasonal fluctuation as well as good empirical evidence for seasonality before corrections are made. Higher than average rainfall for two or three Augusts in a row does not justify the belief that there will never be a drought in August, and this idea extends directly to groundwater quality. At least three complete cycles of the seasonal pattern should be observed on a time series plot before attempting the adjustment below. If seasonality is suspected but the pattern is complicated, the user should seek the help of a professional statistician.

PROCEDURE

Step 1. If a time series plot clearly shows at least 3 full cycles of the seasonal pattern, determine the length of time to complete one full cycle. Since the correction presumes a regular sampling schedule, determine the number of observations (k) in each full cycle (this number should be the *same* for each cycle). Then, assuming that N complete cycles of data are available, let x_{ij} denote the raw, unadjusted measurement for the ith sampling event during the jth complete cycle. Note that this could represent monthly data over an annual cycle, quarterly data over a biennial cycle, semi-annual data over a 10-year cycle, *etc.*

Step 2. Compute the mean concentration for sampling event i over the N-cycle period:

$$\bar{x}_i = \left(x_{i1} + x_{i2} + \ldots + x_{iN}\right)/N \qquad [14.21]$$

This is the average of all observations taken in different cycles, but during the same sampling event. For instance, with monthly data over an annual cycle, one would calculate the mean concentrations for all Januarys, the mean for all Februarys, and so on for each of the 12 months.

Step 3. Calculate the grand mean, $\bar{x}$, of all $N \times k$ observations:

$$\bar{x} = \sum_{i=1}^{k}\sum_{j=1}^{N}\frac{x_{ij}}{N \times k} = \sum_{i=1}^{k}\frac{\bar{x}_i}{k} \qquad [14.22]$$

Step 4. Compute seasonally-corrected, adjusted concentrations using the equation:

$$z_{ij} = x_{ij} - \bar{x}_i + \bar{x} \qquad [14.23]$$

Computing $x_{ij} - \bar{x}_i$ removes the average seasonal effect of sampling event i from the data series. Adding back the overall mean, $\bar{x}$, gives the adjusted z_{ij} values the same mean as the raw, unadjusted data. Thus, the overall mean of the corrected values, $\bar{z}$, equals the overall mean value, $\bar{x}$.

►EXAMPLE 14-8

Consider the monthly unadjusted concentrations of an analyte over a 3-year period graphed in **Figure 14-15** and listed in the table below. Given the clear and regular seasonal pattern, use the above method to produce a deseasonalized data set.

	Unadjusted Concentrations				**Adjusted Concentrations**		
	1983	1984	1985	Monthly Average	1983	1984	1985
January	1.99	2.01	2.15	2.05	2.11	2.13	2.27
February	2.10	2.10	2.17	2.12	2.14	2.14	2.21
March	2.12	2.17	2.27	2.19	2.10	2.15	2.25
April	2.12	2.13	2.23	2.16	2.13	2.14	2.24
May	2.11	2.13	2.24	2.16	2.12	2.14	2.25
June	2.15	2.18	2.26	2.20	2.12	2.15	2.23
July	2.19	2.25	2.31	2.25	2.11	2.17	2.23
August	2.18	2.24	2.32	2.25	2.10	2.16	2.24
September	2.16	2.22	2.28	2.22	2.11	2.17	2.23
October	2.08	2.13	2.22	2.14	2.10	2.15	2.24
November	2.05	2.08	2.19	2.11	2.11	2.14	2.25
December	2.08	2.16	2.22	2.16	2.09	2.17	2.23

Overall 3-year average = 2.17

SOLUTION

Step 1. From **Figure 14-15**, there are $N = 3$ full cycles represented, each lasting approximately a year. With monthly data, the number of sampling events per cycle is $k = 12$.

Step 2. Compute the monthly averages across the 3 years for each of the 12 months using equation [14.21]. These values are shown in the fifth column of the table above.

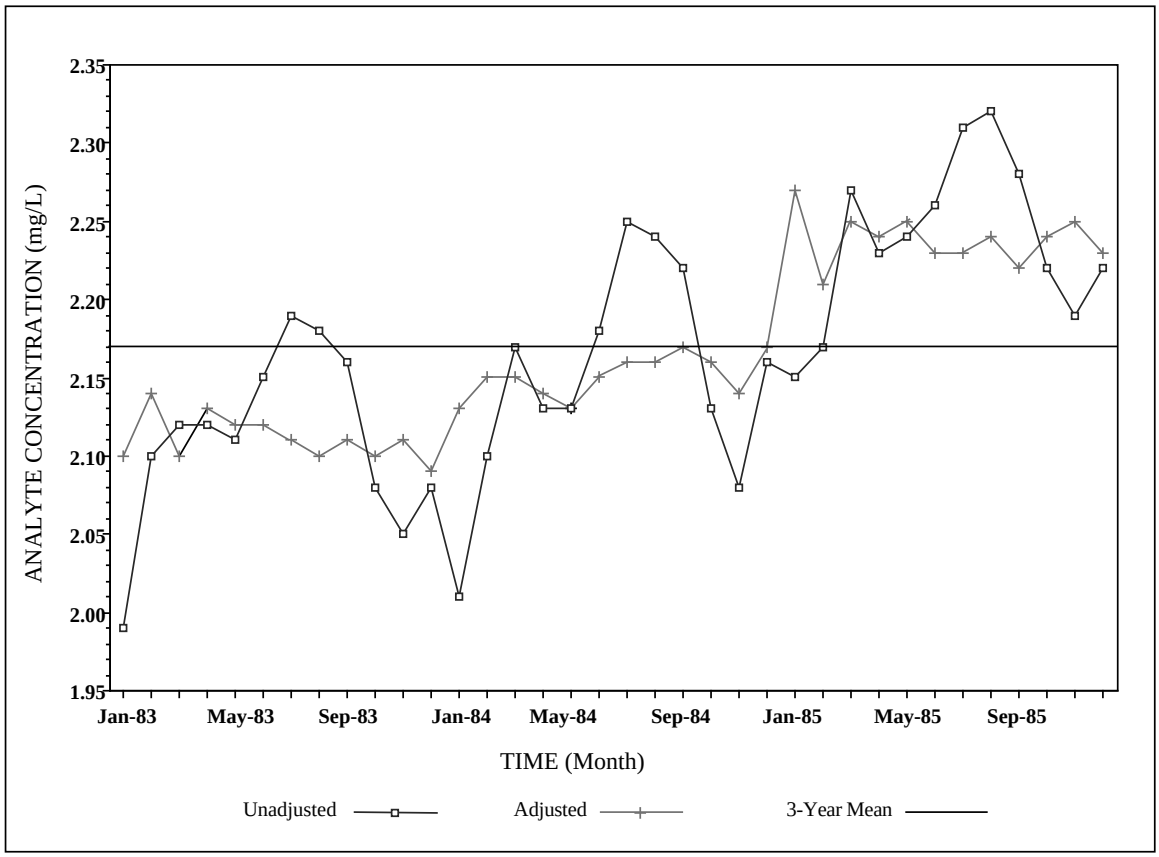
Step 3. Calculate the grand mean over the 3-year period using equation [14.22]:

$$\bar{x} = \frac{1}{3 \cdot 12}\left(1.99 + 2.01 + 2.15 + 2.10 \quad + \ldots + 2.22\right) = 2.17$$

Step 4. Within each month and year, subtract the average monthly concentration for that month and add-in the grand mean, using equation [14.23]. As an example, for January 1983, the adjusted concentration becomes:

$$z_{11} = 1.99 - 2.05 + 2.17 = 2.11$$

Figure 14-15. Seasonal Time Series Over a Three-Year Period



The adjusted concentrations are shown in the last three columns of the table above. The average of all 36 adjusted concentrations equals 2.17, the same as the mean *unadjusted* concentration. **Figure 14-15** shows the adjusted data superimposed on the unadjusted data. The raw data exhibit seasonality, as well as an upward trend. The adjusted data, on the other hand, no longer exhibit a seasonal pattern, although the upward trend still remains. From a statistical standpoint, the trend is much more easily identified by a trend test on the adjusted data than with the raw data. ◄

14.3.3.2 Correcting For A Temporal Effect Across Several Wells

BACKGROUND AND PURPOSE

When a significant temporal dependence or correlation is identified across a group of wells using one-way ANOVA for temporal effects (**Section 14.2.2**), results of the ANOVA can be used to create stationary adjusted data similar to the seasonal correction described in **Section 14.3.3.1**. The difference is that the adjustment is not applied to a data series at a single well, but rather simultaneously to several well sets.

The adjustment works in the same way as a correction for seasonality. First, the mean for each sampling event or season (averaged across wells) is computed along with the grand mean. Then each individual measurement is adjusted by subtracting off the event/seasonal mean and adding the overall or grand mean. In practice, this process is identical to adding the one-way ANOVA residual to the grand mean, so the already-computed results of the ANOVA can be used. By removing or correcting for a significant temporal effect, the adjusted data will have a temporally stationary mean and less overall variation. This allows for more powerful and accurate detection monitoring tests.

Temporal dependence (*e.g.*, seasonality) is sometimes observed as parallel traces on a time series plot across multiple wells (**Section 14.2.1**), although the one-way ANOVA for temporal effects is *non-significant*. This can occur due to the simultaneous presence of strong spatial variability (**Chapter 13**). Differences in mean levels from well to well can be large enough to 'swamp' the added variation due to the temporal dependence. The one-way ANOVA for temporal effects will not identify the dependence because the mean error sum of squares will then include the spatial variation component and not just random error.

Two remedies are possible when the ANOVA for temporal effects is non-significant. First, if a strong parallelism is evident on time series plots, the residuals from the ANOVA can still be used to create a set of adjusted, temporally-stationary measurements. The adjustment will not eliminate or remove any existing spatial variation, but it may not matter. Intrawell tests are needed anyway when such spatial variability is evident, and those tests assume temporal independence of the measurements collected at each well.

A second remedy is to perform a *two-way* ANOVA, testing for both spatial variation and temporal effects. This procedure is discussed in Davis (1994). Not only will a two-way ANOVA more readily identify a significant temporal effect even when there is simultaneous spatial variability, but the *F*-statistic used to test for the temporal dependence can be utilized to further adjust the appropriate degrees of freedom in intrawell background limits, such as prediction limits and control charts.

REQUIREMENTS AND ASSUMPTIONS

The key requirement to correct for a temporal effect using ANOVA is that the same effect must be present in all wells to which the adjustment is applied. Otherwise, the adjustment will tend to skew or bias measurements at wells with no observable temporal dependence. Parallel time series plots (**Section 14.2.1**) should be examined to determine whether all the wells under consideration exhibit a similar temporal pattern.

The parametric one-way ANOVA assumes the data are normal or can be normalized. If the data cannot be normalized, a Kruskal-Wallis non-parametric ANOVA can be conducted to test for the presence of a temporal dependence. In this case, no residuals can be computed since the Kruskal-Wallis test employs ranks of the data rather than the measurements themselves. So the adjustment presented below is only applicable for data sets that can be normalized.

PROCEDURE

Step 1. Given a set of W wells and measurements from each of T sampling events at each well on each of K years, label the observations as x_{ijk}, for i = 1 to W, j = 1 to T, and k = 1 to K. Then x_{ijk} represents the measurement from the ith well on the jth sampling event during the kth year.

Step 2. Using the one-way ANOVA for temporal effects (**Section 14.2.2**), compute the sampling event or seasonal means (whichever is appropriate), along with the grand (overall) mean. Also construct the ANOVA residuals using either equation [14.5] or [14.6].

Step 3. Add each residual to the grand mean to form adjusted values $z_{ijk} = x_{\bullet\bullet\bullet} + r_{ijk}$. Use these adjusted values in subsequent statistical testing instead of the original measurements.

►EXAMPLE 14-9

The manganese data of **Examples 14-1** and **14-2** were found to have a significant temporal dependence using ANOVA for temporal effects. Adjust these data to remove the temporal pattern.

	Manganese Residuals (ppm)				
Qtr	Event Mean	BW-1	BW-2	BW-3	BW-4
1	29.290	-1.15	2.12	-2.14	1.17
2	30.110	-0.78	0.16	0.13	0.49
3	30.780	-0.33	1.79	-1.64	0.18
4	31.620	0.80	1.15	-1.03	-0.92
5	33.747	0.6225	-0.7175	1.1325	-1.0375
6	31.930	1.32	0.25	-1.40	-0.17
7	30.513	0.5075	-1.6625	-0.1825	1.3375
8	30.345	-1.845	2.535	0.075	-0.765
	Grand mean = 31.042				

SOLUTION

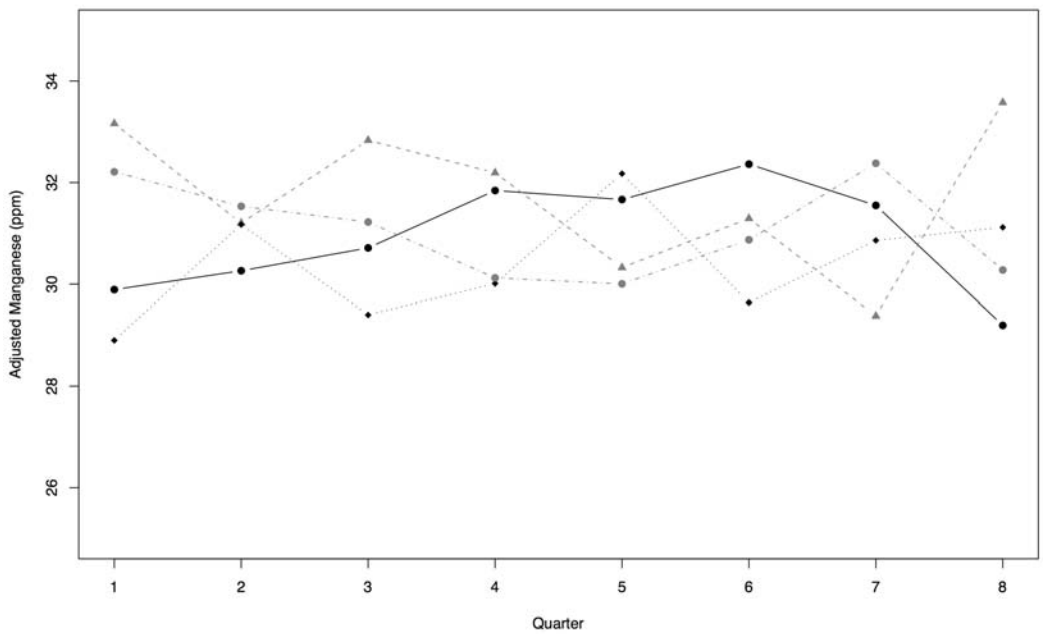
Step 1. The mean of each sampling event taken across the four background wells was computed in **Example 14-2**, along with the grand mean. These results are listed in the table above, along with the individual residuals which were also computed in that example.

Step 2. Add the grand mean to each residual to form the adjusted manganese concentrations, as in the table below.

	Adjusted Manganese (ppm)				
Qtr	Event Mean	BW-1	BW-2	BW-3	BW-4
1	29.290	29.89	33.16	28.90	32.21
2	30.110	30.26	31.20	31.17	31.53
3	30.780	30.71	32.83	29.40	31.22
4	31.620	31.84	32.19	30.01	30.12
5	33.747	31.66	30.32	32.17	30.00
6	31.930	32.36	31.29	29.64	30.87
7	30.513	31.55	29.38	30.86	32.38
8	30.345	29.20	33.58	31.12	30.28
	Grand mean = 31.042				

Step 3. Plot a time series of the adjusted manganese values, as in **Figure 14-16**. The 'hump-like' temporal pattern evident in **Figure 14-2** is no longer apparent. Instead, the overall mean is stationary across the 8 quarters. ◄

Figure 14-16. Parallel Time Series Plot of Adjusted Manganese Concentrations



14.3.3.3 CORRECTING FOR LINEAR TRENDS

If a data series exhibits a linear trend, the sample will exhibit temporal dependence when tested via the sample autocorrelation function (**Section 14.2.3**), the rank von Neumann ratio (**Section 14.2.4**), or similar procedure. These data can be de-trended, much like the data in the previous example were deseasonalized. Probably the easiest way to de-trend observations with a linear trend is to compute a linear regression on the data (**Section 17.3.1**) and then use the regression *residuals* instead of the original measurements in subsequent statistical analysis.

But no matter how tempting it may be to automatically de-trend data of this sort, the user is strongly cautioned to consider what a linear trend may represent. Often, an upward trend is indicative of changing groundwater conditions at a site, perhaps due to the increasing presence of contaminants during a gradual waste release. The trend in this case may itself be statistically significant evidence of groundwater contamination, particularly if it occurs at compliance wells but not at upgradient background wells. The trend tests of **Chapter 17** are useful for such determinations. Trends in background may signal other important factors, including migration of contaminants from off-site sources, changes in the regional aquifer, or possible groundwater mounding.

The overriding point is that data should be deseasonalized when a cyclical pattern might obscure the random deviations around an otherwise stable average concentration level, or to more clearly identify an existing trend. However, a linear trend is inherently indicative of a changing mean level. Such data should not be de-trended before it is determined what the trend likely represents, and whether or not it is itself *prima facie* evidence of possible groundwater contamination.

A similar trend both in direction and slope may be exhibited by background wells *and* compliance wells, perhaps suggestive of sitewide changes in natural groundwater conditions. Residuals from a one-way ANOVA for temporal effects (**Section 14.2.2**) can be used to simultaneously create adjusted values across the well network (**Section 14.3.3.2**). Linear trends are just as easily identified and adjusted in this way as are parallel seasonal fluctuations or other temporal effects.

14.3.4 IDENTIFYING LINEAR TRENDS AMIDST SEASONALITY: SEASONAL MANN-KENDALL TEST

BACKGROUND AND PURPOSE

Corrections for seasonality or other cyclical patterns over time in a single well are discussed in **Section 14.3.3.1**. These adjustments work best when the long-term mean at the well is stationary. In cases where a test for trend is desired and there are also seasonal fluctuations, **Chapter 17** tests may not be sensitive enough to detect a real trend due to the added seasonal variation.

One possible remedy is to use the seasonal correction in **Section 14.3.3.1** and illustrated in **Example 14-8**. The seasonal component of the trend is removed prior to conducting a formal trend test. A second option is the seasonal Mann-Kendall test (Gilbert, 1987).

The seasonal Mann-Kendall is a simple modification to the Mann-Kendall test for trend (**Section 17.3.2**) that accounts for apparent seasonal fluctuations. The basic idea is to divide a longer multi-year data series into subsets, each subset representing the measurements collected on a common sampling event (*e.g.*, all January events or all fourth quarter events). These subsets then represent different points along the regular seasonal cycle, some associated with peaks and others with troughs. The usual Mann-Kendall test is performed on each subset separately and a Mann-Kendall test statistic S_i formed for each. Then the separate S_i statistics are summed to get an overall Mann-Kendall statistic S.

Assuming that the same basic trend impacts each subset, the combined statistic S will be powerful enough to identify a trend despite the seasonal fluctuations.

REQUIREMENTS AND ASSUMPTIONS

The basic requirements of the Mann-Kendall trend test are discussed in **Section 17.3.2**. The only differences with the seasonal Mann-Kendall test are that 1) the sample should be a multi-year series with an observable seasonal pattern each year; 2) each 'season' or subset of the overall series should include at least three measurements in order to compute the Mann-Kendall statistic; and 3) a normal approximation to the overall Mann-Kendall test statistic must be tenable. This will generally be the case if the series has at least 10-12 measurements.

PROCEDURE

Step 1. Given a series of measurements from each of T sampling events on each of K years, label the observations as x_{ij}, for i = 1 to T, and j = 1 to K. Then x_{ij} represents the measurement from the ith sampling event during the jth year.

Step 2. For each distinct sampling event (i), form a seasonal subset by grouping together observations $x_{i1}, x_{i2},...., x_{iK}$. This results in T separate seasons.

Step 3. For each seasonal subset, use the procedure in **Section 17.3.2** to compute the Mann-Kendall statistic S_i and its standard deviation $SD[S_i]$. Form the overall seasonal Mann-Kendall statistic (S) and its standard deviation with the equations:

$$S = \sum_{i=1}^{T} S_i \qquad [14.24]$$

$$SD\left[S\right] = \sqrt{\sum_{i=1}^{T} SD^2\left[S_i\right]} \qquad [14.25]$$

Step 4. Compute the normal approximation to the seasonal Mann-Kendall statistic using the equation:

$$Z = \left(\left|S\right| - 1\right) \Big/ SD\left[S\right] \qquad [14.26]$$

Step 5. Given significance level, α, determine the critical point z_{cp} from the standard normal distribution in **Table 10-1** of **Appendix D**. Compare Z against this critical point. If $Z > z_{cp}$, conclude there is statistically significant evidence at the α-level of an increasing trend. If $Z < -z_{cp}$, conclude there is statistically significant evidence of a decreasing trend. If neither, conclude that the sample evidence is insufficient to identify a trend.

►EXAMPLE 14-10

The data set in **Example 14-8** replicated below indicated both clear seasonality and an apparent increasing trend. Use the seasonal Mann-Kendall procedure to test for a significant trend with α = 0.01 significance.

	Analyte Concentrations				
	1983	1984	1985	S_i	$SD[S_i]$
January	1.99	2.01	2.15	3	1.915
February	2.10	2.10	2.17	2	1.633
March	2.12	2.17	2.27	3	1.915
April	2.12	2.13	2.23	3	1.915
May	2.11	2.13	2.24	3	1.915
June	2.15	2.18	2.26	3	1.915
July	2.19	2.25	2.31	3	1.915
August	2.18	2.24	2.32	3	1.915
September	2.16	2.22	2.28	3	1.915
October	2.08	2.13	2.22	3	1.915
November	2.05	2.08	2.19	3	1.915
December	2.08	2.16	2.22	3	1.915
				S = 35	$SD[S]$= 6.558

SOLUTION

Step 1. Form a seasonal subset for each month by grouping all the January measurements, all the February measurements, and so on, across the 3 years of sampling. This gives 12 seasonal subsets with n = 3 measurements per season. Note there are no tied values in any of the seasons except for February.

Step 2. Use equations [17.30] and [17.31] in **Section 17.3.2** to compute the Mann-Kendall statistic (S_i) for each subset. These values are listed in the table above. Also compute their sum to form the overall seasonal Mann-Kendall statistic, giving S = 35.

Step 3. Use equation [17.28] from **Section 17.3.2** for all months but February to compute the standard deviation of S_i. Since n = 3 for each of these subsets, this gives

$$SD\left[S_i\right] = \sqrt{\frac{1}{18} n(n-1)(2n+5)} = \sqrt{\frac{1}{18} 3 \cdot 2 \cdot 11} = 1.915$$

For the month of February, one pair of tied values exists. Use equation [17.27] to compute the standard deviation for this subset:

$$SD\left[S_i\right] = \sqrt{\frac{1}{18}\left[n(n-1)(2n+5) - \sum_{j=1}^{g} t_j\left(t_j - 1\right)\left(2t_j + 5\right)\right]} = \sqrt{\frac{1}{18}\left[3 \cdot 2 \cdot 11 - 2 \cdot 1 \cdot 9\right]} = 1.633$$

List all the subset standard deviations in the table above. Then use equation [14.25] to compute the overall standard deviation:

$$SD\left[S\right] = \sqrt{\sum_{i=1}^{12} SD^2\left[S_i\right]} = \sqrt{11 \cdot (1.915)^2 + (1.633)^2} = 6.558$$

Step 4. Compute a normal approximation to S with equation [17.29]:

$$Z = \left(35 - 1\right) / 6.558 = 5.18$$

Step 5. Compare Z against the 1% critical point from the standard normal distribution in **Table 10-1** of **Appendix D**, $z_{.01}$ = 2.33. Since Z is clearly larger than $z_{.01}$, the increasing trend evidence in **Figure 14-15** is highly significant. ◄

CHAPTER 15. MANAGING NON-DETECT DATA

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This chapter considers strategies for accommodating non-detect measurements in groundwater data analysis. Five particular methods are described for incorporating non-detects into parametric statistical procedures. These include:

- Simple substitution (**Section 15.2**);
- Kaplan-Meier (**Section 15.3**);
- Robust Regression on Order Statistics (**Section 15.4**);
- Cohen's Method (**Section 15.5.1**); and
- Parametric Regression on Order Statistics (**Section 15.5.2**).

15.1 GENERAL CONSIDERATIONS FOR NON-DETECT DATA

Non-detects commonly reported in groundwater monitoring are statistically known as "left-censored" measurements, because the concentration of any non-detect either cannot be estimated or is not reported directly. Rather, it is known or assumed only to fall within a certain range of concentration values (*e.g.*, between zero and the *quantitation limit* [QL]). The direct estimate has been censored by the limitations of the measurement process or analytical technique, and is deemed too uncertain to be considered reliable. Groundwater non-detect data are censored on the low or left end of a sample concentration range. Other kinds of threshold data, particularly survival rates in the medical literature, are often reported as right-censored values.

Historically, there has been inconsistent treatment of non-detects in groundwater analysis. Often, easily applied techniques have been favored over more sophisticated methods of handling non-detects. This may primarily be due to the lack of familiarity and difficulties with software that can incorporate such methods. Even at present, most statistical packages include analysis routines for right-censored values but not left-censored ones (Helsel, 2005). Left-censored data needs to be converted to right-censored data for analysis and then back again. Despite these limitations, the more sophisticated methods are almost always superior to the methods of simple substitution.

The past twenty years has seen considerable research on statistical aspects of non-detect data analysis. Helsel (2005) provides a detailed summary of available methods for non-detects, and

concludes that simple substitution usually leads to greater statistical bias and inaccuracy than with better technical methods. Gibbons (1994b) and Gibbons & Coleman (2001) offer a broad review of some of the same research, not all of it directly relating to groundwater data. Both Gibbons and McNichols & Davis (1988) note that most of the existing studies focus on an *estimation of parameters* such as the mean and variance of an underlying population from which the censored and detected data originate. For these tasks, simple substitution methods tend to perform poorly, especially when the non-detect percentages are high (Gilliom & Helsel, 1986).

Much less attention has been given to how left-censored data impact the *results of statistical tests*, the actual data-based conclusions that are drawn when using detection, compliance, or corrective action monitoring tests. Closely estimating the true mean and variance of the underlying background population may be important, but does not directly answer how well a given test performs (in achieving the nominal false positive error rate and correctly identifying true significant differences). McNichols & Davis (1988) performed a limited study to address these concerns. They found that simple substitution methods were among the best performers in simulated prediction limit tests even with fairly high rates of censoring, *so long as* the prediction limit procedure incorporated a verification resample.

Gibbons (1994b; also Gibbons and Coleman, 2001) conducted a similar limited simulation of prediction limit testing performance incorporating a verification resample. They, too, found that a type of simple substitution was one of the best performers when either an average of 20% or 50% of the data was non-detect. The Gibbons study concluded that substituting zero for each non-detect worked better to keep the false positive rate low than by substituting half the method detection limit [MDL].

Both studies primarily focused on the achievable false positive rate when censored data are present, rather than the statistical power of these tests to identify contaminated groundwater. In addition, both only considered parametric prediction limits. For data sets with fairly low detection frequencies (*e.g.*, <50%), parametric prediction limits may not accurately accommodate left-censored measurements, with or without retesting. The McNichols & Davis study in particular found that *none* of the simpler methods for handling non-detects did well when the underlying data came from a skewed distribution and the non-detect percentage was over 50%.

On balance, there are four general strategies for handling non-detects: 1) employing a test specifically designed to accommodate non-detects, such as the Tarone-Ware two-sample alternative to the *t*-test (**Section 16.3**); 2) using a rank-based, non-parametric test such as the Kruskal-Wallis alternative to analysis of variance [ANOVA] (**Section 17.1.2**) when the non-detects and detects can be jointly sorted and ordered (except for tied values); 3) estimating the mean and standard deviation of samples containing non-detects by means of a *censored estimation technique*; and 4) *imputing an estimated value* for each non-detect prior to further statistical manipulation.

The first two strategies mentioned above are discussed in **Chapters 16** and **17** of the Unified Guidance as alternative testing procedures for evaluating left-censored data when parametric distribution assumptions cannot be made. Tests that can accommodate non-detects are typically non-parametric and thus carry both the advantages and disadvantages of non-parametric methods. The third and fourth strategies — presented in this chapter — are often employed as an *intermediate step* in parametric analyses. Estimates of the background mean and standard deviation are needed to construct parametric prediction and control chart limits, as well as confidence intervals. Imputed values for individual non-detects can be used as an alternate way to construct mean and standard deviation estimates, which are

needed to update the *cumulative sum* [CUSUM] portion of control charts or to compute the means of order *p* that get compared against prediction limits.

The guidance generally favors the use of the more sophisticated Kaplan-Meier or Robust ROS methods which can address the problem of multiple detection limits. Two older techniques-- Cohen's method and parametric ROS-- are also included as somewhat easier methods which can work in some circumstances. Applying any of the four estimation techniques as well as simple substitution does rely on a fundamental underlying assumption. Both the detectable and non-detect portions of a data set are assumed to arise from a single distribution, and in particular this underlying population is expected to be stable or *stationary* during the period of the sampling record.

However, if an underlying distribution is subject to a trend over time, applying any of these techniques including simple substitution is more problematic. If data indicating a decreasing trend also happen to contain multiple detection limit data (perhaps the result of improved analytical methods), it may be very difficult to determine whether there is truly a trend or analytical problems are the apparent cause of the observed decreases. None of the techniques provided in this chapter can directly address this issue. As discussed in **Chapter 5**, careful exploratory review of the historical data sets, particularly those which might serve as background, need to consider which data including non-detects are most representative of present or near-term future conditions. In some cases, removal of the older, less reliable data may also resolve multiple detection limit problems. If non-detect values higher than other quantified data at reasonable detection limits are included in a data set (especially if dictated by reporting policy rather than analytical considerations), these will almost invariably need to be removed. Even sophisticated multiple detection limit techniques cannot realistically address these particular information-limited data values. But presuming valid and reliable data are selected, the four estimation techniques are provided to address the management of non-detects.

A data set may also not be defined by a single distribution. If observed data are the result of two or more different generative processes and indicate one or more separate peaks, it is referred to as a mixture distribution. One example might be trace organics data in a release subject to changes in the flow direction of the aquifer, which can result in very high to absent values. The subject is a complex one and generally beyond the scope of this guidance. Aitchison's method can be used in limited situations where detectable data form one discrete distribution, and the remainder are non-detect. The following discussion also addresses when Aitchison's method might be appropriate. The non-detect data are simply considered as some single value, another form of simple substitution.

15.2 IMPUTING NON-DETECT VALUES BY SIMPLE SUBSTITUTION

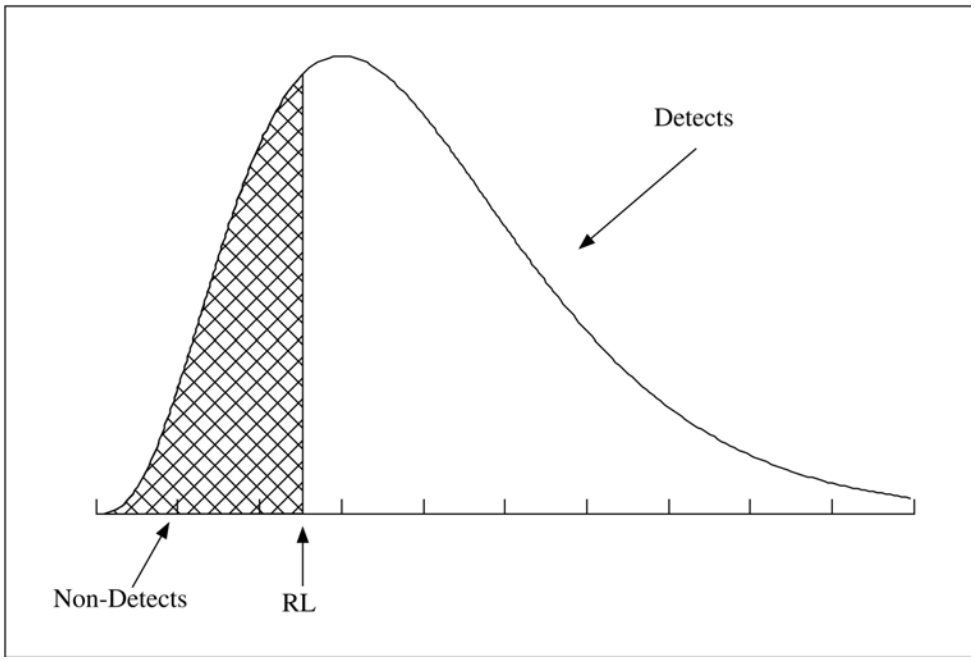
The simplest approach in managing non-detects is to substitute an *imputed value* for each prior to subsequent statistical analysis. The imputation is intended to be a ‘reasonable estimate’ of the true, but unknown concentration, usually a fraction (*e.g.*, 0, ½, 1) of the reporting limit [RL]. If non-detects represent an *absence* of the contaminant being measured, replacing a reported ‘less than’ value by zero may make sense. If the true concentration is completely unknown, but believed to be between zero and the RL, half the RL, or RL/2, may be a reasonable substitution, since this choice is the *maximum likelihood estimate* [MLE] of the mean or median for a population of measurement values *uniformly*

distributed along the interval [0, RL].[1] In other cases, a conservative choice might be made to maximize the *possible* concentration levels present in non-detects by selecting the RL itself as the imputation.

Any of these substitution choices is imperfect since they ignore two realities about left-censored measurements. First, non-detects are a product of *both* the underlying distribution of actual concentrations *and* the measurement process used to estimate these concentrations. In particular, the measurement technique may impart random or not so random bias to the 'true' concentration levels, causing the reported values to be 'shifted away from' the true values. As an example, simple substitution of zero for each non-detect ignores the fact that only the *measurements* can be observed and analyzed, not the actual concentration levels. Physical groundwater samples that are completely devoid of a given chemical may not receive *measurements* of zero, even if the actual amount is zero. Simple substitution by zero thereby ignores the *measurement distribution* in favor of an *a priori* assumption about what non-detects might represent.

A second reality is that non-detects must be considered with respect to other, detected measurements, as well as the physical process that generated the data. In many cases, the entire sample is drawn from a single statistical distribution (representing a common physical process) but some portion of the lower tail has been censored during measurement, as illustrated in **Figure 15-1**. In this situation, the overall distribution (and especially the shape of the lower tail) dictates how likely it is that a given non-detect would have an *uncensored* measurement close to zero or close to the RL. Substitution by half the RL or by the RL itself ignores the larger distributional pattern, especially since this distribution will rarely be uniform in the interval [0, RL].

Figure 15-1. Single Distributional Model For Detects and Non-Detects



[1] The uniform distribution places equal probability along every point of a finite concentration or measurement range. This model implies that a true value close to zero is just as likely as a true value close to RL or any other point along the interval.

These realities can lead to severe biases in statistical parameter estimates made from censored data when simple substitution methods are used (Helsel, 2005). Even if only 20% of the data are censored, Gibbons (1994b) found that the false positive rate of a prediction limit test was far above the *nominal* (*i.e.*, expected or targeted) rate of $\alpha = .05$ when a simple imputation strategy was employed. For that reason, the Unified Guidance recommends imputation by simple substitution only in select circumstances described below:

❖ ***When the sample size is too small to do anything else***.

With only a handful of measurements (*e.g.*, 5 or less), it will be almost impossible to accurately apply a censored estimation technique, such as those described in **Sections 15-3 to 15-5**. Instead, simple substitution of half the RL is recommended, perhaps until enough data has been collected to allow a more sophisticated analysis. Three situations where simple substitution might commonly be needed include:

1. Plotting cumulative sums [CUSUM] on control charts (**Chapter 20**). While there should be enough background data to allow for a more sophisticated estimate of the control limit, the CUSUM must be updated with each single new compliance observation ($n = 1$). If the new measurement is a non-detect, the value must be imputed for the CUSUM to be calculated.

2. Constructing future means for prediction limits (**Chapter 19**). Again, if censored data exist in background, the prediction limit for a future mean can be computed with the help of a censored estimation technique. But with only 2 or 3 new measurements per compliance well ($p = 2, 3$), the same strategy will not work for computing a mean of order p.

3. Construction of confidence intervals in compliance monitoring or corrective action. Especially in the early months or years after the onset of compliance monitoring or a corrective action plan, there may be too few compliance point measurements to allow for a statistically refined treatment of non-detects. Until more data has been collected that is representative of the conditions under which these phases of monitoring have been triggered, simple substitution of non-detects will probably be needed. Furthermore, if groundwater conditions are in a state of flux, it may be impossible — even with a larger sample size — to postulate a single, stationary distributional model (similar to **Figure 15-1**) on which to base a censored estimation technique.

❖ ***When non-detects comprise no more than 10-15% of the total sample***.

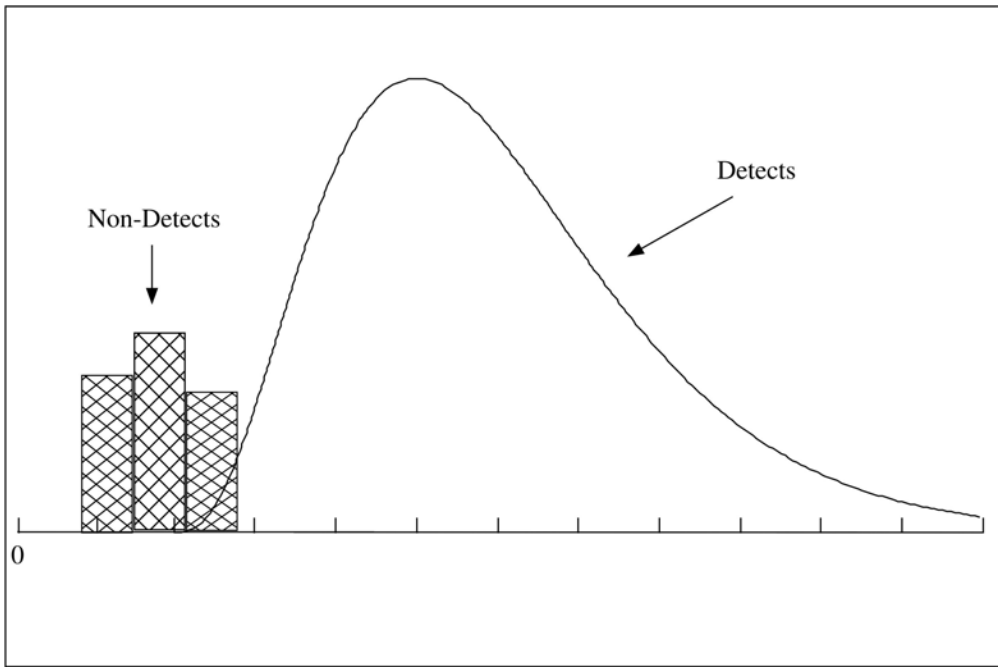
If the percentage of non-detects is small enough, results of parametric *t*-tests and ANOVA are usually not significantly affected if non-detects are first replaced by half their reporting limits [RLs]. A similar statement can be made for parametric prediction limits, tolerance limits, control charts, and confidence intervals. However, because *t*-tests and ANOVA involve a comparison of means utilizing multiple data points per mean estimate,[2] while prediction limits for individual observations, tolerance limits, and control charts focus on single measurements, it is important that *retesting* be included in the statistical procedure whenever simple substitution is utilized with these latter methods.

[2] Parametric confidence intervals around the mean also involve an estimate of the population average using multiple data points.

- ***When non-detects are generated by a different physical process than the detected values, and thus represent a distinct statistical distribution***.

One non-detect treatment recommended in past EPA guidance — Aitchison's method (1955), as applied to groundwater[3] — assumed that non-detects were actually *free* of the contaminant being measured, so that all non-detects could be regarded as zero concentrations. In some cases, if an analyte has been detected infrequently or not at all in background measurements, and/or all non-detects are qualified as "U" (*i.e.*, undetected) values, this assumption may be practical, even if it cannot be directly verified. Another example might be seasonal changes in groundwater elevation at wells located on the edges of a contaminant plume. Parameters detectable at certain times of the year may be non-detect during other seasons, even within the same well. Such non-detects may result from a different data-generating mechanism, due to seasonal changes in groundwater chemistry, and so may not follow the same distribution as detects.

Figure 15-2. Modified Delta Model For Mixture Distribution of Detects/Non-Detects



More generally, Aitchison's original model posited a 'spike' of zero-valued measurements, combined with a lognormal distribution governing the detected values. A modification to Aitchison's model known as the *modified delta method*[4] (USEPA, 1993) has been found to be more practical and realistic in many circumstances (**Figure 15-2**). Instead of assuming that all non-detects represent zero

[3] Aitchison's model was not originally applied to concentration data. More typical applications were in the fields of economics and demographics.

[4] The original Aitchison model was termed the *delta-lognormal*, so called because it presumed that the data consisted of a mixture of two distinct populations: 1) a lognormal distribution representing the observed continuous measurements, and 2) a 'spike' of values, known as a delta function, located at zero.

concentrations, the modified delta method assumes that non-detects constitute a separate, discrete distribution. When combined with the detected values, a *mixture distribution* is formed consisting of a continuous detected portion (usually the normal or lognormal distribution) and a discrete non-detect portion. Rather than assuming that all non-detects are zeros, the modified delta model assigns all non-detects at half the reporting limit [RL]. (Note: this might be a method detection limit [MDL], a quantitation limit [QL], or a contract RL). This method can accommodate multiple reporting limits since each non-detect is assigned half of its possibly sample-specific RL. It can also accommodate low-valued detects *intermingled* with the non-detects, since the non-detects and detects are modeled by distinct distributions.

15.3 ESTIMATION BY KAPLAN-MEIER

BACKGROUND AND PURPOSE

When a sample contains both detects and non-detects generated by a common process and governed by a single underlying distribution (**Figure 15-1**), a more reliable strategy is to attempt to fit the sample to a known distribution (*e.g.*, normal, lognormal) and then to estimate the mean and standard deviation of this distribution via a *censored estimation technique*. These adjusted estimates can be input into standard equations for parametric prediction, tolerance, and control chart limits, as well parametric confidence intervals around the mean.

Two censored estimation methods which can address the multiple detection limit problem are discussed in the Unified Guidance: the Kaplan-Meier estimator and *robust regression on order statistics* [ROS] (**Section 15.4**). Both involve initially fitting a left-censored sample to a known distribution. After that, the procedures differ. The Kaplan-Meier creates an estimate of the population mean and standard deviation adjusted for data censoring, based on the fitted distributional model, whereas the Robust ROS uses the fitted model to construct a *model-based imputation* for each non-detect. Once the imputations are made, the adjusted mean and standard deviation are estimated using standard equations for the sample mean ($\bar{x}$) and standard deviation (s).

The key to either method is finding a single distributional model that adequately fits the joint sample of detects and non-detects. While each procedure does the fitting in a slightly different fashion, both utilize the notion of *partial ranking*. As discussed in **Section 16.2** on "Handling Non-Detects," the presence of left-censored measurements, particularly when there are multiple RLs and/or an *intermingling* of detects and non-detects, prevents a full and complete ranking of the sample. Both Kaplan-Meier and ROS construct a partial ranking of the data, accounting for the non-detects and assigning explicit ranks to each of the detected values. These detected values can then be graphed on a *censored probability plot* and fitted against a known distribution.

The Kaplan-Meier technique estimates the approximate proportion of concentrations below each observed level by sorting and ordering the distinct sample values, although the exact concentrations of non-detects are unknown. In particular, the probability of observing a concentration no greater than a given level (x_i) depends on the relative proportion of the sample greater than x_i. Any detects larger than x_i obviously fall into this latter proportion, while non-detects with RLs of at most x_i do not. On balance, the proportion of the sample greater than x_i cannot be precisely calculated for every x_i, but it can be estimated.

The Kaplan-Meier estimator for left-censored data thus depends on a series of *conditional* probabilities, where the frequency of lower concentrations depends on how many larger concentrations have already been observed. The final result is an estimate of the *cumulative distribution function* [CDF] for each distinct concentration level in the sample.

In mathematical notation, suppose there are m distinct values in the sample (out of a total of n measurements), including distinct reporting limits. Order these values from least to greatest and denote them as $x_{(1)}, x_{(2)}, \ldots, x_{(m)}$. Let n_i for $i = 1$ to m denote the 'risk set' associated with value $x_{(i)}$. The *risk set* represents the total number of measurements — both detects and non-detects — no greater than $x_{(i)}$. Since a non-detect with a RL larger than $x_{(i)}$ is *potentially* (but not necessarily) larger than $x_{(i)}$, non-detects with RL > $x_{(i)}$ are *not* included in n_i. A further term d_i identifies the number of detected measurements exactly equal to $x_{(i)}$.

With these definitions in place and letting X denote a random variable concentration from the true underlying distribution, the Kaplan-Meier estimator is constructed from the pair of probabilities:

$$\Pr\left(X \le x_{(m)}\right) = 1 \qquad [15.1]$$

$$\Pr\left(X \le x_{(i)} \middle| X \le x_{(i+1)}\right) = 1 - \frac{d_{i+1}}{n_{i+1}} \quad for\ \ i = 1\ \ to\ \ m \qquad [15.2]$$

where $x_{(m+1)} = +\infty$, $d_{m+1} = 0$, and $n_{m+1} = n$ all by definition. Equation [15.2] represents the conditional probability that the concentration does not exceed $x_{(i)}$ given that it does not exceed $x_{(i+1)}$. The final Kaplan-Meier CDF estimate (F_{KM}) for each $i = 1$ to m–1 (each distinct detected value) is given by a product of these conditional probabilities and can be expressed as:

$$F_{KM}\left(x_{(i)}\right) = \Pr\left(X \le x_{(i)}\right) = \left(1 - \frac{d_{i+1}}{n_{i+1}}\right) \times \left(1 - \frac{d_{i+2}}{n_{i+2}}\right) \times \ldots \times \left(1 - \frac{d_m}{n_m}\right) = \prod_{k=i}^{m-1}\left(1 - \frac{d_{k+1}}{n_{k+1}}\right) \qquad [15.3]$$

Once the CDF is estimated using equation [15.3], two additional steps are made possible. One is to use the distinct values ($x_{(i)}$) and their corresponding CDF values (F_{KM}) to construct censored probability plots. The other is to use the Kaplan-Meier CDF to estimate the population mean and standard deviation.

REQUIREMENTS AND ASSUMPTIONS

The Kaplan-Meier estimator is a non-parametric procedure originally devised to estimate *survival probabilities* for right-censored samples (Kaplan and Meier, 1958), such as in medical studies of cancer treatments. Because it is non-parametric, there is no requirement that the underlying population be normal or transformable to normality. However, in adapting the technique to left-censored data (*i.e.*, samples containing non-detects), the Unified Guidance recommends that the Kaplan-Meier procedure be utilized to estimate the mean and variance of a normal or normalized distribution for use in *parametric* statistical tests.

The Kaplan-Meier assumes that all detected and non-detect data arise from the same population, but that non-detect values have been 'censored' at their RLs. This implies that the contaminant of

concern is *actually present* in non-detect samples, but that the analytical method cannot accurately measure, or is not sufficiently sensitive to, concentrations lower than the RL.

To construct a censored probability plot, a normal quantile or z-score needs to be computed for each value of the Kaplan-Meier CDF (F_{KM}). Doing so is straightforward except for the CDF value of the sample maximum, which is assigned a value of one. The z-score associated with a cumulative probability of one is infinite. To surmount this difficulty, the Unified Guidance recommends temporarily setting the CDF value for the sample maximum equal to $(n - .375)/(n + .25)$. This value is the Blom plotting position often utilized in standard probability plots (Helsel, 2005). It is close to one for large n, but allows for a finite z-score.

Estimation of the Kaplan-Meier mean and standard deviation using equations [15.4] and [15.5] below will tend to be slightly biased, typically with the mean on the high side and the standard deviation on the low side. This occurs because the Kaplan-Meier CDF levels corresponding to distinct RLs are treated as if they were known measurements rather than the upper bounds on possible values. As long as the total proportion of censored measurements is not too high, the degree of bias will tend to be small. Larger biases are more likely whenever the detection rate is less than 50%.

PROCEDURE

Step 1. Given a sample of size n containing left-censored measurements, identify and sort the $m < n$ distinct values, including distinct RLs. Label these as $x_{(1)}, x_{(2)}, \ldots, x_{(m)}$.

Step 2. For each $i = 1$ to m, calculate the *risk set* (n_i) as the total number of detects and non-detects no greater than $x_{(i)}$. Also compute d_i as the number of *detected* values exactly equal to $x_{(i)}$.

Step 3. Using equation [15.3], compute the Kaplan-Meier CDF estimate $F_{KM}\left(x_{(i)}\right)$ for $i = 1, \ldots, m-1$. Also let $F_{KM}\left(x_{(m)}\right) = 1$.

Step 4. Construct censored probability plots using the estimated CDF. First temporarily set $F_{KM}\left(x_{(m)}\right) = (n - .375)/(n + .25)$ so that a finite normal quantile (or z-score; see **Chapter 9**) can be associated with $x_{(m)}$. Then compute normal quantiles (*i.e.*, z-scores) for each value of F_{KM} from **Step 3** as $z_{(i)} = \Phi^{-1}\left[F_{KM}\left(x_{(i)}\right)\right]$, where $\Phi^{-1}[\cdot]$ is the inverse of the standard normal distribution function as discussed in the construction of probability plots in **Chapter 9**. Plot the values $z_{(i)}$ against the unique detected concentrations $x_{(i)}$ to form a *normal* censored probability plot. Plot the $z_{(i)}$'s against a transformation of the $x_{(i)}$'s (*e.g.*, log, square root, inverse, *etc.*) to form a *normalized* censored probability plot.

Step 5. For each attempted transformation $f(\cdot)$ including the unchanged observations as one option, compute the *correlation coefficient* between the pairs $[f(x_{(i)}), z_{(i)}]$ (**Chapter 3**). The transformation with the highest correlation coefficient and also a linear appearance on the censored probability plot, is one that optimally normalizes the left-censored sample. Estimate the mean and standard deviation in Step 6 on the transformed scale and use these estimates in subsequent statistical analysis.

If no transformation results in an adequately linear censored probability plot, conclude that the sample cannot be normalized. Mean and standard deviation estimates of the original concentrations can still be computed, but they will not correspond to a known probability distribution.

Step 6. If the raw concentration data are approximately normal, compute mean and standard deviation estimates adjusted for censoring using the equations:

$$\hat{\mu}_{KM} = \sum_{i=1}^{m} x_{(i)} \cdot \left[F_{KM}\left(x_{(i)}\right) - F_{KM}\left(x_{(i-1)}\right)\right] \qquad [15.4]$$

$$\hat{\sigma}_{KM} = \sqrt{\sum_{i=1}^{m} \left(x_{(i)} - \hat{\mu}_{KM}\right)^2 \cdot \left[F_{KM}\left(x_{(i)}\right) - F_{KM}\left(x_{(i-1)}\right)\right]} \qquad [15.5]$$

where $x_{(0)} = 0$ and $F_{\mathrm{KM}}\left(x_{(0)}\right) = F_{\mathrm{KM}}\left(0\right) = 0$ by definition. Otherwise, compute the adjusted mean and standard deviation after applying the normalizing transformation $f(\cdot)$ with the equations:

$$\hat{\mu}_{KM} = \sum_{i=1}^{m} f\left(x_{(i)}\right) \cdot \left[F_{KM}\left(x_{(i)}\right) - F_{KM}\left(x_{(i-1)}\right)\right] \qquad [15.6]$$

$$\hat{\sigma}_{KM} = \sqrt{\sum_{i=1}^{m} \left(f\left(x_i\right) - \hat{\mu}_{KM}\right)^2 \cdot \left[F_{KM}\left(x_{(i)}\right) - F_{KM}\left(x_{(i-1)}\right)\right]} \qquad [15.7]$$

Estimates from equations [15.4] and [15.5] can then be used in place of the sample mean ($\bar{x}$) and standard deviation (s) in parametric equations for prediction and control limits, and for confidence intervals. If a normalizing transformation is required, equations [15.6] and [15.7] can be used to construct similar statistical limits and intervals on the *transformed* scale.

►EXAMPLE 15-1

Use the Kaplan-Meier technique on the following manganese concentration data to construct estimates of the population mean and standard deviation that are adjusted for censoring.

	Manganese Concentrations (ppb) in Background				
Sample	Well 1	Well 2	Well 3	Well 4	Well 5
1	<5.0	<5.0	<5.0	6.3	17.9
2	12.1	7.7	5.3	11.9	22.7
3	16.9	53.6	12.6	10.0	3.3
4	21.6	9.5	106.3	<2.0	8.4
5	<2.0	45.9	34.5	77.2	<2.0

SOLUTION

Step 1. From the combined sample of n = 25 measurements, identify and sort the 21 distinct values including distinct RLs as in the table below. Compute the risk set (n_i) for each distinct level ($x_{(i)}$) as the total number of detects and non-detects no greater than $x_{(i)}$. Also calculate the exact number of detects (d_i) equal to each level.

Step 2. Compute the Kaplan-Meier estimate of the CDF using equations [15.1] and [15.3], shown in column 5 of the table below. Two example calculations are given by:

$$F_{\mathrm{KM}}(22.7) = \left(1-\frac{1}{21}\right)\left(1-\frac{1}{22}\right)\left(1-\frac{1}{23}\right)\left(1-\frac{1}{24}\right)\left(1-\frac{1}{25}\right) = 0.8$$

$$F_{KM}(3.3) = \left(1-\frac{0}{7}\right)\cdot\left(1-\frac{1}{8}\right)\cdot\left(1-\frac{1}{9}\right)\cdot\ldots\cdot\left(1-\frac{1}{24}\right)\cdot\left(1-\frac{1}{25}\right) = 0.28$$

i	$x_{(i)}$	At Risk (n_i)	d_i	CDF
1	<2.0	3	0	0.21
2	3.3	4	1	0.28
3	<5.0	7	0	0.28
4	5.3	8	1	0.32
5	6.3	9	1	0.36
6	7.7	10	1	0.40
7	8.4	11	1	0.44
8	9.5	12	1	0.48
9	10.0	13	1	0.52
10	11.9	14	1	0.56
11	12.1	15	1	0.60
12	12.6	16	1	0.64
13	16.9	17	1	0.68
14	17.9	18	1	0.72
15	21.6	19	1	0.76
16	22.7	20	1	0.80
17	34.5	21	1	0.84
18	45.9	22	1	0.88
19	53.6	23	1	0.92
20	77.2	24	1	0.96
21	106.3	25	1	1.00

Step 3. Compute normal quantiles or z-scores for each value of F_{KM} in the above table. First re-set the last entry to (n – .375)/(n + .25) = 0.9752 so that a finite quantile can be associated with the sample maximum.

Step 4. Plot the z-scores against the distinct manganese levels to form a normal censored probability plot (**Figure 15-3**). The probability plot correlation coefficient is r = 0.902. The plot itself shows substantial curvature, suggesting that the sample is non-normal.

Step 5. Plot the *z*-scores against one or more transformations of the manganese levels. First attempt a log transformation, as shown in **Figure 15-4**. In this case, the correlation coefficient improves to $r = 0.989$ and the normalized censored probability plot looks fairly linear. Conclude that the sample is approximately normal on the log-scale, that is, the manganese concentrations are lognormal in distribution.

Step 6. Compute Kaplan-Meier log-mean ($\hat{\mu}_{y,KM}$) and log-standard deviation ($\hat{\sigma}_{y,KM}$) estimates for the manganese data using equations [15.6] and [15.7], taking $f(\cdot)$ as the natural logarithm. This gives for the log-mean:

$$\hat{\mu}_{y,KM} = \log(2)\cdot[.21-0] + \log(3.3)\cdot[.28-.21] + \ldots + \log(106.3)\cdot[1-.96] = 2.31\log(ppb)$$

and for the log-standard deviation:

$$\hat{\sigma}_{y,KM} = \sqrt{(\log(2)-2.31)^2\cdot[.21-0]+\ldots+(\log(106.3)-2.31)^2\cdot[1-.96]} = 1.18\ \log(ppb)$$

These adjusted mean and standard deviation estimates can then be used in place of the sample log-mean and log-standard deviation in parametric prediction and control limits, or in parametric confidence intervals. ◀

Figure 15-3. Censored Probability Plot of Manganese Concentrations

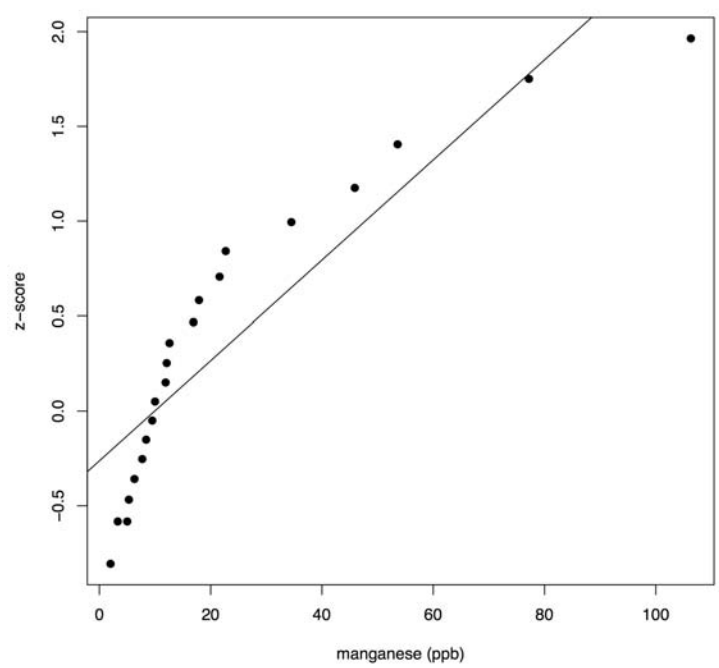
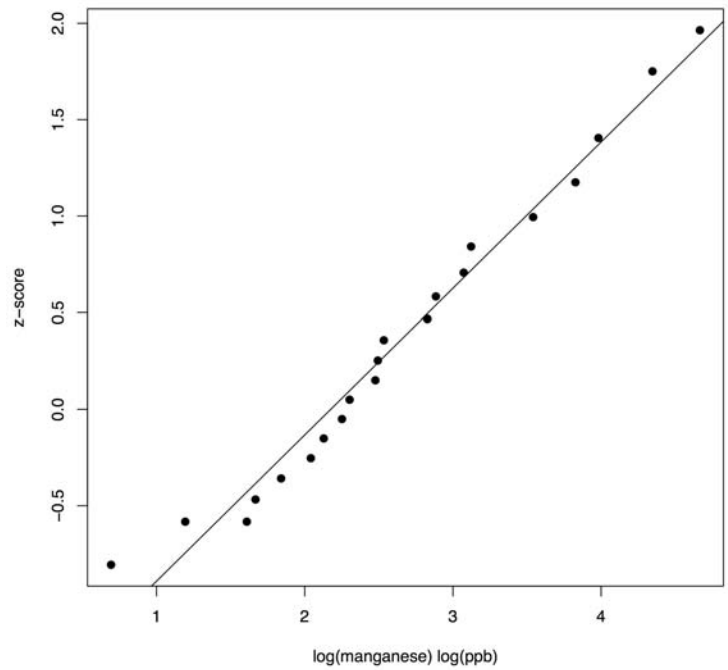


Figure 15-4. Censored Probability Plot of Logged Manganese Sample



15.4 ROBUST REGRESSION ON ORDER STATISTICS

BACKGROUND AND PURPOSE

Robust regression on order statistics [ROS] differs from Kaplan-Meier in that it uses the fitted model to construct a ***model-based imputation*** for each non-detect. Once the imputations are made, the adjusted mean and standard deviation are estimated using standard equations for the sample mean ($\bar{x}$) and standard deviation (s).

The first step in using Robust ROS is to find a single distributional model that adequately fits the joint sample of detects and non-detects. Standard probability plots (**Chapter 9**) and normality tests (**Chapter 10**) rely on a full ranking or ordering of the sample in order to fit candidate distributions. With left-censored data, the true concentrations of non-detects are unknown, so only a *partial ranking* is possible. Like Kaplan-Meier, the Robust ROS technique constructs a partial ranking of the data, accounting for the non-detects and assigning explicit ranks to each of the detected values. These detected values can be graphed on a *censored probability plot* to check the fit of possible distributional models.

Once an adequate distribution is found, Robust ROS determines the approximate cumulative probability associated with each distinct RL. The method then arbitrarily distributes non-detects with a common RL so that each one accounts for an equal share of the estimated cumulative probability

assigned to that RL. Once non-detects are ranked in this manner, the fitted distributional model is used to impute a value for each non-detect. This last task is accomplished by conducting a linear regression (Chapter 17) between the detected values and the z-scores from the censored probability plot. The parameters of the regression line (i.e., intercept and slope) can be used to estimate the mean and standard deviation of the distributional model, which in turn will generate imputed values for the non-detects.

The mathematics behind Robust ROS can be expressed as follows. First suppose there are k distinct RLs in the sample. Order these from least to greatest. Define A_i as the number of detected values between the ith and (i+1)th RLs for i = 1 to k–1. Let A_k = number of detects above the highest RL, and take A_0 = number of detects below the lowest RL. Also define B_i as the total number of observations, both detects and non-detects, with values below the ith RL. Define B_0 = 0. Then the number of non-detects below the ith RL can be written as:

$$C_i = B_i - B_{i-1} - A_{i-1} \qquad for\ i = 1\ to\ k \qquad [15.8]$$

With these definitions in place, exceedance probabilities can be assigned to each of the k RLs, representing the proportion of the sample greater than or equal to each distinct RL. These probabilities can be written as:

$$pe_i = pe_{i+1} + \frac{A_i}{A_i + B_i}\left(1 - pe_{i+1}\right) \qquad [15.9]$$

where pe_j denotes the proportion of the sample exceeding the ith RL. Equation [15.9] can be interpreted in the following manner. The exceedance probability associated with a given RL is equal to the exceedance probability assigned to the next highest RL combined with a fraction of the remaining, non-exceedance probability (*i.e.*, 1 – pe_{i+1}). The specific fraction depends on the relative occurrence of detects between the ith and (i+1)th RLs. When $i = k$, define pe_{i+1} = 0; when i = 0, define pe_0 = 1.

Once the exceedance probabilities are computed, plotting positions for the detects — *i.e.*, cumulative probabilities on a probability plot — can be calculated with the equation

$$pd_{ij} = \left(1 - pe_i\right) + \left(\frac{j}{A_i + 1}\right) \cdot \left(pe_i - pe_{i+1}\right) \qquad for\ j = 1\ to\ A_i;\quad and\ i = 0\ to\ k \qquad [15.10]$$

for each set of detected values falling between the ith and (i+1)th RLs. Note that this equation also applies to any detects below the lowest RL [i = 0] or above the highest RL [i = k]). Similarly, plotting positions for each group of non-detects can be written as:

$$pc_{ij} = \left(\frac{j}{C_i + 1}\right) \cdot \left(1 - pe_i\right) \qquad for\ j = 1\ to\ C_i;\ and\ i = 1\ to\ k \qquad [15.11]$$

With plotting positions for the detects, a normal quantile or z-score can be computed for each value of pd_{ij}. Then censored probability plots can be constructed using either the detected concentrations (x_{ij}) or some normalizing transformation of the detected values, say f(x_{ij}). If a linear probability plot can be identified, a linear regression (**Chapter 17**) can be calculated for the pairs (z_{ij}, $f(x_{ij})$) and used to impute values for the non-detects in the sample.

REQUIREMENTS AND ASSUMPTIONS

Robust ROS was originally devised to account for non-detects in water quality data (Helsel, 2005). Robust ROS is an extension of a technique termed *regression on order statistics [ROS]* (Gilliom and Helsel, 1986), described in **Section 15.5.** That procedure assumes the joint sample of detects and non-detects follows an underlying lognormal distribution. The fitted lognormal is used to estimate the population mean and standard deviation as a parametric technique. Robust ROS by contrast only relies on a parametric model to impute values for the non-detects. It can be applied to any normal or normalized distribution, rather than just the lognormal distribution. It may also be regarded as quasi-non-parametric since estimates for the sample are computed from the combined group of observed detects and imputed non-detects, rather than from the mean and standard deviation of the underlying distributional model, as in the original formulation.

In practice, because Robust ROS is not fully non-parametric, a known distribution must be fitted to the entire sample in order to construct imputed values for the non-detects. Closely related to this, Robust ROS assumes that both detected and non-detect data arise from the same population, with non-detect values censored at their respective RLs. Like Kaplan-Meier, this implies that the contaminant of concern is *present* in non-detect samples, but that the analytical method cannot accurately measure concentrations lower than the RL.

PROCEDURE

Step 1. Given a left-censored sample with a total of n measurements, identify and sort the k distinct RLs. Following the discussion above, count the number of detected values below the lowest RL (A_0), the number of detected values at least as great as the highest RL (A_k), and the number of detects between the ith and (i+1)th RLs (A_i for i = 1 to k–1). Also let $B_0 = 0$ and count the total number of detects and non-detects below the ith RL (B_i for i = 1 to k). Then use equation [15.8] to calculate the number of non-detects (C_i for i = 1 to k) below the ith RL.

Step 2. Let $pe_0 = 1$ and $pe_{k+1} = 0$. For i = 1 to k, compute the probability of exceeding the ith distinct RL (pe_i) using equation [15.9].

Step 3. With the exceedance probabilities from Step 2, sort each group of detects associated with A_i and then compute plotting positions (*i.e.*, cumulative probabilities) for these detects — pd_{ij} — using equation [15.10].

Step 4. Form normal quantiles (*i.e.*, z-scores) associated with the detected measurements and plotting positions pd_{ij} by computing $z_{ij}^d = \Phi^{-1}\left(pd_{ij}\right)$, where $\Phi^{-1}\left(\cdot\right)$ is the inverse standard normal CDF.

Step 5. Construct censored probability plots using the z-scores from Step 4. Plot the values z_{ij}^d against the detected concentrations x_{ij}^d to form a *normal* censored probability plot. Plot the z_{ij}^d's against a transformation of the x_{ij}^d's (*e.g.*, log, square root, inverse, *etc.*) to form a *normalized* censored probability plot.

Step 6. For each attempted transformation $f(\cdot)$ including the unchanged observations as one option, compute the *correlation coefficient* between the pairs $\left[f\left(x_{ij}^{d}\right), z_{ij}^{d} \right]$ (**Chapter 3**). The transformation with the highest correlation coefficient and also a linear appearance on the censored probability plot, is the one that optimally normalizes the left-censored sample. If no transformation results in an adequately linear censored probability plot, conclude that the sample cannot be normalized and that the Robust ROS may not provide reasonable imputations for the non-detects.

Step 7. If a normalizing transformation can be identified, compute a linear regression (**Chapter 17**) of the values $f\left(x_{ij}^{d}\right)$ on the *z*-scores, z_{ij}^{d}, to form the regression equation $f(X) = \hat{a} + \hat{b} \cdot Z$. The slope and intercept can be estimated using the equations

$$\hat{b} = \sum_{i=0}^{k} \sum_{j=1}^{A_i} \left(z_{ij}^{d} - \bar{z}_d\right) \cdot f\left(x_{ij}^{d}\right) \Big/ \left(n_d - 1\right) \cdot s_{z_d}^{2} \qquad [15.12]$$

$$\hat{a} = \bar{x}_d - \hat{b} \cdot \bar{z}_d \qquad [15.13]$$

where $\bar{z}_d$ is the mean of the *z*-scores associated with the detected values, n_d = number of detects, $s_{z_d}^{2}$ is the sample variance of the detected *z*-scores, and $\bar{x}_d$ is the mean of the detected measurements. The regression intercept ($\hat{a}$) is an estimate of the population mean of the normalized distribution, while the slope ($\hat{b}$) is an estimate of the population standard deviation.

Step 8. Compute plotting positions (pc_{ij}) for the non-detects (*i.e.*, censored observations) associated with each distinct RL using equation [15.11]. Then form a second set of *z*-scores, this time associated with the non-detects, by computing $z_{ij}^{c} = \Phi^{-1}\left(pc_{ij}\right)$ for j = 1 to C_i; and i = 1 to k.

Step 9. Form imputed values $f\left(\hat{x}_{ij}^{c}\right) = \hat{a} + \hat{b} \cdot z_{ij}^{c}$ using the slope and intercept from **Step 7** and the censored z-scores from **Step 8**. Combine these (transformed) imputed values for the non-detects with the (transformed) detected measurements $f\left(x_{ij}^{d}\right)$ to get censored estimates of the population mean and standard deviation by computing the overall sample mean ($\hat{\mu} = \bar{x}$) and sample standard deviation ($\hat{\sigma} = s$).

These censored estimates can be used in place of the unadjusted sample mean ($\bar{x}$) and standard deviation (s) in parametric equations for prediction and control limits, and for confidence intervals. If a normalizing transformation $f(\cdot)$ is needed, the censored estimates should be used to construct statistical limits and intervals on the *transformed* scale.

►EXAMPLE 15-2

In **Example 15-1**, the Kaplan-Meier technique was used on a sample of background manganese concentrations to compute the log-mean and log-standard deviation, adjusted for the presence of non-detects. Apply Robust ROS to these same data to compare the estimates.

SOLUTION

Step 1. The n = 25 manganese observations include 2 distinct RLs (<2 and <5). Count the number of detected measurements below the lowest RL, above the highest RL, and between the two RLs, denoted by A_i in the table below. Also count the total number of measurements — both detected and non-detect — below each RL, denoted below by B_i. Use equation [15.8] to count the number of non-detects associated with each RL, denoted below by C_i.

i	RL	A_i	B_i	C_i
0		0	0	0
1	<2	1	3	3
2	<5	18	7	3

Step 2. Compute the probability of exceeding each RL using equation [15.9] and noting that $pe_3 = 0$:

$$pe_2 = pe_3 + \frac{A_2}{A_2 + B_2}\left(1 - pe_3\right) = \frac{18}{18+7} = 0.72$$

$$pe_1 = pe_2 + \frac{A_1}{A_1 + B_1}\left(1 - pe_2\right) = 0.72 + \frac{1}{1+3}\left(1 - 0.72\right) = 0.79$$

Step 3. Sort the detects associated with each A_i and compute plotting positions for these detects using equation [15.10], as listed in the table below. For instance, $A_1 = 1$, corresponding to the detected value 3.3. The plotting position for this observation equals

$$pd_{11} = \left(1 - pe_1\right) + \left(\frac{1}{A_1 + 1}\right)\left(pe_1 - pe_2\right) = 0.21 + 0.5\left(0.79 - 0.72\right) = 0.245$$

Also form the normal quantiles (i.e., z-scores) associated with the detected observations, as listed below:

Detected Value (ppb)	Plotting Position	z-score
3.3	0.245	−0.690
5.3	0.318	−0.474
6.3	0.356	−0.370
7.7	0.394	−0.270
8.4	0.432	−0.172
9.5	0.469	−0.077
10.0	0.507	0.018
11.9	0.545	0.114
12.1	0.583	0.210
12.6	0.621	0.308
16.9	0.659	0.410
17.9	0.697	0.515
21.6	0.735	0.627
22.7	0.773	0.748
34.5	0.811	0.880
45.9	0.848	1.030
53.6	0.886	1.207
77.2	0.924	1.434
106.3	0.962	1.776

Step 4. Plot the *z*-scores against the detected manganese levels to form a normal censored probability plot (**Figure 15-5)**. The probability plot correlation coefficient is $r = 0.901$, almost identical to the Kaplan-Meier censored probability plot constructed in **Example 15-1**. The plot also shows substantial curvature, suggesting that the sample is non-normal. Also plot the *z*-scores against a log transformation of the detected manganese values (**Figure 15-6**). Not only does the normalized probability plot appear linear, but the correlation coefficient increases to $r = 0.994$. Conclude as in **Example 15-1** that the sample is approximately normal on the log-scale, so that the manganese concentrations are lognormal in distribution.

Step 5. Compute a linear regression of the $n_d = 19$ logged manganese detects against their corresponding *z*-scores using equations [15.12] and [15.13]. The sample mean and variance of the detected *z*-scores are $\bar{z}_d = 0.3802$ and $s^2_{z_d} = 0.4577$. Also, the log-mean of the detected observations equals $\overline{\log(x_d)} = 2.80$. The slope and intercept of the resulting line are:

$$\hat{b} = \frac{1}{18 \times .4577}\left[1.194 \cdot (-.690 - .3802) + \ldots + 4.666(1.776 - .3802)\right] = 1.372$$

$$\hat{a} = \bar{x}_d - \hat{b} \cdot \bar{z}_d = 2.80 - 1.372 \times .3802 = 2.278$$

Figure 15-5. Robust ROS Censored Probability Plot of Manganese Concentrations

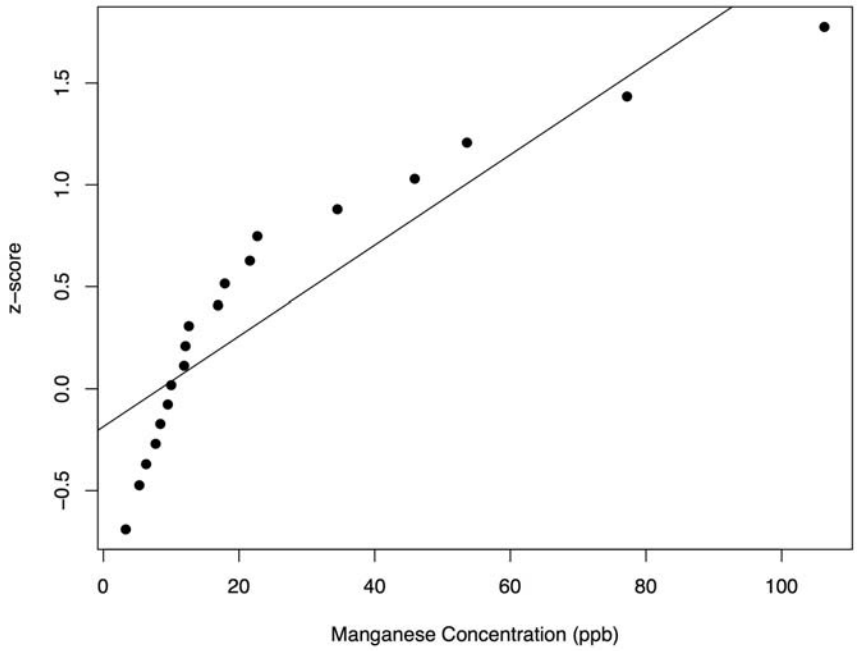
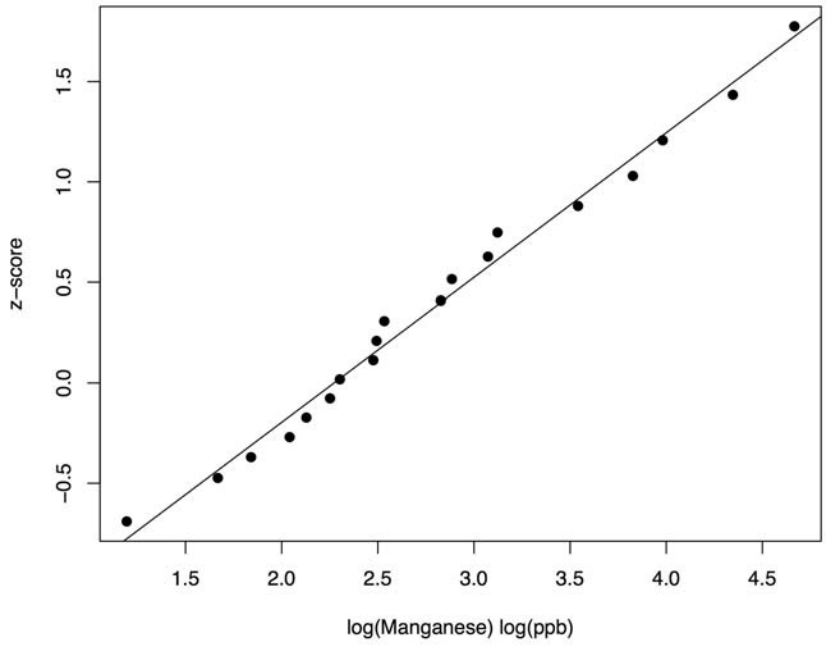


Figure 15-6. Robust ROS Censored Probability Plot of Logged Manganese



Step 6. Compute plotting positions for the non-detects (*i.e.*, censored observations) associated with each distinct RL using equation [15.11], listed in the table below. Form a second set of *z*-scores, this time associated with the non-detects, also listed below. Note that each non-detect is given a distinct plotting position, even though they cannot be ordered. This is done to 'fill in' the unknown portion of the underlying distribution, but should *not* be interpreted as a legitimate 'estimate' for any particular non-detect observation. The positions for the first pair of the 3 non-detects with RLs of 2 (*i.e.*, <2) are

$$pc_{11} = \left(\frac{1}{C_1+1}\right)\left(1-pe_1\right) = \left(\frac{1}{3+1}\right)\left(1-0.79\right) = 0.0525$$

$$pc_{12} = \left(\frac{2}{C_1+1}\right)\left(1-pe_1\right) = \left(\frac{2}{3+1}\right)\left(1-0.79\right) = 0.105$$

RL	Plotting Position	z-score	Imputed Value
<2	0.0525	−1.621	0.054
<2	0.1050	−1.254	0.558
<2	0.1575	−1.005	0.899
<5	0.0700	−1.476	0.253
<5	0.1400	−1.080	0.796
<5	0.2100	−0.806	1.172

Step 7. Form a second set of z-scores associated with the censored plotting positions from **Step 6**. These are listed in the table above. Then, using the regression parameters from **Step 5**, form a prediction for each non-detect using the equation $\log\left(x_{ij}^c\right) = \hat{\alpha} + \hat{\beta} \cdot z_{ij}^c$. Take these predictions as the imputed values for the set of non-detects, as listed above. The first two imputed values are computed as:

$$\log\left(x_{11}^c\right) = 2.278 + 1.372 \cdot \left(-1.621\right) = 0.054$$

$$\log\left(x_{12}^c\right) = 2.278 + 1.372 \cdot \left(-1.254\right) = 0.558$$

Step 8. Combine the logged detected manganese values with the imputed values from Step 7. Then compute the sample mean and standard deviation using the adjusted sample. These calculations give $\hat{\mu} = 2.28 \log(ppb)$ and $\hat{\sigma} = 1.26 \log(ppb)$. By comparison, the Kaplan-Meier method in **Example 15-1** gives very similar corresponding estimates of 2.31 log(ppb) and 1.18 log(ppb). ◄

SECTION 15.5 OTHER METHODS FOR A SINGLE CENSORING LIMIT

The two preferred methods using Kaplan-Meier or Robust ROS provided above for multiple detection limits are computationally intensive. Helsel (2005) indicates that public software is available for the Robust ROS method. Although the more common situation encountered in evaluating data sets is the presence of multiple detection limits (hence the UG recommendations), two older techniques are still applicable in some situations. The Cohen method and the parametric ROS techniques are both simpler to apply, but depend on the use of a single censoring limit. One needs to evaluate the prospects before applying them. If detectable data sets are large enough (e.g., $n > 50$) and detection percentages near or greater than 50%, most of these methods will work comparably.

15.5.1 COHEN'S ADJUSTMENT

Cohen's adjustment (Cohen, 1959) can be useful when a significant fraction (up to 50%) of the observed measurements in a data set are reported as non-detects. The technique assumes that all the measurements, detects and non-detects alike, arise from a common population, but that the lowest valued observations have been *censored* at the QL. Using the censoring point (*i.e.*, QL) and the pattern in the detected values, Cohen's method attempts to reconstruct the key features of the original population, providing explicit estimates of the population mean and standard deviation. These in turn can be used in certain statistical interval estimates, where Cohen's adjusted estimates are used as replacements for the sample mean and sample standard deviation.

REQUIREMENTS AND ASSUMPTIONS

Cohen's adjustment assumes that the common underlying population has a normal distribution. The technique should only be used when the observed sample data approximately fit a normal model including transformations to normality. Because the presence of a large fraction of non-detects will make explicit normality testing difficult, if not impossible, the most helpful diagnostic aid may be to construct a censored probability plot on the detected measurements. If the censored probability plot is clearly linear on the original measurement scale but not on the log-scale, assume normality for purposes of computing Cohen's adjustment. If, however, the censored probability plot is clearly linear on the log-scale, but not on the original scale, assume instead that the common underlying population is lognormal. Then compute Cohen's adjustment to the estimated mean and standard deviation on the log-scale measurements and construct the desired statistical interval using the algorithm for lognormally-distributed observations.

When the detection rate is less than 50%, the accuracy of Cohen's method worsens as the percentage of non-detects increases. The guidance does not generally recommend the use of Cohen's adjustment when more than half the data are non-detect. In such circumstances, one should consider an alternate statistical method, for instance a non-parametric interval or perhaps the Wilcoxon rank-sum test for small samples.

One other requirement of Cohen's original method is that there should be just a single censoring point. Data sets with multiple RLs will usually require a more sophisticated treatment such as Kaplan-Meier or Robust ROS methods or via maximum likelihood techniques (Cohen, 1963) or perhaps a multiply-censored probability plot technique (Helsel and Cohn, 1988). If only 2 or 3 RLs do not substantially differ and few detected intermingled data are lost, the censoring point (*QL*) can be set to

the highest RL. Cohen's method requires explicit definition of the censoring limit, and is somewhat sensitive to variation in this parameter.

PROCEDURE

Step 1. Divide the data set into two groups, detects and non-detects. If the total sample size equals n, let m represent the number of detects and (n–m) represent the number of non-detects. Denote the ith detected measurement by x_i. Then compute the mean and sample variance of the set of detects using the equations:

$$\bar{x}_d = \frac{1}{m}\sum_{i=1}^{m} x_i \quad \text{and} \quad s_d^2 = \frac{1}{m-1}\left[\sum_{i=1}^{m} x_i^2 - m\bar{x}_d^2\right]$$

Step 2. Denote the single censoring point by QL. Then compute the two intermediate quantities, h and γ, necessary to derive Cohen's adjustment via the following equations:

$$h = 100 \cdot (n-m)/n = ND\% \quad \text{and} \quad \gamma = s_d^2 \Big/ \left(\bar{x}_d - QL\right)^2$$

Step 3. Use the intermediate quantities h and γ to determine Cohen's adjustment parameter λ from the table below.

Values of Lamba (λ) for Cohen's Adjustment

γ\ND%	1	5	10	15	20	25	30	35	40	45	50
.01	.0102	.0530	.1111	.1747	.2443	.3205	.4043	.4967	.5989	.7128	.8403
.05	.0105	.0547	.1143	.1793	.2503	.3279	.4130	.5066	.6101	.7252	.8540
.10	.0110	.0566	.1180	.1848	.2574	.3366	.4233	.5184	.6234	.7400	.8703
.20	.0116	.0600	.1247	.1946	.2703	.3525	.4422	.5403	.6483	.7678	.9012
.30	.0122	.0630	.1306	.2034	.2819	.3670	.4595	.5604	.6713	.7937	.9300
.40	.0128	.0657	.1360	.2114	.2926	.3803	.4755	.5791	.6927	.8179	.9570
.50	.0133	.0681	.1409	.2188	.3025	.3928	.4904	.5967	.7129	.8408	.9826
.60	.0137	.0704	.1455	.2258	.3118	.4045	.5046	.6133	.7320	.8625	1.0070
.70	.0142	.0726	.1499	.2323	.3206	.4156	.5180	.6291	.7502	.8832	1.0303
.80	.0146	.0747	.1540	.2386	.3290	.4261	.5308	.6441	.7676	.9031	1.0527
.90	.0150	.0766	.1579	.2445	.3370	.4362	.5430	.6586	.7844	.9222	1.0743
1.00	.0153	.0785	.1617	.2502	.3447	.4459	.5548	.6725	.8005	.9406	1.0951
1.25	.0162	.0828	.1705	.2636	.3627	.4687	.5825	.7053	.8385	.9841	1.1443
1.50	.0170	.0868	.1786	.2758	.3793	.4897	.6081	.7357	.8738	1.0245	1.1901
1.75	.0177	.0905	.1861	.2873	.3948	.5094	.6321	.7641	.9069	1.0625	1.2332
2.00	.0184	.0940	.1932	.2981	.4093	.5279	.6547	.7909	.9382	1.0984	1.2739
2.25	.0191	.0973	.1999	.3082	.4231	.5454	.6761	.8164	.9679	1.1325	1.3127
2.50	.0197	.1005	.2062	.3179	.4363	.5621	.6965	.8407	.9962	1.1651	1.3498
2.75	.0203	.1035	.2123	.3272	.4489	.5781	.7161	.8639	1.0234	1.1963	1.3854
3.00	.0209	.1063	.2182	.3361	.4609	.5935	.7348	.8863	1.0495	1.2264	1.4197
3.50	.0219	.1118	.2292	.3529	.4838	.6226	.7704	.9287	1.0990	1.2835	1.4847
4.00	.0229	.1168	.2395	.3687	.5052	.6498	.8038	.9685	1.1455	1.3371	1.5458
4.50	.0239	.1216	.2492	.3836	.5253	.6755	.8353	1.0060	1.1895	1.3878	1.6037
5.00	.0248	.1262	.2585	.3977	.5445	.7000	.8653	1.0418	1.2312	1.4359	1.6587
5.50	.0256	.1305	.2673	.4111	.5628	.7233	.8938	1.0758	1.2711	1.4820	1.7113
6.00	.0264	.1346	.2757	.4240	.5803	.7456	.9212	1.1085	1.3094	1.5262	1.7617

Step 4. Using the adjustment parameter λ found in Step 3, compute adjusted estimates of the population mean and standard deviation with the equations:

$$\hat{\mu} = \bar{x}_d - \lambda(\bar{x}_d - QL) \quad \text{and} \quad \hat{\sigma} = \sqrt{s_d^2 + \lambda \cdot (\bar{x}_d - QL)^2}$$

Step 5. Once the adjusted estimates for the population mean and standard deviation are derived, these values can be substituted for the sample mean and standard deviation in equations for the statistical intervals.

15.5.2 PARAMETRIC REGRESSION ON ORDER STATISTICS (ROS)

A second useful method (EPA, 2004) for estimating mean and standard deviation parameters for data sets with non-detect values censored at a single limit is a parametric Regression on Order Statistics (ROS). The same assumptions apply as with Cohen's method. Both the detected and non-detect portions of the data are presumed to arise from a single population. That population should either be normal or transformable to a normal distribution. The parametric ROS method performs similarly to Cohen's method, and offers two principal advantages. The procedure can easily be implemented on almost any statistical software, and the method is not sensitive to the exact censoring limit.

If variable X originates from a normal distribution with mean μ and standard deviation σ $[X \succ N(\mu,\sigma)]$ and Z is the standard normal distribution $[Z \succ N(0,1)]$, statistical theory indicates that $X = \mu + \sigma \cdot Z$ when X and Z are at the same percentiles in their respective distributions. For a given observation or sample x above a detection limit, the order statistic (i.e., the proportion of observations less than x) can be estimated. This order statistic is an estimate of the percentile. The corresponding Z-value can be obtained from reference tables or a computer algorithm. For a list of ordered observations above the detection limit $(x_1, x_2, \ldots\ldots \text{ to } x_m)$ of m detectable samples out of a total n and a corresponding set of Z-values $(Z_1, \quad Z_2, \ldots\ldots \text{ to } Z_m)$ at the same percentiles, regression analysis of X against Z will provide estimates of the mean and standard deviation of distribution X. The intercept is the mean estimate and the slope of the regression is the standard deviation estimate.

When sample data better fit a lognormal or other normal transformable distribution, the regression is performed on the transformed data. The mean and standard deviation estimates are also for the transformed data (e.g., logarithmic mean and standard deviation). One may also use the regression results to "fill in" or quantify the values below the detection limit. When the Z-distribution is developed for the full set of total n sample values, the Z-values for the detectable portion are separated from those for the remaining $n - m$ non-detect percentiles. Estimates for the non-detect values are obtained from the equation $X = \hat{\mu} + \hat{\sigma} \cdot Z$, using $\hat{\mu}$ the intercept mean estimate, $\hat{\sigma}$ the slope standard deviation estimate and the non-detect Z-values. These can then be aggregated with the sample detectable values to obtain the overall mean and standard deviation estimate.

PROCEDURE

Step 1. Determine the appropriate normal transformation and convert the data if necessary. Divide the data set into two groups, detects and non-detects. If the total sample size equals n, let m represent the number of detects and $(n - m)$ represent the number of non-detects. Denote the ith detected measurement by x_i. Order the m detected data from smallest to largest.

Step 2. Define the normal percentiles for the total n sample set as follows. For a set of *i* values from 1 to *n*, $p_i = (i - .375)/(n + .25)$. Then convert to Z-values using the inverse normal distribution $Z_i = \Phi^{-1}(p_i)$. Separate the Z_i values into two groups: the larger *m* detected and *n - m* non-detected portions.

Step 3. Use linear regression of the ordered *m* data values against the corresponding Z-values. Obtain the intercept and slope of the regression as the estimated mean and standard deviation estimates, $\hat{\mu}$ and $\hat{\sigma}$. These can be used directly as the distributional parameter estimates or Step 4 can be followed.

Step 4. Using equation $X_{n-m} = \hat{\mu} + \hat{\sigma} \cdot Z_{n-m}$ with $\hat{\mu}$ the intercept mean estimate, $\hat{\sigma}$ the slope standard deviation estimate and the non-detect Z_{n-m} values, calculate the remaining x_{n-m} values and combine with the x_m detected data. Use the combined direct sample mean and standard deviation calculations as the final parameter estimates:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

►EXAMPLE 15-3

Use Cohen's and the parametric ROS methods for the data in **Example 15-1** and compare the results to the Kaplan-Meier and Robust ROS Methods. A single overall logarithmic distribution can be assumed. In the example, it is possible to utilize the higher detection limit (<5) as the censoring limit, with the loss of only a single detected point of information. The detection frequency is still 72%.

For Cohen's method, h = .28 and γ = .465 for the logarithmic data. The adjustment parameter from the above table is interpolated as λ = .445. The resulting mean and standard deviation estimates for the full data set are $\hat{\mu}$ = 2.32 log(ppb) and $\hat{\sigma}$ = 1.22 log(ppb).

Mean and standard deviation estimates for the parametric ROS method are $\hat{\mu}$ = 2.33 log(ppb) and $\hat{\sigma}$ = 1.21 log(ppb) following regression of the ordered detectable log values against the corresponding Z- values of the standard normal distribution. With such few non-detects near the lowest end of the sample distribution, the results are quite similar to the Robust ROS and Kaplan-Meier methods. For higher non-detect percentages and more heavily intermingled non-detect data, the results using these methods can differ considerably. ◄

15.6 USE OF THE 15% AND 50% NON-DETECT RULE

In this chapter and elsewhere in the Unified Guidance, it is recommended that imputing arbitrary values be limited to data sets with 10-15% or fewer non-detects and that parametric procedures be applied when there are 50% or fewer non-detects. The guidance continues to suggest this basic non-detects rule for both historical and conservative reasons. The same approach was found in both the earlier RCRA 1989 and 1992 RCRA statistical guidance documents, although it was recognized in the

first as a guideline "based on judgment". It was also noted that "there is no general procedure that is applicable in all cases." The 10-15% rule using direct substitution of arbitrary values is believed adequate for many applications, but one of the censoring estimation techniques provided in this chapter can be used instead. For a skewed distribution like the lognormal, the latter approach would be preferable. We have cited studies above by Davis and others indicating that parameter estimation and test performance can suffer when more than 50% of the data are non-detects. Most of the common parameters (i.e., mean, median, standard deviation, etc.) can be estimated with tolerable bias and error when no more than 50% of the values are originally non-detect and the superior non-detect fitting techniques used. Statistical test performance using these limitations appears to be reasonable for most applications. However, it should be recognized that they are only "rules of thumb", not absolute criteria.

Other authors (e.g., Helsel 2005) have suggested that certain tests will perform adequately even with higher non-detect rates in data. The criterion of non-detect percentage is not the only factor. For example with very large data sets (e.g., 100-300), quite reasonable fits can be made to the detectable portion using techniques found in **Chapter 15** even with non-detect percentages greater than 50%. Having a sufficient number of detectable data is also an important consideration, applying equally to small data sets. One should have a fairly good idea that the detect data themselves follow one or another parametric distributions. To do so, one should have a sufficiently large number of detected data points for comparison.

A second factor is the potential application for fitted non-detect data. As an example, fits of high non-detect percentage larger data sets using the lognormal distribution can provide decent parameter estimates (log mean and log standard deviation) for use with upper prediction limit detection monitoring tests. Generally, the fits accurately describe the upper portions of the observed data sets. At the same time, these estimated logarithmic parameters may result in considerably larger errors when estimating the true arithmetic mean and standard deviation (the bias problem in transformations), such as with compliance level tests. In this case, the 50% rule is best followed.

The guidance generally recommends non-parametric options when non-detect data exceed 50%. However, even this suggestion comes with caveats. For example, if a number of wells to be compared using Kruskal-Wallis non-parametric ANOVA had mostly or all well data sets greater than 50% non-detects, the outcome would be ambiguous. This is because the test involves comparisons of medians, which would lie below the detection limit. At very high non-detect percentages, fewer options are available. Upper non-parametric prediction limits can work with very few detectable values, but the assumption of any distributional pattern is increasingly tenuous. In some cases, a binomial test of proportions (found in the 1989 guidance) may be the only realistic option. As a final suggestion, we recommend that users take these factors into account and consider recommendations of other statistical literature in the field as well, when considering non-detect limitations to specific test procedures.

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PART III. DETECTION MONITORING TESTS

This third part of the Unified Guidance presents core procedures recommended for formal detection monitoring at RCRA-regulated facilities. **Chapter 16** describes two-sample tests appropriate for some small facilities, facilities in interim status, or for periodic updating of background data. These tests include two varieties of the *t*-test and two non-parametric versions-- the Wilcoxon rank-sum and Tarone-Ware procedures. **Chapter 17** discusses one-way analysis of variance [ANOVA], tolerance limits, and the application of trend tests during detection monitoring. **Chapter 18** is a primer on several kinds of prediction limits, which are combined with retesting strategies in **Chapter 19** to address the statistical necessity of performing multiple comparisons during RCRA statistical evaluations. Retesting is also discussed in **Chapter 20**, which presents control charts as an alternative to prediction limits.

As discussed in **Section 7.5**, any of these detection-level tests may also be applied to compliance/assessment and corrective action monitoring, where a background groundwater protection standard [GWPS] is defined as a critical limit using *two- or multiple-sample* comparison tests. Caveats and limitations discussed for detection monitoring tests are also relevant to this situation. To maintain continuity of presentation, this additional application is presumed but not repeated in the following specific test and procedure discussions.

Although other users and programs may find these statistical tests of benefit due to their wider applicability to other environmental media and types of data, the methods described in **Parts III** and **IV** are primarily tailored to the RCRA setting and designed to address formal RCRA monitoring requirements. In particular, the series of prediction limit tests found in **Chapter 18** is designed to address the range of interpretations of the sampling rules in §264.97(g), §264.98(d) and §258.54. Further, *all* of the regulatory tests listed in §264.97(i) and §258.53(h) are discussed, as well as the Student's *t*-test requirements of §265.93(b).

Taken as a whole, the set of detection monitoring methods presented in the Unified Guidance should be appropriate for almost all the situations likely to be encountered in practice. Professional statistical consultation is recommended for the rest.

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CHAPTER 16. TWO-SAMPLE TESTS

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This chapter describes statistical tests between two groups of data, known as two-sample tests. These tests may be appropriate for the smallest of RCRA sites performing upgradient-to-downgradient comparisons on a very limited number of wells and constituents. They may also be required for certain facilities in interim status, and can be more generally used to compare older versus newer data when updating background.

Two versions of the classic Student's *t*-test are first discussed: the *pooled variance t-test* and *Welch's t-test*. Since both these tests expect approximately normally-distributed data as input, two non-parametric alternatives to the *t*-test are also described: the *Wilcoxon rank-sum test* (also known as the *Mann-Whitney*) and the *Tarone-Ware test*. The latter is particularly helpful when the sample data exhibit a moderate to larger fraction of non-detects and/or multiple detection/reporting limits.

16.1 PARAMETRIC T-TESTS

BACKGROUND AND PURPOSE

A statistical comparison between two sets of data is known as a two-sample test. While several varieties of two-sample tests exist, the most common is the parametric *t*-test. This test compares two distinct statistical populations. The goal of the two-sample *t*-test is to determine whether there is any statistically significant difference between the mean of the first population when compared against the mean of the second population, based on the results observed in the two respective *samples*.

In groundwater monitoring, the typical hypothesis at issue is whether the average concentration at a compliance point is the same as (or less than) the average concentration in background, or whether the compliance point mean is larger than the background mean, as represented in equation [16.1] below:

$$H_0 : \mu_C \le \mu_{BG} \;\; vs. \;\; H_A : \mu_C > \mu_{BG} \qquad [16.1]$$

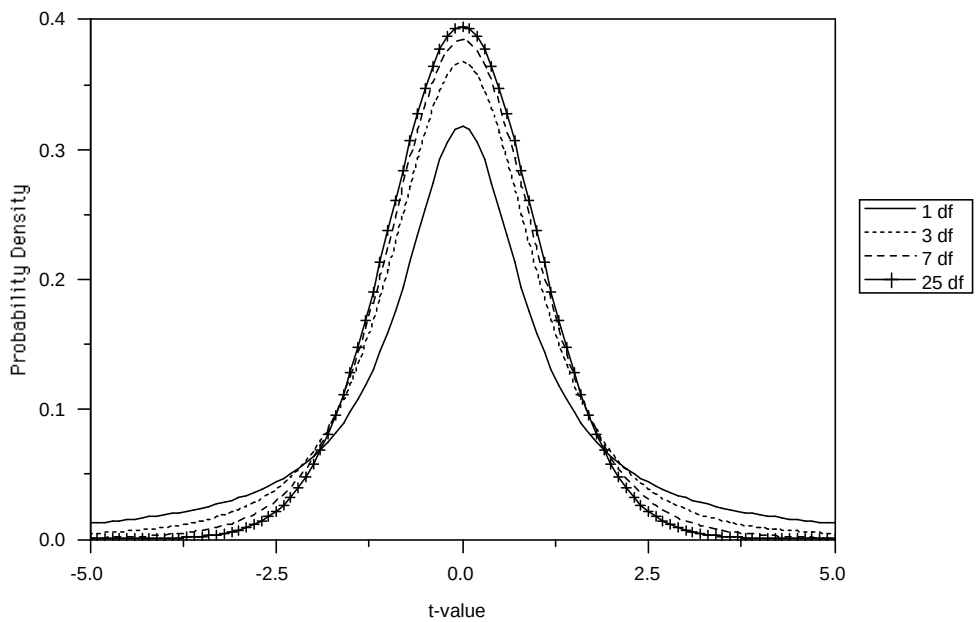
A natural statistic for comparing two population means is the difference between the sample means, $\left(\bar{x}_C - \bar{x}_{BG}\right)$. When this difference is small, a real difference between the respective population means is considered unlikely. However, when the sample mean difference is large, the null hypothesis is rejected, since in that case a real difference between the populations seems plausible. Note that an observed difference between the *sample* means does *not* automatically imply a true population difference. Sample means can vary for many reasons even if the two underlying parent populations are

identical. Indeed, the Student's *t*-test was invented precisely to determine when an observed sample difference should be considered significant (*i.e.*, more than a chance fluctuation), especially when the sizes of the two samples tend to be small, as is the usual case in groundwater monitoring.

Although the null hypothesis (H_0) represented in equation [16.1] allows for a true compliance point mean to be less than background, the behavior of the *t*-test statistic is assessed at the point where H_0 is most difficult to verify — that is, when H_0 is true and the two population means are identical. Under the assumption of equal population means, the test statistic in any *t*-test will tend to follow a Student's *t*-distribution. This fact allows the selection of critical points for the *t*-test based on a pre-specified Type I error or false positive rate (α). Unlike the similarly symmetric normal distribution, however, the Student's *t*-distribution also depends on the number of independent sample values used in the test, represented by the *degrees of freedom* [*df*].

The number of degrees of freedom impacts the shape of the *t*-distribution, and consequently the magnitude of the critical (percentage) points selected from the *t*-distribution to provide a basis of comparison against the *t*-statistic (see **Figure 16-1**). In general, the larger the sample sizes of the two groups being compared, the larger the corresponding degrees of freedom, and the smaller the critical points (in absolute value) drawn from the Student's *t*-distribution. In a one-sided hypothesis test of whether compliance point concentrations exceed background concentrations, a smaller critical point corresponds to a more powerful test. Therefore, all other things being equal, the larger the sample sizes used in the two-sample *t*-test, the more protective the test will be of human health and the environment.

Figure 16-1. Student's *t*-Distribution for Varying Degrees of Freedom



In groundwater monitoring, *t*-tests can be useful in at least two ways. First, a *t*-test can be employed to compare background data from one or more upgradient wells against a single compliance

well. If more than one background well is involved, all the upgradient data would be pooled into a single group or sample before applying the test.

Second, a *t*-test can be used to assess whether updating of background data is appropriate (see **Chapter 5** for further discussion). Specifically, the two-sample *t*-test can be utilized to check whether the more recently collected data is consistent with the earlier data assigned initially as the background data pool. If the *t*-test is non-significant, both the initial background and more recent observations may be considered part of the same statistical population, allowing the overall background data set to grow and to provide more accurate information about the characteristics of the background population.

The Unified Guidance describes two versions of the parametric *t*-test, the pooled variance Student's *t*-test and a modification to the Student's *t*-test known as Welch's *t*-test. This guidance prefers the latter *t*-test to use of Cochran's Approximation to the Behrens-Fisher (CABF) Student's *t*-test. Initially codified in the 1982 RCRA regulations, the CABF *t*-test is no longer explicitly cited in the 1988 revision to those regulations. Both the pooled variance and Welch's *t*-tests are more standard in statistical usage than the CABF *t*-test. When the parametric assumptions of the two-sample *t*-test are violated, the Wilcoxon rank-sum or the Tarone-Ware tests are recommended as non-parametric alternatives.

REQUIREMENTS AND ASSUMPTIONS

The two-sample *t*-test has been widely used and carefully studied as a statistical procedure. Correct application of the Student's *t*-test depends on certain key assumptions. First, every *t*-test assumes that the observations in each data set or group are statistically independent. This assumption can be difficult to check in practice (see **Chapter 14** for further discussion of statistical independence), especially if only a handful of measurements are available for testing. As noted in **Chapter 5** in discussing data mixtures, lab replicates or field duplicates are not statistically independent and should not be treated as independent water quality samples. That section discussed the limited conditions under which certain replicate data might be applicable for *t*- testing. Incorrect usage of replicate data was one of the concerns that arose in the application of the CABF *t*-test.

Second, all *t*-tests assume that the underlying data are approximately normal in distribution. Checks of this assumption can be made using one of the tests of normality described in **Chapter 10**. The *t*-test is a reasonably robust statistical procedure, meaning that it will usually provide accurate results even if the assumption of normality is partially violated. This robustness of the *t*-test provides some insurance against incorrect test results if the underlying populations are non-normal. However, the robust assumption is dubious when the parent population is heavily skewed. For data that are lognormal and positively skewed, the two-sample *t*-test can give misleading results unless the data are first log-transformed. Similarly, a transformation may be needed to first normalize data from other non-normal distributions.

Another assumption particularly relevant to the use of *t*-tests in groundwater monitoring is that the population means need to be ***stable*** or ***stationary*** over the time of data collection and testing. As discussed in **Part II** of the guidance, many commonly monitored groundwater parameters exhibit mean changes in both space and time. Consequently, correct application of the *t*-test in groundwater requires an implicit assumption that the two populations being sampled (***e.g.***, a background well and a

compliance point well) have average concentrations that are not trending with time. Time series plots and diagnostic trend tests (**Chapter 14**) can sometimes be used to check this assumption.

The *t*-test does an excellent job of identifying a stable mean level difference between two populations. However, if one or both populations have trends observable in the sample measurements, the *t*-test may have difficulty correctly identifying a difference between the two groups. For instance, if earlier samples in a compliance well were uncontaminated but later samples are increasing with time, the *t*-test may still provide a non-significant result. With compliance point concentrations increasing relative to background, the *t*-test may not be the appropriate method for identifying this change. Some form of trend testing will provide a better evaluation.

Another concern in applying the *t*-test to upgradient-downgradient interwell comparisons is that the null hypothesis is assumed to be true *unless* the downgradient well becomes contaminated. Absent such an impact, the population means are implicitly assumed to be identical. *Spatial variability* in background and compliance well groundwater concentrations for certain monitoring constituents do not allow clear conditions for comparisons intended to identify a release at a downgradient compliance well. Natural or pre-existing synthetic mean differences among background wells will be confused with a potential release. In such cases, neither the two-sample *t*-test nor *any* interwell procedure comparing upgradient against downgradient measurements is likely to give a correct conclusion.

One final requirement for running any *t*-test is that each group should have an adequate sample size. The *t*-test will have minimal statistical power to identify any but the largest of concentration differences if the sample size in each group is less than four. Four measurements per group should be considered a *minimum* requirement, and much greater power will accrue from larger sample sizes. Of course, the attractiveness of larger data sets must be weighed against the need to have statistically independent samples and the practical limitation of semi-annual or annual statistical evaluations. These latter requirements often constrain the frequency of sampling so that it may be impractical to secure more than 4 to 6 or possibly 8 samples during any annual period.

16.1.1 POOLED VARIANCE T-TEST

BACKGROUND AND PURPOSE

In the case of two independent samples from normal populations with common variance, the Student's *t*-test statistic is expressed by the following equation:

$$t = \left(\bar{x}_C - \bar{x}_{BG}\right) \Bigg/ \sqrt{\left[\frac{\left(n_{BG}-1\right)s_{BG}^2 + \left(n_C-1\right)s_C^2}{\left(n_{BG}+n_C-2\right)}\right]\left(\frac{1}{n_{BG}}+\frac{1}{n_C}\right)} \qquad [16.2]$$

The first bracketed quantity in the denominator is known as the *pooled variance*, a weighted average of the two sample variances. The entire denominator of equation [16.2] is labeled the *standard error of the difference* (SE_{diff}). It represents the probable chance fluctuation likely to be observed between the background and compliance point sample means when the null hypothesis in equation [16.1] is true. Note that the formula for SE_{diff} depends on both the pooled variance and the sample size of each group.

When the null hypothesis (H_0) is satisfied and the two populations are truly identical, the test statistic in equation [16.2] behaves according to an exact Student's *t*-distribution. This fact enables critical points for the *t*-test to be selected based on a pre-specified Type I error rate (α) and an appropriate degrees of freedom. In equation [16.2], the joint degrees of freedom is equal to $\left(n_{BG} + n_C - 2\right)$, the sum of the background and compliance point sample sizes less two degrees of freedom (one for each mean estimate).

REQUIREMENTS AND ASSUMPTIONS

Along with the general requirements for *t*-tests, the pooled variance version of the test assumes that the population variances are equal in both groups. Since only the sample variances will be known, this assumption requires a formal statistical test of its own such as Levene's test described in **Chapter 11**. An easier, descriptive method is to construct side-by-side box plots of both data sets. If the population variances are equal, the interquartile ranges represented by the box lengths should also be comparable. If the population variances are distinctly different, on the other hand, the box lengths should also tend to be different, with one box much shorter than the other.

When variances are unequal, the Unified Guidance recommends Welch's *t*-test be run instead. Welch's *t*-test does not require the assumption of equal variances across population groups. Furthermore, the performance of Welch's *t*-test is almost always equal or superior to that of the usual Student's *t*-test. Therefore, one may be able to skip the test of equal variances altogether before running Welch's *t*-test.

All t-tests require approximately normally-distributed data. If a common variance (σ^2) exists between the background and compliance point data sets, normality in the pooled variance *t*-test can be assessed by examining the combined set of background and compliance point ***residuals***. A residual can be defined as the difference between any individual value and its sample group mean (***e.g.***, $x_i - \bar{x}_{BG}$ for background values x_i). Not only will the combined set of residuals allow for a more powerful test of normality than if the two samples are checked separately, but it also avoids a difficulty that can occur if the sample measurements are naively evaluated with the ***Shapiro-Wilk multiple group test***. The multiple group normality test allows for populations with different means ***and*** different variances. If an equal variance check has not already been made, the multiple group test could register both populations as being normal even though the two population variances are distinctly different. The latter would violate a key assumption of the pooled variance t-test. To avoid this potential problem, either always check explicitly for equal variances before running the pooled variance *t*-test, or consider running Welch's *t*-test instead.

PROCEDURE

Step 1. To conduct the two-sample Student's *t*-test at an α-level of significance, first compute the sample mean ($\bar{x}$) and standard deviation (s) of each group. Check for equal variances using a test from **Chapter 11**. If there is no evidence of heteroscedasticity, check normality in both samples, perhaps by calculating the residuals from each group and running a normality test on the combined data set.

Step 2. Once the key assumptions have been checked, calculate the two-sample *t*-statistic in equation [16.2], making use of the sample mean, sample standard deviation, and sample size of each group.

Step 3. Set the degrees of freedom to $df = n_{BG} + n_C - 2$, and look up the (1–α) × 100th percentage point from the *t*-distribution in **Table 16-1** in **Appendix D**. Compare this α-level critical point against the *t*-statistic. If the *t*-statistic does not exceed the critical point, conclude there is insufficient evidence of a significant difference between the two population means. If, however, the *t*-statistic is greater than the critical point, conclude that the compliance point population mean is significantly greater than the background mean.

►EXAMPLE 16-1

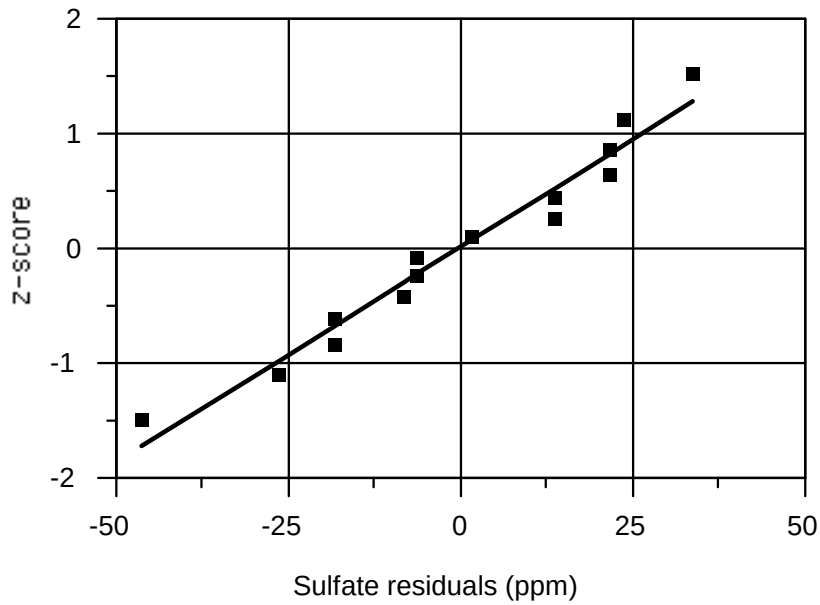
Consider the quarterly sulfate data in the table below collected from one upgradient and one downgradient well during 1995-96. Use the Student's *t*-test to determine if the downgradient sulfate measurements are significantly higher than the background values at an α = 0.01 significance level.

	Sulfate Concentrations (ppm)			
Quarter	Background	Downgradient	Background Residuals	Downgradient Residuals
1/95	560		23.75	
4/95	530		−6.25	
7/95	570	600	33.75	−8.33
10/95	490	590	−46.25	−18.33
1/96	510	590	−26.25	−18.33
4/96	550	630	13.75	21.67
7/96	550	610	13.75	1.67
10/96	530	630	−6.25	21.67
Mean	536.25	608.33		
SD	26.6927	18.3485		

SOLUTION

Step 1. Compute the sample mean and standard deviation in each well, as listed in the table above. Then compute the sulfate residuals by subtracting the well mean from each individual value. These differences are also listed above. Comparison of the sample variances shows no evidence that the population variances are unequal. Further, a probability plot of the combined set of residuals (**Figure 16-2**) indicates that the normal distribution appears to provide a reasonable fit to these data.

Figure 16-2. Probability Plot of Combined Sulfate Residuals



Step 2. Compute the two-sample *t*-statistic on the raw sulfate measurements using equation [16.2]. Note that the background sample size is n_{BG} = 8 and the downgradient sample size is n_C = 6.

$$t = \left(608.33 - 536.25\right) \Big/ \sqrt{\left[\frac{7\left(26.6927\right)^2 + 5\left(18.3485\right)^2}{8+6-2}\right]\left(\frac{1}{8}+\frac{1}{6}\right)} = 5.66$$

Step 3. Compute the degrees of freedom as $df = 8 + 6 - 2 = 12$. Since $\alpha = .01$, the critical point for the test is the upper 99th percentile of the *t*-distribution with 12 *df*. **Table 16-1** in **Appendix D** then gives the value for $t_{cp} = 2.681$. Since the *t*-statistic is clearly larger than the critical point, conclude the downgradient sulfate population mean is significantly larger than the background population mean at the 0.01 level. ◀

16.1.2 WELCH'S T-TEST

BACKGROUND AND PURPOSE

The pooled variance Student's *t*-test in **Section 16.1.1** makes the explicit assumption that both populations have a common variance, σ^2. For many wells and monitoring constituents, local geochemical conditions can result in both different well means and variances. A contamination pattern at a compliance well can have very different variability than its background counterpart.

Welch's *t*-test was designed as a modification to the Student's *t*-test when the population variances might differ between the two groups. The Welch's *t*-test statistic is defined by the following equation:

$$t = \left(\bar{x}_C - \bar{x}_{BG}\right) \Big/ \sqrt{\frac{s_{BG}^2}{n_{BG}} + \frac{s_C^2}{n_C}} \qquad [16.3]$$

The denominator of equation [16.3] is also called the *standard error of the difference* (SE_{diff}), similar to the pooled variance *t*-test. But it is a different weighted estimate based on the respective sample variances and sample sizes, reflecting the fact that the two population variances may not be the same.

The most difficult part of Welch's *t*-test is deriving the correct degrees of freedom. Under the assumption of a common variance, the pooled variance estimate incorporated into the usual Student's *t*-test has $df = \left(n_{BG} + n_C - 2\right)$ degrees of freedom, representing the number of independent "bits" of sample information included in the variance estimate. In Welch's *t*-test, the derivation of the degrees of freedom is more complicated, but can be approximately computed with the following equation:

$$\hat{df} = \left[\frac{s_{BG}^2}{n_{BG}} + \frac{s_C^2}{n_C}\right]^2 \Bigg/ \left[\frac{\left(s_{BG}^2/n_{BG}\right)^2}{n_{BG}-1} + \frac{\left(s_C^2/n_C\right)^2}{n_C-1}\right] \qquad [16.4]$$

Despite its lengthier calculations, Welch's *t*-test has several practical advantages. Best and Rayner (1987) found that among statistical tests specifically designed to compare two populations with different variances, Welch's *t*-test exhibited comparable statistical power (for $df \geq 5$) and was much easier to implement in practice than other tests they examined. Moser and Stevens (1992) compared Welch's *t*-test against the usual pooled variance *t*-test and determined that Welch's procedure was the more appropriate in almost every case. The only advantage registered by the usual Student's *t*-test in their study was in the case where the sample sizes in the two groups were unequal *and* the population variances were *known* to be essentially the same. In practice, the population variances will almost never be known in advance, so it appears reasonable to use Welch's *t*-test in the majority of cases where a two-sample *t*-test is warranted.

REQUIREMENTS AND ASSUMPTIONS

Welch's *t*-test is also a reasonably robust statistical procedure, and will usually provide accurate results even if the assumption of normality is partially violated. This robustness of the *t*-test provides some insurance against incorrect test results if the underlying populations are non-normal. But heavily skewed distributions do require normalizing transformations. Certain limitations apply when using transformed data, discussed in the following section.

Unlike the pooled variance *t*-test, Welch's procedure does not require that the population variances be equal in both groups. Other general requirements of *t*-tests, however, such as statistical independence of the sample data, lack of spatial variability when conducting an interwell test, and stationarity over time, are applicable to Welch's t-test and needs to be checked prior to running the procedure.

Because the variances of the tested populations may not be equal, an assessment of normality cannot be made under Welch's t-test by combining the residuals (as with the pooled variance t-test), unless an explicit check for equal variances is first conducted. The reason is that the combined residuals from normal populations with different variances may not test as normal, precisely because of the

heteroscedasticity. Since this latter variance check is not required for Welch's test, it may be easier to input the sample data directly into the *multiple group test of normality* described in **Chapter 10**.

PROCEDURE

Step 1. To run the two-sample Welch's *t*-test, first compute the sample mean $(\bar{x})$, standard deviation (s), and variance (s^2) in each of the background (BG) and compliance point (C) data sets.

Step 2. Compute Welch's *t*-statistic with equation [16.3].

Step 3. Compute the approximate degrees of freedom in equation [16.4] using the sample variance and sample size from each group. Since this quantity often results in a fractional amount, round the approximate $d\hat{f}$ to the nearest integer.

Step 4. Depending on the α significance level of the test, look up an appropriate critical point (t_{cp}) in **Table 16-1** in **Appendix D**. This entails finding the upper $(1-\alpha)\times 100th$ percentage point of the Student's *t*-distribution with df degrees of freedom.

Step 5. Compare the *t*-statistic against the critical point. If $t \le t_{cp}$, conclude there is no statistically significant difference between the background and compliance point population means. If, however, $t > t_{cp}$, conclude that the compliance point population mean is significantly greater than the background mean at the α level of significance.

►EXAMPLE 16-2

Consider the following series of monthly benzene measurements (in ppb) collected over 8 months from one upgradient and one downgradient well. What significant difference, if any, does Welch's *t*-test find between these populations at the α = .05 significance level?

	Benzene (ppb)	
Month	BG	DG
Jan	0.5	0.5
Feb	0.8	0.7
Mar	1.6	4.6
Apr	1.8	2.0
May	1.1	16.7
Jun	16.1	12.5
Jul	1.6	26.3
Aug	0.6	186.0
N	8	8
Mean	3.0	31.2
SD	5.31	63.22
Variance	28.204	3997.131

Step 1. Compute the sample mean, standard deviation, and variance of each group as in the table above.

Step 2. Use equation [16.3] to compute Welch's *t*-statistic:

$$t = \left(31.2 - 3.0\right) \Big/ \sqrt{\frac{28.204}{8} + \frac{3997.131}{8}} = 1.257$$

Step 3. Compute the approximate degrees of freedom using equation [16.4]:

$$\hat{df} = \left[\frac{28.204}{8} + \frac{3997.131}{8}\right]^2 \Bigg/ \left[\frac{(28.204/8)^2}{7} + \frac{(3997.131/8)^2}{7}\right] = 7.1 \quad \approx 7$$

Step 4. Using **Table 16-1** in **Appendix D** and given α = .05, the upper 95% critical point of the Student's *t*-distribution with 7 *df* is equal to 1.895.

Step 5. Compare the *t*-statistic against the critical point, t_{cp}. Since $t < t_{cp}$, the test on the raw concentrations provides insufficient evidence of a true difference in the population means. However, given the order of magnitude difference in the sample means and the fact that several of the downgradient measurements are substantially larger than almost all the background values, we might suspect that one or more of the *t*-test assumptions was violated, possibly invalidating the result. ◄

16.1.3 WELCH'S T-TEST AND LOGNORMAL DATA

Users should recall that if the underlying populations are *lognormal* instead of normal and Welch's *t*-test is run on the logged data, the procedure is not a comparison of arithmetic means but rather between the population *geometric means*. In the case of a lognormal distribution, the geometric means are equivalent to the population *medians*. In effect, a test of the log-means is equivalent to a test of the medians in terms of the raw concentrations. Both the population geometric mean and the lognormal median can be estimated from the logged measurements as $\exp(\bar{y})$, where $y = \log x$ represents a logged value and $\bar{y}$ is the log-mean. On the other hand, the (arithmetic) lognormal mean on the concentration scale would be estimated as $\exp\left(\bar{y} + s_y^2/2\right)$, a quantity larger than the geometric mean or median due to the presence of the term involving s_y^2, the log-variance.

Although a *t*-test conducted in the logarithmic domain is not a direct comparison of the arithmetic means, there are situations where that comparison can be *inferred* from the test results. For instance, consider using the pooled variance two-sample Student's *t*-test on logged data with a common (*i.e.*, equal) population log-variance (σ_y^2) in each group. In that case, finding a larger geometric mean or median in a compliance well population when compared to background also implies that the compliance point *arithmetic mean* is larger than the background *arithmetic mean*. However, when using Welch's *t*-test, the assumption of equal variances is not required. Because of this, on rare occasions one might find

a larger compliance point geometric mean or median when testing the log-transformed data, even though the compliance point population arithmetic mean is ***smaller*** than the background arithmetic mean.

Fortunately, such a reversal can only occur in the unlikely situation that the background population log-variance is distinctly larger than the compliance point log-variance. Factors contributing to an increase in the log-mean concentration level in lognormal populations often serve, if anything, to also increase the log-variance, and almost never to ***decrease*** it. Consequently, *t*-test results indicating a compliance point geometric mean higher than background should very rarely imply a less-than-background compliance point log-variance. This in turn will generally ensure that the compliance point arithmetic mean is also larger than the background arithmetic mean, so that a test of the log-transformed measurements can be used to infer whether a difference exists in the population ***concentration means***.

One caution in this discussion is for cases where the Welch's *t*-test is ***not significant*** on the log-transformed measurements. Because the log-variances (σ_y^2) are not required to be equal in the two populations when running Welch's *t*-test, yet the arithmetic lognormal mean depends on both the population log-mean (μ_y) and the log-variance through the quantity $\exp\left(\mu_y + \sigma_y^2/2\right)$, it should ***not*** be inferred that a non-significant comparison on the log-scale between a compliance point and background is equivalent to finding ***no difference*** between the lognormal arithmetic ***means***. If the log-variances differ but the log-means do not, the lognormal arithmetic ***means*** will still be different even though the lognormal ***medians*** might be identical.

Therefore, if a comparison of arithmetic means is required, but the statistical populations are lognormal, care must be taken in interpreting the results of Welch's *t*-test. Two possible remedies would include: 1) only running a *t*-test on lognormal data if the log-variances can be shown to be approximately equivalent (this would allow use of the pooled variance *t*-test); and 2) using a non-parametric two-sample bootstrap procedure on the original (non-logged) measurements to compare the arithmetic means directly. Consultation with a professional statistician may be required in this second case.

►EXAMPLE 16-3

The benzene data from **Example 16-2** indicated no significant upgradient-to-downgradient difference in population means when tested on the raw measurement scale. Check to see whether the same data more closely approximate a lognormal distribution and conduct Welch's *t*-test under that assumption.

	Benzene (ppb)		Log(Benzene) log(ppb)	
Month	BG	DG	BG	DG
Jan	0.5	0.5	−0.693	−0.693
Feb	0.8	0.7	−0.223	−0.357
Mar	1.6	4.6	0.470	1.526
Apr	1.8	2.0	0.588	0.693
May	1.1	16.7	0.095	2.815
Jun	16.1	12.5	2.779	2.526
Jul	1.6	26.3	0.470	3.270
Aug	0.6	186.0	−0.511	5.226
N	8	8	8	8
Mean	3.0	31.2	0.372	1.876
SD	5.31	63.22	1.0825	1.9847
Variance	28.204	3997.131	1.1719	3.9392

SOLUTION

Step 1. First check normality of the original measurements. To do this, compute the Shapiro-Wilk statistic (*SW*) separately for each well. *SW* = 0.505 for the background data, and *SW* = 0.544 for the downgradient well. Combining these two values using the equations in **Section 10.7**, the multiple group Shapiro-Wilk statistic becomes $G = -6.675$, which is significantly less than the 5% critical point of –1.645 from the standard normal distribution.[1] Thus, the assumption of normality was violated in **Example 16-2**.

Step 2. Compute the log-mean, log-standard deviation, and log-variance of each group, as listed above. Then compute the multiple group Shapiro-Wilk test to check for (joint) normality on the log-scale. The respective *SW* statistics now increase to 0.818 for the background data and 0.964 for the downgradient well. Combining these into an overall test, the multiple group Shapiro-Wilk statistic becomes –0.721 which now exceeds the $\alpha = 0.05$ standard normal critical point. A log transformation adequately normalizes the benzene data — suggesting that the underlying populations are lognormal in distribution — so that Welch's *t*-test can be run on the logged data.

Step 2. Using the logged measurements and equation [16.3], the *t*-statistic becomes:

$$t = \left(1.876 - 0.372\right) \Big/ \sqrt{\frac{1.1719}{8} + \frac{3.9392}{8}} = 1.88$$

[1] Note that $\alpha = 5\%$ is used in this example because the total sample size (BG and DG) is $n = 16$. Nevertheless, the test would also fail at $\alpha = 1\%$ or just about any significance level one might choose.

Step 3. Again using the log-variances and equation [16.4], the approximate *df* works out to:

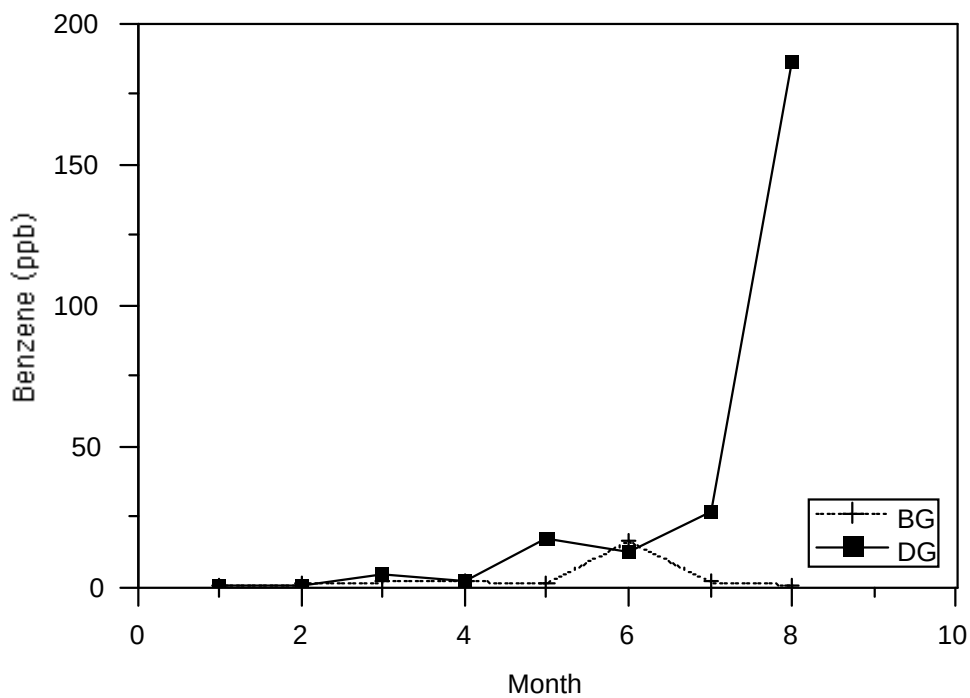
$$df = \left[\frac{1.1719}{8} + \frac{3.9392}{8}\right]^2 \Bigg/ \left[\frac{\left[1.1719/8\right]^2}{7} + \frac{\left[3.9392/8\right]^2}{7}\right] = 10.8 \approx 11$$

Note that the approximate *df* in Welch's *t*-test is somewhat less than the value that would be computed for the two-sample pooled variance Student's *t*-test. In that case, with 8 samples per data set, the *df* would have been 14 instead of 11. The reduction in degrees of freedom is due primarily to the apparent difference in variance between the two groups.

Step 4. Using **Table 16-1** in **Appendix D** and given α = .05, the upper 95% critical point of the Student's *t*-distribution with 11 *df* is equal to 1.796.

Step 5. Comparing *t* against t_{cp}, we find that 1.88 exceeds 1.796, suggesting a statistically significant difference between the background and downgradient population log-means, at least at the 5% level of significance. This means that the downgradient geometric mean concentration — and equivalently for lognormal populations, the median concentration — is statistically greater than the same statistical measure in background. Further, since the downgradient sample log-variance is over three times the magnitude of the background log-variance, it is also probable that the downgradient arithmetic mean is larger than the background arithmetic mean.

Figure 16-3. Benzene Time Series Plot



A note of caution in this example is that the same test run at the $\alpha = 0.01$ level would yield a *non-significant* result, since the upper 99% Student's *t* critical point in that case would be 2.718. The fact that the conclusion differs based on a small change to the significance level ought to prompt review of other *t*-test assumptions. A check of the downgradient sample measurements indicates an upward (non-stationary) trend over the sample collection period (**Figure 16-3**). This reinforces the fact that the *t*-test can be ill-suited for measuring differences between populations when trends over time cause instability in the underlying population means. It might be necessary to either perform a formal test of trend at the downgradient well or to limit the compliance data included in the evaluation only to those most representative of current conditions at the downgradient well (*e.g.*, the last four measurements). ◄

16.2 WILCOXON RANK-SUM TEST

BACKGROUND AND PURPOSE

When the underlying distribution of a data set is unknown and cannot be readily identified as normal or normalized via a transformation, a non-parametric alternative to the two-sample *t*-test is recommended. Probably the best and most practical substitute is the *Wilcoxon rank-sum test* (Lehmann, 1975; also known as the two-sample *Mann-Whitney U test*), which can be used to compare a single compliance well or data group against background. Like many non-parametric methods, the Wilcoxon rank-sum test is based on the ranks of the sample measurements rather than the actual concentrations. Some statistical information contained in the original data is lost when switching to the Wilcoxon test, since it only uses the relative magnitudes of data values.

The benefit is that the ranks can be used to conduct a statistical test even when the underlying population has an unusual form and is non-normal. The parametric *t*-test depends on the population being at least approximately normal; when this is not the case, the critical points of the *t*-test can be highly inaccurate. The Wilcoxon rank-sum test is also a statistically *efficient* procedure. That is, when compared to the *t*-test using normally-distributed data especially for larger sample sizes, it performs nearly as well as the *t*-test. Because of this fact, some authors (*e.g.*, Helsel and Hirsch, 2002) have recommended routine use of the Wilcoxon rank-sum even when the parametric *t*-test might be appropriate.

Although a reasonable strategy for larger data sets, one should be careful about automatically preferring the Wilcoxon over the *t*-test on samples as small as those often available in groundwater monitoring. For instance, a Wilcoxon rank-sum test of four samples in each of a background and compliance well and an $\alpha = 0.01$ level of significance can *never* identify a significant difference between the two populations. This is true no matter what the sample concentrations are, even if *all* four compliance measurements are larger than any of the background measurements. This Wilcoxon test will require at least five samples in at least one of the groups, or a higher level of significance (say $\alpha = 0.05$ or 0.10) is needed.

The Wilcoxon test statistic (W) consists of the sum of the *ranks* of the compliance well measurements. The rationale of the test is that if the ranks of the compliance data are quite large relative to the background ranks, then the hypothesis that the compliance and background values came from the same population ought to be rejected. Large values of the W statistic give evidence of possible

contamination in the compliance well. Small values of *W*, on the other hand, suggest there is little difference between the background and compliance well measurements.

REQUIREMENTS AND ASSUMPTIONS

The Wilcoxon rank-sum test assumes that both populations being compared follow a common, though unknown, parent distribution under the null hypothesis (Hollander and Wolfe, 1999). Such an assumption is akin to that used in the two-sample pooled variance Student's *t*-test, although the form of the common distribution need not be normal. The Wilcoxon test assumes that both population variances are equal, unlike Welch's *t*-test. Side-by-side box plots of the two data groups can be compared (**Chapter 9**) to examine whether or not the level of variability appears to be approximately equal in both samples. Levene's test (**Chapter 11**) can also be applied as a formal test of heteroscedasticity given its relative robustness to non-normality. If there is a substantial difference in variance between the background and compliance point populations, one remedy is the *Fligner-Policello test* (Hollander and Wolfe, 1999), a more complicated rank-based procedure.

The Wilcoxon procedure as described in the Unified Guidance is generally used as an *interwell* test, meaning that it should be avoided under conditions of significant natural spatial variability. Otherwise, differences between background and compliance point wells identified by the test may be mistakenly attributed to possible contamination, instead of natural differences in geochemistry, *etc.* At small sites, the Wilcoxon procedure can be adapted for use as an *intrawell* test, involving a comparison between intrawell background and more recent measurements from the same well. However, the per-comparison false positive rate in this case should be raised to either $\alpha = 0.05$ or $\alpha = 0.10$. More generally, a significance level of at least 0.05 should be adopted whenever the sample size of either group is no greater than $n = 4$.

In addition to spatial stationarity (*i.e.*, lack of natural spatial variability), the Wilcoxon rank-sum test assumes that the tested populations are stationary *over time*, so that mean levels are not trending upward or downward. As with the *t*-test, if trends are evident in time series plots of the sample data, a formal trend test might need to be employed instead of the Wilcoxon rank-sum, or the scope of the sample may need to be limited to only include data representative of current groundwater conditions.

HANDLING TIES

When ties are present in a combined data set, adjustments need to be made to the usual Wilcoxon test statistic. Ties will occur in two situations: 1) detected measurements reported with the same numerical value and 2) non-detect measurements with a common RL. Non-detects are considered ties because the actual concentrations are unknown; presumably, every non-detect has a concentration somewhere between zero and the quantitation limit [QL]. Since these measurements cannot be ordered and ranked explicitly, the approximate remedy in the Wilcoxon rank-sum procedure is to treat such values as ties.

One may be able to partially rank the set of non-detects by making use of laboratory-supplied analytical qualifiers. As discussed in **Section 6.3**, there are probable concentration differences between measurements labeled as undetected (*i.e.*, given a "U" qualifier), non-detect (usually reported without a qualifier), or as estimated concentrations (usually labeled with "J" or "E"). One reasonable strategy is to group all U values as the lowest set of ties, other non-detects as a higher set of ties, and to rank all J

and/or E values according to their estimated concentrations. In situations where estimated values for J and E samples are not provided, treat these measurements as the highest group of tied non-detects. Always give the highest ranks to explicitly quantified or estimated concentration measurements. In this way, a more detailed partial ranking of the data will be possible.

Tied observations in the Wilcoxon rank-sum test are handled as follows. All tied observations in a particular group should receive the same rank. This rank called the *midrank* (Lehmann, 1975) is computed as the average of the ranks that would be assigned to a group of ties if the tied values actually differed by a tiny amount and could be ranked uniquely. For example, if the first four ordered observations are all the same, the midrank given to each of these samples would be equal to (1 + 2 + 3 + 4)/4 = 2.5. If the next highest measurement is a unique value, its rank would be 5, and so on until all observations are appropriately ranked. A more detailed example is illustrated in **Figure 16-4**.

Figure 16-4. Computation of Midranks for Groups of Tied Values

Order	Concentration	Mid-Rank	
1	<1	1.5	$\Rightarrow \frac{1}{2}(1+2)$
2	<1	1.5	
3	1.2	3	
4	1.3	5	$\Rightarrow \frac{1}{3}(4+5+6)$
5	1.3	5	
6	1.3	5	
7	1.5	7.5	$\Rightarrow \frac{1}{2}(7+8)$
8	1.5	7.5	
9	1.6	9	

HANDLING NON-DETECTS

If either of the samples contains a substantial fraction of non-detect measurements (say more than 20-30%), identification of an appropriate distributional model (*e.g.*, normality) may be difficult, effectively ruling out the use of parametric tests like the *t*-test. Even when a normal or other parametric model can be fit to such left-censored data, a *t*-test cannot be run without imputing estimated values for each non-detect. Past guidance has recommended the Wilcoxon rank-sum test as an alternative to the *t*-test in the presence of non-detects, with all non-detects at a common RL being treated as tied values.

If the combined data set contains a single, common RL, that limit is smaller than any of the detected/quantified values, and the proportion of censored data is small (say no more than 10-15% of the total), it may be reasonable to treat the non-detects as a set of tied values and to apply the Wilcoxon rank-sum test adjusted for ties (described below). More generally, however, the statistical behavior of the Wilcoxon statistic depends on a full and accurate ranking of all the measurements. Groups of left-censored values cannot be ranked with certainty, even if each such measurement possesses a common RL. The problem is compounded in the presence of multiple RLs and/or quantified values less than the

RL(s). What is the relative ranking, for instance, of the pair of measurements (<1, <5)? A higher RL does not guarantee that the second observation is larger in magnitude than the first. A similar uncertainty plagues the pair of values (4, <10). And there is no guarantee either that the pair (<2, <2) is actually tied. One may be able to partially rank the set of non-detects by making use of laboratory-supplied analytical qualifiers as described in the previous section.

Because non-detects generally prevent a complete ranking of the measurements, the Wilcoxon rank-sum test is not recommended for most censored data sets. Instead, a modified version of the Tarone-Ware test (Hollander and Wolfe, 1999) is presented in **Section 16.3**. The Tarone-Ware test is essentially a generalization of the Wilcoxon test specifically designed to accommodate censored values.

PROCEDURE

Step 1. To conduct the Wilcoxon rank-sum test, first combine the compliance and background data into a single data set. Sort the combined values from smallest to largest, and — if there are no tied values or non-detects with a common RL — rank the ordered values from 1 to N. Assume there are n compliance well samples and m background samples so that $N = m + n$. Denote the ranks of the compliance samples by C_i and the ranks of the background samples by B_i.

Step 2. If there are groups of tied values (including non-detects with a common RL), form the midranks of the combined data set by assigning to each set of ties the average of the potential ranks the tied members would have been given if they could be uniquely ranked.

Step 3. Sum the ranks of the compliance samples to get the Wilcoxon statistic W:

$$W = \Sigma_{i=1}^{n} C_i \qquad [16.5]$$

Step 4. Find the α-level critical point of the Wilcoxon test, making use of the fact that the sampling distribution of W under the null hypothesis, H_0, can be approximated by a normal curve. By standardizing the statistic W (*i.e.*, subtracting off its mean or expected value and dividing by its standard deviation), the standardized statistic or z-score, Z, can be approximated by a standard normal distribution. Then an appropriate critical point (z_{cp}) can be determined as the upper $(1-\alpha) \times 100$th percentage point of the standard normal distribution, listed in **Table 10-1** in **Appendix D**.

Step 5. To compute Z when there are no ties, first compute the expected value and standard deviation of W, given respectively by the following equations:

$$E(W) = \frac{1}{2} n(N+1) \qquad [16.6]$$

$$SD(W) = \sqrt{\frac{1}{12} mn(N+1)} \qquad [16.7]$$

Then compute the approximate z-score for the Wilcoxon rank-sum test as:

$$Z = \frac{W - E(W) - 1/2}{SD(W)} \qquad [16.8]$$

The factor of 1/2 in the numerator serves as a continuity correction since the discrete distribution of the Wilcoxon statistic W is being approximated by a continuous normal distribution.

Step 6. If there are tied values, compute the expected value of W using [16.6] and the standard deviation of W adjusted for the presence of ties with the equation:

$$SD^*(W) = \sqrt{\frac{mn(N+1)}{12}\left(1 - \Sigma_{i=1}^{g} \frac{t_i^3 - t_i}{N^3 - N}\right)} \qquad [16.9]$$

where g equals the number of different groups of tied observations and t_i represents the number of tied values in the ith group.

Then compute the approximate z-score for the Wilcoxon rank-sum test as:

$$Z = \frac{W - E(W) - 1/2}{SD^*(W)} \qquad (16.10)$$

Step 7. Compare the approximate z-score against the critical point, z_{cp}. If Z exceeds z_{cp}, conclude that the compliance well concentrations are significantly greater than background at the α level of significance. If not, conclude that the null hypothesis of equivalent background and compliance point distributions cannot be rejected.

►EXAMPLE 16-4

The table below contains copper concentrations (ppb) found in groundwater samples at a Western monitoring facility. Wells 1 and 2 denote background wells while Well 3 is a single downgradient well suspected of being contaminated. Calculate the Wilcoxon rank-sum test on these data at the α = .01 level of significance.

	Copper Concentration (ppb)		
	Background		Compliance
Month	Well 1	Well 2	Well 3
1	4.2	5.2	9.4
2	5.8	6.4	10.1
3	11.3	11.3	14.5
4	7.0	11.5	16.1
5	7.0	10.1	21.5
6	8.2	9.7	17.6

SOLUTION

Step 1. Sort the N = 18 observations from least to greatest. Since there are 3 pairs of tied values, compute the midranks as in the table below. Note that m = 12 and n = 6.

Step 2. Compute the Wilcoxon statistic by summing the compliance well ranks, so that W = 84.5.

Step 3. Using α = .01, find the upper 99th percentage point of the standard normal distribution in **Table 10-1** of **Appendix D**. This gives a critical value of z_{cp} = 2.326.

	Midranks of Copper Concentrations		
	Background		Compliance
Month	Well 1	Well 2	Well 3
1	1	2	8
2	3	4	10.5
3	12.5	12.5	15
4	5.5	14	16
5	5.5	10.5	18
6	7	9	17

Step 4. Compute the expected value and adjusted standard deviation of W using equations [16.6] and (16.10), recognizing there are 3 groups of ties with t_i = 2 measurements in each group:

$$E(W) = \frac{1}{2} \cdot 6 \cdot 19 = 57$$

$$SD(W) = \sqrt{\frac{1}{12} \cdot 12 \cdot 6 \cdot (18+1)\left[1 - 3 \cdot \left(\frac{2^3 - 2}{18^3 - 18}\right)\right]} = \sqrt{113.647} = 10.661$$

Then compute the standardized statistic or *z*-score, *Z*, using equation (16.10):

$$Z = \frac{84.5 - 57 - 0.5}{10.661} = 2.533$$

Step 5. Compare the observed *z*-score against the critical point z_{cp}. Since $Z = 2.533 > 2.326 = z_{.99}$, there is statistically significant evidence of possible contamination in the compliance well at the $\alpha = .01$ significance level. ◄

16.3 TARONE-WARE TWO-SAMPLE TEST FOR CENSORED DATA

BACKGROUND

In statistical terms, non-detect measurements represent left-censored values, in which the 'true' magnitude is known only to exist somewhere between zero and the RL, i.e., within the concentration interval [0, RL). The uncertainty introduced by non-detects impacts the applicability of other two-sample comparisons like the t-test and Wilcoxon rank-sum test. Because the Student's t-test cannot be run unless a specific magnitude is assigned to each observation, estimated or imputed values need to be assigned to the non-detects. The Wilcoxon procedure requires that every observation be ranked in relation to other values in the combined sample, even though non-detects allow at best only a partial ranking, as discussed in **Section 16.2**.

The Tarone-Ware two-sample test can be utilized to overcome these limitations for many groundwater data with substantial fractions of non-detects along with multiple RLs. Tarone and Ware (1977) actually proposed a family of tests to analyze censored data. One variant of this family is the logrank test, frequently used in survival analysis for right-censored data. Another variant is known as Gehan's generalized Wilcoxon test (Gehan, 1965). The Unified Guidance presents the variant recommended by Tarone and Ware, slightly modified to account for left-censored measurements.

The key benefit of the Tarone-Ware procedure is that it is designed to provide a valid statistical test, even with a large fraction of censored data. As a non-parametric test, it does not require normally-distributed observations. In addition, non-detects do not have to be imputed or even fully ranked. Instead, for each detected concentration (*c*), a simple count needs to be made within each sample of the number of detects and non-detects no greater in magnitude than *c*. These counts are then combined to form the Tarone-Ware statistic.

REQUIREMENTS AND ASSUMPTIONS

The null hypothesis (H_0) under the Tarone-Ware procedure assumes that the populations in background and the compliance well being tested are identical. This implies that the variances in the two distributions are the same, thus necessitating a check of equal variances. With many non-detect data sets, it can be very difficult to formally test for heteroscedasticity. Often the best remedy is to make an informal, visual check of variability using side-by-side box plots (**Chapter 9**), setting each non-detect to half its RL.

The Tarone-Ware test will typically be used as an *interwell* test, meaning that it should be avoided under conditions of significant natural spatial variability. In addition, the tested populations should be stationary *over time*, so that mean levels are not trending upward or downward. Both assumptions can be more difficult to verify with censored data. Spatial variation can sometimes be checked with a non-parametric Kruskal-Wallis analysis of variance (**Chapter 17**). Trends with censored data can be identified with the Mann-Kendall test (**Chapter 14**).

As with other two-sample tests, if a trend is identified in one or both samples, a formal trend test may be needed instead of the Tarone-Ware, or the scope of the sample may need to be limited to only include data representative of current groundwater conditions.

Because the Tarone-Ware test presented in the Unified Guidance depends on counts of observations with magnitudes no greater than each detected concentration, and in that sense generalizes the ranking process used by the Wilcoxon rank-sum procedure, it is recommended that estimated concentrations (*i.e.*, sample measurements assigned unique magnitudes but labeled with qualifiers "J" or "E") be treated as detections for the purpose of computing the Tarone-Ware statistic. Such observations provide valuable statistical information about the relative ranking of each censored sample, even if estimated concentrations possess larger measurement uncertainty than fully quantified values.

PROCEDURE

Step 1. To compare a background data set against a compliance well using the Tarone-Ware test, first combine the two samples. Locate and sort the k distinct detected values and label these as:

$$w_{(1)} \langle w_{(2)} \langle \ldots \langle w_{(k-1)} \langle w_{(k)}$$

Note that the set of w's will not include any RLs from non-detects. Also, if two or more detects are tied, k will be less than the total number of detected measurements.

Step 2. For the combined sample, count the number of observations (described by Tarone & Ware as 'at risk') for each distinct detected concentration. That is, for $i = 1,\ldots,k$, let n_i = the number of detected values no greater than $w_{(i)}$ plus the number of non-detects with RLs no greater than $w_{(i)}$. Also let d_i = the number of detects with concentration equal to $w_{(i)}$. This value will equal 1 unless there are multiple detected values with the same reported concentration.

Step 3. For the compliance sample, count the observations 'at risk', much as in Step 2. For $i = 1$ to k, let n_{i2} = the number of detected compliance values no greater than $w_{(i)}$ plus the number of compliance point non-detects with RLs no greater than $w_{(i)}$. Also let d_{i2} = the number of compliance point detects with concentration equal to $w_{(i)}$. Note that $d_{i2} = 0$ if $w_{(i)}$ represents a detected value from background. Also compute n_{i1}, the number 'at risk' in the background sample.

Step 4. For $i = 1$ to k, compute the expected number of compliance point detections using the formula:

$$E_{i2} = d_i n_{i2} / n_i \tag{16.11}$$

Also compute the variance of the number of compliance point detections, using the equation:

$$V_{i2} = \frac{d_i\left(n_i - d_i\right)n_{i1}n_{i2}}{n_i^2\left(n_i - 1\right)} \tag{16.12}$$

Note in equation (16.12) that if $n_i = 1$ for the smallest detected value, the numerator of V_{i2} will necessarily equal zero (since $d_i = 1$ in that case), so compute $V_{i2} = 0$.

Step 5. Construct the Tarone-Ware statistic (*TW*) with the equation:

$$TW = \frac{\sum_{i=1}^{k}\sqrt{n_i}\left(d_{i2} - E_{i2}\right)}{\sqrt{\sum_{i=1}^{k} n_i V_{i2}}} \tag{16.13}$$

Step 6. Find the α-level critical point of the Tarone-Ware test, making use of the fact that the sampling distribution of *TW* under the null hypothesis, H_0, is designed to approximately follow a standard normal distribution. An appropriate critical point (z_{cp}) can be determined as the upper (1–α) × 100th percentage point of the standard normal distribution, listed in **Table 10-1** of **Appendix D**.

Step 7. Compare *TW* against the critical point, z_{cp}. If *TW* exceeds z_{cp}, conclude that the compliance well concentrations are significantly greater than background at the α level of significance. If not, conclude that the null hypothesis of equivalent background and compliance point distributions cannot be rejected.

►EXAMPLE 16-5

A heavily industrial site has been historically contaminated with tetrachloroethylene [PCE]. Using the Tarone-Ware procedure at an α = .05 significance level, test the following PCE measurements collected from one background and one compliance well.

PCE (ppb)	
Background	Compliance
<4	6.4
1.5	10.9
<2	7
8.7	14.3
5.1	1.9
<5	10.0
	6.8
	<5

SOLUTION

Step 1. Combine the background and compliance point samples. List and sort the distinct detected values (as in the table below), giving $k = 10$. Note that the 4 non-detects comprise 28% of the combined data.

Step 2. Compute the number of measurements (n_i) in the combined sample 'at risk' for each distinct detected value ($w_{(i)}$), indexed from $i = 1,\ldots, 10$, by adding the number of detects and non-

detects no greater than $w_{(i)}$, as listed in column 6 of the table below. Also list in column 3 the number of detected values (d_i) exactly equal to $w_{(i)}$.

Step 3. For the compliance point sample, compute the number (n_{i2}) 'at risk' for each distinct detected value, as listed in column 5 below. Also compute the number (n_{i1}) 'at risk' for the background sample (column 4) and the number of compliance point measurements exactly equal to $w_{(i)}$ (column 2).

Step 4. Use equations (16.11) and (16.12) to compute the expected value (E_{i2}) and variance (V_{i2}) of the number of compliance point detections at each $w_{(i)}$ (columns 7 and 8 below).

$w_{(i)}$	d_{i2}	d_i	n_{i1}	n_{i2}	n_i	E_{i2}	V_{i2}
1.5	0	1	1	0	1	0	0
1.9	1	1	1	1	2	0.5	0.25
5.1	0	1	5	2	7	0.2857	0.2041
6.4	1	1	5	3	8	0.375	0.2344
6.8	1	1	5	4	9	0.4444	0.2469
7.0	1	1	5	5	10	0.5	0.25
8.7	0	1	6	5	11	0.4545	0.2479
10.0	1	1	6	6	12	0.5	0.25
10.9	1	1	6	7	13	0.5385	0.2485
14.3	1	1	6	8	14	0.5714	0.2449

Step 5. Calculate the Tarone-Ware statistic (TW) using equation (16.13):

$$TW = \frac{\sqrt{1}\cdot(0-0)+\sqrt{2}\cdot(1-0.5)+\sqrt{7}\cdot(0-.2857)+\ldots+\sqrt{14}\cdot(1-.5714)}{\sqrt{1\cdot 0+2\cdot .25+7\cdot .2041+\ldots+14\cdot .2449}} = 1.85$$

Step 6. Determine the 0.05 level critical point from **Table 10-1** in **Appendix D** as the upper 95th percentage point from a standard normal distribution. This gives z_{cp} = 1.645.

Step 7. Compare the Tarone-Ware statistic against the critical point. Since TW = 1.85 > 1.645 = z_{cp}, conclude that the PCE concentrations are significantly greater at the compliance well than in background at the 5% significance level. ◄

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CHAPTER 17. ANOVA, TOLERANCE LIMITS, AND TREND TESTS

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This chapter describes two statistical procedures — analysis of variance [ANOVA] and tolerance limits — explicitly allowed within §264.97(h) and §258.53(g) for use in groundwater monitoring. The Unified Guidance does not generally recommend either technique for formally making regulatory decisions about compliance wells or regulated units, instead focusing on prediction limits, control charts, and confidence intervals. But both ANOVA and tolerance tests are standard statistical procedures that can be adapted for a variety of uses. ANOVA is particularly helpful in both identifying on-site spatial variation and in sometimes aiding the computation of more effective and statistically powerful intrawell prediction limits (see **Chapters 6** and **13** for further discussion).

This chapter also presents selected trend tests as an alternative statistical method that can be quite useful in groundwater detection monitoring, particularly when groundwater populations are not *stationary* over time. Although trend tests are not explicitly listed within the RCRA regulations, they possess advantages in certain situations and can meet the performance requirements of §264.97(i) and §258.53(h). They can also be helpful during diagnostic evaluation and establishment of historical background (**Chapter 5**) and in verifying key statistical assumptions (**Chapter 14**).

17.1 ANALYSIS OF VARIANCE [ANOVA]

17.1.1 ONE-WAY PARAMETRIC F-TEST

BACKGROUND AND PURPOSE

The parametric one-way ANOVA is a statistical procedure to determine whether there are statistically significant differences in mean concentrations among a set of wells. In groundwater applications, the question of interest is whether there is potential contamination at one or more compliance wells when compared to background. By finding a significant difference in means and specifically *higher* average concentrations at one or more compliance wells, ANOVA results can sometimes be used to identify unacceptably high contaminant levels in the absence of natural spatial variability.

Like the two-sample *t*-test, the one-way ANOVA is a comparison of population means. However, the one-way parametric ANOVA is a comparison of *several* populations, not just two: one set of

background data versus at least two compliance wells. The *F*-statistic that forms the heart of the ANOVA procedure is actually an extension of the *t*-statistic; an *F*-statistic formed in a comparison of only two datasets reduces to the square of the usual pooled variance Student's *t*-statistic. Like the *t*-statistic, the *F*-statistic is a ratio of two quantities. The numerator is a measure of the average squared difference observed between the pairs of sample means, while the denominator represents the average variability found in each well group.

Under the null hypothesis that all the wells or groups have the same population mean, the *F*-statistic follows the *F*-distribution. Unlike the *t*-distribution with a single degrees of freedom *df*, there are two *df* quantities associated with *F*. One is for the numerator and the other for the denominator. When critical points are needed from the *F*-distribution, one must specify both degrees of freedom values.

Computation of the *F*-statistic is only the first step of the full ANOVA procedure, when used as a formal compliance test. It can only determine whether *any* significant mean difference exists between the possible pairs of wells or data groups, and not whether or what specific compliance wells differ from background. To accomplish this latter task when a significant *F*-test is registered, individual tests between each compliance well and background needs to be conducted, known as individual *post-hoc comparisons* or *contrasts*. These individual tests are a specially constructed series of *t*-tests, with critical points chosen to limit the *test-wise* or *experiment-wise* false positive rate.

REQUIREMENTS AND ASSUMPTIONS

The parametric ANOVA assumes that the data groups are normally-distributed with constant variance. This means that the group *residuals* should be tested for normality (**Chapter 10**) and that the groups have to be tested for equality of variance, perhaps with Levene's test (**Chapter 11**). Since the *F*-test used in the one-way ANOVA is reasonably robust to small departures from normality, the first of these assumptions turns out to be less critical than the second. Research (Milliken and Johnson, 1984) has shown that the statistical power of the *F*-test is strongly affected by inequality in the population variances. A noticeable drop in power is seen whenever the ratio of the largest to smallest group variance is at least 4. A severe drop in power is found whenever the ratio of the largest to smallest group variance is at least a factor of 10. These ratios imply that the *F*-test will lose some statistical power if any of the group population standard deviations is at least twice the size of any other group's standard deviation, and that the power will be greatly curtailed if any standard deviation is at least 3 times as large as any other group's.

If the hypothesis of equal variances is rejected or if the group residuals are found to violate an assumption of normality (especially at the .01 significance level or less), one should consider a transformation of the data, followed by testing of the ANOVA assumptions on the transformed scale. If the residuals from the transformed data still do not satisfy normality or if there are too many non-detect measurements to adequately test the assumptions, a non-parametric ANOVA (called the *Kruskal-Wallis test*) using the ranks of the observations is recommended instead (see **Section 17.1.2**).

Since ANOVA is inherently an interwell statistical method, a critical point in using ANOVA for compliance testing is that the well field should exhibit *minimal spatial variability*. Interwell tests also require the groundwater well populations to be spatially *stationary*, so that absent a release the population well means are stable over time. Because spatial variation is frequently observed in many

groundwater constituents, especially for common inorganic constituents and some metals, ANOVA may not be usable as compliance testing tool. Yet it can be utilized on the same data sets to help *identify* the presence of spatial variability. In this capacity, the same procedure and formulas are utilized as described below (with the exception of the post-hoc contrasts, which are unnecessary for assessing spatial variation). The results are then employed to guide the appropriate choice of a compliance test (*e.g.*, intrawell or interwell prediction limits).

For formal ANOVA testing under §264.97(i) and §258.53(h), the *experiment-wise* or *test-wise* false positive rate (α) needs to be at least 5% during any statistical evaluation for each constituent tested. Furthermore, the individual *post-hoc contrasts* used to test single compliance wells against background need to be run at a significance level of at least $\alpha^* = 1\%$ per well. Combined, these regulatory constraints imply that if there are more than five post-hoc contrasts that need to be tested (*i.e.*, more than 5 compliance wells are included in the ANOVA test), the overall, maximal false positive rate of the procedure will tend to be greater, and perhaps substantially so, than 5%. Also, since $\alpha = 5\%$ is the minimum significance level per monitoring constituent, running multiple ANOVA procedures to accommodate a list of constituents will lead to a minimum *site-wide false positive rate* [SWFPR] greater than the Unified Guidance recommended target of 10% per statistical evaluation.

In addition, if a contaminated compliance well exists but too many uncontaminated wells are also included in the same ANOVA, the *F*-statistic may result in a non-significant outcome. Performing ANOVA with more than 10 to 15 well groups can "swamp" the procedure, causing it to lose substantial power. It therefore will be necessary to consider one of the retesting strategies described in **Chapters 18** and **20** as an alternative to ANOVA in the event that either the expected false positive rate is too large, or if more than a small number of wells need to be tested.

Another drawback to the one-way ANOVA is that the *F*-test accounts for *all possible paired comparisons* among the well groups. In some cases, the *F*-statistic may be *significant* even though all of the contrasts between compliance wells and background are *non-significant*. This does not mean that the *F*-test has necessarily registered a false positive. Rather, it may be that two of the compliance wells significantly differ from each other, but neither differs from background. This could happen, for instance, if a compliance well has a lower mean concentration than background while other compliance wells have near background means. The *F*-test looks for all possible differences between pairs of well groups, not just those comparisons against background.

In order to run a valid one-way *F*-test, a minimum number of observations are needed. Denoting the number of data groups by p, at least $p > 2$ groups must be compared (*e.g.*, two or more compliance wells versus background). Each group should have at least three to four statistically independent observations and the total sample size, N, should be large enough so that $N–p > 5$. As long as $p \geq 3$ and there are at least 3 observations per well, this last requirement will always be met. But the statistical power of an ANOVA to identify differences in population means tends to be minimal unless there are at least 4 or more observations per data group. It is also helpful to have at least 8 measurements in background for the test.

Similarly to the two-sample *t*-test, it may be very difficult to verify that the measurements are statistically independent with only a handful of observations per well. One should additionally ensure that the samples are collected far enough apart in time to avoid significant *autocorrelation* (see **Chapter 14** for further discussion). A periodic check of statistical independence in each may be possible after a

few testing periods, when enough data has been collected to enable a statistical assessment of this assumption.

PROCEDURE

Step 1. Combine all the relevant background data collected from multiple wells into one group. These wells should have insignificant mean differences under prior ANOVA testing. If the regulated unit has (p–1) compliance wells, there will then be a total of p data groups. Because there may be different numbers of observations per well, denote the sample size of the ith group by n_i and the total number of data points across all groups by N.

Step 2. Denote the observations in the ith well group by x_{ij} for i = 1 to p and j = 1 to n_i. The first subscript designates the well, while the second denotes the jth value in the ith well. Then compute the mean of each well group along with the overall (grand) mean of the combined dataset using the following formulas:

$$\bar{x}_{i\bullet} = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij} \qquad [17.1]$$

$$\bar{x}_{\bullet\bullet} = \frac{1}{N}\sum_{i=1}^{p}\sum_{j=1}^{n_i} x_{ij} \qquad [17.2]$$

Step 3. Compute the sum of squares of differences between the well group means and the grand mean, denoted SS_{wells}:

$$SS_{wells} = \sum_{i=1}^{p} n_i\left(\bar{x}_{i\bullet} - \bar{x}_{\bullet\bullet}\right)^2 = \sum_{i=1}^{p} n_i\bar{x}_{i\bullet}^2 - N\bar{x}_{\bullet\bullet}^2 \qquad [17.3]$$

The formula on the far right is usually the most convenient for calculation. This sum of squares has (p–1) degrees of freedom associated with it and is a measure of the variability *between* wells. It constitutes the numerator of the F-statistic.

Step 4. Compute the corrected total sum of squares, denoted by SS_{total}:

$$SS_{total} = \sum_{i=1}^{p}\sum_{j=1}^{n_i}\left(x_{ij} - \bar{x}_{\bullet\bullet}\right)^2 = \sum_{i=1}^{p}\sum_{j=1}^{n_i} x_{ij}^2 - N\bar{x}_{\bullet\bullet}^2 \qquad [17.4]$$

The far right equation is convenient for calculation. This sum of squares has (N–1) degrees of freedom associated with it and is a measure of the variability in the entire dataset. In fact, if SS_{total} is divided by (N–1), one gets the overall sample variance.

Step 5. Compute the sum of squares of differences between the observations and the well group means. This is known as the within-wells component of the total sum of squares or, equivalently, as the sum of squares due to error. It is easiest to obtain by subtraction using the far right side of equation [17.5] and is denoted SS_{error}:

$$SS_{error} = \sum_{i=1}^{p}\sum_{j=1}^{n_i}\left(x_{ij} - \bar{x}_{i\bullet}\right)^2 = SS_{total} - SS_{wells} \qquad [17.5]$$

SS_{error} is associated with (N–p) degrees of freedom and is a measure of the variability *within* well groups. This quantity goes into the denominator of the F-statistic.

Step 6. Compute the mean sum of squares for both the between-wells and within-wells components of the total sum of squares, denoted by MS_{wells} and MS_{error}. These quantities are simply obtained by dividing each sum of squares by its corresponding degrees of freedom:

$$MS_{wells} = SS_{wells} \big/ \left(p - 1\right) \qquad [17.6]$$

$$MS_{error} = SS_{error} \big/ \left(N - p\right) \qquad [17.7]$$

Step 7. Compute the F-statistic by forming the ratio between the mean sum of squares for wells and the mean sum of squares due to error, as in **Figure 17-1**. This layout is known as a one-way parametric ANOVA table and illustrates the sum of squares contribution to the total variability, along with the corresponding degrees of freedom, the mean squares components, and the final F-statistic calculated as $F = MS_{wells}/MS_{error}$. Note that the first two rows of the one-way table sum to the last row.

Figure 17-1. One-Way Parametric ANOVA Table

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	F-Statistic
Between Wells	SS_{wells}	p–1	$MS_{wells} = SS_{wells}/(p-1)$	$F = MS_{wells}/MS_{error}$
Error (within wells)	SS_{error}	N–p	$MS_{error} = SS_{error}/(N-p)$	
Total	SS_{total}	N–1		

Step 8. To test the hypothesis of equal means for all p wells, compare the F-statistic in **Figure 17-1** to the α-level critical point found from the F-distribution with (p–1) and (N–p) degrees of freedom in **Table 17-1** of **Appendix D**. α is usually set at 5%, so that the needed comparison value equals the upper 95th percentage point of the F-distribution. The numerator (p-1) and denominator (N-p) degrees of freedom for the F-statistic are obtained from the above table. If the observed F-statistic exceeds the critical point ($F_{.95,\, p-1,\, N-p}$), reject the hypothesis of equal well group population means. Otherwise, conclude that there is insufficient evidence of a significant difference between the concentrations at the p well groups and thus no evidence of potential contamination in any of the compliance wells.

Step 9. In the case of a significant F-statistic that exceeds the critical point in Step 8, determine which compliance wells have elevated concentrations compared to background. This is done by comparing each compliance well individually against the background measurements. Tests to assess concentration differences between a pair of well groups are called *contrasts* in a multiple comparisons ANOVA framework. Since the contrasts are a series of individual

statistical tests, each run at a fixed significance level α^*, the Type I error accumulates across the tests as the number of contrasts increases.

To keep the overall false positive rate close to the targeted rate of 5%, the individual contrasts should be set up as follows: Given (p–1) separate background-compliance contrasts, if (p–1) ≤ 5, run each contrast at a significance level equal to $\alpha^* = .05/(p-1)$. However, if (p–1) > 5, run each contrast at a significance level equal to $\alpha^* = .01$. Note that when there are more than 5 compliance wells, this last provision will tend to raise the overall false positive rate above 5%.

Step 10. Denote the background data set as the first well group, so that the number of background samples is equal to n_b. Then for each of the remaining (p–1) well groups, compute the standard error of the difference between each compliance well and background:

$$SE_i = \sqrt{MS_{error} \cdot \left(\frac{1}{n_b} + \frac{1}{n_i} \right)} \qquad [17.8]$$

Note that MS_{error} is taken from the one-way ANOVA table in **Figure 17-1**. The standard error here is an extension of the standard error of the difference involving the pooled variance in the Student's t-test of **Chapter 16**.

Step 11. Treat the background data as the first well group with the average background concentration equal to $\bar{x}_b$. Compute the Bonferroni t-statistic for each of the (p–1) compliance wells from i = 2 to p, dividing the standard error in Step 10 into the difference between the average concentration at the compliance well and the background average, as shown below:

$$t_i = \left(\bar{x}_i - \bar{x}_b \right) / SE_i \qquad [17.9]$$

Step 12. The Bonferroni t-statistic in equation [17.9] is a type of t-test. Since the estimate of variability used in equation [17.8] has (N–p) degrees of freedom, the critical point can be determined from the Student's t-distribution in **Table 16-1** of **Appendix D**. Let the Bonferroni critical point (t_{cp}) be equal to the upper $(1-\alpha^*) \times 100$th percentage point of the t-distribution with (N–p) degrees of freedom.

Step 13. If any of the Bonferroni t-statistics (t_i) exceed the critical point t_{cp}, conclude that these compliance wells have population mean concentrations significantly greater than background and thus exhibit evidence of possible contamination. Compliance wells for which the Bonferroni t-statistic does not exceed t_{cp} should be regarded as similar to background in mean concentration level.

►EXAMPLE 17-1

Lead concentrations in ground water at two background and four compliance wells were tested for normality and homoscedasticity. These data were found to be best fit by a lognormal distribution with approximately equal variances. The two background wells also indicated insignificant log mean differences. The natural logarithms of these lead values are shown in the table below. Use the one-way parametric ANOVA to determine whether there are any significant concentration increases over background in any of the compliance wells.

	Log(Lead) log(ppb)					
	Background		Compliance			
Date	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6
Jan 1995	4.06	3.83	4.61	3.53	4.11	4.42
Apr 1995	3.99	4.34	5.14	4.54	4.29	5.21
Jul 1995	3.40	3.47	3.67	4.26	5.50	5.29
Oct 1995	3.83	4.22	3.97	4.42	5.31	5.08
Well Mean	3.82	3.96	4.35	4.19	4.80	5.00
Well SD	0.296	0.395	0.658	0.453	0.704	0.143
	$\bar{x}_{BG}$ = 3.89	s_{BG} = 0.333	Grand Mean = 4.35			

SOLUTION

Step 1. Combine the two background wells into one group, so that the background average becomes 3.89 log(ppb). Then $n_b = 8$, while $n_i = 4$ for each of the other four well groups. Note that the total sample size is $N = 24$ and $p = 5$.

Step 2. Compute the (overall) grand mean and the sample mean concentrations in each of the well groups using equations [17.1] and [17.2]. These values are listed (along with each group's standard deviation) in the above table.

Step 3. Compute the sum of squares due to well-to-well differences using equation [17.3]:

$$SS_{wells} = \left[8 \cdot (3.89)^2 + 4 \cdot (4.35)^2 + \ldots + 4 \cdot (5.00)^2\right] - 24 \cdot (4.35)^2 = 4.289$$

This quantity has (5–1) = 4 degrees of freedom.

Step 4. Compute the corrected total sum of squares using equation [17.4] with (N–1) = 23 *df*:

$$SS_{total} = \left[(4.06)^2 + \ldots + (5.08)^2\right] - 24 \cdot (4.35)^2 = 8.934$$

Step 5. Obtain the within-well or error sum of squares by subtraction using equation [17.5]:

$$SS_{error} = 8.934 - 4.289 = 4.646$$

This quantity has (N–p) = 24–5 = 19 degrees of freedom.

Step 6. Compute the mean sums of squares using equations [17.6] and [17.7]:

$$MS_{wells} = 4.289/4 = 1.072$$

$$MS_{error} = 4.646/19 = 0.245$$

Step 7. Construct the F-statistic and the one-way ANOVA table, using **Figure 17-1** in **Appendix D** as a guide:

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	*F*-Statistic
Between Wells	4.289	4	1.072	*F* = 1.072/0.245
Error (within wells)	4.646	19	0.245	= 4.39
Total	8.934	23		

Step 8. Compare the observed *F*-statistic of 4.39 against the critical point taken as the upper 95th percentage point from the *F*-distribution with 4 and 19 degrees of freedom. Using **Table 17-1**, this gives a value of $F_{.95,4,19}$ = 2.90. Since the *F*-statistic exceeds the critical point, the hypothesis of equal well means is rejected, and post-hoc Bonferroni *t*-test comparisons should be conducted.

Step 9. Determine the number of individual contrasts needed. With four compliance wells, (p–1) = 4 comparisons need to be made against background. Therefore, run each Bonferroni *t*-test at the α^* = .05/4 = .0125 level of significance.

Step 10. Compute the standard error of the difference between each compliance well average and the background mean using equation [17.8]. Since the number of observations is the same in each compliance well, the standard error in all four cases will be equal to:

$$SE_i = \sqrt{0.245\left(\frac{1}{8}+\frac{1}{4}\right)} = 0.303$$

Step 11. Compute the Bonferroni *t*-statistic for each compliance well using equation [17.9]:

$$\text{Well 3:} \quad t_2 = (4.35 - 3.89)/0.303 = 1.52$$
$$\text{Well 4:} \quad t_3 = (4.19 - 3.89)/0.303 = 0.99$$
$$\text{Well 5:} \quad t_4 = (4.80 - 3.89)/0.303 = 3.00$$
$$\text{Well 6:} \quad t_5 = (5.00 - 3.89)/0.303 = 3.66$$

Note that because Wells 1 and 2 jointly constitute background, the subscripts above correspond to the well groups and not the actual well numbers. Thus, subscript 2 in the Bonferroni *t*-statistic corresponds to Well 3, subscript 3 corresponds to Well 4, and so forth.

Step 12. Look up the critical point from the *t*-distribution in **Table 16-1** of **Appendix D** using a significance level of α^* = .0125 and (N–p) = 19 *df*. This gives t_{cp} = 2.433.

Step 13. Compare each Bonferroni *t*-statistic from Step 11 against the critical point from Step 12. Because the *t*-statistics at compliance wells 5 and 6 both exceed 2.433, while those at wells 3 and 4 do not, conclude that the population averages in compliance wells 5 and 6 are significantly higher than background. ◄

17.1.2 KRUSKAL-WALLIS TEST

BACKGROUND AND PURPOSE

The parametric one-way ANOVA makes a key assumption that the data residuals are normally-distributed. If this assumption is inappropriate or cannot be tested because of a large fraction of non-detects, a non-parametric ANOVA can be conducted using the *ranks* of the observations rather than the original observations. In **Chapter 16**, the Wilcoxon rank-sum test is presented as a non-parametric alternative to the Student's *t*-test when comparing two groups. The Kruskal-Wallis test is offered as a non-parametric alternative to the one-way *F*-test when several groups need to be simultaneously compared, for instance when assessing patterns of spatial variability. Instead of a test of means, the Kruskal-Wallis tests differences among average population ranks equivalent to the *medians*.

The Kruskal-Wallis test statistic, *H*, does not have the intuitive form of the Student's *t*-test. Under the null hypothesis that all the sample measurements come from identical parent populations, the Kruskal-Wallis statistic follows the well-known *chi-square* statistical distribution. Critical points for the Kruskal-Wallis test can be found as upper percentage points of the chi-square ($\chi^2_{1-\alpha,df}$) distribution in **Table 17-2** of **Appendix D**.

If *H* indicates a significant difference between the populations, individual post-hoc comparisons between each compliance well and background need to be conducted if the Kruskal-Wallis is being used for formal compliance testing. Post-hoc contrasts are not generally necessary for identifying spatial variability. Rather than Bonferroni *t*-statistics, contrasts are based on the data ranks and approximately follow a standard normal distribution. The critical points for these contrasts can be obtained from the standard normal distribution in **Table 10-1** of **Appendix D**.

REQUIREMENTS AND ASSUMPTIONS

While the Kruskal-Wallis test does not require the underlying populations to be normally-distributed, statistical independence of the data is still assumed. Under the null hypothesis of no difference among the groups, the observations are assumed to arise from identical distributions with equal population variances (Hollander and Wolfe, 1999). However, the form of the distribution need not be specified.

A non-parametric ANOVA can be used in any situation that the parametric ANOVA can be used. The minimum data requirements are similar: the sample size for each group in the Kruskal-Wallis procedure should generally be at least four to five observations per group. Despite this similarity, it is often true that non-parametric tests require larger sample sizes than their parametric test counterparts to ensure a similar level of statistical power or *efficiency*. Non-parametric tests make fewer assumptions concerning the underlying data distribution and so more observations are often needed to make the same judgment that would be rendered by a parametric test. However, the greater efficiency of parametric tests is only achieved when the parent population follows certain known statistical distributions. When the distribution is unknown, non-parametric tests may have much greater power than their parametric counterparts.

Even when a known statistical distribution is considered, rank-based non-parametric tests like the Wilcoxon rank-sum and Kruskal-Wallis often perform reasonably well compared to the t-test and ANOVA. The relative *efficiency* of two procedures is defined as the ratio of the sample sizes needed by each to achieve a certain level of power against a specified alternative hypothesis. As sample sizes get larger, the efficiency of the Kruskal-Wallis test relative to the parametric ANOVA approaches a limit that depends on the underlying distribution of the data, but is always at least 86 percent. This means roughly that, in the worst case, if 86 measurements are available for a parametric ANOVA, only 100 sample values are needed to have an equivalently powerful Kruskal-Wallis test. In many cases, the increase in sample size necessary to match the power of a parametric ANOVA is much smaller or not needed at all. The efficiency of the Kruskal-Wallis test is 95% if the underlying data are really normal, and can be much larger than 100% in other cases (*e.g.*, it is 150% if the data residuals follow a distribution called the *double exponential*). When the efficiency exceeds 100%, the Kruskal-Wallis actually needs fewer observations than the parametric ANOVA to achieve a certain power.

These results imply that the Kruskal-Wallis test is reasonably powerful for detecting concentration differences despite the fact that the original data have been replaced by their ranks. The test can be used with fair success even when the data are normally-distributed and the Kruskal-Wallis is not needed. When the data are not normal or a normalizing transformation cannot be found, the Kruskal-Wallis procedure tends to be more powerful for detecting differences than the usual parametric approach.

ADJUSTING FOR TIED OBSERVATIONS

The Kruskal-Wallis procedure will frequently be used when the sample data contain a significant fraction of non-detects. However, the presence of non-detects prevents a unique and complete ranking of the concentration values since the exact values of non-detects are unknown.

To address this problem, two steps are necessary. Since they cannot be uniquely ranked, all non-detects are to be treated statistically as 'tied' values. This is an imperfect remedy, since non-detects represent left-censored values and are not necessarily tied. Unfortunately, there is no straightforward, easily implemented alternative to the Kruskal-Wallis for comparing three or more groups containing left-censored observations, unlike the Tarone-Ware alternative to the Wilcoxon rank-sum test discussed in **Chapter 16**. So in the presence of ties (*e.g.*, non-detects or quantified concentrations rounded to the same value), all tied observations should receive the same midrank (discussed in **Section 16.3**). This rank is computed as the *average* of the ranks that would be given to each group of ties if the tied values actually differed by a tiny amount and could be ranked.

To account for multiple reporting limits, all non-detects should be treated as if censored at the highest reporting limit [RL] in the overall sample. Thus, a non-detect reported as <5 would be treated as 'tied' with a non-detect reported as <1, due to the impossibility of knowing which value is actually larger. The only exception to this strategy is when laboratory qualifiers can be used to rank some non-detects as probably greater in magnitude than others. A reasonable strategy discussed in **Section 16.3** is to group all "U" values as the lowest set of ties, other non-detects as a higher set of ties, and to rank all "J" and/or "E" values according to their estimated concentrations. In situations where estimated values for J and E samples are not provided, treat these measurements as the highest group of tied non-detects. Always give the highest ranks to explicitly quantified or estimated concentration measurements.

The second step for handling ties is to compute the Kruskal-Wallis statistic as described below, using for each tied value its corresponding midrank. Then an adjustment to the Kruskal-Wallis statistic needs to be made to account for the presence of ties. This adjustment requires computation of the formula:

$$H^* = H \Big/ \left[1 - \left(\sum_{i=1}^{g} \frac{t_i^3 - t_i}{N^3 - N} \right) \right] \qquad [17.10]$$

where g equals the number of distinct groups of tied observations, N is the total sample size across all groups, and t_i is the number of observations in the ith tied group. Unless there are a substantial number of ties in the overall dataset, the adjustment in equation [17.10] will tend to be small. Still, it is important to properly account for the presence of tied values.

PROCEDURE

Step 1. To run the Kruskal-Wallis test, denote the total sample size across all well groups by N. Temporarily combine all the data into one group and rank the observations from smallest to largest. Treat all non-detects as tied at the lowest possible concentration value, unless using lab qualifiers to distinguish between 'undetected' and other non-detects. Combine all background wells into a single group where appropriate. Denote this set of background data as group 1. Then let R_{ij} denote the jth rank from the ith well group, and let k equal the total number of groups (*i.e.*, one group of background values and (k–1) groups of compliance wells).

Step 2. Compute the sum of the ranks and the average rank in each well group, letting n_i equal the sample size in the ith group and using the following formulas:

$$R_{i\bullet} = \sum_{j=1}^{n_i} R_{ij} \qquad [17.11]$$

$$\bar{R}_{i\bullet} = \frac{1}{n_i} R_{i\bullet} \qquad [17.12]$$

Step 3. Calculate the Kruskal-Wallis test statistic H and the adjustment for ties, if necessary, using equation [17.10], where H is given by:

$$H = \left[\frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_{i\bullet}}{n_i} \right] - 3(N+1) \qquad [17.13]$$

Step 4. Given the level of significance (α), determine the Kruskal-Wallis critical point (χ^2_{cp}) as the upper (1–α) × 100th percentage point from the chi-square distribution with (k–1) degrees of freedom (**Table 17-2** in **Appendix D**). Usually α is set equal to 0.05, so that the upper 95th percentage point of the chi-square distribution is needed.

Step 5. Compare the Kruskal-Wallis test statistic, H, against the critical point χ^2_{cp}. If H is no greater than the critical point, conclude there is insufficient evidence of significant differences between any of the well group populations. If $H > \chi^2_{cp}$, however, conclude there is a significant difference between at least one pair of the well groups. Post-hoc comparisons are then necessary to determine whether any of the compliance wells significantly exceeds background (note that post-hoc comparisons are not necessary if using the Kruskal-Wallis test to merely identify spatial variability).

Step 6. In the case of a significant H-statistic that exceeds the critical point in Step 5, determine which compliance wells have elevated concentrations compared to background. This is done by comparing each compliance well against background, using a set of *contrasts* (as described for the parametric one-way ANOVA in **Section 17.1.1**).

To keep the test-wise or experiment-wise false positive rate close to the targeted (*i.e.*, nominal) rate of 5%, the individual contrasts should be set up as follows: Given (k–1) separate background-compliance contrasts, if (k–1) ≤ 5, run each contrast at a significance level equal to $\alpha^* = .05/(k-1)$. However, if (k–1) > 5, run each contrast at a significance level equal to $\alpha^* = .01$. Note that when there are more than 5 downgradient wells, this last provision will tend to raise the overall false positive rate above 5%.

Step 7. Since the background data is the first well group, the number of background observations is equal to n_1. For each of the remaining (k–1) well groups, compute the approximate rank-based standard error of the difference between each compliance well and background using equation [17.14]:

$$SE_i = \sqrt{\frac{N(N+1)}{12}\left(\frac{1}{n_1}+\frac{1}{n_i}\right)} \qquad [17.14]$$

Step 8. Let the average background rank be identified as $\overline{R}_b$. Compute the post-hoc Z-statistic for each of the (k–1) compliance wells for i = 2 to k, dividing the standard error in step 7 into the difference between the average rank at the compliance well and the background rank average, as shown below:

$$Z_i = \left(\overline{R}_i - \overline{R}_b\right)/SE_i \qquad [17.15]$$

Step 9. The Z-statistic in equation [17.15] has an approximate standard normal distribution under the null hypothesis that the ith compliance well is identical in distribution to background. The critical point (z_{cp}) can be found as the upper (1–α) × 100th percentage point of the normal distribution in **Table 10-1** of **Appendix D**.

Step 10. Compare the post-hoc Z-statistics for each of the (k–1) compliance wells against the critical point (z_{cp}). Any Z-statistic that exceeds the critical point provides significant evidence of an elevation over background in that compliance well at the α level of significance.

►EXAMPLE 17-2

Use the non-parametric Kruskal-Wallis test on the following data to determine whether there is evidence of possible toluene contamination at a significance level of α = 0.05.

	Toluene Concentration (ppb)				
	Background Wells		Compliance Wells		
Month	Well 1	Well 2	Well 3	Well 4	Well 5
1	<5	<5	<5	<5	<5
2	7.5	<5	12.5	13.7	20.1
3	<5	<5	8.0	15.3	35.0
4	<5	<5	<5	20.2	28.2
5	6.4	<5	11.2	25.1	19.0

SOLUTION

Step 1. Since non-detects account for 48% of these data, it would be very difficult to verify the assumptions of normality and equal variance necessary for a parametric ANOVA. Use the Kruskal-Wallis test instead, pooling both background wells into one group and treating each compliance well as a separate group. Note that $N = 25$ and $k = 4$.

Compute ranks for all the data including tied observations (*e.g.*, non-detects) as in the following table. Note that each non-detect is given the same midrank, equal to the average of the first 12 unique ranks.

	Toluene Ranks				
	Background Wells		Compliance Wells		
Month	Well 1	Well 2	Well 3	Well 4	Well 5
1	6.5	6.5	6.5	6.5	6.5
2	14	6.5	17	18	21
3	6.5	6.5	15	19	25
4	6.5	6.5	6.5	22	24
5	13	6.5	16	23	20
Group Size	$n_1 = 10$		$n_2 = 5$	$n_3 = 5$	$n_4 = 5$
Rank Sum	$R_{1\bullet} = 79$		$R_{2\bullet} = 61$	$R_{3\bullet} = 88.5$	$R_{4\bullet} = 96.5$
Rank Mean	$\bar{R}_{1\bullet} = 7.9$		$\bar{R}_{2\bullet} = 12.2$	$\bar{R}_{3\bullet} = 17.7$	$\bar{R}_{4\bullet} = 19.3$

Step 2. Calculate the sum and average of the ranks in each group using equations [17.11] and [17.12]. These results are given in the above table.

Step 3. Compute the Kruskal-Wallis statistic H using equation [17.13]:

$$H = \frac{12}{25 \cdot 26}\left[\frac{79^2}{10} + \frac{61^2}{5} + \frac{88.5^2}{5} + \frac{96.5^2}{5}\right] - (3 \cdot 26) = 10.56$$

Also compute the adjustment for ties with equation [17.10]. There is only one group of distinct tied observations — the non-detects — containing 12 samples. Thus, the adjusted Kruskal-Wallis statistic is given by:

$$H^* = 10.56 \Big/ \left[1 - \left(\frac{12^3 - 12}{25^3 - 25} \right) \right] = 11.87$$

Step 4. Determine the critical point of the Kruskal-Wallis test: with α = .05, the upper 95th percentage point of the chi-square distribution with (k–1) = 4–1 = 3 degrees of freedom [df] is needed. **Table 17-2** of **Appendix D** gives $\chi^2_{cp} = \chi^2_{.95,3} = 7.81$.

Step 5. Since the observed Kruskal-Wallis statistic of 11.87 is greater than the chi-square critical point, there is evidence of significant differences between the well groups. Therefore, post-hoc pairwise comparisons are necessary.

Step 6. To determine the significance level appropriate for post-hoc comparisons, note there are three compliance wells that need to be tested against background. Therefore, each of these contrasts should be run at the α^* = 0.05/3 = 0.0167 significance level.

Step 7. Calculate the standard error of the difference for the three contrasts using equation [17.14]. Since the sample size at each compliance well is five, the *SE* will be identical for each comparison, namely,

$$SE_i = \sqrt{\frac{25 \cdot 26}{12} \left(\frac{1}{10} + \frac{1}{5} \right)} = 4.031$$

Step 8. Form the post-hoc Z-statistic for each contrast using equation [17.15]:

$$\text{Well 3: } Z_2 = (12.2 - 7.9) \big/ 4.031 = 1.07$$
$$\text{Well 4: } Z_3 = (17.7 - 7.9) \big/ 4.031 = 2.43$$
$$\text{Well 5: } Z_4 = (19.3 - 7.9) \big/ 4.031 = 2.83$$

Step 9. Find the upper (1–α^*) × 100th percentage point from the standard normal distribution in **Table 10-1** in **Appendix D**. With α^* = .0167, this gives a critical point (by linear interpolation) of $z_{cp} = z_{.9833} = 2.127$.

Step 10. Since the *Z*-statistics at wells 4 and 5 exceed the critical point, there is significant evidence of increased concentration levels at wells 4 and 5, but not at well 3. ◄

17.2 TOLERANCE LIMITS

A *tolerance interval* is a concentration range designed to contain a pre-specified proportion of the underlying population from which the statistical sample is drawn (*e.g.*, 95 percent of all possible population measurements). Since the interval is constructed from random sample data, a tolerance interval is expected to contain the specified population proportion only with a certain level of statistical

confidence. Two coefficients are thus associated with any tolerance interval. One is the population proportion that the interval is supposed to contain, called the coverage (γ). The second is the degree of confidence with which the interval reaches the specified coverage. This is sometimes known as the tolerance coefficient or more simply, the confidence level ($1-\alpha$). A tolerance interval with 95% coverage and a tolerance coefficient of 90 percent is constructed to contain, on average, 95% of the distribution of all possible population measurements with a confidence probability of 90%.

A *tolerance limit* is a one-sided tolerance interval. The upper limit is typically of most interest in groundwater monitoring. Tolerance limits are a standard statistical method that can be useful in groundwater data analysis, especially as an alternative to *t*-tests or ANOVA for interwell testing. The RCRA regulations allow greater flexibility in the choice of α when using tolerance and prediction limits and control charts, so a larger variety of data configurations may be amenable to one of these approaches. The Unified Guidance still recommends prediction limits or control charts over tolerance limits for formal compliance testing in detection monitoring, and confidence intervals over tolerance limits in compliance/assessment monitoring when a background standard is needed.

An interwell tolerance limit constructed on background data is designed to cover all but a small percentage of the background population measurements. Hence background observations should rarely exceed the upper tolerance limit. By the same token, when testing a null hypothesis (H_0) that the compliance point population is identical to background, *compliance point* measurements also should rarely exceed the upper tolerance limit, unless H_0 is false. The upper tolerance limit thus gauges whether or not concentration measurements sampled from compliance point wells are too extreme relative to background.

17.2.1 PARAMETRIC TOLERANCE LIMITS

BACKGROUND AND PURPOSE

To test the null hypothesis (H_0) that a compliance point population is identical to that of background, an upper tolerance limit with high coverage (γ) can be constructed on the sample background data. Coverage of 95% is usually recommended. In this case, random observations from a distribution identical to background should exceed the upper tolerance limit less than 5% of the time. Similarly, a tolerance coefficient or confidence level of at least 95% is recommended. This gives 95% confidence that the (upper) tolerance limit will contain at least 95% of the distribution of observations in background or in any distribution similar to background. Note that a tolerance coefficient of 95% corresponds to choosing a significance level (α) equal to 5%. Hence, as with a one-way ANOVA, the overall false positive rate for a tolerance interval is set to approximately 5%.

Once the limit is constructed on background, each compliance point observation (perhaps from several different wells) is compared to the upper tolerance limit. This is different from the comparison of sample means in an ANOVA test. If any compliance point measurement exceeds the limit, the well from which it was drawn is flagged as showing a significant increase over background. Note that the factors κ used to adjust the width of the tolerance interval (**Table 17-3** in **Appendix D**) are designed to provide *at least* 95% coverage of the parent population. Applied over many data sets, the *average* coverage of these intervals will often be close to 98% or more (see Guttman, 1970). Therefore, it would be unusual to find

more than 2 or 3 samples out of every 100 exceeding the tolerance limit under the null hypothesis. This fits with the purpose behind the use of a tolerance interval, which is to establish an upper limit on background that will rarely be exceeded, unless some change in the groundwater causes concentration levels to rise significantly at one or more compliance points.

Testing a large number of compliance point samples against such a background tolerance limit even under conditions of no releases practically ensures a few measurements will occasionally exceed the limit. The Unified Guidance therefore recommends that tolerance limits be used in conjunction with verification resampling of those wells suspected of possible contamination, in order to either verify or disconfirm the initial round of sampling and to avoid false positive results.

REQUIREMENTS AND ASSUMPTIONS

Standard parametric tolerance limits assume normality of the sample background data used to construct the limit. This assumption is critical to the statistical validity of the method, since a tolerance limit with high coverage can be viewed as an estimate of a *quantile* or *percentile* associated with the *tail probability* of the underlying distribution. If the background sample is non-normal, a normalizing transformation should be sought. If a suitable transformation is found, the limit should be constructed on the transformed measurements and can then be *back-transformed* to the raw concentration scale prior to comparison against individual compliance point values.

If no transformation will work, a non-parametric tolerance limit should be considered instead. Unfortunately, non-parametric tolerance limits generally require a much larger number of observations to provide the same levels of coverage and confidence as a parametric limit. It is recommended that a parametric model be fit to the data if at all possible.

A tolerance limit can be computed with as few as three observations from background. However, doing so results in a high upper tolerance limit with limited statistical power for detecting increases over background. Usually, a background sample size of at least eight measurements will be needed to generate an adequate tolerance limit. If multiple background wells are screened in equivalent hydrostratigraphic positions and the data can reasonably be combined (**Chapter 5**), one should consider using pooled background data from multiple wells to increase the background sample size.

Like many tests described in the Unified Guidance, tolerance limits as applied to groundwater monitoring assume *stationarity* of the well field populations both temporally (*i.e.*, over time) and spatially. The data also needs to be statistically *independent*. Since an adequately-sized background sample will have to be amassed over time (in part to maintain enough temporal spacing between observations so that independence can be assumed), the background data should be checked for apparent *trends* or *seasonal effects*. As long the background mean is stable over time, the amassed data from a longer span of sampling will provide a better statistical description of the underlying background population.

As a primarily interwell technique, tolerance limits should only be utilized when there is minimal *spatial variability*. Explicit checks for spatial variation should be conducted using box plots and/or ANOVA.

In the usual test setting, one new compliance point observation from each distinct well is compared against the tolerance limit during each statistical evaluation. Under the null hypothesis of identical

populations, the compliance point measurements are assumed to follow the same distribution as background. Further, the compliance data are assumed to be mutually statistically independent. Such assumptions are almost impossible to check with only one new value per compliance well. However, periodic checks of the key assumptions are recommended after accumulating several sampling rounds of compliance data.

PROCEDURE

Step 1. Calculate the mean $\bar{x}$, and the standard deviation s, from the background sample.

Step 2. Construct the one-sided upper tolerance limit as

$$TL = \bar{x} + \kappa\left(n, \gamma, 1-\alpha\right) \cdot s \qquad [17.16]$$

where $\kappa(n,\gamma,1-\alpha)$ is the one-sided normal tolerance factor found in **Table 17-3** of **Appendix D** associated with a sample size of n, coverage coefficient of γ, and confidence level of $(1-\alpha)$.

Equation [17.16] applies to normal data. If a transformation is needed to normalize the sample, the tolerance limit needs to be constructed on the transformed measurements and the limit back-transformed to the original concentration scale. If the limit was constructed, for example, on the logarithms of the original observations, where $\bar{y}$ and s_y are the log-mean and log-standard deviation, the tolerance limit can be back-transformed to the concentration scale by exponentiating the limit. The tolerance limit is computed as:

$$TL = \exp\left[\bar{y} + \kappa\left(n, \gamma, 1-\alpha\right) \cdot s_y\right] \qquad [17.17]$$

Step 3. Compare each observation from the compliance well(s) to the upper tolerance limit found in Step 2. If any observation exceeds the tolerance limit, there is statistically significant evidence that the compliance well concentrations are elevated above background. Verification resampling should be conducted to verify or disconfirm the initial result.

►EXAMPLE 17-3

The table below consists of chrysene concentration data (ppb) found in water samples obtained from two background wells (Wells 1 and 2) and three compliance wells (Wells 3, 4, and 5). Compute the upper tolerance limit on background for coverage of 95% with 95% confidence and determine whether there is evidence of possible contamination at any of the compliance wells.

	Chrysene Concentration (ppb)				
Month	Well 1	Well 2	Well 3	Well 4	Well 5
1	19.7	10.2	68.0	26.8	47.0
2	39.2	7.2	48.9	17.7	30.5
3	7.8	16.1	30.1	31.9	15.0
4	12.8	5.7	38.1	22.2	23.4
Mean	19.88	9.80	46.28	24.65	28.98
SD	13.78	4.60	16.40	6.10	13.58

SOLUTION

Step 1. A Shapiro-Wilk test of normality on the pooled set of eight background measurements gives W = 0.7978 on the original scale and W = 0.9560 after log-transforming the data, suggesting that the data are better fit by a lognormal distribution. Therefore, construct the tolerance limit on the logged observations, listed below along with the log-means and log-standard deviations.

	Log Chrysene log(ppb)				
Month	Well 1	Well 2	Well 3	Well 4	Well 5
1	2.981	2.322	4.220	3.288	3.850
2	3.669	1.974	3.890	2.874	3.418
3	2.054	2.779	3.405	3.463	2.708
4	2.549	1.740	3.640	3.100	3.153
Mean	2.813	2.204	3.789	3.181	3.282
SD	0.685	0.452	0.349	0.253	0.479
BG Mean	2.509				
BG SD	0.628				

Step 2. Compute the upper tolerance limit on the pooled background data using the logged chrysene concentration data. The tolerance factor for a one-sided upper normal tolerance limit with 95% coverage and 95% probability and n = 8 observations is equal to (from **Table 17-3** of **Appendix D**) κ = 3.187. Therefore, the upper tolerance limit is computed using equation [17.17] as:

$$TL = \exp\left[2.509 + 3.187 \times 0.628\right] = 90.96 \text{ ppb}$$

Step 3. Compare the measurements at each compliance well to the upper tolerance limit, that is TL = 90.96 ppb. Since none of the original chrysene concentrations exceeds the upper TL, there is insufficient evidence of chrysene contamination in these data. ◄

17.2.2 NON-PARAMETRIC TOLERANCE INTERVALS

BACKGROUND AND PURPOSE

When an assumption of normality cannot be justified especially with a significant portion of non-detect observations, the use of non-parametric tolerance intervals should be considered. The upper tolerance limit in a non-parametric setting is usually chosen as an order statistic of the sample data (Guttman, 1970), commonly the maximum value or maybe the second or third largest value observed.

Because the maximum observed background value is often taken as the upper tolerance limit, non-parametric tolerance intervals are easy to construct and use. The sample data needs to be ordered, but no

ranks need be assigned to the concentration values other than to determine the largest measurements. This also means that non-detects do not have to be uniquely ordered or handled in any special manner.

One advantage to using a maximum concentration instead of assigning ranks to the data (Wilcoxon rank-sum or Kruskal-Wallis tests) is that non-parametric tolerance intervals are reflective of actual concentration magnitudes. Another advantage is that unless all the background data are non-detect, the maximum value will be a detected concentration leading to a well-defined upper tolerance limit. If all the sample data are non-detect, an RL (*e.g.*, the lowest achievable *quantitation limit* [QL]) may serve as an approximate upper tolerance limit.

REQUIREMENTS AND ASSUMPTIONS

Unlike parametric tolerance intervals, the desired coverage (γ) or confidence level ($1-\alpha$) cannot be pre-specified using a non-parametric limit. Instead, the *achieved* coverage and/or confidence level depends entirely on the background sample size (n) and the order statistic chosen as the upper tolerance limit (*e.g.*, the maximum value). Guttman (1970) has shown that the *coverage* of the limit follows a *beta probability density* with cumulative distribution:

$$I_t\left(n-m+1,m\right)=\int_{u=0}^{t}\frac{\Gamma\left(n+1\right)}{\Gamma\left(n-m+1\right)\Gamma\left(m\right)}u^{n-m}\left(1-u\right)^{m-1}du \qquad [17.18]$$

where n = sample size and m = [(n+1)–(rank of upper tolerance limit value)]. If the background maximum is selected as the tolerance limit, its rank is equal to n and so m = 1. If the second largest value is chosen as the limit, its rank would be equal to (n–1) giving m = 2.

As a non-parametric procedure, no distributional model must be fit to the background measurements. It is assumed, however, that the compliance point data follow the same distribution as background — even if unknown — under the null hypothesis. Even though no distributional model is assumed, order statistics of any random sample follow certain probability laws as noted above. Since the beta distribution is closely related to the more familiar *binomial distribution*, Guttman showed that in order to construct a non-parametric tolerance interval with at least γ coverage and ($1-\alpha$) confidence probability, the number of (background) samples should be chosen such that:

$$\sum_{t=m}^{n}\binom{n}{t}\left(1-\gamma\right)^{t}\gamma^{n-t}\geq 1-\alpha \qquad [17.19]$$

If the background maximum is selected as the upper tolerance limit, so that m = 1, this inequality reduces to the simpler form

$$1-\gamma^{n}\geq 1-\alpha\,. \qquad [17.20]$$

Table 17-4 in **Appendix D** provides minimum coverage levels with 95% confidence for various choices of n, using either the maximum sample value or the second largest measurement as the tolerance limit. As an example, with n = 16 background measurements, the minimum coverage is γ = 83% if the background maximum is designated as the upper tolerance limit and γ = 74% if the tolerance limit is

taken as the second largest background value. In general, **Table 17-4** of **Appendix D** illustrates that if the underlying distribution is unknown, *more background samples are needed compared to the parametric setting in order to construct a tolerance interval with sufficiently high coverage.* Parametric tolerance intervals do not require as many background samples precisely because the form of the underlying distribution is assumed to be known.

An alternate way to construct an appropriate tolerance limit is to calculate the maximum confidence level for various choices of n guaranteeing at least 95% coverage. With n = 8 background measurements, the approximate confidence level is at most 34% when the largest value is taken as the tolerance limit and only 6% if the second largest value is taken as the tolerance limit. Clearly, it is advantageous to fit a parametric distributional model to the data if at all possible unless n is fairly large.

Although non-parametric tolerance limits do not require an assumption of normality, other assumptions of tolerance limits apply equally to the parametric and non-parametric versions. Specifically, the sample data should be statistically *independent* and show no evidence of *autocorrelation*, *trends*, or *seasonal effects* in background. Applied as an interwell test, there should also be minimal to no natural on-site *spatial variation*.

By construction, outliers in background can be a particular problem for non-parametric tolerance limits, especially if the background maximum is chosen as the upper limit. A limit based on a large, extreme outlier will result in a test having little power to detect increases in compliance wells. Consequently, the background sample should be screened ahead of time for possible *outliers* (**Chapter 12**). Confirmed outliers should be removed from the data set before setting the tolerance limit.

An important caveat to this advice is that almost all statistical outlier tests depend crucially on the ability to fit the remaining data (minus the suspected outlier(s)) to a known statistical distribution. In those cases where a non-parametric tolerance limit is selected because of a large fraction of non-detects, fitting the data to a distributional model may be difficult or impossible, negating formal outlier tests. As an alternative, the non-parametric upper tolerance limit could be set to a different order statistic in background (*i.e.*, other than the maximum), to provide some insurance against possible large outliers. This strategy will work provided there are enough background measurements to allow for adequately high coverage and confidence in the resulting limit.

PROCEDURE

Step 1. Sort the set of background data into ascending order and choose either the largest or second largest measurement as the upper *TL*. Use **Table 17-4** in **Appendix D** to determine the coverage γ associated with 95% or 99% confidence. Note also that if the largest or second largest measurement is a non-detect, the upper tolerance limit should be set to the RL most appropriate to the data (*e.g.*, the lowest achievable practicable quantification limit [PQL]).

Step 2. Compare each compliance point measurement against the upper tolerance limit. Identify significant evidence of possible contamination at any compliance well in which one or more measurements exceed the upper tolerance limit. If the upper tolerance limit equals the RL, a violation should be flagged anytime a detected value is quantified above the RL.

Step 3. Because the risk of false positive errors is greatly increased if either the confidence level or coverage drop substantially below 95%, both of these parameters should be routinely reported

and noted as being below the target levels. One should also strongly consider comparing one or more verification resamples against the upper tolerance limit before identifying a clear violation.

►EXAMPLE 17-4

Use the following copper background data to establish a non-parametric upper tolerance limit and determine if either compliance well shows evidence of copper contamination.

	Copper Concentration (ppb)				
	Background Wells			Compliance Wells	
Month	Well 1	Well 2	Well 3	Well 4	Well 5
1	<5	9.2	<5		
2	<5	<5	5.4		
3	7.5	<5	6.7		
4	<5	6.1	<5		
5	<5	8.0	<5	6.2	<5
6	<5	5.9	<5	<5	<5
7	6.4	<5	<5	7.8	5.6
8	6.0	<5	<5	10.4	<5

SOLUTION

Step 1. The pooled background data in Wells 1, 2, and 3 have a maximum observed value of 9.2 ppb. Set the 95% confidence upper tolerance limit equal to this value. Because 24 background samples are available, **Table 17-4** in **Appendix D** indicates that the minimum coverage is equal to 88%. To increase either the coverage, more background samples would have to be collected.

Step 2. Compare each sample in compliance Wells 4 and 5 to the upper tolerance limit. Since none of the measurements at Well 5 is above 9.2 ppb, while one sample from Well 4 is above the limit, conclude that there may be significant evidence of copper contamination at Well 4 but not Well 5.

Step 3. Note that with only 88% coverage and 24 background samples, the risk of a false positive result is more than 10%. Well 4 should be resampled to determine whether the exceedance is replicated. ◄

17.3 TREND TESTS

The Unified Guidance recommends *trend testing* as an intrawell alternative to prediction limits or control charts when those methods are not suitable. Prediction limits and control charts (as well as *t*-tests and ANOVA) all involve a comparison of compliance and background populations under the key assumption that the underlying concentration distributions are *stationary over time*. That is, the populations are presumed to have stable (*i.e.*, roughly constant) means over the period of sampling prior to statistical evaluation.

Unfortunately, there is no guarantee that groundwater populations will remain stable during long-term monitoring. Because sampling at many sites is generally done on a quarterly, semi-annual, or annual basis, it will generally take one to two years or more to collect enough background data to run the statistical tests discussed in the Unified Guidance. Over this length of time, the statistical characteristics of groundwater may or may not change in significant ways.

If background groundwater conditions are in a state of flux, trend tests provide a significant advantage over both intrawell prediction limits and control charts. Both of the latter methods involve a designation of some portion of the historical sampling record as the intrawell background for a given compliance well. Ideally, this intrawell background should consist of measurements known to be uncontaminated and which represent a random sample from a stable underlying population, just as with *t*-tests and ANOVA. If the mean and/or standard deviation of the underlying population *changes* while intrawell background is being compiled, results of either prediction limit or control chart tests against more recently collected data can be severely biased or altogether inaccurate.

One drawback to the Shewhart-CUSUM control charts presented in **Chapter 20** is that they are somewhat sensitive to the parametric assumption of underlying normality. If the measurements are lognormal rather than normal, for instance, the nominal performance characteristics (*i.e.*, Type I error rate and statistical power) of control charts are significantly affected. By the same token, control charts are impacted if the intrawell background contains a large fraction of non-detects. Non-detect adjustments can sometimes be made to the baseline data via methods discussed in **Chapter 15**, but if a normalizing transformation or adjustment is not successful, no straightforward non-parametric control chart exists.

Consequently, neither prediction limits nor control charts are appropriate for every circumstance where an intrawell comparison may be warranted or necessary. Thus, ***the Unified Guidance recommends that users consider trend testing as an alternative to prediction limits or control charts when those methods are not suitable as intrawell techniques***. Tests for trend are specifically designed to identify those groundwater populations whose mean concentrations are not stationary over time, but rather are increasing (or decreasing) by measurable amounts. Ultimately, the goal of any reasonable detection or compliance/assessment monitoring program is to determine whether or not the concentration levels of key contaminants or indicator parameters have significantly increased during the period of monitoring and, if so, whether the increase is attributable to facility waste management practices.

The detection of trends is a complex subject. Whole textbooks are devoted to the more general topic of *time series analysis*, including the identification and modeling of time trends — step functions, linear and quadratic trends, exponential growth, *etc.* The Unified Guidance only attempts to identify the simplest kind of linear increases, not the specification or testing of more complex models. The methods described below are all designed to effectively test for (increasing) linear trends, though they will also identify simple increases over time when a trend is present but does not follow a strictly linear pattern.

The Unified Guidance recommends using trend tests in detection monitoring to measure the extent and nature of an apparent concentration increase, especially to determine whether or not the increase occurs consistently over time. Two questions are of particular interest: 1) is there a statistically significant, (positive) trend over the period of monitoring? and 2) what is the nature (*i.e.*, slope and intercept) of the trend? By identifying a positive trend, one can show that contaminant levels have gotten worse compared to early measurements from the well being tested. Furthermore, by measuring the nature

of the trend, including the average rate of increase per unit of time, one can estimate how rapidly concentration levels are increasing and the current mean- or median-level magnitude of contamination. Such information can provide an invaluable portrait of the changes occurring on-site and probably offers the most compelling evidence — under these conditions — for demonstrating that the basic null hypothesis of detection monitoring has been violated.

17.3.1 LINEAR REGRESSION

BACKGROUND AND PURPOSE

The most common way to measure a linear trend is to compute a *linear regression* of concentration data when plotted against the time or date of sample collection. By way of interpretation, each point along a linear regression trend line is an estimate of the true mean concentration *at that point in time*. Thus, a linear regression can be used to assess whether or not the population mean at a compliance well has significantly increased or decreased.

Linear regression is a standard technique in statistics textbooks and many data analysis software packages. It is more generally applicable to linear relationships between any pair of random variables and not simply to time trends. Good references for performing linear regression and for checking and verifying its assumptions include Draper and Smith (1998) and Cook and Weisberg (1999).

Unlike prediction limits or control charts which are constructed using only the background data, trend tests including linear regression are computed with all available earlier and more recent data at the compliance well of interest. One then might incorrectly assume that a comparison against intrawell background is not being conducted. But an intrawell comparison does occur with a trend test. Statistical identification of a structured pattern of increase from the first portion of the sampling record to more recent data indicates that concentration levels are no longer similar to intrawell background, but have risen more than expected by chance.

Statistical identification of a positive trend involves testing the estimated slope coefficient from the linear regression trend line. A specially constructed *t*-test is used to make this determination, as described below. If this test is significant, the slope is judged to be different from zero, indicating that a change in concentration levels has occurred over the period of sampling represented by the data set.

REQUIREMENTS AND ASSUMPTIONS

Linear regression as a parametric statistical technique makes a number of underlying assumptions. Among the most important of these are that the regression residuals (*i.e.*, the difference between each concentration measurement and its predicted value from the regression equation) are approximately normal in distribution, homoscedastic (*i.e.*, equal in variance at different times and for different mean concentration levels), and statistically independent. Significant skewness or the presence of outliers can bias or invalidate the results of a trend test based on linear regression. Furthermore, standard linear regression methods do not account for non-detects or missing data values at selected sampling events.

Because the key assumptions for linear regression depend not on the original measurements but rather on the regression residuals, a tentative trend line needs to first be constructed before its

assumptions can be checked. Once a linear regression on time is fitted to the data, the residuals around the trend line need to be computed and then tested for normality, apparent skewness, and equal variance over time. This last assumption is particularly important to testing whether the slope of an apparent trend is statistically different from zero (a zero slope indicating that well concentrations have not changed over time).

Inferences around a linear regression are generally appropriate when three conditions hold: 1) the residuals from the regression are approximately normal or at least reasonably symmetric in distribution; 2) a scatter plot of residuals versus concentrations indicates a scatter cloud of essentially uniform *vertical thickness or width* (i.e., the scatter cloud does not tend to increase in width with the level of concentration which would suggest a proportional effect between the underlying population mean and variance); and 3) a scatter plot of residuals versus time also exhibits a uniformly thick scatter cloud. If the thickness or width is substantially different at distinct time points, the assumption of equal variances over time may not be true.

If any of these conditions is substantially violated, it may indicate that the basic trend is either non-linear or the magnitude of the variance is not independent of the mean concentration level and/or the time of sampling. One possible remedy is to try a transformation of the concentration data and re-estimate the linear regression. This will change the interpretation of the estimated regression from a linear trend of the form $y = a + bt$, where y and t represent concentration and time respectively, to a non-linear pattern. As an example, if the concentration data are log-transformed, the regression equation will have the form $\log y = a + bt$. Back-transformed to the original concentration scale, the trend function will then have the form $y = \exp\left(a + bt\right)$.

In transforming the regression data this way, the estimated trend in the concentration domain (after back-transforming) no longer represents the original mean. Rather, the transformation induces a *bias* when converted back to the raw-scale data. If a log transformation is used, for instance, the back-transformed trend line will represent the raw-scale *geometric mean* and not the *arithmetic mean*. As with Student's t-tests on lognormal data (**Chapter 16**), demonstrating that the *geometric* mean is increasing also implies that the *arithmetic* mean has risen so long as the regression residuals are homoscedastic.

A minimum of 8 to 10 measurements is generally necessary to compute a linear regression, especially to estimate the variance around the trend line (known as the *mean squared error* or MSE). The regression residuals should be statistically independent, an assumption that can be approximately verified via one of the autocorrelation tests of **Chapter 14**.

One last assumption is that there should be few if any non-detects when computing a linear regression. As a matter of common sense, a significant increasing or decreasing trend should be based on reliably quantified measurements. If this is not the case, the user should check to see whether the "trend" may be an artifact induced by changes in detection and/or quantitation limits over time. The concentration levels of a series of non-detects may appear to be decreasing, for instance, simply because analytical methods have improved over the years leading to lower RLs. Such artifacts of plotting and data reporting should not be considered real trends.

When the assumptions of linear regression cannot be verified at least approximately, a non-parametric trend method should be considered instead. **Sections 17.3.2** and **17.3.3** discuss the *Mann-*

Kendall test for trend and the *Theil-Sen trend line*. These methods can be particularly valuable when constructing trends on data sets containing non-detects.

PROCEDURE

Step 1. Construct a time series plot of the compliance point measurements. If a discernible trend is evident, compute a linear regression of concentration against sampling date (time), letting x_i denote the *i*th concentration value and t_i denote the *i*th sampling date. Estimate the linear slope $\hat{b}$ with the formula:

$$\hat{b} = \sum_{i=1}^{n} (t_i - \bar{t}) \cdot x_i \Big/ (n-1) \cdot s_t^2 \qquad [17.21]$$

This estimate then leads to the regression equation, given by:

$$\hat{x}_t = \bar{x} + \hat{b} \cdot (t - \bar{t}) \qquad [17.22]$$

where $\bar{t}$ denotes the mean sampling date, s_t^2 is the variance of sampling dates, $\bar{x}$ is the mean concentration level, and $\hat{x}_t$ represents the estimated mean concentration at time *t*.

Note: though the variable *t* above represents time, it could just as easily signify another variable, perhaps a second constituent for which an association with *x* is estimated.

Step 2. Compute the regression residual at each sampling event *i* with equation [17.23]:

$$r_i = x_i - \hat{x}_i \qquad [17.23]$$

Check the set of residuals for lack of normality and significant skewness using the techniques in **Chapter 10**. Also, plot the residuals against the estimated regression values ($\hat{x}_i$) to check for non-uniform vertical thickness in the scatter cloud. Make a similar check by plotting the residuals against the sampling dates (t_i).

If the residuals are non-normal and substantially skewed and/or the scatter clouds appear to have a definite pattern (*e.g.*, funnel-shaped; "U"-shaped; or, residuals mostly positive on one end of graph and mostly negative on the other end, instead of randomly scattered around the horizontal line $r = 0$), repeat **Steps 1** and **2** after first attempting a normalizing transformation.

Step 3. Calculate the estimated variance around the regression line (also known as the *mean squared error* [MSE]) with equation [17.24]:

$$s_e^2 = \frac{1}{n-2} \sum_{i=1}^{n} r_i^2 \qquad [17.24]$$

Step 4. Compute the standard error of the linear regression slope coefficient using the s^2_e result from Step 3 in equation [17.25]:

$$se(\hat{b}) = \sqrt{s^2{}_e \Big/ \sum_{i=1}^{n} (t - \bar{t})^2} \qquad [17.25]$$

Step 5. Test whether the trend is significantly different from zero by forming the *t*-statistic ratio in equation [17.26]:

$$t_b = \hat{b} \big/ se(\hat{b}) \qquad [17.26]$$

This *t*-statistic (t_b) has n–2 degrees of freedom [*df*]. Given a level of significance (α), choose the critical point (t_{cp}) for the test as the (1– α) × 100th percentage point of the Student's *t*-distribution with (n–2) *df* or $t_{cp} = t_{1-\alpha,n-2}$. Compare t_b against the critical point. If $t_b > t_{cp}$, conclude that the slope of the trend is both positive and significantly different from zero at the α-level of significance. If $t_b < -t_{cp}$, conclude there is a significant decreasing trend. If neither exists, there is insufficient evidence of an increasing or decreasing trend.

►EXAMPLE 17-5

The following groundwater chloride measurements (n = 19) were collected over a five-year period at a solid waste landfill. Test for a significant trend at the α = 0.01 level using linear regression.

Sample Date	Chloride (ppm)	Elapsed Days	Residuals
2002-03-18	11.5	76	–0.25
2002-05-14	12.6	133	0.67
2002-08-22	13.8	233	1.56
2003-02-12	12.3	407	–0.48
2003-05-29	12.8	513	–0.30
2003-08-18	13.2	594	–0.15
2003-11-20	14.1	688	0.45
2004-02-19	13.3	779	–0.63
2004-04-26	13.1	846	–1.04
2004-07-29	13.2	940	–1.23
2004-11-09	15.3	1043	0.56
2005-02-24	15.0	1150	–0.08
2005-06-14	15.2	1260	–0.22
2005-08-23	15.8	1330	0.17
2005-10-17	16.1	1385	0.30
2006-02-08	15.1	1499	–1.06
2006-04-27	16.4	1577	0.00
2006-08-10	17.7	1682	0.98
2006-10-26	17.7	1759	0.74

SOLUTION

Step 1. Check for an apparent trend on a time series plot (**Figure 17-2**). Since the chloride values are increasing in reasonably linear fashion, compute the tentative regression line using equations [17.21] and [17.22]. To compute the slope estimate, first convert the sample dates to elapsed days using a starting date prior to the first event. In this case, choose an arbitrary starting date of 2002-01-01 as zero and compute the elapsed days as listed in the table above.

Using elapsed days as the time variable, compute the sample mean and variance to get:

$$\bar{t} = 941.79 \text{ days}$$
$$s_t^2 = 279374.3 \text{ days}^2$$

Then compute the tentative slope as:

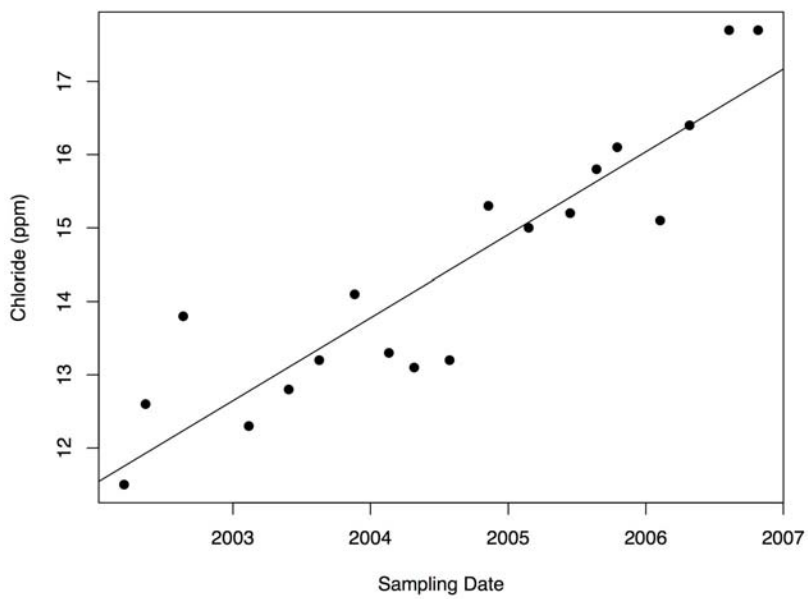
$$\hat{b} = [(76-941.79)\cdot 11.5 + \ldots + (1759-941.79)\cdot 17.7]/[(19-1)\cdot 279374.3] = .0031$$

and the regression line itself as:

$$\hat{x}_t = \bar{x} + \hat{b}\cdot(t-\bar{t}) = 14.432 + .0031\cdot(t-941.79)$$

where the mean chloride value is $\bar{x} = 14.432 \text{ ppm}$. The regression line is overlaid on the scatter plot in **Figure 17-2**.

Figure 17-2. Time Series Plot of Chloride (ppm) Overlaid With Linear Regression



Step 2. Calculate the regression residual at each sampling event using equation [17.23]. This involves computing an estimated concentration along the regression line for each sampled time (*t*) and then subtracting from the observed concentration. For example, the residual at t = 407 is

$$x_t - \hat{x}_t = 12.3 - 12.78 = -0.48$$

All the residuals are listed in the table above. Then check the residuals for normality, homoscedasticity, and lack of association with the predicted values from the regression line.

Figure 17-3 is a probability plot of the residuals, indicating good agreement with normality. **Figure 17-4** is a scatter plot of the residuals versus sampling date and **Figure 17-5** is a scatter plot of the residuals versus predicted values from the trend line. Both of these last plots do not exhibit any particular trends or patterns with sampling date or the trend line predicted values; the residuals are fairly randomly scattered.

Step 3. Compute the MSE of the regression using the squared residuals in equation [17.24] to get

$$s_e^2 = \frac{1}{n-2} \cdot \sum_{i=1}^{n} r_i^2 = \frac{1}{17} \cdot \left[(-.25)^2 + (.67)^2 + \ldots + (.74)^2\right] = 0.5628$$

Step 4. Calculate the standard error of the regression slope coefficient using equation [17.25]:

$$se(\hat{b}) = \sqrt{s^2{}_e \Big/ \sum_{i=1}^{n} (t-\bar{t})^2} = \sqrt{.5628 \Big/ \left[(76-941.79)^2 + \ldots + (1759-941.79)^2\right]} = .00033$$

Step 5. Form the *t*-statistic ratio with formula [17.26] to get:

$$t_b = \hat{b} \big/ se(\hat{b}) = 0.0031/0.00033 = 9.39$$

Since α = 0.01, compare this value to a critical point equal to the 99th percentile of a Student's *t*-distribution with (*n*–2) = 17 degrees of freedom, that is, $t_{cp} = t_{.99,17} = 2.567$. Since the *t*-statistic is substantially larger than the critical point, conclude the upward trend is significant at the 1% α-level. ◀

Figure 17-3. Probability Plot of Chloride Regression Residuals

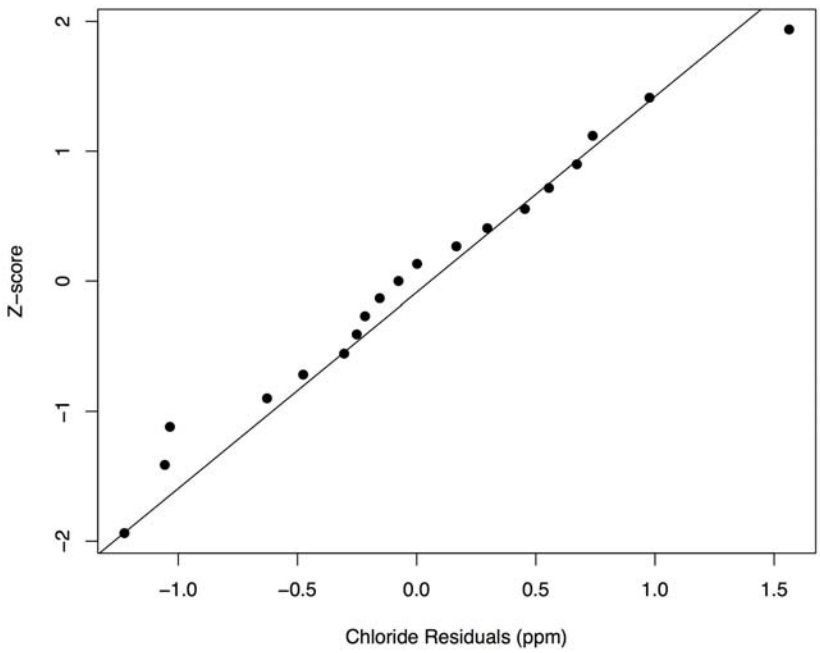


Figure 17-4. Scatter Plot of Chloride Residuals vs. Sampling Date

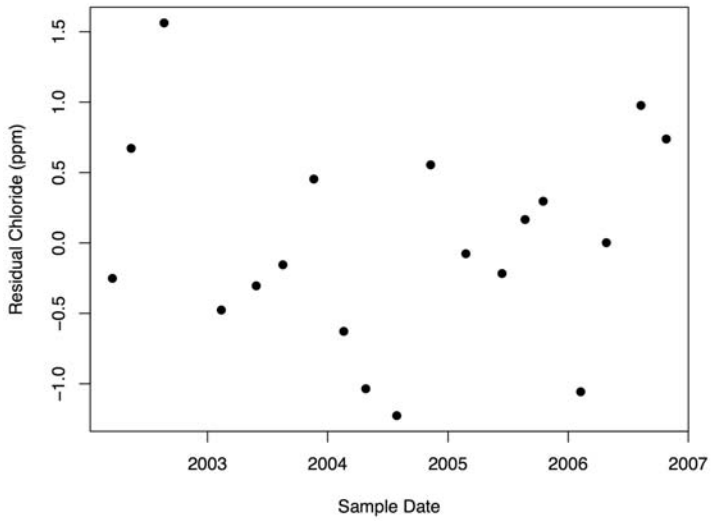
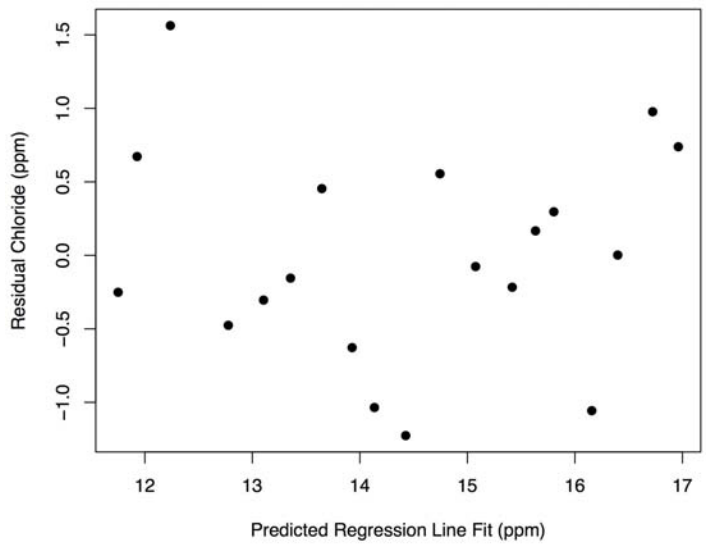


Figure 17-5. Scatter Plot of Chloride Residuals vs. Predicted Regression Fits



17.3.2 MANN-KENDALL TREND TEST

BACKGROUND AND PURPOSE

The Mann-Kendall test (Gilbert, 1987) is a non-parametric test for linear trend, based on the idea that a lack of trend should correspond to a time series plot fluctuating randomly about a constant mean level, with no visually apparent upward or downward pattern. If an *increasing* trend really exists, the sample taken first from any randomly selected pair of measurements should on average have a *lower* concentration than the measurement collected at a later point. The Mann-Kendall statistic is computed by examining all possible pairs of measurements in the data set and scoring each pair as follows. An earlier measurement less in magnitude than a later one is assigned a value of 1. If an earlier value is greater in magnitude than a later sample, the pair is tallied as –1; two identical measurement values are assigned 0.

After scoring each pair in this way and adding up the total to get the Mann-Kendall statistic (S), a positive value of S implies that a majority of the differences between earlier and later measurements are positive, suggestive of an upward trend over time. Likewise, a negative value for S implies that a majority of the differences between earlier and later values are negative, suggestive of a decreasing trend. A value near zero indicates a roughly equal number of positive and negative differences. This would be expected if the measurements were randomly fluctuating about a constant mean with no apparent trend.

To account for randomness and inherent variability in the sample, the Mann-Kendall test is based on the critical ranges of the statistic S likely to occur under stationary conditions. The larger the absolute

value of *S*, the stronger the evidence for a real increasing or decreasing trend. The critical points for identifying a trend get larger as the level of significance (α) drops. Only if the absolute value of the test statistic (*S*) is larger than the critical point is a statistically significant increasing or decreasing trend indicated.

REQUIREMENTS AND ASSUMPTIONS

As a non-parametric procedure, the Mann-Kendall test does not require the underlying data to follow a specific distribution. Ranks of the data are not explicitly used in forming the test statistic as with the Wilcoxon rank-sum. Only the relative magnitudes of the concentration values are needed to compute *S*, not the actual concentrations themselves. Non-detects can be treated by assigning them a common value lower than any of the detected measurements. Any pair of tied values or any pair of non-detects is simply given a score of 0 in the calculation of the Mann-Kendall statistic *S*.

This treatment of non-detects is an imperfect remedy since it is usually impossible to know whether censored values are actually tied in magnitude. Further complications are introduced when there are multiple RLs and/or an intermingling of detected values and RLs. Lab qualifiers may be used to aid the scoring of pairs that involve non-detects or estimated concentrations. Instead of treating all non-detects as tied, consider 'undetected or U' values as the lowest in magnitude, other non-detects as higher in magnitude than U's but lower than estimated concentrations ('J' or 'E' values). In this way, a richer scoring of the sample pairs may be possible.

When the sample size *n* becomes large, exact critical values for the statistic *S* are not readily available. However, as a sum of identically-distributed random quantities, the behavior of *S* for larger *n* tends to approximate the normal distribution by the Central Limit Theorem. Therefore a normal approximation to *S* can be used for $n > 10$[1]. In this case, a standardized *Z*-statistic is formed by first computing the expected mean value and standard deviation of *S*. From the discussion above, when no trend is present, positive differences in randomly selected pairs of measurements should balance negative differences, so the expected mean value of *S* under the null hypothesis of no trend is simply zero. The standard deviation of *S* can be computed using equation [17.27]:

$$SD\left[S\right] = \sqrt{\frac{1}{18}\left[n(n-1)(2n+5) - \sum_{j=1}^{g} t_j\left(t_j - 1\right)\left(2t_j + 5\right)\right]} \qquad [17.27]$$

where *n* is the sample size, *g* represents the number of groups of ties in the data set (if any), and t_j is the number of ties in the *j*th group of ties. If no ties or non-detects are present, equation [17.27] reduces to the simpler form:

$$SD\left[S\right] = \sqrt{\frac{1}{18}n(n-1)(2n+5)} \qquad [17.28]$$

[1] Guidance **Table 17-5** contains exact confidence levels up to $n = 10$. Exact confidence levels for $n \leq 20$ have been developed in (Hollander & Wolfe, 1999), Table A.30. These might be preferentially used if sample sizes are fairly small and the data contain non-detect values.

Once the standard deviation of S has been derived, the standardized Z-statistic for an increasing (or decreasing) trend is formed using the equation:

$$Z = \left(|S| - 1\right) / SD\left[S\right] \qquad [17.29]$$

Note that although the expected mean value of S is zero, applying the continuous normal to the discrete S distribution is an approximation. Therefore, a *continuity correction* is made to Z by first subtracting 1 from the absolute value of S. The final Z-statistic can then be compared to an α-level critical point taken from **Table 10-1** in **Appendix D** to complete the test.

PROCEDURE

Step 1. Order the data set by sampling event or time of collection, x_1, x_2, to x_n. Then consider all possible differences between distinct pairs of measurements, $(x_j - x_i)$ for $j > i$. For each pair, compute the *sign* of the difference, defined by:

$$\operatorname{sgn}\left(x_j - x_i\right) = \begin{cases} 1 & \text{if } \left(x_j - x_i\right) > 0 \\ 0 & \text{if } \left(x_j - x_i\right) = 0 \\ -1 & \text{if } \left(x_j - x_i\right) < 0 \end{cases} \qquad [17.30]$$

Pairs of tied values including non-detects, will receive scores of zero using equation [17.30].

Step 2. Compute the Mann-Kendall statistic S using equation [17.31]:

$$S = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \operatorname{sgn}\left(x_j - x_i\right) \qquad [17.31]$$

In equation [17.31] the summation starts with a comparison of the very first sampling event against each of the subsequent measurements. Then the second event is compared with each of the samples taken after it (*i.e.*, the third, fourth, fifth, *etc.*). Following this pattern is probably the most convenient way to ensure that all distinct pairs are tallied in forming S. For a sample of size n, there will be $n \cdot (n-1)/2$ distinct pairs.

Step 3. If $n \leq 10$, and given the level of significance (α), determine the critical point s_{cp} from **Table 17-5 of Appendix D.** If $S > 0$ and $|S| > s_{cp}$, conclude there is statistically significant evidence of an increasing trend at the α significance level. If $S < 0$ and $|S| > s_{cp}$, conclude there is statistically significant evidence of a decreasing trend. If $|S| \leq s_{cp}$, conclude there is insufficient evidence to identify a significant trend.

Step 4. If $n > 10$, determine the number of groups of ties (g) and the number of tied values in each group of ties (t_j). Then use equation [17.27] to compute the standard deviation of S and equation [17.29] in turn to compute the standardized Z-statistic.

Step 5. Given the significance level (α), determine the critical point z_{cp} from the standard normal distribution in **Table 10-1** in **Appendix D**. Compare Z against this critical point. If $Z > z_{cp}$, conclude there is statistically significant evidence at the α-level of an increasing trend. If $Z < -z_{cp}$, conclude there is statistically significant evidence of a decreasing trend. If neither exists, conclude that the sample evidence is insufficient to identify a trend.

►EXAMPLE 17-6

Test for a significant upward trend using the Mann-Kendall procedure in the following set of sulfate measurements (ppm) collected over several years.

Sample No.	Sampling Date (yr.mon)	Sulfate Conc. (ppm)	Sample No.	Sampling Date (yr.mon)	Sulfate Conc. (ppm)
1	89.6	480	13	93.1	590
2	89.8	450	14	93.6	550
3	90.1	490	15	94.1	600
4	90.3	520	16	94.6	700
5	90.6	485	17	95.1	570
6	90.8	510	18	95.6	610
7	91.1	510	19	95.8	650
8	91.3	530	20	96.1	620
9	91.6	510	21	96.3	830
10	91.8	560	22	96.6	720
11	92.1	560	23	96.8	590
12	92.6	540			

SOLUTION

Step 1. Construct a time series plot of the sulfate observations to check for a possible trend as in **Figure 17-6**. A clearly rising concentration pattern is seen, although the variability in the measurements appears greater toward the end of the sampling record than at the beginning.

Step 2. Compute the difference between each distinct pair of measurements and determine the sign of the difference, using equation [17.30]. Then sum up the signs with equation [17.31]. Note that to make sure all the distinct pairs have been summed, begin with the first listed observation and compare it to each of values below it. Then take the second listed value and compare it to each of the remaining ones below it, *etc*. The Mann-Kendall statistic becomes:

$$S = \text{sgn}(450-480) + \text{sgn}(490-480) + \ldots + \text{sgn}(590-720) = 196$$

Step 3. Since the sample size $n = 23 > 10$, form the normal approximation to the Mann-Kendall statistic. Because there are some ties in the data, use equation [17.27] to compute the approximate standard deviation. Among the sulfate measurements, there are three groups of ties with 3, 2, and 2 tied values in each set respectively (at values 510, 560, and 590). The adjusted standard deviation is then:

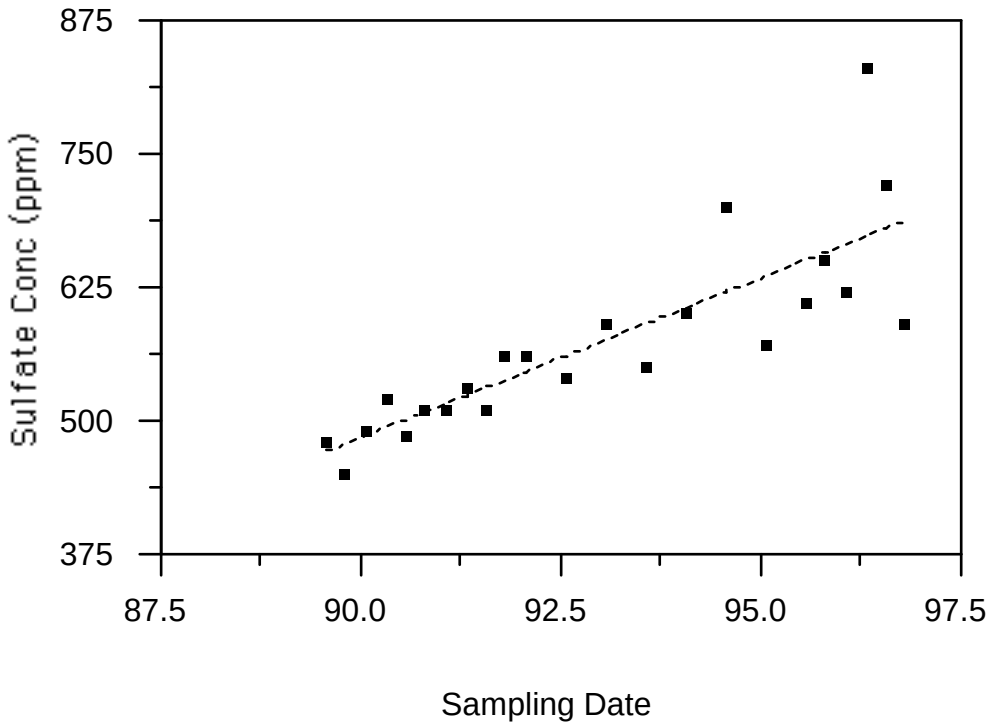
$$SD[S] = \sqrt{\frac{1}{18}\cdot\left[23\cdot(23-1)(2\cdot 23+5) - \{3\cdot(3-1)(2\cdot 3+5) + \ldots + 2\cdot(2-1)(2\cdot 2+5)\}\right]} = 37.79$$

Finally, using equation [17.29], the normalized Mann-Kendall statistic is:

$$Z = \left(|196| - 1\right) / 37.79 = 5.16$$

Step 4. The Z statistic can be compared to a critical point from the standard normal distribution in **Table 10-1** in **Appendix D**. As large as it is, the test statistic is bigger than the critical point for any usual significance level, suggesting that the trend appears to be real and not just a chance artifact of the sample. ◄

Figure 17-6. Time Series Plot of Sulfate Concentrations (ppm)



17.3.3 THEIL-SEN TREND LINE

BACKGROUND AND PURPOSE

The Mann-Kendall procedure is a non-parametric test for a significant slope in a linear regression of the concentration values plotted against time of sampling. But the Mann-Kendall statistic S does not indicate the *magnitude* of the slope or estimate the trend line itself even when a trend is present. This is slightly different from parametric linear regression, where a test for a significant slope follows naturally from the estimate of the trend line. Even a relatively modest slope can be statistically distinguished from zero with a large enough sample. It is best to first identify whether or not a trend exists, and then determine how steeply the concentration levels are increasing over time for a significant trend. The *Theil-Sen trend line* (Helsel, 2005) is a non-parametric alternative to linear regression which can be used in conjunction with the Mann-Kendall test.

The Theil-Sen method handles non-detects in almost exactly the same manner as the Mann-Kendall test. It assigns each non-detect a common value less than any other detected measurement (*e.g.*,

half the RL). Unlike the Mann-Kendall test, however, the actual concentration values are important in computing the slope estimate in the Theil-Sen procedure. The essential idea is that if a ***simple slope estimate*** is computed for every pair of distinct measurements in the sample (known as the set of ***pairwise slopes***), the average of this series of slope values should approximate the true slope. The Theil-Sen method is non-parametric because instead of taking an ***arithmetic average*** of the pairwise slopes, the ***median*** slope value is determined. By taking the median pairwise slope instead of the mean, extreme pairwise slopes — perhaps due to one or more outliers or other errors — are ignored and have little if any impact on the final slope estimator.

The Theil-Sen trend line is also non-parametric because the median pairwise slope is combined with the median concentration value and the median sample date to construct the final trend line. As a consequence of this construction, the Theil-Sen line estimates the change in ***median*** concentration over time and not the ***mean*** as in linear regression.

REQUIREMENTS AND ASSUMPTIONS

The Theil-Sen procedure does not require normally-distributed trend residuals as in a linear regression. It is also not critical that the residuals be homoscedastic (***i.e.***, having equal variance over time and with increasing average concentration level). It ***is*** important to have at least 4 and preferably at least 8 or more observation on which to construct the trend. But trend residuals are assumed to be statistically independent. Approximate checks of this assumption can be made using the techniques of **Chapter 14**, once the estimated trend has been removed and the number of non-detect data is limited. Sampling events should also be spaced far enough apart relative to the site-specific groundwater velocity so that an assumption of ***physical*** independence of consecutive sample volumes is reasonable.

A more difficult problem is encountered when a large fraction of the data is non-detect. As long as less than half the measurements are non-detects occurring in the lower part of the observed concentration range, the median concentration value will be quantified and the median pairwise slope will generally be associated with a pair of detects. Larger proportions of non-detect data make computation of the Theil-Sen trend line more difficult and uncertain. The reason is that each time a non-detect is paired with a quantified measurement, the pairwise slope is known only within a range of values. One end of the range results from supposing the true non-detect concentration is equal to zero; the other when the non-detect concentration is equal to the RL.

PROCEDURE

Step 1. Order the data set by sampling event or time of collection, x_1, x_2, to x_n. Then consider all possible distinct pairs of measurements, (x_i, x_j) for $j > i$. For each pair, compute the simple pairwise slope estimate:

$$m_{ij} = \left(x_j - x_i\right) \Big/ \left(j - i\right) \qquad [17.32]$$

With a sample size of n, there should be a total of $N = n(n–1)/2$ such pairwise estimates m_{ij}. If a given observation is a non-detect, use half the RL as its estimated concentration.

Step 2. Order the N pairwise slope estimates (m_{ij}) from least to greatest and rename them as $m_{(1)}$, $m_{(2)}$, …, $m_{(N)}$. Then determine the Theil-Sen estimate of slope (Q) as the median value of this list.

Finding this value will depend on whether N is even or odd, but the following equation can be used:

$$Q = \begin{cases} m_{(\lceil N+1 \rceil/2)} & \text{if } N \text{ is odd} \\ \left(m_{(N/2)} + m_{(\lceil N+2 \rceil/2)}\right)/2 & \text{if } N \text{ is even} \end{cases} \qquad [17.33]$$

Step 3. Order the sample by concentration magnitude from least to greatest, $x_{(1)}$, $x_{(2)}$, to $x_{(n)}$. Determine the median concentration with the formula:

$$\tilde{x} = \begin{cases} x_{(n+1)/2} & \textit{if n is odd} \\ \left(x_{n/2} + x_{(n+2)/2}\right)/2 & \textit{if n is even} \end{cases} \qquad [17.34]$$

Again replace each non-detect by half its RL during this calculation. Also find the median sampling date ($\tilde{t}$) using the ordered times t_1, t_2, to t_n by a similar computation.

Step 4. Compute the Theil-Sen trend line with the equation:

$$x = \tilde{x} + Q \cdot \left(t - \tilde{t}\right) = \left(\tilde{x} - Q \cdot \tilde{t}\right) + Q \cdot t \qquad [17.35]$$

Using equation [17.35], an estimate can be made at any time (t) of the expected median concentration (x).

►EXAMPLE 17-7

Use the following sodium measurements to compute a Theil-Sen trend line. Note that the sample dates are recorded as the year of collection (2-digit format) plus a fractional part indicating when during the year the sample was collected. This allows an annual slope estimate, since 1 unit = 1 year.

Sample Date (yr)	Sodium Conc. (ppm)
89.6	56
90.1	53
90.8	51
91.1	55
92.1	52
93.1	60
94.1	62
95.6	59
96.1	61
96.3	63

SOLUTION

Step 1. Compute the pairwise slopes for each distinct pair of measurements using equation [17.32]. With n = 10 observations, there will be a total of 10(9)/2 = 45 such pairs. The first few are listed below:

$$m_{12} = (53-56)/(90.1-89.6) = -6$$
$$m_{13} = (51-56)/(90.8-89.6) = -4.17$$
$$m_{14} = (55-56)/(91.1-89.6) = -.667$$

Step 2. Since the total number of distinct pairs is odd, sort the list of pairwise slopes as in the table below and let Sen's estimated slope equal the middle or 23rd largest value in this list. This gives an estimate of Q = 1.33 ppm increase *per year*, an estimate in line with the time series plot of **Figure 17-7**.

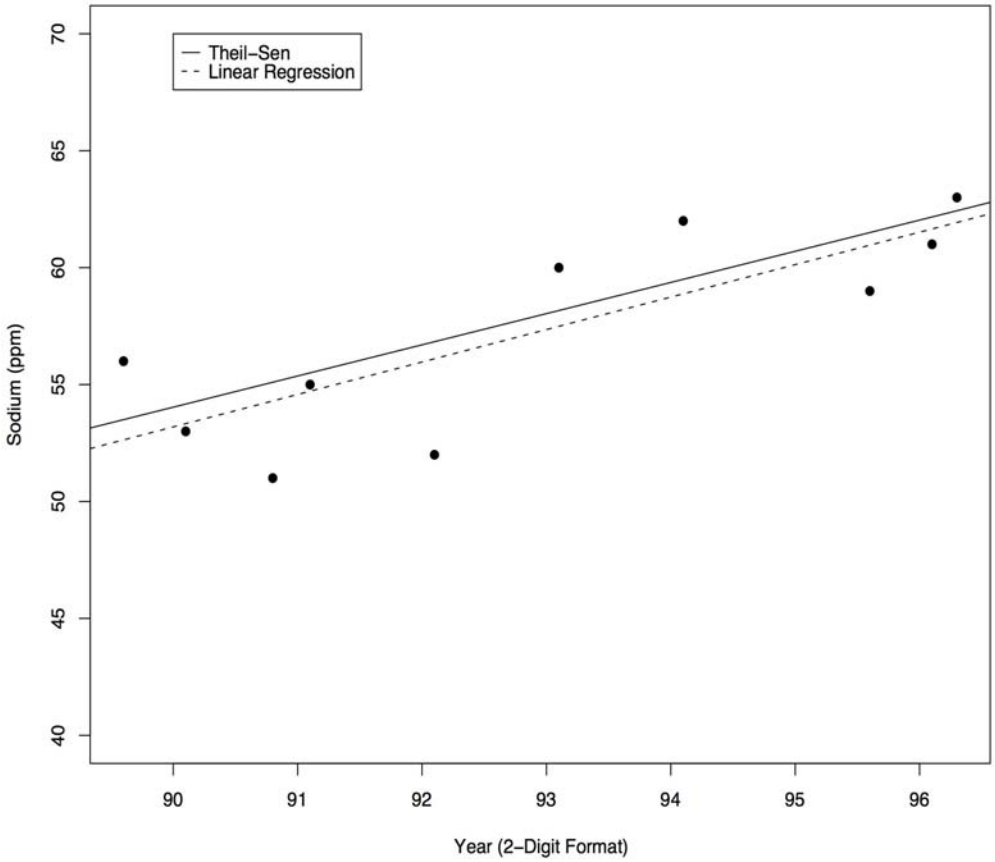
Step 3. Compute the median concentration value $\tilde{x} = 57.5$ and the median sample date $\tilde{t} = 92.6$ from the table above. Then calculate the Theil-Sen trend line using the slope estimate from Step 2:

$$x = 57.5 + 1.333(t - 92.6) = -65.97 + 1.333t$$

This trend line can be used to estimate the predicted median concentration (x) at any desired time in years (t). For example, at the beginning of 1998 (t = 98), the trend line would predict a median sodium concentration of approximately x = 64.7 ppm. ◄

Rank	Pairwise Slope	Rank	Pairwise Slope
1	-6	24	1.538
2	-4.167	25	1.613
3	-3	26	1.667
4	-2.857	27	1.887
5	-2	28	2
6	-1.6	29	2
7	-0.667	30	2
8	-0.5	31	2.182
9	-0.5	32	2.25
10	-0.4	33	2.25
11	0.333	34	2.333
12	0.455	35	2.333
13	0.5	36	2.5
14	0.769	37	2.619
15	0.769	38	3.333
16	0.889	39	3.913
17	0.938	40	4
18	1.045	41	5
19	1.091	42	5.714
20	1.143	43	8
21	1.2	44	10
22	1.333	45	13.333
23	1.333		

Figure 17-7. Time Series Plot of Sodium Concentrations (ppm)



CHAPTER 18. PREDICTION LIMIT PRIMER

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This chapter introduces the concept of statistical intervals and focuses on several types of prediction limits useful for detection monitoring. The requirements and common assumptions of such limits are explained, as well as specific descriptions of:

- Prediction limits for *m* future values (**Section 18.2.1**)
- Prediction limits for future means (**Section 18.2.2**)
- Non-parametric prediction limits for *m* future values (**Section 18.3.1**)
- Non-parametric prediction limits for a future median (**Section 18.3.2**)

18.1 INTRODUCTION TO PREDICTION LIMITS

First discussed in **Chapter 6,** *prediction limits* belong to a class of methods known as *statistical intervals*. Statistical intervals represent concentration or measurement ranges computed from a sample that are designed to estimate one or more characteristics of the parent population. In groundwater monitoring, statistical intervals offer a convenient and statistically valid way to test for significant differences between background versus compliance point groundwater measurements.

The statistical interval accounts for variability inherent not only in future measurements, but also additional uncertainty in the prediction limit itself. The latter is derived from a relatively small background sample with an associated level of variability in estimating the true characteristics of the underlying groundwater population.

Prediction limits are generally easy to construct and have a straightforward interpretation. Background data are used to construct a concentration limit *PL*, which is then compared to one or more observations from a compliance point population. The acceptable range of concentrations includes all values no greater than the prediction limit. The appropriate *prediction interval* will generally have the form [0, *PL*], with the upper limit *PL* as the comparison of importance. Unless pH or a similar parameter is being monitored, a one-sided upper prediction limit is used in detection monitoring.

A significant advantage to prediction limits is their *flexibility*, which can accommodate a wide variety of groundwater monitoring networks. Prediction limits can be constructed so that as few as *one*

new measurement per compliance well may suffice for a test. Prediction limits may be based on a comparison of means, medians, or individual compliance point measurements, depending on the characteristics of the monitoring network and the constituents being tested.

Prediction limits can also be designed to accommodate a wide range of *multiple statistical comparisons* or *tests*. Each periodic statistical evaluation (*e.g.*, semi-annually) under RCRA and other regulations involves separate tests at all compliance well locations for each monitoring constituent. Often, the number of separate statistical tests can be quite sizeable. Prediction limits can be constructed to precisely account for the number of tests to be conducted, so as to limit the *site-wide false positive rate* [SWFPR] and ensure an adequate level of *statistical power* (see discussion in **Chapter 6**).

This and the following chapter present basic concepts and procedures for using prediction limits as detection monitoring tests. The intent is to provide a relatively simple framework for using prediction limits in RCRA or CERCLA groundwater monitoring. **Chapter 18** describes the construction of prediction limits for tests involving a single constituent at one well. It describes the basic mechanics of each type of prediction limit and how they differ from one another.

Chapter 19 expands this discussion to cover multiple *simultaneous* prediction limit tests (*i.e.*, all occurring during a single statistical evaluation or during a single year of monitoring). Cumulative SWFPRs and statistical power are considered, including how these criteria impact the expected performance of a given prediction limit strategy. Examples are provided to illustrate these procedures, as well as explanations of associated tables and software.

Specific strategies in **Chapter 19** apply the concept of *retesting*. Generally speaking, ***almost any prediction limit procedure in detection monitoring should be combined with an appropriate retesting strategy***. The reason is that when testing a large number of compliance point samples, it is almost guaranteed that one or more measurements will exceed an upper prediction limit. *Resampling* of those wells where an exceedance has occurred can either verify the initial evidence of a release or disconfirm it, while avoiding unnecessary false positives.

Chapter 6 introduced a number of key terms used in the Unified Guidance, especially for prediction limit and control chart tests. The guidance applies the term *comparison* to <u>individual</u> future measurements or sample statistics evaluated against a prediction limit (or *control chart limit*), and the term *test* to represent a series of future data comparisons that ultimately result in a statistical decision. A 1-of-*m* retesting procedure (described below), for instance, might involve comparison of up to *m* distinct sample measurements against the prediction limit. Each of these individual samples involves a *comparison*, but only after all the necessary individual comparisons have been made is the *test* complete. This distinction becomes particularly important when properly determining SWFPRs, a subject discussed both in **Chapter 6** and **Chapter 19**.

One or more *future* observations are collected for purposes of testing compliance well data, as distinct from the *background* sample from which the prediction limit is constructed. Background data can be obtained from upgradient wells or in combination with historical, uncontaminated compliance well data. In intrawell testing, data from an individual compliance well constitute both the background and future samples. The two data sets need to be distinct and may not overlap, even if the historical

background data is periodically updated with previously evaluated future samples. The key idea is that at any given point in time, background and future data sets are clearly distinguished.

Formally, prediction limits are constructed to contain one or more future observations or sample statistics generated from the background population with a specified probability equal to (1–α). The probability (1–α) is known as the confidence level of the limit. It represents the chance — over repeated applications of the limit to many similar data sets — that the prediction limit will contain future observations or statistics drawn from its background population.

A sample of *n* background measurements is used to construct the prediction limit. Under the null hypothesis that the compliance point population is identical to background, a set of *m* independent compliance point observations or a statistic like the mean based on those observations (*i.e.*, the future data) is then compared against the prediction limit. For the prediction limit to serve as a valid statistical test, the future observations are initially presumed to follow the same distribution as background.

Only background values are used to construct the prediction limit. But the probability that the limit contains all *m* future observations or sample statistics derived from those future data does not depend solely on the observed background. It is also based on the number of future measurements or sample statistics used in the comparison and *how* the individual comparisons are conducted. To underscore this point, consider the general equation for a prediction limit based on normal or transformably normal populations, given by

$$PL = \bar{x} + \kappa s \qquad [18.1]$$

where $\bar{x}$ is the sample mean in background, s is the background standard deviation, and κ is a multiplier depending on the type of prediction limit under construction. The simplest type of prediction limit test compares a specific number of individual future observations to the limit (*PL*). For example, do all three compliance measurements collected during a 6-month period fall within the prediction interval? The multiplier κ and hence the prediction limit itself, changes depending on whether one, two or three compliance observations will be compared against *PL*. More generally, the κ-multiplier is selected to account not only for the number of future comparisons, but also for the *rules of the comparison strategy* and the number of simultaneous tests to be conducted (*e.g.*, the number of monitoring constituents times the number of compliance wells).

In the simplest case of a successive comparison of *m* individual future measurements against *PL*, the test is labeled as an *m*-of-*m* prediction limit. All *m* of the future observations need to fall within the prediction interval for the test to 'pass' — that is, be no greater than *PL*. If any one or more of the future values exceed the PL, the test fails and the well is deemed to have a *statistically significant increase* [SSI] or constitute an exceedance.

The κ-multiplier appropriate for an *m*-of-*m* prediction limit test is different from the multipliers that would be computed for other kinds of comparison rules. Another simple type is a comparison of a single future *mean of order p*. Here, *p* future measurements are collected and *averaged* before comparing against *PL*. If the order-*p* mean is no greater than *PL*, the test passes; otherwise, it fails. A test following this rule is labeled a *1-of-1 prediction limit on a future mean*. The important thing to remember is that the κ-multiplier and thus the prediction limit will differ depending on whether or not the *p* future values are first averaged or simply compared against *PL* one-by-one. The choice to use one rule versus the other

impacts the magnitude of the prediction limit and ultimately its expected statistical power and false positive rate.

Other comparison rules of substantial benefit in groundwater monitoring are 1-of-*m* prediction limit on future observations or a statistic like the mean or median. This test requires at least one of *m* successive observations or statistic to fall within the prediction interval in order to pass. Operationally this means that if an initial compliance well measurement is no greater than *PL*, the test is complete and no further sampling need be done. If the initial value exceeds *PL*, one or more of (*m*–1) *resamples* need to be obtained. Since these additional measurements are collected sequentially over sufficiently long time periods to maintain approximate statistical independence (**Chapter 3**), the first resample to fall within the prediction interval also ends the test as 'inbounds' or passing, frequently obviating the need to gather all *m* measurements.

Another comparison rule of some use is known as the California strategy, first developed for the State of California RCRA program. The California strategy can be construed as a *conditional* rule: if an initial future observation is no greater than *PL*, further comparisons are not needed and the test passes. However, if the initial observation exceeds the *PL*, 2-of-2 or 3-of-3 resamples *all need to not exceed the PL* in order for the well to remain in compliance. A slight modification to this rule termed the *modified California* approach has better statistical power and false positive rate characteristics than the original California strategies, and is therefore included as a potential prediction limit test.

18.1.1 BASIC REQUIREMENTS FOR PREDICTION LIMITS

All prediction limits share certain basic assumptions when applied as tests of groundwater. Further, *parametric* prediction limits as presented in the Unified Guidance require the sample data to be either normally-distributed or normalized via a transformation. The key points can be summarized as follows:

1. background and future sample measurements need to be identically and independently distributed (the *i.i.d.* presumption; see **Chapter 3**);
2. sample data do not exhibit temporal non-stationarity in the form of trends, autocorrelation, or other seasonal or cyclic variation;
3. for interwell tests (*e.g.*, upgradient-to-downgradient comparisons), sample data do not exhibit non-stationary distributions in the form of significant natural spatial variability;
4. background data do not include statistical outliers (a form of non-identical distributions);
5. for parametric prediction limits, background data are normal or can be normalized using a transformation; and
6. a minimum of 8 background measurements is available; more for non-parametric limits or when accounting for multiple, simultaneous prediction limit tests.

The first assumption implies that background data are randomly drawn from a single common parent population, especially if aggregated from more than one source well. As discussed in **Chapter 5**, analysis of variance [ANOVA] can be used to determine the appropriateness of pooling data from

different background wells. There is also a presumption that the compliance point measurements follow the same distribution as background in the absence of a release.

The second assumption is corollary to the first, and requires that the background data are *stationary over time* (**Chapter 3**). This can be evaluated with one or more techniques described in **Chapter 14** on temporal variability. These account for trends, autocorrelation, or other variation, perhaps by utilizing *data residuals* instead of the raw measurements. If the background residuals meet the basic points above, they can be used to construct an adjusted prediction limit. Residuals of the future observations would also need to be computed and compared against the adjusted prediction limit to ensure a valid and consistent test.

The second assumption also requires that there be only a single source of variation in the data, when using the usual sample standard deviation (s) to compute the prediction limit. If there are other sources of variation such as seasonal patterns or temporal variation in lab analytical performance, these should be included in the estimate of variability. Otherwise s is likely to be *biased*. One method to accomplish this is by use of an appropriate ANOVA model to include temporal factors affecting the variability (**Chapter 14**). Determination of the components of variance in more complicated models is beyond the scope of this guidance and may require consultation with a professional statistician.

The third assumption requires that background and compliance point populations be identical in distribution, absent a release, for interwell tests. Spatial variation violates this assumption since the well population means (μ) will be different, making it impossible to know whether an apparent upgradient-to-downgradient difference is attributable to a release or simply variations in natural groundwater concentration levels. The assumption also requires that each population share a common variance (σ^2). Tests of equal variance (*i.e.*, homoscedasticity) when using prediction limits may be possible either by examining groups of historical background and compliance point data or by performing periodic tests when enough compliance point measurements have been accumulated to make a diagnostic test possible.

The fourth assumption implies that background data should be screened for outliers using the techniques in **Chapter 12**. Statistical outliers can potentially inflate a prediction limit and severely limit its statistical power and accuracy by over-inflating both the sample background mean ($\bar{x}$) and especially the background standard deviation (s). The Unified Guidance discourages automated removal of outliers from background samples, but all possible outliers should be examined to determine whether a cause can be identified (see discussion in **Chapter 6**). In some cases, an apparent outlier may represent a valid portion of the underlying background population that has not yet been sampled or observed. It also could represent evidence that conditions in background have changed or are changing.

The fifth assumption of normality for parametric prediction limits can be evaluated using the diagnostic techniques described in **Part II** of the guidance. If skewed background data can be normalized via a transformation (*e.g.*, the natural logarithm), the prediction limit should be constructed on the transformed background values. The resulting limit should either be: 1) back-transformed to the concentration domain (*e.g.*, by exponentiation) when comparing future individual compliance observations; or 2) left in the transformed scale when compared to future mean compliance data also based on *the same transformation*. In the latter case, use of a logarithmic transformation results in evaluating population medians or geometric means and *not* the arithmetic means.

When normality cannot be justified, a non-parametric prediction limit should be considered instead. A non-parametric limit assumes only that all the data come from the same, usually unknown, continuous population. Non-parametric prediction limits generally require a much larger number of background observations in order to provide the same level of confidence (1–α) as a comparable parametric limit. Consequently, the Unified Guidance recommends that a parametric model be fit to the data if at all possible.

The last assumption concerns sufficient background sample sizes. A prediction interval can be computed with as few as three observations from background. However, this can result in an unacceptably large upper prediction limit and a test with very limited statistical power. A sample size of eight or more is generally needed to derive an adequate parametric prediction limit, especially if a retesting strategy is not employed. The exact requirements depend on the number of simultaneous tests (*i.e.*, number of wells times number of constituents per well) to be made against the prediction limit and the type of retesting strategy adopted (see **Chapter 19** for more discussion of retesting strategies).

If a minimum schedule of quarterly sampling is being followed and there is only one background well, at least two years of data will be needed before constructing the prediction limit.[1] If data from multiple background wells screened in comparable hydrologic conditions can reasonably be combined (see **Chapter 5**), pooling background data to increase background sample sizes is encouraged.

18.1.2 PREDICTION LIMITS WITH CENSORED DATA

When a sample contains a substantial fraction of non-detects or left-censored measurements, it may be impossible to even approximately normalize the data A sample data set may originate from a normal or transformable-to-normal population, but the uncertainty surrounding both the censored values and the consequent shape of the lower tail of the distribution prevents a clear identification. If the apparent underlying distribution is not normal or transformable to normality, a non-parametric prediction limit (**Section 18.3**) should be used.

Given that non-parametric prediction limits typically have much steeper background data requirements than their parametric counterparts, one remedy is to attempt a fit to normality by using censored probability plots (**Chapter 15**) in conjunction with either the *Kaplan-Meier* or *robust regression on order statistics* [ROS] techniques (**Chapter 15**) for left-censored data. Censored observations prevent a full and complete ordering of the sample, making it difficult to assess normality with standard probability plots (**Chapter 9**). Censored probability plots, on the other hand, only graph the detected values, but do so based on a *partial ordering and ranking* of the sample. Data that appear distinctly non-normal on a standard probability plot (where non-detects are perhaps replaced by half their reporting limits [RLs] to allow plotting) can sometimes appear reasonably normal on a censored probability plot. Transformations can also be applied and the censored probability plot reconstructed to see if the data can be normalized in that fashion.

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[1] The Unified Guidance does not recommend that only one background well be used in any kind of interwell or upgradient-to-downgradient comparison. Multiple background wells are always preferred so that tests for spatial variability may be made and the exact nature of background better understood.

If the censored probability plot is close to linear and the sample approximately normalized, an estimated mean and standard deviation should be computed. These estimates will not be the same if each non-detect were replaced by half its RL, and the sample mean calculated from the resulting imputed sample. To properly account for the censoring, the estimated mean (denoted as $\hat{\mu}$) and the estimated standard deviation ($\hat{\sigma}$) needs to be derived as parameters from the normal distribution providing the closest fit to a partial ordering of the sample (as on a censored probability plot). The Unified Guidance describes two slightly different techniques for accomplishing this task.

Once $\hat{\mu}$ and $\hat{\sigma}$ estimates have been computed, an adjusted parametric prediction limit is constructed by substituting $\hat{\mu}$ for $\bar{x}$ and $\hat{\sigma}$ for s in the equations of **Section 18.2** or **Chapter 19**. For example, the adjusted equation for a general parametric prediction limit would become:

$$PL = \hat{\mu} + \kappa \cdot \hat{\sigma} \qquad [18.2]$$

Another potential difference between the adjusted prediction limit in equation [18.2] and the unadjusted prediction limit in equation [18.1] is the number of *degrees of freedom* [*df*] used in selecting the κ-multiplier. Absent any censored measurements, a background sample of size n would normally have (n–1) *df*. With censoring, there is greater statistical uncertainty surrounding each non-detect than surrounding the detected values. Because of this, the actual degrees of freedom is somewhere between d (the number of detects) and (n–1) (the total sample minus one). Unfortunately, there is no straightforward, general method to determine the true *df*. To be conservative, the *df* should be set equal to d, since the value of each detect is known with reasonable certainty. Setting a lower *df* tends to raise the κ-multiplier and thus the prediction limit over what would be selected with an uncensored sample of the same size. This is consistent with the greater uncertainty associated with non-detect measurements. However, it is at best an approximate remedy. Further consultation with a professional statistician may be warranted to arrive at a better choice of the degrees of freedom.

18.2 PARAMETRIC PREDICTION LIMITS

18.2.1 PREDICTION LIMIT FOR M FUTURE VALUES

BACKGROUND AND PURPOSE

A prediction limit test for m future values is constructed so that m compliance point observations are evaluated by determining whether or not they fall within a prediction interval derived from background. As mentioned in **Chapter 2**, some State programs may require up to 4 successive sampling events per evaluation period for testing, which can be addressed by the prediction limit approach described below.

If the distributions of background and compliance point data are identical as assumed under the null hypothesis H_0, *all m of the compliance point observations* should be no greater than the upper prediction limit [*PL*]. If any of the future observations exceeds *PL*, there is statistical evidence that the

compliance data do not come from the same distribution as background, but instead are elevated above background.[2]

With intrawell comparisons, a prediction limit can be computed on historical data or intrawell background to contain a specified number (*m*) of future (*i.e.*, more recent) observations from the same well. If any of the future values exceeds the upper prediction limit, there is evidence of recent contamination at the well.

REQUIREMENTS AND ASSUMPTIONS

As noted in **Section 18.1**, the prediction limit test on *m* future values is designated as an *m*-of-*m* test. *Each* of the *m* individual future observations need to be compared to the prediction limit [*PL*]. All should be no greater than *PL* for the test to pass. The number of future observations to be collected (*m*) need to be specified *in advance* in order to correctly compute the κ-multiplier from equation [18.1]. Consequently, if compliance data are collected on a regular schedule, the prediction interval can be constructed to cover a specified *time period* of future sampling. Usually this period will coincide with the time between statistical evaluations specified in the site permit (*e.g.*, on a semi-annual or annual basis). Keep in mind also that *m* denotes the number of consecutive sampling events being compared to the prediction limit at a given well for a given constituent.

As discussed in more detail in **Chapter 6**, a new prediction limit should be constructed prior to each statistical evaluation for *interwell* tests, when additional background data have been collected along with the new compliance point measurements. Unless there is evidence of characteristic changes within background groundwater quality (*e.g.*, as demonstrated by observable trends in background), background data should be amassed or accumulated over time. Earlier background measurements need not be discarded, both to maintain an adequate background sample size and also because a larger span of sampling results will provide a better statistical description of the underlying background population. The revised prediction limit will then reflect a larger background sample size, *n*, but possibly the same number, *m*, of future values to be predicted at the next statistical evaluation.

For *intrawell* tests, the prediction limits should be revised only after intrawell background has been updated (**Chapter 5**). Such updating may not coincide with the regular schedule of statistical evaluations if done, for instance, every two years or so. In that case, the same intrawell prediction limit might be used for multiple evaluations before being revised.

PROCEDURE

Step 1. Calculate the sample mean $\bar{x}$, and standard deviation *s*, from the set of *n* background measurements.

Step 2. Specify the number of individual future observations (*m*) from the compliance well to be included in the prediction interval for an *m*-of-*m* test. For an upper prediction limit with an overall (1−α) confidence test level for the *m* comparisons, use the equation:

[2] In the context of the Unified Guidance, *m* represents the number of consecutive samples being compared in the prediction limit test for a given well and constituent.

$$PL = \bar{x} + t_{1-\alpha/m,n-1} s \sqrt{1+\frac{1}{n}} \qquad [18.3]$$

It is assumed that exactly *m* consecutive sample values from the compliance point will be compared against the upper *PL*. Note that the quantile from a Student's *t*-distribution used in equation [18.3] has two parameters: the degrees of freedom (*n*–1) and a joint comparison confidence level $(1-\alpha/m)$. Most Student's *t*-quantiles can be found directly or approximated through interpolation by looking in **Table 16-1** of **Appendix D**.

Note: equation [18.3] assumes the prediction limit is applied to only one constituent at a single well. If multiple tests need to be performed (e.g., on multiple wells and/or multiple constituents), the prediction limit takes the form:

$$PL = \bar{x} + \kappa s \qquad [18.4]$$

where the κ-multiplier is determined using one of the strategies described in **Chapter 19**.

If a log transformation is applied to the data to bring about approximate normality, the upper *PL* should be constructed using the log-mean ($\bar{y}$) and log-standard deviation (s_y), using the equation:

$$PL = \exp\left(\bar{y} + t_{1-\alpha/m,n-1} s_y \sqrt{1+\frac{1}{n}}\right) \qquad [18.5]$$

If multiple tests must be conducted and a log transformation has been applied to the data, the upper *PL* will have the form:

$$PL = \exp\left(\bar{y} + \kappa s_y\right) \qquad [18.6]$$

Note: other transformations besides the natural logarithm are handled in a similar manner; compute the prediction limit on the transformed data, then back-transform the limit to the original concentration scale prior to comparison with any future observations.

Step 3. Once the prediction limit (*PL*) has been calculated, compare each of *m* compliance point future values against *PL*. If all of these measurements are no greater than *PL*, the test passes and the well is deemed to be in compliance. If, however, any compliance point concentration exceeds *PL*, there is statistically significant evidence of an increase over background.

►EXAMPLE 18-1

The data in the table below represent quarterly arsenic concentrations measured in a single well at a solid waste landfill. Calculate an intrawell upper prediction limit for 4 future samples with 95% confidence and determine whether there is evidence at the annual statistical evaluation of a possible release during Year 4 of monitoring.

Intrawell Background		Compliance Data	
Sampling Period	Arsenic (ppb)	Sampling Period	Arsenic (ppb)
Year 1	12.6	Year 4	48.0
	30.8		30.3
	52.0		42.5
	28.1		15.0
Year 2	33.3		
	44.0		
	3.0		
	12.8		
Year 3	58.1		
	12.6		
	17.6		
	25.3		
	n = 12		
	Mean = 27.52		
	SD = 17.10		

SOLUTION

Step 1. First check the sample data for the key points identified in **Section 18.1.1**. As an example, a Shapiro-Wilk test on the background data gives a test statistic of SW = 0.947. The critical point at the α = .05 level for the Shapiro-Wilk test on n = 12 observations is 0.859. Since the test statistic exceeds the critical point, there is insufficient evidence to reject an assumption of normality.

Step 2. Compute the prediction interval using the raw background data. The sample mean and standard deviation of the 12 background samples are 27.52 ppb and 17.10 ppb, respectively.

Step 3. A single future year of compliance data then is compared to the prediction limit, leading to a test of m = 4 individual measurements. Setting the overall confidence level to $(1-\alpha)$ = 95%, the probability used to determine an appropriate Student's t-quantile needs to be set to $(1-\alpha/m) = 1-.05/4 = .9875$. The t-distribution with probability .9875 and (n–1) = 11 degrees of freedom in **Table 16-1** of **Appendix D** results in a t-quantile of 2.593. Using equation [18.3], the upper prediction limit can be computed as:

$$PL = 27.52 + t_{.9875,11}(17.10)\sqrt{1+\frac{1}{12}} = 27.52 + 2.593(17.10)\sqrt{1.0833} = 73.67 \text{ ppb}$$

Step 4. Compare the upper PL to each compliance measurement in Year 4. None of the four observations exceeds 73.67 ppb. Consequently, there is no statistically significant evidence of arsenic contamination during that year. ◄

18.2.2 PREDICTION LIMIT FOR A FUTURE MEAN

BACKGROUND AND PURPOSE

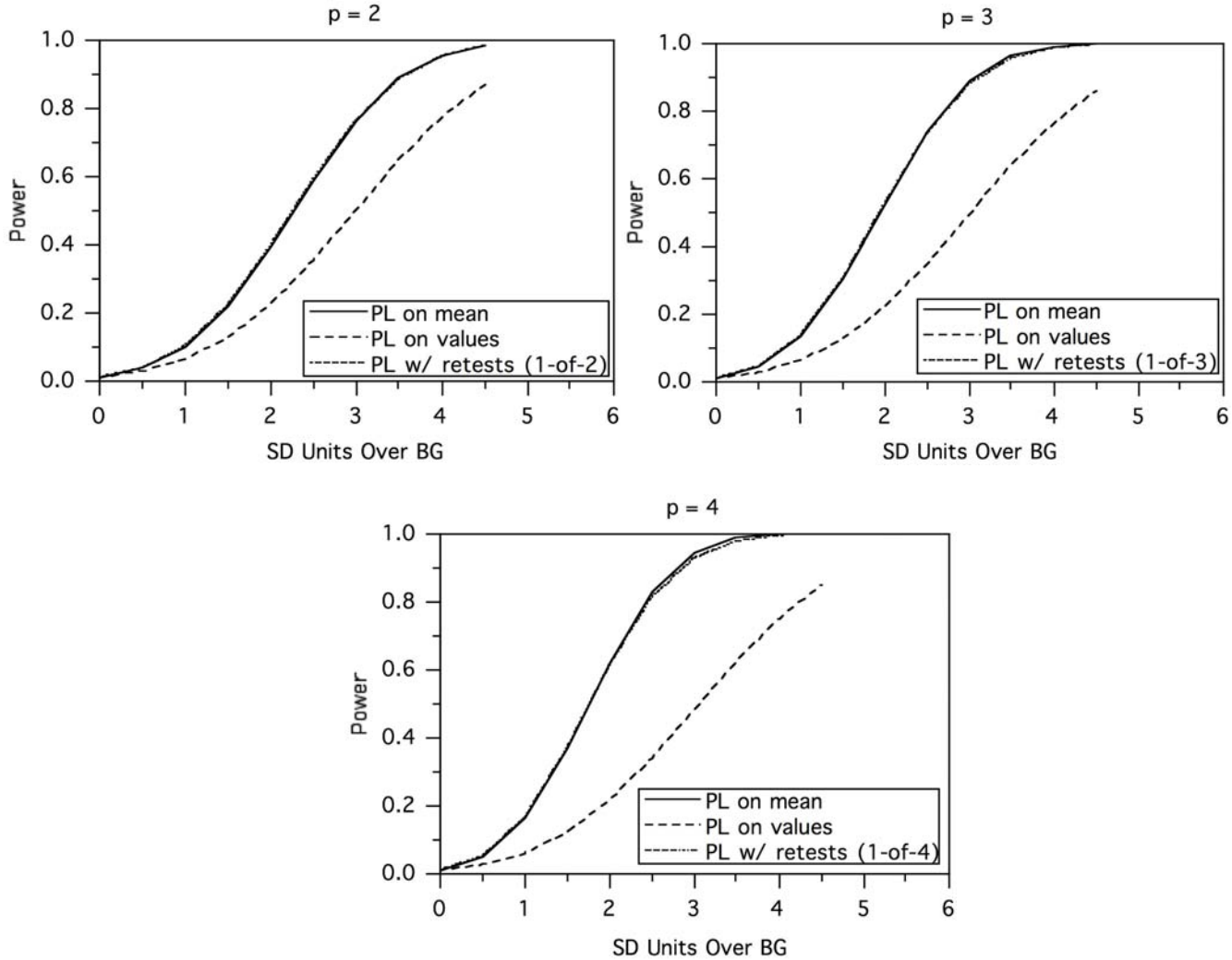
Although prediction limits are often constructed as bounds on extreme individual measurements, they can also be formulated to predict an acceptable range of concentrations for the *mean* of *p* future values. The comparison rule for the test is then different: instead of requiring all of a set of m individual values to fall within the prediction interval for the test to pass, only the *average* of the (*p*) future values should not exceed the prediction limit.

In this setting, the prediction limit for a future mean is more nearly akin to a *t*-test or parametric ANOVA, since the mean of the compliance point well is compared to a limit based on the background mean. The principal differences in using a prediction limit as opposed to those tests are: first that the variability of the compliance point population is *assumed* to be identical to that in background. With a *t*-test or ANOVA, each distinct well group contributes to the overall estimate of variability, not merely the background values. Secondly, t-tests and especially ANOVA are typically utilized as interwell tests, whereas prediction limits for a future mean can be constructed for either interwell or intrawell testing.

The hypothesis being tested when using a prediction limit for a future mean in detection monitoring is exactly the same as that posited for a prediction limit for *m* future values, namely, H_0: background population is identical to compliance population (implying $\mu_C \leq \mu_{BG}$) *vs*. H_A: compliance mean is greater than background mean (*i.e.*, $\mu_{DG} > \mu_{BG}$). However, the statistical properties of the two prediction interval formulations are somewhat different.

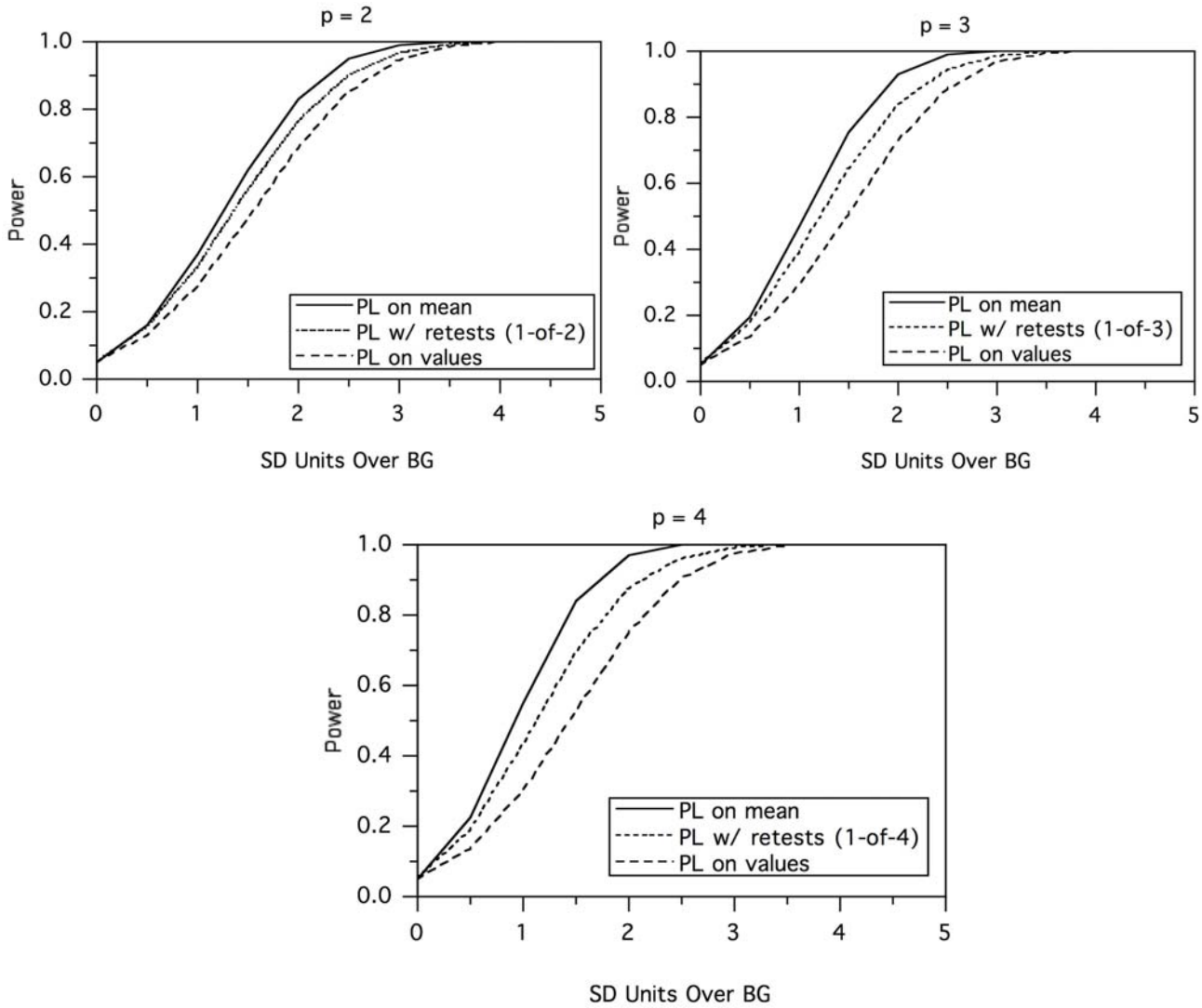
For the same background sample size (*n*), false positive rate (α), and number of future samples where $p = m$, the power of the prediction limit for a future mean of order *p* with normally-distributed data is generally higher than for a prediction limit of the next *m* individual future observations. *This suggests that* ***when feasible and appropriately implemented,*** *a prediction limit strategy based on future means may be more environmentally protective than a strategy based on predicting individual future measurements*. A few examples of the power differences are presented in **Figures 18-1** and **18-2**.

Figure 18-1. Comparison of Prediction limits (BG = 8, α = .01, 1 test)



Even when a retesting strategy is employed, such as the 1-of-*m* schemes for prediction limits on individual values described in **Chapter 19**, the statistical power at best *matches* that of a prediction limit on a single future mean with no retesting, when the same numbers of background and compliance point measurements are used. As **Figure 18-2** illustrates, for some cases the 1-of-*m* power is comparatively lower. Under background conditions, 1-of-*m* strategies provide an earlier indication of *uncontaminated* groundwater, since a single observation can indicate uncontaminated groundwater. By contrast, all $p = m$ individual samples need to be collected to form a mean of order $p = m$ when using a prediction limit test for a single future mean. With a groundwater release, no such potential time savings exists. In that case, *all p or m* samples need to be collected with either type of prediction limit.

Figure 18-2. Comparison of Prediction limits (BG = 20, α = .05, 1 test)



REQUIREMENTS AND ASSUMPTIONS

Although a prediction limit for a future mean is generally preferable in terms of statistical power for identifying potential contamination, it is not always practical to implement. To accommodate the large number of statistical tests that all but the smallest sites must contend with, the Unified Guidance recommends that almost any prediction limit be implemented in conjunction with a retesting strategy (**Chapter 19**). Otherwise, the prediction limit formulations provided in this chapter will likely fall short of providing an adequate balance between false negative and positive decision errors. Retesting with a prediction limit for a future mean will necessitate the collection of *p* additional measurements to form the resampled mean, whenever the initial future mean exceeds the prediction limit. Since all prediction limit tests assume that both the background and compliance data are statistically independent, there needs to generally be enough temporal spacing between sampling events to avoid introducing significant autocorrelation in the series of compliance point values.

If semi-annual evaluation of groundwater quality is required, and depending on data characteristics (see **Chapter 14** discussions on temporal variability), there may not be sufficient time for collecting at least 4 independent groundwater measurements from a given well over a six-month period. This would be the minimum needed to form an initial mean and potentially a resample mean of order 2. To avoid this dilemma, the guidance discusses an alternate approach in **Chapter 19** for using 1-of-1 prediction limit tests on means.

Like the parametric prediction limit for *m* future values, the prediction limit on a future mean assumes that the background data used to construct the limit are either normally-distributed or can be normalized. If a transformation is used (*e.g.*, the natural logarithm) and the limit built on the transformed values, the prediction limit should *not* be back-transformed before comparing to the compliance point data. Rather, because of transformation bias in the mean, the compliance point data should also be transformed, and the future mean computed from the *transformed* compliance measurements. Then the *mean of the transformed values* (*e.g.*, log-mean) should be compared to the prediction limit in the transformed domain. As previously mentioned, the prediction limit in the logarithmic domain is not a test of the arithmetic mean, but rather of the *geometric mean* or *median* (also see **Chapter 16**). In most situations, a decision that the lognormal median at the compliance point exceeds background will also imply that the lognormal arithmetic mean exceeds background.

PROCEDURE

Step 1. Calculate the sample mean, $\bar{x}$, and the standard deviation, *s*, from the set of *n* background measurements.

Step 2. Specify the order (*p*) of the mean to be predicted (*i.e.*, the number of individual compliance observations to be averaged). If the background data are approximately normal and an upper prediction limit with confidence level (1−α) is desired, use the equation:

$$PL = \bar{x} + t_{1-\alpha,n-1} s \sqrt{\frac{1}{p} + \frac{1}{n}} \qquad [18.7]$$

where it is assumed that an average of *p* consecutive sample values from the compliance point will be compared against *PL*. Note that the Student's *t*-quantile used in the equation has two parameters: the degrees of freedom (*n*–1) and the cumulative probability (1−α). Most Student's *t*-quantile values can be found directly or approximated through interpolation by using **Table 16-1** in **Appendix D**.

Note also that equation [18.7] assumes that the prediction limit is applied to only one constituent at a single well. If multiple tests are to be conducted and a retesting procedure is employed, the prediction limit will take the form of equation [18.4] where the κ-multiplier is determined using the tables described in **Chapter 19**.

Step 3. If a log transformation is applied to normalize the background sample, the upper *PL* on the log-scale should be constructed using the log-mean ($\bar{y}$) and log-standard deviation (s_y), using equation [18.8]:

$$PL = \bar{y} + t_{1-\alpha,n-1} s_y \sqrt{\frac{1}{p} + \frac{1}{n}} \qquad [18.8]$$

Note that unlike the lognormal prediction limit for future values, the limit in equation [18.8] is not exponentiated back to the concentration domain. Also, equation [18.8] only applies to a single test (*i.e.*, one constituent at a single well). If multiple tests are to be performed, the prediction limit will have the form:

$$PL = \bar{y} + \kappa s_y \qquad [18.9]$$

where the κ-multiplier is again determined from the tables described in **Chapter 19**.

Other transformations are handled similarly: construct the prediction limit on the transformed background, but do *not* back-transform the limit.

Step 4. Once the limit has been computed, compare the compliance point mean against the prediction limit. If the compliance point mean is below the upper *PL*, the test passes. If the mean exceeds the *PL*, there is statistically significant evidence of an increase over background.

►EXAMPLE 18-2

The table below contains chrysene concentration data found in water samples obtained from two background wells (Wells 1 and 2) and a compliance well (Well 3). Compute the upper prediction limit for a future mean of order 4 with 99% confidence and determine whether there is evidence of possible chrysene contamination.

	Chrysene (ppb)			
	Background			Compliance
Month	Well 1	Well 2	Joint	Well 3
1	6.9	15.1		68.0
2	27.3	7.2		48.9
3	10.8	48.4		30.1
4	8.9	7.8		38.1
Mean	13.47	19.62	16.55	46.28
SD	9.35	19.52	14.54	16.40
Log-mean	2.451	2.656	2.533	3.789
Log-SD	0.599	.881	.706	0.349

SOLUTION

Step 1. Before constructing the prediction limit, check the key assumptions. Assuming there is no substantial natural spatial variability and it is appropriate to combine the background wells into a single data pool, the algorithm for a parametric prediction limit presumes that the background data jointly originate from a single normal population. Running the Shapiro-Wilk test on the pooled set of eight background measurements gives *SW* = 0.7289 on the original

scale and SW = 0.8544 after log-transforming the data. Since the critical point for the test at the α = .10 level of significance is $sw_{.10,8}$ = 0.851 (from **Table 10-3** of **Appendix D**), the results suggest that the data should be fit to a lognormal model. The log-transformed statistics for the joint background and compliance well are also found in the above table.

Step 2. Construct the prediction limit on the pooled and logged background observations. Then n = 8, the log-mean is 2.533, and the log-standard deviation is 0.706. Since there are 4 observations in the compliance well, take p = 4 as the order of the mean to be predicted. Then setting (1–α) = .99, the Student's t-quantile with (n–1) = 7 degrees of freedom and cumulative probability of .99 is found from **Table 16-1** in **Appendix D** to be 2.998. Using equation [18.8], the upper prediction limit on the log-scale is computed as:

$$PL = 2.533 + (2.998)(0.706)\sqrt{\frac{1}{4} + \frac{1}{8}} = 3.83 \text{ log(ppb)}$$

Step 3. Compare the log-mean of the chrysene measurements at Well 3 against the upper prediction limit. Since it is less than the limit, there is insufficient evidence of chrysene contamination at this well at the α = 0.01 significance level. ◄

18.3 NON-PARAMETRIC PREDICTION LIMITS

Two basic remedies are available when a data set cannot be even approximately normalized, often due to the presence of a significant fraction of non-detects. If the sample includes left-censored data (*e.g.*, non-detects), a fit to normality can be attempted using censored probability plots (**Chapter 15**) in conjunction with either the *Kaplan-Meier* or *Robust Regression on Order Statistics* [Robust ROS] techniques (**Chapter 15**). If a reasonable normality fit can be found, a parametric prediction limit can be applied. Otherwise, a non-parametric prediction limit can be considered. A non-parametric upper prediction limit is constructed by setting the limit as a large *order statistic* selected from background (*e.g.*, the maximum or second-largest background value).

As with their parametric counterparts, non-parametric prediction limits have an associated *confidence level* (1–α) which indicates the probability that the prediction interval [0, PL] will accurately contain all m of a set of m future values over repeated application on many similar data sets. Unlike parametric limits, the confidence level for non-parametric limits is not *adjustable*. Despite being easily constructed for a fixed background sample size and the number of comparisons, the confidence level associated with the any maximal value used as the prediction limit is also fixed. To increase the confidence level, the primary choices are to *decrease* the number of future values to be predicted, or *increase* the number of background observations.

If existing background can be supplemented with data collected from other background wells (e.g., in interwell testing), a non-parametric test confidence level can be increased. Larger samples also provide a better characterization of site spatial variability. Unfortunately, it may always not be possible to supplement background. In these cases, another option to achieve a desired confidence level and

correspondingly control the false positive rate is to incorporate a retesting strategy as outlined in **Chapter 19**.

Although non-parametric prediction limits do not require a presumption of normality, other assumptions apply equally to both parametric and non-parametric limits. Checks should be made of statistical independence, identical distributions (under the null hypothesis), and stationarity over time and space as discussed in **Chapter 3** and **Part II** of the guidance. One particular caution for non-parametric limits is that background should ideally be screened ahead of time for possible outliers, since the upper prediction limit may be set to the background maximum or second highest observed value. Unfortunately, this often cannot be accomplished with a formal statistical test. Outlier tests are rather sensitive to the underlying distribution of the data. If this distribution cannot be adequately determined due to the presence of non-detects, an outlier test is not likely to give reliable results.

Instead of a formal test, it may be possible to screen for outliers using box plots (**Chapter 12**). Even with non-detects, the box plot 'whiskers' delineating the concentration range associated with possible outliers are computed from the sample lower and upper quartiles (*i.e.*, the 25th and 75th percentiles), which may or may not be impacted by data censoring, or perhaps mildly so when computing the lower quartile. For large fractions of non-detects, the best that can usually be done is to identify a suspected outlier through close examination of laboratory results and chain-of-custody reports.

One of two steps can be taken in the event a possible outlier is flagged. If an error has occurred, it should be corrected before constructing the prediction limit. If an error is merely suspected but cannot be proven, the prediction limit can be constructed as another order statistic from background instead of the maximum (*e.g.*, the second largest value). This will prevent the suspected outlier from being adopted as the upper prediction limit without ignoring the possibility that it may be a real measurement.

18.3.1 PREDICTION LIMIT FOR M FUTURE VALUES

BACKGROUND AND PURPOSE

Given n background measurements and a desired confidence level $(1-\alpha)$, a non-parametric prediction limit test for m future values is an m-of-m comparison rule. *All m* future samples need to not exceed the upper prediction limit for the test to pass. Thus the procedure is an exact parallel to the parametric prediction limit for future values. Because the method is non-parametric, no distributional model needs to be fit to the background measurements. It is assumed that the compliance point data follow the same distribution as background under the null hypothesis — even if this distribution is unknown. Although no distributional model is assumed, order statistics of any random sample follow certain probability laws which allow the statistical properties of the non-parametric prediction limit to be determined.

Once an order statistic of the sample data (*e.g.*, the maximum value) is selected as the upper prediction limit, Guttman (1970) has shown that the statistical *coverage* of the interval — that is, the fraction of the background population actually contained within the prediction interval — when constructed repeatedly over many data sets, has a *beta probability density* with cumulative distribution equal to

$$I_t\left(j, n-j+1\right) = \int_{u=0}^{t} \frac{\Gamma\left(n+1\right)}{\Gamma\left(n-j+1\right)\Gamma\left(j\right)} u^{j-1}\left(1-u\right)^{n-j}\, du \qquad [18.10]$$

where n = sample size, j = (rank of prediction limit value), and $\Gamma(n) = (n-1)! = (n-1)\times(n-2)\ldots\times 2\times 1$ denotes the gamma function. If the maximum is selected as the prediction limit, its rank is equal to n and so $j = n$. If the second largest value is chosen as the limit, its rank would be equal to (n–1) and so j = (n–1). The confidence probability for predicting that one future observation (*i.e.*, m = 1) from a compliance well does not exceed the prediction limit is equal to the *expected or average coverage* of the non-parametric prediction limit.

Because of these properties, the confidence probability for a prediction limit on one future measurement can be shown to equal $(1-\alpha) = j/(n+1)$. If the background maximum is taken as the upper prediction limit, the confidence level thus becomes n/(n+1). Gibbons (1991a) has shown that the probability of having m future samples all not exceed such a limit is $(1-\alpha) = n/(n+m)$. More generally, the same probability when the jth order statistic is taken as the upper prediction limit becomes (Davis and McNichols, 1999):

$$1-\alpha = \frac{(j+m-1)\cdot(j+m-2)\ldots\cdot(j+1)\cdot j}{(n+m)\cdot(n+m-1)\ldots\cdot(n+2)\cdot(n+1)} \qquad [18.11]$$

Table 18-1 in **Appendix D** lists these confidence levels for various choices of j, n, and m. The false positive rate (α) associated with a given prediction limit can be computed as one minus the confidence level. As this table illustrates, the penalty for not knowing the form of the underlying distribution can be severe. If a non-parametric prediction limit is to be used, *more background observations are needed compared to the parametric setting in order to construct a prediction interval with sufficiently high confidence*. As an example, to predict m = 2 future samples with 95% confidence, at least 38 background samples are needed. Parametric prediction intervals do not require as many background measurements precisely because the form of the underlying distribution is assumed to be known.

It is possible to create an approximate non-parametric limit with background data containing all non-detects, by using the RL (often a quantitation limit) as the PL. A quantified value above the PL would constitute an exceedance. A superior procedure is recommended in this guidance, using the Double Quantification Rule described in **Chapter 6**.

PROCEDURE

Step 1. Sort the background data into ascending order and set the prediction limit equal to the maximum, the second-largest observed value or another large background order statistic. Then use **Table 18-1** of **Appendix D** to determine the confidence level ($1-\alpha$) associated with predicting the next m future samples.

Step 2. Compare each of the m compliance point measurements to the upper prediction limit [*PL*]. Identify significant evidence of possible contamination at the compliance well if one or more measurements exceed the *PL*.

Step 3. Because the risk of false positive decision errors is greatly increased if the confidence level drops substantially below a target rate of at least 90% to 95%, the actual confidence level (as identified by equation [18.11]) needs to be routinely reported and noted whenever it is below the target level.

Note that equation [18.11] assumes the prediction limit is applied to only one constituent at a single well. If multiple tests must be conducted and a retesting procedure is employed, the confidence level of the prediction limit must be determined using the tables described in **Chapter 19**.

►EXAMPLE 18-3

Use the following trichloroethylene data to compute a non-parametric upper prediction limit for the next m = 4 monthly measurements from a downgradient well and determine the level of confidence associated with the prediction limit.

	Trichloroethylene Concentrations (ppb)			
	Background Wells			Compliance
Month	BW-1	BW-2	BW-3	CW-4
1	<5	7	<5	
2	<5	6.5	<5	
3	8	<5	10.5	7.5
4	<5	6	<5	<5
5	9	12	<5	8
6	10	<5	9	14

SOLUTION

Step 1. Determine the background maximum and use this value to estimate the non-parametric prediction limit. In this case, the maximum value of the n = 18 pooled background observations is 12 ppb. Set PL = 12 ppb.

Step 2. Compare each of the downgradient measurements against the prediction limit. Since the value of 14 ppb for Month 6 exceeds PL, conclude that there is statistically significant evidence of an increase over background at CW-4.

Step 3. Compute the confidence level and false positive rate associated with the prediction limit. Since four future samples are being predicted and n = 18, the confidence level equals $n/(n + m)$ = 18/22 = 82%. Consequently, the Type I error or false positive rate is at most (1 – 0.82) = 18% and the test is significant at the α = 0.18 level. This means there is nearly a one in five chance that the test has been falsely triggered. Only additional background data and/or use of a retesting strategy would lower the false positive rate. ◄

18.3.2 PREDICTION LIMIT FOR A FUTURE MEDIAN

BACKGROUND AND PURPOSE

A prediction limit for a future median is a non-parametric alternative to a parametric prediction limit for a future mean (**Section 18.2.2**) when the sample cannot be normalized. In groundwater monitoring, the most practical application for this kind of limit is for medians of order 3 (*i.e.*, the median of three consecutive measurement values), although the same procedure could theoretically be employed for medians of any odd order (*e.g.*, 5, 7, *etc.*). The comparison rule in this case is that the test passes only if the *median* of a set of 3 compliance point measurements does not exceed the upper prediction limit. Note that this is also the same as a 2-of-3 test, whereby the well is deemed in compliance if at least 2 of 3 consecutive observations fall within the prediction interval. Therefore, only 2 independent observations will generally be needed to complete the test at uncontaminated wells. The third measurement will be irrelevant if the first two pass and so will not need to be collected.

Given n background measurements and a desired confidence level $(1-\alpha)$, a non-parametric prediction limit for a future median involves a confidence probability that the median of the next p future observations will not exceed the limit. As noted in **Section 18.3.1**, order statistics of any random sample follow certain probability laws. In particular, the statistical coverage (C) of a prediction limit estimated by the jth order statistic (that is, the jth largest value) in background will follow a *beta distribution* with parameters j and $(n+1-j)$. Following the notation of Davis and McNichols (1987), the *conditional probability* that the median of 3 independent future values will not exceed the non-parametric prediction limit can be shown to equal

$$\Pr\left\{\text{Future median inbounds} \,\middle|\, X_{j:n}\right\} = 3C^2 - 2C^3 \qquad [18.12]$$

where $X_{j:n}$ denotes that the prediction limit equals the jth largest order statistic in a sample of n observations and a conditional probability denotes the chance that an event will occur given the observance of another event (in this case, after having observed $X_{j:n}$). The (unconditional) confidence probability $(1-\alpha)$ can then be derived by taking the *expected value* of equation [18.12] with respect to the random variable C. Using standard properties of the beta distribution, this probability becomes:

$$1-\alpha = \frac{(3n-2j+5)(j+1)j}{(n+3)(n+2)(n+1)} \qquad [18.13]$$

Thus the confidence level associated with a prediction limit for a future median of order 3 depends simply on the sample size of background (n) and the order statistic selected as the upper prediction limit (j). **Table 18-2** in **Appendix D** provides values of the confidence level for various n and choices of the order statistic. Like the non-parametric prediction limit for m future values, ease of construction comes with a price. More background measurements are required to achieve the same levels of confidence attainable via a parametric prediction limit for a future mean. For instance, to achieve 99% confidence in predicting a median of order 3 in a single test, at least 22 background observations are needed if the maximum is selected as the upper prediction limit, and at least 40 background observations are needed if the prediction limit is set to the second largest measurement. Parametric prediction intervals do not

require as many background samples precisely because the form of the underlying distribution is assumed to be known.

REQUIREMENTS AND ASSUMPTIONS

Once an order statistic (of rank *j*) is selected as the upper prediction limit, the confidence level is fixed by the number of background samples (*n*). The confidence level can only be increased by enlarging background. However, equation [18.13] is only applicable for the case of predicting a future median of ***a single constituent at a single well***. To account for multiple tests and to incorporate a retesting strategy (both of which are usually needed), the specific strategies and tables of confidence levels presented in **Chapter 19** should be consulted.

PROCEDURE

Step 1. Sort the background data into ascending order and set the upper prediction limit [*PL*] equal to one of the following: the background maximum, the second largest value, or another large order statistic in background. If the largest background measurement is a non-detect, set an approximate upper prediction limit as the RL most appropriate to the data (usually the lowest achievable quantitation limit [QL]).

Step 2. Compute the median of the next three consecutive compliance point measurements. Compare this statistic to the upper prediction limit. Identify significant evidence of possible contamination at the compliance well if the median exceeds *PL*. If *PL* equals the RL, identify an exceedance, if the median is quantified above the reporting limit.

Step 3. Based on the background sample size (*n*), use **Table 18-2** of **Appendix D** to determine the confidence level ($1-\alpha$) associated with predicting the median of the next $p = 3$ future measurements. Because the risk of false positive errors is greatly increased if the confidence level drops much below a targeted rate of at least 90% to 95%, the actual confidence level (as identified in equation [18.13]) should be routinely reported and noted whenever it is below the target level.

Note that equation [18.13] assumes the prediction limit is applied to only one constituent at a single well. If multiple tests are conducted and a retesting procedure is employed, the confidence level of the prediction limit needs to be determined using the tables described in **Chapter 19**.

►EXAMPLE 18-4

Use the following xylene background data to establish a non-parametric upper prediction limit for a future median of order 3. Then determine if the compliance well shows evidence of excessive xylene contamination.

	Xylene Concentrations (ppb)			
		Background		Compliance
Month	Well 1	Well 2	Well 3	Well 4
1	<5	9.2	<5	
2	<5	<5	5.4	
3	7.5	<5	6.7	
4	<5	6.1	<5	
5	<5	8.0	<5	
6	<5	5.9	<5	<5
7	6.4	<5	<5	7.8
8	6.0	<5	<5	10.4

SOLUTION

Step 1. The maximum value in the set of pooled background measurements is 9.2. Assign this value as the non-parametric upper prediction limit, $PL = 9.2$.

Step 2. Compute the median of the three compliance measurements. This statistic equals 7.8 ppb. Since the median does not exceed PL, there is insufficient evidence of xylene contamination at Well 4, despite the fact that the ***maximum*** at Well 4 is larger than the maximum observed in background.

Step 3. Compute the confidence level and false positive rate associated with this prediction limit. Given that $n = 24$ and the order statistic selected is the maximum (*i.e.*, $j = n$), use **Table 18-2** in **Appendix D** to determine that the confidence level for predicting a future median of order 3 equals 99.1% and therefore the Type I error or false positive rate is at most 0.9%. ◄

CHAPTER 19. PREDICTION LIMIT STRATEGIES WITH RETESTING

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This chapter is a core part of the recommended statistical approach to detection monitoring. Even the smallest of facilities will perform enough statistical tests on an annual basis to justify use of a retesting strategy. Such strategies are described in detail in this chapter in conjunction with prediction limits. First, the Unified Guidance considers the concept and computation of site-wide false positive rates [SWFPR]. Then different retesting strategies useful for groundwater monitoring are presented, including:

- Parametric prediction limits with retesting (**Section 19.3**), and
- Non-parametric prediction limits with retesting (**Section 19.4**)

19.1 RETESTING STRATEGIES

Retesting is a statistical strategy designed to efficiently solve the problem of *multiple comparisons* (*i.e.*, multiple, simultaneous statistical tests). An introduction to multiple comparisons is presented in **Chapter 6**. At first glance, formal retesting seems little different than a repackaged form of *verification resampling*, a practical technique used for years to double-check or verify the results of initial groundwater sampling. Indeed, all retesting schemes are predicated on the idea that when the initial groundwater results indicate the presence of potentially contaminated groundwater, one or more additional groundwater samples should be collected and tested to determine whether or not the first results were accurate.

The difference between formal retesting schemes and verification resampling found in the regulations is that the former *explicitly incorporates the resample(s) into the calculation of the statistical properties of the overall test*. A statistical "test" then needs to be redefined to include not only the statistical manipulation of the initial groundwater sampling results, but also that for any further resamples. Both the initial samples and the resamples are integral components of any retesting method.

The principal advantage of retesting is that very large monitoring networks can be statistically tested without necessarily sacrificing either an acceptable false positive rate or adequately high *effective power*. Data requirements for a typical retesting scheme are often less onerous than those required for an analysis of variance (ANOVA). Instead of having to sample each well perhaps four times during any

given evaluation period, many of the retesting strategies discussed below involve a minimum of one new sample at each compliance well. Resamples are collected only at wells where the initial results exceed a limit, and no explicit *post-hoc* testing of individual wells is necessary as with ANOVA in order to identify a contaminated well.

Since a statistical test utilizing retesting is not complete until all necessary resamples have been evaluated, it is important to outline the formal *decision rules* or *scheme* associated with each retesting strategy. Retesting schemes presented in the Unified Guidance fall into two types: 1-of-*m* and the modified California approach. The 1-of-*m* approach was initially suggested by Davis and McNichols (1987) as part of a broader method termed "*p*-of-*m*." The 1-of-*m* scheme assumes that as many as *m* samples might be collected for a particular constituent at a given well, including the initial groundwater sample and up to (*m*–1) resamples.

1-of-*m* schemes are particularly attractive as retesting strategies. If the initial groundwater observation is in-bounds, the test is complete and no resamples need to be collected. Only when the first value exceeds the background prediction limit, does additional sampling come into play. For practical reasons, only 1-of-*m* schemes with *m* no greater than 4 are considered in the Unified Guidance. A 1-of-4 retesting plan implies that *up to* 4 groundwater measurements may have to be collected at each compliance well, including the initial observation and 3 possible resamples. For the test to be valid, all of these sample measurements need to be statistically independent. This generally requires that sufficient time elapses *between resample collection* so that the assumption of statistical independence or lack of autocorrelation is reasonable (see the discussion in **Chapter 14**). Because many groundwater evaluations are conducted on a semi-annual basis, three will generally be a practical upper bound on the number of independent resamples that might be collected. Thus the 1-of-2, 1-of-3, and 1-of-4 retesting schemes are included below.

The second type of retesting scheme is known as the modified California approach. The decision rules for this test are slightly different from the 1-of-*m* schemes, although the test passes as before if the initial groundwater measurement is inbounds. If it exceeds the background limit, two of the three resample need to be inbounds for the test to pass. The modified California strategy thus requires a *majority* of the resamples to be inbounds for a compliance well test to be deemed 'in bounds'. A 1-of-4 scheme could have both the initial value and the first two resamples be out-of-bounds, yet pass the test with an inbounds result from the third resample. Although the modified California test appears to be more stringent, the prediction limit for a 1-of-4 test under the same input conditions will be lower and hence be more likely to trigger single comparison exceedances. With the prediction limits correctly defined, both will have identical false positive errors for any specific monitoring design. The guidance also provides the same four non-parametric versions of the 1-of-*m* and modified California tests for future values.

A useful variation on the 1-of-*m* retesting scheme for individual measurements is the 1-of-*m* strategy for *means or medians*. Instead of testing a series of individual values, a series of means or medians of order *p* is tested. The *order* of the mean or median refers to the number of individual measurements used to compute the statistic. For example, 1-of-2 retesting with means of order 2 requires that a pair of initial observations be averaged and the resulting mean compared against the background limit. If that initial mean is out-of-bounds, a second pair of observations (*i.e.*, two resamples) would be

collected and averaged to form the resample mean. The test would fail only if both the initial mean and the resample mean exceeded the background limit.

Retesting schemes for means or medians have steeper data requirements than retesting strategies for individual measurements and may not be practical at many sites. Nevertheless, the statistical properties (*e.g.*, power and false positive rate) associated with the testing of means and medians are superior to comparable plans on individual observations. The Unified Guidance provides five mean retesting plans: 1-of-1, 1-of-2, or 1-of-3 for means of order 2; and 1-of-1 and 1-of-2 for means of order 3. The guidance also provides 1-of-1 and 1-of-2 tests of medians of size 3 as non-parametric options.

These plans were chosen to limit the maximum possible number of distinct and independent sampling measurements per compliance well during a single evaluation period to six. In fact, the data requirements vary substantially by scheme. With means of order 2, the 1-of-1 plan requires a maximum of two new sample measurements; the 1-of-2 plan requires as many as four; while only the 1-of-3 plan might need a total of six. For means of order 3, the 1-of-1 plan requires three new measurements to form the single mean; the 1-of-2 plan might require up to six. But for higher order 1-of-*m* mean or median tests, only the initial samples may be needed to identify a 'passing' test outcome under most background conditions.

The three 1-of-1 mean and median plans provided in the guidance are technically not retesting schemes. The decision rule for these plans merely requires a comparison of a single mean or median against the background limit. If the initial mean or median comparison is inbounds, the test passes. If not, the test fails. The fact that each average is computed from multiple individual measurements implies that an implicit retest or verification resampling is built into these strategies. The statistical properties of the 1-of-1 plans can often be better than comparable 1-of-*m* schemes for individual values, with fairly similar sampling requirements.

The Unified Guidance provides 1-of-1 and 1-of-2 non-parametric prediction limit tests for future medians of order 3. By ‘median of order 3’, it means that the median or ‘middle value’ of a set of three consecutive sampling events. In the 1-of-2 case, the test passes if either the initial median is inbounds or, if not, when the resample median is inbounds. The 1-of-1 scheme does not involve any resampling, but does require at least two distinct sampling measurements to determine whether the initial median is inbounds.[1]

As discussed in **Chapter 6**, proper design of a groundwater detection monitoring program will generally require an initial choice of a retesting scheme *before* future or compliance sampling data have been collected. As a practical matter, sample collection should be spaced far enough apart in time to ensure that any potentially needed resamples are statistically independent. Thus, the maximum number of resamples need to be known in advance in order to structure a feasible sampling plan for a particular retesting strategy. Each retesting scheme also involves a different set of decision rules for evaluating the status of any given compliance well. The rules will determine how the background limit will be computed. Given the same background sample and group of compliance wells, different retesting schemes lead to *different* background limits on the *same* data.

[1] As noted in **Chapter 18**, the 1-of-1 retesting scheme for medians of order 3 is equivalent as a decision rule to a 2-of-3 scheme for individual measurements.

If parametric prediction limits are used, the general formula for the limit introduced in **Chapter 18** is $\bar{x} + \kappa s$. The κ-multiplier and thus the prediction limit will vary depending on which 1-of-*m* or modified California plan is chosen. The κ-multipliers also depend on the monitoring evaluation schedule in place at the facility. In typical applications, it is expected that the background sample used in statistical evaluations from any given year will either be *static* or substantially overlap from one evaluation to the next. The same background observations are likely to be utilized or will substantially overlap if newer background data are added to the existing pool. Since at least a subset of the background measurements will be commonly employed in all the evaluations, there will be a *statistical dependence* exhibited between distinct evaluations (see **Section 19.2** below). The number of evaluations per year against a common background will affect the correct identification of prediction limits. Consequently, the evaluation schedule (*i.e.*, annual, semi-annual, quarterly) also needs to be known or specified in advance.[2]

19.2 COMPUTING SITE-WIDE FALSE POSITIVE RATES [SWFPR]

As discussed in **Chapter 6**, the fundamental purpose of detection monitoring is to accurately identify a significant change in groundwater relative to background conditions. To meet this objective, statistical monitoring programs should be designed with the twin goals of ensuring adequate statistical power to flag well-constituent pairs elevated above background levels and limiting the risk of *falsely* flagging uncontaminated wells across an entire facility. The latter is accomplished by addressing the *site-wide false positive rate* [SWFPR]. Both goals contribute to accurate evaluation of groundwater and to the validity of statistical groundwater monitoring programs.

Retesting significantly aids this process of meeting *both* criteria. However, it can be much easier to design and implement an appropriate retesting scheme if one understands how the SWFPR is derived. The SWFPR is based on the assumptions that no contamination is actually present at on-site monitoring wells, and that each well-constituent pair in the network behaves independently of other constituents and wells from a statistical standpoint. If *Q* denotes the probability that a particular well-constituent pair will be falsely declared an exceedance (a *false positive* error), the probability of at least one such false positive error among *r* independent tests is given by:

$$\alpha = SWFPR = 1 - \left(1 - Q\right)^r \qquad [19.1]$$

(1–*Q*) equals the chance that the test will correctly identify the well-constituent pair as 'inbounds.' The value of *Q* itself will depend on the type of retesting scheme being used.

[2] The Unified Guidance distinguishes between the statistical *evaluation* (or testing) schedule and the *sampling* schedule. Regularly scheduled sampling events might occur quarterly, even though a statistical evaluation of the data only occurs semi-annually or annually. Further, resamples do not constitute regular sampling events, since they are only collected at wells with initial exceedances, but they *are* associated with the data for a particular evaluation. By separately identifying the evaluation schedule, there is 1) less confusion about the role of resamples in the testing process, and 2) opportunity to design monitoring programs, so as to allow for multiple individual observations to be collected prior to each evaluation.

Consider a 1-of-3 retesting plan for future observations. A false positive at a given well-constituent pair will be registered only if all three observations — the initial groundwater measurement and two resamples — exceed the background prediction or control limit. If ω represents the probability that one of these observations exceeds the background limit, *Q* can be calculated as $\omega \times \omega \times \omega$ (since the initial measurement and resamples are statistically independent) and the SWFPR as:

$$\alpha = SWFPR = 1 - \left(1 - \omega^3\right) \qquad [19.2]$$

By setting the target site-wide α equal to 0.10 and solving for ω, one could potentially compute the individual comparison false positive rate ($\alpha_{comp} = \omega$) associated with any single comparison against the background limit. This would identify the individual *per-comparison* confidence level ($1 - \alpha_{comp}$) necessary to compute the background limit in the first place.[3] If the background limit is computed as a prediction limit for the next single future measurement (*i.e.*, *m* = 1 in a 1-of-*m* scheme), then ω equals the probability that a single new observation (independent of background) exceeds the prediction limit, and (1–ω) equals the confidence level of that prediction limit. Further, since ω can be obtained from equation [19.2] as:

$$\omega = \sqrt[3]{1 - \left(1 - \alpha\right)^{1/r}} \qquad [19.3]$$

the upper prediction limit for a site involving 500 tests (for instance, 50 wells and 10 constituents per well) and 20 background samples could be computed using an individual, per-comparison confidence level of

$$1 - \omega = 1 - \sqrt[3]{1 - \left(1 - .10\right)^{1/500}} = 1 - .0595 = 94.0\%$$

leading to a final prediction limit of

$$PL = \bar{x} + t_{.94,19} s \sqrt{1 + \frac{1}{20}}$$

where $\bar{x}$ and *s* are the background sample mean and standard deviation.

Unfortunately, certain statistical dependencies render the foregoing calculations somewhat inaccurate. Whether or not a resample exceeds the background limit for any constituent depends partly on whether the initial observation for that test *also* eclipsed the limit. This is because the *same background data* are used in the comparison of both the initial groundwater measurement and the resamples. This creates a statistical dependence between the *comparisons*, even when the compliance point observations themselves are statistically *independent*. If the background data sample mean happens to be low relative to the true population mean, the background limit will tend to be low. Each of the compliance point observations (whether the first measurement or subsequent resamples) will have a

[3] Note that α_{comp} does not represent the false positive rate for the complete 1-of-3 test, but is being treated for the sake of argument as a one of a series of 3 individual and independent tests.

greater than expected chance of exceeding it. Likewise, if the background sample mean is substantially higher than the population mean, the background limit will tend to be high, resulting in a lower-than-expected chance of exceedance for each of the compliance measurements.

A similar dependence occurs for each well-constituent pair tested against a single background across evaluation periods (see discussions in **Chapter 5** and **Section 19.1**). A further dependence occurs when well-constituent pairs from many compliance wells are compared to a common interwell background. The tests during each statistical evaluation again share a common (or nearly common) background, thus impacting the individual test false positive rate (α_{test}) and the SWFPR (α) in turn. Three common evaluation strategies are considered in the Unified Guidance: quarterly, semi-annual, and annual. The SWFPR is computed on a cumulative, annual basis, with the assumption that background and the associated background limit will not be updated or recomputed (especially for intrawell tests) more often than every one to two years.[4]

These dependencies between successive comparisons and tests against the background limit during retesting means that the derivation above will generally *not* result in a background limit with the targeted annual SWFPR of 10%. The actual false positive rate (α) will be somewhat higher and can be substantially higher if the background sample size (n) is small to moderate (say less than 50 samples). In part, this is because the correlation between successive comparisons against a common background limit is on the order of $1/(1+n)$. That is, the smaller the background size, the greater the correlation between the resamples and test comparisons. The impact on the SWFPR is also greater if this dependence is ignored.

Fortunately, as Gibbons (1994) has noted, the solution suggested in the previous example will be approximately valid for large background data sets (say $n > 50$), since then the correlation between successive resamples and/or tests is minimal. In fact, an approximate solution for the modified California and more general 1-of-m retesting schemes can also be derived. In the case of 1-of-m schemes, the probability Q of a false positive (for m = 1 to 4) is ω^m, leading to a SWFPR of :

$$\alpha = SWFPR = 1 - \left(1 - \omega^m\right)^r \qquad [19.4]$$

Solving for ω in equation [19.4] leads to an approximate individual comparison false positive rate ($\alpha_{comp} = \omega$) of:

$$\omega = \sqrt[m]{1 - \left(1 - \alpha\right)^{1/r}} \qquad [19.5]$$

For the modified California plan, a false positive for a given well-constituent pair during a single evaluation will be registered only if both the initial measurement and at least two of three resamples are

[4] Even with these assumptions, not all the statistical dependence will be accounted for at every site or for all constituents. Even when background is updated with new measurements, some of the already existing background values are likely to be used in re-computing the background limit. Some well-constituent pairs may be correlated, contradicting the assumption of independence between tests at the same well or for the same constituent at different wells. The Unified Guidance also does not presume to compute the SWFPR for other multi-year periods or for the life of the facility.

out-of-bounds (*i.e.*, exceed the background limit). Consequently, the probability Q of a false positive for that pair may be expressed as:

$$Q = \omega\left[3\omega^2\left(1-\omega\right)+\omega^3\right] = \omega^3\left(4-3\omega\right) \qquad [19.6]$$

As before, ω represents the probability of any single observation exceeding the background limit. Both the initial and any resample comparisons against the limit are assumed to be statistically independent. Given Q, the approximate overall false positive rate then becomes:

$$\alpha = SWFPR = 1-\left[1-\omega^3\left(4-3\omega\right)\right]^r \qquad [19.7]$$

Since ω will always be small in practice, one can usually ignore the term ω^4 when expanding the right-hand side of equation [19.7]. Then the approximate SWFPR becomes:

$$\alpha \approx 1-\left[1-4\omega^3\right]^r \qquad [19.8]$$

Leading to a solution for ω:

$$\omega \approx \sqrt[3]{1-\left(1-\alpha\right)^{1/r}}\sqrt[3]{\frac{1}{4}} \qquad [19.9]$$

which can again be used to construct a background limit for a single new observation.

As an example, if the target SWFPR is 10% and one must test r = 200 comparisons using the modified California plan, ω would become:

$$\omega \approx \sqrt[3]{1-.90^{1/200}}\sqrt[3]{\frac{1}{4}} = .0508 = 5.1\%$$

If the background limit is a prediction limit for the next future value, a confidence level of approximately 94.9% would be needed to achieve the desired overall false positive rate of 10%. This assumes that the background sample size is sufficiently large (say n > 50) to make the correlation between retests negligible. In similar fashion, the respective single comparison error rates for the 1-of-2 through 1-of-4 tests of future observations in this example would respectively be: ω = .0229, .0808, and .1515.

19.2.1 BASIC SUBDIVISION PRINCIPLE

The previous section highlighted certain dependencies in statistical tests due to comparisons of one or more samples or sample sets against a common background. In the sitewide design of a facility detection monitoring system, the overall target design SWFPR is proportionately divided among all relevant tests conducted in an annual period. Depending on the type of testing (e.g., interwell versus

intrawell, or a parametric versus non-parametric), the target error rates for a portion of the total set of potential tests may need to be calculated.

Identifying false positive target rates is important when considering non-parametric prediction limit tests. The cumulative target error rate for a group of annual tests against a single constituent is needed to compare with the achievable levels in **Tables 19-19 through 19-24** in **Appendix D**. The latter achievable rates take into account the dependencies previously discussed. ***κ***-multiple **Tables 19-1 to 19-18** in **Appendix D** for parametric prediction limit tests have already made use of target false positive rate calculations which are generally not needed for identifying the appropriate multipliers. The various dependencies against a common background are accounted for in the ***κ***-multiple tables to meet the nominal target rates. **R**-script software for certain parametric prediction limit tests discussed in a following section and in **Appendix C** also makes use of a target per-test false positive error rate as input.

In assigning target rates, the Unified Guidance uses a basic *subdivision principle* which makes certain assumptions. First and foremost, it is assumed that the total suite of tests can be subdivided into mutually exclusive, independent[5] tests. Each relevant annual statistical test is assigned the same single test error rate (α_{test}). Using the properties of the Binomial distribution, the target single test error rate can be obtained using equation [19.10] for r total annual tests. The total number of annual tests r is the product of the number of compliance wells (w), the number of valid constituents (c), and the number of evaluations per year (n_E) or $r = w \times c \times n_E$, with α = SWFPR:

$$\alpha_{test} = 1 - (1-\alpha)^{1/r} \qquad [19.10]$$

Then a cumulative false positive rate can be assessed for any appropriate subset of tests. This principle would apply, for instance, if there is more than one regulated unit at a site and each regulated unit can be treated independently. A consistent portion of the overall targeted false positive rate α would be assigned to each regulated unit (α_{unit}), using a rearrangement of equation [19.10]. If a facility with three units B, C, and D had 120 total annual tests ($b + c + d = 120 = r$), the cumulative target error rate for Unit B would be: $a_{UnitB} = 1 - (1-\alpha_{test})^b$ and similarly for Units C and D. These three cumulative error rates will *approximately* (but not exactly) sum to a total sitewide value close to the SWFPR. However, as joint independent tests taken together, the annual SWFPR is in fact exactly 10%. The Bonferroni assumption makes use of the approximately linearity of such error rates for SWFPR calculations (discussed below).

The ways in which the overall SWFPR might be partitioned will vary with each site, considering units, types of tests, number of wells, constituents and evaluations per year. If unit-specific cumulative false positive rates were established, the group of tests associated with each monitoring constituent within each unit might be separately considered. Each group might potentially be further subdivided into intrawell versus interwell tests, or prediction limits versus control charts, *etc*., assuming a mixture of statistical methods is employed. By using the subdivision principle in a consistent way, the targeted SWFPR can be accurately maintained.

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[5] The Unified Guidance does not presume that every statistical test is in fact independent. Tests or groups are treated as if independent, however, to allow the computation of nominal target false positive rates and/or to be consistent with regulatory constraints (*e.g.*, all constituents must be tested separately).

One important use when calculating SWFPR rates is to account for multiple constituents. In particular, non-parametric test theory is applied to only a single constituent at a time. Since each constituent has its own set of background data and presuming the constituents behave independently of one another, the dependence caused by using a common background pertains only to those comparisons made against the background for that constituent. To clarify this concept, suppose a total set of r tests consists of c separate chemicals each monitored at w wells annually (*i.e.*, $r = c \times w \times n_E$ and $n_E = 1$). For each constituent, the dependence caused by a common background only applies to the w comparisons (one for each well) made for that monitoring parameter. This means that the overall target α = SWFPR needs to be apportioned into a fraction for each constituent, called the per-constituent false positive rate or α_c. This can be done using the Binomial formula based on the single test error rate for w wells as: $a_c = 1 - (1 - \alpha_{test})^{w \cdot n_E}$ or by partitioning the overall α to each constituent c:

$$\alpha_c = 1 - (1 - \alpha)^{1/c}$$

The two calculations are equivalent under these conditions, with the latter equation somewhat easier to use.

A similar situation occurs at sites requiring a combination of interwell and intrawell tests. Computation of the SWFPR can be appropriately handled using the basic subdivision principle. For interwell tests, measurements collected at each compliance well are compared against a common interwell background, creating a degree of statistical dependence not only between successive individual test comparisons (*i.e.*, initial sample and any resamples) at a given well, but also between tests at different compliance wells. With intrawell tests, each well supplies its own background. This implies that the component of between-well test dependence is eliminated, changing the way κ-multipliers for intrawell background limits with retesting are computed.

For a given set of r well-constituent pairs, l tests to be conducted on an interwell basis, and the remaining ($r - l$) tests conducted as intrawell, two cumulative false positive rates need to be computed. The single test false positive error rate α_{test} approach can be used: $a_{\text{int}\,er} = 1 - (1 - \alpha_{test})^l$ for the subset of l interwell tests, and $a_{\text{int}\,ra} = 1 - (1 - \alpha_{test})^{r-l}$ for the subset of $r - l$ intrawell tests, in order to correctly maintain the SWFPR equal to α. A somewhat more direct approach can also be used: $\alpha_{\text{int}\,er} = 1 - (1 - \alpha)^{l/r}$ for the interwell tests and $\alpha_{\text{int}\,ra} = 1 - (1 - \alpha)^{(r-l)/r}$ for the intrawell tests. The two sets of equations are consistent.

In general, the subdivision principle works as follows. If a group of r tests with targeted false positive rate, α, is divided into s distinct and mutually exclusive independent subsets, the false positive rate for each subset (α_{sub}) can be computed as:

$$\alpha_{\text{sub}} = 1 - \left(1 - \alpha\right)^{1/s} \qquad [19.11]$$

The basic subdivision principle does not guarantee that the resulting detection monitoring program will have sufficient effective power to match the EPA reference power curve (ERPC). The foregoing calculations merely point to the correct overall false positive rate.

As discussed in Section 6.2.2 of **Chapter 6**, a simpler approach would be to partition the overall SWFPR among a facility's annual number of tests, and can make use of the Bonferroni approximation. With low false positive rates characteristic of detection monitoring design, the total SWFPR can be divided by the number of annual tests for any of the various combinations of constituents, separate units, or interwell versus intrawell tests. The Bonferroni approach results in slightly different false positive values than by directly using the Binomial formula, as described above.

As an overall example, assume a facility with w = 20 wells monitored twice per year (n_E = 2) for c = 8 constituents Further, assume that 5 of the constituents can be monitored interwell and 3 need to be handled as intrawell comparisons. Non-parametric prediction limits will be considered for all tests. Calculate the target cumulative false positive error rates for interwell and intrawell comparisons, with the SWFPR = α = .1.

This site has a total of $r = w \times c \times n_E$ = 20×8×2 = 320 tests per year. For the five interwell constituents, there are 20×2×5 = 200 tests, with 20×2×3 = 120 intrawell tests. Each of the 5 interwell constituents will have 20×2 = 40 tests against a common background, while 2 semi-annual sample tests will be made against each of the 20×3 = 60 intrawell backgrounds.

From equation [19.10], the single test false positive error rate is: $\alpha_{test} = 1-(1-\alpha)^{1/r}$ = $1-(1-.1)^{1/320} = .0003292$. Each set of interwell constituent tests will have a cumulative false positive error rate α_c for the 40 annual tests as: $\alpha_c = 1-(1-\alpha)^{1/c}$ = $1-(1-.1)^{1/8} = .01308$. Note that *all 8* constituents are used in the equation, since the same false positive error rate is uniformly applied to all distinct subgroup tests. The result can be obtained using the single test error rate equation: $a_c = 1-(1-\alpha_{test})^{w \cdot n_E} = 1-(1-.0003292)^{40} = .01308$. This target value would be used to compare with achievable non-parametric test error rates for the same input conditions. The cumulative interwell error rate for all five constituents can be calculated as: $a_{\text{int}\,er} = 1-(1-\alpha_c)^c = 1-(1-.01308)^5 = .06371$.

For the intrawell tests, the simplest approach uses the single test error rate for two tests: $a_{2-\text{int}\,ra} = 1-(1-\alpha_{test})^{w \cdot n_E} = 1-(1-.0003292)^{1 \cdot 2} = .0006583$. This would be the cumulative error rate to consider with non-parametric intrawell tests. The overall intrawell cumulative error rate for the sixty tests would then be: $a_{60-\text{int}\,ra} = 1-(1-\alpha_{2-\text{int}\,ra})^{w \cdot c} = 1-(1-.0006583)^{60} = .03873$.

If the two overall interwell and intrawell cumulative error rates were added, the sum is .1024, quite close to the nominal 10% SWFPR. It is exactly that value when considered jointly. By comparison the single test error rate using the Bonferroni approximation would be .1/320 = .0003125, while the exact Binomial value is .0003292. The estimated interwell cumulative error for a single constituent would be 40 times the single test value or .0125 (versus the calculated .01308). For many non-parametric test considerations, these differences are relatively minor.

19.3 PARAMETRIC PREDICTION LIMITS WITH RETESTING

BACKGROUND AND PURPOSE

Upper prediction limits for *m* future observations and for future means were described in **Chapter 18**. Applied to a network of statistical comparisons in detection monitoring, these procedures can be considered an extension to Dunnett's *multiple comparison with control* [MCC] procedure (Dunnett, 1955). These procedures explicitly incorporate retesting that is applicable to a wider variety of cases than addressed by Dunnett.

Retesting can be incorporated with either interwell or intrawell prediction limits. Depending on which approach is adopted, there is a distinct difference in the κ-multipliers of the general prediction limit formula. In an interwell retesting strategy, there are at least two forms of statistical dependence that impact the SWFPR. One is that each initial measurement or resample at a given compliance well is compared against the same background. A second is the dependence among compliance wells and number of annual evaluations, all of which are compared against a common upgradient background. In intrawell retesting, this second form of dependence is either essentially eliminated if there is only one annual statistical evaluation or else substantially reduced in the event of multiple evaluations.[6] The remaining dependence is among successive resamples at each well.

To account for the basic differences between interwell and intrawell prediction limit tests, an extensive series of tables is provided in **Appendix D** listing a wide combination of background sample sizes, numbers of wells, numbers of constituents, and distinctions between interwell and intrawell tests. In conjunction with an evaluation schedule (*i.e.*, annual, semi-annual, or quarterly), these tables can be used to design and implement specific parametric retesting strategies in this chapter. All of the κ-multiplier tables for parametric prediction limits are structured to meet an annual SWFPR of 10% per year and to accommodate groundwater networks ranging in size from one to 8,000 total statistical tests per year. The Unified Guidance tables are more extensive than similar tables in Gibbons (1994b). Further, each table is designed to indicate the effective power of the *κ*-multiplier entries.

If a particular network configuration is not directly covered in the **Appendix D** tables, two basic options are available. First, bilinear interpolation can be used to derive an approximate *κ*-multiplier (see below for guidance on table interpolation). Second, the free-of-charge, open source, and widely available **R** statistical programming package (www.r-project.org) can be employed to compute an exact *κ*-multiplier. Further instructions and the two template codes used to compute the Unified Guidance κ-multiplier tables are provided in **Appendix C**. After installing the **R** package, these template codes can be run by supplying specific parameters for the network of interest (*e.g.*, number of wells, constituents, background sample size, *etc.*). Some familiarity with properly installing a program like **R** is helpful. **Appendix C** explains how to execute a pre-batched set of commands. No other technical programming experience is needed.

[6] If multiple evaluations occur each year, new compliance samples each evaluation period are tested against the common intrawell background.

REQUIREMENTS AND ASSUMPTIONS

The basic assumptions of parametric prediction limits were described in **Chapter 18**. These include data that are normal or can be normalized (via a transformation), lack of outliers, homogeneity of variance between the background and compliance point populations, absence of trends over time, stationarity, and statistical independence of the observations.

The Unified Guidance provides *separate* κ-tables for interwell and intrawell limits. One of these approaches should be justified before computing prediction limits. To use *interwell* prediction limits, there should be no significant natural spatial variation among the mean concentrations at different well locations. Otherwise, a prediction limit test could give meaningless results, since average downgradient levels might naturally be higher than background even in the absence of a contaminant release. The assumption of spatial variability should therefore be checked using the methods in **Chapter 13**.

While *intrawell* testing eliminates the problem of natural spatial variability, intrawell background often is developed using the first n samples from each compliance point well. Since historical data from compliance wells need to be utilized to do this, these groundwater measurements should be uncontaminated. The number of intrawell background samples available may also be rather limited. n will tend to be initially small prior to any updating of background. Such constraints will limit the intrawell retesting schemes that can both minimize the SWFPR yet maintain effective power similar to the ERPCs.

One possible way to overcome this limitation is to estimate a *pooled standard deviation* across many wells along the lines suggested by Davis (1998). Such a calculation is no more difficult than a one-way ANOVA (**Chapter 13**) for identifying on-site spatial variability. The *mean squared error* [MSE] component of the F-statistic in ANOVA gives an estimate of the average per-well variability. To the extent that mean levels vary by well location *but the population standard deviation does not*, a one-way ANOVA can be run on a collection of wells (both background and compliance) to estimate the average within-well variance, and hence, the common *intrawell* standard deviation (see **Chapter 13** for further details and examples).

Instead of a standard deviation estimate based solely on intrawell background at a single well with its attendant limits in size and degrees of freedom, the *mean* concentration level can be estimated on a well-specific basis, while the *standard deviation* is estimated utilizing a collection of wells leading to much larger degrees of freedom. Although the intrawell background size for a given well might be small (*e.g.*, n = 4 or 8), the κ-multiplier used to construct the prediction limit is based on both the *effective sample size* (*i.e.*, degrees of freedom plus one) *and* the intrawell sample size (n).

The pooled standard deviation for intrawell comparisons can be utilized if the population standard deviation is *approximately constant across wells*. Many data sets may not appear so initially; however, any transformation to normality must first be taken into account. The standard deviation is only assumed to be constant *on the transformed scale*. Furthermore, once any transformation is applied, the collection of wells should explicitly be tested for homogeneity of variance using the tools in **Chapter 11**. *Only if the assumption of equal variances across wells seems reasonable should the pooled standard deviation estimate be used*.

With little or no spatial variability among well locations, an interwell test might be considered. However, the sample standard deviation (s) computed from background may not adequately estimate true background variability. This can happen when there is a temporal component to the variability affecting all wells at a site or regulated unit in parallel fashion, or when there is a significant degree of autocorrelation between successive samples.

A random, temporal component to the variability can result from changes to the laboratory analytical method or field sampling methodology, periodic re-calibration of lab instruments, or other sample handling or preparation artifacts that tend to impact all observations collected during a given sampling event. Such a temporal component can sometimes be identified through the use of parallel time series plots (**Section 14.2.1**) or through a one-way ANOVA using time-of-sampling as the factor (**Section 14.2.2**). Results of the ANOVA can be used to derive a better estimate of the background population standard deviation (σ), along with adjusted degrees of freedom for use in constructing the upper prediction limit (see **Chapter 14** for further details and an example).

When autocorrelation is present, methods to adjust the standard deviation estimate and degrees of freedom entail possibly modeling the autocorrelation function. This issue is beyond the scope of the Unified Guidance and consultation with a professional statistician is recommended. The most practical way to avoid significant autocorrelation between samples is to allow enough time to lapse between sampling events. Precisely how much time will vary from site to site, but Gibbons (1994a) and others (for instance, American Society for Testing and Materials, 2005) recommend that the frequency of sampling be no more frequent than quarterly. Alternatively, a pilot study can be run on two or three wells with the sample autocorrelation function estimated from the results (**Sections 14.3.1** and **14.2.3**). The minimum lag (***i.e.***, time) between sampling events at which the autocorrelation is effectively zero can be used as an appropriate sampling interval.

APPENDIX TABLES FOR PARAMETRIC RETESTING PLANS

The Unified Guidance provides tables of κ-multipliers for both interwell and intrawell prediction limits with retesting. It also provides separate tables for predicting individual future values versus future means. Four distinct retesting schemes are presented in the case of prediction limits for individual values: 1-of-2, 1-of-3, 1-of-4, and the modified California plan schemes. Five distinct schemes are presented for the case of future means: 1-of-1, 1-of-2, and 1-of-3 for means of order 2, and 1-of-1 and 1-of-2 for means of order 3.

Both the **Appendix D** interwell retesting tables (**Tables 19-1** through **19-9**) and the intrawell retesting tables (**Tables 19-10** through **19-18**) are similarly structured. Separate sub-tables are provided for a range of possible monitoring constituents (c = 1 to 40) and for each of the retesting schemes mentioned above. Each table is divided into three parallel sections, one section applicable to annual statistical evaluations, one to semi-annual evaluations, and one to quarterly evaluations. Within each section, κ-multipliers are listed for all combinations of background sample size (from n = 4 to 150) and number of wells (from w = 1 to 200). These κ-multipliers are computed to meet a target annual SWFPR of 10%, as discussed in **Chapter 6**.

The **Appendix** tables also list those κ-multipliers which achieve adequate effective power compared to the ERPCs. The κ-multipliers are **bolded** when the effective power consistently exceeds the appropriate ERPC for mean level increases above background of 3 or more standard deviations

(designated as 'good' power). The multipliers are *italicized and shaded* when the effective power is somewhat less, but still consistently exceeds the ERPC at mean level increases of 4 or more standard deviations above background (designated as 'acceptable' power). Non-bolded, non-italicized entries achieve the target SWFPR, but have *low power*.

To use the tables, certain key statistical parameters should be known or identified. These include whether the prediction limit tests are interwell or intrawell, the evaluation schedule (annual, semi-annual, or quarterly), the number of constituents (c), the size of the background sample (n), and the number of compliance wells to be tested (w). In the interwell case, it is presumed that there are n (upgradient) background measurements for each constituent (c). The listed κ-multiplier would then be applied to each of c prediction limits, one for each monitoring constituent. The intrawell case presumes that there are n well-specific background measurements designated at each well-constituent pair, thus giving $w \times c$ separate sets of intrawell background. Here, the κ-multiplier would be applied to each of $w \times c$ distinct prediction limits.

In situations where a mixture of test types is needed (*e.g.*, intrawell testing for some constituents, interwell for others), the Unified Guidance tables can still be employed. The κ-multipliers are computed to apportion an *equal share* of the overall cumulative SWFPR to each of the $w \times c$ tests that need to be run during a given statistical evaluation. Because of this fact, if r of the constituents are analyzed using interwell tests, but $(c - r)$ of the constituents are handled using intrawell limits, correct prediction limits can be developed by first selecting an interwell κ-multiplier based on all c constituents, and then selecting an intrawell κ-multiplier *also* based on c constituents. This will ensure that the target SWFPR is met, although each multiplier is respectively applied only to a subset of the monitoring list.

Some background samples might be of different sizes, either for different constituents or at distinct wells (*e.g.*, when using intrawell background). Again the Unified Guidance tables can be inspected to select a different κ-multiplier for each distinct n. However, each multiplier should be chosen as if the background sample sizes were equal for all $w \times c$ tests. Thus, while a multiplier based on n_1 background observations is applied only to those tests involving that sample size, it should be selected from the **Appendix D** tables as if it will be applied to *all* the tests.

For network configurations not listed in **Tables 19-1** to **19-18** in **Appendix D**, an appropriate κ-multiplier can be estimated using bilinear interpolation. Such interpolation will be fairly accurate as long as adjacent table entries are used, representing the closest values to the desired combination of number of wells (w) and background sample size (n).

In general, to calculate a κ_{w^*, n^*}, where w^* and n^* are the desired input points that lie between the closest table entries as: $w_1 < w^* < w_2$ and $n_1 < n^* < n_2$, first calculate the fractional terms:

$$f_w = \frac{(w^* - w_1)}{(w_2 - w_1)} \quad \text{and} \quad f_n = \frac{(n^* - n_1)}{(n_2 - n_1)}$$

The interpolated κ-multiplier can then be computed as:

$$\kappa_{w^*,n^*} = (1 - f_w)(1 - f_n) \cdot \kappa_{w_1,n_1} + f_w(1 - f_n) \cdot \kappa_{w_2,n_1} + (1 - f_w) \cdot f_n \cdot \kappa_{w_1,n_2} + f_w \cdot f_n \cdot \kappa_{w_2,n_2} \qquad [19.12]$$

For example, suppose a κ-multiplier is needed for a 1-of-3 interwell prediction limit test for individual values using an annual evaluation schedule. Assume the monitoring network consists of c = 5 constituents monitored at w = 28 compliance wells, using n = 17 upgradient background measurements on which to base the prediction limit. From **Table 19-2** in **Appendix D**, the closest table entries, $\kappa_{,w,n}$ to the desired combination are $\kappa_{20,16}$ = 1.59, $\kappa_{30,16}$ = 1.70, $\kappa_{20,20}$ = 1.52, and $\kappa_{30,20}$ = 1.62. The interpolated value, $\kappa_{25,18}$, can then be found using the equations in [19.12]:

$$f_w = \frac{(28-20)}{(30-20)} = .8 \qquad f_n = \frac{(17-16)}{(20-16)} = .25$$

$$\kappa_{25,18} = (1-.8)(1-.25)\cdot\kappa_{20,16} + .8(1-.25)\cdot\kappa_{30,16} + (1-.8)\cdot .25\cdot\kappa_{20,20} + .8\cdot .25\cdot\kappa_{30,20}$$

$$= .15\cdot 1.59 + .60\cdot 1.70 + .05\cdot 1.52 + .20\cdot 1.62 \quad = 1.659$$

Important considerations in designing a reasonable retesting scheme for detection monitoring are discussed in **Chapter 6**. Given a background sample and a particular network configuration and size, parametric 1-of-m plans tend to increase in statistical power as the order of m increases. All of the schemes have greater power with larger background sample sizes (n). Furthermore, plans involving prediction limits for future means tend to be more powerful than similar plans using prediction limits for individual observations. So if the κ-multiplier for a particular plan is not **bolded** or *italicized*, another plan can be sought to achieve sufficient effective power using more resamples or perhaps changing to a mean prediction limit. Alternatively, the background sample size might need to be augmented if feasible, prior to implementing the retesting procedure.

19.3.1 TESTING INDIVIDUAL FUTURE VALUES

The advantages to using a prediction limit for future individual values include: 1) the ability to explicitly control the SWFPR across a series of well-constituent pairs; and 2) greater flexibility than that provided by prediction limits for future means (**Section 19.3.2**) to handle temporal autocorrelation. In those cases when the sampling frequency needs to be reduced to maximize statistical independence of the observations, the method can be applied to evaluations of a single new measurement (plus possible resamples) at each compliance point well.

To properly implement a prediction limit strategy for future values with retesting, it needs to be feasible to collect 2 to 4 independent measurements at each compliance well during a given evaluation period. All initial and any resamples are assumed to be statistically independent and thus should exhibit no autocorrelation.

If statistical evaluations are done annually, it may be possible to collect data on a quarterly basis and meet the minimal sampling requirements of any of the resampling schemes discussed in the Unified Guidance. However, more frequent evaluations (say semi-annual or quarterly) will require that new samples be collected perhaps monthly or every six weeks. In these cases, explicit tests for autocorrelation may need to be conducted before adopting a 1-of-m retesting scheme with m > 2 or a

modified California plan. If significant autocorrelation is identified, the sampling frequency may need to be reduced and/or an alternate strategy utilizing fewer resamples may need to be adopted instead.

PROCEDURE

Step 1. Identify the overall targeted annual false positive rate (SWFPR = α = 0.10). Determine the number of wells (w) to be monitored and the number of constituents (c) to be sampled at each well. Also determine whether the evaluation schedule at the unit or facility is *annual, semi-annual or quarterly*.

Step 2. Decide on the number of observations (m) to be predicted. To incorporate retesting, a maximum of two independent measurements should be collected from every compliance well during each evaluation period to use a 1-of-2 retesting scheme, three independent measurements if a 1-of-3 plan is desired, and four independent measurements if either a 1-of-4 plan or a modified California plan is employed.

Step 3. For interwell prediction limits given a background sample of n measurements, compute the background sample mean ($\bar{x}$) and standard deviation (s) for each constituent. Then, based on the evaluation schedule (annual, semi-annual or quarterly), c, n, w, and the specific retesting scheme chosen, use **Tables 19-1** to **19-4** in **Appendix D** to determine a κ-multiplier possessing acceptable statistical power. Interpolate within the tables to find the closest multiplier if an exact value is not available.

For intrawell prediction limits, designate n early measurements as intrawell background for each well-constituent pair; compute the intrawell background mean ($\bar{x}$) and standard deviation (s) for each case. Given the evaluation schedule, c, n, w, and the chosen retesting scheme, use **Tables 19-10** to **19-13** in **Appendix D** to determine an acceptably powerful κ-multiplier. Note: if the intrawell background sample size varies by well, a series of κ-multipliers should be computed, one for each distinct n.

For each κ-multiplier, calculate the upper prediction limit with (1– α) confidence as:

$$PL_{1-\alpha} = \bar{x} + \kappa s \qquad [19.13]$$

If data were transformed prior to constructing the prediction interval, *back-transform* the prediction limit *before* making comparisons against the compliance point data. Unlike a prediction limit for future means, the formula for predicting m future values *does not involve any transformation bias* if the comparison is made in the original measurement domain.

Step 4. Collect an initial measurement from each well-constituent pair being tested. Compare each value against either 1) the upper prediction limit based on upgradient background in the interwell case or 2) the intrawell prediction limit specific to that well-constituent pair. Depending on the retesting scheme chosen, if any initial compliance point concentration exceeds the limit, collect 1 to 3 additional resamples at that well. If feasible, analyze only for those constituents which exhibited initial exceedances. Compare these values sequentially against the upper prediction limit. If the test 'passes' prior to collection of all the scheduled resamples, the remaining resamples do not need to be gathered or compared against PL.

Step 5. Decide that the test at a given well passes (*i.e.*, the well is in-compliance) if any one or more of the resamples does not exceed *PL* when using a 1-of-*m* scheme or when at least 2 resamples do not exceed PL when using the modified California scheme. Identify the well as failing when either (1) *all* resamples using a 1-of-*m* plan also exceed the prediction limit, or (2) at least two of three resamples using a modified California plan exceed *PL*.

►EXAMPLE 19-1

A large hazardous waste facility with 50 compliance wells is to monitor 10 naturally-occurring inorganic parameters in addition to 30 non-naturally occurring volatile organic compounds that have never been detected on-site. Groundwater evaluations are performed on a semi-annual basis. If the regulating authority will allow up to two resamples per exceedence of the background concentration limit, construct an interwell prediction limit with adequate statistical power and false positive rate control on the following pooled set (n = 25) of background sulfate measurements.

BG Well	Sampling Date	Sulfate (mg/l)	Log (Sulfate) log(mg/l)
GW-01	07-08-99	63	4.143
	09-12-99	51	3.932
	10-16-99	60	4.094
	11-02-99	86	4.454
GW-04	07-09-99	104	4.644
	09-14-99	102	4.625
	10-12-99	84	4.431
	11-15-99	72	4.277
GW-08	10-12-97	31	3.434
	11-16-97	84	4.431
	01-28-98	65	4.174
	04-20-99	41	3.714
	06-04-02	51.8	3.947
	09-16-02	57.5	4.052
	12-02-02	66.8	4.202
	03-24-03	87.1	4.467
GW-09	10-16-97	59	4.078
	01-28-98	85	4.443
	04-12-98	75	4.317
	07-12-98	99	4.595
	01-30-00	75.8	4.328
	04-24-00	82.5	4.413
	10-24-00	85.5	4.449
	12-01-02	188	5.236
	03-24-03	150	5.011

SOLUTION

Step 1. Assume for purposes of the example that there are no significant spatial differences among the well locations, either upgradient or downgradient. A check of normality of the pooled background sulfate measurements indicates that the interwell prediction limit should be constructed on the logged sulfate measurements rather than the raw concentrations.

Step 2. Groundwater evaluations must be conducted semi-annually (S). By excluding never-detected organic chemicals from the SWFPR calculation, the number of constituents that are to be considered is c = 10 at each of w = 50 wells.

Step 3. Since a maximum of two resamples will be allowed during any given evaluation period, neither the 1-of-4 nor the modified California retesting plan are an option. Consequently, only a 1-of-2 or 1-of-3 retesting strategy is appropriate. With n = 25 background measurements, **Tables 19-1** and **19-2** in **Appendix D** should be examined for a semi-annual evaluation schedule to determine κ-multipliers with adequate power. The multiplier of κ = 2.75 for a 1-of-2 plan has 'acceptable' power compared to the semi-annual ERPC, but the multiplier of κ = 2.00 for a 1-of-3 plan has 'good' power. Use the latter value to construct the interwell prediction limit.

Step 4. The sample log-mean and log-standard deviation of the sulfate background measurements are $\bar{y} = 4.32$ and s_y = 0.376, respectively. Use these values and the κ-multiplier to compute the prediction limit on the log-scale as

$$PL = \bar{y} + \kappa s_y = 4.32 + 2.00 \times 0.376 = 5.072$$

Then exponentiate the limit to back-transform it to the original measurement domain, for a final sulfate prediction limit of $PL = e^{5.072}$ = 159.5 mg/l.

Step 5. Compare the final prediction limit against one new sulfate measurement from each of the 50 compliance point wells. For any exceedence, compare the first of two resamples to the prediction limit. If the limit is still exceeded, test the second resample. If all three measurements (initial plus two resamples) are above the prediction limit at any specific well, declare that a statistically significant exceedence for sulfate has been identified. If, however, neither of the resamples exceeds the limit, judge the evidence to be insufficient to declare the well to be out-of-compliance. ◄

►EXAMPLE 19-2

Due to significant natural spatial variability, an intrawell testing scheme needs to be adopted at a solid waste landfill that monitors for 5 inorganic constituents at each of 10 compliance wells. If only one year's worth of quarterly sampling data is available at each well, but no recent contamination is suspected, develop an appropriate modified California intrawell retesting plan for the following chloride measurements. Assume that one statistical evaluation must be conducted each year.

Well ID	Chloride (mg/l)	Well Mean ± SD (mg/l)	Well ID	Chloride (mg/l)	Well Mean ± SD (mg/l)
GW-09	22	28.5 ± 10.021	GW-16	31	43.6 ± 13.392
	18.4			34.6	
	39.9			60.1	
	33.7			48.7	
GW-12	78	68.7 ± 7.208	GW-24	23.4	33.98 ± 9.083
	70			36.4	
	61			31.1	
	65.8			45	
GW-13	75.1	65.75 ± 8.128	GW-25	33.5	31.38 ± 6.533
	65.6			30.2	
	67			23.1	
	55.3			38.7	
GW-14	59.2	51.28 ± 8.427	GW-26	79.8	60.92 ± 14.447
	57.1			61.3	
	41.1			57.8	
	47.7			44.8	
GW-15	35	50.72 ± 15.672	GW-28	37.7	38.0 ± 8.273
	56.8			26.6	
	69.8			45.7	
	41.3			42	

SOLUTION

Step 1. With c = 5 constituents, w = 10 wells, one annual evaluation, and an intrawell background size for each well of only n = 4, **Table 19-13** in **Appendix D** can be examined to locate a possible κ-multiplier, leading to an interpolated κ = 4.33. Although this multiplier will adequately control the annual SWFPR to 10% or less, it yields low power for identifying contamination. As an alternative, try computing a pooled standard deviation across the compliance wells for chloride.

Step 2. Side-by-side box plots (**Section 11.1**) of the chloride values exhibit no obvious differences in spread or variation. The F-statistic for Levene's test (**Section 11.2**) is also non-significant (F = 1.0673) at the α = 5% level, suggesting that the variances are not unequal and that a pooled standard deviation can be appropriately formed.

Step 3. Conduct a one-way ANOVA on all chloride measurements from the 10 compliance wells, using Wells as the main factor (**Section 13.2.2**). The ANOVA table is presented below.

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	*F*-Statistic
Between Wells	7585.25	9	842.81	7.55
Error (within wells)	3350.37	30	111.68	
Total	10935.62	39		

Step 4. Compute the square root of the Error Mean Squares (also called the *root mean squared error* or RMSE) component in the ANOVA table to derive an estimate of the pooled intrawell standard deviation of s_p = 10.568. This estimate of the average intrawell variation has 30 degrees of freedom [*df*], computed by multiplying (4–1) = 3 degrees of freedom per well times the number of wells, or $df = 3 \times 10 = 30$.

Step 5. The **Appendix D** tables are not used to derive κ-multipliers when a pooled standard deviation estimate is used for intrawell prediction limits. **R** script listed in **Appendix C** is used (see **Section 13.3**). For a modified California retesting strategy with n = 4 and *df* = 30, the κ-multiplier becomes κ = 1.98.[7] This value not only controls the SWFPR but also has good statistical power. So use this multiplier along with the pooled intrawell standard deviation to compute an intrawell prediction limit for each compliance well. As an example, since the mean for chloride at well GW-09 is 28.5, the intrawell prediction limit would be:

$$PL = 28.5 + 1.98 \times 10.568 = 49.4 \text{ mg/l}$$

Prediction limits for the other compliance wells would be computed similarly. ◄

19.3.2 TESTING FUTURE MEANS

BACKGROUND AND REQUIREMENTS

The background, requirements, and assumptions for a prediction limit on future means of order *p* are essentially identical to those for prediction limits for future values (**Section 19.3**). For a comparable level of sampling effort, predicting a future mean offers *increased effective power* compared to a strategy that uses prediction limits for individual future values. To properly implement a prediction limit strategy for future means with retesting, *it must be feasible to collect 2 to 6 independent measurements at each compliance well during a given evaluation period*. All initial and resample measurements are assumed to be statistically independent.

To include explicit retesting, it should be feasible to collect either 2*p* or 3*p* independent measurements per well during each evaluation. The initial *p* observations are used to form the initial mean, while the remaining values are used to form either one or two resample means. If statistical evaluations are done annually, it may be possible to collect quarterly data and meet the minimal sampling requirements for *p* = 2 and a 1-of-2 retesting scheme. For more frequent semi-annual or quarterly evaluations, a larger order *p* or a retesting scheme entailing two resample means will require that new samples be collected perhaps monthly or every six weeks. An explicit test for autocorrelation should be made before adopting the strategy presented here. If significant autocorrelation exists, the frequency of sampling may need to be reduced and alternate prediction limit strategies considered such as a 1-of-1 plan for a future mean (see **Section 19.1**) or individual future values (**Section 19.3.1**).

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[7] The EPA Region 8 approximation equation described in **Chapter 13, Section 13.3** provides a κ-multiple estimate of 1.99 for individual wells at *n* = 4. The annual κ-factor for *w* = 10 and *c* =5 and *n* = 31 in **Table 19-13** of **Appendix D** is interpolated as κ = 1.508. Using the appropriate *A*, *b* & *c* coefficients from Chapter 13, Note 2 for the modified California plan, results are quite close to that generated from **R**-script.

An important difference between testing means versus individual values is that in some cases it may not be necessary to implement a retest at all. As noted above, for the same degree of sampling effort, a prediction limit for a mean of two or more observations can provide greater effective power than a prediction limit for the same number of individual values, ***even if a resampled mean is not collected***. In other words, when a 1-of-2 retesting plan for individual observations is compared to a 1-of-1 plan for means of order 2, the 1-of-1 mean-based scheme generally has greater power for identifying real concentration increases if background samples sizes are $n > 10$ (compare κ-multiple power ratings at higher n, c, and w in **Tables 19-1** and **19-5 *of* Appendix D**) A similar comparison holds between a 1-of-3 retesting plan for individual observations and a 1-of-1 plan for a mean of order 3 (**Table 19-2** versus **Table 19-8** in **Appendix D**).

Even more powerful prediction limits for future means are possible when explicit retesting is added to the procedure. However, the minimum sampling increases substantially. With a 1-of-2 retesting plan for means of order 2, as many as four independent groundwater measurements needs to be collected and analyzed per evaluation period. With a 1-of-3 plan for means of order 2 or a 1-of-2 plan for means of order 3, the sampling increases to as many as six independent observations per period. The latter plans may only be feasible for a single annual evaluation.

A problem common to all future mean prediction limits arises if the data have to be normalized via a transformation. In this case, all comparisons need to be made on the transformed data in order to avoid a transformation bias. As a consequence, the procedure is not a direct test of the background and compliance point ***arithmetic*** means. The test is still valid as a measure of significant mean differences in the transformed domain (***e.g.***, a test of geometric mean differences for logarithmic data). To the extent that the populations being compared share a common variance in the transformed domain, it may also indicate that a significant difference on the transformed scale also corresponds to a significant difference in the arithmetic means of the original populations.

A final potential drawback is that although a 1-of-m plan for future observations and a 1-of-1 plan for means of order $p = m$ seem to require the same total sampling effort, a prediction limit for observations can actually entail ***less*** sampling. For a future mean test of order $p = m$, m individual measurements will always need to be collected and analyzed. With a prediction limit for individual observations, the first sample is analyzed and compared to the limit. If it passes (***i.e.***, does not exceed the limit) there is no need to test the second or subsequent observations. Any subsequent resample that passes, also indicates that no further resample comparisons are needed for that test.

Under typical conditions at a site where most or all tested well-constituent pairs are likely to be at background conditions, there is a substantial savings in the number of samples for future observations versus means of the same size. It can also be noted that the same principle is true for a 1-of-2 test of a mean of order 2. Under background conditions, the two initial mean samples may be all that is required. When groundwater is contaminated, both the 1-of-m retesting plan for observations and the 1-of-1 plan for a mean of order $p = m$ require exactly the same amount of sampling and analysis to identify a significant exceedance.

PROCEDURE

Step 1. Identify the number of wells (w) to be monitored and the number of constituents (c) to be sampled at each well. Also identify the evaluation schedule as annual (A), semi-annual (S), or quarterly (Q).

Step 2. Decide on the order (p) of the future mean to be predicted. To incorporate retesting, it needs to be possible to collect $2p$ independent samples during each evaluation period to use a 1-of-2 retesting scheme, or $3p$ independent samples if a 1-of-3 plan is desired.

Step 3. If an *interwell* prediction limit is needed, use the common sample of n (upgradient) background measurements to compute the background sample mean ($\bar{x}$) and standard deviation (s). Given the n background measurements, w, c, p, and the evaluation schedule (annual, semi-annual or quarterly), use **Tables 19-5 to 19-9** in **Appendix D** to determine a κ-multiplier possessing acceptable statistical power. Calculate the upper prediction limit on background as:

$$PL = \bar{x} + \kappa s \qquad [19.14]$$

If *intrawell* prediction limits are needed, designate n early measurements at each compliance well as intrawell background. Compute the background sample mean ($\bar{x}$) and standard deviation (s) for each well. Then, based on n, w, c, p, and the number of evaluations per year, use **Tables 19-14** to **19-18** in **Appendix D** to determine an adequately powerful κ-multiplier. Compute an intrawell prediction limit for each compliance well using equation [19.14]. Note: if the intrawell background sample sizes vary by well, a series of κ-multipliers will need to be identified in these **Appendix D** tables, one for each distinct n.

If the background data were transformed prior to constructing the prediction limit, *also transform any compliance point data before making comparisons against the prediction limit*. In particular, compute the comparison mean of order p using the *transformed values*, rather than transforming the sample mean of the raw concentrations.

Step 6. Collect p initial measurements from each compliance well. Compute the mean of order p for each well, *first transforming the data if necessary using the same function applied to background*. Then compare each mean against the upper prediction limit. If retesting is desired, for any compliance point mean that exceeds the limit, collect either p or $2p$ additional resamples at that well, depending on the retesting scheme chosen. Form either one or two resample means of order p from these data; compare these means sequentially to the upper prediction limit.

Step 7. Identify the well as potentially contaminated when either 1) the initial mean of order p exceeds the limit in a 1-of-1 plan, or 2) the initial mean and *all* resample means using a 1-of-2 or 1-of-3 plan also exceed the prediction limit. Deem the well to be in-compliance if either 1) the initial mean does not exceed the prediction limit, or 2) *any* of the resample means do not exceed the limit.

►EXAMPLE 19-3

Suppose a large facility with minimal natural spatial variation is to monitor for 20 separate naturally-occurring inorganic chemicals along with a number of other never detected organic constituents. If 100 compliance wells are to be tested every six months and 25 background sample measurements are available, which resampling plans can control the SWFPR, providing acceptable statistical power? Assume that the data for each inorganic compound can be normalized and that the temporal autocorrelation between successive samples at the same well is minimal, *provided* that no more than four samples are collected during any semi-annual period.

SOLUTION

Step 1. The frequency of statistical evaluations is semi-annual (S). Excluding never-detected compounds from the SWFPR calculation leaves c = 20 constituents that need to be explicitly tested at each of w = 100 wells. For each of these constituents, since the data can be normalized, assume that an interwell prediction limit can be constructed using n = 25 background measurements.

Step 2. Determine κ-multipliers and power ratings for seven possible prediction limit retesting plans excluding the 1-of-3 mean order 2 and the 1-of-2 mean order 3 tests. Use the sub-tables identified as "*20 COCs, Semi-Annual*" for n =25 and w = 100 in interwell **Tables 19-1** through **10-9** in **Appendix D**, to obtain the following:

Prediction Limit Plan	κ-Multiplier	Power	Total Samples
1-of-2, observations	3.13	Low	2
1-of-3, observations	2.31	Good	3
1-of-4, observations	1.81	Good	4
Mod. California, observations	2.54	Good	4
1-of-1, mean order 2	3.56	Acceptable	2
1-of-2, mean order 2	2.29	Good	4
1-of-1, mean order 3	2.95	Good	3

Step 3. Compare the various plans in terms of statistical power and typical sampling effort. The only plan with low power is the 1-of-2 scheme for observations. The 1-of-1 mean order 2 has acceptable power. The other plans all have good power (*i.e.*, ones consistently meeting or bettering the ERPC for mean-level increases above background of 3 or more standard deviations), but potentially require either 2 or 3 resamples.

Restricting attention to those with good power, the least potential sampling effort is required by the 1-of-1 plan for a mean of order 3 or a 1-of-3 plan for observations. These two plans would requires *less total* sampling than the 1-of-4 plan for observations, the 1-of-2 mean order 2 plan and the *same or less* sampling than the modified California plan for observations in identifying a contaminant release.

If groundwater is not contaminated, the 1-of-m plans for observations require a minimum of 1 measurement to demonstrate that the well is in-bounds (*i.e.*, when the initial measurement does not exceed the background limit) as does the modified California plan. The 1-of-2 plan for a mean of order 2 requires a minimum of 2 measurements, and the 1-of-1 plan for a mean of order 3 requires a minimum of 3 measurements. On balance, the 1-of-3 plan for individual observations or the 1-of-2 plan for a mean of order 2 may provide the best compromise

between minimizing sampling effort and offering a higher probability of identifying contaminated groundwater. ◄

►EXAMPLE 19-4

Use the chloride data of **Example 19-2** to compute and contrast prediction limits for a future mean of order 2, with and without explicit retesting. Assume as before that 10 wells are monitored for 5 inorganic constituents, and evaluated on an annual basis.

SOLUTION

Step 1. The chloride data in **Example 19-2** showed significant spatial variability, suggesting the use of intrawell prediction limits. Furthermore, a one-way ANOVA evaluation of the w = 10 compliance wells indicated that a pooled standard deviation estimate of s_p = 10.568 with 30 degrees of freedom could be used to build intrawell prediction limits, instead of using individual variance estimates from each compliance well.

Step 2. With c = 5 constituents, w = 10 wells to be monitored, one annual evaluation (A), and a pooled degrees of freedom of df = 30, the **R** script in **Appendix C** can be repeatedly run to determine κ-multipliers for each retesting scheme for prediction limits on means of order 2. Since the sample size for each of the 10 wells is the same n = 4, the following multiples were generated from the **R**-script for the 1-of-1 to 1-of-3 tests of mean order 2: κ = 2.68, 1.88 and 1.51, respectively.[8] The prediction limits can then be constructed using equation [19.15], as shown for the first five compliance wells in the table below.

$$PL = \bar{x} + \kappa s_p \qquad [19.15]$$

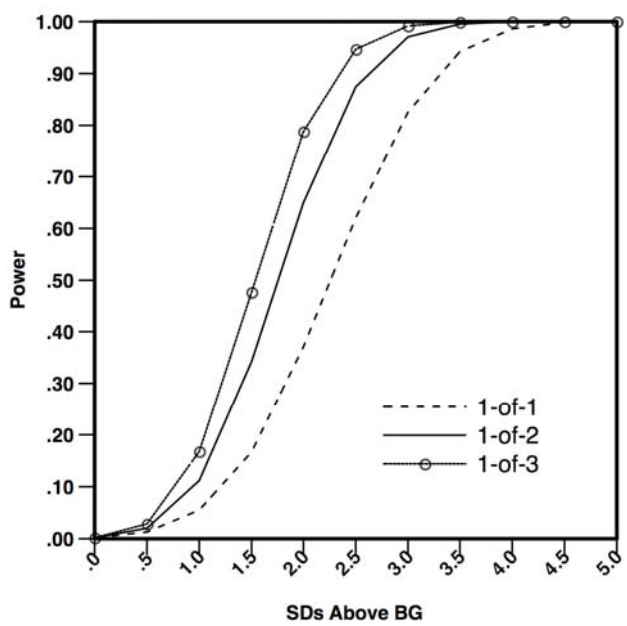
Step 3. While the power of each retesting plan is rated 'good' compared to the annual-evaluation ERPC, the prediction limits are obviously higher when less (or no) explicit retesting is conducted. Depending on conditions at the site, the range of approximately 13 mg/l of chloride in the well-specific prediction limits may or may not be important in deciding which strategy to use. The 1-of-1 plan for a mean of order 2 requires fewer total samples than the other plans. In some situations, the higher initial limits may be outweighed by the savings in sampling cost.

On the other hand, the ERPC provides a minimal standard for assessing statistical power. There can be a range of power curves even among plans all rated as 'good' seen in **Figure 19-1** below, where the full effective power curves for these three strategies are presented. Clearly, the 1-of-2 and 1-of-3 plans for means of order 2 have visibly higher power than the 1-of-1 retesting scheme. If site conditions permit, it may be beneficial to incorporate the 1-of-2 plan as a reasonable compromise between the gain in statistical power versus the increase in sampling requirements (for contaminated wells). ◄

[8] Using the Region 8 approximation equation in **Chapter 13**, the corresponding κ-multiples were 2.69, 1.89 and 1.52, respectively, based on tabular values at n = 31 of 2.258, 1.364 & .946 and using the appropriate A, b & c coefficients for each test. Results are very comparable to the R-script values.

Well ID	Retesting Plan	κ Multiplier	Power Rating	Well Mean (mg/l)	Prediction Limit
GW-09	1-of-1	2.68	Good	28.50	56.82
	1-of-2	1.88	Good	28.50	48.37
	1-of-3	1.51	Good	28.50	44.46
GW-12	1-of-1	2.68	Good	68.70	97.02
	1-of-2	1.88	Good	68.70	88.57
	1-of-3	1.51	Good	68.70	84.66
GW-13	1-of-1	2.68	Good	65.75	94.07
	1-of-2	1.88	Good	65.75	85.62
	1-of-3	1.51	Good	65.75	81.71
GW-14	1-of-1	2.68	Good	51.28	79.60
	1-of-2	1.88	Good	51.28	71.15
	1-of-3	1.51	Good	51.28	67.24
GW-15	1-of-1	2.68	Good	50.72	79.04
	1-of-2	1.88	Good	50.72	70.59
	1-of-3	1.51	Good	50.72	66.68

Figure 19-1. Comparison of Power Curves for 1-of-m Plans for Mean of Order 2



19.4 NON-PARAMETRIC PREDICTION LIMITS WITH RETESTING

BACKGROUND AND PURPOSE

When parametric prediction limits are not appropriate, either due to a large fraction of non-detects or data that cannot be normalized, retesting can be conducted using *non-parametric* prediction limits. The Unified Guidance discusses retesting schemes for both individual future values and for future medians (in parallel to the parametric options discussed in **Section 19.3**). Tests on individual observations include the three 1-of-*m* plans and modified California plan approaches. Tests on future medians include the 1-of-1 and 1-of-2 plans for medians of order 3. The basic strategy is to establish a non-parametric prediction limit for each monitoring constituent based on background measurements so that it accounts for the number of well-constituent tests in the overall network. Instead of determining a κ-multiplier, a non-parametric limit is computed as an *order statistic* from the background sample. The term order statistic refers to one of the values in a sorted (or *ordered*) data set.

In order to maintain adequate statistical power while minimizing the overall false positive rate, retesting will almost always be needed as part of the detection monitoring system design. As in the parametric case, a specific number of additional, *independent* resamples will potentially need to be collected for each compliance well test. The initial and subsequent resamples are then compared against the non-parametric prediction limit.

The largest or second-largest value in background is often selected as a non-parametric limit, representing the nth or (n–1)th order statistics. With higher level 1-of-m tests of observations, an even lower order statistic may be more appropriate in achieving an optimal balance between the desired SWFPR and adequate statistical power. This can be particularly true if the background sample size is large, but depends on the overall network design requirements. Although the Unified Guidance provides tables of non-parametric limits only for the largest and second-largest order statistics, EPA Region 8 has released software written in Visual Basic® labeled the *Optimal Rank Values Calculator* that computes the optimal choice of order statistic for 1-of-m retesting plans for m = 1 to 4. The program also provides approximate statistical power estimates based on user inputs of a target cumulative false positive rate, background sample size, and number of simultaneous tests to be conducted. The software and explanatory narrative will be provided on the EPA website.[9]

REQUIREMENTS AND ASSUMPTIONS

When more independent data are added to the testing procedure, retesting with non-parametric prediction limits leads to more powerful and more accurate assessments of possible contamination. As with parametric retesting schemes, a balance must be struck between 1) quick identification and confirmation of contaminated groundwater and 2) statistical independence of successive resamples. All retesting strategies depend on the assumption of statistical independence between successive resamples. This trade-off is typically resolved by allowing enough time between resamples to allow both the well to

[9] The calculator, an accompanying narrative, fact sheet and this guidance will be located on the EPA website: http://www.epa.gov/hazard/correctiveaction/resources/guidance/sitechar/gwstats/index.htm. If the calculator cannot be accessed, contact Mike Gansecki for assistance (e:mail: gansecki.mike@epa.gov; or phone: 303- 312-6150.)

recharge and additional groundwater to flow past the well screen, and by limiting the number of possible resamples to 2 or 3.

Non-parametric retesting schemes offer somewhat less flexibility than their parametric counterparts. As with other non-parametric statistical intervals, the same SWFPR control afforded by a parametric interval based on a small n cannot usually be attained in a non-parametric interval; larger sample sizes are almost always necessary. κ-multipliers for parametric prediction limits are continuous statistical parameters that can be adjusted to match a desired false positive rate for even the smallest sample sizes. By contrast, the bounds of non-parametric intervals are restricted to values in the observed background sample. For a given sample size and number of tests to be run, any order statistic selected from background as the non-parametric prediction limit results in a ***discrete probability*** of false positive error. Altering the prediction limit by selecting a different order statistic changes the false positive rate only in discrete probability steps, providing a less efficient means of controlling the SWFPR.

The non-parametric prediction limit tests provided in the Unified Guidance do not require the underlying distribution to be normal. One potentially attractive application is for background data sets containing higher percentages of non-detects which cannot be normalized. For some constituent data sets, it may be possible to pool data from several upgradient and historical compliance wells to generate much larger total background sizes. A non-parametric Kruskal-Wallis test of medians can establish that these data are appropriate for pooling.

Since larger background sample sizes are needed because no distributional model is posited, the non-parametric testing schemes are ***most applicable to interwell*** comparisons. Small ***intrawell*** background sample sizes make it difficult for any of the non-parametric test options to be applied which can meet the SWFPR cumulative false positive design objective. Unlike parametric intrawell tests, effective sample sizes cannot be expanded by estimating a common pooled standard deviation across a number of wells. This conclusion is generally true no matter what order statistic is used to estimate the non-parametric prediction limit. But there are other considerations which might allow intrawell testing using non-parametric alternatives. For a given sample size, target false positive, a fixed maximum and number of total tests, the higher 1-of-m tests of future observations will have lower achievable false positive errors, with the 1-of-4 test the lowest. If the background sample size is increased through periodic additions, this false positive will continue to drop. The power of these tests using the maximum with small sample sizes is almost always greater than the EPA reference levels. A temporary strategy might be to utilize the highest order 1-of-m test for intrawell purposes until larger sample sizes are available. However, the target cumulative false positive rate may not initially be met. With larger sample sizes, it may also be possible to decrease the m of the test and still achieve the target false positive rate.

Even interwell comparisons between upgradient and downgradient wells are acceptable only if the degree of spatial variability is insignificant. Fortunately, spatial variability may be less of a problem in those cases where a non-parametric retesting scheme might be implemented, ***i.e.***, when the detection rate of the chemical being monitored is fairly low. High constituent non-detect rates tend to result in more uniform spatial distribution across site wells, allowing for similar median concentrations.

APPENDIX TABLES FOR NON-PARAMETRIC PREDICTION LIMITS

To design appropriate non-parametric prediction limits with retesting, the Unified Guidance provides separate tables for predicting individual future values versus future medians. Four distinct retesting schemes are presented in the case of prediction limits for individual values: 1-of-2, 1-of-3, 1-of-4, and modified California plan schemes. Two distinct schemes are presented for the case of future medians: 1-of-1 and 1-of-2 for medians of order 3.

Unlike the tables for parametric prediction limits discussed in **Section 19.3**, non-parametric prediction limits do not involve κ-multipliers. Instead, the entries in **Tables 19-19** to **19-24** of **Appendix D** consist of *per-constituent significance levels*. These levels represent the *achievable* false positive rate (α_{const}) associated with each tested constituent for a given retesting scheme, choice of non-parametric prediction limit, and network configuration (*i.e.*, number of wells [w] and background sample size [n]).[10] The non-parametric prediction limit can be estimated via any order statistic from the background sample. However, the most practical limits are usually either the maximum observed background value or the second-highest value. Consequently, the Unified Guidance provides tables for these two options.

Each table for the six specific non-parametric tests contains two sub-tables. One uses a limit based on the background maximum and the other the second-highest background value. All the tables are otherwise similarly structured. Within each table and sub-tables, per-constituent significance levels are given for all combinations of background sample size (n = 4 to 200) and number of wells (w = 1 to 200). These significance levels can be used to meet a target annual SWFPR of 10%, discussed in **Chapter 6**.

Correct use of these tables involves a few important considerations. First, if an *interwell* prediction limit is desired, the *target per-constituent* false positive rate (α_{const}) needs to be computed. Any prediction limit strategy selected should have a table entry no greater than α_{const} in order to ensure that the annual SWFPR is no greater than 10%. To compute this target rate, use the formula:

$$\alpha_{\text{const}} = 1 - \left(1 - \alpha\right)^{1/c} \qquad [19.16]$$

where c equals the number of monitoring constituents and α is the SWFPR = 0.10.

Unlike the tables for parametric prediction limits, separate tables are not provided for each of the three most common evaluation schedules (*i.e.*, annual, semi-annual, and quarterly). The number of 'wells' in each non-parametric table must be regarded as the actual number of compliance wells (w) times the number of annual statistical evaluations (n_E = 1, 2, or 4). For using these tables, let $w^* = w \times n_E$. This adjustment is necessary because on each evaluation, w wells should be compared against a prediction limit computed from a common interwell background. A site with w^* wells tested annually is statistically equivalent to a site having w distinct well locations tested n_E times per year ($w \times n_E$ tests).

[10] Per-constituent rates instead of network-wide false positive rates are given in these tables and those of Davis and McNichols (1994; 1999) for computational reasons. Although the mathematical algorithm is exact, it is difficult to compute with accuracy for a large number of tests (r). Hence the decomposition of r into constituents (c) times wells (w). By calculating the per-constituent false positive rate, only the number of wells (w) need be varied.

Once w^* is computed in this way, the table entry corresponding to w^* and n represents the *achievable* annual false positive rate per constituent. As noted, this rate should not exceed the target rate (α_{const}) in order to meet the overall SWFPR. If α_{const} is exceeded for a given choice of retesting scheme and choice of non-parametric prediction limit, a different limit or scheme should be considered. In general, selecting a 1-of-*m* retesting scheme with larger *m* will lead to a lower achieved false positive rate. Also, per-constituent significance levels for the modified California approach are generally larger than those for the 1-of-*m* plans.

If *intrawell* prediction limits are needed, a somewhat different method needs to be employed to correctly use the per-constituent significance levels in **Tables 19-19** through **19-24** of **Appendix D**. In this case, a target *per well-constituent pair* false positive rate ($\alpha_{w \cdot c}$) needs to be first computed using the equation:

$$\alpha_{w \cdot c} = 1 - (1-\alpha)^{1/(w \cdot c)} \qquad [19.17]$$

where α is the SWFPR, w equals the actual number of compliance wells and c is the number of monitoring constituents. Then the placeholder w^* for the non-parametric tables is to be equated with the number of annual statistical evaluations ($w^* = n_E$ = 1, 2, or 4). w^* represents the number of times per year that the common intrawell background at any given well-constituent pair will be compared against new compliance measurements from that well. The table entry corresponding to w^* and the intrawell background sample size n may be regarded as the achievable false positive rate per well-constituent pair. This rate should not exceed the target rate, $\alpha_{w\text{-}c}$, if the overall SWFPR is to be met.

The same approach presented in **Section 19.3** is used if a mixture of test methods is needed (*e.g.*, parametric prediction limits for some constituents, and non-parametric limits for other constituents). By construction, the target SWFPR is evenly proportioned across the list of monitored constituents. As long as the significance level per constituent (interwell case) or per well-constituent pair (intrawell case) is computed using <u>all</u> c constituents and not just those for which a non-parametric prediction limit test will be applied, the SWFPR will not exceed α = 0.10 on an annual basis.

Tables 19-19 through **19-24** in **Appendix D** provide the same **bold**, *italicized* or plain text used to identify 'good', 'acceptable' and 'low' power ratings following the ERPC 3 and 4 standard deviation reference criteria as in the parametric prediction limit tables.

As final technical notes about these tables, the significance levels listed as table entries are presented using a short-hand notation in order to compactly present a wide range of false positive rates. In this notation, the first four non-zero digits of the significance level are given, followed if necessary, by the symbol –*d*. The value *d* represents the number of leading zeros to the right of the decimal point. This is equivalent to taking the non-zero portion of the entry and multiplying it by 10^{-d} to get the actual significance level. As an example, if the entry is .4251–4, the equivalent significance level is .00004251. Entries without the –*d* symbol are the actual fractional significance levels where no adjustment is needed.

For network configurations (number of wells [w] and background sample size [n]) not listed in **Tables 19-19** through **19-24** in **Appendix D**, bilinear interpolation can be used to approximate the significance level associated with the desired configuration. As discussed in **Section 19.3**, interpolation

should be restricted to the closest four adjacent table entries. The shorthand significance level notations in the tables should first be converted to actual fractions before interpolating.

19.4.1 TESTING INDIVIDUAL FUTURE VALUES

BACKGROUND AND REQUIREMENTS

The Unified Guidance recommends two variations of non-parametric prediction limits for use in groundwater detection monitoring. The first is the prediction limit for individual future values, introduced in **Section 18.3.1**. The other is the prediction limit for future medians, detailed in **Section 18.3.2**. Basic requirements for non-parametric prediction limits are outlined in those sections.

The main advantage to a prediction limit for future values is its overall flexibility and ease of implementation. Fewer data from each compliance well are needed to implement the test compared to a prediction limit for a future median. Only an initial observation from each compliance point may be needed to identify a well-constituent pair 'in-bounds'; initial exceedances can be followed by up to a maximum of three additional individual resamples. Once the non-parametric upper prediction limit has been selected from background as a large order statistic (often the maximum or second-largest value), each compliance point measurement is compared directly against this upper limit.

The user should decide which retesting scheme to use and how many resamples per well are feasible, given that the measurements from any well during a given evaluation period need to be statistically independent. **Tables 19-19** through **19-22** in **Appendix D** can be employed to compare the *achievable false positive rates* of different schemes and to determine whether they exhibit adequate effective power. The user can also explore EPA Region VIII's *Optimal Rank Values Calculator* software to consider order statistics other than the maximum or second-largest.

PROCEDURE

Step 1. For an *interwell* test, use the number of monitoring constituents (c) in equation [19.16] to determine the target per-constituent false positive rate (α_{const}). Also multiply the number of yearly statistical evaluations (n_E) by the actual number of compliance wells (w) to determine the look-up table entry, w^*. Then depending on the background sample size n and w, choose a type of non-parametric prediction limit (*i.e.*, maximum or 2nd highest value in background) and a retesting scheme for individual observations using **Tables 19-19** through **19-22** in **Appendix D**. The final plan should have an achieved significance level no greater than α_{const} and also should be labeled with 'acceptable' or 'good' power in the **Appendix** tables.

Step 2. For an *intrawell* test, use the number of constituents (c) and the actual number of compliance wells (w) in equation [19.17] to compute the target significance level per well-constituent pair ($\alpha_{w\text{-}c}$). Set w^* in the look-up table equal to the number of yearly evaluations, n_E. Based on $w^* = n_E$ and the intrawell background sample size n, choose a non-parametric prediction limit and retesting scheme so that the achieved well-constituent pair significance level (*i.e.*, the selected table entry) does not exceed the target significance level, $\alpha_{w\text{-}c}$, and also is labeled with 'acceptable' or 'good' statistical power.

Step 3. Sort the background data into ascending order and set the upper prediction limit equal to an appropriate order statistic of the data (*e.g.*, the maximum or the second-largest observed value). If all constituent measurements in a background sample are non-detect, use the Double Quantification rule in **Chapter 6**. The constituent should not be included in calculations for identifying the target false positive.

Step 4. Collect one initial measurement per compliance well. Then compare each initial measurement against the upper prediction limit. Depending on the retesting scheme chosen, for any compliance point value that exceeds the limit, collect one to three additional resamples from that well. Again compare the resamples against the upper prediction limit.

Step 5. Identify any well with an initial exceedance as potentially contaminated when either (1) *all* resamples using a 1-of-2, 1-of-3, or 1-of-4 plan also exceed the prediction limit, or (2) at least two resamples exceed the limit using a modified California retesting scheme. Conversely, declare a well to have 'passed' the test if either 1) the initial measurement does not exceed the prediction limit, 2) *any* resamples from a 1-of-*m* scheme do not exceed the limit, or 3) at least 2 of 3 resamples from a modified California approach do not exceed the limit.

19.4.2 TESTING FUTURE MEDIANS

BACKGROUND AND REQUIREMENTS

Prediction limits for a future median based on either a single or with one repeat (1of-1 or 1-of-2 tests) are two non-parametric procedures recommended as retesting methods in the Unified Guidance. Compared to a prediction limit for future individual values, the prediction of a median (**Chapter 18**) often requires more data to be collected from each compliance well particularly if resampling is included. Slightly greater statistical manipulation is also needed once the data are in hand. For the 1-of-1 test, the initial median to be predicted requires at least two initial observations from each compliance point, and any resample medians will require additional sets of up to three measurements, all of which needs to be statistically independent.

Given equal amounts of data and the same input conditions, a prediction limit for a future median tends to be more statistically powerful than a prediction limit for individual values. This is true whether one uses a fixed order statistic or selects across a range of order statistics to form the prediction limit. Because of this and the fact that both spatial variability and autocorrelation may be less of a problem (or at least less easily assessed) when the detection rate is low and a non-parametric strategy is needed, the Unified Guidance includes **Appendix D** tables for both a 1-of-1 scheme and a 1-of-2 scheme to predict medians of order 3. The 1-of-2 median test will have a lower achievable false positive rate than the 1-of-1 version, with all other conditions equal.

Depending on the number of annual evaluations and the test configuration, care needs to be taken that potentially needed samples are far enough apart in time. The series of observations from any well is assumed to be uncorrelated. If autocorrelation is a problem, a prediction limit for future values (**Section 19.4.1**) should be considered in which the per-well sampling requirements with explicit retesting are more modest.

PROCEDURE

Step 1. For an *interwell* test, use the number of monitoring constituents (c) in equation [19.16] to determine the target per-constituent false positive rate (α_{const}). Also multiply the number of yearly statistical evaluations (n_E) by the actual number of compliance wells (w) to determine the look-up table margin value, w^*. Then, depending on the background sample size n and w^*, choose a type of non-parametric prediction limit (*i.e.*, maximum or 2nd highest value in background) and a retesting scheme for future medians using **Tables 19-23** to **19-24** in **Appendix D**. The final plan should have an achieved significance level no greater than α_{const}, and also should be labeled with 'acceptable' or 'good' power in the **Appendix** tables.

Step 2. For an *intrawell* test, use the number of constituents (c) and the actual number of compliance wells (w) in equation [19.17] to compute the target significance level per well-constituent pair ($\alpha_{w\text{-}c}$). Set w^* in the look-up table margin equal to the number of yearly evaluations, n_E. Based on $w^* = n_E$ and the intrawell background sample size (n), choose a non-parametric prediction limit and retesting scheme for future medians so that the achieved well-constituent pair significance level (*i.e.*, the selected table entry) does not exceed the target significance level, $\alpha_{w\text{-}c}$, and also is labeled with 'acceptable' or 'good' statistical power.

Step 3. Sort background into ascending order and set the upper prediction limit equal to a large background order statistic (*e.g.*, the maximum or second largest value). If all constituent measurements in a background sample are non-detect, use the Double Quantification rule in **Chapter 6**. The constituent should not be included in calculations identifying the target false positive rate.

Step 4. Collect two initial measurements per compliance well. If both do not exceed the upper prediction limit, the test passes since the median of order 3 will also not exceed the limit. There is no need to collect the third initial observation or any resamples. If both exceed the prediction limit, the median will also exceed the limit. There is no need to collect the third initial measurement. If using a 1-of-1 plan, move to Step 5. Otherwise, collect up to three resamples in order to assess the resample median.

If one initial measurement is above and one below the limit, collect a third observation to determine the position of the median relative to the prediction limit. In all cases, if two or more of the compliance point observations are non-detect, set the median equal to the quantification level (QL).

Step 5. Compare the median value for each compliance well against the upper prediction limit. If a 1-of-2 retesting scheme is selected and any compliance point median exceeds the limit, collect up to three additional resamples from that well. Compute the resample median and compare this value to the upper prediction limit.

Identify a compliance well as potentially contaminated when either the initial median exceeds the upper prediction limit for a 1-of-1 plan, or both the initial median and the resample median exceed the prediction limit in a 1-of-2 plan. Conversely, declare a well to have passed the test if the initial median does not exceed the prediction limit, or the resample median in a 1-of-2 scheme does not exceed it.

►EXAMPLE 19-5

The following trace mercury data have been collected in the past year from a site with four background wells and 10 compliance wells (two of which are shown below). The facility must monitor for five constituents, including mercury. Assuming that the percentage of non-detects in background is too high to make a parametric analysis appropriate or feasible, compare interwell non-parametric prediction limits for both observations and medians at the annual statistical evaluation, and determine whether either compliance well indicates significant evidence of mercury contamination. Further assume that the sequentially reported compliance well data below are obtained as needed for the different test comparisons.

	Mercury Concentrations (ppb)					
Event	BG-1	BG-2	BG-3	BG-4	CW-1	CW-2
1	.21	<.2	<.2	<.2	.22	.36
2	<.2	<.2	.23	.25	.20	.41
3	<.2	<.2	<.2	.28	<.2	.28
4	<.2	.21	.23	<.2	.25	.45
5	<.2	<.2	.24	<.2	.24	.43
6					<.2	.54

SOLUTION

Step 1. Using a target SWFPR of 10%, compute the target per-constituent false positive rate, noting that the monitoring list consists of five parameters. This implies that $\alpha_{const} = 1-(1-.1)^{1/5} = .021$ using equation [19.16]. Since the detection rate in background is only 35%, it is reasonable to consider non-parametric prediction limits with retesting. The background sample size of n = 20 is to be used to construct an interwell prediction limit for all w = 10 compliance wells. Since there is only one annual evaluation (n_E = 1), the look-up table margin value of w^* equals $w \times n_E$ = 10.

Step 2. Determine potentially applicable retesting plans. First consider non-parametric prediction limits for individual observations with n = 20 and w = 10. Consulting **Tables 19-19** through **19-22** in **Appendix D**, only the 1-of-3, 1-of-4, and modified California plans meet (*i.e.*, do not exceed) the target false positive rate of 2.1%. To use the 1-of-3 or modified California plans, the prediction limit needs to be set to the maximum background measurement. In the 1-of-4 plan, the prediction limit can be set to either the maximum or second-highest value in background using the **Appendix D** tables. A final 1-of-4 plan determined with the *Optimal Rank Values Calculator* allows the use of the 3rd highest value. All of these plans boast good power compared to the annual ERPC. Both the 1-of-4 and modified California schemes may require as many as 3 separate and independent resamples in addition to the initial observation.

Consider tests for future medians of order 3 in **Tables 19-23** and **19-24** in **Appendix D**. Only the 1-of-2 plan using the maximum background value as the prediction limit meets the α_{const} target. It also has good power, but requires 3 initial measurements and up to 3 additional individual resamples.

Step 3. Sort the combined background data and compute the possible prediction limits as $PL_{(n)}$ = .28 ppb, $PL_{(n-1)}$ = .25 ppb, and $PL_{(n-2)}$ = .24 ppb, respectively representing the maximum, second-largest, and third-largest background values.

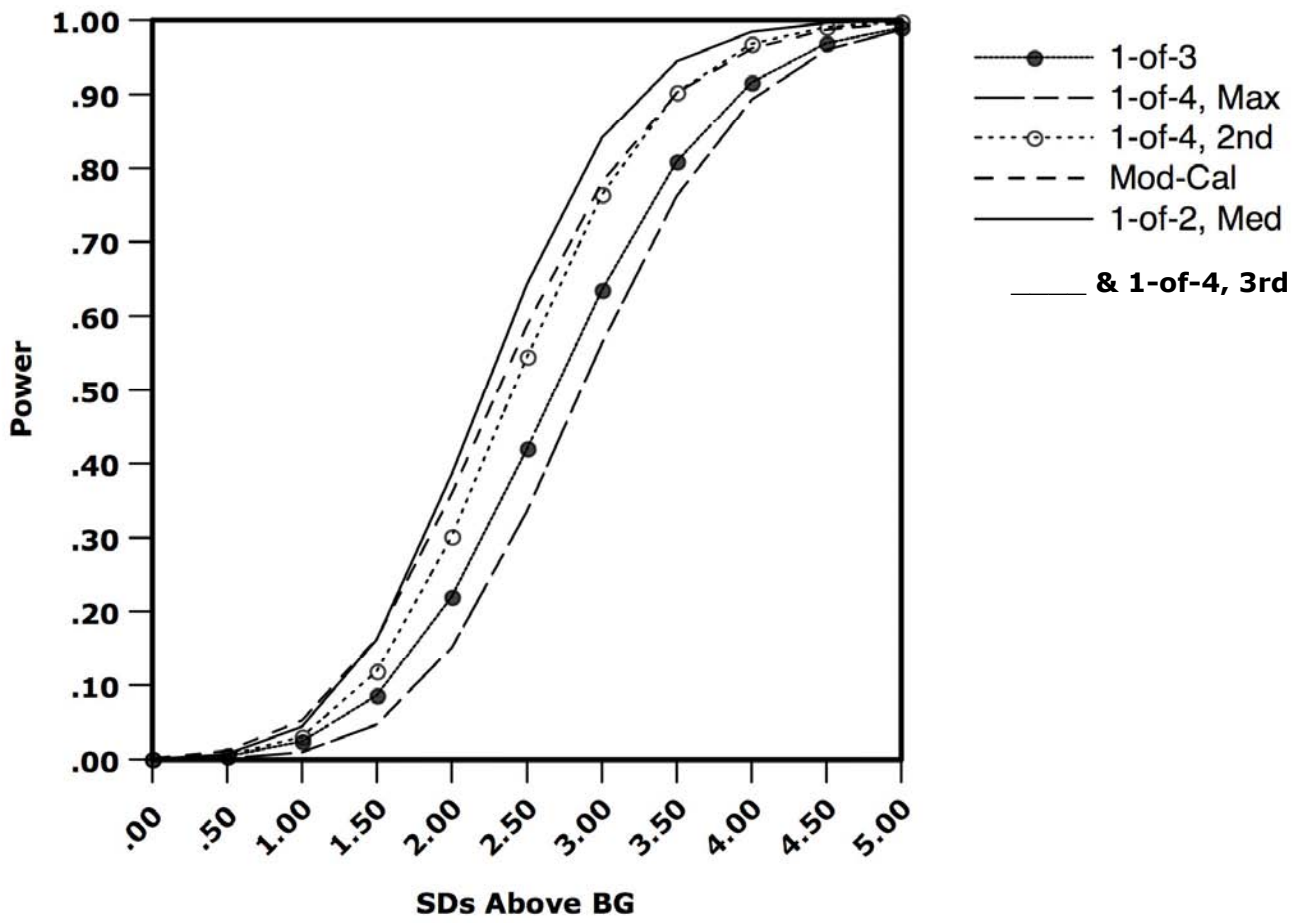
Step 4. Determine the test outcomes at each compliance well using the various retesting plans, as shown in the table below. For the prediction limits on individual observations, the first sample collected during Event 1 is used as the initial screen to determine if any resampling is necessary. The first 3 measurements at each compliance well are used to form the initial comparison. The median at CW-1 is .20 ppb, while that at CW-2 is .41 ppb.

Compliance Well	Retesting Plan	Achieved α	# Initial Samples	Resamples Required	BG Limit	Result
CW-1	1-of-3	.0055	1	0	.28	Pass
	1-of-4, Max	.0009	1	0	.28	Pass
	1-of-4, 2nd	.0046	1	0	.25	Pass
	1-of-4, 3rd	.0135	1	0	.24	Pass
	Mod-Cal	.0140	1	0	.28	Pass
	1-of-2, Med	.0060	3	0	.28	Pass
CW-2	1-of-3	.0055	1	2	.28	Pass
	1-of-4, Max	.0009	1	2	.28	Pass
	1-of-4, 2nd	.0046	1	3	.25	Fail
	1-of-4, 3rd	.0135	1	3	.24	Fail
	Mod-Cal	.0140	1	3	.28	Fail
	1-of-2, Med	.0060	3	3	.28	Fail

All of the acceptable plans indicate that CW-1 is not statistically different from background, although more initial sampling is required for the 1-of-2 retesting plan with medians. For CW-2, the results are more problematic. The 1-of-3 and 1-of-4 plans based on the maximum background value allow the well to pass, while the other four plans indicate a significant difference from background. The least degree of sampling is required by the 1-of-3 plan; at some facilities, greater sampling efforts may not be feasible. When a well is likely to be contaminated, the number of samples required to actually make a decision about the well is similar across the plans with the exception of the 1-of-2 prediction limit on a median.

A further consideration is that although the power of each plan exceeds the annual ERPC when additional resampling is possible, it is helpful to compare the full power curves of multiple plans to determine whether a particular plan offers greater power than the rest. **Figure 19-2** displays an overlay of the six power curves associated with the retesting plans in this example. For these inputs, the 1-of-2 retesting plan for a median of order 3 using the background maximum and the 1-of-4 plan on individual observations using the 3rd highest background value achieve the best overall power (shown as a single curve on **Figure 19-2**).

Figure 19-2. Comparison of Full Power Curves



As seen in **Figure 19-2**, the two plans that pass the second compliance well have visibly lower power — especially in the range of 2 to 3.5 standard deviations above background — than the four plans that failed CW-2. In such a situation, the user needs to carefully balance the risks and benefits of each acceptable resampling plan. In some cases, the cost of greater amounts of resampling may be outweighed by the added sensitivity of the test to evidence of groundwater contamination. ◄

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CHAPTER 20. MULTIPLE COMPARISONS USING CONTROL CHARTS

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This chapter describes control charts, a second recommended core strategy for detection monitoring. Control charts are a useful and powerful alternative to prediction limits. The Unified Guidance is the first EPA document to discuss retesting and simultaneous testing of multiple wells and/or constituents as they relate to control charts. Research of these topics is still ongoing.

20.1 INTRODUCTION TO CONTROL CHARTS

Control charts are a viable alternative to parametric prediction limits for testing groundwater in detection monitoring. They are similar to prediction limits for future observations in that a control chart limit is estimated from background and then compared to a sequence of compliance point measurements. If any of these values exceeds the control limit, there is initial evidence that the compliance point concentrations exceed background.

Control charts can be constructed as either interwell or intrawell tests. The main difference is how background is defined and what measurements are utilized to build the control limit. Interwell control charts establish the control limit from designated upgradient and potentially other background wells. Intrawell control charts, on the other hand, employ historical measurements from a compliance point well as background. Intrawell tests can only be appropriately applied if the historical compliance well background is uncontaminated.

An advantage of control charts over prediction limits is that a control chart graphs the compliance data over time. Certain varieties can also evaluate gradual increases above background over the period of monitoring. Trends and changes in concentration levels can be easily seen since the sample observations are consecutively plotted on the chart. This provides the analyst an historical overview of the pattern of measurement levels. Prediction limits are typically constructed to allow only ***point-in-time comparisons*** between the most recent compliance data and background, making long-term trends more difficult to identify.[1]

[1] Long-term results from repeated application of a prediction limit can be plotted over time, creating a graph similar in nature to a control chart. But this has been infrequently done in practice.

As a well-established statistical methodology, there are many kinds of control charts. Historically, control charts have been put to great use in quality engineering and manufacturing, but have more recently been adapted for use in groundwater monitoring. The specific control chart recommended in the Unified Guidance is known as a combined Shewhart-CUSUM control chart (Lucas, 1982). It is a 'combined' chart because it simultaneously utilizes two separate control chart evaluation procedures. The Shewhart portion is almost identical to a prediction limit in that compliance measurements are individually compared against a background limit. The cumulative sum [CUSUM] portion sequentially analyzes each new measurement with prior compliance data. Both portions are used to assess the similarity of compliance data to background in detection monitoring.

The Shewhart-CUSUM control chart works as follows. Appropriate background data are first collected from the specific compliance well for intrawell comparisons or from separate background wells for interwell tests. The baseline parameters for the chart, estimates of the mean and standard deviation, are obtained from these background data. These baseline measurements characterize the expected background concentrations at compliance wells.

As future compliance observations are collected, the baseline parameters are used to standardize the newly gathered data. After these measurements are standardized and plotted, a control chart is declared *out-of-control* if future concentrations exceed the baseline control limit. This is indicated on the control chart when either the Shewhart or CUSUM plot traces begins to exceed a control limit. The limit is based on the rationale that if the well remains uncontaminated as it was during the baseline period, new standardized observations should not deviate substantially from the baseline mean. If a release occurs, the standardized values will deviate significantly from baseline and tend to exceed the control limit. The historical baseline parameters then no longer accurately represent current well concentration levels.

Combined Shewhart-CUSUM control charts initially featured two control limits, one for testing the Shewhart portion of the chart, one for testing the CUSUM portion of the chart. Later research on control charts (Davis, 1999; Gibbons, 1999) indicated that having separate control limits for the Shewhart and CUSUM procedures is generally not important. Both control chart traces can instead be compared to a *single* control limit. This modification not only makes the control chart method slightly easier to apply, but also aids in measuring the statistical performance of control charts over a variety of monitoring networks.

20.2 BASIC PROCEDURE

The basic procedure for constructing a control chart is presented below. Requirements and assumptions for control charts are discussed in later sections:

Step 1. Given n background measurements (x_{jB}), estimate the baseline parameters by computing the sample mean ($\bar{x}_B$) and standard deviation (s_B).

Step 2. For a compliance point measurement (x_i) collected on sampling event T_i, compute the standardized concentration Z_i:

$$Z_i = \left(x_i - \bar{x}_B\right) / s_B \qquad [20.1]$$

Step 3. For each sampling event T_i, use the standardized concentrations from **Step 2** to compute the standardized CUSUM S_i. Set $S_0 = 0$ when computing the first CUSUM S_1.

$$S_i = \max\left[0,\ \left(Z_i - k\right) + S_{i-1}\right] \qquad [20.2]$$

The notation max[A, B] in equation [20.2] refers to picking the maximum of quantities A and B. Furthermore, the parameter k designates half the ***displacement*** or shift in standard deviations that should be quickly detected on a control chart. Often k is set equal to 1, meaning that the control chart will be designed to rapidly detect upward concentration shifts of at least two standard deviations. Since Z_i is standardized by the estimated baseline standard deviation, an increase of r units in Z_i corresponds to an increase of r standard deviations above the baseline mean in the domain of concentrations x_i.

Step 4. To plot the control chart in concentration units, compute the ***non-standardized*** CUSUMs S_i^c with the equation:

$$S_i^c = \bar{x}_B + S_i \cdot s_B \qquad [20.3]$$

Step 5. Calculate the non-standardized control limit used to assess compliance of both future measurements (x_i) and non-standardized CUSUMs (U_i). Traditionally, two parameters were used to compute standardized limits: the decision internal value (h) and the Shewhart Control Limit (SCL). The Unified Guidance instead recommends only one standardized control limit (h). Compute the non-standardized control limit (h_c) as:

$$h_c = \bar{x}_B + h \cdot s_B \qquad [20.4]$$

Step 6. Construct the control chart by plotting both the compliance measurements (x_i) and the non-standardized CUSUMs (S_i^c) on the y-axis against the sampling events T_i along the x-axis. Also draw a horizontal line at the concentration value equal to the control limit, h_c.

Step 7. Moving forward in time from the first plotted sampling event T_1, declare the control chart to be potentially out-of-control if either of two situations occurs: 1) the trace of non-standardized concentrations exceeds h_c; or 2) the CUSUMs become too large, exceeding h_c.

The first case signifies a rapid increase in concentration level among the most recent sample data. The second can represent either a sudden rise in concentration levels or a gradual increase over time. A gradual increase or trend is particularly indicated if the CUSUM exceeds the control limit but the compliance concentrations do not. The reason for this is that several consecutive, small, increases in x_i will not trigger the control limit, but may cause a large enough increase in the CUSUM. As such, a control chart can indicate the onset of either sudden or gradual contamination at the compliance point.

►EXAMPLE 20-1

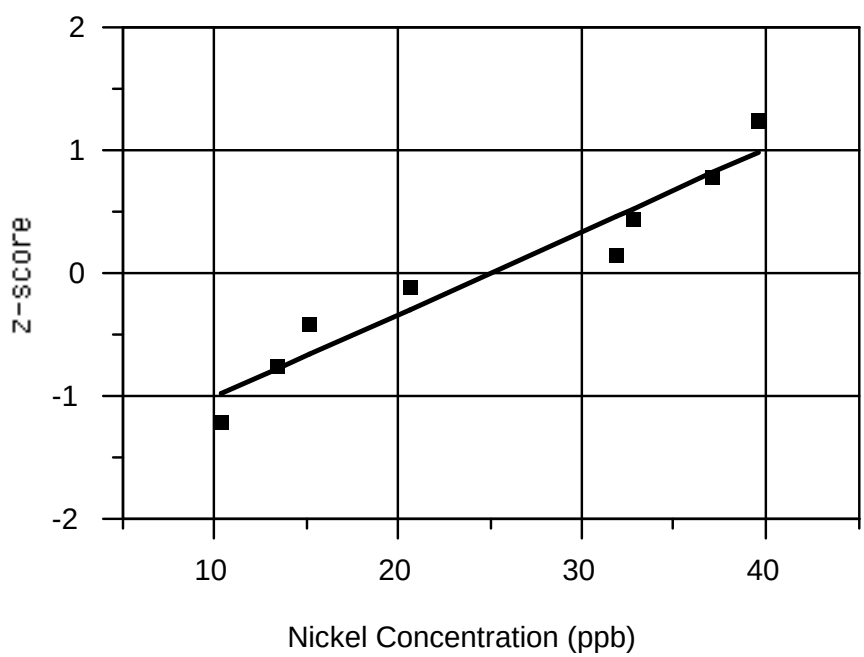
For background nickel data collected during 8 months in 1995 shown below, construct an intrawell control chart and compare it with the first 8 months of the compliance period (1996):

Month	Nickel Concentration (ppb)	
	Baseline Period (1995)	Compliance Period (1996)
1	32.8	19.0
2	15.2	34.5
3	13.5	17.8
4	39.6	23.6
5	37.1	34.8
6	10.4	28.8
7	31.9	23.7
8	20.6	81.8

SOLUTION

Step 1. As discussed in **Section 20.3.3**, control charts are a parametric procedure requiring normal or normalized data. Test the n = 8 baseline measurements for normality. A probability plot of these data provided in **Figure 20-1** exhibits a mostly linear trend. The Shapiro-Wilk test statistic computed for these data is W = 0.896. Compared to the α = .10 level critical point of $w_{.10,8}$ = 0.851 (**Table 10-3** of **Appendix D**), the Shapiro-Wilk test indicates that the baseline data are approximately normal. Construct the control chart using the original nickel measurements.

Figure 20-1. Probability Plot of Baseline Nickel Data



Step 2. Use the 1995 baseline nickel data to compute the sample mean and standard deviation: $\bar{x}_B$ = 25.14 ppb and s_B = 11.518 ppb. Then compute the standardized concentration Z_i for each 1996 compliance period sampling event using equation [20.1]. These values are listed in the fourth column of the table below.

Month	T_i	Nickel (ppb)	Z_i	$Z_i - k$	S_i	S_i^c
1	1	19.0	−0.53	−1.53	0.00	25.14
2	2	34.5	0.81	−0.19	0.00	25.14
3	3	17.8	−0.64	−1.64	0.00	25.14
4	4	23.6	−0.13	−1.13	0.00	25.14
5	5	34.8	0.84	−0.16	0.00	25.14
6	6	28.8	0.32	−0.68	0.00	25.14
7	7	43.7	1.61	0.61	0.61	32.16
8	8	81.8	4.92	3.92	4.53	77.31

Step 3. Compute the standardized CUSUMs as follows. First let the shift displacement parameter k = 1 and set $S_0 = 0$. After subtracting k from each Z_i, calculate the CUSUM using equation [20.2] . Note that none of the CUSUMs are positive until the first occurrence of a positive quantity ($Z_i - k$). As shown in the sixth column above, the standardized CUSUMs for the 6th, 7th and 8th events are calculated as:

$$S_6 = \max\left[0, (0.32 - 1) + 0\right] = 0$$
$$S_7 = \max\left[0, (1.61 - 1) + 0\right] = 0.61$$
$$S_8 = \max\left[0, (4.92 - 1) + 0.61\right] = 4.53$$

Step 4. Calculate the non-standardized CUSUMs (S_i^c) using the individual Z_i, baseline mean and standard deviation parameters in equation [20.3]. These values are listed in the last column of the table above. For the 8th sampling event, this calculation gives:

$$S_8^c = 25.14 + 11.518(4.53) = 77.31$$

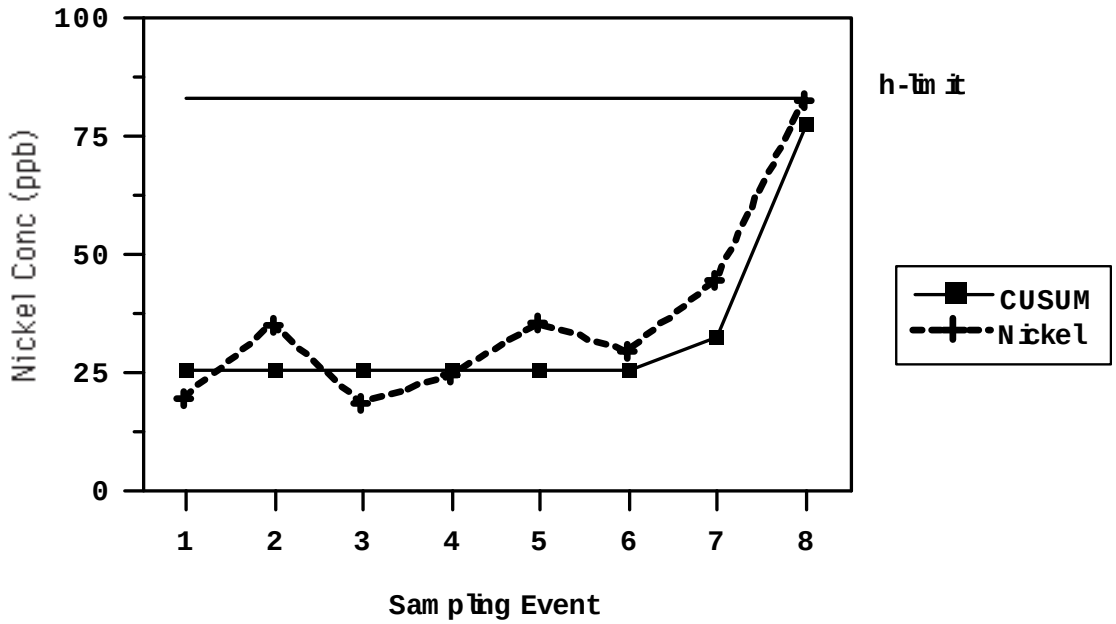
Step 5. Compute the non-standardized control limit using equation [20.4]. For purposes of this example, set h = 5; the non-standardized limit becomes:

$$h_c = 25.14 + 11.518(5) = 82.73 \text{ ppb}$$

Step 6. Using the compliance period nickel concentrations and the non-standardized CUSUMs, plot the control chart as in **Figure 20-2**. The combined chart indicates there is insufficient evidence of groundwater contamination in 1996 because neither the nickel concentrations nor the CUSUM statistics exceed the control limit for the months examined. However, both traces nearly exceed h_c, and conceivably might do so in future sampling events if the apparent trend continues. If that were to happen, retesting can be performed to better determine whether the

increase was one or a series of chance fluctuations or an actual mean-level change in nickel concentrations. ◄

Figure 20-2. Shewhart-CUSUM Control Chart for Nickel Measurements



20.3 CONTROL CHART REQUIREMENTS AND ASSUMPTIONS

As with other statistical methods, control charts are based on certain assumptions about the sample data. There are also some minimum requirements for constructing them. None of the assumptions or requirements are unique to control charts, although there are some special issues.

20.3.1 STATISTICAL INDEPENDENCE AND STATIONARITY

The methodology for control charts assumes that the sample data are statistically independent. A control chart can give misleading results if consecutive sample measurements are serially correlated (*i.e.*, autocorrelated). For this reason, it is important to design a sampling plan so that distinct volumes of groundwater are analyzed at each sampling event (**Section 14.3.1**). Duplicate laboratory analyses (*i.e.*, aliquot or field splits) should also not be treated as independent observations when constructing a control chart. Gibbons (1999) recommends that control chart observations be collected no more frequently than quarterly. Since physical independence does generally not guarantee statistical independence (**Section 14.1**), a test of autocorrelation using the sample autocorrelation function or rank von Neumann ratio tests (**Section 14.2**) should be performed to determine whether the current sampling interval affords uncorrelated measurements.

If the background data exhibit a clear seasonal cyclical pattern, the values should be deseasonalized before computing the control chart baseline parameters. For a seasonal pattern at a single well, the

method of **Section 14.3.3.1** can be used to create adjusted measurements having a stable mean. At several or a group of wells indicating a common seasonal pattern, the adjusted values can be computed using a one-way analysis of variance [ANOVA] for temporal effects (**Section 14.3.3.2**). When baseline data are deseasonalized, it is essential that newly collected compliance measurements also be deseasonalized in the same manner. It is presumed that the same pattern or physical cause will impact future data in the same manner as for the baseline measurements.

To deseasonalize compliance point measurements, simply use the seasonal and grand means estimated from background in computing the adjusted compliance point values. If the control chart remains in control following deseasonalizing, the existing background can be updated with the newer measurements. However, the revised background set should be checked again for seasonality and the seasonal and grand means re-computed, in order to more accurately adjust future measurements.

Control charts also assume that the background mean is *stationary over time*. This means there should be no apparent upward or downward trend in the background measurements. A trend imparts greater-than-expected variation to the background data, increasing the baseline standard deviation and ultimately the control limit. The net result is a control chart that has less power to identify groundwater contamination. Tests for trend described in **Chapter 17** can be used to check the assumption of no background trends. Should an upward or downward trend be verified, *the background data should not be de-trended*. While it is possible to construct and use a control chart with de-trended background and future data, the assumption that the trend will continue indefinitely is very problematic. The trend should first be investigated to ensure that background has been properly designated. Other monitoring wells should be checked to see if the same trend is occurring, indicating either evidence of an earlier release or possibly a sitewide change in the aquifer. In any case, a switch should be made to a trend test rather than a control chart.

As noted, control charts can be employed as either interwell or intrawell tests. However, interwell control charts require a spatially stationary mean across the monitoring network. If spatial variability exists among background wells for certain constituents, interwell control charts will be no more interpretable than prediction limits. A related problem can plague intrawell control charts if there is *prior* spatial variability (*i.e.*, some compliance wells are already contaminated prior to selection of intrawell backgrounds). *Historical observations should be used as baseline data in intrawell tests only if the compliance wells are known to be unaffected by a release from the monitored unit.* Otherwise, the control limit based on the greater-than-expected background values may be set too high to identify current contamination.

20.3.2 SAMPLE SIZES AND UPDATING BACKGROUND

Both background mean and standard deviation estimates are needed to construct a control chart limit. The Unified Guidance recommends at least n = 8 measurements for the defining the baseline, particularly to ensure an accurate standard deviation estimate. *Baseline* observations are traditionally not plotted on the chart, although it may be visually helpful to include background values on the plot using a distinct symbol (*e.g.*, hollow instead of filled symbol).

Whether baseline observations are obtained from upgradient background wells for interwell testing or from individual compliance well historical data for intrawell use, these data are only small random samples used to estimate the true background population characteristics. Any particular sample set may not be adequately representative. Because of this likelihood, the background sample size requirements suggested above for constructing a control chart should be regarded as a minimum. More background observations should preferably be added to the initial set to improve the characterization of the background distribution.

For interwell control charts, periodic updating of background (**Chapter 5**) poses no difficulty. New observations should be collected at background wells on each sampling event. Then, every 1-2 years, the newly collected background should be added to the existing background pool after testing/checking for statistical similarity. The revised background can be used to re-compute the baseline parameters and, in turn, the control limit.

Updating background for intrawell control charts depends on the control chart remaining 'in-control' for several consecutive sampling events. As long as a confirmed exceedance does not occur, the in-control compliance measurements collected since the last background update can be tested against the existing background for statistical similarity using a Student's *t*- or Wilcoxon rank-sum test (**Section 5.3**). ASTM Standard D6312-98 (1999) recommends testing the newly revised background set for trends, using trend tests including those in **Chapter 17**. The ASTM methodology is intended to avoid incorporating a subtle trend into the control chart background, which influences the re-computed baseline parameters and weakens the statistical power of the control chart to identify contaminant releases.

If the comparison of recent in-control measurements against existing background indicates a statistically significant difference, it may reflect changes in natural groundwater conditions unrelated to contamination events. In these circumstances, it is possible to update background by creating a 'moving window.' The background sample size n remains fixed, with only the most recent n measurements included as background for computing baseline parameters. Earlier sampling events are excluded. The overriding goal is to ensure that background reflects the most current and representative groundwater conditions (**Chapter 3**).

Despite the apparent benefits, the statistical performance of control charts is only partially known when background is periodically updated. Davis (1999) has performed the most extensive simulations of this question. He suggests that substantially different simulation results occur with the CUSUM portion when background is periodically updated (especially early on) and combined with either a small maximum *run length* or a 'warm-up' period or both (see **Section 20.4.1**).

Two other issues affect both control charts and prediction limits when updating intrawell background. First, if background is periodically augmented by adding new measurements (either from upgradient background wells or from recent in-control compliance measurements), the overall background sample size is increased. This in turn should cause the prediction or control chart limit to decrease.

For instance, prediction limit tables in **Chapter 19** demonstrate that as the background sample size increases, lower prediction limit κ–multipliers are appropriate. The expanded background sample is used to re-compute the prediction limit, provided that the measurements added to background do not indicate an adverse change in groundwater quality. New compliance measurements are then tested against the revised prediction limit. But the same cannot be done with control charts unless the CUSUM is *reset to zero*. The reason is that the CUSUM will have *already been affected* by those compliance measurements now being added to intrawell background. An independent comparison between compliance point values and background is thus precluded. Consequently, the Unified Guidance recommends that the CUSUM portion of the control chart be reset after each periodic update of intrawell background.[2]

The second issue is how to update intrawell background when an initial measurement has exceeded the control or prediction limit, but one or more resamples disconfirm the exceedance. Routine detection monitoring continues in this situation. No confirmed exceedance is registered for a prediction limit test and the control chart remains in-control. Should the initial exceedance be included or excluded when later updating intrawell background?

The Unified Guidance recommends a strategy parallel to the handling of outliers (**Chapter 12**). If the exceedance can be shown to be a measurement in error or a confirmed outlier, it should be excluded from the revised background. Otherwise, any disconfirmed exceedances (including any resamples that exceed the background limit but are disconfirmed by other resamples) should probably be included when updating the background. The reason is that background limits designed to incorporate retesting are computed as low as possible to ensure adequate statistical power. The trade-off is that compliance measurements legitimately similar to background but drawn from the upper tail of the distribution, sometimes exceed the limit and have to be disconfirmed with a resample. Any exceedance not documented as an error or outlier is most likely representative of some portion of the background population that previously had gone unsampled or unobserved.

20.3.3 NORMALITY AND NON-DETECT DATA

The combined Shewhart-CUSUM control chart is a parametric procedure. This implies that background used to estimate the baseline parameters should either be normal or normalized via a transformation. Normality can be tested on either the raw measurement or transformed scale using one of the goodness-of-fit techniques described in **Chapter 10**. If the hypothesis of normality is accepted,

[2] The same 'overlapping' dependence between the CUSUM and revised background will also be true when background is updated using a 'moving window' approach. The CUSUM should therefore be reset in these cases too. However, since the background sample size is kept fixed, the standardized control limit (h) will not decrease as it does when background is augmented.

construct the control chart on the raw measurements. If it is rejected, try a transformation and retest the transformed data for normality. If the transformation works to normalize background, construct the control chart on the transformed measurements, being sure to use the same transformation on both background and the compliance values to be plotted.

Unlike prediction limits, no non-parametric version of the combined Shewhart-CUSUM control chart exists. If the background sample cannot be normalized perhaps due to a large fraction of non-detects, a non-parametric prediction limit should be considered (**Section 19.4**). Control charts will be most appropriate for those constituents with a reasonably high detection frequency. These include many inorganic constituents (*e.g.*, certain trace elements, indicators and geochemical monitoring parameters) that occur naturally in groundwater, or for other persistently detected, site-specific organic chemicals.

If no more than 10-15% of the data are non-detect, it may be possible to normalize the data via simple substitution (**Section 15.2**) of half the reporting limit [RL] for each background non-detect. A normalizing transformation can sometimes be found using a *censored probability plot* (**Chapter 15**) for background data containing a substantial fraction of non-detects up to 50%. A censored estimation technique such as Kaplan-Meier or Robust Regression on Order Statistics [Robust ROS] (**Chapter 15**) can then be used to compute estimates of the baseline mean ($\hat{\mu}_B$) and standard deviation ($\hat{\sigma}_B$) that account for the left-censored measurements. These adjusted estimates should replace the background sample mean ($\bar{x}_B$) and standard deviation (s_B) in the control chart equations of **Section 20.2**. The Unified Guidance differs somewhat from the recommended approach in ASTM Standard D6312-98 (ASTM, 1999), which is to set all non-detects identically to zero.

No matter how background non-detects are treated, control charts require an additional step for future observations that isn't needed with prediction limits. Each new compliance point measurement statistic must be added to the CUSUM associated with previous sampling events. If the new observation is a non-detect, some value (typically a fraction of the RL) needs to be imputed for the censored measurement in order to update the CUSUM. The Unified Guidance recommends that half the RL be substituted for these measurements.[3]

[3] If an intrawell control chart is constructed and it remains 'in-control' until the next background update, any non-detects observed in the meantime should be treated as left-censored measurements for purposes of updating the baseline mean and standard deviation estimates. In other words, the simple substitution of RL/2 should only apply temporarily to compute an updated CUSUM.

20.4 CONTROL CHART PERFORMANCE CRITERIA

A significant difference exists between control charts and prediction limits in setting statistical performance criteria. Standard equations described in previous chapters allow the user to generate an exact confidence level (1–α) for prediction limits. Obtaining similar confidence levels for the Shewhart-CUSUM control charts needs to be done experimentally through varying the two background control chart limits (*h*) and the displacement parameter (*k*), as well as the retesting options. The control chart parameter limits in the two previous EPA RCRA statistical guidance documents were based on work by Lucas (1982), Hockman & Lucas (1987), and Starks (1988). Monte Carlo simulations for various combinations of control chart parameters (without retests) were used to develop the overall recommendations in their papers.

The specific parameter choices were not fixed, but appeared to work best in simulations at a single well. Starks (1988) recommended setting $h = 5$ and $k = 1$ for standardized measurements, especially in the early stages of monitoring. He further suggested that after 12 consecutive in-control measurements, the baseline mean and standard deviation be updated to include more recent sampling measurements. The values of k and SCL (the separate Shewhart control limit) could then be reduced to $k = 0.75$ and $SCL = 4.0$. In effect, this tightens the control chart limits to reflect that additional data are available to better characterize the baseline population.

More recent research (notably Gibbons, 1999) has demonstrated that control charts from the quality control literature do not account for several important characteristics of groundwater monitoring networks. The most important is the problem of multiple comparisons (*i.e.*, the need to simultaneously conduct testing of many well-constituent pairs during an evaluation period described in **Chapter 6**). Control chart performance is typically assessed on an individual well basis, rather than over a network of simultaneous tests. The recommended control limits have no obvious connection to the expected false positive rate (α), nor is the traditional control limit adjustable like the κ-factor in prediction limits. There is a need to account for differences in background sample sizes, a desired false positive rate, and the number of monitoring network tests in similar fashion to prediction limits. Moreover, early research and guidance did not address the issue of retesting in control charts. Retesting provides substantial improvements in prediction limit performance, and its potential needs to be evaluated for control charts.

It is standard practice to discuss the performance of prediction limits in terms of statistical power and false positive rates. However, statistical performance of control charts is usually measured via the *average run length* [ARL]. The ARL is the average number of sampling events before the control limit is first exceeded, identifying an 'out-of-control' process. Ideally, the ARL should be large when the mean concentration of the tested constituent is at or near the baseline average, but increasingly smaller as the true mean is gradually shifted above baseline.

Put in standard statistical terms, the control chart should not easily or quickly signal false evidence of a release when a release has *not* occurred. To have a low false positive rate when the null hypothesis of no contamination is true, the chart should stay 'in-control' for a long time indicated by a large ARL. The statistical power for detecting a release when it occurs should be as high as possible. A short ARL will indicate that a control chart is quickly determined to be out-of-control.

False positive rates (α) for CUSUM control charts cannot be equated precisely with ARLs. But it has been found that the ARLs closely follow a geometric distribution pattern with a mean equal to ($1/\alpha$). Thus, a control chart with an ARL of 100 would have an associated false positive rate of roughly 1%. The relationship is not exact, especially for combined Shewhart-CUSUM control charts. It is also affected by the randomness in the background data used to establish the control chart baseline.

Thus, the Unified Guidance offers a new framework for measuring control chart statistical performance. It is suggested that measuring false positive rates in control charts be conducted by establishing a time frame or run length of interest, specifically, a period of one year. A false positive is counted if the chart has a confirmed exceedance sometime during the year, under the assumption of no contaminant release. Statistical power is similarly evaluated for a fixed time interval (*e.g.*, one year) by measuring the proportion of run lengths with confirmed exceedances *during that interval*. In this way, both the false positive rate and power are tied to a specific one-year time frame.

This framework is consistent with the guidance recommendations that prediction limit performance be measured according to an annual, cumulative 10% site-wide false positive rate [SWFPR] and that cumulative, annual effective power be comparable to the EPA reference power curves [ERPC]. The suggested framework for control charts allows a direct comparison with prediction limits when designing alternate statistical approaches.

20.4.1 CONTROL CHARTS WITH MULTIPLE COMPARISONS

Until recently, control charts were not designed to address the SWFPR when testing multiple well-constituent pairs. Furthermore, it was not clear to a user how to adjust for multiple tests using fixed control limits (SCL, k and h). Because of these problems, Gibbons (1999) performed a series of Monte Carlo simulations to gauge intrawell control chart performance for up to 500 simultaneous tests. Gibbons also examined the outcomes when the single Shewhart and CUSUM decision limit was allowed to vary between h = {4.5, 5.0, 5.5, and 6.0}. He found that control charts could be designed with both high power and a low SWFPR, as long as retesting was incorporated into the methodology.

Additional Monte Carlo simulation work was performed by Davis (1999). He found that control charts perform similarly to prediction limits when both use retests. But he also noted that certain favorable outcomes in Gibbons (1999) were the result of combining frequent updating of background and a 'warm-up' period for the chart. In the latter period, any control limit exceedances were ignored. The simulations were based on small *maximum run lengths.*

Other researchers have noted (for instance, Luceño and Puig-Pey, 2000) that the run length distribution of CUSUM control charts is often close to geometric. This implies that even when the ARL is large, there can be significant probability of an early failure. The difficulty in a real-life setting is that one will not know whether an early exceedance of the control limit is due to contaminated groundwater or simply a false positive exceedance for an otherwise in-control chart. This guidance recommends against the use of 'warm-up' periods when implementing or assessing the performance of Shewhart-CUSUM control charts.

Gibbons (1999) provides results for a number of control chart limit options, but does not determine limits which can provide exact false positive rate control. A number of potential commonly applied retesting strategies are also not evaluated. In contrast, both Gibbons (1994) and the Unified Guidance (**Chapter 19**) do provide such control for prediction limits using a wider array of retesting strategies.

Facilities may need to conduct theirown specific Monte Carlo simulations if the published literature options cannot be applied at their site. Simulations might be needed for either intrawell or interwell control charts or both. Overall methodologies for Monte Carlo simulations are provided below. The first step for either type test is a simulation of the cumulative annual false positive rate. Then a second simulation measures the cumulative, annual statistical power.

To perform an *intrawell* simulation, repeat the following steps for a large number of simulations (*e.g.*, N_{sim} = 10,000):

1. Determine the total number of well-constituent pairs for which statistical testing is required, as well as the number of pairs at which intrawell control charts will be constructed. Use the basic subdivision principle (**Section 19.2.1**) to determine the per-test false positive rate (α_{test}) associated with each control chart that meets the target SWFPR.

2. Determine the intrawell background sample size (n). Generate n standard normal measurements. Then form baseline estimates by computing the sample mean ($\bar{x}_B$) and standard deviation (s_B).

3. Pick a set of possible standardized control limits (h). Choose a maximum run length (M), based on the number of sampling events conducted each year (*e.g.*, M = 4 for quarterly sampling).

4. For each potential control limit (h), compute the non-standardized control limit using equation [20.4]. Then simulate the behavior of the control chart from sampling event 1 to sampling event M by generating standard normal compliance measurements for each event. Generate enough random measurements to account for resamples potentially needed with a selected retesting strategy.

5. Test the initial measurement associated with each sampling event against the non-standardized control limit. Also form the CUSUM for events 1 to M using equations [20.2] and [20.3]. Compare the non-standardized CUSUM against the control limit.

6. If either the initial measurement or the CUSUM exceeds h_c, use the resample(s) for that sampling event to perform a retest (see below). If the retest confirms the initial exceedance, record a false positive for that particular simulation (out of N_{sim}).

7. After all N_{sim} runs have been conducted, compute the observed false positive rate (α_h) associated with each possible *standardized* control limit (h) by dividing N_{sim} into the number of observed false positives. Set the final control limit equal to that value of h for which α_h is closest to α_{test}.

The simulation for an *interwell* control chart is similar to the intrawell case, with a few key differences. First, instead of a per-test false positive rate, the basic subdivision principle must be used to compute a *per-constituent* false positive rate (α_{const}). The reason is that the same background measurements for a given constituent are used to test each of the compliance wells in the network.

Secondly, when generating standard normal compliance point measurements in **Step 4** of the intrawell simulation, a set of such random observations needs to be generated for ***each*** of the w wells in the network. The behavior of w control charts must be simulated using a common set of background data and single control limit for each one.

Once a control limit meeting the target SWFPR has been established, a second Monte Carlo simulation is run to determine the statistical power of the control chart. Since effective power is defined as the ability to flag a single contaminated well-constituent pair, the basic steps are the same for either interwell or intrawell control charts. Repeat the following over a large number of simulations (N_{sim}).

1. Determine the background sample size (n). Generate n standard normal measurements. From these, form baseline estimates by computing the sample mean ($\bar{x}_B$) and standard deviation (s_{B}).

2. Using the standardized control limit (h) chosen in the first Monte Carlo simulation, compute a non-standardized control limit using equation [20.4]. Then simulate the behavior of the control chart from sampling event 1 to sampling event M by generating sets of normal N(Δ,1) compliance measurements for each event, where Δ varies from 1 to 5 by unit steps. Generate enough random measurements in each set to account for resamples potentially needed with a selected retesting strategy.

3. For each set of successively higher-valued compliance measurements, test the initial measurement associated with each sampling event against the non-standardized control limit. Also form the CUSUM for events 1 to M using equations [20.2] and [20.3]. Compare the non-standardized CUSUM against the control limit.

4. If either the initial measurement or the CUSUM exceeds h_{c}, use the resample(s) for that sampling event to perform a retest (see below). If the retest confirms the initial exceedance, record a true detection for that particular mean-level Δ and simulation (out of N_{sim}).

5. After all N_{sim} runs have been conducted, compute the observed power (1–β) associated with each true mean level (Δ) by dividing N_{sim} into the number of observed detections. The simulated effective power curve for standardized control limit (h) is a plot of (1–β) versus Δ for Δ = 1 to 5.

If the standardized control limit identified during Monte Carlo simulation has effective power comparable to the appropriate ERPC (matching the site-specific sampling frequency to one of the three curves in **Chapter 6**: quarterly, semi-annual, or annual), h can be used to form site-specific control limits. For interwell limits, compute the (upgradient) background mean and standard deviation for each monitoring constituent and use equation [20.4] to form the final, non-standardized control limits. For intrawell limits, use the same equation only with intrawell background at each well-constituent pair.

20.4.2 RETESTING IN CONTROL CHARTS

Control chart and prediction limit tests are only practical for most monitoring networks if retesting is part of the procedure, demonstrated both by Gibbons (1999) and Davis (1999). A key issue is to decide how control chart retesting should be conducted. Practical retesting strategies for prediction

limits on future observations are described in **Section 19.1**, including both 1-of-*m* (for *m* = 2, 3, 4) and modified California plans.

ASTM Standard D6312-98 (1999) recommends a 1-of-2 retesting strategy: whenever an exceedance of the control limit occurs on a given sampling event, the next quarterly sampling event is used as the resample. Furthermore, if the exceedance is not confirmed by the resample, the ASTM standard recommends that the initial exceedance be *replaced* in the CUSUM by the follow-up sampling event, thus implicitly assuming that the initial observation was an error.

Gibbons (1999) considers the performance of other retesting plans, including 1-of-2, 1-of-3, and the original Cal-3 plan (see **Section 19.1** and **Appendix B**). For each plan, resampling is triggered when the most recent observation either by itself exceeds or causes the CUSUM to exceed the limit. Then, each resample (if more than one) is compared against *h*. The initial exceedance measurement is removed from the CUSUM computation, replaced by the resample, and then re-compared to the control limit. A statistically significant increase [SSI] is declared only if the resample verifies the initial exceedance (or both resamples for a 1-of-3 plan).

Gibbon's study and ASTM Standard D6312-98 raises an important concern as to the most statistically powerful treatment of the CUSUM when an initial exceedance is *not* confirmed by retesting. A second concern addresses when resamples should be collected.

The Unified Guidance suggests two practical possibilities to address the first concern. The initial exceedance can be removed from the CUSUM altogether, re-setting the CUSUM to its value from the previous sampling event. As noted above, this is essentially assuming the first sampling event was in error. Another option is to replace the initial exceedance by the first resample which disconfirms the exceedance, and then re-compute the CUSUM with that resample.

In either strategy, the effects on statistical power and accuracy should be simulated when constructing site-specific control limits as in the procedure outlined above. Both the false positive rate and power depend on a faithful simulation of *all* aspects of the control chart testing procedure. This includes background sample size, the number of well-constituent pairs evaluated, the retesting strategy and how the CUSUM is adjusted for resampling.

The second issue concerns when resamples should be collected. The Unified Guidance does not recommend using the next scheduled sampling event as a resample. If the exceedance were due to a laboratory analytical error or calculation mistake, a more quickly retrieved resample can resolve the discrepancy without waiting until the next quarterly or semi-annual monitoring event.

Where multiple resamples are used (a 1-of-3 plan, for instance), one would have to wait two additional sampling rounds simply to collect the resamples. These in turn could not be plotted on the control chart as regular sampling events without intermingling the roles of resamples and non-resamples, thereby complicating the interpretation and assessment of control chart performance. The common guidance recommendation is to identify an intermediate period or periods for resampling between regularly scheduled evaluations for both control charts and prediction limits.

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PART IV: COMPLIANCE/ASSESSMENT AND CORRECTIVE ACTION TESTS

This last part of the Unified Guidance addresses statistical methods useful in compliance/assessment and corrective action monitoring, where single-sample testing is required against a fixed groundwater protection standard [GWPS]. These standards include not only health- or risk-based limits, but also those derived from background as a fixed standard. The full subject of background GWPS testing is treated in **Section 7.5,** but any of the procedures in the following chapters might be applied to single-sample background tests.

The primary tool for both stages of monitoring is the confidence interval. Several varieties of confidence intervals are presented in **Chapter 21**, including confidence intervals around means, medians, and upper percentiles for stationary populations, and confidence bands around a trend for cases where groundwater concentrations are actively changing.

Strategies to implement confidence interval tests are discussed in **Chapter 22**. In particular, the focus is on designing tests with reasonable statistical performance in terms of power and per-test false positive rates.

Chapter 7 of **Part I** provides a discussion of the overall compliance/assessment and corrective action monitoring network design. Program elements such as the appropriate hypothesis structure, selecting the appropriate parameter for comparison to a fixed limit GWPS, sampling frequency, statistical power, and confidence levels are covered. These final two chapters present the tests in greater detail.

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CHAPTER 21. CONFIDENCE INTERVALS

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Confidence intervals are the recommended general statistical strategy in compliance/assessment or corrective action monitoring. Groundwater monitoring data must typically be compared to a fixed numerical limit set as a GWPS. In compliance/assessment, the comparison is made to determine whether groundwater concentrations have increased above the compliance standard. In corrective action, the test determines whether concentrations have decreased below a clean-up criterion or compliance level. In compliance/assessment monitoring, the lower confidence limit [LCL] is of primary interest, while the upper confidence limit [UCL] is most important in corrective action. For single-sample background GWPS testing, the hypothesis structures are the same as for fixed-limit health-based standards. Where a GWPS is based on two- or multiple sample testing, a somewhat different hypothesis structure is used (**Section 7.5**) and detection monitoring test procedures in **Part III** are applicable.

General strategies for using confidence intervals in compliance/assessment or corrective action monitoring are presented in **Chapter 7**, including discussion of how regulatory standards should be matched to particular statistical parameters (*e.g.*, mean or upper percentile). More specific strategies and examples are detailed in **Chapter 22**. In this chapter, basic algorithms and equations for each type of confidence interval are described, along with an example of the calculations involved.

21.1 PARAMETRIC CONFIDENCE INTERVALS

Confidence intervals are designed to estimate statistical characteristics of some parameter of a sampled population. *Parametric* confidence intervals do this for known distributional models, *e.g.*, normal, lognormal, gamma, Weibull, *etc*. Given a statistical parameter of interest such as the population mean (μ), the lower and upper limits of a confidence interval define the most probable concentration range in which the true parameter ought to lie.

Like any estimate, the true parameter may not be located within the confidence interval. The frequency with which this error tends to occur (based on repeated confidence intervals on different samples of the same sample size and from the same population) is denoted α, while its complement ($1-\alpha$) is known as the *confidence level*. The confidence level represents the percentage of cases where a confidence interval constructed according to a fixed algorithm or equation will actually contain its

intended target, *e.g.*, the population mean. **Section 7.2** discusses the difference between one- and two-sided confidence intervals and how the α error is assigned.

A point worth clarifying is the distinction between α as the complement of the confidence level when constructing a confidence interval and the significance level (α) used in hypothesis testing. Confidence intervals are often used strictly for estimation of population quantities. In that case, no test is performed, so α does not represent a false positive rate. Rather, it is simply the fraction of similar intervals that do not contain their intended target.

The Unified Guidance focuses on confidence interval limits compared to a fixed standard as a formal test procedure. In this case, the complement (α) of the confidence level used to generate the confidence interval is equivalent to the significance level (α) of the test. This assumes that the true population parameter under the null hypothesis is no greater than the standard in compliance/assessment monitoring or not less than the standard in corrective action.[1]

The parametric confidence intervals presented in the Unified Guidance share some common statistical assumptions. The most basic is that measurements used to construct a confidence interval be independent and identically distributed [*i.i.d.*]. Meeting this assumption requires that there be no outliers (**Chapter 12**), a stationary mean and variance over the period during which observations are collected (**Chapters 3** and **14**), and no autocorrelation between successive sampling events (**Chapter 14**). In particular, sampling events should be spaced far enough apart so that approximate statistical independence can be assumed (at many sites, observations should not be sampled more often than quarterly). Sample data should also be examined for trends. The mean is not stationary under a significant trend, as assumed in applying the other methods of this section. An apparent trend may need to be handled by computing a confidence band around the trend line (**Section 21.3**).

Another common assumption is that the sample data are either normal in distribution or can be normalized via a transformation (**Chapter 10**). Normality can be difficult to check if the sample contains a significant number of left-censored measurements (i.e., non-detects). The basic options for censored samples are presented in **Chapter 15**. If the non-detect percentage is no more than 10-15%, it may be possible to assess normality by first substituting one-half of the reporting limit [RL] for each non-detect. For higher non-detect percentages up to 50%, the Unified Guidance recommends computing a censored probability plot using either the Kaplan-Meier or Robust Regression on Order Statistics [Robust ROS] techniques (both in **Chapter 15**).

If a censored probability plot suggests that the sample (or some transformation of the sample) is normal, either Kaplan-Meier or Robust ROS can be used to construct estimates of the mean ($\hat{\mu}$) and standard deviation ($\hat{\sigma}$) adjusted for the presence of non-detects. These estimates should be used *in place of* the sample mean ($\bar{x}$) and standard deviation (s) in the parametric equations below.

[1] Technically, α represents the *maximum* possible false positive rate associated with the composite null hypothesis H_0: $\mu \leq$ GWPS or H_0: $\mu \geq$ GWPS.

21.1.1 CONFIDENCE INTERVAL AROUND NORMAL MEAN

BACKGROUND AND PURPOSE

When compliance point data is to be compared to a fixed standard (*e.g.*, a maximum concentration limit [MCL]) and the standard in question is interpreted to represent an average or true mean concentration, a confidence interval around the mean is the method of statistical choice. A confidence interval around the mean is designed to estimate the true average of the underlying population, while at the same time accounting for variability in the sample data.

REQUIREMENTS AND ASSUMPTIONS

Confidence intervals around the mean of a normal distribution should only be constructed if the data are approximately normal or at least are reasonably symmetric (*i.e.*, the skewness coefficient is close to zero). An inaccurate confidence interval is likely to result if the sample data are highly non-normal, particularly for right-skewed distributions. If the observations are better fit by a lognormal distribution, special equations or methods need to be used to construct an accurate confidence interval on the arithmetic mean with a specified level of confidence (**Section 21.1.3**). *Therefore, checking for normality is an important first step*.

A confidence interval should not be constructed with less than 4 measurements per compliance well, and preferably 8 or more. The equation for a normal-based confidence interval around the mean involves estimating the population standard deviation via the sample standard deviation (s). This estimate can often be imprecise using a small sample size (*e.g.*, $n \leq 4$). The equation also involves a Student's t-quantile based on n–1 degrees of freedom [df], where n equals the sample size. The t-quantile is large for small n, leading to a much wider confidence interval than would occur with a larger sample size. For a 99% confidence level, the appropriate t-quantile would be t = 31.82 for n = 2, t = 4.54 for n = 4, and t = 3.00 for n = 8.

This last consideration is important since statistically significant evidence of a violation during compliance/assessment or success during corrective action is indicated only when the entire confidence interval is to one side of the standard (*i.e.*, it does not *straddle* the fixed standard; see **Chapter 7**). For a small sample size, the confidence interval may be so wide that a statistical difference is unlikely to be identified. This can happen *even if* the true mean groundwater concentration *is* different from the compliance or clean-up standard, due to the statistical uncertainty associated with the small number of observations. More specific recommendations on appropriate sample sizes are presented in **Chapter 22**, where the *statistical power* of the confidence interval tests is explored.

PROCEDURE

Step 1. Check the basic statistical assumptions of the sample as discussed above. Assuming a normal distributional model is acceptable, calculate the sample mean ($\bar{x}$) and standard deviation (s).

Step 2. Given a sample of size n and the desired level of confidence ($1-\alpha$), for each compliance well calculate either the lower confidence limit (for compliance/assessment monitoring) with the equation:

$$LCL_{1-\alpha} = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} \qquad [21.1]$$

or the upper confidence limit (for corrective action) with the equation:

$$UCL_{1-\alpha} = \bar{x} + t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} \qquad [21.2]$$

where $t_{1-\alpha,n-1}$ is obtained from a Student's *t*-table with (n–1) degrees of freedom (**Table 16-1** in **Appendix D**). To construct a two-sided interval with overall confidence level equal to (1–α), substitute α/2 for α in the above equations.

Step 3. Compare the limit calculated in **Step 2** to the fixed compliance or clean-up standard (***e.g.***, the MCL or alternate concentration limit [ACL]. For compliance/assessment monitoring, the LCL in equation [21.1] should be used to compute the test. For corrective action, the UCL in equation [21.2] should be used instead.

►EXAMPLE 21-1

The table below lists concentrations of the pesticide Aldicarb in three compliance wells. For illustrative purposes, the health-based standard in compliance monitoring for Aldicarb has been set to 7 ppb. Determine at the α = 5% significance level whether or not any of the wells should be flagged as being out of compliance.

	Aldicarb Concentration (ppb)		
Sampling Date	Well 1	Well 2	Well 3
January	19.9	23.7	5.6
February	29.6	21.9	3.3
March	18.7	26.9	2.3
April	24.2	26.1	6.9
Mean	23.10	24.65	4.52
SD	4.93	2.28	2.10
Skewness (γ_1)	0.506	−0.234	0.074
Shapiro-Wilk (W)	0.923	0.943	0.950

SOLUTION

Step 1. First test the data for non-normality and/or significant skewness. Based on four samples per well, the skewness coefficients and Shapiro-Wilk statistics have been computed and are listed above. None of the skewness coefficients are significantly different from zero. In addition, the α = .10 critical point for the Shapiro-Wilk test with n = 4 (as presented in **Chapter 10**) is 0.792, less than each of the Shapiro-Wilk statistics; consequently, there is no significant evidence of non-normality. Construct a normal-based confidence interval around the mean.

Step 2. Calculate the sample mean and standard deviation of the Aldicarb concentrations for each compliance well. These statistics are listed above.

Step 3. Since α = 0.05, the confidence level must be set to (1–α) = 0.95. Obtain the upper 95th percentile of the *t*-distribution with (*n*–1) = 3 degrees of freedom from **Table 16-1** in **Appendix D**, namely $t_{.95,3}$ = 2.353. Then calculate the lower confidence limit [LCL] for each well's mean concentration, using equation [21.1]:

$$\text{Well 1: } LCL_{.95} = 23.10 - \left(2.353 \times 4.93\right)\Big/\sqrt{4} = 17.30 \text{ ppb}$$

$$\text{Well 2: } LCL_{.95} = 24.65 - \left(2.353 \times 2.28\right)\Big/\sqrt{4} = 21.97 \text{ ppb}$$

$$\text{Well 3: } LCL_{.95} = 4.52 - \left(2.353 \times 2.10\right)\Big/\sqrt{4} = 2.05 \text{ ppb}$$

Step 4. Compare each LCL to the compliance standard of 7 ppb. The LCLs for Well 1 and Well 2 lie above 7 ppb, indicating that the mean concentration of Aldicarb in both of these wells significantly exceeds the compliance standard. However, the LCL for Well 3 is below 7 ppb. providing insufficient evidence at the α = 0.05 level that the mean in Well 3 is out of compliance. ◄

21.1.2 CONFIDENCE INTERVAL AROUND LOGNORMAL GEOMETRIC MEAN

PURPOSE AND BACKGROUND

For many groundwater monitoring constituents, neither the assumption of normality nor approximate symmetry holds for the original concentration data. Often the underlying population is heavily right-skewed, characterized by a majority of lower level concentrations combined with a long right-hand tail of infrequent but extreme values. A model such as the lognormal distribution is commonly used to analyze such data.

The lognormal is traditionally designated by the notation $\Lambda(\mu, \sigma)$ (Aitchison and Brown, 1976), where μ and σ denote parameters controlling the *location* and *scale* of the population. Typically designated as $N(\mu, \sigma)$, a normal distribution also has parameters μ and σ which denote the true mean and standard deviation. These two parameters play different roles in lognormal distributions. The key distinction is between the *arithmetic* domain (or the original measurement scale of the data) and the *logarithmic* domain. The latter denotes the mathematical space following a logarithmic transformation. Transformed lognormal data are *normally-distributed* in the logarithmic domain. In this new domain, μ represents the true mean of the log-transformed measurements--- that is, the *log-mean*. Likewise, σ represents the true standard deviation of the log-transformed values or the *log-standard deviation*.

A common misperception is to assume that a standard equation for a normal-based confidence interval can be applied to log-transformed data, with the interval endpoints then back-transformed (*i.e.*, exponentiated) to the arithmetic domain to get a confidence interval around the lognormal *arithmetic* mean. Invariably, such an interval will underestimate the true mean. The Student t- confidence interval applies to a *geometric* mean of the lognormal population when back-transformed, rather than the higher-valued *arithmetic* mean. The reason is that the sample log-mean gives an estimate of the lognormal parameter μ. When this estimate is back-transformed to the arithmetic domain, one has an estimate of

exp(μ) — the lognormal geometric mean — not an estimate of the lognormal arithmetic mean, which is expressed as $\exp(\mu + .5\sigma^2)$.

Although a confidence interval around the lognormal geometric mean is *not* an accurate estimate of the arithmetic mean, there are instances where such an interval may be helpful. While many GWPSs are interpreted to represent long-term arithmetic averages, some (as detailed in **Chapter 7**) can better represent medians or percentiles of the underlying distribution. Because the lognormal geometric mean is equivalent to the median, a geometric mean may in some cases be a better statistical parameter of comparison than the lognormal arithmetic mean. Furthermore, when the lognormal coefficient of variation is large, the arithmetic mean is substantially larger than the geometric mean, mostly due to infrequent but extreme individual measurements. The bulk of individual observations are located much closer to the geometric mean. It may be that a comparison of the GWPS to the geometric mean rather than to the arithmetic mean will provide a more reasonable test of long-term concentration levels.

Special equations or computational methods are used to construct an accurate confidence interval with a specified level of confidence (**Section 21.1.3**) when an estimate of the *arithmetic* mean is needed and the observations are approximately normal. There is another factor to consider when estimating an *upper* confidence limit on the lognormal arithmetic mean using Land's procedure (described in **Section 21.1.3**) or other possible procedures (see for instance Singh et al., 1997). When used with highly variable data, it can lead to severely-biased, high estimates of the confidence limit. This can make it very difficult to evaluate the success of corrective action measures.

In these cases, precise parametric estimation of the arithmetic mean may have to be foregone in favor of an alternate statistical procedure. One such alternative is a non-parametric confidence interval around the median (**Section 21.2**). Another alternative when the sample is approximately lognormal is an estimate around the geometric mean which is equivalent to the population median. A third more computationally intensive option is a *bootstrap confidence interval* around the lognormal arithmetic mean (see discussion in **Section 21.1.3**). Unlike the first two options, this last alternative allows a direct estimate of the arithmetic mean.

REQUIREMENTS AND ASSUMPTIONS

Confidence intervals around the geometric mean of a lognormal distribution should only be constructed if the log-transformed data are approximately normal or at least reasonably symmetric (*i.e.*, the skewness coefficient in the logarithmic domain is close to zero). The methods of **Chapter 10** can be used to test normality of the log-transformed values. If the log-transformed sample contains non-detects, normality *on the log-scale* should be assessed using a censored probability plot. Adjusted estimates of the mean and standard deviation on the log-scale can then be substituted for the log-mean ($\bar{y}$) and log-standard deviation (s_y) in the equations below. Like a normal arithmetic mean, a confidence interval around the lognormal geometric mean should not be constructed without a minimum of 4 measurements per compliance well, and preferably with 8 or more.

PROCEDURE

Step 1. Take the logarithm of each measurement, denoted as y_i, and check the n log-transformed values for normality. If the log-transformed measurements are approximately normal, calculate

the log-mean ($\bar{y}$) and log-standard deviation (s_y). If the normal model is rejected, consider instead a non-parametric confidence interval (**Section 21.2**).

Step 2. Given the desired level of confidence (1–α), calculate either the LCL (for compliance/assessment monitoring) with the equation:

$$LCL_{1-\alpha} = \exp\left(\bar{y} - t_{1-\alpha,n-1}\frac{s_y}{\sqrt{n}}\right) \qquad [21.3]$$

or the UCL (for corrective action) with the equation:

$$UCL_{1-\alpha} = \exp\left(\bar{y} + t_{1-\alpha,n-1}\frac{s_y}{\sqrt{n}}\right) \qquad [21.4]$$

where $t_{1-\alpha,n-1}$ is obtained from a Student's *t*-table with (n–1) degrees of freedom (**Table 16-1** in **Appendix D**). In order to construct a two-sided interval with the overall confidence level equal to (1–α), substitute α/2 for α in the above equations.

Step 3. Compare the limits calculated in **Step 2** to the fixed compliance or clean-up standard (*e.g.*, the MCL or ACL). For compliance/assessment, use the LCL in equation [21.3]. For corrective action, use the UCL in equation [21.4].

Note in either case that the regulatory authority will have to approve the use of the geometric mean as a reasonable basis of comparison against the compliance standard. In some cases, there may be few other statistical options. However, stakeholders should understand that the geometric and arithmetic means estimate two distinct statistical characteristics of the underlying lognormal population.

►EXAMPLE 21-2

Suppose the following 8 sample measurements of benzene (ppb) have been collected at a landfill that previously handled smelter waste and is now undergoing remediation efforts. Determine whether or not there is statistically significant evidence at the α = 0.05 significance level that the true geometric mean benzene concentration has fallen below the permitted MCL of 5 ppb.

Sample Month	Benzene (ppb)	Log Benzene log(ppb)
1	0.5	−0.693
2	0.5	−0.693
3	1.6	0.470
4	1.8	0.588
5	1.1	0.095
6	16.1	2.779
7	1.6	0.470
8	<0.5	−1.386

SOLUTION

Step 1. To estimate an upper confidence bound on the geometric mean benzene concentration with 95% confidence, first test the skewness and normality of the data set. Since the one non-detect concentration is unknown but presumably between 0 ppb and the RL of 0.5 ppb, a reasonable compromise is to impute this value at 0.25 ppb, half the RL. The skewness is computed as γ_1 = 2.21, a value too high to suggest the data are normal. In addition, a Shapiro-Wilk test statistic on the raw measurements works out to SW = 0.521, failing an assumption of normality at far below a significance level of α = 0.01.

On the other hand, transforming the data via natural logarithms gives a smaller skewness coefficient of γ_1 = 0.90 and a Shapiro-Wilk statistic of W = 0.896. Because these values are consistent with normality on the log-scale (the critical point for the Shapiro-Wilk test with n = 8 and α = 0.10 is 0.818), the data set should be treated as lognormal for estimation purposes. As a consequence, equation [21.4] can be used to construct a one-sided UCL on the geometric mean.

Step 2. Compute the sample log-mean and log-standard deviation. This gives $\bar{y}$ = 0.2037 log(ppb) and s_y = 1.2575 log(ppb).

Step 3. Apply the log-mean and log-standard deviation into equation [21.4] for a UCL with α = .05, n = 8, and 7 degrees of freedom. This gives an estimated limit of:

$$UCL_{.95} = \exp\left(\bar{y} + t_{.95,7}\frac{s_y}{\sqrt{8}}\right) = \exp\left(.2037 + 1.895 \times .4446\right) = 2.847 \text{ ppb}$$

Step 4. Compare the UCL to the MCL of 5 ppb. Since the limit is less than the fixed standard, there is statistically significant evidence that the benzene geometric mean, and consequently, the median benzene concentration, is less than 5 ppb. However, this calculation does *not* show that the benzene *arithmetic* mean is less than the MCL. Extreme individual benzene measurements could show up with enough regularity to cause the arithmetic mean to be higher than 5 ppb. ◄

21.1.3 CONFIDENCE INTERVAL AROUND LOGNORMAL ARITHMETIC MEAN

PURPOSE AND BACKGROUND

Estimation of a lognormal arithmetic mean is not completely straightforward. As discussed in **Section 21.1.2**, applying standard equations for normal-based confidence limits around the mean to log-transformed measurements and then exponentiating the limits, results in confidence intervals that are invariably underestimate the arithmetic mean.

Inferences on arithmetic means for certain kinds of skewed populations can be made either exactly or approximately through the use of special techniques. In particular, if a confidence interval on the arithmetic mean is desired, Land (1971; 1975) developed an exact technique along with extensive tables

for implementing it when the underlying population is lognormal. Land also developed a more complicated approximate technique (for a full description and examples see EPA, 1997) when the population can be transformed to normality via any other increasing, 1-1, and twice differentiable transformation (*e.g.*, square, square root, cube root, *etc.*).

Although the core of Land's procedure is a correction for the so-called 'transformation bias' that occurs when making back-transforming estimates from the logarithmic domain to the raw concentration domain, it can produce unacceptable results, particularly with UCLs. The Unified Guidance advises caution when applying Land's procedure, particularly when the lognormal population has a high coefficient of variation. In those cases, the user may want to consider alternate techniques, such as those discussed in Singh, *et al* (1997 and 1999). One option is to use EPA's free-of-charge **Pro-UCL** software Version 4.0 (www.epa.gov/esd/tsc/software.htm). It computes a variety of upper confidence limits, including a bootstrap confidence interval around the arithmetic mean. This technique can be applied to lognormal data to get a direct, non-parametric UCL that tends to be less biased and to give less extreme results than Land's procedure.

For cases or sample sizes not covered by **Tables 21-1** through **21-8** in **Appendix D** when using Land's procedure, Gibbons and Coleman (2001) describe a method of approximating the necessary *H*-factors. The same authors review other alternate parametric methods for computing UCLs.

REQUIREMENTS AND ASSUMPTIONS

Confidence intervals around the arithmetic mean of a lognormal distribution should be constructed only if the data pass a test of approximate normality *on the log-scale*. While many groundwater and water quality populations tend to follow the lognormal distribution, the data should first be tested for normality on the original concentration scale. If such a test fails, the sample can be log-transformed and re-tested. If the log-transformed sample contains non-detects, normality *on the log-scale* should be assessed using a censored probability plot (**Chapter 15**). If a lognormal model is tenable, adjusted estimates of the mean and standard deviation on the log-scale can be substituted for the log-mean ($\bar{y}$) and log-standard deviation (s_y) in the equations below.

As with normal-based confidence intervals, the confidence interval here should not be constructed with fewer than 4 measurements per compliance well, and preferably with 8 or more. The reasons are similar: the equation for a lognormal-based confidence interval around the arithmetic mean depends on the sample log-standard deviation (s_y), used as an estimate of the underlying log-scale population standard deviation. This estimate can be quite imprecise when fewer than 4 to 8 observations are used. A special factor (*H*) was developed by Land to account for variability in a skewed population. These factors are larger for smaller samples sizes, and need to be exponentiated to estimate the final confidence limits (see below). Consequently there is a significant penalty associated with estimating the arithmetic mean using a small sample size, occasionally seen in remarkably wide confidence limits. The effect is especially noticeable when computing an UCL for corrective action monitoring.

PROCEDURE

Step 1. Test the log-tranformed sample for normality. If the lognormal model provides a reasonable fit, denote the log-transformed measurements by y_i and move to Step 2.

Step 2. Compute the sample log-mean ($\bar{y}$) and log-standard deviation (s_y).

Step 3. Obtain the correct bias-correction factor(s) (H_α) from Land's (1975) tables (**Tables 21-1 through 21-8** in **Appendix D**), where the correct factor depends on the sample size (n), the sample log-standard deviation (s_y), and the desired confidence level (1–α).

Step 4. Plug these factors into one of the equations given below for the LCL or UCL (depending on whether the comparison applies to compliance/assessment monitoring or to corrective action). Note that to construct a two-sided interval with an overall confidence level of (1–α), the equations should be applied by substituting α/2 for α.

$$LCL_{1-\alpha} = \exp\left(\bar{y} + .5s_y^2 + \frac{s_y H_\alpha}{\sqrt{n-1}}\right) \qquad [21.5]$$

$$UCL_{1-\alpha} = \exp\left(\bar{y} + .5s_y^2 + \frac{s_y H_{1-\alpha}}{\sqrt{n-1}}\right) \qquad [21.6]$$

Step 5. Compare the confidence limit computed in **Step 4** to the fixed compliance or clean-up standard. In compliance/assessment monitoring, use the LCL of equation [21.5]. In corrective action, use equation [21.6] for the UCL.

►EXAMPLE 21-3

Determine whether the benzene concentrations of **Example 21-2** indicate that the benzene arithmetic mean is below the permitted MCL of 5 ppb at the α = 0.05 significance level.

SOLUTION

Step 1. From **Example 21-2**, the benzene data were found to fail a test of normality, but passed a test of lognormality (*i.e.*, they were approximately normal on the log-scale). As a consequence, Land's equation in [21.6] should be used to construct a one-sided UCL on the arithmetic mean.

Step 2. Compute the log-mean and log-standard deviation from the log-scale data. This gives $\bar{y}$ = 0.2037 log(ppb) and s_y = 1.2575 log(ppb).

Step 3. Using **Table 21-6** in **Appendix D**, pick the appropriate *H*-factor for estimating confidence limits around a lognormal arithmetic mean, noting that to achieve 95% confidence for a one-sided UCL, one must use (1–α) = 0.95. With a sample size of n = 8 and a standard deviation on the log-scale of 1.2575 log(ppb), $H_{.95}$ = 4.069.

Step 4. Plug these values along with the log-mean of 0.2037 log(ppb) into equation [21.6] for the UCL. This leads to a 95% one-sided confidence limit equal to:

$$UCL_{.95} = \exp\left(.2037 + .5(1.5813) + \frac{(1.2575)(4.069)}{\sqrt{7}} \right) = 18.7 \text{ ppb}$$

Step 5. Compare the UCL against the MCL of 5 ppb. Since the UCL is greater than the MCL, evidence is not sufficient at the 5% significance level to conclude that the true benzene arithmetic mean concentration is now below the MCL. This conclusion holds despite the fact that all but one of the benzene measurements is less than than 5 ppb. In lognormal populations, it is not uncommon to see one or two seemingly extreme measurements coupled with a majority of much lower concentrations. Since these extreme measurements help determine the location of the arithmetic mean, it is not unreasonable to expect that the true mean might be larger than 5 ppb.

The contrast in this result to **Example 21-2** is noteworthy. In that case, the UCL on the geometric mean was only 2.85 ppb. The estimated lognormal coefficient of variation with these data (**Chapters 3** and **10**) is $CV = 1.965$, somewhat on the high side. It is no surprise that results for the arithmetic and geometric means on the same sample are rather different. Neither estimator is necessarily invalid, but a decision needs to be made as to whether the MCL for benzene in this setting should be better compared to an arithmetic mean or to a geometric mean/ median for lognormal distributions. ◄

21.1.4 CONFIDENCE INTERVAL AROUND UPPER PERCENTILE

BACKGROUND AND PURPOSE

Although most MCLs and ACLs appear to represent arithmetic or long-term averages (**Chapter 7**), they can also be interpreted as standards not to be exceeded with any regularity. Other fixed standards like nitrate/nitrite attempt to limit short-term risks and thus represent upper percentiles instead of means. In these cases, the appropriate confidence interval is one built around a specific upper percentile.

The particular upper percentile chosen will depend on what the fixed compliance standard represents or is intended to represent. If the standard is a concentration that represents the 90th percentile, the confidence interval should be built around the upper 90th percentile. If the standard is meant to be a ***maximum***, 'not to be exceeded,' concentration, a slightly different strategy should be used. Since there is no maximum value associated with continuous distributions like normal and lognormal, it is not possible to construct a confidence interval around the population maximum. Instead, one must settle for a confidence interval around a sufficiently high percentile, one that will exceed nearly all of the population measurements. Possible choices are the upper 90th to 95th percentile. By estimating the location of these percentiles, one needs to determine whether a sufficiently small fraction (***e.g.***, at most 1 in 10 or 1 in 20) of the possible measurements will ever exceed the standard. For even greater protection against exceedances, the upper 99th percentile could be selected, implying that at most 1 in 100 measurements would ever exceed the standard. But as noted in **Chapter 7**, selection of very high percentiles using non-parametric tests can make it extremely difficult to demonstrate corrective action success.

REQUIREMENTS AND ASSUMPTIONS

The equations for constructing parametric confidence intervals around an upper percentile assume that the data are normally distributed, at least approximately. If the data can be normalized via a transformation, the observations should first be transformed before computing the confidence interval. Unlike confidence intervals around an arithmetic mean for transformed data, no special equations are required to construct similar intervals around an upper percentile. The same equations used for normal data can be applied to data in the transformed domain. The only additional step is that the confidence interval limits must be back-transformed prior to comparing them against the fixed standard.

The confidence interval presented here should not be constructed with fewer than 4 measurements per compliance well, and preferably with 8 or more. Too small a sample size leads to imprecise estimates of the sample standard deviation (s). Another reason is that the confidence interval equation involves a special multiplier τ, which depends on both the desired confidence level ($1-\alpha$) and the sample size (n). When n is quite small, the τ multiplier is much greater. This leads to a much wider confidence interval than that obtained with a larger n, and therefore much greater statistical uncertainty. For example, at a confidence level of 95%, the appropriate τ multiplier for an upper one-sided limit on the 95th percentile is $\tau = 52.559$ when $n = 2$, $\tau = 6.602$ when $n = 4$, and $\tau = 3.640$ when $n = 8$.

When determining the τ factor(s) needed for a confidence interval around an upper percentile, it should be noted that unlike the symmetric Student's t-distribution, separate τ factors need to be determined for the LCL and UCL. Since an upper percentile like the 95th is generally larger than the population mean, the equations for ***both*** the lower (***i.e.***, LCL) and upper (***i.e.***, UCL) limits involve ***adding*** a multiple of the standard deviation to the sample mean. The only difference is that a smaller multiple τ_{LCL} is used for the LCL, while a larger τ_{UCL} is used for the upper confidence limit. For certain choices of n, P and $1-\alpha$, the multiple τ_{LCL} can even be negative.

PROCEDURE

Step 1. Test the raw data for normality. If approximately normal, construct the interval on the original measurements. If the data can be normalized via a transformation, construct the interval on the transformed values.

Step 2. For a normal sample, compute the sample mean ($\bar{x}$) and standard deviation (s). If the data have been transformed, compute the mean and standard deviation of the transformed measurements.

Step 3. Given the percentile (P) to be estimated, sample size (n), and the desired confidence level ($1-\alpha$), use **Tables 21-9** and **21-10** in **Appendix D** to determine the τ factor(s) needed to construct the appropriate one-sided or two-sided interval. A one-sided LCL is then computed with the equation:

$$LCL_{1-\alpha} = \bar{x} + s \cdot \tau\left(P; n, \alpha\right) \qquad [21.7]$$

where $\tau(P;\ n,\ \alpha)$ is the lower α factor for the Pth percentile given n sample measurements. A one-sided UCL is given similarly by the equation:

$$UCL_{1-\alpha} = \bar{x} + s \cdot \tau\left(P; n, 1-\alpha\right) \qquad [21.8]$$

Finally, a two-sided confidence interval is computed by the pair of equations for the LCL and UCL:

$$LCL_{1-\alpha/2} = \bar{x} + s \cdot \tau\left(P; n, \alpha/2\right) \qquad [21.9]$$

$$UCL_{1-\alpha/2} = \bar{x} + s \cdot \tau\left(P; n, 1-\alpha/2\right) \qquad [21.10]$$

Step 4. If the data have been transformed, the equations of Step 3 would be used but with two changes: 1) the mean and standard deviation of the transformed values are substituted for $\bar{x}$ and s; and 2) the resulting limits back-transformed to get final confidence limits in the concentration domain. If a logarithmic transformation has been employed, the log-mean and log-standard deviation would be substituted for the sample mean and standard deviation. The resulting limit(s) must be exponentiated to get the final confidence limits, as in the equations below:

$$LCL_{1-\alpha} = \exp\left[\bar{y} + s_y \cdot \tau\left(P; n, \alpha\right)\right] \qquad [21.11]$$

$$UCL_{1-\alpha} = \exp\left[\bar{y} + s_y \cdot \tau\left(P; n, 1-\alpha\right)\right] \qquad [21.12]$$

Step 5. Compare the confidence limit(s) computed in Step 3 (or Step 4) versus the fixed compliance or clean-up standard. In compliance/assessment, use the LCL of equation [21.7]. In corrective action, use equation [21.8] for the UCL.

Note that although the above equations differentiate between the α-error used with the LCL and 1-α for the UCL, **Tables 21-9** and **21-10** in **Appendix D** are constructed identically. The α-error is represented by its confidence complement 1-α in **Table 21-10** of **Appendix D**.

►EXAMPLE 21-4

Assume that a facility permit has established an ACL of 30 ppb that should not be exceeded more than 5% of the time. Use the Aldicarb concentrations and diagnostic statistical information from **Example 21-1** to evaluate data from the three compliance wells. Determine whether any of the wells should be flagged as being out of compliance.

SOLUTION

Step 1. From **Example 21-1**, all of the wells pass a normality test. Use the sample mean and standard deviation for each compliance well, from the tabular information in **Example 21-1**.

Step 2. Select the correct τ factor from **Table 21-10** of **Appendix D** to construct a 99% LCL on the upper 95th percentile. The upper 95th percentile is needed because the permitted ACL cannot be exceeded more than 5% of the time, implying that 95% of all the Aldicarb measurements should fall below the fixed standard. With n = 4 observations per well, this leads to τ (P; n, α) = τ (.95; 4, .01) = 0.443.

Step 3. Compute the LCL for each well as follows using equation [21.7]:

$$Well1: LCL_{.99} = 23.10 + (0.443)(4.93) = 25.28\,ppb$$
$$Well2: LCL_{.99} = 24.65 + (0.443)(2.28) = 25.66\,ppb$$
$$Well3: LCL_{.99} = 4.52 + (0.443)(2.10) = 5.45\,ppb$$

Step 4. Compare each LCL against the ACL of 30 ppm. Since each well LCL is less than the ACL, there is insufficient statistical evidence that the upper 95th percentile of the Aldicarb distribution exceeds the fixed standard. Consequently, there is no conclusive evidence that more than 5% of the Aldicarb concentrations will exceed the ACL.

If the site were in corrective action instead of compliance/assessment monitoring, UCLs around the 95th percentile would be needed instead of LCLs. In that case, with n = 4 observations per well, τ (P; n, 1–α) = τ (.95; 4, .99) = 9.083 from **Table 21-9** of **Appendix D**. Then, the respective well UCLs would be:

$$Well1: UCL_{.99} = 23.10 + (9.083)(4.93) = 67.88\,ppb$$
$$Well2: UCL_{.99} = 24.65 + (9.083)(2.28) = 45.36\,ppb$$
$$Well3: UCL_{.99} = 4.52 + (9.083)(2.10) = 23.59\,ppb$$

In this case, two of the three wells would not meet the corrective action limit of 30 ppb. ◄

21.2 NON-PARAMETRIC CONFIDENCE INTERVALS

BACKGROUND AND PURPOSE

A non-parametric confidence interval should be considered when a sample is non-normal and cannot be normalized, perhaps due to a significant fraction of non-detects. Non-parametric confidence interval endpoints are generally chosen as *order statistics* of the sample data. The specific order statistics selected will depend on the sample size (n), the desired confidence level (1–α), and the population characteristic being estimated.

Since the data are not assumed to follow a particular distribution, it is generally not possible to construct a confidence interval around the population mean. One fairly rare exception would be if it were already known that the distribution is symmetric (where the mean is also the median). Sample order statistics represent, by definition, concentration levels exceeded by a certain number and hence a *fraction* of the sample values. They are excellent estimators of the *percentiles* of a distribution, but *not* of quantities like the arithmetic mean. The latter entails summing the data values and averaging the result. In positively-skewed populations, not only is the arithmetic mean greater than the median, it also may not correspond to any particular percentile.

Non-parametric confidence intervals can be developed either around a measure of the center of the population (*i.e.*, the population median or 50th percentile) or around an upper or lower percentile (*e.g.*, the upper 90th). The choice of percentile affects which order statistics are selected as interval endpoints.

The sample median is generally estimated using a smaller order statistic than that used for an upper 95th percentile.

Despite the distinction between non-parametric confidence intervals around the median and similar intervals around an upper or lower percentile, the mathematical algorithm used to construct both types is essentially identical. Given an unknown $P \times 100$th percentile of interest (where P is between 0 and 1) and a sample of n concentration measurements, the probability that any randomly selected measurement will be less than the $P \times 100$th percentile is simply P. Then the probability that the measurement will exceed the $P \times 100$th percentile is $(1–P)$. Hence the number of sample values falling below the $P \times 100$th percentile out of a set of n should follow a ***binomial distribution*** with parameters n and ***success probability*** P, where 'success' is defined as the event that a sample measurement is below the $P \times 100$th percentile.

Because of this connection, the binomial distribution can be used to determine the probability that the interval formed by a given pair of order statistics will contain the percentile of interest. This kind of probability calculation makes repeated use of the cumulative binomial distribution, often denoted $Bin(x;n,p)$. It represents the probability of x or fewer successes occurring in n trials with success probability p. The computational equation for this expression[2] can be written as:

$$Bin(x;n,p) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i} \qquad [21.13]$$

To make statistical inferences about the $P \times 100$th percentile, P (expressed as a fraction) would be substituted for p in equation [21.13]. It can be seen why the same basic algorithm applies both to confidence intervals around the median and around upper percentiles like the 95th. If an interval around the median is desired, one would set $P = 0.50$. For an interval needed around the upper 95th percentile, one would set $P = 0.95$ and perform similar calculations.

When constructing non-parametric confidence intervals, the type of confidence interval needs to be matched against the kind of fixed standard to which it will be compared. Since the arithmetic mean cannot be estimated directly, a confidence interval around the ***median*** should be used for those cases where the compliance standard represents an average. Some fixed standards can, of course, be directly interpreted as ***median*** concentration levels, but even for those standards representing arithmetic averages, the confidence interval on the median will give the 'next best' comparison when a non-parametric method is used.

The interpretation of a confidence interval on the median is similar to that of a parametric confidence interval around the mean. In compliance/assessment monitoring, if the LCL with confidence level $(1–\alpha)$ exceeds the compliance standard, there is statistically significant evidence that the true

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[2] The mathematical expression $\binom{n}{i}$ refers to the combination of n events taken i at a time. It can be calculated as: $n!/(i! \times [n-i]!)$, where $n! = \{n \times (n-1) \times \ldots \times 2 \times 1\}$. By convention, $0! = 1$.

population *median* is higher than the standard. In corrective action monitoring, if the UCL is *below* the clean-up standard, one can conclude that the true population median is less than the standard with α-level significance.

REQUIREMENTS AND ASSUMPTIONS

Because a non-parametric confidence interval does not assume a specific distributional form for the underlying population, there is no need to fit a probability model to the data. If a significant portion of the data are non-detect, it may be impossible to adequately fit such a model.. The non-parametric confidence interval method only requires the ability to rank the sample data values and pick out selected order statistics as the interval endpoints. Unfortunately, this ease of construction comes with a price. As opposed to parametric intervals, non-parametric confidence intervals tend to be wider and generally require larger sample sizes to achieve comparably high confidence levels. To compute the LCL around the median with 99% confidence, at least 7 compliance point measurements are needed in the non-parametric case. Therefore, sample data should be fit to a specific probability distribution whenever possible.

The general method for constructing non-parametric confidence intervals involves an *iterative testing procedure*, where potential endpoints are selected from the sorted data values (*i.e.*, order statistics) and then tested to determine what confidence level is associated with those endpoints. If the initial choice of order statistics gives an interval with insufficient confidence, the interval needs to be widened and tested again. Clearly, the greatest confidence will be associated with an interval defined by the minimum and maximum observed sample values. But if the sample size n is small, even the largest possible confidence level may be less than the desired target confidence (*e.g.*, $(1-\alpha) = 0.99$). As such, the *actual* or *achieved* confidence level needs to be listed when reporting results of a non-parametric confidence interval test.

It may be especially difficult to achieve target confidence levels around upper percentiles even when the sample size is fairly large. An instructive example is when estimating an upper 95th percentile with a sample size of $n = 20$. In that case, the highest possible two-sided confidence level is approximately 64%, achieved when the minimum and maximum data values are taken as the interval endpoints. The confidence level is substantially less than the usual targets of 90% or more, and has very limited value as a decision basis.

The *width* of a confidence interval (which expands as the level of confidence increases) should be balanced against the desire to construct an interval *narrow enough* to provide useful information about the probable location of the underlying population characteristic (*e.g.*, the P = 95th percentile in the above example). A reasonable goal is to construct the shortest interval possible that still approaches the highest confidence level. In the example, a confidence level of almost 63% could be achieved by setting the 17th and 20th ordered sample values as the confidence interval endpoints. The 20th ordered value is obviously the maximum observation and cannot be changed. However, if any ranked value less than the 17th is taken as the lower endpoint, the confidence level will increase only slightly, but the overall interval will be unnecessarily widened.

An iterative process is used to construct non-parametric confidence limits. It is recommended that a *stopping rule* be used to decide when the improvement in the confidence level brought about by picking more extreme order statistics is outweighed by the loss of information from making the interval

too wide. A reasonable stopping rule might be to end the iterative computations if the confidence level changes by less than 1 or 2 percent when a new set of candidate ranks is selected.

Repeated calculation of cumulative binomial distribution probabilities $Bin(x;n,p)$ are quite tedious when performed manually. One can make use of either an extensive table of binomial probabilities or a software package that computes them. Almost all commercial statistical packages will compute binomial probabilities. For small sample sizes up to $n \leq 20$, **Table 21-11** in **Appendix D** provides achievable confidence levels for various choices of the sample order statistic endpoints such as the median and common upper percentiles.

Tied values do not affect the procedure for constructing non-parametric confidence intervals. All tied values (including any non-detects treated as ties) should be regarded as distinct measurements. Because of this, ties can be arbitrarily broken when ranking the data. For example, a list of 6 values including 3 non-detects would be ordered as [<5, <5, <5, 8, 12, 20] and given the set of ranks [1, 2, 3, 4, 5, 6]. Note that it is possible for the LCL to be set equal to the RL used for non-detects.

PROCEDURE FOR A CONFIDENCE INTERVAL AROUND THE MEDIAN

Step 1. Given a sample of size n, order the measurements from least to greatest. Denote the ordered values by $x_{(1)}, x_{(2)},\ldots, x_{(n)}$, where $x_{(i)}$ is the ith concentration value in the ordered list and numbers 1 through n represent the data ranks.

Step 2. Given P = .50, pick candidate interval endpoints by choosing ordered data values with ranks as close to and as symmetrical as possible around the product of $(n+1) \times 0.50$. If this last quantity is a fraction (an even-numbered sample size), the ranks immediately above and below it can be selected as candidate endpoints. If the product $(n+1) \times 0.50$ is an integer (an odd-numbered sample size), add 1 and subtract 1 to get the upper and lower candidate endpoints. Once the candidate endpoints have been selected, denote the ranks of these endpoints by L^* and U^*.

Step 3. For a two-sided confidence interval, compute the confidence level associated with the tentative endpoints L^* and U^* by taking the difference in the cumulative binomial probabilities given by the equation:

$$1-\alpha = Bin\left(U^* - 1;\ n,.50\right) - Bin\left(L^* - 1;\ n,.50\right) = \sum_{x=L^*}^{U^*-1} \binom{n}{x} \left(\frac{1}{2}\right)^n \qquad [21.14]$$

For a one-sided LCL, compute the confidence level associated with endpoint L^* using the equation:

$$1-\alpha = 1 - Bin\left(L^* - 1;\ n,.50\right) = \sum_{x=L^*}^{n} \binom{n}{x} \left(\frac{1}{2}\right)^n \qquad [21.15]$$

For a one-sided UCL, compute the confidence level associated with endpoint U^* using the equation:

$$1-\alpha = Bin\left(U^{*}-1;\ n,.50\right) = \sum_{x=0}^{U^{*}-1}\binom{n}{x}\left(\frac{1}{2}\right)^{n} \qquad [21.16]$$

To minimize the amount of direct computation needed, these equations have been used to compute selected cases over a range of sample sizes for the median in **Table 21-11** of **Appendix D**.

Step 4. If the candidate endpoint(s) do not achieve the desired confidence level, compute new candidate endpoints (L^{*}–1) and (U^{*}+1) and re-calculate the achieved confidence level. Repeat this process until the target confidence level is achieved. If one candidate endpoint already equals the data minimum or maximum, only change the rank of the other endpoint. If neither endpoint rank can be changed, set either: 1) the minimum concentration value as a one-sided LCL; 2) the maximum concentration value as a one-sided UCL; or 3) the interval spanned by the range of the sample as a two-sided confidence interval around the median. In each case, report the achieved confidence level associated with the chosen confidence limit(s).

Step 5. Compare the confidence limit(s) computed in **Step 4** versus the fixed compliance or clean-up standard. In compliance/assessment monitoring, use the LCL derived as the order statistic with rank L^{*}. In corrective action monitoring, use the UCL derived as the order statistic with rank U^{*}.

►EXAMPLE 21-5

Use the following four years of well beryllium concentrations, collected quarterly for a total of n = 16 measurements, to compute a non-parametric LCL on the median concentration with (1–α) = 99% confidence.

SAMPLE DATA		ORDERED DATA	
Date	**Beryllium (ppb)**	**Be**	**Rank**
2002, 1st Q	3.17	2.32	(1)
2002, 2nd Q	2.32	3.17	(2)
2002, 3rd Q	7.37	3.39	(3)
2002, 4th Q	4.44	3.65	(4)
2003, 1st Q	9.50	3.74	(5)
2003, 2nd Q	21.36	4.44	(6)
2003, 3rd Q	5.15	5.15	(7)
2003, 4th Q	15.70	5.58	(8)
2004, 1st Q	5.58	6.15	(9)
2004, 2nd Q	3.39	6.94	(10)
2004, 3rd Q	8.44	7.37	(11)
2004, 4th Q	10.25	8.44	(12)
2005, 1st Q	3.65	9.50	(13)
2005, 2nd Q	6.15	10.25	(14)
2005, 3rd Q	6.94	15.70	(15)
2005, 4th Q	3.74	21.36	(16)

SOLUTION

Step 1. Order the 16 measurements from least to greatest and determine the rank associated with each value (listed above in the last two columns). The smallest observation, 2.32 ppb, receives the smallest rank, while the largest value, 21.36 ppb, receives a rank of 16.

Step 2. Since a confidence interval on the median must be constructed, the desired percentile is the 50th (*i.e.*, $P = 0.50$). Therefore the quantity $(n+1) \times P = 17 \times 0.50 = 8.5$. The data ranks closest to this value are $L^* = 8$ and $U^* = 9$, so these are used as initial candidate endpoints.

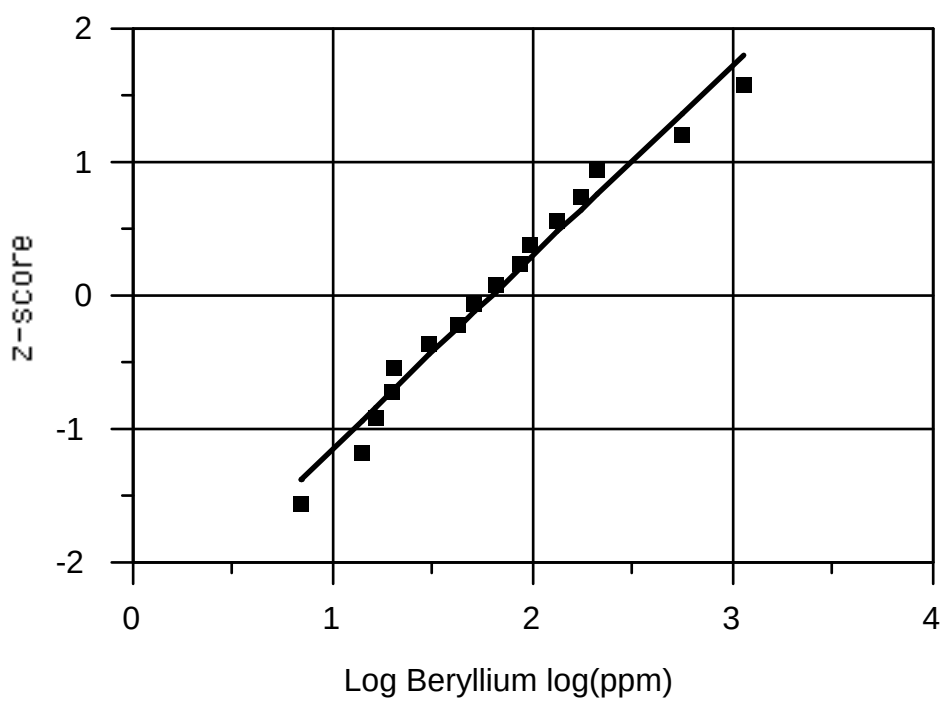
Step 3. Using the cumulative binomial distribution, and recognizing that only a lower confidence limit is needed, use equation [21.15] to calculate the actual confidence level associated with the order statistic $x_{(8)}$:

$$1-\alpha = 1 - Bin\left(L^* - 1;\ \text{n,P}\right) = 1 - Bin\left(7;\ 16,.50\right) = 1 - \sum_{x=0}^{7}\binom{16}{x}\left(.50\right)^{16} = 0.4018$$

Since the achieved confidence level is much less than 99%, subtract 1 from L^* and recompute the confidence level. Repeat this process until the confidence level is at least 99%. Since the achieved confidence when $L^* = 4$ is equal to .9894 or approximately 99%, the LCL should be selected as $x_{(4)}$ (*i.e.*, the 4th order statistic in the data set, also equal to the fourth smallest measurement), which equals 3.65 ppm. With statistical confidence of 98.94%, one can assert that the true median beryllium concentration in the underlying population is no less than 3.65 ppm.

Step 4. In this example, a lognormal model could also have been fit to the sample. Indeed the probability plot in **Figure 21-2** below indicates good agreement with a lognormal fit, enabling a comparison between the non-parametric LCL with that derived from assuming a parametric model for the same data.

Figure 21-2. Probability Plot on Logged Beryllium Data



Step 5. Since the non-parametric LCL was constructed around the population median, the fairest comparison is to construct a lognormal-based confidence interval around the *median* and not the arithmetic mean. As discussed in **Section 21.1.2**, this is equivalent to constructing a confidence interval around the lognormal *geometric mean*. This can be built via a normal-based confidence interval around the mean using the log-transformed measurements and then exponentiating the interval limits. Thus, using equation [21.3] with the log-mean and log-standard deviation given by $\bar{y}$ = 1.8098 log(ppm) and s_y = 0.60202 log(ppm) respectively, one can compute the 99% LCL as:

$$LCL_{.99} = \exp\left(\bar{y} - t_{.99,n-1}\frac{s_y}{\sqrt{n}}\right) = \exp\left[1.8098 - (2.602)(.60202)/\sqrt{16}\right] = 4.13 \text{ ppm}$$

The non-parametric LCL around the median is slightly lower than the limit computed by assuming an underlying lognormal distribution. Given the apparent lognormal fit, the parametric LCL is probably a slightly better estimate, but the non-parametric method performs well nonetheless.

The chief virtue of using a parametric confidence interval is the ability to generate estimates at any confidence level even with small sample sizes. On the other hand, if the data are lognormally-distributed, a confidence interval on the arithmetic mean may be preferred for comparisons to a fixed standard, depending on the type of standard. The advantage of a non-parametric interval around the median is its greater flexibility to define confidence intervals on non-normal data sets. ◄

PROCEDURE FOR A CONFIDENCE INTERVAL AROUND A PERCENTILE

Step 1. Given a sample of size n, order the measurements from least to greatest. Denote the ordered values by $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$, where $x_{(i)}$ is the ith concentration value in the ordered list and numbers 1 through n represent the data ranks.

Step 2. Given the desired percentile P, pick candidate interval endpoints by choosing ordered data values with ranks as close to and as symmetrical as possible around the product $(n+1) \times P$, where n is the sample size and P is expressed as a fraction. If this last quantity is a fraction (even-numbered sample size), the ranks immediately above and below it can be selected as candidate endpoints (unless the fraction is larger than n, in which case the maximum rank n would be chosen as the upper endpoint). If the product $(n+1) \times P$ is an integer (odd-numbered sample size), add 1 and subtract 1 to get the upper and lower candidate endpoints. Once the candidate endpoints have been selected, denote these by L^* and U^*.

Step 3. For a two-sided confidence interval, compute the confidence level associated with the tentative endpoints L^* and U^* by taking the difference in the cumulative binomial probabilities given by the equation:

$$1-\alpha = Bin\left(U^{*}-1;\ n,P\right) - Bin\left(L^{*}-1;\ n,P\right) = \sum_{x=L^{*}}^{U^{*}-1}\binom{n}{x}P^{x}\left(1-P\right)^{n-x} \qquad [21.17]$$

For a one-sided LCL, compute the confidence level associated with the endpoint L^* using the equation:

$$1-\alpha = 1 - Bin\left(L^{*}-1;\ n,P\right) = \sum_{x=L^{*}}^{n}\binom{n}{x}P^{x}\left(1-P\right)^{n-x} \qquad [21.18]$$

For a one-sided UCL, compute the confidence level associated with the endpoint U^* using the equation:

$$1-\alpha = Bin\left(U^{*}-1;\ n,P\right) = \sum_{x=0}^{U^{*}-1}\binom{n}{x}P^{x}\left(1-P\right)^{n-x} \qquad [21.19]$$

To minimize the amount of direct computation, these equations have been used to compute selected cases over a range of sample sizes and for certain percentiles in **Table 21-11** of **Appendix D**.

Step 4. If the candidate endpoint(s) do not achieve the desired or target confidence level, compute new candidate endpoints, (L^*–1) and (U^*+1), and re-calculate the achieved confidence level. Repeat this process until the target confidence level is achieved. If one candidate endpoint already equals the data minimum or maximum, only change the rank of the other endpoint. If neither endpoint rank can be changed, set either: 1) the minimum concentration value as a one-sided LCL; 2) the maximum concentration value as a one-sided UCL; or 3) the interval spanned by the range of the sample data as a two-sided confidence interval around the *P*th percentile. In each case, report the achieved confidence level associated with the chosen confidence limit(s).

Step 5. Compare the confidence limit(s) computed in **Step 4** versus the fixed compliance or clean-up standard. In compliance/assessment monitoring, use the LCL derived as the order statistic with rank L^*. In corrective action monitoring, use the UCL derived as the order statistic with rank U^*.

►EXAMPLE 21-6

Use the following 12 measurements of nitrate at a well used for drinking water to determine with 95% confidence whether or not the infant-based, acute risk standard of 10 mg/L has been violated. Assume that the risk standard represents an upper 95th percentile limit on nitrate concentrations.

Sampling Date	Nitrate (mg/L)	Rank
7/28/99	<5.0	(1)
9/3/99	12.3	(11)
11/24/99	<5.0	(2)
5/3/00	<5.0	(3)
7/14/00	8.1	(7)
10/31/00	<5.0	(4)
12/14/00	11.0	(10)
3/27/01	35.1	(12)
6/13/01	<5.0	(5)
9/16/01	<5.0	(6)
11/26/01	9.3	(8)
3/2/02	10.3	(9)

SOLUTION

Step 1. Half of the sample concentrations are non-detects, making a test of normality extremely difficult. One could attempt to fit these data via the *Kaplan-Meier* or *Robust ROS* adjustments (see **Chapter 15**), but here a non-parametric confidence interval around the upper 95th percentile will be constructed.

Step 2. Order the data values from least to greatest and assign ranks as in the last column of the table above. Note that the apparent ties among the non-detects have been arbitrarily broken in order to give a unique rank to each measurement.

Step 3. Using **Table 21-11** in **Appendix D** for n = 12, there is approximately 88% confidence associated with using L^* = 11 as the rank of the lower confidence bound and approximately 98% confidence associated with using L^* = 10. Since the target confidence level is 95%, it can only be achieved by using a rank of 10 or less. Thus the non-parametric LCL needs to be set to the 10th smallest observation or $x_{(10)}$. Scanning the list of nitrate measurements, the LCL = 11.0 ppm.

Step 4. Since the order statistic $x_{(10)}$ achieves a confidence level of 98%, one can conclude that the true upper 95th percentile nitrate concentration is no smaller than 11.0 ppm with 98% confidence. Even by this more stringent confidence level, the acute risk standard for nitrate is violated and there is statistically significant evidence that at least 1 of every 20 nitrate measurements from the well will exceed 10 mg/L.

Step 5. If the well was being remediated under corrective action monitorign, the fixed standard would be compared against a one-way UCL around the upper 95th percentile. In that case, for n = 12, **Table 21-11** of **Appendix D** indicates that the maximum observed value of 35.1 mg/L taken as the UCL achieves a confidence level of only 46%. 95% confidence could not be achieved unless at least 59 sample measurements were available and the UCL was set to the maximum of those values. The remedial action would be considered successful only if *all* 59 measurements were below the fixed standard of 10 mg/L. ◄

21.3 CONFIDENCE INTERVALS AROUND TREND LINES

It was assumed that the underlying population is stable, (*i.e.*, characteristics like the mean, median, or upper percentiles are stationary over the period of sampling) for the confidence intervals so far presented in this chapter. In some cases, however, the concentration data will exhibit a trend. Examples might include successful remediation efforts that serve to gradually drive down a well's concentration levels, or interception of an intensifying plume of contaminated groundwater.[3]

The problem with ignoring a discernible trend when building a confidence interval is that the interval will incorporate not only the natural variability in the underlying population, but also additional variation induced by the trend itself. The net result is a confidence interval that can be much wider than expected for a given confidence level and sample size (n). A wider confidence interval makes it more difficult to demonstrate an exceedance or return to compliance versus a fixed standard in compliance/assessment or corrective action monitoring. The confidence interval will have less statistical power to identify compliance violations, or to judge the success of remedial efforts.

When a linear trend is present, it is possible to construct an appropriate confidence interval built around the estimated trend. A continuous series of confidence intervals is estimated at each point along the trend, termed a ***simultaneous confidence band***. An upper or lower confidence band will tend to follow the estimated trend line whether the trend is increasing or decreasing. It is computed once the trend line has been estimated.

Construction of a confidence interval around a trend line presumes that a trend actually exists. The algorithms presented in this section ***assume*** that a trend is readily discernible on a time series plot of the measurements and that it is essentially linear. Otherwise, the results may be less than credible.

21.3.1 PARAMETRIC CONFIDENCE BAND AROUND LINEAR REGRESSION

BACKGROUND AND PURPOSE

A standard method for estimating a linear trend is ***linear regression***, introduced in **Chapter 17**. In this section, equations for constructing a linear regression are extended to form a ***confidence band*** around the trend. Although a parametric technique, there is no requirement that the concentration measurements be normal or transformable to normality. Instead, the ***residual concentrations*** after subtracting out the estimated trend line should be roughly normal in distribution or at least symmetric.

By way of interpretation, each point along the trend line is an estimate of the true mean concentration ***at that point in time***. As the underlying population mean either increases or decreases, the confidence band similarly increases or decreases to reflect this change.

Although the equations presented below can be used to simultaneously construct a confidence interval around each point on the trend line, in practice, the user will want to compute a confidence

[3] This might occur if the well screen first intercepts the leading edge of the plume, followed by the more heavily contaminated core.

interval for a few or several of the most recent sampling events. Because the individual confidence intervals comprising the simultaneous confidence band have a joint confidence level of (1–α), no matter how many confidence intervals are constructed, the overall false positive rate associated with the entire set of tests against the fixed standard will be no greater than a pre-specified α.

REQUIREMENTS AND ASSUMPTIONS

To accurately estimate a confidence band, the sample variance should be stationary or constant as a function of time. Although the mean level may be increasing or decreasing with time, the level of variation about the mean should be essentially the same.

Once a linear regression is fitted to the data, the residuals around the trend line should be tested for normality and apparent skewness. Inferences concerning a linear regression are generally appropriate when two conditions hold: 1) the residuals from the regression are approximately normal or at least reasonably symmetric in distribution; and 2) a plot of residuals versus concentrations indicates a scatter cloud of essentially uniform *vertical thickness or width*. That is, the scatter cloud does not tend to increase in width with the level of concentration or exhibit any kind of regular pattern other than looking like a random scatter of points.

If one or both of these conditions is seriously violated, it may indicate that the basic trend is either non-linear, or the size of the variance is not independent of the mean level. If the variance is roughly proportional to mean concentrations, one possible remedy is to try a transformation of the measurements and re-estimate the linear regression. This will change the interpretation of the estimated regression from a linear trend of the form $y = a + bt$, where y and t represent concentration and time respectively, to a non-linear pattern. As an example, if the concentration data are transformed via logarithms, the regression equation will have the form $\log y = a + bt$. On the original concentration scale, the trend function will then have the form $y = \exp\left(a + bt\right)$.

When the regression data are transformed in this way, the estimated trend in the concentration domain (after back-transforming) no longer represents the original mean. The transformation induces a *bias* in the confidence intervals comprising the confidence band when converted back to the original scale as in the case of samples with no trend. If a log transformation is used, for instance, the back-transformed confidence band around the trend line represents confidence intervals around the original-scale *geometric means* and not the *arithmetic means*. If a comparison of an estimated geometric mean or similar quantity to the fixed standard makes sense, computing a trend line on the transformed data should be acceptable. However, if a confidence interval around an arithmetic mean is required, consultation with a professional statistician may be necessary.

The technique presented here produces a confidence interval around the *mean* as a function of time and not an *upper percentile*. Thus, we recommend that the use of this method be restricted to cases where the fixed standard represents a mean concentration and not an explicit upper percentile or a 'not-to-exceed' limit.

At least 8 to 10 measurements should be available when computing a confidence band around a linear regression. There must be enough data to not only estimate the trend function but also to compute the variance around the trend line. In the simplest case when no trend is present, there are (n–1) *degrees*

of freedom [*df*] in a sample of size *n* with which to estimate the population variance. With a linear trend, however, the available degrees of freedom *df* is reduced to (*n*–2). For moderate to large samples, loss of one or two degrees of freedom makes little difference. But for the smallest samples, the impact on the resulting confidence limits can be substantial.

One last assumption is that there should be few if any non-detects when computing the regression line and its associated confidence band. As a matter of common sense, a readily discernible trend in a data set (either increasing or decreasing) should be based on quantified measurements. Changes in detection and/or RLs over time can appear as a declining trend, but may actually be an artifact of improved analytical methods. Such artifacts of plotting and data reporting should generally *not* be considered real trends.

PROCEDURE

Step 1. Construct a time series plot of the measurements. If a discernible trend is evident, compute a linear regression of concentration against sampling date (time), letting x_i denote the *i*th concentration value and t_i denote the *i*th sampling date. Estimate the linear slope with the equation:

$$\hat{b} = \sum_{i=1}^{n} (t_i - \bar{t}) \cdot x_i / (n-1) \cdot s_t^2 \qquad [21.20]$$

This estimate leads to the regression equation, given by:

$$\hat{x} = \bar{x} + \hat{b} \cdot (t - \bar{t}) \qquad [21.21]$$

where $\bar{t}$ denotes the mean sampling date, s_t^2 is the variance of the sampling dates, $\bar{x}$ is the mean concentration level, and $\hat{x}$ represents the estimated mean concentration at time *t*.

Step 2. Compute the regression residual at each sampling event with the equation:

$$r_i = x_i - \hat{x}_i \qquad [21.22]$$

Check the set of residuals for lack of normality and significant skewness using the techniques in **Chapter 10**. Also, plot the residuals against the estimated regression values ($\hat{x}_i$) to check for non-uniform vertical thickness in the scatter cloud. If the residuals are non-normal and substantially skewed and/or the scatter cloud appears to have a definite pattern (*e.g.*, funnel-shaped; 'U'-shaped; or, residuals mostly positive on one end of the graph and mostly negative on the other end, instead of randomly scattered around the horizontal line $r = 0$), repeat Steps 1 and 2 after first transforming the concentration data.

Step 3. Calculate the estimated variance around the regression line (also known as the *mean squared error* [MSE]) with the equation:

$$s_e^2 = \frac{1}{n-2} \sum_{i=1}^{n} r_i^2 \qquad [21.23]$$

Step 4. Given confidence level (1–α) and a point in time (t_0) at which a confidence interval around the trend line is desired, compute the lower and upper confidence limits with the respective equations:

$$LCL_{1-\alpha} = \hat{x}_0 - \sqrt{2s_e^2 \cdot F_{1-2\alpha,2,\,n-2} \cdot \left[\frac{1}{n} + \frac{(t_0 - \bar{t})^2}{(n-1)\cdot s_t^2}\right]} \qquad [21.24]$$

$$UCL_{1-\alpha} = \hat{x}_0 + \sqrt{2s_e^2 \cdot F_{1-2\alpha,2,\,n-2} \cdot \left[\frac{1}{n} + \frac{(t_0 - \bar{t})^2}{(n-1)\cdot s_t^2}\right]} \qquad [21.25]$$

where $\hat{x}_0$ is the estimated mean concentration at time t_0 from the regression using equation [21.21], and $F_{1-2\alpha,\,2,\,n-2}$ is the upper (1–2α)th percentage point from an F-distribution with 2 and (n–2) degrees of freedom. Values for F can be found in **Table 17-1** of **Appendix D**.

Step 5. Depending on whether the regulated unit is in compliance/assessment or corrective action monitoring, compare the appropriate confidence limit against the GWPS. Multiple confidence limits can be computed at a single compliance point well without increasing the significance level (α) of the comparison. It is possible to estimate at what point in time (if ever) the confidence limit first lies completely to one side of the fixed comparison standard, without risking an unacceptable false positive rate increase for that well.

►EXAMPLE 21-7

Trichloroethylene [TCE] concentrations are being monitored at a site undergoing remediation. If the GWPS for TCE has been set at 20 ppb, test the following 10 measurements collected at a compliance point well over the last two and a half years to determine if the clean-up goal has been reached at the α = 0.05 level of significance.

Month Sampled	TCE Concentration (ppb)	Regression Estimates	Residuals
2	54.2	51.735	2.465
4	44.3	48.530	-4.230
8	45.4	42.119	3.281
11	38.3	37.311	0.989
13	27.1	34.106	-7.006
16	30.2	29.298	0.902
20	28.3	22.888	5.412
23	17.6	18.080	-0.480
26	14.7	13.272	1.428
30	4.1	6.861	-2.761

SOLUTION

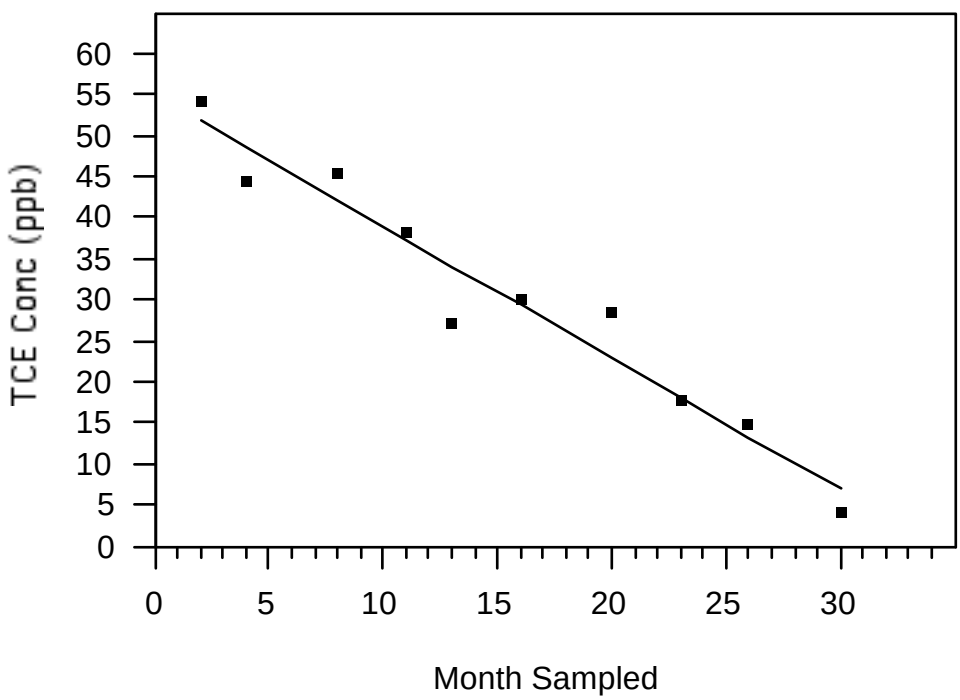
Step 1. Construct a time series plot of the TCE measurements as in the graph below (see **Figure 21-3**). A general downward, linear trend is evident. Then compute the estimated regression line

using equations [21.20] and [21.21], first determining that the mean time value is $\bar{t} = 15.3$, the variance of time values is $s_t^2 = 88.2333$, and the mean TCE measurement is $\bar{x} = 30.42$ ppb:

$$\hat{b} = [(2-15.3)\cdot 54.2 + (4-15.3)\cdot 44.3 + \ldots + (30-15.3)\cdot 4.1]/(9\times 88.2333) = -1.603\ ppp/month$$

$$\hat{y} = 30.42 - 1.603\cdot(t-15.3)$$

Figure 21-3. Time Series Plot and Regression Line of TCE Measurements



Step 2. Compute the regression residuals using equation [21.22] (listed in the table above). Note that the residuals are found by first computing the regression line estimate for each sampled month (*i.e.*, t = 2, 4, 8, *etc.*) and then subtracting these estimates from the actual TCE concentrations. A probability plot of the regression residuals appears reasonably linear (**Figure 21-4**) and the Shapiro-Wilk statistic computed from these data yields SW = 0.962, well above the α = 0.05 critical point for n = 10 of $sw_{.05,10}$ = 0.842. Thus, normality of the residuals cannot be rejected.

In addition, a plot of the residuals versus the regression line estimates (**Figure 21-5**) exhibits no unusual pattern, merely random variation about the residual mean of zero. Therefore, proceed to compute a confidence interval around the trend line.

Figure 21-4. Probability Plot of TCE Residuals

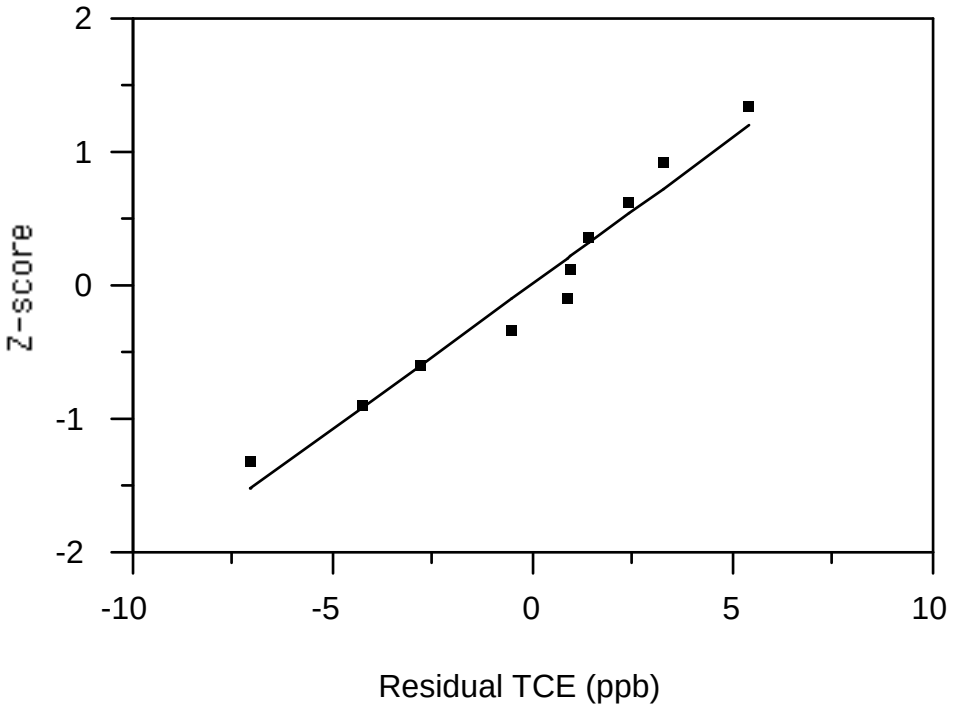
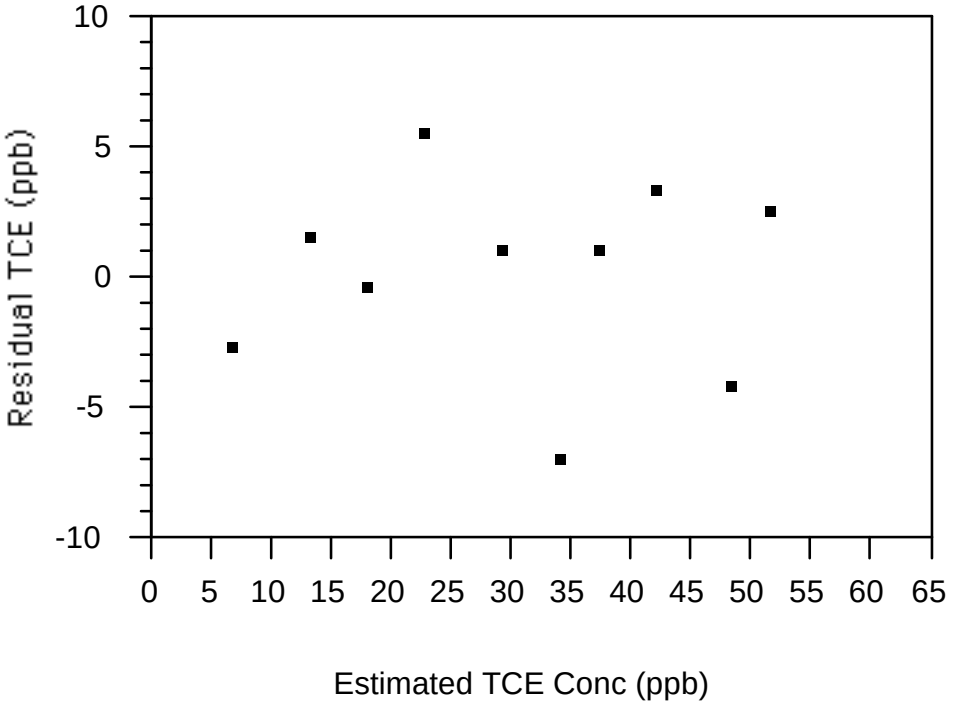


Figure 21-5. Scatterplot of TCE Residuals vs. Regression Line Estimates



Step 3. Compute the variance around the estimated trend line using equation [21.23]:

$$s_e^2 = \frac{1}{8} \cdot \left[(2.465)^2 + (-4.230)^2 + \ldots + (-2.761)^2 \right] = 15.60$$

Step 4. Since the comparison to the GWPS of 20 ppb is to be made at the $\alpha = 0.05$ significance level, the confidence limit is $(1-\alpha) = 95\%$ confidence. Since the remediation effort aims to demonstrate that the true mean TCE level has dropped below 20 ppb, a one-way UCL needs to be determined using equation [21.25]. A logical point along the trend to examine is the last sampling event at $t_0 = 30$. Using the estimated regression value at $t_0 = 30$, and the fact that $F_{.90,2,8} = 3.1131$, the UCL on the mean TCE concentration at this point becomes:

$$UCL_{.95} = 6.861 + \sqrt{2 \times 15.60 \times 3.1131 \times \left[\frac{1}{10} + \frac{(30-15.3)^2}{8 \times 88.2333} \right]} = 13.14 \text{ ppb}$$

Since this upper limit is less than the GWPS for TCE, conclude that the remediation goal has been achieved by $t_0 = 30$. In fact, other times can also be tested using the same equation. At the next to last sampling event ($t_0 = 26$), the UCL is:

$$UCL_{.95} = 13.272 + \sqrt{2 \times 15.60 \times 3.1131 \times \left[\frac{1}{10} + \frac{(26-15.3)^2}{8 \times 88.2333} \right]} = 18.32 \text{ ppb}$$

which also meets the remediation target at the $\alpha = 0.05$ level of significance.

Step 5. If the linear trend is ignored, a onew-way UCL of the mean might have been used. The overall TCE sample mean $\bar{x} = 30.42$, the TCE standard deviation $s = 15.508$, the upper 95th percentage point of the t-distribution with 9 degrees of freedom is $t_{.95,9} = 1.8331$. Using equation [21.2] with the same data yields the following:

$$UCL_{.95} = 30.42 + (1.8331)(15.508)/\sqrt{10} = 39.41 \text{ ppb}$$

Had the linear trend been ignored when computing the UCL, the remediation target would not have been achieved. The downward trend induces the largest part of the variation observed over the two and a half years of sampling and needs to be taken into account.◀

21.3.2 NON-PARAMETRIC CONFIDENCE BAND AROUND THEIL-SEN LINE

BACKGROUND AND PURPOSE

The Theil-Sen trend line is introduced in **Section 17.3.3** as a non-parametric alternative to linear regression. Whether due to the presence of non-detects or trend residuals that cannot be normalized, the

Theil-Sen method can usually construct a trend estimate without some of the assumptions needed by linear regression.

The Theil-Sen trend line is non-parametric because it combines the median pairwise slope (**Section 17.3.3**) with the median concentration value and the median sample date to construct the trend. Because of this construction, the Theil-Sen line estimates the change in *median* concentration over time and not the *mean* as in linear regression.

There are no simple formulas to construct a confidence band around the Theil-Sen line. However, a more computationally-intensive technique — bootstrapping — can be employed instead. The conceptual algorithm is fairly simple. First consider the set of n pairs of measurements used to construct the Theil-Sen trend. Each pair consists of a sample date (t_i) and the concentration value measured on that date (x_i) as a statistical sample. Next, repeatedly draw samples of size n *with replacement* from the original sample of pairs. These artificially constructed samples are known as *bootstrap samples*. At least 500 to 2,000 bootstrap samples are generated in order to improve the accuracy of the final confidence band. Note that a bootstrap sample is not precisely the same as the original because pairs are sampled with replacement. This means that a given pair might show up multiple times in any particular bootstrap sample.

For each bootstrap sample, use the Theil-Sen algorithm to construct an associated trend line (**Section 17.3.3**). Each of these trend lines is known as a *bootstrap replicate*. Finally, determine the distribution of the bootstrap replicates and select certain percentiles of this distribution to form lower and upper confidence limits. These limits can be constructed to represent a non-parametric simultaneous confidence band around the Theil-Sen trend line with (1–α) confidence.

REQUIREMENTS AND ASSUMPTIONS

The key requirements for constructing a confidence band around a Theil-Sen trend are the same as for the Theil-Sen procedure itself (**Section 17.3.3**). As a non-parametric procedure, the trend residuals do not have to be normal or have equal variance across the data range. But the residuals are assumed to be statistically independent. Approximate checks of this assumption can be made using the techniques of **Chapter 14**, after removing the estimated Theil-Sen trend and as long as there aren't too many non-detects. It is also important to have at least 8-10 observations from which to construct the bootstrap samples.

Non-detects can be accommodated by the Theil-Sen method as long as the detection frequency is at least 50%, and the censored values occur in the lower part of the observed concentration range. Then the median concentration value and the median pairwise slope used to compute the Theil-Sen trend will be based on clearly quantified values.

Since there are no simple mathematical equations which can construct the Theil-Sen confidence band, a computer software program is essential for performing the calculations. Perhaps the best current solution is to use the open-source, free-of-charge, statistical computing package **R** (www.r-project.org). A template program (or script) written in **R** to compute a Theil-Sen confidence band is listed in **Appendix C**. This script can be adapted to any site-specific data set and used as many times as necessary, once the **R** computing environment has been installed.

PROCEDURE

Step 1. Given the original sample of n measurements, form a sample of n pairs (t_i, x_i), where each pair consists of a sample date (t_i) and the concentration measurement from that date (x_i).

Step 2. Form B bootstrap samples by repeatedly sampling n pairs at random with replacement from the original sample of pairs in Step 1. Typically, set $B \geq 500$.

Step 3. For each bootstrap sample, construct a Theil-Sen trend line using the algorithm in **Section 17.3.3**. Denote each of these B trend lines as a bootstrap replicate.

Step 4. Determine a series of equally spaced time points (t_j) along the range of sampling dates represented in the original sample, j = 1 to m. At each time point, use the Theil-Sen trend line associated with each bootstrap replicate to compute an estimated concentration ($\hat{x}_j^B$). There will be B such estimates at each of the m equally-spaced time points when this step is complete.

Step 5. Given a confidence level (1–α) to construct a two-sided confidence band, determine the lower (α/2)th and the upper (1–α/2)th percentiles, denoted $\hat{x}_j^{[\alpha/2]}$ and $\hat{x}_j^{[1-\alpha/2]}$ from the distribution of estimated concentrations at each time point (t_j). The collection of these lower and upper percentiles along the range of sampling dates (t_j, j = 1 to m) forms the bootstrapped confidence band. To construct a lower confidence band, follow the same strategy. But determine the lower αth percentile $\hat{x}_j^{[\alpha]}$ from the distribution of estimated concentrations at each time point (t_j). For an upper confidence band, compute the upper (1–α)th percentile, $\hat{x}_j^{[1-\alpha]}$ at each time point (t_j).

Step 6. Depending on whether the regulated unit is in compliance/assessment or corrective action monitoring, compare the appropriate confidence band against the GWPS. Estimate at what point in time (if ever) the confidence band first sits completely to one side of the fixed comparison standard.

►EXAMPLE 21-8

In **Example 17-7**, a Theil-Sen trend line was estimated for the following sodium measurements. Note that the sample dates are recorded as the year of collection (2-digit format), plus a fractional part indicating when during the year the sample was collected. Construct a two-sided 95% confidence band around the trend line.

Sample Date (yr)	Sodium Conc. (ppm)
89.6	56
90.1	53
90.8	51
91.1	55
92.1	52
93.1	60
94.1	62
95.6	59
96.1	61
96.3	63

SOLUTION

Step 1. Designate the n = 10 (sample date, concentration) pairs as the original sample for purposes of bootstrapping. Set the number of bootstrap samples to N_B = 500.

Step 2. Sample at random and with replacement N_B = 500 times from the original sample to form the bootstrap samples. Compute a bootstrap replicate Theil-Sen trend line for each bootstrap sample. This gives 500 distinct linear trend lines.

Step 3. Divide the observed range of sampling dates from 89.6 to 96.3 into m = 101 equally-spaced time points, t_j (note: choice of m is arbitrary, depending on how often along the time range an estimate of the confidence band is needed). At each time point, compute the Theil-Sen concentration estimate using each bootstrap replicate trend. This leads to 500 estimates of the form:

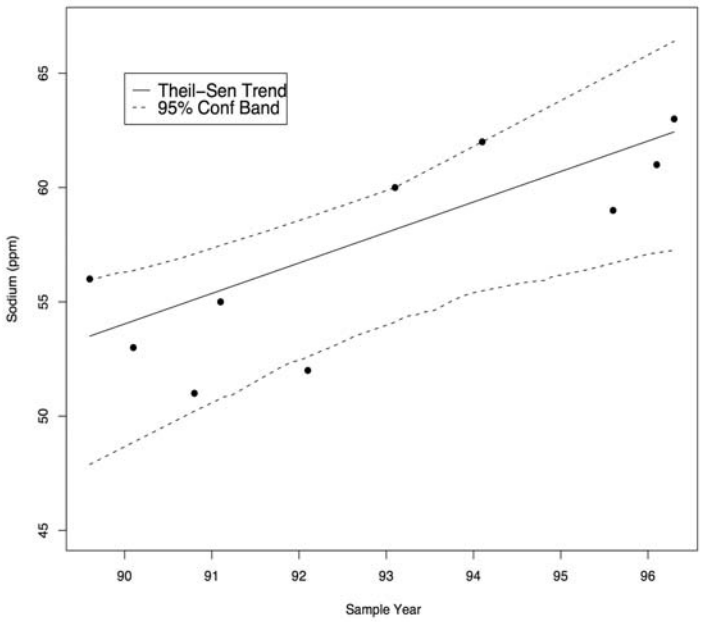
$$\hat{x}_j^B = \tilde{x}^B + Q^B \cdot \left(t_j - \tilde{t}^B\right)$$

where $\tilde{x}^B$ is the median concentration of the Bth bootstrap sample, Q^B is the Theil-Sen slope of the Bth bootstrap sample, and $\tilde{t}^B$ is the median sampling date of the Bth bootstrap sample.

Step 4. Given a two-way confidence level of 95%, compute the lower $\alpha/2 = 0.05/2 = 0.025$ and upper $(1-\alpha/2) = (1-0.05/2) = 0.975$ sample percentiles (**Chapter 3**) for the set of 500 concentration estimates associated with each time point (t_j). This entails sorting each set and finding the value closest to rank $(n+1) \times p$, where p = desired percentile. In a list of n = 500, find the sorted values closest to the ranks $501 \times 0.025 = 12.525$ for the lower percentile and $501 \times 0.975 = 488.475$ for the upper percentile. Collectively, the lower and upper percentiles plotted by the time points give an approximation to the 95% two-sided confidence band.

Step 5. Plot the lower and upper confidence bands as well as the original Theil-Sen trend line and the raw sodium measurements, as in **Figure 21-6**. The fact that the trend is increasing over time is confirmed by the rising confidence band. ◄

Figure 21-6. 95% Theil-Sen Confidence Band on Sodium Measurements



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CHAPTER 22. COMPLIANCE/ASSESSMENT AND CORRECTIVE ACTION TESTS

22.1 CONFIDENCE INTERVAL TESTS FOR MEANS 22-1
22.1.1 Pre-Specifying Power In Compliance/Assessment 22-2
22.1.2 Pre-Specifying False Positive Rates in Corrective Action 22-9
22.2 CONFIDENCE INTERVAL TESTS FOR UPPER PERCENTILES 22-18
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22.2.2 Upper Percentile Tests in Corrective Action 22-20

Chapter 7 lays out general strategies for statistical testing in compliance/assessment and corrective action monitoring via the use of confidence intervals. Procedures for constructing confidence intervals are described in **Chapter 21**. This chapter discusses potential methods for developing confidence interval tests so that adequate statistical power is maintained in compliance/assessment monitoring and false positive rates are minimized in corrective action monitoring.

22.1 CONFIDENCE INTERVAL TESTS FOR MEANS

As discussed in **Chapter 7**, EPA's primary concern in compliance/assessment and corrective action monitoring is the identification and remediation of contaminated groundwater. The basic statistical hypotheses are reversed in these two phases of monitoring as described in **Chapter 21** and earlier. The lower confidence limit [LCL] is of most interest in compliance/assessment, while the upper confidence limit [UCL] is used in corrective action. Statistical power is also of greater concern to the regulatory agency in compliance/assessment— representing the probability that contamination above a fixed standard will be identified. A sufficiently conservative false positive rate during corrective action is important from a regulatory standpoint, since a false positive implies that contaminated groundwater has been falsely declared to meet a compliance standard. The reverse of these risks is generally true for a regulated entity.

To ensure that contaminated groundwater is treated in ways that are statistically sound, the two specific strategies which follow separately address compliance/assessment monitoring and formal testing in corrective action. The latter occurs after the completion of remedial activities or when potential compliance can be anticipated. Each strategy is designed to allow stakeholders on both sides of the regulator/regulated divide to understand the expected statistical performance of a given confidence interval test.

The two strategies which follow are based on the behavior of the *normal mean* confidence interval. They especially assume that the monitoring data are stationary over the period of record. Other important assumptions were discussed in **Chapter 21**. In the discussion which follows, consideration is given to data that is normal following a logarithmic transformation and the possible tests which can be applied.

22.1.1 PRE-SPECIFYING POWER IN COMPLIANCE/ASSESSMENT

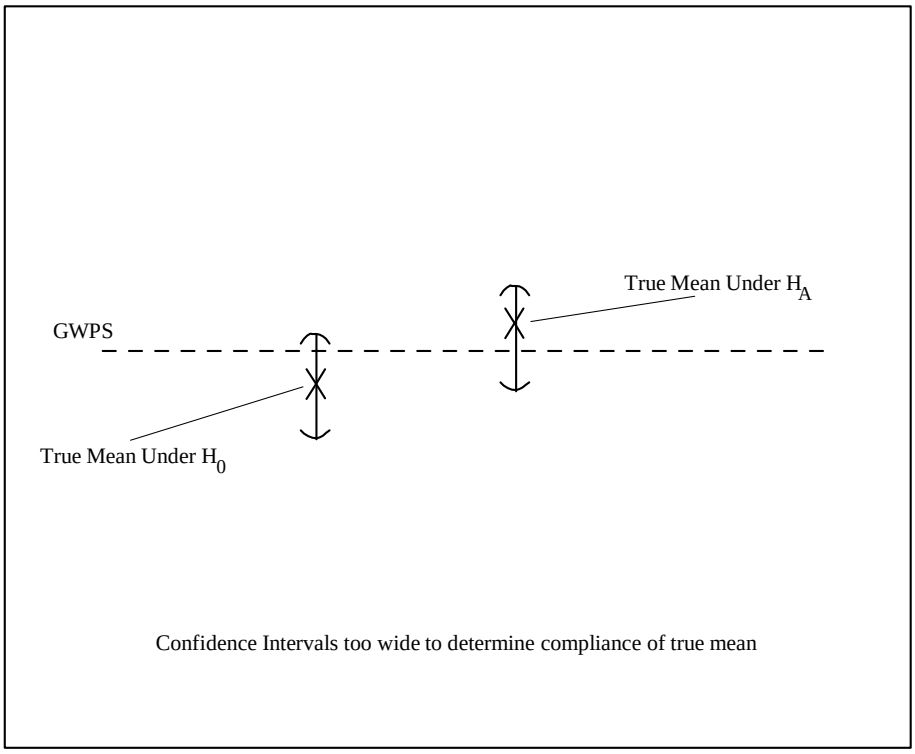
In most statistical literature including Gibbons & Coleman (2001) comparing a confidence interval against a fixed standard, a low false positive error rate (α) is chosen or recommended without respect to the power of the test. However, the power to detect increases above a fixed standard using a lower confidence limit around the mean can be negligible when contaminant variability is high and the sample size is small (**Chapter 7**). To remedy this problem, the Unified Guidance suggests an alternate strategy. That is, instead of pre-specifying the false positive rate α prior to computing confidence interval limits, a desired level of power ($1-\beta$) should be set as an initial target.

Ideally one would like to simultaneously minimize α and maximize power by also minimizing β (*i.e.*, the false negative rate). However, this is generally impossible given a fixed sample size (**Chapter 3**), since there is a trade-off between power and the false positive rate. Especially for small sample sizes, fixing a low α often leads to less than desirable power. Conversely, pre-specifying a high power necessitates a higher than typical false positive rate. Larger sample sizes are needed if both power and α are pre-specified. High variability at a fixed sample size both lowers power and/or increases the need for a larger false positive error rate.

A number of considerations are relevant when constructing mean confidence limits to achieve adequate statistical power. In most Agency risk assessment evaluations, chronic risk levels are generally *proportional* to the average concentration. Development of MCLs followed similar proportional risk methodologies. Fixed health-based limits which can serve as groundwater protection standards [GWPS] also cover an enormous concentration range when both carcinogenic and non-carcinogenic constituents are included.

Another relevant factor pertains to those situations where the true mean concentrations lie quite close to either side of a compliance standard. The difference between complying and not complying with the GWPS in terms of the true mean concentration level may be so small as to make a clear determination of compliance very difficult (**Figure 22-1**). Only sufficiently large differences relative to a standard are likely to be determined with a high level of certainty (*i.e.*, statistical power).

Figure 22-1. True Means Too Close to Standard to Clearly Identify Violation



With the wide range of GWPS in place and recognizing that risk factors are proportional or multiplicative rather than additive (***e.g.***, a 10^{-6} cancer risk), it would be appropriate to use a consistent measure of increased risk that is ***independent*** of the actual GWPS concentration level. While ultimately the decision of the regulatory authority, the Unified Guidance suggests a proportional increase (***i.e.***, a ratio) above the GWPS, which is identified at some predetermined level of statistical power to judge the appropriateness of any specific mean confidence interval test.

For compliance/assessment monitoring purposes, increases in the true concentration mean of 1.5 and 2 times a fixed standard are evaluated at a range of confidence levels. While this is not quite the same as evaluating an ***absolute*** mean increase for a given constituent, the use of a risk ratio (***R***) does in fact define a specific increase in concentration level. For example, a risk ratio of 1.5 would identify a critical increase above the 15 µg/l MCL standard for lead of 22.5 – 15 = 7.5 µg/l, while for chromium with an MCL = 100 ug/l, the absolute increase would be 50 ug/l. Each represents a 50% increase in risk relative to the GWPS.

Two approaches for assessing statistical power in compliance/assessment monitoring are provided using these critical risk ratios, based on different assumptions regarding sample variability. In the first approach, a constant population variance is assumed, equal to the standard (***i.e.***, GWPS) being tested. Under the null hypothesis that the true population mean is no greater than the GWPS, this assumption corresponds to having a coefficient of variation [*CV*] of 1 when the true mean equals the standard. Although observed sample variability is ignored, this case can be considered a relatively conservative approach.

Assuming CV = 1, the relationship between the risk ratio (R), statistical power (1–β), sample size (n), and the false positive rate (α) can be obtained using the following equation:

$$1-\beta = G_{T,n-1}\left(t_{1-\alpha,n-1}\middle|\Delta=\sqrt{n}\left(R-1\right)\right) \qquad [22.1]$$

where $t_{1-\alpha,n-1}$ is the (1–α)th Student's t-quantile with (n–1) degrees of freedom and $G_{T,n-1}\left(\bullet\middle|\Delta\right)$ represents the cumulative *non-central* t-distribution with (n–1) degrees of freedom and non-centrality parameter Δ. By fixing a desired or target power level, equation [22.1] can be used to choose the necessary α based on the available sample size n. Alternatively, the equation can be used to determine the sample size (n) needed to allow for a pre-determined choice of α.

Numerical tabulations of equation [22.1] are found in **Tables 22-1** and **22-2** in **Appendix D**. These tables cover a practical range of n = 3 to 40 and α = .001 to .20, and offer combinations of the minimum false positive rate (α) and sample size (n) for several fixed levels of power. These can be used to construct lower confidence limits having a pre-specified level of power. It is important to note that the listed combinations are the *smallest* α-values resulting in the targeted power. For a fixed n, use of an α-value *larger* than that listed in the tables will provide even greater power than the target. Similarly, for given α, use of a larger sample size than that listed in the tables will also result in greater power than the target.

Minimum parameter values are presented in **Tables 22-1** and **22-2** of **Appendix D** to document how the desired power level can be achieved with as few observations and as small a false positive error rate as possible. It is also true that an assumption of CV = 1 should be somewhat conservative at many sites. Actual power will be higher than that listed in these tables if the coefficient of variation is smaller. Not every power level is achievable in every combination of n and α, so some of the entries in these two tables are left blank.

The second approach requires an estimate of the population coefficient of variation. In this case, the required (but approximate) false positive rate of the test can be directly obtained from equation [22.2], where R is the desired risk ratio, n is the sample size, $C\hat{V}$ is the estimated sample coefficient of variation, $t_{1-\beta,n-1}$ is the (1–β)th Student's t-quantile with (n–1) degrees of freedom, and $F_{T,n-1}\left(\bullet\right)$ is the cumulative (central) Student's t-distribution function:

$$a \cong 1 - F_{T,n-1}\left(\frac{\left(R-1\right)\cdot\sqrt{n}}{R\cdot C\hat{V}} - t_{1-\beta,n-1}\right) \qquad [22.2]$$

Equation [22.2] was evaluated for sample sizes varying from n = 4 to 12 and for CVs ranging from 0.1 to 3.0 at two target combinations of power and risk ratio — R = 1.5 at 50% power and R = 2 at 80% power. Results of these calculations are provided in **Table 22-3** of **Appendix D**. Similar to the critical power targets recommended by the Unified Guidance in detection monitoring (*i.e.*, 55-60% power at 3σ above background, and 80-85% power at 4σ over background), two high power targets at proportionally increasing risk ratios were also chosen for this setting.

Table 22-3 in **Appendix D** provides the approximate minimum false positive rate (α) necessary to achieve each power target in a single confidence interval test. The shaded and italicized entries in the table represent those cases where the minimum α is below the RCRA regulatory limitation of α = .01 from §264.97(i)(2) for an individual test false positive error rate. For these situations, the user would need to set α = 0.01, which in turn would provide even greater statistical power than the target.

For higher estimated *CV*s, many of the entries in this table exceed α = .5 (bolded entries). These cases illustrate the difficulty of simultaneously attaining the recommended level of power while controlling the false positive rate, especially for small sample sizes and highly variable data. Setting a lower α, results in insufficient statistical power. On the other hand, setting $\alpha \geq .5$ amounts to a simple comparison of the sample mean against the fixed standard, with essentially no adjustment for sample variability or uncertainty. Similar to the first approach, a maximum false positive rate of α = .2 is a reasonable upper bound which implies at most a 1-in-5 chance of an error.

Generally speaking, setting 80% power at a risk ratio of R = 2 in **Table 22-3** of **Appendix D** is more constraining (requiring higher α's) than 50% power at a risk ratio of R = 1.5, although the effect can be reversed for low *CV*s and sample sizes. To meet both targets simultaneously for a given n, the larger of the corresponding significance levels (α) should be selected. Guidance users may choose either of the two approaches described above. Other ratio and power options not covered in **Tables 22-1** through **22-3** of **Appendix D** can be handled by direct computation using either equation [22.1] or equation [22.2]. The first method makes an *a priori* assumption about the *CV*. The second method is approximate, depending on a sample *CV* estimate which might be erratic at small sample sizes and larger true population *CV*s especially if the compliance data are non-normal.

Both approaches are directly applicable to the normal mean LCL test in **Section 21.1.1**. While the *CV* can be directly estimated using $s/\bar{x}$ on the original concentration data, this statistic will underestimate the likely variability when data are lognormal. In that case, the logarithmic CV estimate in **Chapter 10**, **Section 10.4** should be used. If the data best fit a lognormal distribution, a number of considerations follow:

- It is possible to misapply the normal mean confidence interval test using the original concentration data, even when the data stem from a lognormal distribution. The mean is relatively robust with respect to departures from normality as long as the CV variability is not too great. If the predetermined false positive error α is selected based on the normal power criteria above, the resulting LCL test will be at least as powerful as the normal test. The actual false positive error rate will also differ.

- If a geometric mean test in **Section 21.1.2** is used, the LCL should be computed from the logarithmically transformed data. **Tables 22-1** to **22-3** in **Appendix D** are based on normal distribution assumptions and the error rates are very conservative with respect to the achievable power. As an example, given a data set from a lognormal distribution with n = 10, and an estimated CV = .8, an alpha value of .151 can be identified from **Table 22-3** in **Appendix D**. The actual power to detect a doubling above a GWPS at 80% confidence

would result in a power level of 94.5%. The false positive needed to detect a geometric mean doubling for this example to meet the above criteria would be α = .026.[1]

- The Land lower confidence interval test from **Section 21.1.3** can also be used. But since there are limited α-choices in the tables, the guidance option is to select a fixed limit of .01, .05, or .1. If data are truly lognormal, the power of this test is at least as great as would be predicted by equations [22.1] and [22.2]. Otherwise, professional statistical assistance may be necessary.

Since compliance data will often be pooled over time to increase the eventual sample size (**Chapter 7**), the two approaches can be combined by determining the false positive rate for a given risk ratio and power level during the first year with **Tables 22-1** and **22-2** of **Appendix D**. Tests in subsequent years might use the second power approach when a better *CV* estimate (using more data) can be derived. Overall, each approach should provide a reasonable manner of adjusting the individual test false positive rate (α) to ensure adequate power to detect real contaminant increases. As a general guide, the Unified Guidance suggest formulating power in terms of risk ratios no higher than R = 2. There should be at least 70-80% statistical power for detecting increases of that magnitude during compliance/assessment monitoring.

►EXAMPLE 22-1

Compliance monitoring recently began at a solid waste landfill. Measurements for vinyl chloride during detection monitoring are listed below for two compliance wells. If a value of 5 ppb vinyl chloride is used as the GWPS and a confidence interval test must have 80% power for detecting an increase in mean vinyl chloride levels of twice the GWPS, how should the confidence interval bounds be constructed and what do they indicate? Assume that compliance monitoring began with Year 2 of the sampling record and that annual groundwater evaluations are required.

Vinyl Chloride Concentrations (ppb)

Sample	GW-1	GW-2	Sample	GW-1	GW-2
Q1, Yr1	6.3	5.9	Q1, Yr3	8.4	13.8
Q2, Yr1	9.5	3.0	Q2, Yr3	6.4	5.6
Q3, Yr1	8.1	8.8	Q3, Yr3	8.9	11.0
Q4, Yr1	11.9	12.0	Q4, Yr3	4.9	9.8
Q1, Yr2	7.3	11.2	Q1, Yr4	9.6	6.3
Q2, Yr2	11.2	8.6	Q2, Yr4	9.7	10.4
Q3, Yr2	6.0	12.6	Q3, Yr4	8.7	7.5
Q4, Yr2	7.5	7.2	Q4, Yr4	8.7	9.7

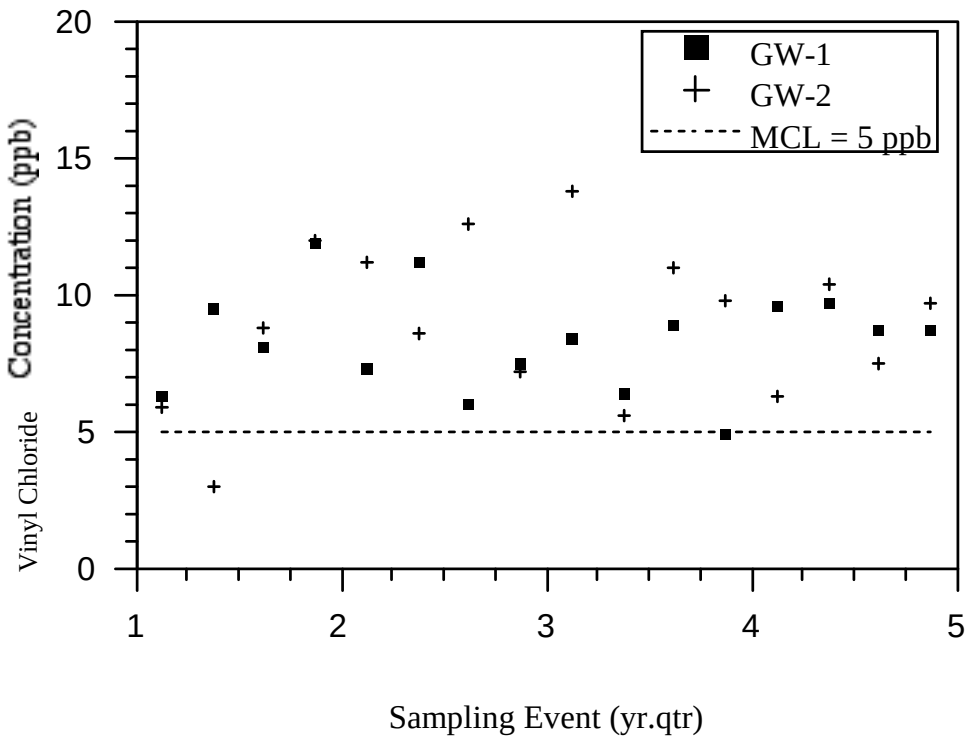
SOLUTION

Step 1. Assume for purposes of this example that the vinyl chloride data are approximately normal. In practice, this should be explicitly checked. Also evaluate potential trends in the vinyl chloride

[1] For users with access to statistical software containing the cumulative non-central t-distribution, the inverse non-central t CDF can be used to identify the appropriate false positive level. For sample size df =n -1= 9, and a non-central t parameter = $\delta = \sqrt{n} \cdot \log(R) / s_y$, the appropriate central t-value can be obtained from $F^{-1}(df, \beta, \delta)$. The confidence level of this t-value is 1 - α. For the example, df = 9, β= .2, δ = 3.115, and the central t-value is 2.23 with α = .0264.

measurements over time, as in the time series plot of **Figure 22-2**. Despite apparent fluctuations, no obvious trend is observed. So treat these data as if the population has a stable mean at least for the time frame indicated in the sampling record.

Figure 22-2. Vinyl Chloride Time Series Plot



Step 2. Given that compliance monitoring began in Year 2, use the four measurements available from each well to construct lower confidence limits. Since 80% power is desired for detecting vinyl chloride increases of two times the 5 ppb GWPS, **Table 22-2** in **Appendix D** indicates that for n = 4, a false positive rate of α = 0.163 must be used to guarantee the desired power. This corresponds to a Student's t-quantile of $t_{1-\alpha,n-1} = t_{.837,3} = 1.1714$. Then using the sample means and standard deviations of the Year 2 vinyl chloride measurements, the lower confidence limits can be computed as:

$$LCL_{GW-1} = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} = 8.0 - 1.1714\left(2.2346/\sqrt{4}\right) = 6.7 \text{ ppb}$$

$$LCL_{GW-2} = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} = 9.9 - 1.1714\left(2.4468/\sqrt{4}\right) = 8.5 \text{ ppb}$$

Step 3. Since both lower confidence limits exceed the GWPS, there is statistically significant evidence of an increase in vinyl chloride at these wells above the compliance limit. Such a conclusion also seems reasonable from **Figure 22-2**. However, the chance is better than 15% (*i.e.*, α =

16.3%) that the apparent exceedance is merely a statistical artifact. If power criteria are ignored and a fixed minimum rate of α = .01 is used, the lower confidence limits would be:

$$LCL_{GW-1} = \bar{x} - t_{\alpha,n-1}\frac{s}{\sqrt{n}} = 8.0 - 4.541\left(2.2346/\sqrt{4}\right) = 2.9 \text{ ppb}$$

$$LCL_{GW-2} = \bar{x} - t_{\alpha,n-1}\frac{s}{\sqrt{n}} = 9.9 - 4.541\left(2.4468/\sqrt{4}\right) = 4.3 \text{ ppb}$$

Neither limit now exceeds the GWPS, so the vinyl chloride concentrations would be judged in compliance with this test, illustrating the lack of power in lowering the false positive rate (α).

Step 4. To increase the confidence level (*i.e.*, by lowering α) of the tests at the end of the first year of compliance monitoring (*i.e.*, Year 2 in the preceding table of vinyl chloride values) ***without*** losing statistical power, combine the measurements from Years 1 and 2, where Year 1 samples represent the final measurements from detection monitoring prior to the start of compliance monitoring. In this case, n = 8, and the minimum false positive rate from **Table 22-2** of **Appendix D** can be lowered to α = .046 or approximately 4.5%. Then the re-computed lower confidence limits $LCL_{GW-1} = 7.0\ ppb$ and $LCL_{GW-2} = 6.4\ ppb$ again both exceed the GWPS, indicating significant evidence of a compliance violation.

Step 5. If the strategy presented in **Step 4** of combining measurements from detection monitoring and compliance monitoring is considered untenable, additional confirmation of the results can be made at the end of Year 3 by combining the first two years of compliance monitoring samples and ignoring the measurements from Year 1. Again with n = 8, the minimum false positive rate guaranteeing at least 80% power will be α = .046. The lower confidence limits are then:

$$LCL_{GW-1} = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} = 7.575 - 1.9512\left(1.9521/\sqrt{8}\right) = 6.2 \text{ ppb}$$

$$LCL_{GW-2} = \bar{x} - t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} = 9.975 - 1.9512\left(2.7473/\sqrt{8}\right) = 8.1 \text{ ppb}$$

Step 6. An even lower false positive rate can be achieved after the first three years of compliance monitoring. Pooling these measurements gives n = 12. Then **Table 22-2** in **Appendix D** identifies a minimum false positive rate of α = .013 or less than 1.5%. In this case, the lower confidence limits $LCL_{GW-1} = 6.8\ ppb$ and $LCL_{GW-2} = 7.6\ ppb$ again exceed the GWPS, confirming the previous vinyl chloride exceedances from either Year 2, Years 1 and 2 combined, or Years 2 and 3 combined. Furthermore, not only is the false positive rate quite low, but the power of the test still meets the pre-specified target. ◄

22.1.2 PRE-SPECIFYING FALSE POSITIVE RATES IN CORRECTIVE ACTION

As noted earlier, the primary regulatory concern in formal corrective action testing is false declarations of remedial success. If groundwater is truly contaminated above a regulatory standard yet a statistical test result indicates the concentrations are no longer so elevated, then on-going contamination has been missed and the remedial process should not be exited. Statistically, this idea translates into a desire to minimize the corrective action false positive rate (α). False positives in corrective action are precisely those decisions where the true concentration mean is falsely identified to be below the regulatory standard, when in fact it still exceeds the standard.

Constructing confidence interval tests by fixing a low target false positive rate is straightforward. All of the confidence interval tests presented in **Chapter 21** can be calibrated for choice of α. What is not straightforward is how best to incorporate statistical power in corrective action. As with any confidence interval test, selecting a low α when the sample size is small typically results in a confidence limit with low power. Power under corrective action monitoring represents the probability that the upper confidence limit [UCL] will fall below the fixed standard when in fact the true population mean is also less than the standard. Facilities undergoing remediation clearly have an interest in demonstrating the success of those clean-up efforts. They therefore may want to maximize the power of the confidence interval tests during corrective action, under the constraint that α must be kept low.

What statistical power criteria might a facility reasonably define in corrective action testing? Because of the orders of magnitude range found among various GWPS, a risk ratio approach similar to what is suggested in **Section 22.1.1;** only in this case, the target ratios (R) are *less than one*. While a true mean at a level of $R = 0.9$ times a given standard might be declared in compliance very infrequently, one at $R = 0.5$ times or $R = 0.25$ times the standard should meet the compliance requirements much more often. By using a consistent risk ratio across a variety of constituents, absolute decreases in the mean concentration are consistent with an assumed level of risk.

Unlike the risk ratio method detailed for compliance/assessment monitoring, where power was pre-specified but a combination of the false positive rate (α) and sample size (n) might be varied to meet that power level, in corrective action both power and α are likely to be pre-specified (power by the facility and α by the regulatory authority). The remaining component is how large a sample size is needed to attain the desired level of power, given a pre-specified false positive rate (α).

The normal distribution can be used to estimate sample size requirements for such risk ratios, given a specific false positive rate (α) and desired level of power (1–β). There is likely to be uncertainty, however, in the degree of sample variation, as expressed by the *CV*. Since the constituents in a contaminated aquifer may be modified by remedial actions, it can be difficult to estimate *future* variability (and the *CV*) from pre-treatment data. In some situations, a decrease in the mean over time might be paralleled by a decrease in total variation. If proportional, the *CV* would remain relatively constant. However, the *CV* could decrease or increase depending on aquifer conditions, constituent behavior, *etc*. The best that can be recommended is to develop an estimate of the expected future *CV* under conditions of aquifer stability.

As with compliance/assessment testing, future year estimates of the *CV* could be developed from the accumulated previous years' data. Sample sizes necessary to meet specific power targets (1–β) can

then be generated from the following approximate equation, where R = fractional risk ratio (less than 1.0), (1–α) is the desired confidence level, and $\widetilde{CV}$ = estimated coefficient of variation:

$$n = \left[R \cdot \left(t_{1-\alpha,n-1} + t_{1-\beta,n-1}\right) \cdot C\hat{V} \big/ (1-R)\right]^2 \qquad [22.3]$$

Since n appears on both sides of equation [22.3], it has to be solved iteratively for trial-and-error choices of n, making it difficult to calculate without a proper computing environment. **Tables 22-4** to **22-6** in **Appendix D** provide requisite sample sizes (n) based on equation [22.3] for three specific risk ratios (R = .75, .50, and .25) over a variety of inputs of α, β, and CV.

These tables can be consulted when designing a remedial program, especially when determining a sampling frequency adequate for generating the minimally needed sample size over a specific period of time. For example, to detect a drop in the true mean down to 0.75 × GWPS (*i.e.*, R = 0.75) with 80% power when CV = 0.6, **Table 22-4** in **Appendix D** indicates that a minimum of n = 16 observations are needed to have a false positive rate (α) no greater than 10%. Demonstrating such a reduction over the next two years might then require the collection of 8 measurements per year (or two per quarter) from the compliance well involved.[2]

While **Tables 22-4** to **22-6** in **Appendix D** identify the sampling requirements needed to simultaneously meet pre-specified targets for power (1–β) and the false positive rate (α), they come with some limitations. First, many of the minimum sample sizes are prohibitively large when sample variation as measured by the CV is substantial. Proving the success of any remedial program will be difficult when the compliance data exhibit significant relative variability. Less sampling is required to demonstrate a more substantial concentration drop below the compliance standard than to demonstrate a slight decrease (*e.g.*, compare the sample sizes for R = 0.75 to R = 0.25). This fact mirrors the statistical truth in both detection and compliance/assessment monitoring that highly contaminated wells are more easily identified (and require fewer observations to do so) than are only mildly contaminated wells.

Another limitation of equation [22.3] is that it assumes all n measurements are statistically independent. This assumption puts practical limits on the amount of sampling at a compliance well that can reasonably be achieved over a specific time period. Samples obtained too frequently may be autocorrelated and thus violate statistical independence. Minimum sample sizes do not apply to data exhibiting an obvious trend, and are appropriate only when the aquifer is in a relatively steady-state. Alternate methods to construct confidence bands around trends are presented in **Chapter 21**. However, equation [22.3] cannot be used to plan sample sizes in this setting. Finally, **Tables 22-4** to **22-6** in **Appendix D** are based on an assumption of normally-distributed data. Although non-normal data sets might be approximated to some degree by the range of CVs considered, more sophisticated methods might be needed to compute sample size requirements for such data. This might entail consultation with a professional statistician.

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[2] A slightly more approximate direct calculation using the standard normal distribution instead of Student t-values will also provide the needed sample size estimate as: $n = \left[R \cdot \left(z_{1-\alpha} + z_{1-\beta}\right) \cdot C\hat{V} \big/ (1-R)\right]^2$. The recommended sample size in the example above is rounded to n = 15 using the z-normal equation. The estimate can be improved and made more conservative by adding an additional sample.

►EXAMPLE 22-2

Suppose elevated levels of specific conductance (μmho) shown in the table below must be remediated at a hazardous waste facility. If the clean-up standard has been set at L = 1000 μmho, at what point should remediation efforts be declared a success for the two compliance well data in the table below? Assume that the risk of false positive error needs to be no greater than $\alpha = 0.05$ at either well.

Well ID	Date	Spec. Cond.	Well ID	Date	Spec. Cond.
GW-12	10-16-87	2100	GW-13	10-16-87	2200
GW-12	01-28-88	2550	GW-13	01-27-88	1463
GW-12	04-13-88	2360	GW-13	04-13-88	935
GW-12	06-15-88	2405	GW-13	07-12-88	809
GW-12	10-12-88	2560	GW-13	10-12-88	469
GW-12	12-20-88	1163	GW-13	12-19-88	465
GW-12	04-19-89	1880	GW-13	01-31-89	374
GW-12	10-12-89	1650	GW-13	04-19-89	499
GW-12	04-25-90	2410	GW-13	07-10-89	503
GW-12	07-19-90	862	GW-13	10-10-89	590
GW-12	10-23-90	1114	GW-13	01-29-90	403
GW-12	02-13-91	1346	GW-13	04-25-90	527
GW-12	06-27-91	909	GW-13	07-23-90	513
GW-12	09-10-91	888	GW-13	10-24-90	451
GW-12	12-06-91	749	GW-13	02-13-91	622
GW-12	03-18-92	515	GW-13	06-27-91	495
GW-12	06-03-92	180	GW-13	09-12-91	420
GW-12	09-16-92	526	GW-13	12-04-91	634
GW-12	12-02-92	610	GW-13	03-20-92	526
GW-12	03-24-93	570	GW-13	06-04-92	472
			GW-13	09-17-92	442
			GW-13	12-01-92	530
			GW-13	03-24-93	625

SOLUTION

Step 1. First consider the data in well GW-12. A time series plot of the most recent 20 specific conductance values is shown in **Figure 22-3**. This plot indicates a fairly linear downward trend, suggesting that a trend line should be fit to the data, along with an upper confidence bound around the trend.

Figure 22-3. Time Series Plot of Specific Conductance Measurements at GW-12

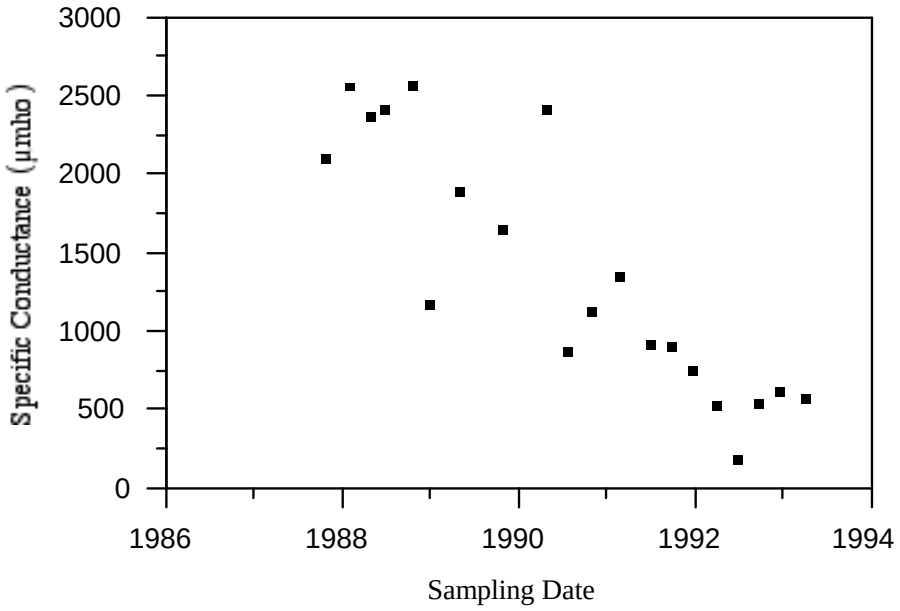
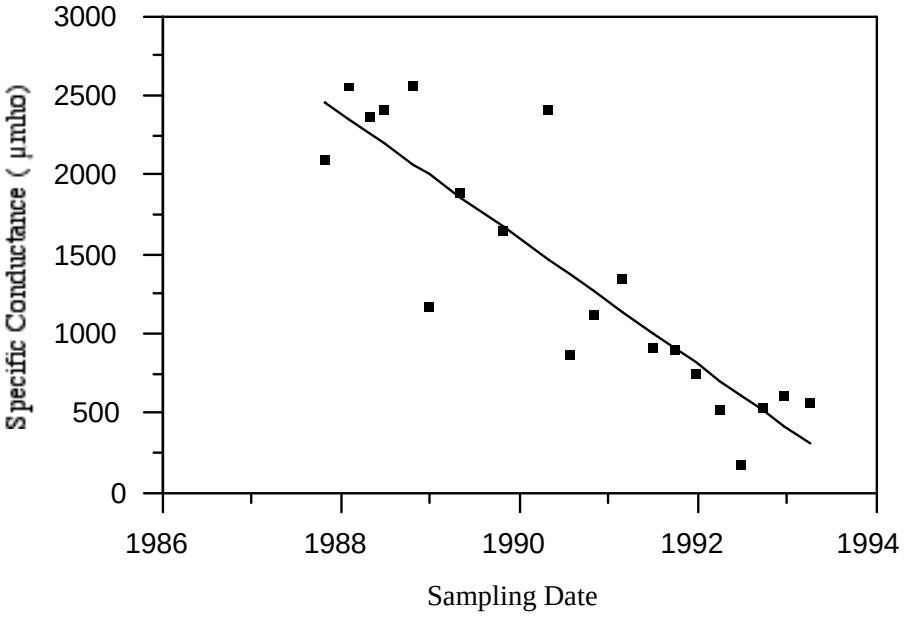


Figure 22-4. Regression of Specific Conductance *vs*. Sampling Date

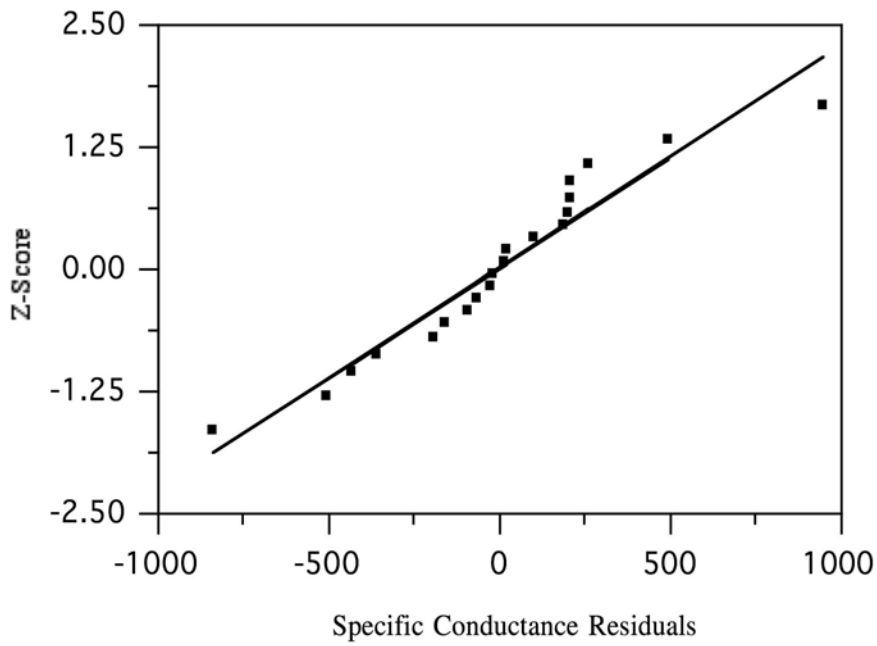


Step 2. Fit a regression line of specific conductance versus sampling date using the formulas in **Section 21.3**. The equation of estimated trend line shown on **Figure 22-4** is:

$$\hat{y} = 790360 - 396.36 \cdot t$$

Step 3. Examine the trend residuals. A probability plot of the residuals is given in **Figure 22-5**. Since this plot is reasonably linear and the Shapiro-Wilk test statistic for these residuals (SW = .9622) is much larger than the 1% critical point for n = 20 ($sw_{.01,\,20}$ = 0.868), there is no reason to reject the assumption of normality.

Figure 22-5. Probability Plot of Specific Conductance Residuals at GW-12



Also plot the residuals against sampling date (**Figure 22-6**). As no unusual pattern is evident on this scatter plot (*e.g.*, trend, funnel-shape, *etc.*) and the variability of the residuals is reasonably constant across the range of sampling dates, the key assumptions of the linear regression appear to be satisfied.

Step 4. Since the false positive error rate must be no greater than 5%, use α = .05 when constructing an upper confidence band around the regression line. Using the formulas in **Section 21.3** at each observed sampling date, both a 95% *upper* confidence band and a 95% *lower* confidence band are computed and shown in **Figure 22-7**. Only the upper confidence band is needed to measure the success of the remedial effort. Note that the formula uses an F-confidence level of 1-α or .95 for a one-sided confidence interval. The lower 95% confidence band is shown for illustrative purposes and the confidence level between the upper and lower bands is actually 90%.

Figure 22-6. Plot of Specific Conductance Residuals *vs*. Sampling Date

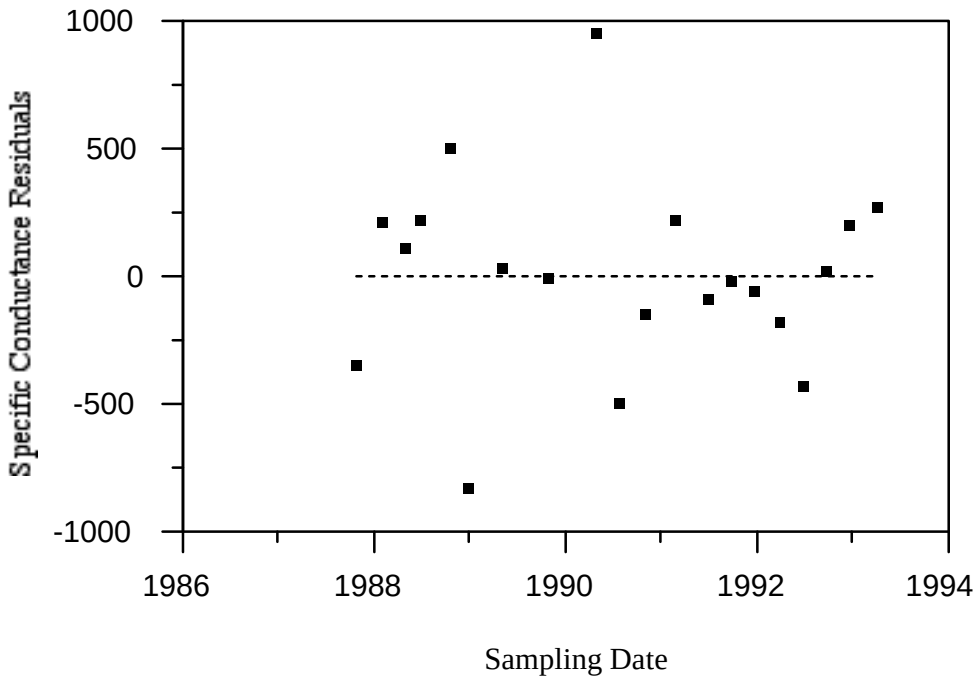
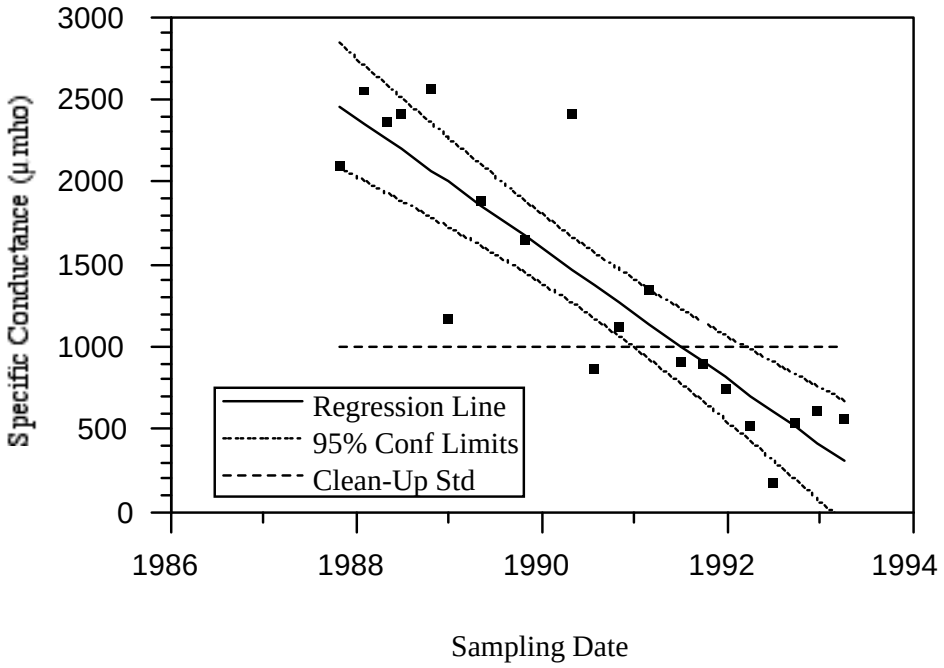


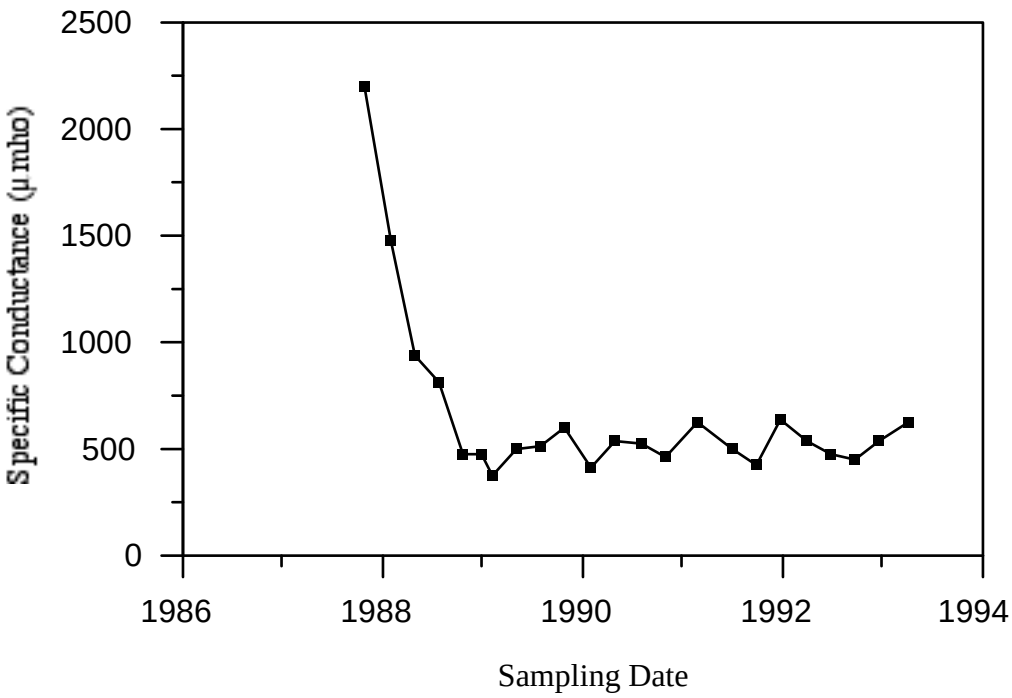
Figure 22-7. 95% Confidence Bounds Around Trend Line at GW-12



Step 5. Determine the first time point at which the remediation effort should be judged successful. In **Figure 22-7**, the upper confidence band drops below the clean-up standard of L = 1000 μmho in the second quarter of 1992, so well GW-12 could be declared in compliance at this point.

Step 6. Now consider compliance well GW-13. A time series plot of the specific conductance measurements in this case (**Figure 22-8**) shows an initially steep drop in conductance level, followed by a more or less stable mean for the rest of the sampling record. The best strategy in this situation is to remove the four earliest measurements and then compute an upper confidence limit on the remaining values.

Figure 22-8. Time Series Plot of Specific Conductance Measurements at GW-13



Step 7. Before computing an upper confidence limit, test normality of the data. If the entire sampling record is included, the Shapiro-Wilk test statistic is only .5804, substantially below the 1% critical point with n = 23 of $sw_{.01,23}$ = 0.881, indicating a non-normal pattern. Certainly, a transformation of the data could be attempted. But simply removing the first four values (representing the steep drop in conductance levels) gives a Shapiro-Wilk statistic equal to .9536, passing the normality test easily. Further confirmation is found by comparing the probability plots in **Figures 22-9** and **22-10**. In the first plot, all the data from GW-13 are included, while in the second the first four values have been removed.

Step 8. Another instructive comparison is to compute the upper confidence limits on the same data with and without the first four values. Consider the initial 8 conductance measurements, representing the first two years of quarterly data under corrective action. If all 8 values are used to compute the upper 95% confidence bound (taking 95% so that α = .05) and the formula for a confidence interval around a normal mean from **Section 21.1** is applied, the limit becomes:

$$UCL_{.95} = \bar{x} + t_{1-\alpha,n-1}\frac{s}{\sqrt{n}} = 901.75 + 1.8946 \times \frac{635.7126}{\sqrt{8}} = 1327.6\ \mu\text{mho}$$

While this limit exceeds the clean-up standard of L = 1000 μmho, the same limit excluding the first four measurements is easily below the compliance standard:

$$UCL_{.95} = 451.75 + 2.3534 \times \frac{54.0085}{\sqrt{4}} = 515.3\ \mu\text{mho}$$

Figure 22-9. Probability Plot at GW-13 Using Entire Sampling Record

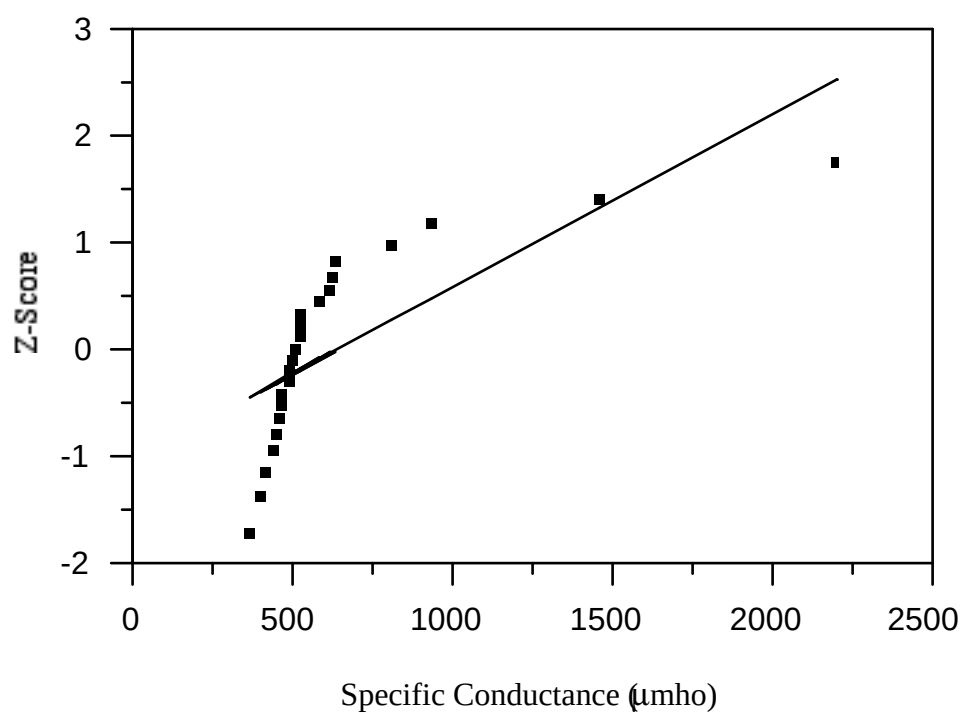
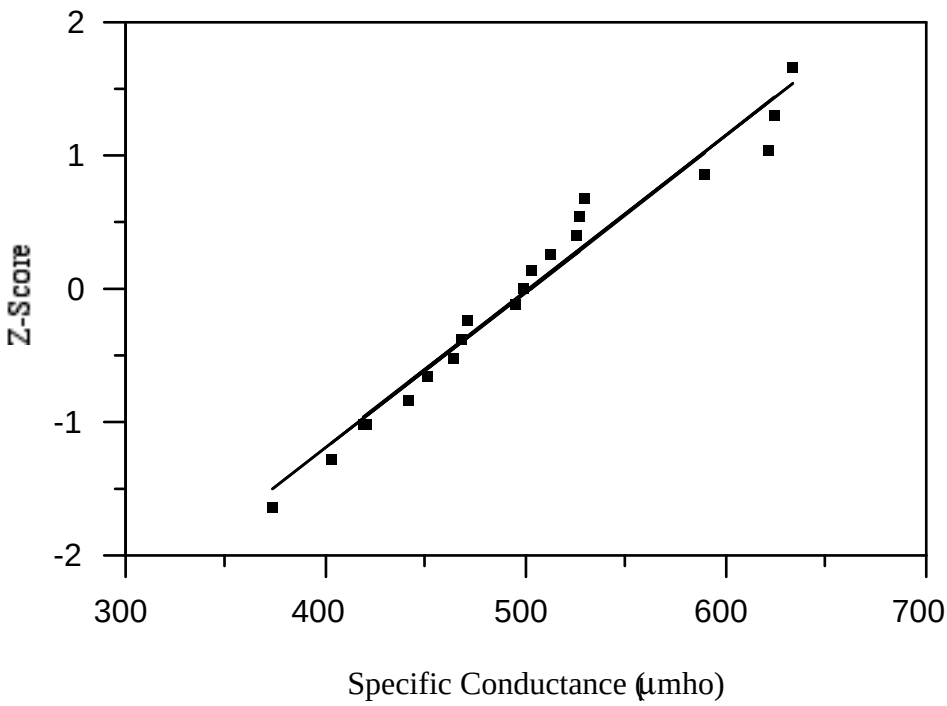


Figure 22-10. Probability Plot at GW-13 Excluding First Four Measurements



Step 9. Based on the calculation in **Step 8**, the clean-up standard is certainly met by early 1990 at GW-13. However, it is also instructive to examine the confidence bounds on larger sets of data from the stable portion of the sampling record. For instance, if the initial 4 measurements are excluded and then the next 8 values are used, the upper 95% confidence bound is:

$$UCL_{.95} = 478.75 + 1.8946 \times \frac{68.3452}{\sqrt{8}} = 524.5\ \mu\text{mho}$$

If all of the last 19 specific conductance values are used, a similar 95% confidence bound becomes:

$$UCL_{.95} = 503.158 + 1.7341 \times \frac{74.2760}{\sqrt{19}} = 532.7\ \mu\text{mho}$$

Step 10. Both of the limits in **Step 9** easily meet the clean-up standard of L = 1000 µmho. However, the amount of data used in the latter case is more than double than that of the former, which can impact the relative statistical power of the upper confidence limit for detecting decreases below the fixed standard. Given that the specific conductance seems to level off at close to 500 µmho, or one-half the clean-up standard, and given that the CV is approximately equal to .15, **Table 22-5** in **Appendix D** (looking under CV = 0.2) indicates that at least 6 measurements are needed to have a 95% chance of detecting a drop in conductance level to half the standard. So in this example, both UCLs are sufficiently powerful for detecting such a decrease. ◄

22.2 CONFIDENCE INTERVAL TESTS FOR UPPER PERCENTILES

For fixed standards which represent an upper percentile or maximum, the proper comparison in compliance/assessment monitoring utilizes the *lower* confidence limit around an *upper* percentile tested against the GWPS. In formal corrective action testing, the appropriate comparison employs an *upper* confidence limit around an upper percentile. Parametric and non-parametric confidence intervals around percentiles are presented in **Chapter 21**.

While the basic comparison is similar to confidence intervals around a mean, two points should be noted. First, any numerical standard identified as a maximum concentration 'not to be exceeded' needs to be treated statistically as an upper percentile. The reason is that while every observed data set has a finite maximum, there is no way to estimate the confidence bounds around the maximum of a continuous population. The true 'maximum' is always positive infinity, illustrating a point of breakdown between mathematical models and physical reality. Nonetheless, confidence limits around an upper 90th to 99th percentile can be used as a close approximation to a maximum or some limit likely to only be infrequently exceeded.

Secondly, computing statistical power for an interval around an upper percentile is similar to but not quite the same as, statistical power for an interval around the mean. Statistical power for a compliance/assessment test of the upper 90th percentile is derived by considering whether more than 10% of all the population measurements exceed the GWPS. If so, the 90th percentile must also exceed the standard. In corrective action testing, the equivalent question is whether *less* than 10% of the measurements exceed the GWPS. In that case, the true 90th percentile must also be less than the standard.

Statistically, each observation is set equal to 0 or 1 depending on whether the measured concentration is less than or greater than the fixed standard. Then the percentage of measurements exceeding the GWPS is given by the *average* of the set of zeros and ones. In other words, the problem is similar to estimating an arithmetic *mean*.

The similarity ends, however, when it comes to setting power targets. For mean-based evaluations, power at true mean concentration levels is equivalent to a fixed multiple or fraction of the GWPS (*e.g.*, 1.5 or 2 times the standard; 0.25 or 0.5 times the standard). But for upper percentile power, the alternative hypothesis is defined in terms of the *actual percentage of measurements* either *exceeding the standard* in compliance/assessment monitoring (*e.g.*, 20% or 30% instead of the null hypothesis value of 10%) or *exceeding the clean-up level* in corrective action monitoring (*e.g.*, 2% or 5% instead of 10%). In both hypothesis frameworks, the actual fraction of measurements above the standard can be denoted by p. Furthermore, the power formulas rely on a normal approximation to the binomial distribution. If p is the probability that an individual observation exceeds the GWPS, and p_0 is the percentage of values exceeding the GWPS when the $(1-p_0)$th upper percentile concentration equals the standard, the quantity:

$$Z = \left(p - p_0\right) \Big/ \sqrt{p_0\left(1 - p_0\right)/n} \qquad [22.4]$$

has an approximately standard normal distribution under either the compliance/assessment null hypothesis $p \leq p_0$ or the corrective action null hypothesis $p \geq p_0$.

Under the compliance/assessment alternative hypothesis (H_A), the true fraction exceeding the standard is greater than the null value ($p = p_1 > p_0$). With the corrective action alternative hypothesis, the true fraction is less than the null value ($p = p_1 < p_0$). p_1 can be specified as a multiple of p_0, say $p_1 = k \cdot p_0$, where k can either be greater or less than one. Then it is possible to compute the sample size (n) necessary to simultaneously achieve a pre-specified level of power (1–β) and false positive rate (α) with the equation:

$$n = \left[\frac{z_{1-\alpha}\sqrt{p_0\left(1-p_0\right)} + z_{1-\beta}\sqrt{kp_0\left(1-kp_0\right)}}{\left(k-1\right)p_0} \right]^2 \qquad [22.5]$$

where z_c represents the cth percentile from a standard normal distribution.

Equation [22.5] can be used for designing and constructing confidence interval tests around upper percentiles in both compliance/assessment and corrective action monitoring. However, the interpretation and practical approach to its use differ depending on the stage of monitoring.

22.2.1 UPPER PERCENTILE TESTS IN COMPLIANCE/ASSESSMENT

In compliance/assessment monitoring, the alternative hypothesis (*i.e.*, that the well is contaminated above the compliance standard) is expressed in terms of the relative percentage of concentration values that will exceed the GWPS compared to an uncontaminated well. To illustrate, if the compliance standard represents the 95th percentile so that no more than p_0 = 5% of the individual measurements exceed this level, the percentage exceeding under the alternative hypothesis might be taken as $p_1 = 2 \times p_0$ = 10%. Then a power level would be targeted so that exceedances of the standard occurring as frequently as p_1 would be identified with a probability equal to (1–β).

If n measurements are used to construct a lower confidence bound on the upper percentile ($1-p_0$) of interest (*e.g.*, the 95th), there will be a (1–β) × 100% chance of showing that the lower confidence limit exceeds the GWPS when in fact at least $kp_0 \times 100\%$ of the measurements actually exceed the standard. Furthermore, equation [22.5] also implies that the LCL will *falsely* exceed the GWPS with probability α. That is, when the true percentage of measurements exceeding the standard is actually p_0 or less, the test will identify a compliance violation α × 100% of the time.

Because EPA's primary concern in compliance/assessment monitoring is having adequate statistical power to detect groundwater contaminated above the regulatory standard, a high power level (1–β) should first be pre-specified. Then α can be varied in equation [22.5] until the resulting minimum sample size (n) matches the available sample or a feasible sample size for future sampling is found. In other words, power should always be kept high (*e.g.*, at least 70-75%), even at the expense of the false positive rate (α). However, there may be sites where a feasible sample size can be calculated such that *both* β and α are minimized.

Values of n for various choices of power level (1–β), Type I error rate (α), and upper percentile ($1-p_0$) are tabulated in **Table 22-7** in **Appendix D**. These can be used to maintain a specific level of power when employing a confidence interval around an upper percentile in compliance/assessment monitoring. The percentiles covered in this table include the 90th, 95th, 98th, and 99th. Levels of statistical power (1–β) provided include .50, .60, .70, .80, .90, .95, and .99, while the false positive rate (α) ranges from .20 down to .01. Specific cases not covered by **Table 22-7** in **Appendix D** can be computed directly with equation [22.5].

►EXAMPLE 22-3

Suppose a compliance limit for the pressure under which chlorine gas is stored in a moving container (for instance, a rail car) is designed to protect against acute, short-term exposures due to ruptures or leaks in the container. If the compliance limit represents an upper 90th percentile of the possible range of pressures that might be used to seal a series of such containers, how many containers should be sampled/tested to ensure that if in fact 30% or more of the container pressures exceed the limit, violation of the standard will be identified with 90% probability and exhibit only a 5% chance of false positive error?

SOLUTION

Step 1. Since the compliance limit on chlorine gas pressure represents the 90th percentile, at most 10% of the container pressures should exceed this limit under normal operations. In statistical notation, $p_0 = 0.10$ and $(1-p_0) = 0.90$. If there is a problem with the process used to seal the containers and 30% of the pressures instead exceed the limit, this amounts to considering a multiple of $k = 3$ times the nominal exceedance amount.

Step 2. Since a violation of the pressure standard by at least $3p_0$ or 30% needs to be identified with 90% probability, the target power is (1–β) = 0.90. Also, the chance of constructing a lower confidence limit on the true 90th percentile gas pressure that *falsely* identifies an exceedance of the standard must be kept to α = .05.

Step 3. Looking in **Table 22-7** in **Appendix D** under the 90th percentile and $k = 3$, the necessary minimum sample size is $n = 30$. Thus, 30 similarly-sealed containers should be tested for gas pressure so that a confidence interval around the 90th percentile can be constructed on these 30 measurements using either the parametric or non-parametric formulas in **Chapter 21**. ◄

22.2.2 UPPER PERCENTILE TESTS IN CORRECTIVE ACTION

Equation [22.5] can also be used in formal corrective action testing. In this setting, an upper confidence limit [UCL] around an upper percentile is of interest and the false positive rate (α) needs to be minimized to ensure a low probability of falsely or prematurely declaring remedial success. In practice, α should be pre-specified to a low value. Then, different values for power (1–β) can be input into equation [22.5] until the resulting minimum sample size (n) either matches the available amount of sampling data or is feasible to collect in future sampling.

Once the minimum sample size is computed and these n measurements are used to construct a UCL on the upper percentile ($1-p_0$) of interest (*e.g.*, the 95th), there will be a (1–β) × 100% chance that

the UCL will be less than the clean-up standard when in fact no more than $kp_0 \times 100\%$ of the measurements actually exceed the standard. For instance, if $k = 1/2$, $(1-\beta)$ will be the power of the test when in fact half as many of the measurements exceed the standard as are nominally allowed.

Equation [22.5] also implies that the UCL will *falsely* drop below the clean-up standard with probability α. That is, when the true percentage of measurements exceeding the standard is actually p_0 or greater — indicating that the clean-up standard has not been met — the test will still declare the remedial effort successful $\alpha \times 100\%$ of the time.

Values of n for various choices of power level $(1-\beta)$, Type I error rate (α), and upper percentile $(1-p_0)$ are tabulated in **Table 22-8** in **Appendix D**. This table can be used to determine or adjust the feasible power level based on a pre-specified α when employing a confidence interval around an upper percentile in corrective action. Note that the minimum sample sizes in **Table 22-8** of **Appendix D** are generally quite large, especially for small error rates (α). Because of the regulatory interest in minimizing the risk of prematurely exiting remediation, statistical comparisons in corrective action are likely to initially have fairly low power. As the clean-up process continues, enough additional data can be accumulated to adequately raise the odds of declaring the remediation a success when in fact it is.

►EXAMPLE 22-4

Suppose excessive nitrate levels must be remediated in a rural drinking water supply. If the clean-up standard for infant nitrate exposure represents an upper 95th percentile of the concentration distribution, what sample size (n) should be selected to ensure that if true nitrate levels drop below the clean-up standard, the remediation effort will be judged successful with at least 80% probability?

SOLUTION

Step 1. Examining **Table 22-8** in **Appendix D** under the 95th percentile and power = $(1-\beta) = .80$, a choice of n cannot be made until two other statistical parameters are fixed: the false positive rate (α) and the relative fraction of exceedances (p). The false positive rate governs the likelihood that the upper confidence limit on nitrate will be below the clean-up standard, even though *more* than 5% of all nitrate measurements are above the compliance standard (so that the *true* 95th percentile for nitrate still exceeds the clean-up criterion). The relative fraction of exceedances (p) sets the true percentage of individual nitrate concentrations that exceed the clean-up standard under the alternative hypothesis (H_A); that is, what fraction of nitrate values are exceedances when the clean-up standard is truly met.

Unfortunately, no matter what choices of α and p are selected in **Table 22-8** of **Appendix D**, the smallest required sample size is $n = 55$, when $\alpha = .20$ and $p = .25$. Even if it is practical and affordable to test 55 samples of groundwater for nitrate, the chance of falsely declaring the remediation effort a success will still be 20%. To cut that probability in half to $\alpha = .10$, $n = 99$ samples needs to be tested.

Step 2. To lessen the required sampling effort, consider the alternatives. Lower sample sizes are needed if the percentile of interest is less extreme, for instance if the clean-up standard represents a 90th percentile instead of the 95th. In this case, only n = 48 samples are needed for 80% power and a 10% false positive rate with p = .25. Of course, more frequent exceedances of the compliance limit are then allowed (*i.e.*, 10% versus 5% of the largest nitrate concentrations).

Another less desirable option is to raise the α level of the test. This raises the risk of falsely declaring the remediation effort to be a success. One could also lower p. At p = .25 for the 95th percentile, 80% power is guaranteed only when the true nitrate exceedance frequency is one-fourth the maximum allowable rate--- i.e., when the true rate of exceedances is $.25 \times 5\%$ = 1.25%. Exceedance rates greater than this will be associated with *less* than 80% power. But while lowering p and keeping other parameters constant will indeed decrease n, it also has the effect of requiring a very low actual exceedance rate before the power of the test will be sufficiently high. At p = .10 for the 95th percentile, for instance, the true exceedance rate then needs to be only $.10 \times 5\%$ = 0.5% to maintain the same level of power.

The final option is to lower the desired power. Power in this setting is the probability that the UCL on nitrate will be below the clean-up standard, when the groundwater is no longer contaminated above the standard. When the true nitrate levels are sufficiently low to meet the compliance standard, demonstrating this fact will only occur with high probability (*i.e.*, high power) when the sample size is fairly large. By taking a greater chance that the status of the remediation will be declared inconclusive (*i.e.*, when the UCL still exceeds the clean-up standard even though the true nitrate levels have dropped), power could be lowered to 70% or 60% for instance, with a corresponding reduction in the required n. To illustrate, if the power is set at 60% instead of 80% for the 95th percentile and the false positive rate is set at α = .10, the required sample size would drop from n = 99 to n = 68.

Step 3. In many groundwater contexts, the minimum sample sizes of **Table 22-8** in **Appendix D** may seem excessive. Certainly, the sampling requirements associated with upper percentile clean-up standards are substantially greater than those needed to test mean-based standards. However, remediation efforts often last several years, so it may be possible to accumulate larger amounts of data for statistical use than is possible in, say, detection or compliance monitoring. In any event, it is important to recognize how the type of standard and the statistical parameters associated with a confidence interval test impact the amount of data necessary to run the comparison. Each parameter should be assessed and interpreted in the planning stages of an analysis, so that the pros and cons of each choice can be weighed.

Step 4. Once a sample size has been selected and the data collected, either a parametric or non-parametric upper confidence limit should be constructed on the nitrate measurements and compared to the clean-up standard. ◄

STATISTICAL ANALYSIS OF GROUNDWATER MONITORING DATA AT RCRA FACILITIES

UNIFIED GUIDANCE

APPENDICES

OFFICE OF RESOURCE CONSERVATION AND RECOVERY

PROGRAM IMPLEMENTATION AND INFORMATION DIVISION

U.S. ENVIRONMENTAL PROTECTION AGENCY

MARCH 2009

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A.2 GLOSSARY

Alpha (α) level	Decimal level of significance or false positive error of a statistical test
1-of-*m* Plan	Retesting plan consisting of an initial sample followed by up to (m–1) resamples; resamples are collected only if initial sample exhibits a statistical difference
Accuracy	Closeness of a measured or computed value to its "true" value, where the true value is obtained with perfect information.
ACL	Alternate Concentration Limit; a fixed standard or clean-up action level alternative to prescribed RCRA regulatory health- or background limits
Aliquot replicates	Physical splits of a single water quality sample for multiple analyses
ANOVA	Analysis of Variance; a statistical method for identifying differences among several population means or medians
Appendix I	40 CFR Part 258 chemical parameter list for Subtitle D detection monitoring programs
Appendix II	40 Part 258 CFR chemical parameter list for Subtitle D compliance or assessment monitoring programs
Autocorrelation	Correlation of values of a single variable data set over successive time intervals
Background	Natural or baseline groundwater quality at a site; can be characterized by upgradient, historical, or sometimes sidegradient water quality
Beta (β) level	Decimal value representing a false negative error rate in a statistical test
Bias	Systematic deviation between a measured (i.e., observed) or computed value and its true value. Bias is affected by faulty instrument calibration and other measurement errors, systematic errors during data collection, and sampling errors such as incomplete spatial randomization during the design of sampling programs.
Box Plot	Plot of selected descriptive statistics at a monitoring point (*e.g.*, mean, median, upper and lower quartiles)
Calibration	Comparison of a measurement standard, instrument, or item with a standard or instrument of higher precision and lower bias to detect and quantify inaccuracies and to report or eliminate those inaccuracies by adjustments. Also used to quantify instrument measurements of a given concentration in a given sample.
CERCLA	Comprehensive Environmental Response, Compensation and Liability Act (or Superfund); statute for non-active hazardous waste site management and remediation
Confidence Interval	Statistical interval designed to bound the true value of a population parameter such as the mean or an upper percentile
Confidence Level	Degree of confidence associated with a statistical estimate or test, denoted as $(1 - \alpha)$

Coverage	Fraction of a population expected to be contained within a tolerance interval
Critical value	Predetermined decision level for a test of statistical hypotheses
Degrees of freedom	The number of ways which members of a data set or sets can be independently varied
Descriptive Statistics	Statistics used to organize and summarize sample data
Effective Power	In a groundwater network of statistical tests, the power of the test method to identify a single well contaminated by a single constituent
EPA Reference Power Curves (ERPC)	Recommended standards for comparing performance of RCRA statistical methods in detection monitoring; based on individual prediction limit using n = 10 background samples and α = .01
False Negative	Finding of no statistically significant difference when there is, in fact, a physical difference in the underlying populations or between a single population and a fixed compliance standard; also known as beta (β) or Type II error
False Positive	Finding a statistically significant difference when there is, in fact, no physical difference in the underlying populations or between a single population and a fixed compliance standard; also known as alpha (α), significance level, or Type I error
GWPS	(Ground Water Protection Standards) Concentration limits set by the regulatory agency as a standard to be attained in groundwater monitoring. These may be fixed health- or risk-based limits (e.g. MCLs) or a background level.
Heterogeneous	Non-uniform in structure or composition throughout
Histogram	Graphical representation of frequency with data values grouped into specified numerical ranges
Homogeneous	Uniform in structure and composition throughout
Homoscedasticity	Equality of variance among sets of data
Hypothesis	One of two statements made about potential outcomes of a statistical test. The null and alternative hypothesis statements refer to the condition of a population parameter. The null hypothesis is favored, unless the statistical test demonstrates the greater likelihood of the alternative hypothesis.
Independent & Identically Distributed (i.i.d)	Groundwater measurements having the same statistical distribution and exhibiting no statistical dependence or correlation
Indicator Parameters	Chemical parameters whose presence or elevation is possibly indicative of a facility release
Interwell	Comparisons between distinct monitoring wells
Intrawell	Comparisons over time at a given monitoring well between early and later measurements
Mann-Kendall Test	Non-parametric test of trend

MCL	Maximum Contaminant Level; a fixed water quality standard defined under the Safe Drinking Water Act and used in 40 CFR 258.40(e)(3)
MDL	Method Detection Limit—the minimum concentration of a substance that can be measured and reported with 99% confidence that the analyte concentration is greater than zero in a specific matrix.
Modified California Plan	Retesting plan consisting of an initial sample followed by three resamples; if initial value exhibits a statistical difference, two of three resamples must *not* exhibit a difference for the test to 'pass'
Non-detects (NDs)	Observations below the MDL, RL, or QL
Non-parametric Test	Statistical test that does not depend on knowledge of the distribution of the sampled population
Normal distribution	A family of symmetric continuous probability distributions defined by two finite parameters, the mean and variance
Outlier	Value unusually discrepant from rest of a series of observations
Parametric Test	Statistical test that depends upon or assumes observations from a particular probability distribution or distributions
Percentile	The specific value of a distribution that divides the distribution such that p percent of the distribution is equal to or below that value. If the 95th percentile is X, it means that 95 percent of the values in the statistical sample are less than or equal to X.
Population	All possible measurements/values over a period of time at a given location, series of locations, or over a spatial or volumetric extent
PQL or QL	Practical Quantification Limit—lowest concentration level for an analytical method which can be reliably achieved within specified limits of precision and accuracy under routine laboratory operating conditions
Precision	A measure of mutual agreement among individual measurements of the same property, usually under prescribed similar conditions, expressed generally in terms of the sample standard deviation.
Prediction Interval	Statistical interval constructed from background data on the next 'future' sample or samples arising from the same population
Prediction Limit	Upper or lower limit of a prediction interval
Probability	Quantitative measure of uncertainty about the occurrence of a random or uncertain event
Probability Distribution	Numerical statistical pattern associated with a population of measurements; many common patterns can be described using mathematical formulas
Proportion	A population proportion (p) is the ratio of the number of units of a population that have the specified characteristic or attribute (M) to the total number of units in the population (N).
Random sample	Collected data which are based only on their probability of occurrence in random fashion

Ranking	Assignment of numbers to an ordered data set indicating their relative position, generally integer values from 1 to *n* for the smallest to largest values in a sample of size *n* (unless specified in reverse rank order)
RCRA	Resource Conservation and Recovery Act; statutory provisions for active facility hazardous (Subtitle C) and non-hazardous waste (Subtitle D) definition, storage, treatment and disposal
Reporting Limit	Reporting Limit—lowest concentration level for an analytical method which can be reliably measured by a laboratory
Residual	Typically, the difference of a value in a data set from its mean
ROS	Regression on order statistics, either parametric or robust; techniques for fitting non-detect data to a single distribution
Sample	Set of measurements from a population (can be as few as one)
SDWA	Safe Drinking Water Act; statute under which drinking water standards are promulgated and water treatment sites regulated
Seasonality	The presence of seasonal effects on ground water quality observations; effects may be natural or man-made.
Sen's Slope Estimator	Non-parametric method to estimate the rate of change of concentration levels over time
SWFPR	Site Wide False Positive Rate; design probability of at least one statistically significant finding among a network of statistical test comparisons at a group of uncontaminated wells
Spearman's Test	Non-parametric test of trend using data ranks
Statistical Parameter	A numerical characteristic of a statistical population or probability distribution
Statistical Power	Strength of a test to identify an actual release of contaminated groundwater or difference from a compliance standard
Statistically Significant Difference (or Increase)	Statistical difference exceeding a test limit large enough to account for data variability and chance
Time Series Plot	Graphical plot of individual concentration values over time
Tolerance Interval	Statistical interval constructed to 'cover' a specified proportion of the underlying population of measurements
Tolerance Limit	The upper or lower limit of a tolerance interval
Trace Value	Measured value close to, but above the limit of detection; may lie between the MDL and the QL
Random Variable	A numerical value or characteristic that can assume different values on different sampling events or at different locations
Variance	A measure of spread or dispersion calculated as the average of squared differences from the mean in a set of data or a population
Verification Resampling or Retesting Plan	A plan to collect an additional resample or resamples to confirm or disconfirm an initial statistically significant finding

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APPENDIX B. HISTORICAL NOTES

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B.1 PAST GUIDANCE FOR CHECKING NORMALITY

The 1989 ***Interim Final Guidance*** [IFG] outlined three different methods for checking normality: the coefficient of variation [CV] test, probability plots, and the chi-square test. Of these three, only probability plots are recommended within the Unified Guidance. The coefficient-of-variation and chi-square tests each have potential problems or are inferior to alternate methods. These alternatives include the coefficient of skewness, the Shapiro-Wilk or Shapiro-Francía tests, and Filliben's probability plot correlation coefficient.

The coefficient of variation [CV] test in the original 1982 RCRA Part 264 groundwater monitoring regulations was recommended within the IFG because it is easy to calculate and amenable to small sample sizes. To ensure that a normal model with a significant fraction of negative concentration values was not fit to positive data, the IFG recommended that a sample CV be less than one to indicate 'normality.' The test was inexact since the distribution of sample CV's from a truly normal population itself is a function of both sample size and the true coefficient of variation. Truly normal populations of positive-valued data are likely to have a CV of 0.3 or lower, although individual sample CV's will occasionally exceed one, depending on the sample size. It was also possible to incorrectly reject normality using this criterion even when the population was really normal.

While the coefficient of variation indirectly offers an estimate of skewness and hence normality/non-normality, there are better formal tests to accomplish both goals. The Unified Guidance recommends estimating skewness of a data set using the coefficient of skewness (**Section 10.4**), along with other tests of normality in **Chapter 10**. Nevertheless, the coefficient of variation provides a measure of intrinsic variability in positive-valued data sets. Although approximate, the coefficient of variation can indicate the relative variability of certain data, especially with small sample sizes and in the absence of other formal tests.

The CV is also a valid measure of the multiplicative relationship between the mean and the standard deviation for positively-valued random variables. The estimator $\hat{CV} = s/\bar{x}$ reasonably approximates the true CV for non-negative normal populations. In lognormal populations, the coefficient of variation can also be used in evaluations of statistical power. For the lognormal distribution, the population coefficient of variation works out to be:

$$CV = \sqrt{\exp\left(\sigma_y^2\right) - 1}$$

where σ_y is the population log-standard deviation. Because of this, instead of a ratio between the standard deviation and the mean, the lognormal coefficient of variation is usually estimated by

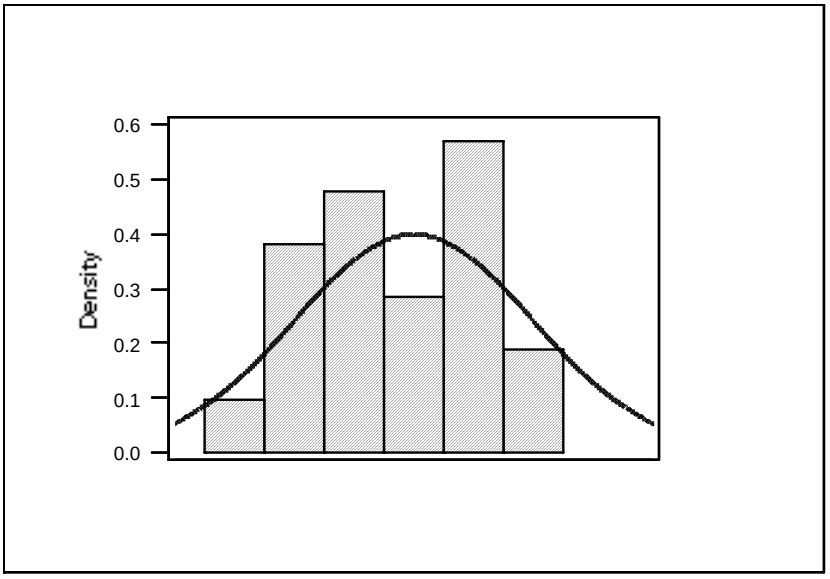
$$CV = \sqrt{\exp\left(s_y^2\right) - 1}$$

where s_y is the log-standard deviation. This last estimate is usually more accurate than the simple ratio of standard deviation-to-mean, especially when the underlying population coefficient of variation is high. However, neither coefficient of variation estimator is a satisfactory test as to whether a data set is truly normal or lognormal.

The chi-square test was also recommended within the IFG. Though an acceptable goodness-of-fit test, it is not considered the most sensitive or powerful test of normality (Gan and Koehler, 1990). The downside to the chi-square test can be explained by considering the behavior of parametric tests based on the normal distribution. Most tests, like the *t*-test or parametric prediction limits, which assume that the underlying data are normal, give fairly robust results when the normality assumption fails over the middle ranges of the data distribution. That is, if the extreme tails are approximately normal in shape even if the middle part of the density is not, these parametric tests will still tend to produce valid results. However, if the extreme tails are non-normal in shape (*e.g.*, highly skewed), normal-based tests can lead to false conclusions, meaning that either a data transformation or a non-parametric technique should be used instead.

The chi-square test entails a division of the sample data into 'bins' or 'cells' representing distinct, non-overlapping ranges of the data (**Figure B-1**). In each bin, an expected value is computed based on the number of data points that would be found if the normal distribution provided an appropriate model. The squared difference between the expected number and observed number is then computed and summed over all the bins to calculate the chi-square test statistic.

Figure B-1. How the Chi-Square Goodness-of-Fit Test Works



If the chi-square test indicates that the data are not normal, it may not be clear what ranges of the data most violate the normality assumption. Departures from normality in the middle bins are given nearly the same weight as departures in bins representing the extreme tails, and all the departures are summed together to form the test statistic. As such, the chi-square test is not as powerful for detecting departures from normality in the extreme tails of the distribution, the areas most crucial to the validity of parametric tests like the *t*-test or ANOVA (Miller, 1986). This implies that if there are departures in the tails, but the middle portion of the data distribution is approximately normal, the chi-square test may not register as statistically significant even when better tests of normality would.

The IFG also suggested that the original data should be presumed to be normal prior to testing the distributional assumption. If a statistical test rejected the model of normality, the data could be checked instead for lognormality by evaluating their natural logarithms. The 1992 ***Addendum to Interim Final Guidance*** [Addendum] noted that many data sets in environmental monitoring are better fit by a lognormal than by a normal distributional model. Primarily on that basis, it was recommended that the lognormal distribution replace the normal as the default model for groundwater analysis, especially since for small data sets, the available tests of normality have limited statistical power to reject the null hypothesis of normality, even if the data arise from a lognormal distribution. The Unified Guidance brings this argument around almost full circle by arguing that the normal model is a slightly better default for small samples, but that distributional testing is recommended in any case in order to establish the most appropriate model (**Section 10.3**).

B.2 THE CABF PROCEDURE

Facilities operating under a RCRA permit specifying Cochran's Approximation to the Behrens-Fisher Student's *t*-test [CABF] may change this method to a more appropriate procedure at the time of State or Regional permit review and update. Owners and operators may also apply for a permit modification under §270.41(a)(3). This change is considered a Class 1 permit modification, which must be made with prior approval from the Director.[1] Depending on the nature of the permit conditions, it may also be appropriate, on a facility-specific basis, for an oversight agency to approve a change of method without a formal permit modification.

Under appropriate circumstances, an owner or operator may wish to continue using a *t*-test type procedure. However, instead of the CABF method, it is recommended that either a pooled variance Student's *t*-test or a variant of this test due to Welch (1937) be employed (**Chapter 16**). Not only is Welch's test a more standard type of *t*-test than the CABF procedure, but research has shown it to be equivalent or preferable to other varieties of the *t*-test (Moser and Stevens, 1992).

Circumstances appropriate for the use of a *t*-test procedure might include facilities with very few monitoring wells (*e.g.*, three or less) and that monitor for a very limited number of constituents (*e.g.*, one or two). As long as no more than 5 to 10 statistical comparisons are being made each year, running a *t*-test at the 0.01 level of significance in each case should result in at most a 10% annual probability of any comparison registering as a false positive when there is no actual contamination.

One of the problems with the CABF procedure in practice was the use of aliquot replicate samples to bolster the total sample size (**Section 2.2.4**). Both the pooled variance *t*-test and Welch's *t*-test make the assumption that the sample observations are statistically independent. Though aliquot replicate sampling increases the number of available measurements, aliquot replicate samples mostly provide information about *analytical* variability and accuracy, and tend to be highly correlated. Since the goal of a RCRA groundwater statistical program is to provide data about hydro-geochemical variability in the (uppermost) aquifer below the facility, aliquot replicate sampling (like the CABF procedure itself) should be avoided unless a more sophisticated components of variance model is used to account for the separate effects of analytical variability and natural groundwater variance.

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[1] See 53 FR 37912, September 28, 1988 for more details about the permit modification process.

B.3 PAST GUIDANCE FOR NON-DETECTS

Guidance for handling non-detect measurements was first offered in the 1989 ***Interim Final Guidance*** [IFG]. There the basic recommendations included the following: 1) if less than 15% of all samples are non-detect, replace each non-detect by half its detection or quantitation limit [QL] and proceed with a parametric analysis, such as ANOVA, tolerance limits, or prediction limits; 2) if the percentage of non-detects is between 15 and 50, either use Cohen's adjustment to the sample mean and variance in order to proceed with a parametric analysis, or employ a non-parametric procedure by using the ranks of the observations and treating all non-detects as tied values; 3) if the percentage of non-detects is greater than 50, use the test of proportions.

In the 1992 ***Addendum to Interim Final Guidance*** [Addendum], the recommendation for data sets with small fractions of non-detects (*i.e.*, ≤ 15%) was left unchanged; however, for cases with moderate detection rates (*i.e.*, non-detects comprising 15% to 50% of the data), Cohen's adjustment was supplemented by Aitchison's method for data sets in which non-detects could be regarded as zero concentrations. In addition, the *test of proportions* was deleted from the Addendum. Instead, for large fractions of non-detects, three options were suggested: 1) for two sample comparisons, the Wilcoxon rank-sum test was recommended over the test of proportions; 2) for moderately large background samples, the Addendum recommended non-parametric prediction and tolerance limits; and 3) for extremely low detection rates (*e.g.*, ≥ 90% non-detects) and small background samples, the Addendum recommended the use of Poisson prediction and tolerance limits.

The test of proportions was not recommended in the Addendum, even for detection rates under 50%, for the following reason. Although acceptable as a statistical procedure, the test of proportions does not account for potentially different magnitudes among the concentrations of detected values. Rather, each sample is treated essentially as a '0' or '1' depending on whether the measured concentration is below or above the QL. The test of proportions ignores information about concentration magnitudes, and hence is often less powerful than a non-parametric rank-based test like the Wilcoxon rank-sum, even after adjusting for a large fraction of tied observations (*e.g.*, non-detects). In part, this is because the ranks of a data set preserve additional information about the relative magnitudes of the concentration values, information which is lost when all observations are scored as 0's and 1's.

Furthermore, small-scale Monte Carlo simulations comparing the test of proportions to the Wilcoxon rank-sum test showed that for small to moderately large proportions of non-detects (say 0% to 60%), the Wilcoxon rank-sum procedure adjusted for ties was more powerful in identifying real concentration differences than the test of proportions. When the percentage of non-detects was quite high (at least 70% to 75%), the test of proportions was occasionally more powerful than the Wilcoxon for extremely small group sample sizes (*e.g.*, no more than 4 to 6 measurements per group), but the results of the two tests usually led to the same conclusion. Consequently, the Wilcoxon rank-sum test was recommended in all cases where non-detects constituted more than 15 percent of the samples.

The revised Unified Guidance also places less emphasis on Cohen's method. The reason is that it could only accommodate a single censoring limit (*e.g.*, reporting limit [RL]) in its original formulation and assumed that all quantified values were necessarily greater than this limit. Because many environmental data sets include multiple reporting and/or detection limits and an intermixing of detects and non-detects, two other methods are now recommended that are designed to handle more complex data configurations (**Chapter 15**). Cohen's and the parametric ROS method may have limited

applicability when both detect and non-detect data are expected to stem from a single parametric distribution and a single censoring limit can be used.

B.4 TREND TESTS

The Unified Guidance recommends trend testing as an alternative to prediction limits or control charts when those methods are not suitable. To understand the basis for this recommendation, it may help to consider how intrawell comparisons initially supplemented, and then came in many cases to supplant, interwell comparisons.

In the 1989 IFG and the 1992 Addendum, the recommended statistical methods closely followed the 1988 and 1991 Final Rules published in the Federal Register. Although these methods replaced historical use of the CABF Student's *t*-test, there was still an emphasis on interwell comparisons between background and downgradient wells through the use of *t*-tests and ANOVA. Indeed, where justified, interwell comparisons provide undeniable conceptual advantages over other kinds of tests. When (upgradient) background measurements can be used to establish a reasonable baseline concentration level, such data offer invaluable information about site-specific conditions at uncontaminated locations and the level of variability one should expect to encounter in the absence of events that precipitate groundwater contamination.

Unfortunately, ANOVA and *t*-tests all involve a comparison of population means under the key assumption that the populations *have not changed over time*. The underlying distributions in each group or well are assumed to be stable over the period of monitoring, so that concentration measurements fluctuate randomly around a constant mean level. Stability, of course, is not guaranteed. Several factors can impact the statistical characteristics of the underlying aquifer at either upgradient or downgradient wells, including natural fluctuations in aquifer parameters, migration of contaminants from off-site sources, changes in the mixture of deposited waste and its geochemical interaction with the subsurface environment, and alterations in geochemistry from 'percolation' effects due to past waste disposal practices or land usage.

EPA's hope in the 1989 IFG was that ANOVA-type comparisons would be done quickly enough (*e.g.*, every six months) that the underlying populations could be considered essentially static during each testing period. At some sites, this may be a reasonable assumption. However, in practice, sampling is now done on a quarterly, semi-annual, or annual basis. In order to gather the four to five samples needed— at a minimum — to run a *t*-test or ANOVA, at least one to four years of sampling is necessary. Over this length of time, the statistical characteristics of groundwater may or may not change.

Furthermore, interwell comparisons between upgradient and downgradient well locations are not always appropriate, either due to natural spatial variability, screening of background and downgradient wells in different hydrostratigraphic positions, effects of groundwater mounding, *etc.* In such cases, the appropriate statistical approach is to use an *intrawell* test at each compliance location. Intrawell tests involve a comparison only of data collected at that specific well location, thus eliminating spurious differences that might arise due to natural spatial variability or other background-to-downgradient differences not attributable to the presence of contaminated groundwater.

Two basic intrawell techniques are described in the Unified Guidance: intrawell prediction limits and control charts. Both designate some portion of the historical sampling record as intrawell 'background' for that well. Ideally, this intrawell background should consist of measurements known to

be uncontaminated. Furthermore, both methods assume (unless special adjustments are made) that the intrawell background represents a random sample from a stable population, just as with the *t*-test and ANOVA. If the population mean and/or standard deviation *change* while intrawell background is being compiled, results of either prediction limit or control chart tests against more recent data from the well can be severely biased or altogether inaccurate.

For these reasons, neither prediction limits nor control charts are appropriate for every circumstance where an intrawell test is warranted. The Unified Guidance recommends trend testing as an alternative to prediction limits or control charts when those methods are not suitable as intrawell techniques (**Chapter 17**). Tests for trend are specifically designed to identify groundwater populations whose mean concentration levels are not stable over time, but rather are significantly increasing (or decreasing).

B.5 PREDICTION LIMITS AND RETESTING

B.5.1 RETESTING SCHEMES

Since roughly 1987, several different retesting schemes have been suggested in regulatory documents or published in scientific literature. Classification of these schemes shows that they fall into three basic types: 1-of-*m*, California, and tolerance screens. The 1-of-*m* approach was initially suggested by Davis and McNichols (1987) as part of a broader method termed '*p*-of-*m*.' Essentially the *p*-of-*m* approach assumed that as many as *m* observations would be collected for a particular constituent at a given well, including the initial groundwater measurement and up to (*m*–1) resamples. As long as at least *p* of these observations were below a predetermined upper prediction limit, the constituent would 'pass' the test at that well, allowing detection monitoring to continue.

Davis and McNichols determined how to calculate the necessary prediction limits so that the overall false positive rate would remain below a fixed value (say 5%, as targeted in the 1992 Addendum), even when the same testing procedure was applied over many different testing periods (*r* in their terminology). By applying the same technique to *r* different well-constituent pairs (and assuming mutual statistical independence among constituents and compliance wells) instead of to *r* different testing or evaluation periods, one then has a retesting scheme that can be applied at a large variety of monitoring networks while ensuring that the site-wide false positive rate [SWFPR] is kept to a minimum.

In practice, though the *p*-of-*m* strategy provides a great deal of flexibility in designing a retesting scheme, only those schemes known as 1-of-*m* are typically useful in the current regulatory context of groundwater monitoring. Consider, for example, a 2-of-3 strategy. By definition, if at least two of three groundwater samples are below the upper prediction limit (*i.e.*, are 'in-bounds'), the constituent passes and is not flagged as exceeding background. Since at least two samples must be 'in-bounds,' it is not enough to collect one initial groundwater measurement and show that it is below the prediction limit. At least one additional resample must *always* be collected and measured. 1-of-*m* strategies, by contrast, only require a single groundwater observation to pass. If the initial measurement is below the prediction limit, the constituent passes the overall test and no resamples need be collected.

The second retesting scheme, known as California-style plans, was suggested partly in response to perceived problems with the 1-of-*m* plans. California regulators noted, for instance, that a 1-of-3 retesting scheme would allow a constituent in a given well to pass even if *both* the initial groundwater

measurement *and* one of the two retests exceeded the predetermined prediction limit. The only way for that well-constituent pair to fail would be if all three measurements — the initial and the two resamples — exceeded the prediction limit. To many regulators (and not just those in California) the 1-of-*m* scheme appeared to practically guarantee that contaminated wells would go unidentified, 'passing' the test each time and undermining protection of human health and the environment.

In 1991, California received explicit approval from EPA to use an alternate retesting scheme constructed as follows. For each well-constituent pair, collect an initial groundwater observation. If this initial measurement is in-bounds (*i.e.*, below the prediction limit), the test for that pair passes and no resamples need be collected. If the initial measurement exceeds the prediction limit, two or possibly three resamples must be collected and *each* must be in-bounds for the test to pass. If any of the resamples exceeds the prediction limit (*i.e.*, is 'out-of-bounds'), the test fails and possible groundwater contamination is indicated.

The California strategy was seen as a more environmentally 'conservative' approach to retesting. An initially high groundwater measurement would only be deemed 'spurious' if all the subsequent resamples were below the target prediction limit, providing at least double reconfirmation that the well was 'clean' for that constituent. Unfortunately, the more stringent requirements of the California plans came with unexpected consequences. A California retesting plan typically requires a *larger* target prediction limit (or 'trigger level') than a 1-of-*m* plan with a comparable number of resamples, in order to achieve the same overall SWFPR. Since a larger trigger level corresponds to a *less* statistically powerful test, a given California plan may or may not have adequate effective power even if a similar 1-of-*m* plan does.

The net result is that 1-of-*m* retesting schemes often provide greater statistical power for detecting real groundwater contamination, particularly in large networks, even though not every resample need be below the prediction limit. If the trigger level is low enough, at least one of the resamples may exceed the prediction limit even when there is no contamination. So these cases should not automatically be classified as verified contamination. Conversely, a lower prediction limit increase the odds (*i.e.*, power) that truly contaminated groundwater will be identified, since both the initial observation and any resamples will be more likely to exceed a lower trigger level than one set to a higher benchmark.

B.5.2 TOLERANCE SCREENS

A final type of retesting scheme might be termed the *tolerance screen* approach. First suggested by Gibbons (1991b), this approach was modified and recommended by EPA in the 1992 Addendum, but — for reasons discussed below — is *not* recommended within the Unified Guidance. In contrast to the 1-of-*m* and California-style plans, which make use of repeated *prediction* limits as the trigger levels, the tolerance screen involves a two-stage testing procedure as follows. An initial groundwater measurement is collected from each well in the network and compared to an upper *tolerance* limit with specified coverage and confidence levels. If any measurement exceeds the tolerance limit, one or more resamples are collected from that well and these measurements are compared against an upper *prediction* limit.

Other than the use of a tolerance limit instead of a prediction limit as the 'screen' for the initial groundwater measurement, the rules for passing the test are the same as a modified California approach described in **Section 19.1**. Either the first observation must be below the tolerance limit (*i.e.*, 'in-bounds') or q-of-(m–1) resamples must be below the prediction limit. If both of these conditions are violated, possible groundwater contamination is indicated.

The use of two separate trigger levels (*i.e.*, tolerance limit and prediction limit) for the initial observation versus the resamples may seem an unnecessary complication in developing a retesting procedure. However, there are two advantages to this approach. For one, the tolerance and prediction limits are computed on the same background data and both these calculations are done prior to any data comparisons. Secondly, by allowing two different trigger levels, greater flexibility is gained in designing — for a given sized network of comparisons — a retesting scheme that meets a target SWFPR.

Gibbons' (1991b) original tolerance screen approach advocated constructing a 95% confidence tolerance limit with a degree of coverage that would vary depending on the network size. For 100 tests, Gibbons reasoned that a tolerance limit with 95% coverage would result in as many as 5 exceedances of the initial trigger just by chance (*i.e.*, even when no contamination was present). Any such exceedance would then require that a resample be collected at that well and compared to a prediction limit with 95% confidence for the next 5 future samples (m = 5), in order to maintain an overall 5% SWFPR. The same type of false positive rate control could be achieved by setting the degree of coverage to 99%, so that only 1 exceedance would be expected in 100 tests against the tolerance limit. In this case, the prediction limit would be computed with 95% confidence but m = 1 instead. In all cases, the number of future measurements (m) being predicted would equal the number of measurements possibly expected to exceed the tolerance limit just by chance.

To offer even greater flexibility, EPA recommended a modification to Gibbons' tolerance screen within the 1992 Addendum. To understand why a modified version was adopted, note that the formula for an upper prediction limit on the next m future samples with $(1-\alpha)$ confidence may be expressed as follows:

$$PL_{1-\alpha} = \bar{x} + t_{n-1,1-\alpha/m} s \sqrt{1+\frac{1}{n}}$$

Careful examination of this formula shows that the effect of changing the number of future samples m for a given confidence level is equivalent to changing the *confidence level* associated with a prediction limit for a *single* future observation (m = 1).

Because of this, EPA suggested three alterations to Gibbons' original scheme: 1) instead of fixing the level of confidence and varying the number of future samples m, fix m = 1 and allow the confidence level of the prediction limit to vary; 2) allow more than one resample per comparison up to a practical maximum of three; and 3) use a tolerance limit with *average* coverage instead of *minimum* coverage. While Gibbons offered power comparisons with his scheme against either a single tolerance limit or a single prediction limit, the Addendum offered recommended choices of degrees of coverage and confidence levels that would simultaneously limit the SWFPR to approximately 5% and generate effective power at least as high as the EPA reference power curve.

Unfortunately, as Davis and McNichols (1994) noted, the Monte Carlo simulations used in the Addendum to generate recommended retesting plans based on the tolerance screen approach were partly flawed. Two criticisms were particularly relevant. First, Davis and McNichols noted that the Appendix to the Addendum spoke of networks in terms of number of *wells* rather than the number of *tests*. Since the total number of tests is a product of the number of wells and the number of constituents being

monitored in each well, they suggested that the Addendum recommendations for retesting plans might elevate the SWFPR above 5% (the recommended per-evaluation rate in 1992).

The reason is that if a particular plan in the Addendum was only applicable to a single constituent (albeit across a large number of wells), a similar but separate plan would be needed for each constituent. This in turn would imply that the target overall false positive rate of 5% would only apply *per constituent*, meaning that tests for many constituents in the same network would lead to a sharply elevated SWFPR. Of course, the *text* of the Addendum clearly spoke of tests as a combination of wells and constituents. Still, Davis and McNichols were correct to note that some may have misunderstood the contextual meaning of the phrase 'wells' in the Appendix and also in the table on non-parametric retesting strategies, which was naively used as a simple shorthand for the more awkward 'well-constituent pairs.'

A second criticism related to the algorithm used to simulate the effective power of the tolerance screen plans. Davis and McNichols correctly observed that while effective power was defined in terms of a single well contaminated by a single constituent, the power curves illustrated in the Addendum Appendix mistakenly added those cases where the contaminated well failed the overall testing procedure to those where *uncontaminated* wells failed the procedure (*i.e.*, instances of false positives). The net effect was to slightly raise the stated power above the actual power, especially at lower standard deviation shifts in the mean level above background (*e.g.*, $0 < \Delta < 2$). As a result of this criticism, all calculations in the Unified Guidance with respect to retesting plans have been divided into two components: 1) computation of the SWFPR based on the total number of tests, taken as a product of wells times constituents, and 2) computation of the effective power based on a single contaminated well-constituent pair.

A third criticism can now be added to those offered by Davis and McNichols. Given a fixed background sample, one drawback to both 1-of-*m* and California-style plans is that they have limited flexibility when it comes to controlling the SWFPR below a target level (*e.g.*, 10%) over a variety of network sizes. In some cases, sufficient false positive rate control and adequate power can only be achieved by switching, say, from a 1-of-2 plan to a 1-of-3 plan, or from a California plan to a 1-of-*m* scheme, or by increasing the background sample size. The problem is that using the same trigger level — here a prediction limit — to test both the initial measurement and any resamples restricts the number of simultaneous tests that can be accommodated. The EPA tolerance screen approach uses different trigger levels at each stage, allowing greater manipulation of the statistical parameters used to construct the tolerance and prediction limits and ultimately more flexibility in designing a retesting scheme that can meet a target SWFPR for a fixed background size over a wide variety of networks.

Despite this advantage, new research done in preparing the Unified Guidance indicates that the effective power of any tolerance screen retesting procedure is always *less* than a comparable scheme based on a single repeated trigger value. The gain in flexibility in controlling false positive rates is real, but the most powerful retesting procedures will be of the 1-of-*m* or modified California-style varieties. Because of this loss in effective power, the Unified Guidance recommends an appropriate 1-of-*m* or modified California-style plan (**Chapter 19**).

B.5.3 NON-PARAMETRIC RETESTING SCHEMES

In the Addendum, two basic approaches to non-parametric retesting were described, each suggested by Gibbons (1990; 1991a). Both of these strategies defined the upper prediction limit as the

maximum observed background value. Once in hand, one new observation was collected from each downgradient well and compared against the non-parametric prediction limit. Measurements that exceeded the prediction limit were then retested. In his 1990 article, Gibbons presented tables of approximate network-wide significance levels for the case of 1-of-*m* retesting plans. Gibbons' 1991 article detailed the more stringent non-parametric California plans, giving exact false positive rates, but only in the case where the prediction limit was defined as the maximum of background.

Both of these efforts were superseded by Davis and McNichols (1994), who give exact false positive rates are given for both 1-of-*m* and strict California retesting strategies. In addition, Davis and McNichols compute these false positive rates when the non-parametric prediction limit is taken as either the maximum background value or the second-largest background concentration. The latter calculation is helpful in two ways. First, if a particular background concentration is unusually high and possibly an outlier, one could choose to fix the non-parametric prediction limit as the second-highest (and presumably more representative) background concentration. The statistical characteristics of the retesting scheme would still be assured without having to 'throw out' the suspected background outlier. Secondly, the statistical power of prediction limits based on the second-largest background value is greater than for those prediction limits based on the maximum. For large background samples (n), use of this alternate prediction limit may be the only option at some sites to achieve both the targeted false positive rate and sufficient effective power.

While the tables in the Davis and McNichols article are extremely useful, they do not include results for the modified California retesting scheme with $m = 4$, in which either the initial measurement or two of three resamples must be in-bounds. To complete the tables needed for the Unified Guidance, a variation of the Davis and McNichols algorithm was initially used to calculate the significance levels of the modified California retesting scheme with $m = 4$. Since that time, Davis and McNichols (1999) published an exact algorithm not only for the 1-of-*m* and strict California plans, but also for the modified California plan first suggested in an earlier draft of the Unified Guidance. Following their algorithm with some minor computational adjustments, the Unified Guidance tables have been recomputed, covering first the non-parametric 1-of-*m* plans and then the non-parametric modified California plan (**Chapter 19**). In each case, results are provided for non-parametric prediction limits taken either as the maximum value in background or as the second-largest concentration.

To measure the statistical power of these non-parametric retesting strategies, Davis and McNichols estimated power using Monte Carlo simulation with normally-distributed random variates. They then offered a new measure of power labeled the *Modified Addendum Criterion* or MAC, which rated schemes against an EPA reference power curve with $n = 8$ background samples. Recognizing that a particular power curve might only exceed the EPA reference power curve at large mean concentration shifts (Δ) above background, the MAC evaluated at what percentage power a proposed scheme did in fact begin to exceed the EPA reference power curve (*e.g.*, starting at 30% power, or 50% power, *etc.*).

In the Unified Guidance, effective power of non-parametric retesting schemes is measured in a similar, though not identical, manner. One difference is that the recommended EPA reference power curves are based on 10 rather than 8 background samples. With $n = 8$, there is less than a 50% probability of identifying a mean concentration increase above background of 3 standard deviations and less than an 80% chance of identifying an increase of 4 standard deviations. Another mostly semantic difference is that the schemes in the Unified Guidance are evaluated on whether or not they exceed the EPA reference power curve for concentrations exceeding background by a given number of standard

deviation units (*e.g.*, 3 or 4 standard deviations), instead of at a particular power percentage (*e.g.*, 30%, 70%, *etc.*).

To actually compute effective power, Monte Carlo simulations were not utilized in the Unified Guidance. Rather, since the underlying data were assumed to be normal, a simple modification to the numerical integration algorithms presented in Davis and McNichols (1987) was used to compute the power directly. Of course, if the data are normal in the first place, a parametric retesting scheme would be more appropriate. Non-parametric strategies should only be considered when the data appear to be distinctly non-normal or exhibit too many non-detects to judge normality. Nevertheless, since the true underlying distribution is unknown, the usual method of attack is to measure the statistical power that results when the underlying distribution is taken to be normal.

APPENDIX C. TECHNICAL APPENDIX

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C.1 SPECIAL STUDY: NORMAL VS. LOGNORMAL PREDICTION LIMITS

Section 10.3 outlines the strategy for distributional testing in the Unified Guidance. Among these recommendations is that the normal distribution should be treated as a default model until specific testing indicates otherwise. To establish this recommendation, a special study was conducted for the Unified Guidance to answer two key questions: 1) what are the consequences of incorrectly applying statistical techniques based on one distributional assumption (normal or lognormal), when the underlying distribution is, in fact, the other? and 2) what is the impact on statistical power and accuracy of assuming the wrong underlying distribution? These questions were tested for prediction limit tests in detection monitoring (and, by extension, for control charts).

The general effects of violating test assumptions can be measured in terms of false positive and negative error rates (and therefore power). A series of Monte Carlo simulations was generated for the Unified Guidance to evaluate the impacts on prediction limit false positive error rates and statistical power of using normal and lognormal distributions when applied either correctly or incorrectly to the underlying 'true' distributions. For varying inputs of background sample size, population coefficients of variation and confidence levels, sample data sets were generated and prediction limits computed for a single future observation using either a normal prediction limit [*NorPL*] or a lognormal prediction limit [*LgnPL*], as given in the equations below. $\bar{x}$ and s_x are the mean and standard deviation respectively of the original measurements, while $\bar{y}$ and s_y represent the log-mean and log-standard deviation:

$$NorPL_{1-\alpha} = \bar{x} + t_{n-1,1-\alpha} s_x \sqrt{1+\frac{1}{n}}$$

$$LgnPL_{1-\alpha} = \exp\left[\bar{y} + t_{n-1,1-\alpha} s_y \sqrt{1+\frac{1}{n}}\right]$$

To evaluate prediction limit performance, for each choice of inputs and statistical parameters, one million (N = 1,000,000) simulated normal background data sets and one million lognormal background data sets were generated and tested against each limit. When the underlying distribution was normal, a fixed unit standard deviation was coupled with a series of increasing mean levels to vary the population coefficient of variation. Then, to measure power in each case, new measurements were generated from similar normal models with mean levels incremented by k standard deviation units above the background mean, for k ranging from 0 to 5. A parallel evaluation was also conducted when a retest was added to the procedure. In this case, the prediction limits were constructed using the κ multiples for a 1-of-2 retesting scheme as described in **Chapter 19**. A summary of these results is given in **Figure C-1**.

C.1.1 RESULTS FOR NORMAL DATA

If the underlying population is truly normal, *treating the sample data as lognormal* in constructing a prediction limit can have significant consequences. **Figure C-1** presents key results, either averaged over all the statistical input parameters or broken down by sample size, confidence level, and coefficient of variation. These statistics include the average ratio between the normal prediction limit [*NorPL*] and the lognormal prediction limit [*LgnPL*], the average difference between the *nominal* (*i.e.*, expected) false positive rate (α) of the test and the *observed* false positive rate, and the average percentage of cases

where two statistical power targets were met, those being 50% power at 3 standard deviations above the background mean and 80% power at 4 standard deviations above the background mean.

With no retesting and truly normal data, the lognormal prediction limits were in every case considerably longer and thus less powerful than the normal prediction limits. The discrepancies in performance were smallest for larger sample sizes, lower confidence levels, and smaller coefficients of variation. However, in only one of the category breakdowns ($1–\alpha = 0.995$) did the normal prediction limits fail to meet *both* power targets at least half the time, while the lognormal limits jointly met both power targets less than half the time in all cases except one ($1–\alpha = 0.90$). As to false positives, the lognormal limits consistently exhibited *less* than the expected (nominal) false positive rate. The normal prediction limits tended to have slightly higher than nominal error rates.

When retesting was added to the procedure, the performance of both limits improved. The false positive rates of both were closer to the nominal rates, though the normal prediction limits were relatively closer to the expected rates. While power improved across the board compared to *not* using a retest, the normal limits were on average about 13% shorter than the lognormal limits, leading again to a measurable loss of statistical power for the lognormal prediction limits. Particularly noticeable was the significant difference in power at higher confidence levels, the very kinds of confidence levels needed when designing retesting strategies for multiple tests against a prediction limit.

On balance, *misapplication* of logarithmic prediction limits to normal data consistently resulted in lower power (often considerably) and false positive rates that were lower than expected, unless the population coefficient of variation was quite small, the background sample size was larger, and the confidence level more moderate. Since a lognormal prediction limit will be applied only if the underlying population is thought or assumed to be lognormal, it is helpful to gauge how these factors work in practice. On one hand, the higher confidence levels and consequently lower α values needed for retesting strategies with simultaneous tests (**Chapter 19**) would argue *against* presuming the underlying data to be lognormal without specific goodness-of-fit testing. In other words, if the data are actually normal but the lognormal prediction limit is misapplied, a high price in statistical power may be paid.

In terms of sample size, the greatest penalties from misapplying lognormal prediction limits occur for smaller background sizes. Since goodness-of-fit tests are least able to distinguish between normal and lognormal data with small samples, small background samples be not be presumed to be lognormal *as a default* unless other site-specific evidence suggests otherwise. For larger sample sizes, goodness-of-fit tests have much better discriminatory power, enabling a better indication of which model to use.

With regard to coefficient of variation [CV], the guidelines are less clear cut. Given that groundwater data are generally positive in value, truly normal populations are likely to have population coefficients of variation of 0.3 or lower. Larger coefficients of variation would result in a significant fraction of negative measurements. In addition, the probability of observing a large sample coefficient of variation from a normal population with population coefficient of variation of 0.3 or less is rather small.

However, the measurement and *censoring* of small concentration values complicates the picture. Such values are measured below a reporting limit [RL] and are generally listed as ‘less thans.’ A measurement process that is normal with high coefficient of variation and mean close to the RL can generate a mixture of left-censored and detected values with fairly high coefficient of variation yet not be lognormal. In fact, the cases in **Figure C-1** with higher coefficients of variation were analyzed in essentially this fashion, with negative values imputed to a small, positive reporting limit prior to

calculation of the prediction limits. The results indicate a substantial loss of performance when lognormal limits are misapplied to these left-censored normal data sets, *with or without retesting*. Therefore, the observed CV should not be used as the sole criterion of whether to presume an underlying normal or lognormal data model. Rather, if large fractions of censored data are present, censored probability plots (**Chapter 15**) should be constructed to aid in choosing an appropriate distribution.

C.1.2 RESULTS FOR LOGNORMAL DATA

Do normal-based prediction limits suffer in a similar comparison when the underlying population it really lognormal? The results from applying normal and lognormal prediction limits to underlying lognormal data are presented in **Figure C-2**. There, the summaries are similar to **Figure C-1** with one important exception. As explained in **Chapter 10** and **Appendix Section C.2**, the lognormal distribution is not an additive model. Because of this fact, the distributional alternatives used in assessing the statistical power of a lognormal-based prediction limit usually involve setting the alternative mean to a *multiple* of the background mean while keeping a *constant* lognormal coefficient of variation.

The net effect is that the power of lognormal-based tests depends greatly on the actual level of the coefficient of variation. This is different from normal-based power analyses, where the coefficient of variation only plays a role in terms of the degree of censoring in the data (thus affecting power through the handling of left-censored values, *i.e.*, non-detects). Because the achievable power varies over such a large range — depending on the level of skewness of the specific lognormal distribution — reference statistical power for lognormal models must be tied to the observed background coefficient of variation. However, since a performance comparison *across* coefficient of variation levels was needed for the results of **Figure C-2**, a *single benchmark* was used to assess the comparative power of the normal and lognormal prediction limits. While imperfect for practical use, this benchmark was set at 25% power for alternatives of three times the background mean and 50% power at five times the background mean.

For an underlying lognormal model with no retesting, **Figure C-2** indicates that while the false positive rates of lognormal-based prediction limits are essentially as advertised (*i.e.*, a 95% confidence prediction limit has close to the nominal 5% false positive rate), the false positive rates of normal-based limits are higher than expected, often substantially, especially for higher confidence levels and higher coefficients of variation. The most significant drawback to *misapplying* normal prediction limits to lognormal data would then be an excessive site-wide false positive rate from using such limits on multiple well-constituent pairs.

However, the situation changes dramatically with the addition of even a single retest. In this case, the lognormal prediction limits are still more accurate than the normal limits, in terms of having false positive rates closer to the nominal targets. Nevertheless, with the added retest, the achieved false positive rates for the normal limits tend to be *less* than the expected rates, especially for moderate to larger sample sizes. In addition, except for very skewed lognormal distributions, the power of the normal limits is comparable or greater than the power of the lognormal limits.

Figure C-1. Accuracy and Power of Normal vs. Lognormal Prediction Limits When Underlying Data Are Normal

No Retesting, 1-of-1 Scheme

Category		Assumed Model	Length Ratio	α-Error	Power-50%	Power-80%	Power-Both
ALL		normal	0.6599	0.00611	0.759	0.741	0.741
		lognormal		-0.01965	0.348	0.277	0.277
N	4	normal	0.5729	0.00643	0.500	0.500	0.500
		lognormal		-0.01590	0.286	0.107	0.107
	8	normal	0.6713	0.00607	0.679	0.607	0.607
		lognormal		-0.01962	0.357	0.321	0.321
	12	normal	0.6935	0.00599	0.857	0.857	0.857
		lognormal		-0.02107	0.357	0.321	0.321
	16	normal	0.7020	0.00596	1.000	1.000	1.000
		lognormal		-0.02201	0.393	0.357	0.357
(1−α)	0.900	normal	0.8437	0.00898	1.000	1.000	1.000
		lognormal		-0.03843	1.000	0.857	0.857
	0.950	normal	0.7436	0.00857	1.000	1.000	1.000
		lognormal		-0.02884	0.393	0.250	0.250
	0.990	normal	0.5583	0.00416	0.643	0.607	0.607
		lognormal		-0.00749	0.000	0.000	0.000
	0.995	normal	0.4940	0.00274	0.393	0.357	0.357
		lognormal		-0.00385	0.000	0.000	0.000
CV	0.125	normal	0.9504	-0.00005	0.688	0.688	0.688
		lognormal		-0.00740	0.500	0.438	0.438
	0.250	normal	0.8401	-0.00009	0.688	0.688	0.688
		lognormal		-0.01486	0.438	0.438	0.438
	0.333	normal	0.7439	0.00015	0.688	0.688	0.688
		lognormal		-0.01959	0.438	0.313	0.313
	0.500	normal	0.5794	0.00266	0.750	0.688	0.688
		lognormal		-0.02614	0.250	0.188	0.188
	0.667	normal	0.5112	0.00830	0.813	0.813	0.813
		lognormal		-0.02611	0.250	0.188	0.188
	0.752	normal	0.4989	0.01160	0.813	0.813	0.813
		lognormal		-0.02458	0.250	0.188	0.188
	1.000	normal	0.4954	0.02021	0.875	0.813	0.813
		lognormal		-0.01887	0.313	0.188	0.188

Legend. **Category**: N = Sample size; (1-α) = Nominal confidence level; CV = Coefficient of variation of underlying normal distribution. For each case, results for all simulations with that characteristic were averaged to derive that line of the figure.
Assumed Model: Whether normal or lognormal formulas were used to compute the prediction limits.
Length Ratio: Ratio of the normal prediction limit to the lognormal prediction limit.
α-error: Achieved false positive rate minus nominal false positive rate.
Power-50%: Fraction of simulations in which 50% power target at 3 standard deviations above background was met by the prediction limit.
Power-80%: Fraction of simulations in which 80% power target at 4 standard deviations above background was met by the prediction limit.
Power-Both: Fraction of simulations in which both the 50% and 80% power targets were met.

Retesting, 1-of-2 Scheme

Category		Assumed Model	Length Ratio	α-Error	Power-50%	Power-80%	Power-Both
ALL		normal	0.8712	0.00134	0.911	0.884	0.884
		lognormal		0.00052	0.670	0.625	0.625
n	4	normal	0.7870	0.00254	0.643	0.536	0.536
		lognormal		-0.00302	0.500	0.500	0.500
	8	normal	0.8815	0.00130	1.000	1.000	1.000
		lognormal		0.00070	0.643	0.607	0.607
	12	normal	0.9034	0.00066	1.000	1.000	1.000
		lognormal		0.00157	0.714	0.679	0.679
	16	normal	0.9129	0.00087	1.000	1.000	1.000
		lognormal		0.00283	0.821	0.714	0.714
(1−α)	0.900	normal	1.0370	-0.00134	1.000	1.000	1.000
		lognormal		0.01496	1.000	1.000	1.000
	0.950	normal	0.9543	0.00235	1.000	1.000	1.000
		lognormal		-0.00485	1.000	1.000	1.000
	0.990	normal	0.7807	0.00252	0.893	0.786	0.786
		lognormal		-0.00510	0.500	0.321	0.321
	0.995	normal	0.7129	0.00185	0.750	0.750	0.750
		lognormal		-0.00295	0.179	0.179	0.179
CV	0.125	normal	0.9867	0.00009	0.875	0.875	0.875
		lognormal		0.00023	0.875	0.875	0.875
	0.250	normal	0.9486	0.00003	0.875	0.875	0.875
		lognormal		-0.00048	0.813	0.813	0.813
	0.333	normal	0.9065	0.00016	0.875	0.875	0.875
		lognormal		-0.00164	0.688	0.625	0.625
	0.500	normal	0.8289	0.00095	0.938	0.875	0.875
		lognormal		-0.00190	0.563	0.500	0.500
	0.667	normal	0.8051	0.00242	0.938	0.875	0.875
		lognormal		0.00132	0.563	0.500	0.500
	0.752	normal	0.8048	0.00270	0.938	0.875	0.875
		lognormal		0.00241	0.563	0.500	0.500
	1.000	normal	0.8178	0.00306	0.938	0.938	0.938
		lognormal		0.00371	0.625	0.563	0.563

Legend. **Category**: N = Sample size; (1-α) = Nominal confidence level; CV = Coefficient of variation of underlying normal distribution. For each case, results for all simulations with that characteristic were averaged to derive that line of the figure.
Assumed Model: Whether normal or lognormal formulas were used to compute the prediction limits.
Length Ratio: Ratio of the normal prediction limit to the lognormal prediction limit.
α-error: Achieved false positive rate minus nominal false positive rate.
Power-50%: Fraction of simulations in which 50% power target at 3 standard deviations above background was met by the prediction limit.
Power-80%: Fraction of simulations in which 80% power target at 4 standard deviations above background was met by the prediction limit.
Power-Both: Fraction of simulations in which both the 50% and 80% power targets were met.

Figure C-2. Accuracy and Power of Normal vs. Lognormal Prediction Limits When Underlying Data Are Lognormal

No Retesting, 1-of-1 Scheme

Category		Assumed Model	Length Ratio	α-Error	Power-25%	Power-50%	Power-Both
ALL		normal	0.6082	0.03557	0.806	0.581	0.581
		lognormal		-0.00001	0.500	0.412	0.412
n	4	normal	0.4444	0.04682	0.775	0.550	0.550
		lognormal		0.00000	0.400	0.325	0.325
	8	normal	0.6006	0.03782	0.825	0.600	0.600
		lognormal		-0.00002	0.500	0.425	0.425
	12	normal	0.6723	0.03102	0.825	0.600	0.600
		lognormal		0.00001	0.550	0.450	0.450
	16	normal	0.7153	0.02660	0.800	0.575	0.575
		lognormal		-0.00003	0.550	0.450	0.450
(1–α)	0.900	normal	0.8985	0.02509	1.000	0.750	0.750
		lognormal		-0.00001	0.950	0.650	0.650
	0.950	normal	0.7108	0.04175	0.975	0.675	0.675
		lognormal		-0.00002	0.525	0.475	0.475
	0.990	normal	0.4472	0.04037	0.700	0.475	0.475
		lognormal		0.00000	0.275	0.275	0.275
	0.995	normal	0.3762	0.03505	0.550	0.425	0.425
		lognormal		0.00000	0.250	0.250	0.250
CV	0.125	normal	0.9549	0.00733	1.000	1.000	1.000
		lognormal		-0.00003	1.000	1.000	1.000
	0.250	normal	0.8749	0.01419	1.000	1.000	1.000
		lognormal		-0.00004	1.000	1.000	1.000
	0.500	normal	0.7282	0.02535	1.000	1.000	1.000
		lognormal		-0.00004	0.813	0.813	0.813
	0.750	normal	0.6278	0.03315	1.000	0.938	0.938
		lognormal		0.00006	0.500	0.500	0.500
	1.000	normal	0.5629	0.03829	0.938	0.813	0.813
		lognormal		-0.00008	0.438	0.438	0.438
	1.250	normal	0.5204	0.04216	0.938	0.500	0.500
		lognormal		0.00009	0.375	0.188	0.188
	1.500	normal	0.4915	0.04479	0.750	0.438	0.438
		lognormal		-0.00002	0.250	0.188	0.188
	2.000	normal	0.4566	0.04815	0.500	0.125	0.125
		lognormal		0.00003	0.250	0.000	0.000
	2.500	normal	0.4378	0.05037	0.500	0.000	0.000
		lognormal		-0.00004	0.188	0.000	0.000
	3.000	normal	0.4266	0.05189	0.438	0.000	0.000
		lognormal		-0.00002	0.188	0.000	0.000

Legend. **Category**: N = Sample size; (1-α) = Nominal confidence level; CV = Coefficient of variation of underlying lognormal distribution. For each case, results for all simulations with that characteristic were averaged to derive that line of the figure.
Assumed Model: Whether normal or lognormal formulas were used to compute the prediction limits.
Length Ratio: Ratio of the normal prediction limit to the lognormal prediction limit.
α-error: Achieved false positive rate minus nominal false positive rate.
Power-25%: Fraction of simulations in which 25% power target at 3 times the background mean was met by the prediction limit.
Power-50%: Fraction of simulations in which 50% power target at 5 times the background mean was met by the prediction limit.
Power-Both: Fraction of simulations where both 25% and 50% power targets were met.

Retesting, 1-of-2 Scheme

Category		Assumed Model	Length Ratio	α-Error	Power-25%	Power-50%	Power-Both
ALL		normal	0.9920	-0.00405	0.587	0.544	0.537
		lognormal		0.00009	0.600	0.544	0.531
n	4	normal	0.7334	0.00804	0.625	0.525	0.500
		lognormal		0.00022	0.550	0.425	0.425
	8	normal	0.9830	-0.00354	0.600	0.550	0.550
		lognormal		0.00016	0.600	0.550	0.550
	12	normal	1.0931	-0.00897	0.575	0.550	0.550
		lognormal		-0.00020	0.625	0.600	0.575
	16	normal	1.1586	-0.01175	0.550	0.550	0.550
		lognormal		0.00018	0.625	0.600	0.575
(1–α)	0.900	normal	1.3532	-0.02895	0.800	0.700	0.700
		lognormal		0.00027	1.000	0.850	0.850
	0.950	normal	1.1500	-0.00478	0.700	0.600	0.600
		lognormal		0.00009	0.750	0.625	0.625
	0.990	normal	0.7890	0.00892	0.475	0.475	0.475
		lognormal		0.00000	0.375	0.375	0.375
	0.995	normal	0.6759	0.00860	0.375	0.400	0.375
		lognormal		0.00000	0.275	0.325	0.275
CV	0.125	normal	0.9889	-0.00035	1.000	1.000	1.000
		lognormal		0.00011	1.000	1.000	1.000
	0.250	normal	0.9684	-0.00099	1.000	1.000	1.000
		lognormal		0.00008	1.000	1.000	1.000
	0.500	normal	0.9332	-0.00214	1.000	1.000	1.000
		lognormal		0.00008	0.938	0.938	0.938
	0.750	normal	0.9199	-0.00317	0.938	1.000	0.938
		lognormal		0.00018	0.688	0.813	0.688
	1.000	normal	0.9253	-0.00416	0.688	0.688	0.688
		lognormal		0.00008	0.500	0.500	0.500
	1.250	normal	0.9428	-0.00481	0.500	0.500	0.500
		lognormal		0.00014	0.500	0.438	0.438
	1.500	normal	0.9673	-0.00527	0.438	0.250	0.250
		lognormal		0.00011	0.500	0.375	0.375
	2.000	normal	1.0266	-0.00606	0.188	0.000	0.000
		lognormal		0.00004	0.375	0.188	0.188
	2.500	normal	1.0913	-0.00662	0.063	0.000	0.000
		lognormal		0.00003	0.250	0.188	0.188
	3.000	normal	1.1566	-0.00696	0.063	0.000	0.000
		lognormal		0.00004	0.250	0.000	0.000

Legend. **Category**: N = Sample size; (1-α) = Nominal confidence level; CV = Coefficient of variation of underlying lognormal distribution. For each case, results for all simulations with that characteristic were averaged to derive that line of the figure.
Assumed Model: Whether normal or lognormal formulas were used to compute the prediction limits.
Length Ratio: Ratio of the normal prediction limit to the lognormal prediction limit.
α-error: Achieved false positive rate minus nominal false positive rate.
Power-25%: Fraction of simulations in which 25% power target at 3 times the background mean was met by the prediction limit.
Power-50%: Fraction of simulations in which 50% power target at 5 times the background mean was met by the prediction limit.
Power-Both: Fraction of simulations in which both the 25% and 50% power targets were met.

On balance, adding a retest to the testing procedure significantly minimizes the penalty of misapplying normal prediction limits to lognormal data, as long one uses a sample size of at least 8 and the coefficient of variation is not too large. Consequently, for most situations, there is *less* penalty associated with making a default assumption of *normality* than in making a default assumption of *lognormality*. With highly skewed data, say with large coefficients of variation of 1.5 or more, goodness-of-fit tests tend to better discriminate between the normal and lognormal models. Again such diagnostic testing should be done *explicitly*, rather than simply assuming the data are normal or lognormal.

The most problematic cases occur for very small background sample sizes, where a misapplication of prediction limits in either direction can result in poorer statistical performance, even with retesting. In some situations, testing may have to done on an interim or ad-hoc basis until more data is collected. Still, the Unified Guidance does not recommend an automatic *default* assumption of lognormality.

C.2 CALCULATING STATISTICAL POWER

C.2.1 STATISTICAL POWER OF WELCH'S T-TEST

The statistical power of any test represents the probability that the alternative hypothesis, H_A, will be accepted, given that the null hypothesis, H_0, is actually false. In groundwater monitoring, power usually represents the probability that the compliance point concentrations will be identified as significantly higher than background, when in fact they *are* higher. Of course, statistical power is not a single number, but rather a *function* of the increase in the compliance population mean above the background average. This fact makes the exact power of many tests difficult to calculate, especially since many test statistics have a complicated distributional behavior under the alternative hypothesis.

The critical points or percentage points of any test are computed under the assumption that the null hypothesis is true. In the case of Welch's *t*-test, the *t*-statistic approximately follows a Student's *t*-distribution under H_0. That is not the case, however, when the alternative hypothesis is true; then the *t*-statistic follows what is known as a *non-central* *t*-distribution with non-centrality parameter δ. Essentially, the non-centrality parameter δ governs the average or expected value of the *t*-statistic.

When the null hypothesis is true, so that the two population means are equal, the *t*-statistic should tend to be close to zero. The distribution of the *t*-statistic is in fact centered at zero in this case, meaning that the usual Student's *t*-distribution can be regarded as a non-central *t*-distribution with non-centrality parameter equal to zero.

When H_A is true instead, and the compliance point population mean is larger than the background mean, Welch's *t*-statistic will tend to be positive rather than centered at zero. The actual center of the distribution will depend on precisely how much larger the compliance point mean is compared to background. However, if σ_x represents the standard deviation of the first population and σ_y represents the standard deviation of the second population, it can be shown that the two-sample Welch's *t*-statistic approximately follows a non-central *t*-distribution with degrees of freedom equal to

$$df = \left[\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}\right]^2 \Bigg/ \left[\frac{\sigma_x^4}{n_x^2\left(n_x - 1\right)} + \frac{\sigma_y^4}{n_y^2\left(n_y - 1\right)}\right] \qquad [C.1]$$

and non-centrality parameter equal to

$$\delta = \Delta \Bigg/ \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}} \qquad [C.2]$$

where Δ is the concentration difference separating the background and compliance point population means.

Clearly, the distribution of the *t*-statistic under H_A depends in a complex manner not only on the sample sizes and the true difference between the population means, but also the respective population variances. Since statistical power is the probability that the Welch's *t*-statistic exceeds the original critical point, t_{cp}, yet the population variances are almost always unknown, computation of an exact

power is essentially impossible. Instead, an *approximate power* can be computed by substituting the sample variances for their population counterparts into equations [C.1] and [C.2]. By letting $f = \sigma_y^2 / \sigma_x^2$, the non-centrality parameter becomes

$$\delta = \frac{\Delta}{\sigma_x}\sqrt{\frac{n_x n_y}{n_y + f n_x}} = k\sqrt{\frac{n_x n_y}{n_y + f n_x}} \qquad [C.3]$$

where k represents the increase in standard deviation units above the background mean. The non-centrality parameter can be approximated by substituting $\hat{f} = s_y^2 / s_x^2$ for f in [C.3].

Using this formulation, the approximate statistical power of Welch's *t*-test can be computed by repeatedly increasing k (*e.g.*, in half units starting with 0.5) and determining the probability of exceeding the original critical point, t_{cp}, under the non-central *t*-distribution. A concise summary of the non-central t-distribution can be found in Evans, Hastings, and Peacock (1993). Percentage points of this distribution can be computed in selected standard statistical packages, including the free, open-source statistical software **R** (www.r-project.org).

►EXAMPLE C-1

Determine the approximate power of the *t*-test on benzene data used in **Example 16-1**.

SOLUTION

Step 1. Since Welch's *t*-test was run on the logged benzene measurements, power should also be computed using the logged values. In that case, the degrees of freedom was approximated at df = 11 and the critical point at α = .05 was found to be t_{cp} = 1.796 from the Student's *t*-distribution in **Table 16-1**.

Step 2. Determine the non-centrality parameter δ from equation [C.3], substituting $\hat{f} = s_y^2 / s_x^2$ for f. Since $n_x = n_y = 8$, the sample downgradient log-standard deviation is s_y = 1.9849, and the sample background log-standard deviation is s_x = 1.0826. Plugging these values into $\hat{f}$ gives $\hat{f} = (1.9849)^2 / (1.0826)^2 = 3.362$. The approximate non-centrality parameter becomes

$$\delta = k\sqrt{\frac{8 \times 8}{8 + 3.362 \times 8}} = k(1.354)$$

where k represents the increase above the benzene background log-scale mean in log-standard deviation units.

Step 3. Systematically increase k from 0.5 to 5 in steps of 0.5 to determine the non-centrality parameter δ at each point to be computed on the power curve (presented in the table below). Then determine each power value by calculating from the non-central *t*-distribution, with non-centrality parameter δ and df = 11, the probability of exceeding the original critical point of t_{cp} = 1.796.

k	δ	power
0.5	0.677	0.1565
1.0	1.354	0.3541
1.5	2.031	0.6022
2.0	2.708	0.8135
2.5	3.385	0.9360
3.0	4.062	0.9843
3.5	4.739	0.9973
4.0	5.416	0.9997
4.5	6.093	1.0000
5.0	6.770	1.0000

Step 4. Interpret the power results. The table in **Step 3** shows an approximate probability of 81% for detecting a two log-standard deviation increase above the background mean benzene level. If the data had been analyzed in the original units, a two standard deviation increase would translate into almost 11 ppb (using the sample background standard deviation of 5.31 ppb from **Example 16-1** as an estimate of the true standard deviation).

However, in the logarithmic domain, the interpretation is a bit different. As discussed in **Section C.2.3**, adding $k\sigma$ to the log-scale mean is equivalent to *multiplying* the arithmetic mean by $\exp(k\sigma)$. Therefore, a two log-standard deviation increase in the log-scale background mean is roughly equivalent to multiplying the *original* background mean by a factor of $\exp(2 \times 1.0826) = 8.7$, taking the sample log-scale background standard deviation of 1.0826 as an estimate for the true log-scale standard deviation.

Consequently, if the true background mean for benzene is close to the sample value of 3 ppb, the test will have more than 80% power for detecting a downgradient benzene mean of at least $3 \times 8.7 \approx 26$ ppb or larger. ◀

C.2.2 POWER OF PREDICTION LIMITS FOR FUTURE MEAN VS. OBSERVATIONS

The Unified Guidance discusses two basic kinds of parametric prediction limits: those for individual future observations and those for future means. Analytical expressions for the statistical power of each can be written and compared using the same sample size (n), the same false positive rate (α), and the same number of future measurements ($p = m$).

The power of a prediction limit for a future mean of order p (that is, a mean of p individual future values) with normally-distributed data can be expressed in the equation

$$1-\beta = \Pr\left\{ T_{n-1}\left(\delta = \Delta \Big/ \sqrt{\frac{1}{p}+\frac{1}{n}} \right) > t_{1-\alpha,n-1} \right\} \qquad [C.4]$$

where $(1-\beta)$ is a notation for power, Δ is the true difference (in standard deviation units) between the background and compliance point population means, and

$$T_{n-1}\left(\delta = \Delta \Big/ \sqrt{\frac{1}{p}+\frac{1}{n}}\right) \qquad [C.5]$$

denotes a random variable distributed according to the non-central *t*-distribution with non-centrality parameter δ and (*n*–1) degrees of freedom.

By contrast, the power of a prediction limit for *p* individual future values can be derived using the formulation in Davis and McNichols (1987), leading to the expression

$$1-\beta = 1-\int_0^1 \Pr\left\{T_{n-1}\left(\delta=\sqrt{n}\left[\Phi^{-1}(u)+\Delta\right]\right) \le t_{1-\alpha/p,n-1}\sqrt{n+1}\right\} p u^{p-1} du \qquad [C.6]$$

where in this case $\Phi^{-1}(u)$ denotes the inverse standard normal transformation. The non-central *t*-distribution is required in each case, with further integration of the non-central *t* cumulative distribution function [CDF] needed for the case of *p* individual future measurements. These formulas are utilized in **Chapter 18** to provide graphical power comparisons between prediction limits for future means versus prediction limits for individual values.

C.2.3 COMPUTING POWER WITH LOGNORMAL DATA

The special Monte Carlo study presented in **Section C.1** involved a computation of statistical power when the underlying data are lognormal in distribution rather than normal. In the case of normal data, effective power is computed by adding an upward 'shift' in the mean of the baseline distribution, in order to simulate an increasing compliance point concentration. Adding such a shift does not increase the variance (σ^2) of the shifted distribution, only the mean (μ).

With lognormal data, both the mean and variance depend on the two distributional parameters, μ and σ. Adding a shift to the log-mean μ on the log-scale thus increases *both* the variance and the mean in the concentration domain, confusing the usual interpretation of power as the ability to detect upward changes in the mean level when all other factors (including the variance) are held constant.

In fact, if computations are conducted on the log-scale and a shift (Δ) is added to the log-mean parameter (μ), the effect is to *multiply* the lognormal mean in the arithmetic domain by a factor of $\exp(\Delta)$. To see this, note that the lognormal mean is written as

$$M = \exp\left(\mu + .5\sigma^2\right) \qquad [C.7]$$

An additive shift to the log-mean results in a change to the (arithmetic) lognormal mean of

$$M_A = \exp\left(\mu+\Delta+.5\sigma^2\right) = \exp(\Delta)\exp\left(\mu+.5\sigma^2\right) = \exp(\Delta)M \qquad [C.8]$$

To compute statistical power, one must assess test performance both under background conditions and under increasing levels of contamination. But the power that can be expected with lognormal data varies depending on the lognormal coefficient of variation [*CV*]. For a fixed coefficient of variation, as lognormal concentrations increase, the lognormal standard deviation increases proportionally to the

lognormal mean. Because of this — and in contrast to the case of normal data — a different lognormal power curve could be associated with each unique value of the *CV*.

To sidestep this problem, the Unified Guidance assumes that if background is lognormal, the *same* coefficient of variation [*CV*] will apply to both the background and compliance point populations. This assumption has two important consequences: 1) compliance point data with mean levels higher than background will tend to also be more variable than the background measurements, a common empirical truth in environmental data sets; and 2) on the log-scale, the log-variance parameter (σ^2) will be the *same* in both populations. The reason that this second consequence holds is that the log-standard deviation parameter is solely a function of the coefficient of variation, as expressed in the following equation:

$$\sigma = \sqrt{\log\left(1 + CV^2\right)} \qquad [C.9]$$

Thus if the *CV* is held constant, to will the log-standard deviation parameter (σ).

The upshot of the second consequence is that all power computations for lognormal data can be done in the log-domain, using the fact that the transformed data will be normally distributed and that the background and compliance point populations will have a common standard deviation. Consequently, the computational framework for simulating statistical power of lognormal data is almost precisely the same as the framework for the normal case.

In particular, the power curve for a given test can always be generated — without loss of generality — by assuming that the background data follow (perhaps in the log-domain) a standard normal distribution, and that the compliance point data follow (again in the log-domain for lognormal populations) a normal distribution with unit variance and shifted mean equal to $k\sigma = k$, since σ is assumed for computational purposes equal to 1. Then the multiplier k is typically allowed to range from 0 to 5, as this adequately sketches out the normal power curve in most situations.

The only aspect of the lognormal case that differs from the normal is the scaling of the horizontal axis of the power curve. In the log-domain, the curve documents power at increasing multiples of k log-standard deviations (σ) above the background log-mean (μ). To interpret these values in terms of the original concentrations, the background mean has to be reconstructed using the formula

$$M_{BG} = \exp\left(\mu + 0.5\sigma^2\right) \qquad [C.10]$$

while the compliance point (arithmetic) mean corresponding to the $k\sigma$ log-scale increase becomes

$$M_{CW} = \exp\left(\mu + k\sigma + 0.5\sigma^2\right) = M_{BG}\exp\left(k\sigma\right) \qquad [C.11]$$

or equivalently, a multiple of $\exp(k\sigma)$ times the mean background level.

►EXAMPLE C-2

Suppose that background data are fit best by a lognormal distribution with CV = 0.5. What steps must be taken to simulate the statistical performance of a lognormal prediction limit on observations with a single verification resample?

SOLUTION

Step 1. Compute the background log-standard deviation parameter as:

$$\sigma = \sqrt{\log\left(1 + CV^2\right)} = \sqrt{\log\left(1 + .25\right)} = 0.4724$$

taking multiplier k = 0 to represent the background population.

Step 2. Generate simulated random values from a standard normal distribution with zero mean and unit standard deviation. These values represent simulated and standardized log-domain background measurements.

Step 3. Compute the background prediction limit for lognormal data with a single resample using the formula:

$$PL = \exp\left(\bar{y} + \kappa s_y\right) \qquad [C.12]$$

where $\bar{y}$ and s_y are respectively the log-mean and log-standard deviation, and κ is taken from the **Chapter 19** tables in the **Appendix**, depending on the background sample size, the number of tests to be run, and the type of 1-of-2 retesting plan (interwell or intrawell). Note that the simulated background values do not need to be exponentiated prior to computing the background prediction limit, due to the construction of formula [C.12].

Step 4. For any specific k in the range from 0 to 5 (with increasing steps of 0.5), set the compliance point log-mean equal to $\mu_A = k$. Use this result to generate two normal measurements with shifted mean μ_A and unit standard deviation. The two simulated values represent an original sample and a possible resample from the contaminated compliance point. Exponentiate these two values to get simulated lognormal measurements from the desired (alternative) distribution.

Step 5. Compare the simulated values against the background prediction limit. If both exceed the limit, increment the count of cases associated with k in which a difference from background has been identified. If only one or none exceeds the limit, do not increment the count.

Step 6. Repeat **Steps 2** through **5** a large number of iterations (say 10,000 or more) and determine the fraction of cases for given k at which an exceedance of background is found. This fraction represents the estimated power of the lognormal prediction limit in the log-domain of a $k\sigma$ increase above the background log-mean. Equivalently, with a population CV = 0.5, this represents a compliance point mean level of $\exp\left(k\sigma\right) = \exp\left(k \times .4724\right)$ times the (arithmetic) background mean. Repeat this entire process for each k in the range of 0 to 5 to estimate the full lognormal power curve for that prediction limit. ◄

C.3 R SCRIPTS

Certain calculations in the Unified Guidance cannot easily be done either by hand, with a spreadsheet, or even within many common statistical packages. In some cases, proprietary software tailored to groundwater statistics can be consulted. Barring that, an alternate solution is to download and install the free-of-charge, open source, statistical analysis and programming environment **R** software. It can be utilized to perform or program almost any kind of statistical test or calculation. However, with its power and flexibility comes a somewhat steeper learning curve for new programming language.

One of **R**'s advantages is the ability to run 'scripts,' short pre-written programs that can be run repeatedly to perform specific statistical calculations. Scripts can be easily tailored to data- or site-specific configurations using a simple text editor. Because users of the Unified Guidance may occasionally need calculations not covered in the **Appendix** tables or which are unavailable in standard statistical software, a small number of **R** scripts are listed below. These scripts can be modified as necessary and then run in **R**, once the **R** environment is installed on a personal computer. They are provided as a courtesy to users of the Unified Guidance and are provided without any guarantees or implied warranties.

The scripts provided in the Unified Guidance below cover two specific topics: 1) calculation of parametric intrawell prediction limit κ-multipliers used with retesting, especially in cases where a pooled standard deviation estimate might be used in place of the usual sample standard deviation (**Section 13.3**); and 2) computation of a bootstrapped non-parametric confidence band around a Theil-Sen trend line (**Section 21.3.2**).

It is first necessary to install the R-software. As of this date, the latest version is 2.7.2. The program can be downloaded from the website: http://cran.r-project.org. Versions are available for most current Windows operating systems, as well as other types. Once the program has been downloaded (approximately 30 mb), it can be accessed through a self-installed desktop icon.

The R-scripts should first be transferred to a working directory; copies are provided with the distribution CD. If copied directly from the guidance Acrobat pdf using a text editor such as Notepad, it will be necessary to copy each page of the script separately and combine (avoiding unnecessary margin, header and footer information). Each file should be named and saved with the extension changed to a *xxx.r* format. It may be necessary to add a number of additional comment codes (#) at the beginning of the scripts using the text editor, so that each line of narrative text is first identfied by a comment code. To run the scripts:

1) Open the R-software from the desktop icon; you will be in the R-console window;

2) Click File on the toolbar, select Change_dir and hit [Enter]; set the working directory to the one with your scripts; hit [Enter];

3) Then click File and select Open Script [Enter]; Click on the desired R-script file and hit Enter;

4) In the R-console window; change script inputs as desired; Click Edit on the toolbar and select Run All.

5) The program will run behind the console window. Outputs can be read by minimizing the R-editor. Using the side scrollbar, check the R-script text run to determine if any errors occurred. As noted above, it may be necessary to add the comment code (#) where line length has been exceeded. To run additional inputs within a script, simply modify the inputs in the R-console window and then follow steps 4) and 5). To run other scripts, minimize R, select the new script, adjust as appropriate and follow steps 3) to 5).

6) If an effect size power level is desired for the two prediction limit scripts, change one of the two values in parentheses on the line *del = c(3,4)* and run again.

C.3.1 PARAMETRIC INTRAWELL PREDICTION LIMIT MULTIPLIERS

1-of-m Retesting Plans

```
# R Script for 1-of-m retesting plans
# Compute multiplier for intrawell prediction limit using either regular or pooled SD estimate
# and 1-of-m  retesting for either observations or means of order p
# Solve for kappa given an SWFPR adjusted for nbr of constituents and wells;
# then rate by effective power
# ne = number of yearly evaluations
# Note: ne=4 (quarterly eval), ne=2 (semi-annual), ne=1 (annual)
# n = intrawell BG sample size; w = # wells; coc = # constituents
# df = degrees of freedom associated with variance estimate of prediction limit formula
# Note: if the usual std deviation for a single well is used, set df = (n-1);
#       if using a pooled SD estimate across w equal sized wells, set df= w*(n-1) or
#       df = (sum of well n's) – w, if  w pooled wells are of different sizes
# alph = per-test false positive rate
# m = type of 1-of-m retesting scheme (usually m= 1,2,3,or 4)
# ord = order of the mean to be predicted (for tests on observations, set ord=1)
# swfpr is the targeted network-wide false positive rate, by default set to 10%
# Rate power at 3 and 4 SD units above BG;
# use ERPC power values as the reference power
# user supplied values of n, w, coc, df, evaluation frequency, m, and ord
n= 4
w= 10
coc= 5
df= w*(n-1)
ne= 1
m= 3
ord= 2

swfpr= .1
alph= 1 - (1-swfpr)^(1/(coc*w))
ref= c()
if (ne==1) ref= c(.54,.81)
if (ne==2) ref= c(.59,.85)
if (ne==4) ref= c(.60,.86)

# default tolerance values for convergence
tol= .000001
tol2= .0001

# default lower and upper limits on range for desired multiplier
ll= 0
ul= 15

# recursive function to compute correct multiplier within limits (lo,hi)
```

```
kfind= function(lo,hi,n,alph,ne,tol) {
        if (abs(hi-lo)<tol2) return(lo)
        nc= function(x) sqrt(n)*qnorm(x)/sqrt(ord)
        tt= sqrt(n)*lo
        g= function(x) ne*m*(1-(1-x)^m)^(ne-1)*(1-x)^(m-1)*pt(tt,df,nc(x))
        klo = 1 - alph - integrate(g,0,1)$value
        if (abs(klo)<=tol) return(lo)
        tt= sqrt(n)*hi
        khi = 1 - alph - integrate(g,0,1)$value
        if (abs(khi)<=tol) return(hi)
        tt= sqrt(n)*(mean(c(lo,hi)))
        kmid= 1 - alph - integrate(g,0,1)$value
        if (abs(kmid)<=tol) return(mean(c(lo,hi)))
        if (sign(klo)!=sign(khi)) {
                if (sign(klo)!=sign(kmid)) {
                        kfind(lo,mean(c(lo,hi)),n,alph,ne,tol) }
                else {
                        kfind(mean(c(lo,hi)),hi,n,alph,ne,tol) } }
        else {
                stop('bad limits') }
        }

del= c(3,4)
pow= c()
powrate= c()

kap= kfind(ll,ul,n,alph,ne,tol)

for (jj in 1:length(del)) {
        dc= del[jj]
        tt= sqrt(n)*kap
        nc= function(x) {sqrt(n)*(qnorm(x)/sqrt(ord) + del[jj])}
        h= function(x) {
                if (ne==1) {
                        m*((1-x)^(m-1))*pt(tt,df,nc(x))
                        }
                else {
                        ne*m*((1-(1-x)^m)^(ne-1))*((1-x)^(m-1))*pt(tt,df,nc(x))
                        }
                }
        pow[jj]= 1 - integrate(h,0,1,stop.on.error=F)$value
        }

if ((pow[1] >= ref[1]) && (pow[2] >= ref[2])) powrate= 'GOOD'
if ((pow[1] < ref[1]) && (pow[2] >= ref[2])) powrate= 'ACCEPTABLE'
if ((pow[1] >= ref[1]) && (pow[2] < ref[2])) powrate= 'ACCEPTABLE'
if ((pow[1] < ref[1]) && (pow[2] < ref[2])) powrate= 'LOW'

print(paste('intrawell 1ofm'),quote=F)
print(paste('n,w,coc,ne= ',n,w,coc,ne),quote=F)
print(paste('m,ord=',m,ord),quote=F)
print(paste('ref power from ERPC at 3 and 4 SDs'),quote=F)
print(ref)
print(paste('kappa=',round(kap,2)),quote=F)
print(paste('calculated power at 3 and 4 SDs'),quote=F)
print(round(pow,3))
print(paste('power rating=',powrate),quote=F)
```

Modified California Retesting Plans

```
# R Script for modified California plan
# Compute multiplier for intrawell prediction limit using regular
# or pooled SD estimate and modified Calif retesting for observations
# Solve for kappa given an SWFPR adjusted for number of constituents and wells;
# then rate by effective power
# ne = number of yearly evaluations
# Note: ne=4 (quarterly eval), ne=2 (semi-annual), ne=1 (annual)
# n = intrawell BG sample size; w = # wells; coc = # constituents
# df = degrees of freedom associated with variance estimate of prediction limit formula
# Note: if the usual std deviation is used, set df = (n-1);
#       if using a pooled SD estimate across w wells, set df= w*(n-1)
# alph = per-test false positive rate
# swfpr is the targeted network-wide false positive rate, by default set to 10%
# Rate power at 3 and 4 SD units above BG; use ERPC power values as the reference power
# user supplied values of n, w, coc, df, and evaluation frequency
n= 4
w= 10
coc= 5
df= w*(n-1)
ne= 1

swfpr= .1
alph= 1 - (1-swfpr)^(1/(coc*w))

ref= c()

if (ne==1) ref= c(.54,.81)
if (ne==2) ref= c(.59,.85)
if (ne==4) ref= c(.60,.86)

# default tolerance values for convergence
tol= .000001
tol2= .0001

# default lower and upper limits on range for desired multiplier
ll= 0
ul= 15

# recursive function to compute correct multiplier within limits (lo,hi)
kfind= function(lo,hi,n,alph,ne,tol) {
        if (abs(hi-lo)<tol2) return(lo)
        nc= function(x) sqrt(n)*qnorm(x)
        tt= sqrt(n)*lo
        g= function(x) ne*(x*(1 + 3*x - 5*x^2 + 2*x^3))^(ne-1)*(1 + 6*x - 15*x^2 +
8*x^3)*pt(tt,df,nc(x))
        klo = 1 - alph - integrate(g,0,1)$value
        if (abs(klo)<=tol) return(lo)
        tt= sqrt(n)*hi
        khi = 1 - alph - integrate(g,0,1)$value
        if (abs(khi)<=tol) return(hi)
        tt= sqrt(n)*(mean(c(lo,hi)))
        kmid= 1 - alph - integrate(g,0,1)$value
        if (abs(kmid)<=tol) return(mean(c(lo,hi)))
        if (sign(klo)!=sign(khi)) {
                if (sign(klo)!=sign(kmid)) {
                        kfind(lo,mean(c(lo,hi)),n,alph,ne,tol) }
                else {
                        kfind(mean(c(lo,hi)),hi,n,alph,ne,tol) } }
```

```
        else {
                stop('bad limits') }
        }

del= c(3,4)
pow= c()
powrate= c()

kap= kfind(ll,ul,n,alph,ne,tol)

for (jj in 1:length(del)) {
        dc= del[jj]
        tt= sqrt(n)*kap
        nc= function(x) {sqrt(n)*(qnorm(x) + del[jj])}
        h= function(x) {
                if (ne==1) {
                        (1 + 6*x - 15*x^2 + 8*x^3)*pt(tt,df,nc(x))
                        }
                else {
                        ne*(x*(1 + 3*x - 5*x^2 + 2*x^3))^(ne-1)*(1 + 6*x - 15*x^2 +
8*x^3)*pt(tt,df,nc(x))
                        }
                }
        pow[jj]= 1 - integrate(h,0,1,stop.on.error=F)$value
        }

if ((pow[1] >= ref[1]) && (pow[2] >= ref[2])) powrate= 'GOOD'
if ((pow[1] < ref[1]) && (pow[2] >= ref[2])) powrate= 'ACCEPTABLE'
if ((pow[1] >= ref[1]) && (pow[2] < ref[2])) powrate= 'ACCEPTABLE'
if ((pow[1] < ref[1]) && (pow[2] < ref[2])) powrate= 'LOW'

print(paste('intrawell modCal'),quote=F)
print(paste('n,w,coc,ne= ',n,w,coc,ne),quote=F)
print(paste('ref power from ERPC at 3 and 4 SDs'),quote=F)
print(ref)
print(paste('kappa=',round(kap,2)),quote=F)
print(paste('calculated power at 3 and 4 SDs'),quote=F)
print(round(pow,3))
print(paste('power rating=',powrate),quote=F)
```

C.3.2 THEIL-SEN CONFIDENCE BAND

```
# R script for Theil-Sen Confidence band
# Compute bootstrapped confidence band around Theil-Sen trend line
# user inputs: list of x-values, list of y-values, desired confidence level
# Note: replace numbers in parentheses below with specific x and y values
#       corresponding to data-specific ordered pairs
# x-values should be numeric values representing sampling dates or events
# y-values should be concentration values corresponding to these dates or events
# Script produces a plot of the Theil-Sen trend line, the confidence band around the trend,
# and an overlay of the actual data values

x= c(89.6,90.1,90.8,91.1,92.1,93.1,94.1,95.6,96.1,96.3)
y= c(56,53,51,55,52,60,62,59,61,63)
conf = .90

elimna= function(m){
#
# remove any rows of data having missing values
m= as.matrix(m)
```

```
ikeep= c(1:nrow(m))
for(i in 1:nrow(m)) if (sum(is.na(m[i,])>=1)) ikeep[i]= 0
elimna= m[ikeep[ikeep>=1],]
elimna
}

theilsen2= function(x,y){
#
# Compute the Theil-Sen regression estimator
# Do not compute residuals in this version
# Assumes missing pairs already removed
#
ord= order(x)
xs= x[ord]
ys= y[ord]
vec1= outer(ys,ys,"-")
vec2= outer(xs,xs,"-")
v1= vec1[vec2>0]
v2= vec2[vec2>0]
slope= median(v1/v2)
coef= 0
coef[1]= median(y)-slope*median(x)
coef[2]= slope
list(coef=coef)
}

nb= 1000
temp= matrix(c(x,y),ncol=2)
temp= elimna(temp)                          #remove any pairs with missing values
x= temp[,1]
y= temp[,2]
n= length(x)
ord= order(x)
cut= min(x) + (0:100)*(max(x)-min(x))/100   #compute 101 cut pts
t0= theilsen2(x,y)                          #compute trend line on original data
tmp= matrix(nrow=nb,ncol=101)

for (i in 1:nb) {
        idx= sample(ord,n,rep=T)
        xboot= x[idx]
        yboot= y[idx]
        tboot= theilsen2(xboot,yboot)
        tmp[i,]= tboot$coef[1] + cut*tboot$coef[2]
        }

lb= 0; ub= 0
for (i in 1:101){
        lb[i]= quantile(tmp[,i],c((1-conf)/2))
        ub[i]= quantile(tmp[,i],c((1+conf)/2))
        }
tband= list(xcut=cut,lo=lb,hi=ub,ths0=t0)
yt= tband$ths0$coef[1] + tband$ths0$coef[2]*tband$xcut
plot(yt~tband$xcut,type='l',xlim=range(x),ylim=c(min(tband$lo),max(tband$hi)),xlab='Date',ylab='Conc')
points(x,y,pch=16)
lines(tband$hi~tband$xcut,type='l',lty=2)
lines(tband$lo~tband$xcut,type='l',lty=2)
```

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APPENDIX D: STATISTICAL TABLES

D STATISTICAL TABLES

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Table 10-1. Percentiles of Standard Normal Distribution

P	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031

Table 10-1. Percentiles of Standard Normal Distribution

P	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

Table 10-2. Coefficients [a_{n-i+1}] for Shapiro-Wilk Test of Normality, n = 2(1)50

i/n	2	3	4	5	6	7	8	9	10	
1	0.7071	0.7071	0.6872	0.6646	0.6431	0.6233	0.6052	0.5888	0.5739	
2	----	.0000	.1677	.2413	.2806	.3031	.3164	.3244	.3291	
3	----	----	----	.0000	.0875	.1401	.1743	.1976	.2141	
4	----	----	----	----	----	.0000	.0561	.0947	.1224	
5	----	----	----	----	----	----	----	.0000	.0399	
i/n	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0.5601	0.5475	0.5359	0.5251	0.5150	0.5056	0.4968	0.4886	0.4808	0.4734
2	.3315	.3325	.3325	.3318	.3306	.3290	.3273	.3253	.3232	.3211
3	.2260	.2347	.2412	.2460	.2495	.2521	.2540	.2553	.2561	.2565
4	.1429	.1586	.1707	.1802	.1878	.1939	.1988	.2027	.2059	.2085
5	.0695	.0922	.1099	.1240	.1353	.1447	.1524	.1587	.1641	.1686
6	0.0000	0.0303	0.0539	0.0727	0.0880	0.1005	0.1109	0.1197	0.1271	0.1334
7	----	----	.0000	.0240	.0433	.0593	.0725	.0837	.0932	.1013
8	----	----	----	----	.0000	.0196	.0359	.0496	.0612	.0711
9	----	----	----	----	----	----	.0000	.0163	.0303	.0422
10	----	----	----	----	----	----	----	----	.0000	.0140
i/n	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	0.4643	0.4590	0.4542	0.4493	0.4450	0.4407	0.4366	0.4328	0.4291	0.4254
2	.3185	.3156	.3126	.3098	.3069	.3043	.3018	.2992	.2968	.2944
3	.2578	.2571	.2563	.2554	.2543	.2533	.2522	.2510	.2499	.2487
4	.2119	.2131	.2139	.2145	.2148	.2151	.2152	.2151	.2150	.2148
5	.1736	.1764	.1787	.1807	.1822	.1836	.1848	.1857	.1864	.1870
6	0.1399	0.1443	0.1480	0.1512	0.1539	0.1563	0.1584	0.1601	0.1616	0.1630
7	.1092	.1150	.1201	.1245	.1283	.1316	.1346	.1372	.1395	.1415
8	.0804	.0878	.0941	.0997	.1046	.1089	.1128	.1162	.1192	.1219
9	.0530	.0618	.0696	.0764	.0823	.0876	.0923	.0965	.1002	.1036
10	.0263	.0368	.0459	.0539	.0610	.0672	.0728	.0778	.0822	.0862
11	0.0000	0.0122	0.0228	0.0321	0.0403	0.0476	0.0540	0.0598	0.0650	0.0697
12	----	----	.0000	.0107	.0200	.0284	.0358	.0424	.0483	.0537
13	----	----	----	----	.0000	.0094	.0178	.0253	.0320	.0381
14	----	----	----	----	----	----	.0000	.0084	.0159	.0227
15	----	----	----	----	----	----	----	----	.0000	.0076
i/n	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1	0.4220	0.4188	0.4156	0.4127	0.4096	0.4068	0.4040	0.4015	0.3989	0.3964
2	.2921	.2898	.2876	.2854	.2834	.2813	.2794	.2774	.2755	.2737
3	.2475	.2463	.2451	.2439	.2427	.2415	.2403	.2391	.2380	.2368
4	.2145	.2141	.2137	.2132	.2127	.2121	.2116	.2110	.2104	.2098
5	.1874	.1878	.1880	.1882	.1883	.1883	.1883	.1881	.1880	.1878
6	0.1641	0.1651	0.1660	0.1667	0.1673	0.1678	0.1683	0.1686	0.1689	0.1691
7	.1433	.1449	.1463	.1475	.1487	.1496	.1503	.1513	.1520	.1526
8	.1243	.1265	.1284	.1301	.1317	.1331	.1344	.1356	.1366	.1376
9	.1066	.1093	.1118	.1140	.1160	.1179	.1196	.1211	.1225	.1237
10	.0899	.0931	.0961	.0988	.1013	.1036	.1056	.1075	.1092	.1108

Source: Madansky (1988)
Footnote. The notation n = 2(1)50 is shorthand for n from 2 to 50 in unit steps

Table 10-2. Coefficients [a_{n-i+1}] for Shapiro-Wilk Test of Normality, n = 2(1)50

i/n	31	32	33	34	35	36	37	38	39	40
11	0.0739	0.0777	0.0812	0.0844	0.0873	0.0900	0.0924	0.0947	0.0967	0.0986
12	.0585	.0629	.0669	.0706	.0739	.0770	.0798	.0824	.0848	.0870
13	.0435	.0485	.0530	.0572	.0610	.0645	.0677	.0706	.0733	.0759
14	.0289	.0344	.0395	.0441	.0484	.0523	.0559	.0592	.0622	.0651
15	.0144	.0206	.0262	.0314	.0361	.0404	.0444	.0481	.0515	.0546
16	0.0000	0.0068	0.0131	0.0187	0.0239	0.0287	0.0331	0.0372	0.0409	0.0444
17	----	----	.0000	.0062	.0119	.0172	.0220	.0264	.0305	.0343
18	----	----	----	----	.0000	.0057	.0110	.0158	.0203	.0244
19	----	----	----	----	----	----	.0000	.0053	.0101	.0146
20	----	----	----	----	----	----	----	----	.0000	.0049

i/n	41	42	43	44	45	46	47	48	49	50
1	0.3940	0.3917	0.3894	0.3872	0.3850	0.3830	0.3808	0.3789	0.3770	0.3751
2	.2719	.2701	.2684	.2667	.2651	.2635	.2620	.2604	.2589	.2574
3	.2357	.2345	.2334	.2323	.2313	.2302	.2291	.2281	.2271	.2260
4	.2091	.2085	.2078	.2072	.2065	.2058	.2052	.2045	.2038	.2032
5	.1876	.1874	.1871	.1868	.1865	.1862	.1859	.1855	.1851	.1847
6	0.1693	0.1694	0.1695	0.1695	0.1695	0.1695	0.1695	0.1693	0.1692	0.1691
7	.1531	.1535	.1539	.1542	.1545	.1548	.1550	.1551	.1553	.1554
8	.1384	.1392	.1398	.1405	.1410	.1415	.1420	.1423	.1427	.1430
9	.1249	.1259	.1269	.1278	.1286	.1293	.1300	.1306	.1312	.1317
10	.1123	.1136	.1149	.1160	.1170	.1180	.1189	.1197	.1205	.1212
11	0.1004	0.1020	0.1035	0.1049	0.1062	0.1073	0.1085	0.1095	0.1105	0.1113
12	.0891	.0909	.0927	.0943	.0959	.0972	.0986	.0998	.1010	.1020
13	.0782	.0804	.0824	.0842	.0860	.0876	.0892	.0906	.0919	.0932
14	.0677	.0701	.0724	.0745	.0775	.0785	.0801	.0817	.0832	.0846
15	.0575	.0602	.0628	.0651	.0673	.0694	.0713	.0731	.0748	.0764
16	0.0476	0.0506	0.0534	0.0560	0.0584	0.0607	0.0628	0.0648	0.0667	0.0685
17	.0379	.0411	.0442	.0471	.0497	.0522	.0546	.0568	.0588	.0608
18	.0283	.0318	.0352	.0383	.0412	.0439	.0465	.0489	.0511	.0532
19	.0188	.0227	.0263	.0296	.0328	.0357	.0385	.0411	.0436	.0459
20	.0094	.0136	.0175	.0211	.0245	.0277	.0307	.0335	.0361	.0386
21	0.0000	0.0045	0.0087	0.0126	0.0163	0.0197	0.0229	0.0259	0.0288	0.0314
22	----	----	.0000	.0042	.0081	.0118	.0153	.0185	.0215	.0244
23	----	----	----	----	.0000	.0039	.0076	.0111	.0143	.0174
24	----	----	----	----	----	----	.0000	.0037	.0071	.0104
25	----	----	----	----	----	----	----	----	.0000	.0035

Source: Madansky (1988)
Footnote. The notation n = 2(1)50 is shorthand for n from 2 to 50 in unit steps

Table 10-3. α-Level Critical Points for Shapiro-Wilk Test, n = 3(1)50

n\α	0.01	0.05	0.10
3	0.753	0.767	0.789
4	0.687	0.748	0.792
5	0.686	0.762	0.806
6	0.713	0.788	0.826
7	0.730	0.803	0.838
8	0.749	0.818	0.851
9	0.764	0.829	0.859
10	0.781	0.842	0.869
11	0.792	0.850	0.876
12	0.805	0.859	0.883
13	0.814	0.866	0.889
14	0.825	0.874	0.895
15	0.835	0.881	0.901
16	0.844	0.887	0.906
17	0.851	0.892	0.910
18	0.858	0.897	0.914
19	0.863	0.901	0.917
20	0.868	0.905	0.920
21	0.873	0.908	0.923
22	0.878	0.911	0.926
23	0.881	0.914	0.928
24	0.884	0.916	0.930
25	0.888	0.918	0.931
26	0.891	0.920	0.933
27	0.894	0.923	0.935
28	0.896	0.924	0.936
29	0.898	0.926	0.937
30	0.900	0.927	0.939
31	0.902	0.929	0.940
32	0.904	0.930	0.941
33	0.906	0.931	0.942
34	0.908	0.933	0.943
35	0.910	0.934	0.944
36	0.912	0.935	0.945
37	0.914	0.936	0.946
38	0.916	0.938	0.947
39	0.917	0.939	0.948
40	0.919	0.940	0.949
41	0.920	0.941	0.950
42	0.922	0.942	0.951
43	0.923	0.943	0.951
44	0.924	0.944	0.952
45	0.926	0.945	0.953
46	0.927	0.945	0.953
47	0.928	0.946	0.954
48	0.929	0.947	0.954
49	0.929	0.947	0.955
50	0.930	0.947	0.955

Source: Madansky (1988)
Footnote. The notation n = 3(1)50 is shorthand for n from 3 to 50 in unit steps

Table 10-4. α-Level Critical Points for Shapiro-Francía Test, n = 50(1)99

n\α	0.01	0.05	0.10
50	0.935	0.953	0.963
51	0.935	0.954	0.964
53	0.938	0.957	0.964
55	0.940	0.958	0.965
57	0.944	0.961	0.966
59	0.945	0.962	0.967
61	0.947	0.963	0.968
63	0.947	0.964	0.970
65	0.948	0.965	0.971
67	0.950	0.966	0.971
69	0.951	0.966	0.972
71	0.953	0.967	0.972
73	0.956	0.968	0.973
75	0.956	0.969	0.973
77	0.957	0.969	0.974
79	0.957	0.970	0.975
81	0.958	0.970	0.975
83	0.960	0.971	0.976
85	0.961	0.972	0.977
87	0.961	0.972	0.977
89	0.961	0.972	0.977
91	0.962	0.973	0.978
93	0.963	0.973	0.979
95	0.965	0.974	0.979
97	0.965	0.975	0.979
99	0.967	0.976	0.980

Source: Shapiro & Francía (1972)
Footnote. The notation n = 50(1)99 is shorthand for n from 50 to 99 in unit steps

Table 10-5. α-Critical Pts., Prob. Plot Correlation Coeff. Test, n = 3(1)50(5)100

n\α	0.01	0.025	0.05	0.10
3	0.869	0.872	0.879	0.891
4	0.822	0.845	0.868	0.894
5	0.822	0.855	0.879	0.902
6	0.835	0.868	0.890	0.911
7	0.847	0.876	0.899	0.916
8	0.859	0.886	0.905	0.924
9	0.868	0.893	0.912	0.929
10	0.876	0.900	0.917	0.934
11	0.883	0.906	0.922	0.938
12	0.889	0.912	0.926	0.941
13	0.895	0.917	0.931	0.944
14	0.901	0.921	0.934	0.947
15	0.907	0.925	0.937	0.950
16	0.912	0.928	0.940	0.952
17	0.912	0.931	0.942	0.954
18	0.919	0.934	0.945	0.956
19	0.923	0.937	0.947	0.958
20	0.925	0.939	0.950	0.960
21	0.928	0.942	0.952	0.961
22	0.930	0.944	0.954	0.962
23	0.933	0.947	0.955	0.964
24	0.936	0.949	0.957	0.965
25	0.937	0.950	0.958	0.966
26	0.939	0.952	0.959	0.967
27	0.941	0.953	0.960	0.968
28	0.943	0.955	0.962	0.969
29	0.945	0.956	0.962	0.969
30	0.947	0.957	0.964	0.970
31	0.948	0.958	0.965	0.971
32	0.949	0.959	0.966	0.972
33	0.950	0.960	0.967	0.973
34	0.951	0.960	0.967	0.973
35	0.952	0.961	0.968	0.974
36	0.953	0.962	0.968	0.974
37	0.955	0.962	0.969	0.975
38	0.956	0.964	0.970	0.975
39	0.957	0.965	0.971	0.976
40	0.958	0.966	0.972	0.977
41	0.958	0.967	0.973	0.977
42	0.959	0.967	0.973	0.978
43	0.959	0.967	0.973	0.978
44	0.960	0.968	0.974	0.978
45	0.961	0.969	0.974	0.978
46	0.962	0.969	0.974	0.979
47	0.963	0.970	0.975	0.979
48	0.963	0.970	0.975	0.980
49	0.964	0.971	0.977	0.980
50	0.965	0.972	0.978	0.981
55	0.967	0.974	0.980	0.982
60	0.970	0.976	0.981	0.983
65	0.972	0.977	0.982	0.984
70	0.974	0.978	0.983	0.985
75	0.975	0.979	0.984	0.986
80	0.976	0.980	0.985	0.987
85	0.977	0.981	0.985	0.987
90	0.978	0.982	0.985	0.988
95	0.979	0.983	0.986	0.989
100	0.981	0.984	0.987	0.989

Source: Filliben (1975)

Table 10-6. Shapiro-Wilk Multiple Group Test: Values to Compute G_i for n = 7(1)50

n	γ	δ	ε	n	γ	δ	ε
7	-2.356	1.245	.4533	**31**	-6.248	1.965	.1840
8	-2.696	1.333	.4186	**32**	-6.324	1.976	.1811
9	-2.968	1.400	.3900	**33**	-6.402	1.988	.1781
10	-3.262	1.471	.3660	**34**	-6.480	2.000	.1755
11	-3.485	1.515	.3451	**35**	-6.559	2.012	.1727
12	-3.731	1.571	.3270	**36**	-6.640	2.024	.1702
13	-3.936	1.613	.3111	**37**	-6.721	2.037	.1677
14	-4.155	1.655	.2969	**38**	-6.803	2.049	.1656
15	-4.373	1.695	.2842	**39**	-6.887	2.062	.1633
16	-4.567	1.724	.2727	**40**	-6.961	2.075	.1612
17	-4.713	1.739	.2622	**41**	-7.035	2.088	.1591
18	-4.885	1.770	.2528	**42**	-7.111	2.101	.1572
19	-5.018	1.786	.2440	**43**	-7.188	2.114	.1552
20	-5.153	1.802	.2359	**44**	-7.266	2.128	.1534
21	-5.291	1.818	.2264	**45**	-7.345	2.141	.1516
22	-5.413	1.835	.2207	**46**	-7.414	2.155	.1499
23	-5.508	1.848	.2157	**47**	-7.484	2.169	.1482
24	-5.605	1.862	.2106	**48**	-7.555	2.183	.1466
25	-5.704	1.876	.2063	**49**	-7.615	2.198	.1451
26	-5.803	1.890	.2020	**50**	-7.677	2.212	.1436
27	-5.905	1.905	.1980				
28	-5.988	1.919	.1943				
29	-6.074	1.934	.1907				
30	-6.150	1.949	.1872				

Source: Gibbons (1994)
Footnote. The notation n = 7(1)50 is shorthand for n from 7 to 50 in unit steps

Table 10-7. Shapiro-Wilk Multiple Group Test: Values of G_i for n = 3(1)6

	n = 3 (ε = .7500)		n = 4 (ε = .6297)		n = 5 (ε = .5521)		n = 6 (ε = .4963)	
u	W	G_i	W	G_i	W	G_i	W	G_i
-7.0	.7502	-3.291						
-5.4	.7511	-2.810						
-5.0	.7517	-2.678						
-4.6	.7525	-2.543						
-4.2	.7537	-2.400						
-3.8	.7555	-2.254	.6378	-3.497				
-3.4	.7581	-2.099	.6417	-3.270				
-3.0	.7619	-1.937	.6473	-3.043	.5733	-4.013		
-2.6	.7673	-1.767	.6553	-2.839	.5831	-3.698		
-2.2	.7749	-1.589	.6666	-2.642	.5968	-3.383		
-1.8	.7855	-1.404	.6822	-2.441	.6156	-3.113		
-1.4	.7995	-1.210	.7030	-2.222	.6407	-2.874		
-1.0	.8172	-1.010	.7293	-1.964	.6726	-2.558	.6318	-3.719
-0.6	.8386	-0.805	.7609	-1.664	.7108	-2.181	.6748	-2.878
-0.2	.8625	-0.599	.7964	-1.309	.7537	-1.815	.7230	-2.273
0.0	.8750	-0.496	.8149	-1.122	.7761	-1.635	.7482	-2.068
0.2	.8875	-0.395	.8333	-0.944	.7984	-1.418	.7733	-1.858
0.4	.8997	-0.294	.8514	-0.766	.8203	-1.200	.7979	-1.614
0.6	.9114	-0.195	.8688	-0.573	.8413	-0.970	.8215	-1.383
1.0	.9328	-0.003	.9004	-0.192	.8795	-0.513	.8645	-0.842
1.4	.9505	0.181	.9267	0.148	.9114	-0.057	.9004	-0.349
1.6	.9580	0.268	.9378	0.298	.9248	0.174	.9154	-0.075
1.8	.9645	0.354	.9475	0.451	.9365	0.374	.9285	0.182
2.2	.9751	0.516	.9631	0.739	.9553	0.745	.9498	0.653
2.6	.9827	0.669	.9744	0.998	.9690	1.087	.9652	1.045
3.0	.9881	0.812	.9824	1.202	.9788	1.403	.9761	1.440
3.4	.9919	0.947	.9880	1.426	.9855	1.673	.9837	1.838
3.8	.9945	1.074	.9919	1.660	.9902	1.907	.9890	2.170
4.2	.9963	1.195	.9945	1.847	.9934	2.136	.9926	2.512
4.6	.9975	1.309	.9963	2.028	.9955	2.455	.9950	2.748
5.0	.9983	1.418	.9975	2.193	.9970	2.850	.9966	3.090
5.4	.9989	1.522	.9983	2.341	.9980	3.245	.9977	3.540
5.8	.99925	1.621	.9989	2.483	.9986	3.640		
6.2	.99949	1.717	.9993	2.628				
6.6	.99966	1.809	.9995	2.754				
7.0	.99977	1.899	.9997	2.869				
7.4	.99985	1.985	.9998	2.971				
7.8	.99990	2.068	.9998	3.084				
8.2	.99993	2.149	.9999	3.224				
8.6	.99995	2.226	.9999	3.359				

Source: Wilk & Shapiro (1968)
Footnote. The notation n = 3(1)6 is shorthand for n from 3 to 6 in unit steps

Table 12-1. α-Level Critical Points for Dixon's Outlier Test, n = 3(1)25

n\α	.01	.05	.10
3	0.988	0.941	0.886
4	0.889	0.765	0.679
5	0.780	0.642	0.557
6	0.698	0.560	0.482
7	0.637	0.507	0.434
8	0.683	0.554	0.479
9	0.635	0.512	0.441
10	0.597	0.477	0.409
11	0.679	0.576	0.517
12	0.642	0.546	0.490
13	0.615	0.521	0.467
14	0.641	0.546	0.492
15	0.616	0.525	0.472
16	0.595	0.507	0.454
17	0.577	0.490	0.438
18	0.561	0.475	0.424
19	0.547	0.462	0.412
20	0.535	0.450	0.401
21	0.524	0.440	0.391
22	0.514	0.430	0.382
23	0.505	0.421	0.374
24	0.497	0.413	0.367
25	0.489	0.406	0.360

Source: USEPA (1998)
Footnote. The notation n = 3(1)25 is shorthand for n from 3 to 25 in unit steps

Table 12-2. α-Level Critical Points for Rosner's Outlier Test

	k = 2		k = 3	
n\α	**.05**	**.01**	**.05**	**.01**
20	2.83	3.09	2.88	3.13
	2.52	2.76	2.60	2.83
			2.45	2.68
30	3.05	3.35	3.12	3.41
	2.67	2.92	2.73	3.01
			2.56	2.75
40	3.17	3.52	3.22	3.58
	2.77	2.98	2.81	3.03
			2.62	2.82
50	3.27	3.61	3.34	3.68
	2.85	3.08	2.89	3.15
			2.68	2.89
60	3.34	3.70	3.42	3.75
	2.90	3.17	2.95	3.20
			2.73	2.95
80	3.45	3.80	3.49	3.85
	2.97	3.23	3.03	3.27
			2.81	3.01
100	3.52	3.87	3.60	3.97
	3.03	3.28	3.10	3.34
			2.86	3.06

Source: Barnett & Lewis (1994)
Footnote. *k* = number of suspected outliers. Since *k* critical points are needed for each test, there are 2 values under each *k* = 2 entry, 3 under each *k* = 3 entry, etc.

Table 12-2. α-Level Critical Points for Rosner's Outlier Test (cont.)

	k = 4		k = 5	
n\α	**.05**	**.01**	**.05**	**.01**
20	2.95	3.20	2.97	3.18
	2.63	2.83	2.65	2.89
	2.49	2.68	2.51	2.69
	2.39	2.58	2.42	2.61
			2.37	2.57
30	3.16	3.48	3.19	3.48
	2.77	3.02	2.78	3.03
	2.59	2.79	2.60	2.80
	2.49	2.70	2.51	2.74
			2.45	2.62
40	3.32	3.64	3.31	3.63
	2.86	3.10	2.88	3.13
	2.67	2.87	2.69	2.89
	2.55	2.74	2.55	2.74
			2.47	2.65
50	3.40	3.74	3.45	3.77
	2.93	3.18	2.96	3.21
	2.72	2.92	2.74	2.94
	2.59	2.78	2.61	2.79
			2.52	2.70
60	3.48	3.82	3.51	3.81
	2.98	3.20	3.01	3.24
	2.77	2.97	2.77	2.96
	2.63	2.82	2.65	2.83
			2.56	2.72
80	3.57	3.91	3.61	3.93
	3.05	3.31	3.11	3.36
	2.84	3.04	2.86	3.08
	2.69	2.87	2.72	2.89
			2.62	2.76
100	3.64	3.96	3.70	4.01
	3.13	3.34	3.16	3.42
	2.89	3.06	2.91	3.10
	2.74	2.90	2.77	2.93
			2.67	2.84

Table 14-1. Approximate α-Level Critical Points for Rank von Neumann Ratio Test for n = 4(1)30(2)50(5)100

n\α	.005	.01	.025	.05	.10
4	——	——	——	——	0.60
5	——	——	0.40	0.70	——
6	0.29	0.46	0.63	0.80	0.97
7	0.50	0.54	0.64	0.86	1.11
8	0.55	0.62	0.76	0.93	1.14
9	0.57	0.67	0.82	0.98	1.18
10	0.62	0.72	0.89	1.04	1.23
11	0.67	0.77	0.93	1.08	1.26
12	0.71	0.81	0.96	1.11	1.29
13	0.74	0.84	1.00	1.14	1.32
14	0.78	0.87	1.03	1.17	1.34
15	0.81	0.90	1.05	1.19	1.36
16	0.84	0.93	1.08	1.21	1.38
17	0.87	0.96	1.10	1.24	1.40
18	0.89	0.98	1.13	1.26	1.41
19	0.92	1.01	1.15	1.27	1.43
20	0.94	1.03	1.17	1.29	1.44
21	0.96	1.05	1.18	1.31	1.45
22	0.98	1.07	1.20	1.32	1.46
23	1.00	1.09	1.22	1.33	1.48
24	1.02	1.10	1.23	1.35	1.49
25	1.04	1.12	1.25	1.36	1.50
26	1.05	1.13	1.26	1.37	1.51
27	1.07	1.15	1.27	1.38	1.51
28	1.08	1.16	1.28	1.39	1.52
29	1.10	1.18	1.30	1.40	1.53
30	1.11	1.19	1.31	1.41	1.54
32	1.13	1.21	1.33	1.43	1.55
34	1.16	1.23	1.35	1.45	1.57
36	1.18	1.25	1.36	1.46	1.58
38	1.20	1.27	1.38	1.48	1.59
40	1.22	1.29	1.39	1.49	1.60
42	1.24	1.30	1.41	1.50	1.61
44	1.25	1.32	1.42	1.51	1.62
46	1.27	1.33	1.43	1.52	1.63
48	1.28	1.35	1.45	1.53	1.63
50	1.29	1.36	1.46	1.54	1.64
55	1.33	1.39	1.48	1.56	1.66
60	1.35	1.41	1.50	1.58	1.67
65	1.38	1.43	1.52	1.60	1.68
70	1.40	1.45	1.54	1.61	1.70
75	1.42	1.47	1.55	1.62	1.71
80	1.44	1.49	1.57	1.64	1.71
85	1.45	1.50	1.58	1.65	1.72
90	1.47	1.52	1.59	1.66	1.73
95	1.48	1.53	1.60	1.66	1.74
100	1.49	1.54	1.61	1.67	1.74

Sources: Bartels (1982), Madansky (1988)
Footnote. The notation *n* = 4(1)30(2)50(5)100 is shorthand for *n* from 4 to 30 in unit steps, then from 30 to 50 by 2's, then from 50 to 100 by 5's

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Table 16-1. Percentiles of Student's *t*-Distribution

df\P	.75	.80	.85	.90	.95	.96	.97	.975	.98	.9833	.9875	.99	.995	.999
1	1.000	1.376	1.963	3.078	6.314	7.916	10.579	12.706	15.895	19.043	25.452	31.821	63.657	318.309
2	0.816	1.061	1.386	1.886	2.920	3.320	3.896	4.303	4.849	5.334	6.205	6.965	9.925	22.327
3	0.765	0.978	1.250	1.638	2.353	2.605	2.951	3.182	3.482	3.738	4.177	4.541	5.841	10.215
4	0.741	0.941	1.190	1.533	2.132	2.333	2.601	2.776	2.999	3.184	3.495	3.747	4.604	7.173
5	0.727	0.920	1.156	1.476	2.015	2.191	2.422	2.571	2.757	2.910	3.163	3.365	4.032	5.893
6	0.718	0.906	1.134	1.440	1.943	2.104	2.313	2.447	2.612	2.748	2.969	3.143	3.707	5.208
7	0.711	0.896	1.119	1.415	1.895	2.046	2.241	2.365	2.517	2.640	2.841	2.998	3.499	4.785
8	0.706	0.889	1.108	1.397	1.860	2.004	2.189	2.306	2.449	2.565	2.752	2.896	3.355	4.501
9	0.703	0.883	1.100	1.383	1.833	1.973	2.150	2.262	2.398	2.508	2.685	2.821	3.250	4.297
10	0.700	0.879	1.093	1.372	1.812	1.948	2.120	2.228	2.359	2.465	2.634	2.764	3.169	4.144
11	0.697	0.876	1.088	1.363	1.796	1.928	2.096	2.201	2.328	2.430	2.593	2.718	3.106	4.025
12	0.695	0.873	1.083	1.356	1.782	1.912	2.076	2.179	2.303	2.402	2.560	2.681	3.055	3.930
13	0.694	0.870	1.079	1.350	1.771	1.899	2.060	2.160	2.282	2.379	2.533	2.650	3.012	3.852
14	0.692	0.868	1.076	1.345	1.761	1.887	2.046	2.145	2.264	2.359	2.510	2.624	2.977	3.787
15	0.691	0.866	1.074	1.341	1.753	1.878	2.034	2.131	2.249	2.342	2.490	2.602	2.947	3.733
16	0.690	0.865	1.071	1.337	1.746	1.869	2.024	2.120	2.235	2.327	2.473	2.583	2.921	3.686
17	0.689	0.863	1.069	1.333	1.740	1.862	2.015	2.110	2.224	2.315	2.458	2.567	2.898	3.646
18	0.688	0.862	1.067	1.330	1.734	1.855	2.007	2.101	2.214	2.303	2.445	2.552	2.878	3.610
19	0.688	0.861	1.066	1.328	1.729	1.850	2.000	2.093	2.205	2.293	2.433	2.539	2.861	3.579
20	0.687	0.860	1.064	1.325	1.725	1.844	1.994	2.086	2.197	2.285	2.423	2.528	2.845	3.552
21	0.686	0.859	1.063	1.323	1.721	1.840	1.988	2.080	2.189	2.277	2.414	2.518	2.831	3.527
22	0.686	0.858	1.061	1.321	1.717	1.835	1.983	2.074	2.183	2.269	2.405	2.508	2.819	3.505
23	0.685	0.858	1.060	1.319	1.714	1.832	1.978	2.069	2.177	2.263	2.398	2.500	2.807	3.485
24	0.685	0.857	1.059	1.318	1.711	1.828	1.974	2.064	2.172	2.257	2.391	2.492	2.797	3.467
25	0.684	0.856	1.058	1.316	1.708	1.825	1.970	2.060	2.167	2.251	2.385	2.485	2.787	3.450
26	0.684	0.856	1.058	1.315	1.706	1.822	1.967	2.056	2.162	2.246	2.379	2.479	2.779	3.435
27	0.684	0.855	1.057	1.314	1.703	1.819	1.963	2.052	2.158	2.242	2.373	2.473	2.771	3.421
28	0.683	0.855	1.056	1.313	1.701	1.817	1.960	2.048	2.154	2.237	2.368	2.467	2.763	3.408
29	0.683	0.854	1.055	1.311	1.699	1.814	1.957	2.045	2.150	2.233	2.364	2.462	2.756	3.396
30	0.683	0.854	1.055	1.310	1.697	1.812	1.955	2.042	2.147	2.230	2.360	2.457	2.750	3.385

Table 16-1. Percentiles of Student's *t*-Distribution (cont.)

df\P	.75	.80	.85	.90	.95	.96	.97	.975	.98	.9833	.9875	.99	.995	.999
31	0.682	0.853	1.054	1.309	1.696	1.810	1.952	2.040	2.144	2.226	2.356	2.453	2.744	3.375
32	0.682	0.853	1.054	1.309	1.694	1.808	1.950	2.037	2.141	2.223	2.352	2.449	2.738	3.365
33	0.682	0.853	1.053	1.308	1.692	1.806	1.948	2.035	2.138	2.220	2.348	2.445	2.733	3.356
34	0.682	0.852	1.052	1.307	1.691	1.805	1.946	2.032	2.136	2.217	2.345	2.441	2.728	3.348
35	0.682	0.852	1.052	1.306	1.690	1.803	1.944	2.030	2.133	2.215	2.342	2.438	2.724	3.340
36	0.681	0.852	1.052	1.306	1.688	1.802	1.942	2.028	2.131	2.212	2.339	2.434	2.719	3.333
37	0.681	0.851	1.051	1.305	1.687	1.800	1.940	2.026	2.129	2.210	2.336	2.431	2.715	3.326
38	0.681	0.851	1.051	1.304	1.686	1.799	1.939	2.024	2.127	2.207	2.334	2.429	2.712	3.319
39	0.681	0.851	1.050	1.304	1.685	1.798	1.937	2.023	2.125	2.205	2.331	2.426	2.708	3.313
40	0.681	0.851	1.050	1.303	1.684	1.796	1.936	2.021	2.123	2.203	2.329	2.423	2.704	3.307
41	0.681	0.850	1.050	1.303	1.683	1.795	1.934	2.020	2.121	2.201	2.327	2.421	2.701	3.301
42	0.680	0.850	1.049	1.302	1.682	1.794	1.933	2.018	2.120	2.199	2.325	2.418	2.698	3.296
43	0.680	0.850	1.049	1.302	1.681	1.793	1.932	2.017	2.118	2.198	2.323	2.416	2.695	3.291
44	0.680	0.850	1.049	1.301	1.680	1.792	1.931	2.015	2.116	2.196	2.321	2.414	2.692	3.286
45	0.680	0.850	1.049	1.301	1.679	1.791	1.929	2.014	2.115	2.195	2.319	2.412	2.690	3.281
46	0.680	0.850	1.048	1.300	1.679	1.790	1.928	2.013	2.114	2.193	2.317	2.410	2.687	3.277
47	0.680	0.849	1.048	1.300	1.678	1.789	1.927	2.012	2.112	2.192	2.315	2.408	2.685	3.273
48	0.680	0.849	1.048	1.299	1.677	1.789	1.926	2.011	2.111	2.190	2.314	2.407	2.682	3.269
49	0.680	0.849	1.048	1.299	1.677	1.788	1.925	2.010	2.110	2.189	2.312	2.405	2.680	3.265
50	0.679	0.849	1.047	1.299	1.676	1.787	1.924	2.009	2.109	2.188	2.311	2.403	2.678	3.261
51	0.679	0.849	1.047	1.298	1.675	1.786	1.924	2.008	2.108	2.186	2.310	2.402	2.676	3.258
52	0.679	0.849	1.047	1.298	1.675	1.786	1.923	2.007	2.107	2.185	2.308	2.400	2.674	3.255
53	0.679	0.848	1.047	1.298	1.674	1.785	1.922	2.006	2.106	2.184	2.307	2.399	2.672	3.251
54	0.679	0.848	1.046	1.297	1.674	1.784	1.921	2.005	2.105	2.183	2.306	2.397	2.670	3.248
55	0.679	0.848	1.046	1.297	1.673	1.784	1.920	2.004	2.104	2.182	2.304	2.396	2.668	3.245
56	0.679	0.848	1.046	1.297	1.673	1.783	1.920	2.003	2.103	2.181	2.303	2.395	2.667	3.242
57	0.679	0.848	1.046	1.297	1.672	1.782	1.919	2.002	2.102	2.180	2.302	2.394	2.665	3.239
58	0.679	0.848	1.046	1.296	1.672	1.782	1.918	2.002	2.101	2.179	2.301	2.392	2.663	3.237
59	0.679	0.848	1.046	1.296	1.671	1.781	1.918	2.001	2.100	2.178	2.300	2.391	2.662	3.234
60	0.679	0.848	1.045	1.296	1.671	1.781	1.917	2.000	2.099	2.177	2.299	2.390	2.660	3.232
70	0.678	0.847	1.044	1.294	1.667	1.776	1.912	1.994	2.093	2.170	2.291	2.381	2.648	3.211
80	0.678	0.846	1.043	1.292	1.664	1.773	1.908	1.990	2.088	2.165	2.284	2.374	2.639	3.195
90	0.677	0.846	1.042	1.291	1.662	1.771	1.905	1.987	2.084	2.160	2.280	2.368	2.632	3.183
100	0.677	0.845	1.042	1.290	1.660	1.769	1.902	1.984	2.081	2.157	2.276	2.364	2.626	3.174
110	0.677	0.845	1.041	1.289	1.659	1.767	1.900	1.982	2.078	2.154	2.272	2.361	2.621	3.166
120	0.677	0.845	1.041	1.289	1.658	1.766	1.899	1.980	2.076	2.152	2.270	2.358	2.617	3.160

Table 17-1. Percentiles of *F*-Distribution for $(1-\alpha)$ = .80

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	9.47	12.00	13.06	13.64	14.01	14.26	14.44	14.58	14.68	14.77	14.84	14.90	14.95	15.00	15.04
2	3.56	4.00	4.16	4.24	4.28	4.32	4.34	4.36	4.37	4.38	4.39	4.40	4.40	4.41	4.42
3	2.68	2.89	2.94	2.96	2.97	2.97	2.97	2.98	2.98	2.98	2.98	2.98	2.98	2.98	2.98
4	2.35	2.47	2.48	2.48	2.48	2.47	2.47	2.47	2.46	2.46	2.46	2.46	2.45	2.45	2.45
5	2.18	2.26	2.25	2.24	2.23	2.22	2.21	2.20	2.20	2.19	2.19	2.18	2.18	2.18	2.18
6	2.07	2.13	2.11	2.09	2.08	2.06	2.05	2.04	2.03	2.03	2.02	2.02	2.01	2.01	2.01
7	2.00	2.04	2.02	1.99	1.97	1.96	1.94	1.93	1.93	1.92	1.91	1.91	1.90	1.90	1.89
8	1.95	1.98	1.95	1.92	1.90	1.88	1.87	1.86	1.85	1.84	1.83	1.83	1.82	1.82	1.81
9	1.91	1.93	1.90	1.87	1.85	1.83	1.81	1.80	1.79	1.78	1.77	1.76	1.76	1.75	1.75
10	1.88	1.90	1.86	1.83	1.80	1.78	1.77	1.75	1.74	1.73	1.72	1.72	1.71	1.70	1.70
11	1.86	1.87	1.83	1.80	1.77	1.75	1.73	1.72	1.70	1.69	1.69	1.68	1.67	1.67	1.66
12	1.84	1.85	1.80	1.77	1.74	1.72	1.70	1.69	1.67	1.66	1.65	1.65	1.64	1.63	1.63
13	1.82	1.83	1.78	1.75	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60
14	1.81	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.63	1.62	1.61	1.60	1.59	1.58	1.58
15	1.80	1.80	1.75	1.71	1.68	1.66	1.64	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.56
16	1.79	1.78	1.74	1.70	1.67	1.64	1.62	1.61	1.59	1.58	1.57	1.56	1.55	1.55	1.54
17	1.78	1.77	1.72	1.68	1.65	1.63	1.61	1.59	1.58	1.57	1.56	1.55	1.54	1.53	1.53
18	1.77	1.76	1.71	1.67	1.64	1.62	1.60	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
19	1.76	1.75	1.70	1.66	1.63	1.61	1.58	1.57	1.55	1.54	1.53	1.52	1.51	1.51	1.50
20	1.76	1.75	1.70	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.50	1.50	1.49
21	1.75	1.74	1.69	1.65	1.61	1.59	1.57	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48
22	1.75	1.73	1.68	1.64	1.61	1.58	1.56	1.54	1.53	1.51	1.50	1.49	1.49	1.48	1.47
23	1.74	1.73	1.68	1.63	1.60	1.57	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46
24	1.74	1.72	1.67	1.63	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.47	1.46	1.46
25	1.73	1.72	1.66	1.62	1.59	1.56	1.54	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45
26	1.73	1.71	1.66	1.62	1.58	1.56	1.53	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44
27	1.73	1.71	1.66	1.61	1.58	1.55	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44
28	1.72	1.71	1.65	1.61	1.57	1.55	1.52	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.43
29	1.72	1.70	1.65	1.60	1.57	1.54	1.52	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43
30	1.72	1.70	1.64	1.60	1.57	1.54	1.52	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42
35	1.71	1.69	1.63	1.58	1.55	1.52	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40
40	1.70	1.68	1.62	1.57	1.54	1.51	1.49	1.47	1.45	1.44	1.42	1.41	1.40	1.40	1.39
45	1.69	1.67	1.61	1.57	1.53	1.50	1.48	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38
50	1.69	1.66	1.60	1.56	1.52	1.49	1.47	1.45	1.43	1.42	1.41	1.39	1.38	1.38	1.37
55	1.68	1.66	1.60	1.55	1.52	1.49	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36
60	1.68	1.65	1.60	1.55	1.51	1.48	1.46	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35
70	1.67	1.65	1.59	1.54	1.50	1.47	1.45	1.43	1.41	1.40	1.38	1.37	1.36	1.35	1.35
80	1.67	1.64	1.58	1.53	1.50	1.47	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34
90	1.67	1.64	1.58	1.53	1.49	1.46	1.44	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33
100	1.66	1.64	1.58	1.53	1.49	1.46	1.43	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.33
110	1.66	1.63	1.57	1.52	1.49	1.46	1.43	1.41	1.39	1.38	1.36	1.35	1.34	1.33	1.32
120	1.66	1.63	1.57	1.52	1.48	1.45	1.43	1.41	1.39	1.37	1.36	1.35	1.34	1.33	1.32

Table 17-1. Percentiles of *F*-Distribution for $(1-\alpha)$ = .90

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.47	60.71	60.90	61.07	61.22
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41	9.41	9.42	9.42
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22	5.21	5.20	5.20
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90	3.89	3.88	3.87
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27	3.26	3.25	3.24
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90	2.89	2.88	2.87
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67	2.65	2.64	2.63
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50	2.49	2.48	2.46
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38	2.36	2.35	2.34
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28	2.27	2.26	2.24
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21	2.19	2.18	2.17
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15	2.13	2.12	2.10
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10	2.08	2.07	2.05
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.07	2.05	2.04	2.02	2.01
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99	1.97	1.95	1.94
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96	1.94	1.93	1.91
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.95	1.93	1.92	1.90	1.89
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.93	1.91	1.89	1.88	1.86
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89	1.87	1.86	1.84
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.90	1.87	1.86	1.84	1.83
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86	1.84	1.83	1.81
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.87	1.84	1.83	1.81	1.80
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83	1.81	1.80	1.78
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.84	1.82	1.80	1.79	1.77
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.83	1.81	1.79	1.77	1.76
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.82	1.80	1.78	1.76	1.75
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79	1.77	1.75	1.74
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.80	1.78	1.76	1.75	1.73
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77	1.75	1.74	1.72
35	2.85	2.46	2.25	2.11	2.02	1.95	1.90	1.85	1.82	1.79	1.76	1.74	1.72	1.70	1.69
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.71	1.70	1.68	1.66
45	2.82	2.42	2.21	2.07	1.98	1.91	1.85	1.81	1.77	1.74	1.72	1.70	1.68	1.66	1.64
50	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76	1.73	1.70	1.68	1.66	1.64	1.63
55	2.80	2.40	2.19	2.05	1.95	1.88	1.83	1.78	1.75	1.72	1.69	1.67	1.65	1.63	1.61
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66	1.64	1.62	1.60
70	2.78	2.38	2.16	2.03	1.93	1.86	1.80	1.76	1.72	1.69	1.66	1.64	1.62	1.60	1.59
80	2.77	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.71	1.68	1.65	1.63	1.61	1.59	1.57
90	2.76	2.36	2.15	2.01	1.91	1.84	1.78	1.74	1.70	1.67	1.64	1.62	1.60	1.58	1.56
100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66	1.64	1.61	1.59	1.57	1.56
110	2.75	2.35	2.13	2.00	1.90	1.83	1.77	1.73	1.69	1.66	1.63	1.61	1.59	1.57	1.55
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60	1.58	1.56	1.55

Table 17-1. Percentiles of *F*-Distribution for $(1-\alpha) = .95$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	242.98	243.91	244.69	245.36	245.95
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.42	19.42	19.43
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07	2.04	2.01	1.99	1.96
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	2.01	1.97	1.94	1.92	1.89
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95	1.92	1.89	1.87
55	4.02	3.16	2.77	2.54	2.38	2.27	2.18	2.11	2.06	2.01	1.97	1.93	1.90	1.88	1.85
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89	1.86	1.84	1.81
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88	1.84	1.82	1.79
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.90	1.86	1.83	1.80	1.78
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85	1.82	1.79	1.77
110	3.93	3.08	2.69	2.45	2.30	2.18	2.09	2.02	1.97	1.92	1.88	1.84	1.81	1.78	1.76
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.80	1.78	1.75

Table 17-1. Percentiles of *F*-Distribution for $(1-\alpha) = .98$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1012.55	1249.50	1350.50	1405.83	1440.61	1464.45	1481.80	1494.99	1505.34	1513.69	1520.56	1526.31	1531.20	1535.40	1539.05
2	48.51	49.00	49.17	49.25	49.30	49.33	49.36	49.37	49.39	49.40	49.41	49.42	49.42	49.43	49.43
3	20.62	18.86	18.11	17.69	17.43	17.25	17.11	17.01	16.93	16.86	16.81	16.76	16.72	16.69	16.66
4	14.04	12.14	11.34	10.90	10.62	10.42	10.27	10.16	10.07	10.00	9.94	9.89	9.85	9.81	9.78
5	11.32	9.45	8.67	8.23	7.95	7.76	7.61	7.50	7.42	7.34	7.28	7.23	7.19	7.16	7.12
6	9.88	8.05	7.29	6.86	6.58	6.39	6.25	6.14	6.05	5.98	5.93	5.88	5.83	5.80	5.76
7	8.99	7.20	6.45	6.03	5.76	5.58	5.44	5.33	5.24	5.17	5.11	5.06	5.02	4.98	4.95
8	8.39	6.64	5.90	5.49	5.22	5.04	4.90	4.79	4.70	4.63	4.58	4.53	4.49	4.45	4.42
9	7.96	6.23	5.51	5.10	4.84	4.65	4.52	4.41	4.33	4.26	4.20	4.15	4.11	4.07	4.04
10	7.64	5.93	5.22	4.82	4.55	4.37	4.23	4.13	4.04	3.97	3.92	3.87	3.83	3.79	3.76
11	7.39	5.70	4.99	4.59	4.34	4.15	4.02	3.91	3.83	3.76	3.70	3.65	3.61	3.57	3.54
12	7.19	5.52	4.81	4.42	4.16	3.98	3.85	3.74	3.66	3.59	3.53	3.48	3.44	3.40	3.37
13	7.02	5.37	4.67	4.28	4.02	3.84	3.71	3.60	3.52	3.45	3.39	3.34	3.30	3.26	3.23
14	6.89	5.24	4.55	4.16	3.90	3.72	3.59	3.48	3.40	3.33	3.27	3.23	3.18	3.15	3.11
15	6.77	5.14	4.45	4.06	3.81	3.63	3.49	3.39	3.30	3.23	3.18	3.13	3.09	3.05	3.02
16	6.67	5.05	4.36	3.97	3.72	3.54	3.41	3.30	3.22	3.15	3.09	3.05	3.00	2.97	2.93
17	6.59	4.97	4.29	3.90	3.65	3.47	3.34	3.23	3.15	3.08	3.02	2.97	2.93	2.89	2.86
18	6.51	4.90	4.22	3.84	3.59	3.41	3.27	3.17	3.09	3.02	2.96	2.91	2.87	2.83	2.80
19	6.45	4.84	4.16	3.78	3.53	3.35	3.22	3.12	3.03	2.96	2.91	2.86	2.81	2.78	2.74
20	6.39	4.79	4.11	3.73	3.48	3.30	3.17	3.07	2.98	2.91	2.86	2.81	2.77	2.73	2.70
21	6.34	4.74	4.07	3.69	3.44	3.26	3.13	3.02	2.94	2.87	2.81	2.76	2.72	2.68	2.65
22	6.29	4.70	4.03	3.65	3.40	3.22	3.09	2.99	2.90	2.83	2.77	2.73	2.68	2.65	2.61
23	6.25	4.66	3.99	3.61	3.36	3.19	3.05	2.95	2.87	2.80	2.74	2.69	2.65	2.61	2.58
24	6.21	4.63	3.96	3.58	3.33	3.15	3.02	2.92	2.83	2.77	2.71	2.66	2.62	2.58	2.55
25	6.18	4.59	3.93	3.55	3.30	3.13	2.99	2.89	2.81	2.74	2.68	2.63	2.59	2.55	2.52
26	6.14	4.56	3.90	3.52	3.28	3.10	2.97	2.86	2.78	2.71	2.65	2.60	2.56	2.52	2.49
27	6.11	4.54	3.87	3.50	3.25	3.07	2.94	2.84	2.76	2.69	2.63	2.58	2.54	2.50	2.46
28	6.09	4.51	3.85	3.47	3.23	3.05	2.92	2.82	2.73	2.66	2.61	2.56	2.51	2.48	2.44
29	6.06	4.49	3.83	3.45	3.21	3.03	2.90	2.80	2.71	2.64	2.58	2.54	2.49	2.45	2.42
30	6.04	4.47	3.81	3.43	3.19	3.01	2.88	2.78	2.69	2.62	2.57	2.52	2.47	2.44	2.40
35	5.94	4.38	3.73	3.35	3.11	2.93	2.80	2.70	2.61	2.55	2.49	2.44	2.39	2.36	2.32
40	5.87	4.32	3.67	3.30	3.05	2.88	2.74	2.64	2.56	2.49	2.43	2.38	2.34	2.30	2.26
45	5.82	4.27	3.62	3.25	3.01	2.83	2.70	2.60	2.51	2.44	2.39	2.34	2.29	2.25	2.22
50	5.78	4.23	3.59	3.22	2.97	2.80	2.67	2.56	2.48	2.41	2.35	2.30	2.26	2.22	2.18
55	5.74	4.20	3.56	3.19	2.94	2.77	2.64	2.54	2.45	2.38	2.32	2.27	2.23	2.19	2.16
60	5.71	4.18	3.53	3.16	2.92	2.75	2.62	2.51	2.43	2.36	2.30	2.25	2.21	2.17	2.13
70	5.67	4.14	3.49	3.13	2.88	2.71	2.58	2.48	2.39	2.32	2.26	2.21	2.17	2.13	2.10
80	5.64	4.11	3.47	3.10	2.86	2.68	2.55	2.45	2.37	2.30	2.24	2.19	2.14	2.10	2.07
90	5.61	4.09	3.45	3.08	2.84	2.66	2.53	2.43	2.35	2.28	2.22	2.17	2.12	2.08	2.05
100	5.59	4.07	3.43	3.06	2.82	2.65	2.52	2.41	2.33	2.26	2.20	2.15	2.10	2.07	2.03
110	5.57	4.05	3.41	3.05	2.81	2.63	2.50	2.40	2.32	2.25	2.19	2.14	2.09	2.05	2.02
120	5.56	4.04	3.40	3.04	2.80	2.62	2.49	2.39	2.30	2.23	2.18	2.12	2.08	2.04	2.01

Table 17-1. Percentiles of *F*-Distribution for $(1-\alpha) = .99$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6083.32	6106.32	6125.86	6142.67	6157.28
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.41	99.42	99.42	99.43	99.43
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.13	27.05	26.98	26.92	26.87
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.45	14.37	14.31	14.25	14.20
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89	9.82	9.77	9.72
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.66	7.60	7.56
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47	6.41	6.36	6.31
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67	5.61	5.56	5.52
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	5.05	5.01	4.96
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.65	4.60	4.56
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40	4.34	4.29	4.25
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16	4.10	4.05	4.01
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.91	3.86	3.82
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.75	3.70	3.66
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.61	3.56	3.52
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55	3.50	3.45	3.41
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46	3.40	3.35	3.31
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37	3.32	3.27	3.23
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.24	3.19	3.15
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.18	3.13	3.09
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.24	3.17	3.12	3.07	3.03
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.07	3.02	2.98
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	3.02	2.97	2.93
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.98	2.93	2.89
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	3.06	2.99	2.94	2.89	2.85
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	2.90	2.86	2.81
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.99	2.93	2.87	2.82	2.78
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	2.84	2.79	2.75
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93	2.87	2.81	2.77	2.73
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84	2.79	2.74	2.70
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.80	2.74	2.69	2.64	2.60
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	2.52
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.67	2.61	2.55	2.51	2.46
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.63	2.56	2.51	2.46	2.42
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66	2.59	2.53	2.47	2.42	2.38
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.44	2.39	2.35
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.51	2.45	2.40	2.35	2.31
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.48	2.42	2.36	2.31	2.27
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.45	2.39	2.33	2.29	2.24
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.43	2.37	2.31	2.27	2.22
110	6.87	4.80	3.96	3.49	3.19	2.97	2.81	2.68	2.57	2.49	2.41	2.35	2.30	2.25	2.21
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34	2.28	2.23	2.19

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Table 17-2. Percentiles of Chi-Square Distribution for *df* = 1(1)30(5)100

df \ (1−α)	0.90	0.95	0.975	0.98	0.99
1	2.706	3.841	5.024	5.412	6.635
2	4.605	5.991	7.378	7.824	9.210
3	6.251	7.815	9.348	9.837	11.345
4	7.779	9.488	11.143	11.668	13.277
5	9.236	11.070	12.833	13.388	15.086
6	10.645	12.592	14.449	15.033	16.812
7	12.017	14.067	16.013	16.622	18.475
8	13.362	15.507	17.535	18.168	20.090
9	14.684	16.919	19.023	19.679	21.666
10	15.987	18.307	20.483	21.161	23.209
11	17.275	19.675	21.920	22.618	24.725
12	18.549	21.026	23.337	24.054	26.217
13	19.812	22.362	24.736	25.472	27.688
14	21.064	23.685	26.119	26.873	29.141
15	22.307	24.996	27.488	28.259	30.578
16	23.542	26.296	28.845	29.633	32.000
17	24.769	27.587	30.191	30.995	33.409
18	25.989	28.869	31.526	32.346	34.805
19	27.204	30.144	32.852	33.687	36.191
20	28.412	31.410	34.170	35.020	37.566
21	29.615	32.671	35.479	36.343	38.932
22	30.813	33.924	36.781	37.659	40.289
23	32.007	35.172	38.076	38.968	41.638
24	33.196	36.415	39.364	40.270	42.980
25	34.382	37.652	40.646	41.566	44.314
26	35.563	38.885	41.923	42.856	45.642
27	36.741	40.113	43.195	44.140	46.963
28	37.916	41.337	44.461	45.419	48.278
29	39.087	42.557	45.722	46.693	49.588
30	40.256	43.773	46.979	47.962	50.892
35	46.059	49.802	53.203	54.244	57.342
40	51.805	55.758	59.342	60.436	63.691
45	57.505	61.656	65.410	66.555	69.957
50	63.167	67.505	71.420	72.613	76.154
55	68.796	73.311	77.380	78.619	82.292
60	74.397	79.082	83.298	84.580	88.379
65	79.973	84.821	89.177	90.501	94.422
70	85.527	90.531	95.023	96.388	100.425
75	91.061	96.217	100.839	102.243	106.393
80	96.578	101.879	106.629	108.069	112.329
85	102.079	107.522	112.393	113.871	118.236
90	107.565	113.145	118.136	119.648	124.116
95	113.038	118.752	123.858	125.405	129.973
100	118.498	124.342	129.561	131.142	135.807

Footnote. The notation *df* = 1(1)30(5)100 is a shorthand for *df* from 1 to 30 by unit steps, then from 35 to 100 by 5's

Table 17-3. Upper Tolerance Limit Factors With γ Coverage for *n* = 4(1)30(5)100

	95% Confidence			99% Confidence		
n\γ	**0.90**	**0.95**	**0.99**	**0.90**	**0.95**	**0.99**
4	4.162	5.144	7.042	7.380	9.083	12.387
5	3.407	4.203	5.741	5.362	6.578	8.939
6	3.006	3.708	5.062	4.411	5.406	7.335
7	2.755	3.399	4.642	3.859	4.728	6.412
8	2.582	3.187	4.354	3.497	4.285	5.812
9	2.454	3.031	4.143	3.240	3.972	5.389
10	2.355	2.911	3.981	3.048	3.738	5.074
11	2.275	2.815	3.852	2.898	3.556	4.829
12	2.210	2.736	3.747	2.777	3.410	4.633
13	2.155	2.671	3.659	2.677	3.290	4.472
14	2.109	2.614	3.585	2.593	3.189	4.337
15	2.068	2.566	3.520	2.521	3.102	4.222
16	2.033	2.524	3.464	2.459	3.028	4.123
17	2.002	2.486	3.414	2.405	2.963	4.037
18	1.974	2.453	3.370	2.357	2.905	3.960
19	1.949	2.423	3.331	2.314	2.854	3.892
20	1.926	2.396	3.295	2.276	2.808	3.832
21	1.905	2.371	3.263	2.241	2.766	3.777
22	1.886	2.349	3.233	2.209	2.729	3.727
23	1.869	2.328	3.206	2.180	2.694	3.681
24	1.853	2.309	3.181	2.154	2.662	3.640
25	1.838	2.292	3.158	2.129	2.633	3.601
26	1.824	2.275	3.136	2.106	2.606	3.566
27	1.811	2.260	3.116	2.085	2.581	3.533
28	1.799	2.246	3.098	2.065	2.558	3.502
29	1.788	2.232	3.080	2.047	2.536	3.473
30	1.777	2.220	3.064	2.030	2.515	3.447
35	1.732	2.167	2.995	1.957	2.430	3.334
40	1.697	2.125	2.941	1.902	2.364	3.249
45	1.669	2.092	2.898	1.857	2.312	3.180
50	1.646	2.065	2.862	1.821	2.269	3.125
55	1.626	2.042	2.833	1.790	2.233	3.078
60	1.609	2.022	2.807	1.764	2.202	3.038
65	1.594	2.005	2.785	1.741	2.176	3.004
70	1.581	1.990	2.765	1.722	2.153	2.974
75	1.570	1.976	2.748	1.704	2.132	2.947
80	1.559	1.964	2.733	1.688	2.114	2.924
85	1.550	1.954	2.719	1.674	2.097	2.902
90	1.542	1.944	2.706	1.661	2.082	2.883
95	1.534	1.935	2.695	1.650	2.069	2.866
100	1.527	1.927	2.684	1.639	2.056	2.850

Source of algorithm used to compute table: Odeh & Owen (1980)
Footnote. The notation *n* = 4(1)30(5)100 is a shorthand for *n* from 4 to 30 by unit steps, then from 35 to 100 by 5's

Table 17-4. Minimum Coverage of Non-Parametric Upper Tolerance Limit for n = 4(1)30(5)100

	Maximum		2nd Largest	
n\(1−α)	**0.95**	**0.99**	**0.95**	**0.99**
4	0.473	0.316	0.248	0.140
5	0.549	0.398	0.342	0.222
6	0.607	0.464	0.418	0.294
7	0.652	0.518	0.479	0.356
8	0.688	0.562	0.529	0.410
9	0.717	0.599	0.570	0.455
10	0.741	0.631	0.605	0.495
11	0.762	0.658	0.635	0.530
12	0.779	0.681	0.661	0.560
13	0.794	0.702	0.683	0.587
14	0.807	0.720	0.703	0.610
15	0.819	0.736	0.720	0.632
16	0.829	0.750	0.736	0.651
17	0.838	0.763	0.749	0.668
18	0.847	0.774	0.762	0.683
19	0.854	0.785	0.773	0.698
20	0.861	0.794	0.783	0.711
21	0.867	0.803	0.793	0.723
22	0.873	0.811	0.801	0.734
23	0.878	0.819	0.809	0.744
24	0.883	0.825	0.817	0.753
25	0.887	0.832	0.823	0.762
26	0.891	0.838	0.830	0.770
27	0.895	0.843	0.836	0.778
28	0.899	0.848	0.841	0.785
29	0.902	0.853	0.846	0.792
30	0.905	0.858	0.851	0.798
35	0.918	0.877	0.871	0.824
40	0.928	0.891	0.886	0.845
45	0.936	0.903	0.898	0.861
50	0.942	0.912	0.908	0.874
55	0.947	0.920	0.916	0.885
60	0.951	0.926	0.923	0.894
65	0.955	0.932	0.929	0.902
70	0.958	0.936	0.934	0.908
75	0.961	0.940	0.938	0.914
80	0.963	0.944	0.942	0.919
85	0.965	0.947	0.945	0.924
90	0.967	0.950	0.948	0.928
95	0.969	0.953	0.951	0.932
100	0.970	0.955	0.953	0.935

Footnotes. Maximum, 2nd Largest refer to Largest and next largest sample values
The notation *n* = 4(1)30(5)100 is a shorthand for *n* from 4 to 30 by unit steps, then from 35 to 100 by 5's

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Table 17-5. Significance Levels (α) for Mann-Kendall Trend Test for n = 4(1)10

n = 4		n = 5		n = 6		n = 7		n = 8		n = 9		n = 10	
S	**α**	**S**	**α**	**S**	**α**	**S**	**α**	**S**	**α**	**S**	**α**	**S**	**α**
0	0.6250	0	0.5920	1	0.5000	1	0.5000	0	0.5480	0	0.5400	1	0.5000
2	0.3750	2	0.4080	3	0.3600	3	0.3860	2	0.4520	2	0.4600	3	0.4310
4	0.1670	4	0.2420	5	0.2350	5	0.2810	4	0.3600	4	0.3810	5	0.3640
6	0.0420	6	0.1170	7	0.1360	7	0.1910	6	0.2740	6	0.3060	7	0.3000
		8	0.0420	9	0.0680	9	0.1190	8	0.1990	8	0.2380	9	0.2420
		10	0.0083	11	0.0280	11	0.0680	10	0.1380	10	0.1790	11	0.1900
				13	0.0083	13	0.0350	12	0.0890	12	0.1300	13	0.1460
				15	0.0014	15	0.0150	14	0.0540	14	0.0900	15	0.1080
						17	0.0054	16	0.0310	16	0.0600	17	0.0780
						19	0.0014	18	0.0160	18	0.0380	19	0.0540
						21	0.0002	20	0.0071	20	0.0220	21	0.0360
								22	0.0028	22	0.0120	23	0.0230
								24	0.0009	24	0.0063	25	0.0140
								26	0.0002	26	0.0029	27	0.0083
								28	0.0000	28	0.0012	29	0.0046
										30	0.0004	31	0.0023
										32	0.0001	33	0.0011
										34	0.0000	35	0.0005
										36	0.0000	37	0.0002
												39	0.0001
												41	0.0000
												43	0.0000
												45	0.0000

Source: Gilbert (1987)
Footnote: Notation n = 4(1)10 is shorthand for n from 4 to 10 by unit steps

Table 18-1. Confidence Levels of Non-Parametric Prediction Limits for Next *m* Values (PL = *j*th Order Statistic) for *n* = 4(1)60

	j = n				j = n-1				j = n-2			
n	**m = 1**	**m = 2**	**m = 3**	**m = 4**	**m = 1**	**m = 2**	**m = 3**	**m = 4**	**m = 1**	**m = 2**	**m = 3**	**m = 4**
4	0.800	0.667	0.571	0.500	0.600	0.400	0.286	0.214	0.400	0.200	0.114	0.071
5	0.833	0.714	0.625	0.556	0.667	0.476	0.357	0.278	0.500	0.286	0.179	0.119
6	0.857	0.750	0.667	0.600	0.714	0.536	0.417	0.333	0.571	0.357	0.238	0.167
7	0.875	0.778	0.700	0.636	0.750	0.583	0.467	0.382	0.625	0.417	0.292	0.212
8	0.889	0.800	0.727	0.667	0.778	0.622	0.509	0.424	0.667	0.467	0.339	0.255
9	0.900	0.818	0.750	0.692	0.800	0.655	0.545	0.462	0.700	0.509	0.382	0.294
10	0.909	0.833	0.769	0.714	0.818	0.682	0.577	0.495	0.727	0.545	0.420	0.330
11	0.917	0.846	0.786	0.733	0.833	0.705	0.604	0.524	0.750	0.577	0.453	0.363
12	0.923	0.857	0.800	0.750	0.846	0.725	0.629	0.550	0.769	0.604	0.484	0.393
13	0.929	0.867	0.812	0.765	0.857	0.743	0.650	0.574	0.786	0.629	0.511	0.421
14	0.933	0.875	0.824	0.778	0.867	0.758	0.669	0.595	0.800	0.650	0.535	0.446
15	0.938	0.882	0.833	0.789	0.875	0.772	0.686	0.614	0.812	0.669	0.558	0.470
16	0.941	0.889	0.842	0.800	0.882	0.784	0.702	0.632	0.824	0.686	0.578	0.491
17	0.944	0.895	0.850	0.810	0.889	0.795	0.716	0.648	0.833	0.702	0.596	0.511
18	0.947	0.900	0.857	0.818	0.895	0.805	0.729	0.662	0.842	0.716	0.614	0.530
19	0.950	0.905	0.864	0.826	0.900	0.814	0.740	0.676	0.850	0.729	0.629	0.547
20	0.952	0.909	0.870	0.833	0.905	0.823	0.751	0.688	0.857	0.740	0.644	0.563
21	0.955	0.913	0.875	0.840	0.909	0.830	0.761	0.700	0.864	0.751	0.657	0.578
22	0.957	0.917	0.880	0.846	0.913	0.837	0.770	0.711	0.870	0.761	0.670	0.592
23	0.958	0.920	0.885	0.852	0.917	0.843	0.778	0.721	0.875	0.770	0.681	0.605
24	0.960	0.923	0.889	0.857	0.920	0.849	0.786	0.730	0.880	0.778	0.692	0.618
25	0.962	0.926	0.893	0.862	0.923	0.855	0.794	0.739	0.885	0.786	0.702	0.629
26	0.963	0.929	0.897	0.867	0.926	0.860	0.800	0.747	0.889	0.794	0.712	0.640
27	0.964	0.931	0.900	0.871	0.929	0.865	0.807	0.755	0.893	0.800	0.720	0.651
28	0.966	0.933	0.903	0.875	0.931	0.869	0.813	0.762	0.897	0.807	0.729	0.660
29	0.967	0.935	0.906	0.879	0.933	0.873	0.819	0.769	0.900	0.813	0.737	0.670

Footnotes: Notation *n* = 4(1)60 is shorthand for n from 4 to 60 by unit steps
PL = Prediction Limit

Table 18-1. Confidence Levels of Non-Parametric Prediction Limits for Next *m* Values (PL = *j*th Order Statistic) for *n* = 4(1)60

	j = n				j = n-1				j = n-2			
n	**m = 1**	**m = 2**	**m = 3**	**m = 4**	**m = 1**	**m = 2**	**m = 3**	**m = 4**	**m = 1**	**m = 2**	**m = 3**	**m = 4**
30	0.968	0.938	0.909	0.882	0.935	0.877	0.824	0.775	0.903	0.819	0.744	0.678
31	0.969	0.939	0.912	0.886	0.938	0.881	0.829	0.782	0.906	0.824	0.751	0.687
32	0.970	0.941	0.914	0.889	0.939	0.884	0.834	0.787	0.909	0.829	0.758	0.695
33	0.971	0.943	0.917	0.892	0.941	0.887	0.838	0.793	0.912	0.834	0.764	0.702
34	0.971	0.944	0.919	0.895	0.943	0.890	0.842	0.798	0.914	0.838	0.770	0.709
35	0.972	0.946	0.921	0.897	0.944	0.893	0.846	0.803	0.917	0.842	0.776	0.716
36	0.973	0.947	0.923	0.900	0.946	0.896	0.850	0.808	0.919	0.846	0.781	0.723
37	0.974	0.949	0.925	0.902	0.947	0.899	0.854	0.812	0.921	0.850	0.786	0.729
38	0.974	0.950	0.927	0.905	0.949	0.901	0.857	0.816	0.923	0.854	0.791	0.735
39	0.975	0.951	0.929	0.907	0.950	0.904	0.861	0.821	0.925	0.857	0.796	0.741
40	0.976	0.952	0.930	0.909	0.951	0.906	0.864	0.825	0.927	0.861	0.801	0.746
41	0.976	0.953	0.932	0.911	0.952	0.908	0.867	0.828	0.929	0.864	0.805	0.751
42	0.977	0.955	0.933	0.913	0.953	0.910	0.870	0.832	0.930	0.867	0.809	0.756
43	0.977	0.956	0.935	0.915	0.955	0.912	0.872	0.835	0.932	0.870	0.813	0.761
44	0.978	0.957	0.936	0.917	0.956	0.914	0.875	0.839	0.933	0.872	0.817	0.766
45	0.978	0.957	0.938	0.918	0.957	0.916	0.878	0.842	0.935	0.875	0.820	0.770
46	0.979	0.958	0.939	0.920	0.957	0.918	0.880	0.845	0.936	0.878	0.824	0.774
47	0.979	0.959	0.940	0.922	0.958	0.919	0.882	0.848	0.938	0.880	0.827	0.779
48	0.980	0.960	0.941	0.923	0.959	0.921	0.885	0.851	0.939	0.882	0.831	0.783
49	0.980	0.961	0.942	0.925	0.960	0.922	0.887	0.853	0.940	0.885	0.834	0.786
50	0.980	0.962	0.943	0.926	0.961	0.924	0.889	0.856	0.941	0.887	0.837	0.790
51	0.981	0.962	0.944	0.927	0.962	0.925	0.891	0.859	0.942	0.889	0.840	0.794
52	0.981	0.963	0.945	0.929	0.962	0.927	0.893	0.861	0.943	0.891	0.842	0.797
53	0.981	0.964	0.946	0.930	0.963	0.928	0.895	0.863	0.944	0.893	0.845	0.801
54	0.982	0.964	0.947	0.931	0.964	0.929	0.897	0.866	0.945	0.895	0.848	0.804
55	0.982	0.965	0.948	0.932	0.964	0.930	0.898	0.868	0.946	0.897	0.850	0.807
56	0.982	0.966	0.949	0.933	0.965	0.932	0.900	0.870	0.947	0.898	0.853	0.810
57	0.983	0.966	0.950	0.934	0.966	0.933	0.902	0.872	0.948	0.900	0.855	0.813
58	0.983	0.967	0.951	0.935	0.966	0.934	0.903	0.874	0.949	0.902	0.857	0.816
59	0.983	0.967	0.952	0.937	0.967	0.935	0.905	0.876	0.950	0.903	0.860	0.819
60	0.984	0.968	0.952	0.938	0.967	0.936	0.906	0.878	0.951	0.905	0.862	0.821

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Table 18-2. Confidence Levels for Non-Parametric Prediction Limit on Future Median of Order 3 (PL = *j*th Order Statistic) for n = 4(1)60

n	j = n	j = n-1	j = n-2
4	0.857	0.629	0.371
5	0.893	0.714	0.500
6	0.917	0.774	0.595
7	0.933	0.817	0.667
8	0.945	0.848	0.721
9	0.955	0.873	0.764
10	0.962	0.892	0.797
11	0.967	0.907	0.824
12	0.971	0.919	0.846
13	0.975	0.929	0.864
14	0.978	0.937	0.879
15	0.980	0.944	0.892
16	0.982	0.949	0.903
17	0.984	0.954	0.912
18	0.986	0.959	0.920
19	0.987	0.962	0.927
20	0.988	0.966	0.933
21	0.989	0.968	0.939
22	0.990	0.971	0.943
23	0.991	0.973	0.948
24	0.991	0.975	0.951
25	0.992	0.977	0.955
26	0.993	0.978	0.958
27	0.993	0.980	0.961
28	0.994	0.981	0.963
29	0.994	0.982	0.965
30	0.994	0.983	0.967
31	0.995	0.984	0.969
32	0.995	0.985	0.971
33	0.995	0.986	0.973
34	0.995	0.987	0.974
35	0.996	0.987	0.975
36	0.996	0.988	0.977
37	0.996	0.989	0.978
38	0.996	0.989	0.979
39	0.997	0.990	0.980
40	0.997	0.990	0.981
41	0.997	0.991	0.982
42	0.997	0.991	0.982
43	0.997	0.991	0.983
44	0.997	0.992	0.984
45	0.997	0.992	0.985
46	0.997	0.992	0.985
47	0.998	0.993	0.986
48	0.998	0.993	0.986
49	0.998	0.993	0.987
50	0.998	0.994	0.987
51	0.998	0.994	0.988
52	0.998	0.994	0.988
53	0.998	0.994	0.989
54	0.998	0.994	0.989
55	0.998	0.995	0.989
56	0.998	0.995	0.990
57	0.998	0.995	0.990
58	0.998	0.995	0.990
59	0.998	0.995	0.991
60	0.998	0.995	0.991

Footnotes: Notation *n* = 4(1)60 is shorthand for n from 4 to 60 by unit steps;PL = Prediction Limit

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D STATISTICAL TABLES

D.2 TABLES FROM CHAPTER 19: INTERWELL PREDICTION LIMITS FOR FUTURE VALUES

TABLE 19-1 κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations.....D-34

TABLE 19-2 κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations.....D-43

TABLE 19-3 κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations.....D-52

TABLE 19-4 κ-Multipliers for Mod. Cal. Interwell Prediction Limits on Observations.....D-61

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.78	0.67	0.61	0.59	0.57	0.54	0.53	0.52	0.51	0.51	0.50	0.50	0.50	0.50	0.49	0.49	0.49	0.49	0.49	0.49
2	1.21	1.03	0.95	0.90	0.88	0.84	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77	0.77	0.77	0.76	0.76	0.76
3	1.47	1.23	1.13	1.08	1.05	1.01	0.98	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92	0.92	0.91	0.91	0.91	0.91
4	1.65	1.37	1.26	1.20	1.16	1.12	1.09	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.00	1.00
5	1.79	1.48	1.36	1.29	1.25	1.20	1.17	1.15	1.13	1.12	1.11	1.11	1.10	1.10	1.09	1.09	1.08	1.08	1.08	1.07
8	2.09	1.71	1.56	1.48	1.43	1.37	1.34	1.31	1.29	1.28	1.27	1.26	1.25	1.25	1.24	1.24	1.23	1.23	1.22	1.22
12	2.34	1.90	1.73	1.64	1.58	1.51	1.47	1.44	1.42	1.40	1.39	1.39	1.38	1.37	1.36	1.36	1.35	1.35	1.34	1.34
16	2.52	2.03	1.85	1.75	1.68	1.61	1.56	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42	1.42
20	2.65	2.14	1.94	1.83	1.76	1.68	1.64	1.60	1.57	1.56	1.54	1.53	1.53	1.52	1.51	1.50	1.50	1.49	1.49	1.48
30	2.89	2.32	2.10	1.98	1.90	1.81	1.76	1.72	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.60	1.60	1.59	1.59
40	3.06	2.45	2.21	2.08	2.00	1.90	1.85	1.80	1.78	1.75	1.74	1.73	1.72	1.70	1.69	1.69	1.68	1.67	1.67	1.66
50	3.19	2.54	2.29	2.16	2.08	1.97	1.91	1.87	1.84	1.82	1.80	1.79	1.78	1.76	1.75	1.74	1.74	1.73	1.72	1.72
60	3.29	2.62	2.36	2.22	2.13	2.03	1.97	1.92	1.89	1.86	1.85	1.83	1.82	1.81	1.80	1.79	1.78	1.78	1.77	1.76
75	3.41	2.71	*2.44*	2.30	2.21	2.10	2.03	1.98	1.95	1.92	1.91	1.89	1.88	1.86	1.85	1.84	1.84	1.83	1.82	1.81
100	3.57	2.83	2.55	2.40	2.30	2.18	2.11	2.06	2.02	2.00	1.98	1.96	1.95	1.94	1.92	1.91	1.91	1.90	1.89	1.88
125	3.69	2.92	2.63	*2.47*	2.37	2.25	2.18	2.12	2.08	2.06	2.04	2.02	2.01	1.99	1.98	1.97	1.96	1.95	1.94	1.93
150	3.79	3.00	2.69	*2.53*	*2.42*	2.30	2.23	2.17	2.13	2.10	2.08	2.07	2.05	2.03	2.02	2.01	2.00	1.99	1.98	1.97
175	3.87	3.06	2.75	2.58	*2.47*	2.34	2.27	2.21	2.17	2.14	2.12	2.10	2.09	2.07	2.05	2.04	2.04	2.03	2.02	2.01
200	3.93	3.11	2.79	2.62	*2.51*	2.38	2.30	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.09	2.07	2.07	2.06	2.05	2.04

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.21	1.03	0.95	0.90	0.88	0.84	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77	0.77	0.77	0.76	0.76	0.76
2	1.65	1.37	1.26	1.20	1.16	1.12	1.09	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.00	1.00
3	1.91	1.57	1.44	1.37	1.32	1.27	1.24	1.21	1.19	1.18	1.18	1.17	1.16	1.16	1.15	1.15	1.14	1.14	1.14	1.13
4	2.09	1.71	1.56	1.48	1.43	1.37	1.34	1.31	1.29	1.28	1.27	1.26	1.25	1.25	1.24	1.24	1.23	1.23	1.22	1.22
5	2.23	1.82	1.65	1.57	1.51	1.45	1.41	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29
8	2.52	2.03	1.85	1.75	1.68	1.61	1.56	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42	1.42
12	2.76	2.22	2.01	1.90	1.83	1.74	1.69	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.55	1.54	1.53	1.53
16	2.93	2.35	2.12	2.00	1.93	1.83	1.78	1.74	1.71	1.69	1.68	1.67	1.66	1.64	1.63	1.63	1.62	1.62	1.61	1.60
20	3.06	2.45	2.21	2.08	2.00	1.90	1.85	1.80	1.78	1.75	1.74	1.73	1.72	1.70	1.69	1.69	1.68	1.67	1.67	1.66
30	3.29	2.62	2.36	2.22	2.13	2.03	1.97	1.92	1.89	1.86	1.85	1.83	1.82	1.81	1.80	1.79	1.78	1.78	1.77	1.76
40	3.45	2.74	2.47	2.32	2.23	2.12	2.05	2.00	1.97	1.94	1.92	1.91	1.90	1.88	1.87	1.86	1.85	1.85	1.84	1.83
50	3.57	2.83	2.55	2.40	2.30	2.18	2.11	2.06	2.02	2.00	1.98	1.96	1.95	1.94	1.92	1.91	1.91	1.90	1.89	1.88
60	3.67	2.91	2.61	2.46	2.36	2.24	2.16	2.11	2.07	2.05	2.03	2.01	2.00	1.98	1.97	1.96	1.95	1.94	1.93	1.92
75	3.79	3.00	2.69	2.53	2.42	2.30	2.23	2.17	2.13	2.10	2.08	2.07	2.05	2.03	2.02	2.01	2.00	1.99	1.98	1.97
100	3.93	3.11	*2.79*	2.62	2.51	2.38	2.30	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.09	2.07	2.07	2.06	2.05	2.04
125	4.05	3.19	*2.87*	2.69	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.19	2.17	2.15	2.14	2.12	2.12	2.11	2.10	2.09
150	4.14	3.26	2.93	2.75	2.63	2.49	2.41	2.35	2.30	2.27	2.25	2.23	2.22	2.19	2.18	2.17	2.16	2.15	2.14	2.13
175	4.21	3.32	2.98	*2.79*	2.68	2.54	2.45	2.39	2.34	2.31	2.28	2.27	2.25	2.23	2.21	2.20	2.19	2.18	2.17	2.16
200	4.28	3.37	3.02	*2.84*	2.72	2.57	2.49	2.42	2.37	2.34	2.32	2.30	2.28	2.26	2.24	2.23	2.22	2.21	2.20	2.19

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.65	1.37	1.26	1.20	1.16	1.12	1.09	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.00	1.00
2	2.09	1.71	1.56	1.48	1.43	1.37	1.34	1.31	1.29	1.28	1.27	1.26	1.25	1.25	1.24	1.24	1.23	1.23	1.22	1.22
3	2.34	1.90	1.73	1.64	1.58	1.51	1.47	1.44	1.42	1.40	1.39	1.39	1.38	1.37	1.36	1.36	1.35	1.35	1.34	1.34
4	2.52	2.03	1.85	1.75	1.68	1.61	1.56	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42	1.42
5	2.65	2.14	1.94	1.83	1.76	1.68	1.64	1.60	1.57	1.56	1.54	1.53	1.53	1.52	1.51	1.50	1.50	1.49	1.49	1.48
8	2.93	2.35	2.12	2.00	1.93	1.83	1.78	1.74	1.71	1.69	1.68	1.67	1.66	1.64	1.63	1.63	1.62	1.62	1.61	1.60
12	3.16	2.52	2.28	2.15	2.06	1.96	1.90	1.86	1.83	1.80	1.79	1.78	1.77	1.75	1.74	1.73	1.73	1.72	1.71	1.71
16	3.33	2.65	2.39	2.24	2.16	2.05	1.99	1.94	1.90	1.88	1.86	1.85	1.84	1.82	1.81	1.80	1.80	1.79	1.78	1.78
20	3.45	2.74	2.47	2.32	2.23	2.12	2.05	2.00	1.97	1.94	1.92	1.91	1.90	1.88	1.87	1.86	1.85	1.85	1.84	1.83
30	3.67	2.91	2.61	2.46	2.36	2.24	2.16	2.11	2.07	2.05	2.03	2.01	2.00	1.98	1.97	1.96	1.95	1.94	1.93	1.92
40	3.82	3.02	2.71	2.55	2.44	2.32	2.24	2.19	2.15	2.12	2.10	2.08	2.07	2.05	2.03	2.02	2.02	2.01	2.00	1.99
50	3.93	3.11	2.79	2.62	2.51	2.38	2.30	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.09	2.07	2.07	2.06	2.05	2.04
60	4.03	3.18	2.85	2.68	2.57	2.43	2.35	2.29	2.25	2.22	2.20	2.18	2.16	2.14	2.13	2.12	2.11	2.10	2.09	2.08
75	4.14	3.26	2.93	2.75	2.63	2.49	2.41	2.35	2.30	2.27	2.25	2.23	2.22	2.19	2.18	2.17	2.16	2.15	2.14	2.13
100	4.28	3.37	3.02	2.84	2.72	2.57	2.49	2.42	2.37	2.34	2.32	2.30	2.28	2.26	2.24	2.23	2.22	2.21	2.20	2.19
125	4.39	3.45	3.10	2.90	2.78	2.63	2.54	2.47	2.43	2.39	2.37	2.35	2.33	2.31	2.29	2.28	2.27	2.26	2.24	2.23
150	4.47	3.52	*3.15*	2.96	2.83	2.68	2.59	2.52	2.47	2.44	2.41	2.39	2.37	2.35	2.33	2.32	2.31	2.30	2.28	2.27
175	4.54	3.57	*3.20*	3.00	2.87	2.72	2.63	2.56	2.51	2.47	2.44	2.42	2.41	2.38	2.36	2.35	2.34	2.33	2.31	2.30
200	4.61	3.62	3.24	3.04	2.91	2.75	2.66	2.59	2.54	2.50	2.47	2.45	2.44	2.41	2.39	2.38	2.37	2.36	2.34	2.33

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (2 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.27	1.05	0.97	0.92	0.89	0.85	0.83	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77	0.77	0.77	0.76	0.76
2	1.76	1.42	1.29	1.22	1.18	1.13	1.10	1.08	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.02	1.01	1.01	1.01	1.00
3	2.05	1.63	1.48	1.39	1.34	1.28	1.25	1.22	1.20	1.19	1.18	1.17	1.17	1.16	1.15	1.15	1.14	1.14	1.14	1.13
4	2.27	1.78	1.61	1.51	1.45	1.39	1.35	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
5	2.43	1.90	1.71	1.60	1.54	1.47	1.42	1.39	1.37	1.35	1.34	1.33	1.33	1.32	1.31	1.30	1.30	1.30	1.29	1.29
8	2.79	2.15	1.91	1.79	1.72	1.63	1.58	1.54	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
12	3.10	2.36	2.09	1.95	1.87	1.77	1.71	1.67	1.64	1.62	1.60	1.59	1.58	1.57	1.56	1.55	1.55	1.54	1.54	1.53
16	3.32	*2.51*	2.22	2.07	1.97	1.86	1.80	1.76	1.72	1.70	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61
20	3.48	2.62	2.31	2.15	2.05	1.94	1.87	1.82	1.79	1.77	1.75	1.73	1.72	1.71	1.70	1.69	1.68	1.68	1.67	1.66
30	3.78	2.83	*2.48*	2.31	2.20	2.07	2.00	1.94	1.90	1.88	1.86	1.84	1.83	1.81	1.80	1.79	1.79	1.78	1.77	1.76
40	3.99	2.97	2.60	2.41	2.30	2.16	2.08	2.02	1.98	1.96	1.93	1.92	1.91	1.89	1.87	1.86	1.86	1.85	1.84	1.83
50	4.15	3.08	2.70	*2.50*	2.37	2.23	2.15	2.09	2.04	2.01	1.99	1.98	1.96	1.94	1.93	1.92	1.91	1.90	1.89	1.88
60	4.28	3.17	2.77	*2.56*	*2.44*	2.29	2.20	2.14	2.09	2.06	2.04	2.02	2.01	1.99	1.97	1.96	1.95	1.95	1.93	1.93
75	4.43	3.27	2.86	2.65	*2.51*	2.36	2.27	2.20	2.15	2.12	2.10	2.08	2.06	2.04	2.03	2.01	2.01	2.00	1.99	1.98
100	4.63	3.41	2.97	2.75	2.61	*2.44*	2.35	2.28	2.23	2.19	2.17	2.15	2.13	2.11	2.09	2.08	2.07	2.06	2.05	2.04
125	4.78	3.51	3.06	2.83	2.68	*2.51*	*2.41*	2.34	2.28	2.25	2.22	2.20	2.19	2.16	2.14	2.13	2.12	2.11	2.10	2.09
150	4.90	3.60	3.13	2.89	2.74	*2.56*	*2.46*	2.38	2.33	2.29	2.27	2.25	2.23	2.20	2.19	2.17	2.16	2.15	2.14	2.13
175	5.00	3.67	3.19	2.94	2.79	*2.61*	*2.51*	*2.42*	2.37	2.33	2.30	2.28	2.27	2.24	2.22	2.21	2.20	2.19	2.17	2.16
200	5.08	3.73	3.24	2.99	2.83	2.65	*2.54*	*2.46*	*2.40*	2.37	2.34	2.31	2.30	2.27	2.25	2.24	2.23	2.22	2.20	2.19

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (2 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.76	1.42	1.29	1.22	1.18	1.13	1.10	1.08	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.02	1.01	1.01	1.01	1.00
2	2.27	1.78	1.61	1.51	1.45	1.39	1.35	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
3	2.57	2.00	1.79	1.68	1.61	1.53	1.48	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.36	1.35	1.35	1.34	1.34
4	2.79	2.15	1.91	1.79	1.72	1.63	1.58	1.54	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
5	2.96	2.27	2.01	1.88	1.80	1.71	1.65	1.61	1.58	1.57	1.55	1.54	1.53	1.52	1.51	1.50	1.50	1.49	1.49	1.48
8	3.32	2.51	2.22	2.07	1.97	1.86	1.80	1.76	1.72	1.70	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61
12	3.62	2.71	2.39	2.22	2.12	2.00	1.93	1.88	1.84	1.82	1.80	1.78	1.77	1.76	1.74	1.74	1.73	1.72	1.71	1.71
16	3.83	2.86	2.51	2.33	2.22	2.09	2.02	1.96	1.92	1.89	1.88	1.86	1.85	1.83	1.82	1.81	1.80	1.80	1.79	1.78
20	3.99	2.97	2.60	2.41	2.30	2.16	2.08	2.02	1.98	1.96	1.93	1.92	1.91	1.89	1.87	1.86	1.86	1.85	1.84	1.83
30	4.28	3.17	2.77	2.56	2.44	2.29	2.20	2.14	2.09	2.06	2.04	2.02	2.01	1.99	1.97	1.96	1.95	1.95	1.93	1.93
40	4.48	3.30	*2.89*	2.67	2.53	2.38	2.29	2.21	2.17	2.14	2.11	2.09	2.08	2.06	2.04	2.03	2.02	2.01	2.00	1.99
50	4.63	3.41	2.97	2.75	2.61	2.44	2.35	2.28	2.23	2.19	2.17	2.15	2.13	2.11	2.09	2.08	2.07	2.06	2.05	2.04
60	4.75	3.49	3.05	*2.81*	2.67	2.50	2.40	2.32	2.27	2.24	2.21	2.19	2.18	2.15	2.14	2.12	2.11	2.10	2.09	2.08
75	4.90	3.60	3.13	*2.89*	2.74	2.56	2.46	2.38	2.33	2.29	2.27	2.25	2.23	2.20	2.19	2.17	2.16	2.15	2.14	2.13
100	5.08	3.73	3.24	2.99	*2.83*	2.65	2.54	2.46	2.40	2.37	2.34	2.31	2.30	2.27	2.25	2.24	2.23	2.22	2.20	2.19
125	5.23	3.82	3.33	3.06	*2.90*	2.71	2.60	2.52	2.46	2.42	2.39	2.37	2.35	2.32	2.30	2.29	2.27	2.26	2.25	2.24
150	5.34	3.90	3.39	3.13	*2.96*	*2.76*	2.65	2.56	2.50	2.46	2.43	2.41	2.39	2.36	2.34	2.32	2.31	2.30	2.29	2.27
175	5.43	3.97	3.45	3.18	3.01	*2.81*	2.69	2.60	2.54	2.50	2.47	2.44	2.42	2.39	2.37	2.36	2.35	2.34	2.32	2.31
200	5.52	4.03	3.50	3.22	3.05	*2.85*	2.73	2.64	2.57	2.53	2.50	2.47	2.45	2.42	2.40	2.39	2.37	2.36	2.35	2.33

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (2 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.27	1.79	1.61	1.51	1.45	1.39	1.35	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
2	2.79	2.15	1.91	1.79	1.72	1.63	1.58	1.54	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
3	3.10	2.36	2.09	1.95	1.87	1.77	1.71	1.67	1.64	1.62	1.60	1.59	1.58	1.57	1.56	1.55	1.55	1.54	1.54	1.53
4	3.32	2.51	2.22	2.07	1.97	1.86	1.80	1.75	1.72	1.70	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61
5	3.48	2.62	2.31	2.15	2.05	1.94	1.87	1.82	1.79	1.77	1.75	1.73	1.72	1.71	1.70	1.69	1.68	1.68	1.67	1.66
8	3.83	2.86	2.51	2.33	2.22	2.09	2.02	1.96	1.92	1.89	1.88	1.86	1.85	1.83	1.82	1.81	1.80	1.80	1.79	1.78
12	4.12	3.06	2.68	2.48	2.36	2.22	2.14	2.07	2.03	2.00	1.98	1.97	1.95	1.93	1.92	1.91	1.90	1.89	1.88	1.87
16	4.32	3.20	2.80	2.59	2.46	2.31	2.22	2.15	2.11	2.08	2.06	2.04	2.02	2.00	1.99	1.98	1.97	1.96	1.95	1.94
20	4.48	3.30	2.89	2.67	2.53	2.38	2.29	2.22	2.17	2.14	2.11	2.09	2.08	2.06	2.04	2.03	2.02	2.01	2.00	1.99
30	4.75	3.49	3.05	2.81	2.67	2.50	2.40	2.32	2.27	2.24	2.21	2.19	2.18	2.15	2.14	2.12	2.11	2.10	2.09	2.08
40	4.94	3.63	*3.16*	2.91	2.76	2.58	2.48	2.40	2.35	2.31	2.28	2.26	2.24	2.22	2.20	2.19	2.18	2.17	2.15	2.14
50	5.08	3.72	3.24	2.99	2.83	2.65	2.54	2.46	2.40	2.37	2.34	2.31	2.30	2.27	2.25	2.24	2.23	2.22	2.20	2.19
60	5.20	3.81	3.31	3.05	2.89	2.70	2.59	2.51	2.45	2.41	2.38	2.36	2.34	2.31	2.29	2.28	2.26	2.26	2.24	2.23
75	5.34	3.91	3.39	*3.13*	2.96	2.76	2.65	2.56	2.50	2.46	2.43	2.41	2.39	2.36	2.34	2.32	2.31	2.30	2.29	2.28
100	5.52	4.03	3.50	*3.22*	3.05	2.85	2.73	2.64	2.57	2.53	2.50	2.47	2.45	2.42	2.40	2.39	2.37	2.36	2.35	2.33
125	5.65	4.12	3.58	3.29	3.12	2.91	2.79	2.69	2.63	2.58	2.55	2.52	2.50	2.47	2.45	2.43	2.42	2.41	2.39	2.38
150	5.76	4.20	3.64	3.35	*3.17*	2.96	2.83	2.74	2.67	2.63	2.59	2.56	2.54	2.51	2.49	2.47	2.46	2.45	2.43	2.41
175	5.85	4.26	3.70	3.40	*3.22*	3.00	2.87	2.77	2.71	2.66	2.63	2.60	2.58	2.54	2.52	2.50	2.49	2.48	2.46	2.45
200	5.93	4.32	3.74	3.45	*3.26*	3.04	2.91	2.81	2.74	2.69	2.66	2.63	2.61	2.57	2.55	2.53	2.52	2.50	2.48	2.47

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (5 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.02	1.58	1.42	1.33	1.28	1.22	1.18	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.09	1.09	1.08	1.08	1.08
2	2.62	1.97	1.74	1.63	1.56	1.48	1.43	1.40	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
3	3.00	2.20	1.93	1.80	1.72	1.62	1.57	1.53	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.40
4	3.27	2.37	2.07	1.92	1.83	1.72	1.66	1.62	1.59	1.57	1.56	1.54	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
5	3.49	2.51	2.18	2.01	1.91	1.80	1.74	1.69	1.66	1.64	1.62	1.61	1.60	1.59	1.57	1.57	1.56	1.56	1.55	1.54
8	3.96	2.79	2.40	2.21	2.09	1.96	1.89	1.83	1.80	1.77	1.75	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
12	4.37	3.03	2.59	2.38	2.25	2.10	2.02	1.95	1.91	1.89	1.86	1.85	1.84	1.82	1.80	1.80	1.79	1.78	1.77	1.76
16	4.66	3.20	2.73	2.49	2.35	2.19	2.11	2.04	1.99	1.96	1.94	1.92	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
20	4.88	3.34	2.84	2.59	2.44	2.27	2.17	2.10	2.06	2.02	2.00	1.98	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
30	5.28	3.58	3.03	2.75	2.59	2.40	2.30	2.22	2.17	2.13	2.11	2.09	2.07	2.05	2.03	2.02	2.01	2.00	1.99	1.98
40	5.56	3.75	3.16	2.87	2.69	2.49	2.38	2.30	2.24	2.21	2.18	2.16	2.14	2.12	2.10	2.09	2.08	2.07	2.05	2.04
50	5.77	3.88	3.26	2.96	2.77	2.57	2.45	2.36	2.30	2.26	2.23	2.21	2.19	2.17	2.15	2.14	2.13	2.12	2.10	2.09
60	5.94	3.98	3.35	3.03	2.84	2.62	2.50	2.41	2.35	2.31	2.28	2.26	2.24	2.21	2.19	2.18	2.17	2.16	2.14	2.13
75	6.15	4.11	3.45	3.12	2.92	2.69	2.57	2.47	2.41	2.37	2.34	2.31	2.29	2.26	2.24	2.23	2.22	2.21	2.19	2.18
100	6.42	4.27	3.58	3.23	3.02	2.78	2.65	2.55	2.48	2.44	2.40	2.38	2.36	2.33	2.31	2.29	2.28	2.27	2.25	2.24
125	6.62	4.40	3.68	3.32	3.10	2.85	2.71	2.61	2.54	2.49	2.46	2.43	2.41	2.38	2.36	2.34	2.33	2.32	2.30	2.29
150	6.79	4.50	3.75	3.39	3.16	2.91	2.77	2.66	2.59	2.54	2.50	2.47	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.32
175	6.92	4.58	3.82	3.45	3.22	2.96	2.81	2.70	2.63	2.58	2.54	2.51	2.49	2.45	2.43	2.41	2.40	2.39	2.37	2.35
200	7.04	4.65	3.88	3.49	3.26	3.00	2.85	2.73	2.66	2.61	2.57	2.54	2.52	2.48	2.46	2.44	2.43	2.41	2.39	2.38

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (5 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.62	1.97	1.74	1.63	1.56	1.48	1.43	1.40	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
2	3.27	2.37	2.07	1.92	1.83	1.72	1.66	1.62	1.59	1.57	1.56	1.54	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
3	3.67	2.62	2.26	2.09	1.98	1.86	1.80	1.75	1.71	1.69	1.67	1.66	1.65	1.63	1.62	1.62	1.61	1.60	1.60	1.59
4	3.96	2.79	2.40	2.21	2.09	1.96	1.89	1.83	1.80	1.77	1.75	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
5	4.18	2.92	2.51	2.30	2.18	2.04	1.96	1.90	1.86	1.83	1.82	1.80	1.79	1.77	1.76	1.75	1.74	1.74	1.73	1.72
8	4.66	3.20	2.73	2.49	2.35	2.19	2.11	2.04	1.99	1.96	1.94	1.92	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
12	5.06	3.45	2.92	2.66	2.51	2.33	2.23	2.15	2.11	2.07	2.05	2.03	2.01	1.99	1.98	1.97	1.96	1.95	1.94	1.93
16	5.34	3.62	3.06	2.78	2.61	2.42	2.32	2.24	2.19	2.15	2.12	2.10	2.09	2.06	2.05	2.03	2.02	2.02	2.00	1.99
20	5.56	3.75	3.16	2.87	2.69	2.49	2.38	2.30	2.24	2.21	2.18	2.16	2.14	2.12	2.10	2.09	2.08	2.07	2.05	2.04
30	5.94	3.98	3.35	3.03	2.84	2.62	2.50	2.41	2.35	2.31	2.28	2.26	2.24	2.21	2.19	2.18	2.17	2.16	2.14	2.13
40	6.21	4.15	3.48	3.14	2.94	2.71	2.59	2.49	2.43	2.38	2.35	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.20	2.19
50	6.42	4.27	3.58	3.23	3.02	2.78	2.65	2.55	2.48	2.44	2.40	2.38	2.36	2.33	2.31	2.29	2.28	2.27	2.25	2.24
60	6.59	4.37	3.66	3.30	3.09	2.84	2.70	2.60	2.53	2.48	2.45	2.42	2.40	2.37	2.35	2.33	2.32	2.31	2.29	2.28
75	6.79	4.50	3.75	3.39	3.16	2.91	2.77	2.66	2.59	2.54	2.50	2.47	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.32
100	7.04	4.65	3.88	3.49	3.26	3.00	2.85	2.73	2.66	2.61	2.57	2.54	2.52	2.48	2.46	2.44	2.43	2.41	2.39	2.38
125	7.23	4.77	3.98	3.58	3.34	3.06	2.91	2.79	2.72	2.66	2.62	2.59	2.57	2.53	2.51	2.49	2.47	2.46	2.44	2.43
150	7.38	4.87	4.05	3.65	3.40	3.12	2.96	2.84	2.76	2.70	2.66	2.63	2.61	2.57	2.54	2.52	2.51	2.50	2.48	2.46
175	7.52	4.95	4.12	3.70	3.45	3.17	3.00	2.88	2.80	2.74	2.70	2.67	2.64	2.60	2.58	2.56	2.54	2.53	2.51	2.49
200	7.62	5.02	4.17	3.75	3.50	3.20	3.04	2.91	2.83	2.77	2.73	2.70	2.67	2.63	2.60	2.58	2.57	2.55	2.53	2.52

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (5 COCs, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.27	**2.37**	**2.07**	**1.92**	**1.83**	**1.72**	**1.66**	**1.62**	**1.59**	**1.57**	**1.56**	**1.54**	**1.54**	**1.52**	**1.51**	**1.51**	**1.50**	**1.50**	**1.49**	**1.48**
2	3.96	**2.79**	**2.40**	**2.21**	**2.09**	**1.96**	**1.89**	**1.83**	**1.80**	**1.77**	**1.75**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
3	4.37	**3.03**	**2.59**	**2.38**	**2.25**	**2.10**	**2.02**	**1.95**	**1.91**	**1.89**	**1.86**	**1.85**	**1.84**	**1.82**	**1.80**	**1.80**	**1.79**	**1.78**	**1.77**	**1.76**
4	4.66	3.20	**2.73**	**2.49**	**2.35**	**2.19**	**2.11**	**2.04**	**1.99**	**1.96**	**1.94**	**1.92**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
5	4.88	3.34	**2.84**	**2.59**	**2.44**	**2.27**	**2.17**	**2.10**	**2.06**	**2.02**	**2.00**	**1.98**	**1.97**	**1.95**	**1.93**	**1.92**	**1.91**	**1.91**	**1.89**	**1.89**
8	5.34	3.62	**3.06**	**2.78**	**2.61**	**2.42**	**2.32**	**2.24**	**2.19**	**2.15**	**2.12**	**2.10**	**2.09**	**2.06**	**2.05**	**2.03**	**2.02**	**2.02**	**2.00**	**1.99**
12	5.74	3.86	3.24	**2.94**	**2.76**	**2.55**	**2.44**	**2.35**	**2.29**	**2.25**	**2.22**	**2.20**	**2.19**	**2.16**	**2.14**	**2.13**	**2.12**	**2.11**	**2.09**	**2.08**
16	6.01	4.02	3.37	**3.06**	**2.86**	**2.64**	**2.52**	**2.43**	**2.37**	**2.33**	**2.30**	**2.27**	**2.25**	**2.23**	**2.21**	**2.19**	**2.18**	**2.17**	**2.16**	**2.15**
20	6.21	4.15	3.48	*3.14*	**2.94**	**2.71**	**2.59**	**2.49**	**2.43**	**2.38**	**2.35**	**2.33**	**2.31**	**2.28**	**2.26**	**2.24**	**2.23**	**2.22**	**2.20**	**2.19**
30	6.59	4.37	3.66	3.30	**3.09**	**2.84**	**2.70**	**2.60**	**2.53**	**2.48**	**2.45**	**2.42**	**2.40**	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**	**2.28**
40	6.84	4.53	3.78	3.41	*3.19*	**2.93**	**2.79**	**2.68**	**2.60**	**2.55**	**2.52**	**2.49**	**2.47**	**2.43**	**2.41**	**2.39**	**2.38**	**2.37**	**2.35**	**2.34**
50	7.04	4.65	3.88	3.49	*3.26*	**3.00**	**2.85**	**2.73**	**2.66**	**2.61**	**2.57**	**2.54**	**2.52**	**2.48**	**2.46**	**2.44**	**2.43**	**2.41**	**2.39**	**2.38**
60	7.20	4.75	3.96	3.56	*3.32*	**3.05**	**2.90**	**2.78**	**2.71**	**2.65**	**2.61**	**2.58**	**2.56**	**2.52**	**2.50**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
75	7.38	4.87	4.05	3.65	3.40	*3.12*	**2.96**	**2.84**	**2.76**	**2.70**	**2.66**	**2.63**	**2.61**	**2.57**	**2.54**	**2.52**	**2.51**	**2.50**	**2.48**	**2.46**
100	7.62	5.02	4.17	3.75	3.50	*3.20*	**3.04**	**2.91**	**2.83**	**2.77**	**2.73**	**2.70**	**2.67**	**2.63**	**2.60**	**2.58**	**2.57**	**2.55**	**2.53**	**2.52**
125	7.81	5.13	4.26	3.83	3.57	*3.27*	**3.10**	**2.97**	**2.88**	**2.82**	**2.78**	**2.75**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.60**	**2.58**	**2.56**
150	7.96	5.23	4.34	3.90	3.63	*3.32*	*3.15*	**3.02**	**2.93**	**2.87**	**2.82**	**2.79**	**2.76**	**2.72**	**2.69**	**2.67**	**2.65**	**2.63**	**2.61**	**2.59**
175	8.09	5.30	4.40	3.95	3.68	*3.37*	*3.19*	**3.05**	**2.96**	**2.90**	**2.86**	**2.82**	**2.79**	**2.75**	**2.72**	**2.70**	**2.68**	**2.66**	**2.64**	**2.62**
200	8.19	5.37	4.45	4.00	3.72	3.41	*3.23*	**3.09**	**3.00**	**2.93**	**2.89**	**2.85**	**2.82**	**2.78**	**2.75**	**2.72**	**2.70**	**2.69**	**2.67**	**2.65**

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (10 COCs, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.69	**1.99**	**1.76**	**1.64**	**1.56**	**1.48**	**1.43**	**1.40**	**1.37**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**	**1.30**	**1.30**	**1.29**	**1.29**
2	3.43	**2.42**	**2.09**	**1.93**	**1.84**	**1.73**	**1.67**	**1.62**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.52**	**1.51**	**1.51**	**1.50**	**1.50**	**1.49**	**1.48**
3	3.88	2.68	**2.29**	**2.11**	**2.00**	**1.87**	**1.80**	**1.75**	**1.71**	**1.69**	**1.68**	**1.66**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.60**	**1.59**
4	4.24	2.86	*2.44*	**2.23**	**2.11**	**1.97**	**1.89**	**1.84**	**1.80**	**1.77**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
5	4.51	3.01	2.55	**2.33**	**2.20**	**2.05**	**1.97**	**1.91**	**1.86**	**1.84**	**1.82**	**1.80**	**1.79**	**1.77**	**1.76**	**1.75**	**1.74**	**1.74**	**1.73**	**1.72**
8	5.09	3.33	2.79	*2.53*	**2.38**	**2.21**	**2.11**	**2.04**	**2.00**	**1.97**	**1.94**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
12	5.60	3.60	3.00	2.71	*2.54*	**2.34**	**2.24**	**2.16**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**	**1.97**	**1.96**	**1.95**	**1.94**	**1.93**
16	5.95	3.80	3.14	2.83	2.65	*2.44*	**2.33**	**2.25**	**2.19**	**2.15**	**2.12**	**2.11**	**2.09**	**2.06**	**2.05**	**2.03**	**2.02**	**2.02**	**2.00**	**1.99**
20	6.23	3.96	3.26	2.93	2.73	*2.51*	*2.40*	**2.31**	**2.25**	**2.21**	**2.18**	**2.16**	**2.14**	**2.12**	**2.10**	**2.09**	**2.08**	**2.07**	**2.05**	**2.04**
30	6.73	4.22	3.47	3.10	2.89	2.65	*2.52*	*2.42*	**2.36**	**2.32**	**2.28**	**2.26**	**2.24**	**2.21**	**2.19**	**2.18**	**2.17**	**2.16**	**2.14**	**2.13**
40	7.08	4.42	3.61	3.23	3.00	2.74	*2.61*	*2.50*	*2.44*	*2.39*	**2.35**	**2.33**	**2.31**	**2.28**	**2.26**	**2.24**	**2.23**	**2.22**	**2.20**	**2.19**
50	7.35	4.56	3.72	3.32	3.08	2.82	2.67	*2.56*	*2.49*	*2.45*	*2.41*	*2.38*	**2.36**	**2.33**	**2.31**	**2.29**	**2.28**	**2.27**	**2.25**	**2.24**
60	7.58	4.69	3.81	3.40	3.15	2.88	2.73	*2.61*	*2.54*	*2.49*	*2.45*	*2.43*	*2.40*	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**	**2.28**
75	7.83	4.83	3.93	3.49	3.23	2.95	2.79	2.67	*2.60*	*2.55*	*2.51*	*2.48*	*2.46*	*2.42*	*2.40*	*2.38*	**2.37**	**2.36**	**2.34**	**2.32**
100	8.17	5.02	4.07	3.61	3.34	3.04	2.88	2.75	2.67	*2.62*	*2.58*	*2.55*	*2.52*	*2.49*	*2.46*	*2.44*	*2.43*	*2.42*	*2.40*	*2.38*
125	8.43	5.16	4.17	3.70	3.42	3.11	2.94	2.81	2.73	2.67	*2.63*	*2.60*	*2.57*	*2.54*	*2.51*	*2.49*	*2.47*	*2.46*	*2.44*	*2.43*
150	8.63	5.27	4.27	3.78	3.49	3.17	2.99	2.86	2.77	2.72	*2.67*	*2.64*	*2.61*	*2.57*	*2.55*	*2.53*	*2.51*	*2.50*	*2.48*	*2.46*
175	8.80	5.37	4.34	3.84	3.55	3.22	3.04	2.90	2.81	2.75	2.71	2.67	2.65	2.61	2.58	2.56	2.54	2.53	2.51	2.49
200	8.94	5.46	4.40	3.89	3.60	3.26	3.08	2.94	2.85	2.79	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.53	2.52

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (10 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.43	**2.42**	**2.09**	**1.93**	**1.84**	**1.73**	**1.67**	**1.62**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.52**	**1.51**	**1.51**	**1.50**	**1.50**	**1.49**	**1.48**
2	4.24	2.86	**2.44**	**2.23**	**2.11**	**1.97**	**1.89**	**1.84**	**1.80**	**1.77**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
3	4.72	3.13	**2.65**	**2.40**	**2.27**	**2.11**	**2.02**	**1.96**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.82**	**1.81**	**1.80**	**1.79**	**1.78**	**1.77**	**1.76**
4	5.09	3.33	*2.79*	**2.53**	**2.38**	**2.21**	**2.11**	**2.04**	**2.00**	**1.97**	**1.94**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
5	5.37	3.48	2.90	**2.63**	**2.46**	**2.28**	**2.18**	**2.11**	**2.06**	**2.03**	**2.00**	**1.99**	**1.97**	**1.95**	**1.93**	**1.92**	**1.91**	**1.91**	**1.89**	**1.89**
8	5.95	3.80	3.14	*2.83*	**2.65**	**2.44**	**2.33**	**2.25**	**2.19**	**2.15**	**2.12**	**2.11**	**2.09**	**2.06**	**2.05**	**2.03**	**2.02**	**2.02**	**2.00**	**1.99**
12	6.46	4.08	3.35	3.01	*2.80*	**2.57**	**2.45**	**2.36**	**2.30**	**2.26**	**2.23**	**2.21**	**2.19**	**2.16**	**2.14**	**2.13**	**2.12**	**2.11**	**2.09**	**2.08**
16	6.82	4.27	3.49	3.13	*2.91*	**2.67**	**2.54**	**2.44**	**2.38**	**2.33**	**2.30**	**2.28**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.17**	**2.16**	**2.15**
20	7.08	4.42	3.61	3.23	3.00	**2.74**	**2.61**	**2.50**	**2.44**	**2.39**	**2.35**	**2.33**	**2.31**	**2.28**	**2.26**	**2.24**	**2.23**	**2.22**	**2.20**	**2.19**
30	7.58	4.69	3.81	3.40	3.15	*2.88*	**2.73**	**2.61**	**2.54**	**2.49**	**2.45**	**2.43**	**2.40**	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**	**2.28**
40	7.92	4.88	3.96	3.52	3.26	*2.97*	*2.81*	**2.69**	**2.62**	**2.56**	**2.52**	**2.49**	**2.47**	**2.44**	**2.41**	**2.39**	**2.38**	**2.37**	**2.35**	**2.34**
50	8.17	5.02	4.07	3.61	3.34	3.04	*2.88*	*2.75*	**2.67**	**2.62**	**2.58**	**2.55**	**2.52**	**2.49**	**2.46**	**2.44**	**2.43**	**2.42**	**2.40**	**2.38**
60	8.37	5.13	4.15	3.69	3.41	3.10	*2.93*	*2.80*	**2.72**	**2.66**	**2.62**	**2.59**	**2.56**	**2.52**	**2.50**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
75	8.63	5.27	4.27	3.78	3.49	3.17	*2.99*	*2.86*	*2.77*	**2.72**	**2.67**	**2.64**	**2.61**	**2.57**	**2.55**	**2.53**	**2.51**	**2.50**	**2.48**	**2.46**
100	8.94	5.46	4.40	3.89	3.60	3.26	3.08	*2.94*	*2.85*	*2.79*	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.53**	**2.52**
125	9.19	5.60	4.51	3.98	3.68	3.33	3.14	*3.00*	*2.90*	*2.84*	*2.79*	*2.76*	**2.73**	**2.68**	**2.65**	**2.63**	**2.62**	**2.60**	**2.58**	**2.56**
150	9.39	5.70	4.59	4.06	3.74	3.39	3.19	*3.04*	*2.95*	*2.88*	*2.83*	*2.79*	*2.77*	**2.72**	**2.69**	**2.67**	**2.65**	**2.64**	**2.61**	**2.60**
175	9.56	5.80	4.66	4.12	3.80	3.43	3.24	*3.08*	*2.99*	*2.92*	*2.87*	*2.83*	*2.80*	*2.76*	**2.72**	**2.70**	**2.68**	**2.67**	**2.64**	**2.63**
200	9.70	5.88	4.73	4.17	3.85	3.48	3.27	3.12	*3.02*	*2.95*	*2.90*	*2.86*	*2.83*	*2.78*	*2.75*	**2.73**	**2.71**	**2.69**	**2.67**	**2.65**

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (10 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.24	**2.86**	**2.44**	**2.23**	**2.11**	**1.97**	**1.89**	**1.84**	**1.80**	**1.77**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
2	5.09	3.33	**2.79**	**2.53**	**2.38**	**2.21**	**2.11**	**2.04**	**2.00**	**1.97**	**1.94**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
3	5.60	3.60	**2.99**	**2.71**	**2.54**	**2.34**	**2.24**	**2.16**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**	**1.97**	**1.96**	**1.95**	**1.94**	**1.93**
4	5.94	3.80	*3.15*	**2.83**	**2.65**	**2.44**	**2.33**	**2.24**	**2.19**	**2.15**	**2.12**	**2.11**	**2.09**	**2.06**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
5	6.22	3.96	3.26	**2.93**	**2.73**	**2.52**	**2.40**	**2.31**	**2.25**	**2.21**	**2.19**	**2.16**	**2.14**	**2.12**	**2.10**	**2.09**	**2.08**	**2.07**	**2.05**	**2.04**
8	6.82	4.27	3.50	*3.13*	**2.91**	**2.67**	**2.54**	**2.44**	**2.38**	**2.33**	**2.30**	**2.28**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.17**	**2.16**	**2.15**
12	7.30	4.55	3.70	3.30	**3.06**	**2.80**	**2.66**	**2.55**	**2.48**	**2.44**	**2.40**	**2.37**	**2.35**	**2.32**	**2.30**	**2.28**	**2.27**	**2.26**	**2.24**	**2.23**
16	7.64	4.72	3.84	3.42	*3.18*	**2.90**	**2.74**	**2.63**	**2.56**	**2.51**	**2.47**	**2.44**	**2.42**	**2.39**	**2.36**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**
20	7.92	4.88	3.96	3.52	*3.26*	**2.97**	**2.81**	**2.69**	**2.62**	**2.56**	**2.52**	**2.49**	**2.47**	**2.44**	**2.41**	**2.39**	**2.38**	**2.37**	**2.35**	**2.34**
30	8.37	5.14	4.15	3.69	3.41	**3.10**	**2.93**	**2.80**	**2.72**	**2.66**	**2.62**	**2.59**	**2.56**	**2.52**	**2.50**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
40	8.71	5.31	4.29	3.80	3.52	*3.19*	**3.01**	**2.88**	**2.79**	**2.73**	**2.69**	**2.65**	**2.63**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
50	8.94	5.46	4.41	3.90	3.60	*3.26*	**3.08**	**2.94**	**2.85**	**2.79**	**2.74**	**2.71**	**2.68**	**2.63**	**2.61**	**2.59**	**2.57**	**2.56**	**2.53**	**2.52**
60	9.17	5.57	4.49	3.97	3.66	*3.32*	*3.13*	**2.99**	**2.89**	**2.83**	**2.78**	**2.74**	**2.72**	**2.67**	**2.65**	**2.62**	**2.61**	**2.59**	**2.57**	**2.55**
75	9.39	5.71	4.59	4.05	3.74	*3.39*	*3.19*	**3.04**	**2.95**	**2.88**	**2.83**	**2.79**	**2.77**	**2.72**	**2.69**	**2.67**	**2.65**	**2.64**	**2.61**	**2.60**
100	9.68	5.88	4.72	4.17	3.84	3.47	*3.28*	*3.12*	**3.02**	**2.95**	**2.90**	**2.86**	**2.83**	**2.78**	**2.75**	**2.73**	**2.71**	**2.69**	**2.67**	**2.65**
125	9.90	6.01	4.83	4.26	3.93	3.54	*3.33*	*3.18*	**3.07**	**3.00**	**2.95**	**2.91**	**2.88**	**2.83**	**2.79**	**2.77**	**2.75**	**2.74**	**2.71**	**2.69**
150	10.13	6.11	4.92	4.33	3.98	3.60	*3.39*	*3.22*	*3.12*	**3.04**	**2.99**	**2.95**	**2.91**	**2.86**	**2.83**	**2.81**	**2.79**	**2.77**	**2.74**	**2.73**
175	10.24	6.19	4.97	4.39	4.04	3.64	*3.43*	*3.26*	*3.16*	**3.08**	**3.02**	**2.98**	**2.95**	**2.90**	**2.86**	**2.84**	**2.82**	**2.80**	**2.77**	**2.76**
200	10.41	6.28	5.03	4.44	4.08	3.69	3.47	*3.30*	*3.18*	**3.11**	**3.05**	**3.01**	**2.97**	**2.92**	**2.89**	**2.86**	**2.84**	**2.83**	**2.80**	**2.78**

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (20 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.53	**2.44**	**2.11**	**1.94**	**1.85**	**1.73**	**1.67**	**1.63**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.52**	**1.51**	**1.51**	**1.50**	**1.50**	**1.49**	**1.48**
2	4.41	2.91	*2.46*	**2.24**	**2.11**	**1.97**	**1.89**	**1.84**	**1.80**	**1.77**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
3	5.00	3.20	2.67	*2.43*	**2.28**	**2.11**	**2.03**	**1.96**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.82**	**1.81**	**1.80**	**1.79**	**1.78**	**1.77**	**1.76**
4	5.43	3.42	2.82	*2.55*	**2.39**	**2.21**	**2.12**	**2.05**	**2.00**	**1.97**	**1.94**	**1.93**	**1.92**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
5	5.77	3.57	2.95	2.65	*2.48*	**2.29**	**2.19**	**2.11**	**2.06**	**2.03**	**2.00**	**1.99**	**1.97**	**1.95**	**1.93**	**1.92**	**1.92**	**1.91**	**1.89**	**1.89**
8	6.50	3.93	3.20	2.86	2.67	*2.45*	**2.33**	**2.25**	**2.19**	**2.16**	**2.13**	**2.11**	**2.09**	**2.06**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
12	7.13	4.24	3.43	3.05	2.83	*2.59*	*2.46*	**2.36**	**2.31**	**2.26**	**2.23**	**2.21**	**2.19**	**2.16**	**2.14**	**2.13**	**2.12**	**2.11**	**2.09**	**2.09**
16	7.58	4.46	3.59	3.18	2.94	2.69	*2.55*	*2.45*	**2.38**	**2.33**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
20	7.92	4.63	3.71	3.28	3.03	2.76	*2.62*	*2.51*	*2.44*	*2.39*	**2.36**	**2.33**	**2.31**	**2.28**	**2.26**	**2.25**	**2.23**	**2.22**	**2.21**	**2.19**
30	8.54	4.95	3.94	3.47	3.20	2.90	2.74	*2.62*	*2.55*	*2.50*	*2.46*	*2.43*	*2.41*	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**	**2.28**
40	9.00	5.17	4.10	3.59	3.31	2.99	2.83	2.70	*2.62*	*2.57*	*2.53*	*2.50*	*2.48*	*2.44*	*2.41*	*2.40*	*2.38*	**2.37**	**2.35**	**2.34**
50	9.34	5.34	4.22	3.70	3.40	3.07	2.89	2.76	2.68	*2.62*	*2.58*	*2.55*	*2.52*	*2.49*	*2.46*	*2.44*	*2.43*	*2.42*	*2.40*	*2.38*
60	9.62	5.48	4.32	3.78	3.47	3.13	2.95	2.81	2.73	*2.67*	*2.62*	*2.59*	*2.57*	*2.53*	*2.50*	*2.48*	*2.47*	*2.45*	*2.43*	*2.42*
75	9.96	5.65	4.44	3.88	3.56	3.20	3.01	2.87	2.78	2.72	2.68	*2.64*	*2.62*	*2.57*	*2.55*	*2.53*	*2.51*	*2.50*	*2.48*	*2.46*
100	10.36	5.85	4.59	4.01	3.67	3.30	3.10	2.95	2.86	2.79	2.74	2.71	*2.68*	*2.64*	*2.61*	*2.59*	*2.57*	*2.56*	*2.54*	*2.52*
125	10.70	6.02	4.72	4.11	3.76	3.37	3.17	3.01	2.91	2.84	2.80	2.76	2.73	*2.69*	*2.66*	*2.63*	*2.62*	*2.60*	*2.58*	*2.56*
150	10.92	6.16	4.80	4.19	3.83	3.43	3.22	3.06	2.96	2.89	2.84	2.80	2.77	2.72	2.69	*2.67*	*2.65*	*2.64*	*2.61*	*2.60*
175	11.15	6.28	4.89	4.25	3.89	3.48	3.26	3.10	3.00	2.92	2.87	2.84	2.80	2.76	2.72	2.70	*2.68*	*2.67*	*2.64*	*2.62*
200	11.38	6.36	4.96	4.31	3.94	3.53	3.30	3.13	3.03	2.96	2.90	2.86	2.83	2.79	2.75	2.73	2.71	*2.69*	*2.67*	*2.65*

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (20 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.41	2.91	**2.46**	**2.24**	**2.11**	**1.97**	**1.89**	**1.84**	**1.80**	**1.77**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
2	5.43	3.42	*2.82*	**2.55**	**2.39**	**2.21**	**2.12**	**2.05**	**2.00**	**1.97**	**1.94**	**1.93**	**1.92**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
3	6.05	3.71	3.05	**2.73**	**2.55**	**2.35**	**2.25**	**2.16**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**	**1.97**	**1.96**	**1.95**	**1.94**	**1.93**
4	6.50	3.93	3.20	*2.86*	**2.67**	**2.45**	**2.33**	**2.25**	**2.19**	**2.16**	**2.13**	**2.11**	**2.09**	**2.06**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
5	6.84	4.10	3.32	2.96	*2.76*	**2.52**	**2.40**	**2.31**	**2.26**	**2.21**	**2.19**	**2.16**	**2.14**	**2.12**	**2.10**	**2.09**	**2.08**	**2.07**	**2.05**	**2.04**
8	7.58	4.46	3.59	3.18	*2.94*	**2.69**	**2.55**	**2.45**	**2.38**	**2.33**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
12	8.20	4.78	3.81	3.36	3.11	*2.82*	**2.67**	**2.56**	**2.49**	**2.44**	**2.40**	**2.38**	**2.35**	**2.32**	**2.30**	**2.28**	**2.27**	**2.26**	**2.25**	**2.23**
16	8.66	5.00	3.97	3.49	3.22	*2.92*	*2.76*	**2.64**	**2.56**	**2.51**	**2.48**	**2.44**	**2.42**	**2.39**	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**
20	9.00	5.17	4.10	3.59	3.31	*2.99*	*2.83*	**2.70**	**2.62**	**2.57**	**2.53**	**2.50**	**2.48**	**2.44**	**2.41**	**2.40**	**2.38**	**2.37**	**2.35**	**2.34**
30	9.62	5.48	4.32	3.78	3.47	3.13	*2.95*	*2.81*	**2.73**	**2.67**	**2.62**	**2.59**	**2.57**	**2.53**	**2.50**	**2.48**	**2.47**	**2.45**	**2.43**	**2.42**
40	10.02	5.70	4.48	3.91	3.59	3.23	*3.03*	*2.89*	*2.80*	**2.74**	**2.69**	**2.66**	**2.63**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
50	10.36	5.85	4.59	4.01	3.67	3.30	3.10	*2.95*	*2.86*	*2.79*	**2.74**	**2.71**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
60	10.64	5.99	4.69	4.09	3.74	3.36	3.16	*3.00*	*2.90*	*2.84*	*2.79*	*2.75*	**2.72**	**2.68**	**2.65**	**2.62**	**2.61**	**2.60**	**2.57**	**2.55**
75	10.92	6.16	4.80	4.19	3.83	3.43	3.22	*3.06*	*2.96*	*2.89*	*2.84*	*2.80*	*2.77*	**2.72**	**2.69**	**2.67**	**2.65**	**2.64**	**2.61**	**2.60**
100	11.38	6.36	4.96	4.31	3.94	3.53	3.30	3.13	*3.03*	*2.96*	*2.90*	*2.86*	*2.83*	*2.79*	*2.75*	**2.73**	**2.71**	**2.69**	**2.67**	**2.65**
125	11.66	6.53	5.09	4.41	4.03	3.60	3.37	3.20	*3.08*	*3.01*	*2.96*	*2.91*	*2.88*	*2.83*	*2.80*	*2.77*	*2.75*	**2.74**	**2.71**	**2.69**
150	11.88	6.65	5.17	4.49	4.10	3.66	3.42	3.24	3.13	*3.05*	*3.00*	*2.95*	*2.92*	*2.87*	*2.83*	*2.81*	*2.79*	*2.77*	**2.74**	**2.73**
175	12.11	6.76	5.26	4.56	4.15	3.71	3.47	3.28	3.17	*3.09*	*3.03*	*2.99*	*2.95*	*2.90*	*2.86*	*2.84*	*2.82*	*2.80*	*2.77*	*2.76*
200	12.28	6.84	5.31	4.61	4.20	3.75	3.50	3.32	3.20	3.12	*3.06*	*3.02*	*2.98*	*2.93*	*2.89*	*2.86*	*2.84*	*2.83*	*2.80*	*2.78*

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (20 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.43	3.42	**2.82**	**2.55**	**2.40**	**2.21**	**2.11**	**2.04**	**2.00**	**1.97**	**1.94**	**1.93**	**1.92**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.85**	**1.83**
2	6.50	3.93	*3.20*	**2.86**	**2.67**	**2.45**	**2.34**	**2.25**	**2.19**	**2.16**	**2.13**	**2.11**	**2.09**	**2.06**	**2.04**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
3	7.13	4.24	3.42	**3.05**	**2.82**	**2.60**	**2.45**	**2.37**	**2.30**	**2.26**	**2.23**	**2.21**	**2.19**	**2.16**	**2.14**	**2.13**	**2.11**	**2.11**	**2.09**	**2.09**
4	7.58	4.46	3.59	*3.18*	**2.95**	**2.68**	**2.55**	**2.45**	**2.38**	**2.33**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.19**	**2.19**	**2.17**	**2.16**	**2.14**
5	7.92	4.63	3.70	*3.28*	**3.03**	**2.77**	**2.62**	**2.51**	**2.44**	**2.40**	**2.35**	**2.33**	**2.31**	**2.28**	**2.26**	**2.24**	**2.23**	**2.22**	**2.21**	**2.19**
8	8.66	5.00	3.98	3.50	*3.22*	**2.92**	**2.76**	**2.64**	**2.57**	**2.51**	**2.48**	**2.45**	**2.43**	**2.39**	**2.36**	**2.35**	**2.33**	**2.33**	**2.31**	**2.29**
12	9.22	5.31	4.18	3.67	3.39	**3.06**	**2.88**	**2.75**	**2.67**	**2.61**	**2.57**	**2.54**	**2.51**	**2.48**	**2.45**	**2.43**	**2.42**	**2.41**	**2.39**	**2.38**
16	9.68	5.54	4.35	3.81	3.50	*3.15*	**2.96**	**2.83**	**2.74**	**2.68**	**2.64**	**2.61**	**2.58**	**2.54**	**2.52**	**2.50**	**2.48**	**2.47**	**2.45**	**2.43**
20	10.02	5.71	4.46	3.90	3.59	*3.23*	**3.03**	**2.89**	**2.80**	**2.74**	**2.69**	**2.66**	**2.63**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
30	10.58	5.99	4.69	4.10	3.74	*3.36*	*3.15*	**3.00**	**2.91**	**2.84**	**2.79**	**2.75**	**2.72**	**2.68**	**2.65**	**2.62**	**2.61**	**2.60**	**2.57**	**2.55**
40	11.04	6.22	4.86	4.21	3.86	3.46	*3.23*	**3.08**	**2.98**	**2.91**	**2.85**	**2.82**	**2.78**	**2.74**	**2.71**	**2.68**	**2.67**	**2.65**	**2.62**	**2.61**
50	11.38	6.36	4.97	4.32	3.94	3.53	*3.30*	*3.13*	**3.03**	**2.96**	**2.91**	**2.86**	**2.83**	**2.79**	**2.75**	**2.73**	**2.71**	**2.69**	**2.67**	**2.65**
60	11.60	6.50	5.06	4.39	4.01	3.59	*3.36*	*3.18*	**3.08**	**3.00**	**2.95**	**2.91**	**2.87**	**2.82**	**2.79**	**2.77**	**2.74**	**2.73**	**2.70**	**2.69**
75	11.83	6.67	5.17	4.49	4.10	3.66	*3.42*	*3.25*	*3.13*	**3.06**	**3.00**	**2.95**	**2.92**	**2.87**	**2.84**	**2.81**	**2.79**	**2.77**	**2.74**	**2.72**
100	12.28	6.84	5.31	4.61	4.21	3.76	3.50	*3.32*	*3.20*	*3.12*	**3.06**	**3.02**	**2.98**	**2.93**	**2.89**	**2.86**	**2.84**	**2.83**	**2.80**	**2.78**
125	12.51	7.01	5.43	4.72	4.29	3.83	3.57	*3.37*	*3.26*	*3.18*	**3.11**	**3.06**	**3.03**	**2.97**	**2.94**	**2.91**	**2.89**	**2.87**	**2.84**	**2.82**
150	12.73	7.13	5.54	4.78	4.35	3.88	3.62	*3.42*	*3.30*	*3.22*	*3.15*	**3.11**	**3.06**	**3.01**	**2.97**	**2.94**	**2.92**	**2.90**	**2.87**	**2.85**
175	12.96	7.24	5.60	4.86	4.41	3.93	3.66	*3.46*	*3.34*	*3.25*	*3.19*	*3.13*	**3.10**	**3.04**	**3.00**	**2.97**	**2.95**	**2.93**	**2.90**	**2.88**
200	13.19	7.30	5.65	4.90	4.46	3.97	3.70	3.50	*3.37*	*3.28*	*3.22*	*3.16*	*3.13*	**3.07**	**3.03**	**3.00**	**2.98**	**2.96**	**2.92**	**2.90**

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (40 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.56	2.92	*2.48*	**2.24**	**2.12**	**1.98**	**1.89**	**1.84**	**1.80**	**1.77**	**1.75**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.68**	**1.67**	**1.67**
2	5.69	3.48	2.85	*2.57*	*2.41*	**2.22**	**2.12**	**2.05**	**2.01**	**1.97**	**1.95**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.87**	**1.86**	**1.84**	**1.83**
3	6.34	3.81	3.06	2.73	*2.57*	**2.36**	**2.24**	**2.17**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**2.00**	**1.98**	**1.96**	**1.96**	**1.95**	**1.94**	**1.93**
4	6.91	4.00	3.25	2.88	2.69	*2.45*	**2.34**	**2.24**	**2.20**	**2.16**	**2.12**	**2.10**	**2.09**	**2.07**	**2.05**	**2.03**	**2.03**	**2.02**	**2.01**	**2.00**
5	7.38	4.19	3.37	2.99	2.78	*2.52*	*2.41*	**2.31**	**2.25**	**2.22**	**2.18**	**2.16**	**2.15**	**2.12**	**2.10**	**2.09**	**2.08**	**2.07**	**2.05**	**2.04**
8	8.31	4.61	3.62	3.20	2.97	2.69	*2.55*	*2.45*	**2.38**	**2.34**	**2.30**	**2.28**	**2.27**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
12	9.06	4.94	3.91	3.39	3.13	2.83	2.69	*2.57*	*2.49*	*2.44*	*2.41*	*2.38*	**2.36**	**2.32**	**2.30**	**2.29**	**2.28**	**2.27**	**2.24**	**2.23**
16	9.62	5.22	4.05	3.53	3.25	2.93	2.76	*2.64*	*2.57*	*2.51*	*2.48*	*2.45*	*2.43*	*2.38*	**2.36**	**2.35**	**2.34**	**2.32**	**2.30**	**2.29**
20	10.00	5.41	4.19	3.65	3.34	3.02	2.83	2.71	*2.62*	*2.57*	*2.52*	*2.50*	*2.48*	*2.44*	*2.42*	*2.39*	*2.38*	2.37	2.35	2.34
30	10.75	5.78	4.42	3.86	3.51	3.16	2.96	2.82	2.73	*2.66*	*2.63*	*2.59*	*2.57*	*2.52*	*2.50*	*2.48*	*2.46*	*2.45*	*2.43*	*2.42*
40	11.50	6.06	4.61	4.00	3.62	3.25	3.04	2.90	2.80	2.73	2.69	*2.66*	*2.63*	*2.59*	*2.56*	*2.55*	*2.52*	*2.51*	*2.49*	*2.48*
50	11.88	6.25	4.75	4.09	3.72	3.32	3.11	2.96	2.86	2.79	2.75	2.71	2.69	*2.64*	*2.61*	*2.59*	*2.57*	*2.56*	*2.54*	*2.52*
60	12.25	6.34	4.84	4.19	3.81	3.39	3.17	3.02	2.91	2.84	2.79	2.76	2.72	*2.68*	*2.65*	*2.63*	*2.61*	*2.59*	*2.57*	*2.56*
75	12.62	6.53	4.98	4.28	3.91	3.46	3.24	3.06	2.97	2.90	2.84	2.80	2.77	2.72	2.70	*2.68*	*2.65*	*2.64*	*2.62*	*2.59*
100	13.00	6.81	5.17	4.42	4.00	3.55	3.32	3.16	3.04	2.97	2.91	2.86	2.83	2.78	2.76	2.73	2.71	*2.70*	*2.66*	*2.65*
125	13.38	7.00	5.31	4.54	4.09	3.65	3.39	3.20	3.10	3.02	2.96	2.92	2.89	2.83	2.80	2.77	2.76	2.73	2.71	*2.69*
150	13.75	7.19	5.41	4.61	4.19	3.70	3.44	3.25	3.13	3.06	3.00	2.96	2.92	2.88	2.84	2.80	2.79	2.77	2.75	2.72
175	14.12	7.28	5.50	4.70	4.23	3.74	3.48	3.30	3.18	3.10	3.04	2.99	2.96	2.90	2.86	2.84	2.82	2.80	2.77	2.76
200	14.31	7.38	5.59	4.75	4.30	3.79	3.53	3.34	3.21	3.13	3.06	3.02	2.98	2.93	2.90	2.86	2.84	2.82	2.79	2.78

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (40 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.69	3.48	*2.85*	**2.57**	**2.41**	**2.22**	**2.12**	**2.05**	**2.01**	**1.97**	**1.95**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.87**	**1.86**	**1.84**	**1.83**
2	6.91	4.00	3.25	*2.88*	**2.69**	**2.45**	**2.34**	**2.24**	**2.20**	**2.16**	**2.12**	**2.10**	**2.09**	**2.07**	**2.05**	**2.03**	**2.03**	**2.02**	**2.01**	**2.00**
3	7.66	4.38	3.48	3.06	*2.85*	**2.59**	**2.46**	**2.36**	**2.31**	**2.27**	**2.23**	**2.21**	**2.20**	**2.16**	**2.15**	**2.12**	**2.12**	**2.11**	**2.09**	**2.08**
4	8.31	4.61	3.62	3.20	*2.97*	**2.69**	**2.55**	**2.45**	**2.38**	**2.34**	**2.30**	**2.28**	**2.27**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
5	8.69	4.80	3.77	3.32	3.06	*2.78*	**2.62**	**2.51**	**2.44**	**2.39**	**2.36**	**2.34**	**2.31**	**2.29**	**2.27**	**2.24**	**2.23**	**2.22**	**2.21**	**2.20**
8	9.62	5.22	4.05	3.53	3.25	*2.93*	*2.76*	**2.64**	**2.57**	**2.51**	**2.48**	**2.45**	**2.43**	**2.38**	**2.36**	**2.35**	**2.34**	**2.32**	**2.30**	**2.29**
12	10.38	5.59	4.33	3.74	3.41	3.06	*2.90*	*2.76*	**2.68**	**2.62**	**2.57**	**2.55**	**2.51**	**2.48**	**2.45**	**2.43**	**2.42**	**2.41**	**2.38**	**2.37**
16	10.94	5.83	4.47	3.88	3.53	3.18	*2.98*	*2.83*	**2.75**	**2.69**	**2.64**	**2.61**	**2.58**	**2.55**	**2.51**	**2.50**	**2.48**	**2.46**	**2.44**	**2.43**
20	11.50	6.06	4.61	4.00	3.62	3.25	*3.04*	*2.90*	*2.80*	**2.73**	**2.69**	**2.66**	**2.63**	**2.59**	**2.56**	**2.55**	**2.52**	**2.51**	**2.49**	**2.48**
30	12.25	6.34	4.84	4.19	3.81	3.39	3.17	*3.02*	*2.91*	*2.84*	*2.79*	*2.76*	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.59**	**2.57**	**2.56**
40	12.62	6.62	5.03	4.33	3.91	3.48	3.25	3.09	*2.98*	*2.91*	*2.85*	*2.82*	*2.78*	**2.73**	**2.71**	**2.69**	**2.66**	**2.65**	**2.63**	**2.61**
50	13.00	6.81	5.17	4.42	4.00	3.55	3.32	3.16	*3.04*	*2.97*	*2.91*	*2.86*	*2.83*	*2.78*	*2.76*	**2.73**	**2.71**	**2.70**	**2.66**	**2.65**
60	13.38	7.00	5.27	4.52	4.09	3.62	3.38	3.20	*3.09*	*3.00*	*2.95*	*2.91*	*2.88*	*2.83*	*2.79*	*2.77*	**2.75**	**2.73**	**2.70**	**2.69**
75	13.75	7.19	5.41	4.61	4.19	3.70	3.44	3.25	3.13	*3.06*	*3.00*	*2.96*	*2.92*	*2.88*	*2.84*	*2.80*	*2.79*	*2.77*	**2.75**	**2.72**
100	14.31	7.38	5.59	4.75	4.30	3.79	3.53	3.34	3.21	3.13	*3.06*	*3.02*	*2.98*	*2.93*	*2.90*	*2.86*	*2.84*	*2.82*	*2.79*	*2.78*
125	14.88	7.56	5.69	4.84	4.38	3.88	3.60	3.39	3.27	3.18	*3.12*	*3.06*	*3.03*	*2.98*	*2.93*	*2.91*	*2.89*	*2.88*	*2.84*	*2.82*
150	15.06	7.75	5.78	4.94	4.47	3.93	3.65	3.44	3.32	3.23	3.16	*3.11*	*3.07*	*3.02*	*2.97*	*2.95*	*2.92*	*2.90*	*2.87*	*2.85*
175	15.25	7.84	5.88	5.03	4.52	3.98	3.70	3.48	3.34	3.26	3.19	3.14	*3.10*	*3.04*	*3.00*	*2.97*	*2.95*	*2.93*	*2.90*	*2.88*
200	15.62	7.94	5.97	5.08	4.56	4.02	3.74	3.52	3.39	3.30	3.23	3.17	*3.13*	*3.07*	*3.03*	*3.00*	*2.98*	*2.96*	*2.92*	*2.90*

Table 19-1. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Observations (40 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.91	4.00	3.25	**2.88**	**2.69**	**2.45**	**2.34**	**2.24**	**2.20**	**2.16**	**2.12**	**2.10**	**2.09**	**2.07**	**2.05**	**2.03**	**2.03**	**2.02**	**2.01**	**2.00**
2	8.31	4.61	3.62	*3.20*	**2.97**	**2.69**	**2.55**	**2.45**	**2.38**	**2.34**	**2.30**	**2.28**	**2.27**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
3	9.06	4.94	3.91	3.39	*3.13*	**2.83**	**2.69**	**2.57**	**2.49**	**2.44**	**2.41**	**2.38**	**2.36**	**2.32**	**2.30**	**2.29**	**2.28**	**2.27**	**2.24**	**2.23**
4	9.62	5.22	4.05	3.53	*3.25*	**2.93**	**2.76**	**2.64**	**2.57**	**2.51**	**2.48**	**2.45**	**2.43**	**2.38**	**2.36**	**2.35**	**2.34**	**2.32**	**2.30**	**2.29**
5	10.00	5.41	4.19	3.65	3.34	**3.02**	**2.83**	**2.71**	**2.62**	**2.57**	**2.52**	**2.50**	**2.48**	**2.44**	**2.42**	**2.39**	**2.38**	**2.37**	**2.35**	**2.34**
8	10.94	5.83	4.47	3.88	3.53	*3.18*	**2.98**	**2.83**	**2.75**	**2.69**	**2.64**	**2.61**	**2.58**	**2.55**	**2.51**	**2.50**	**2.48**	**2.46**	**2.44**	**2.43**
12	11.69	6.16	4.75	4.07	3.72	*3.32*	**3.11**	**2.95**	**2.85**	**2.78**	**2.73**	**2.70**	**2.68**	**2.63**	**2.61**	**2.58**	**2.56**	**2.55**	**2.50**	**2.51**
16	12.25	6.44	4.89	4.21	3.84	3.41	*3.18*	**3.03**	**2.92**	**2.85**	**2.80**	**2.77**	**2.73**	**2.69**	**2.66**	**2.64**	**2.62**	**2.61**	**2.58**	**2.57**
20	12.62	6.62	5.03	4.33	3.91	3.48	*3.25*	**3.09**	**2.98**	**2.91**	**2.85**	**2.82**	**2.78**	**2.73**	**2.71**	**2.69**	**2.66**	**2.65**	**2.63**	**2.61**
30	13.38	7.00	5.27	4.52	4.09	3.62	*3.38*	*3.20*	**3.09**	**3.00**	**2.95**	**2.91**	**2.88**	**2.83**	**2.79**	**2.77**	**2.75**	**2.73**	**2.70**	**2.69**
40	13.94	7.19	5.45	4.66	4.21	3.72	3.46	*3.27*	*3.16*	**3.07**	**3.02**	**2.97**	**2.93**	**2.89**	**2.85**	**2.83**	**2.80**	**2.78**	**2.76**	**2.73**
50	14.31	7.38	5.59	4.75	4.30	3.79	3.53	*3.34*	*3.21*	*3.13*	**3.06**	**3.02**	**2.98**	**2.93**	**2.90**	**2.86**	**2.84**	**2.82**	**2.79**	**2.78**
60	14.69	7.56	5.69	4.84	4.38	3.86	3.58	*3.39*	*3.25*	*3.18*	**3.11**	**3.06**	**3.03**	**2.97**	**2.92**	**2.90**	**2.88**	**2.86**	**2.83**	**2.82**
75	15.06	7.75	5.78	4.94	4.47	3.93	3.65	*3.44*	*3.32*	*3.23*	*3.16*	**3.11**	**3.07**	**3.02**	**2.97**	**2.95**	**2.92**	**2.90**	**2.87**	**2.85**
100	15.62	7.94	5.97	5.08	4.56	4.02	3.74	3.52	*3.39*	*3.30*	*3.23*	*3.17*	*3.13*	**3.07**	**3.03**	**3.00**	**2.98**	**2.96**	**2.92**	**2.90**
125	16.00	8.12	6.11	5.17	4.66	4.09	3.81	3.58	*3.44*	*3.34*	*3.27*	*3.21*	*3.18*	*3.12*	**3.07**	**3.04**	**3.02**	**3.00**	**2.97**	**2.95**
150	16.19	8.31	6.20	5.27	4.75	4.16	3.86	3.62	*3.48*	*3.39*	*3.31*	*3.26*	*3.21*	*3.16*	**3.11**	**3.07**	**3.05**	**3.03**	**2.99**	**2.97**
175	16.38	8.41	6.25	5.34	4.80	4.21	3.91	3.67	3.52	*3.41*	*3.34*	*3.30*	*3.25*	*3.18*	*3.13*	**3.11**	**3.07**	**3.06**	**3.03**	**3.00**
200	16.75	8.50	6.34	5.41	4.84	4.26	3.93	3.70	3.55	*3.45*	*3.38*	*3.32*	*3.27*	*3.21*	*3.17*	*3.13*	**3.11**	**3.09**	**3.05**	**3.03**

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.33	0.25	0.21	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10
2	0.67	0.54	0.49	0.45	0.43	0.41	0.39	0.38	0.37	0.36	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34
3	0.87	0.71	0.64	0.60	0.58	0.54	0.53	0.51	0.50	0.49	0.49	0.48	0.48	0.48	0.47	0.47	0.47	0.47	0.46	0.46
4	1.01	0.82	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
5	1.12	0.91	0.83	0.78	0.75	0.71	0.69	0.67	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61
8	1.34	1.09	0.99	0.93	0.90	0.85	0.82	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74	0.73
12	1.54	1.25	1.13	1.06	1.02	0.97	0.94	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.85	0.85	0.85	0.84	0.84	0.84
16	1.68	1.36	1.22	1.15	1.11	1.05	1.02	0.99	0.97	0.96	0.95	0.94	0.94	0.93	0.92	0.92	0.92	0.91	0.91	0.91
20	1.78	1.44	1.30	1.22	1.17	1.11	1.08	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96
30	1.97	1.58	1.43	1.34	1.29	1.22	1.18	1.15	1.13	1.12	1.10	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.05	1.05
40	2.10	1.69	1.52	1.43	1.37	1.30	1.25	1.22	1.20	1.18	1.17	1.16	1.16	1.14	1.14	1.13	1.13	1.12	1.12	1.11
50	2.21	1.76	1.59	1.49	1.43	1.35	1.31	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.18	1.17	1.17	1.16	1.16
60	2.29	1.83	1.64	1.54	1.48	1.40	1.35	1.32	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.21	1.20	1.20
75	2.39	1.90	1.71	1.60	1.54	1.46	1.41	1.37	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.25	1.24
100	2.51	2.00	1.80	1.68	1.61	1.53	1.48	1.44	1.41	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.32	1.31	1.30	1.30
125	2.61	2.07	1.86	1.74	1.67	1.58	1.53	1.49	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.36	1.35	1.34
150	2.68	2.13	1.91	1.79	1.72	1.62	1.57	1.53	1.50	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.40	1.39	1.38	1.38
175	2.75	*2.18*	1.96	1.83	1.76	1.66	1.60	1.56	1.53	1.51	1.49	1.48	1.47	1.45	1.44	1.43	1.43	1.42	1.41	1.41
200	2.80	*2.22*	1.99	1.87	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.51	1.50	1.48	1.47	1.46	1.45	1.45	1.44	1.43

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.67	0.54	0.49	0.45	0.43	0.41	0.39	0.38	0.37	0.36	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34
2	1.01	0.82	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
3	1.21	0.98	0.89	0.84	0.80	0.76	0.74	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.67	0.66	0.66	0.66
4	1.34	1.09	0.99	0.93	0.90	0.85	0.82	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74	0.73
5	1.45	1.18	1.07	1.00	0.96	0.92	0.89	0.87	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79	0.79
8	1.68	1.36	1.22	1.15	1.11	1.05	1.02	0.99	0.97	0.96	0.95	0.94	0.94	0.93	0.92	0.92	0.92	0.91	0.91	0.91
12	1.87	1.50	1.36	1.28	1.22	1.16	1.12	1.09	1.08	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.01	1.01	1.00	1.00
16	2.00	1.61	1.45	1.36	1.31	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07	1.06
20	2.10	1.69	1.52	1.43	1.37	1.30	1.25	1.22	1.20	1.18	1.17	1.16	1.16	1.14	1.14	1.13	1.13	1.12	1.12	1.11
30	2.29	1.83	1.64	1.54	1.48	1.40	1.35	1.32	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.21	1.20	1.20
40	2.41	1.92	1.73	1.62	1.55	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.31	1.29	1.28	1.28	1.27	1.27	1.26	1.25
50	2.51	2.00	1.80	1.68	1.61	1.53	1.48	1.44	1.41	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.32	1.31	1.30	1.30
60	2.59	2.06	1.85	1.73	1.66	1.57	1.52	1.48	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.33
75	2.68	2.13	1.91	1.79	1.72	1.62	1.57	1.53	1.50	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.40	1.39	1.38	1.38
100	2.80	2.22	1.99	1.87	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.51	1.50	1.48	1.47	1.46	1.45	1.45	1.44	1.43
125	2.90	2.29	2.06	1.93	1.84	1.74	1.68	1.64	1.60	1.58	1.56	1.55	1.54	1.52	1.51	1.50	1.50	1.49	1.48	1.47
150	2.97	2.35	2.11	1.97	1.89	1.79	1.72	1.67	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.51
175	3.03	2.40	2.15	2.01	1.93	1.82	1.76	1.71	1.67	1.65	1.63	1.62	1.60	1.59	1.57	1.57	1.56	1.55	1.54	1.53
200	3.08	2.44	2.18	2.05	1.96	1.85	1.79	1.73	1.70	1.68	1.66	1.64	1.63	1.61	1.60	1.59	1.58	1.58	1.57	1.56

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.01	0.82	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
2	1.34	1.09	0.99	0.93	0.90	0.85	0.82	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74	0.73
3	1.54	1.25	1.13	1.06	1.02	0.97	0.94	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.85	0.85	0.85	0.84	0.84	0.84
4	1.68	1.36	1.22	1.15	1.11	1.05	1.02	0.99	0.97	0.96	0.95	0.94	0.94	0.93	0.92	0.92	0.92	0.91	0.91	0.91
5	1.78	1.44	1.30	1.22	1.17	1.11	1.08	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96
8	2.00	1.61	1.45	1.36	1.31	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07	1.06
12	2.19	1.75	1.57	1.48	1.42	1.34	1.30	1.26	1.24	1.23	1.21	1.20	1.20	1.18	1.18	1.17	1.17	1.16	1.15	1.15
16	2.32	1.85	1.66	1.56	1.50	1.42	1.37	1.33	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.23	1.22	1.21	1.21
20	2.41	1.92	1.73	1.62	1.55	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.31	1.29	1.28	1.28	1.27	1.27	1.26	1.25
30	2.59	2.06	1.85	1.73	1.66	1.57	1.52	1.48	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.33
40	2.71	2.15	1.93	1.81	1.73	1.64	1.58	1.54	1.51	1.49	1.47	1.46	1.45	1.44	1.42	1.42	1.41	1.41	1.40	1.39
50	2.80	2.22	1.99	1.87	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.51	1.50	1.48	1.47	1.46	1.45	1.45	1.44	1.43
60	2.88	2.28	2.04	1.92	1.83	1.73	1.67	1.63	1.60	1.57	1.56	1.54	1.53	1.52	1.50	1.49	1.49	1.48	1.47	1.47
75	2.97	2.35	2.11	1.97	1.89	1.79	1.72	1.67	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.51
100	3.08	2.44	2.18	2.05	1.96	1.85	1.79	1.73	1.70	1.68	1.66	1.64	1.63	1.61	1.60	1.59	1.58	1.58	1.57	1.56
125	3.17	2.50	2.24	2.10	2.01	1.90	1.83	1.78	1.74	1.72	1.70	1.68	1.67	1.65	1.64	1.63	1.62	1.62	1.61	1.60
150	3.24	2.56	2.29	2.15	2.05	1.94	1.87	1.82	1.78	1.75	1.73	1.72	1.71	1.69	1.67	1.66	1.66	1.65	1.64	1.63
175	3.30	2.60	2.33	2.18	2.09	1.97	1.90	1.85	1.81	1.78	1.76	1.75	1.73	1.72	1.70	1.69	1.68	1.68	1.66	1.66
200	3.35	2.64	2.37	2.22	2.12	2.00	1.93	1.87	1.84	1.81	1.79	1.77	1.76	1.74	1.73	1.71	1.71	1.70	1.69	1.68

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (2 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.71	0.57	0.50	0.46	0.44	0.41	0.39	0.38	0.37	0.37	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34
2	1.08	0.86	0.77	0.72	0.69	0.65	0.62	0.61	0.59	0.59	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
3	1.30	1.03	0.92	0.86	0.82	0.77	0.75	0.73	0.71	0.70	0.70	0.69	0.69	0.68	0.67	0.67	0.67	0.67	0.66	0.66
4	1.47	1.15	1.02	0.96	0.91	0.86	0.83	0.81	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75	0.74	0.74	0.74	0.74
5	1.59	1.24	1.11	1.03	0.99	0.93	0.90	0.87	0.86	0.84	0.84	0.83	0.82	0.82	0.81	0.81	0.80	0.80	0.79	0.79
8	1.86	1.44	1.27	1.19	1.13	1.07	1.03	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.93	0.92	0.92	0.92	0.91	0.91
12	2.09	1.60	1.42	1.32	1.25	1.18	1.14	1.11	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.01	1.00	1.00
16	2.26	1.72	1.52	1.41	1.34	1.26	1.21	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.08	1.07	1.06
20	2.39	1.81	1.59	1.48	1.41	1.32	1.27	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.13	1.12	1.12	1.11
30	2.62	1.97	1.73	1.60	1.52	1.43	1.38	1.33	1.31	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.20	1.20
40	2.78	2.08	1.83	1.69	1.61	1.50	1.45	1.40	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.27	1.26	1.26
50	2.91	2.17	1.90	1.76	1.67	1.56	1.50	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.32	1.31	1.30
60	3.01	*2.24*	1.96	1.81	1.72	1.61	1.55	1.50	1.46	1.44	1.42	1.41	1.40	1.38	1.37	1.36	1.36	1.35	1.34	1.34
75	3.13	2.33	2.03	1.88	1.78	1.67	1.60	1.55	1.51	1.49	1.47	1.46	1.45	1.43	1.42	1.41	1.40	1.40	1.39	1.38
100	3.29	2.44	2.13	1.96	1.86	1.74	1.67	1.61	1.58	1.55	1.53	1.52	1.50	1.49	1.47	1.46	1.46	1.45	1.44	1.43
125	3.41	2.52	*2.20*	2.03	1.92	1.79	1.72	1.66	1.62	1.60	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.48
150	3.50	2.59	*2.25*	2.08	1.97	1.84	1.76	1.70	1.66	1.63	1.61	1.60	1.59	1.57	1.55	1.54	1.53	1.53	1.52	1.51
175	3.58	2.64	*2.30*	2.12	2.01	1.87	1.80	1.74	1.69	1.67	1.64	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
200	3.65	2.69	2.34	*2.16*	2.04	1.91	1.83	1.76	1.72	1.69	1.67	1.65	1.64	1.62	1.61	1.60	1.59	1.58	1.57	1.56

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (2 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.08	0.86	0.77	0.72	0.69	0.65	0.62	0.61	0.59	0.59	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
2	1.47	1.15	1.02	0.96	0.91	0.86	0.83	0.81	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75	0.74	0.74	0.74	0.74
3	1.70	1.32	1.17	1.09	1.04	0.98	0.95	0.92	0.91	0.89	0.88	0.88	0.87	0.86	0.86	0.85	0.85	0.85	0.84	0.84
4	1.86	1.44	1.27	1.19	1.13	1.07	1.03	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.93	0.92	0.92	0.92	0.91	0.91
5	1.99	1.53	1.35	1.26	1.20	1.13	1.09	1.06	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
8	2.26	1.72	1.52	1.41	1.34	1.26	1.21	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.08	1.07	1.06
12	2.49	1.88	1.65	1.53	1.46	1.37	1.32	1.28	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17	1.17	1.16	1.16	1.15
16	2.66	2.00	1.75	1.62	1.54	1.45	1.39	1.35	1.32	1.30	1.29	1.27	1.27	1.25	1.24	1.23	1.23	1.22	1.22	1.21
20	2.78	2.08	1.83	1.69	1.61	1.50	1.45	1.40	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.27	1.26	1.26
30	3.01	2.24	1.96	1.81	1.72	1.61	1.55	1.50	1.46	1.44	1.42	1.41	1.40	1.38	1.37	1.36	1.36	1.35	1.34	1.34
40	3.17	2.35	2.05	1.90	1.80	1.68	1.61	1.56	1.53	1.50	1.48	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.39
50	3.29	2.44	2.13	1.96	1.86	1.74	1.67	1.61	1.58	1.55	1.53	1.52	1.50	1.49	1.47	1.46	1.46	1.45	1.44	1.43
60	3.38	*2.50*	2.18	2.01	1.91	1.78	1.71	1.65	1.61	1.59	1.57	1.55	1.54	1.52	1.51	1.50	1.49	1.49	1.48	1.47
75	3.50	2.59	2.25	2.08	1.97	1.84	1.76	1.70	1.66	1.63	1.61	1.60	1.59	1.57	1.55	1.54	1.53	1.53	1.52	1.51
100	3.65	2.69	2.34	2.16	2.04	1.91	1.83	1.76	1.72	1.69	1.67	1.65	1.64	1.62	1.61	1.60	1.59	1.58	1.57	1.56
125	3.77	2.77	2.41	2.22	2.10	1.96	1.88	1.81	1.77	1.74	1.72	1.70	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.60
150	3.86	2.84	*2.47*	2.27	2.15	2.00	1.92	1.85	1.81	1.77	1.75	1.73	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
175	3.94	2.89	*2.51*	2.31	2.19	2.04	1.95	1.88	1.84	1.81	1.78	1.76	1.75	1.73	1.71	1.70	1.69	1.68	1.67	1.66
200	4.00	2.94	*2.55*	2.35	2.22	2.07	1.98	1.91	1.86	1.83	1.81	1.79	1.77	1.75	1.73	1.72	1.71	1.70	1.69	1.68

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (2 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.47	1.15	1.02	0.96	0.91	0.86	0.83	0.81	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75	0.74	0.74	0.74	0.74
2	1.86	1.44	1.27	1.19	1.13	1.07	1.03	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.93	0.92	0.92	0.92	0.91	0.91
3	2.09	1.60	1.42	1.32	1.25	1.18	1.14	1.11	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.01	1.00	1.00
4	2.26	1.72	1.52	1.41	1.34	1.26	1.21	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.08	1.07	1.06
5	2.39	1.81	1.59	1.48	1.41	1.32	1.27	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.13	1.12	1.12	1.11
8	2.66	2.00	1.75	1.62	1.54	1.45	1.39	1.35	1.32	1.30	1.29	1.27	1.27	1.25	1.24	1.23	1.23	1.22	1.22	1.21
12	2.88	2.16	1.89	1.75	1.66	1.55	1.49	1.44	1.41	1.39	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.29
16	3.04	2.27	1.98	1.83	1.74	1.63	1.56	1.51	1.48	1.45	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.35
20	3.17	2.35	2.05	1.90	1.80	1.68	1.61	1.56	1.53	1.50	1.48	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.39
30	3.38	2.50	2.18	2.01	1.91	1.78	1.71	1.65	1.61	1.59	1.57	1.55	1.54	1.52	1.51	1.50	1.49	1.49	1.48	1.47
40	3.54	2.61	2.27	2.10	1.98	1.85	1.78	1.72	1.68	1.65	1.63	1.61	1.60	1.58	1.56	1.55	1.55	1.54	1.53	1.52
50	3.65	2.69	2.34	2.16	2.04	1.91	1.83	1.76	1.72	1.69	1.67	1.65	1.64	1.62	1.61	1.60	1.59	1.58	1.57	1.56
60	3.75	2.76	2.40	2.21	2.09	1.95	1.87	1.80	1.76	1.73	1.71	1.69	1.68	1.66	1.64	1.63	1.62	1.61	1.60	1.59
75	3.86	*2.84*	2.47	2.27	2.15	2.00	1.92	1.85	1.81	1.77	1.75	1.73	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
100	4.00	2.94	2.55	2.35	2.22	2.07	1.98	1.91	1.86	1.83	1.81	1.79	1.77	1.75	1.73	1.72	1.71	1.70	1.69	1.68
125	4.11	3.02	2.62	2.41	2.28	2.12	2.03	1.96	1.91	1.88	1.85	1.83	1.81	1.79	1.77	1.76	1.75	1.74	1.73	1.72
150	4.20	3.08	2.67	2.46	2.32	2.16	2.07	1.99	1.95	1.91	1.88	1.86	1.85	1.82	1.81	1.79	1.78	1.77	1.76	1.75
175	4.28	3.13	2.71	2.50	2.36	2.20	2.10	2.03	1.98	1.94	1.91	1.89	1.88	1.85	1.83	1.82	1.81	1.80	1.79	1.78
200	4.34	3.17	2.75	2.53	2.39	2.23	2.13	2.05	2.00	1.97	1.94	1.92	1.90	1.87	1.86	1.84	1.83	1.82	1.81	1.80

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (5 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.27	0.98	0.87	0.81	0.77	0.72	0.70	0.68	0.66	0.65	0.65	0.64	0.64	0.63	0.62	0.62	0.62	0.62	0.61	0.61
2	1.71	1.29	1.13	1.05	1.00	0.94	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
3	1.99	1.47	1.28	1.19	1.13	1.06	1.02	0.99	0.97	0.95	0.94	0.94	0.93	0.92	0.91	0.91	0.90	0.90	0.90	0.89
4	2.19	1.60	1.39	1.29	1.22	1.14	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
5	2.35	1.70	1.48	1.36	1.29	1.21	1.16	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.01	1.01
8	2.70	1.92	1.65	1.52	1.43	1.34	1.28	1.24	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
12	3.00	2.11	1.81	1.65	1.56	1.45	1.39	1.34	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.20	1.20
16	3.22	*2.24*	1.91	1.75	1.65	1.53	1.46	1.41	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
20	3.39	2.35	2.00	1.82	1.71	1.59	1.52	1.47	1.43	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
30	3.70	2.54	2.15	1.95	1.83	1.70	1.62	1.56	1.52	1.50	1.48	1.46	1.45	1.43	1.42	1.41	1.40	1.40	1.39	1.38
40	3.92	2.67	*2.25*	2.05	1.92	1.77	1.69	1.63	1.59	1.56	1.54	1.52	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44
50	4.08	2.77	*2.34*	2.12	1.98	1.83	1.75	1.68	1.64	1.61	1.59	1.57	1.56	1.54	1.52	1.51	1.50	1.50	1.48	1.48
60	4.22	2.86	2.40	*2.18*	2.04	1.88	1.79	1.72	1.68	1.65	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51
75	4.38	2.96	2.48	*2.25*	2.10	1.94	1.84	1.77	1.73	1.69	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	1.56	1.55
100	4.59	3.09	2.59	*2.34*	*2.18*	2.01	1.91	1.84	1.79	1.75	1.73	1.71	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60
125	4.75	3.19	2.67	2.41	*2.25*	2.07	1.96	1.88	1.83	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.65	1.64
150	4.88	3.27	2.73	2.46	*2.30*	2.11	2.01	1.92	1.87	1.83	1.81	1.79	1.77	1.74	1.73	1.71	1.70	1.70	1.68	1.67
175	4.99	3.33	2.79	2.51	*2.34*	*2.15*	2.04	1.96	1.90	1.87	1.84	1.82	1.80	1.77	1.76	1.74	1.73	1.72	1.71	1.70
200	5.09	3.39	2.83	2.55	*2.38*	*2.18*	2.07	1.99	1.93	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.73	1.72

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (5 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.71	1.29	1.13	1.05	1.00	0.94	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
2	2.19	1.60	1.39	1.29	1.22	1.14	1.10	1.06	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
3	2.49	1.79	1.55	1.42	1.35	1.26	1.21	1.17	1.15	1.13	1.11	1.10	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.05
4	2.70	1.92	1.65	1.52	1.43	1.34	1.28	1.24	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
5	2.87	2.03	1.74	1.59	1.50	1.40	1.34	1.30	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.18	1.17	1.17	1.16
8	3.22	2.25	1.91	1.75	1.65	1.53	1.46	1.41	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
12	3.53	2.43	2.07	1.88	1.77	1.64	1.56	1.51	1.47	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.34
16	3.75	*2.57*	2.17	1.97	1.85	1.71	1.64	1.58	1.54	1.51	1.49	1.48	1.46	1.45	1.43	1.42	1.42	1.41	1.40	1.39
20	3.92	2.67	2.25	2.05	1.92	1.77	1.69	1.63	1.59	1.56	1.54	1.52	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44
30	4.22	2.86	2.40	2.18	2.04	1.88	1.79	1.72	1.68	1.65	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51
40	4.43	2.99	*2.51*	2.27	2.12	1.95	1.86	1.79	1.74	1.71	1.68	1.66	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
50	4.59	3.09	*2.59*	2.34	2.19	2.01	1.91	1.84	1.79	1.75	1.73	1.71	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60
60	4.73	3.17	*2.65*	2.39	2.24	2.06	1.95	1.88	1.82	1.79	1.76	1.74	1.73	1.70	1.69	1.67	1.66	1.66	1.64	1.63
75	4.88	3.27	2.73	*2.46*	2.30	2.11	2.01	1.92	1.87	1.83	1.81	1.79	1.77	1.74	1.73	1.71	1.70	1.70	1.68	1.67
100	5.09	3.39	2.83	*2.55*	2.38	2.18	2.07	1.99	1.93	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.73	1.72
125	5.24	3.49	2.91	*2.62*	2.44	2.24	2.12	2.03	1.98	1.94	1.91	1.88	1.87	1.84	1.82	1.80	1.79	1.79	1.77	1.76
150	5.37	3.56	2.97	*2.67*	*2.49*	2.28	2.16	2.07	2.01	1.97	1.94	1.92	1.90	1.87	1.85	1.84	1.82	1.82	1.80	1.79
175	5.47	3.63	3.02	2.72	*2.53*	2.32	2.20	2.11	2.05	2.00	1.97	1.95	1.93	1.90	1.88	1.86	1.85	1.84	1.83	1.81
200	5.56	3.69	3.07	2.76	*2.57*	2.35	2.23	2.13	2.07	2.03	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.86	1.85	1.84

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (5 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.19	1.60	1.39	1.29	1.22	1.14	1.10	1.06	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
2	2.70	1.92	1.65	1.52	1.43	1.34	1.28	1.24	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
3	3.00	2.11	1.81	1.65	1.56	1.45	1.39	1.34	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.20	1.20
4	3.22	2.25	1.91	1.75	1.65	1.53	1.46	1.41	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
5	3.39	2.35	2.00	1.82	1.71	1.59	1.52	1.47	1.43	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
8	3.75	2.57	2.17	1.97	1.85	1.71	1.64	1.58	1.54	1.51	1.49	1.48	1.46	1.45	1.43	1.42	1.42	1.41	1.40	1.39
12	4.05	2.75	2.32	2.10	1.97	1.82	1.74	1.67	1.63	1.60	1.58	1.56	1.55	1.53	1.51	1.50	1.50	1.49	1.48	1.47
16	4.27	2.89	2.43	2.20	2.06	1.90	1.81	1.74	1.69	1.66	1.64	1.62	1.60	1.58	1.57	1.56	1.55	1.54	1.53	1.52
20	4.43	2.99	2.51	2.27	2.12	1.95	1.86	1.79	1.74	1.71	1.68	1.66	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
30	4.73	3.17	2.65	2.39	2.24	2.06	1.95	1.88	1.82	1.79	1.76	1.74	1.73	1.70	1.69	1.67	1.66	1.66	1.64	1.63
40	4.93	3.30	2.75	2.48	2.32	2.13	2.02	1.94	1.89	1.85	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.69	1.68
50	5.09	3.39	*2.83*	2.55	2.38	2.18	2.07	1.99	1.93	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.73	1.72
60	5.21	3.47	*2.90*	2.61	2.43	2.23	2.11	2.03	1.97	1.93	1.90	1.88	1.86	1.83	1.81	1.80	1.79	1.78	1.76	1.75
75	5.37	3.56	2.97	2.67	2.49	2.28	2.16	2.07	2.01	1.97	1.94	1.92	1.90	1.87	1.85	1.84	1.82	1.82	1.80	1.79
100	5.56	3.69	3.07	2.76	2.57	2.35	2.23	2.13	2.07	2.03	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.86	1.85	1.84
125	5.71	3.78	3.14	*2.82*	2.63	2.41	2.28	2.18	2.12	2.07	2.04	2.01	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
150	5.83	3.85	3.20	*2.88*	2.68	2.45	2.32	2.22	2.15	2.11	2.07	2.05	2.03	1.99	1.97	1.95	1.94	1.93	1.91	1.90
175	5.93	3.92	3.25	*2.92*	2.72	2.49	2.35	2.25	2.18	2.14	2.10	2.07	2.05	2.02	2.00	1.98	1.97	1.96	1.94	1.93
200	6.02	3.97	3.30	*2.96*	2.76	2.52	2.38	2.28	2.21	2.16	2.13	2.10	2.08	2.04	2.02	2.00	1.99	1.98	1.96	1.95

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (10 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.76	1.31	1.14	1.06	1.00	0.94	0.91	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
2	2.29	1.63	1.41	1.30	1.23	1.15	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
3	2.62	1.83	1.57	1.44	1.35	1.26	1.21	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.05
4	2.87	1.97	1.68	1.53	1.45	1.35	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
5	3.07	2.08	1.77	1.61	1.52	1.41	1.35	1.30	1.27	1.25	1.24	1.22	1.22	1.20	1.19	1.18	1.18	1.17	1.17	1.16
8	3.50	2.33	1.95	1.77	1.66	1.54	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
12	3.88	2.54	2.12	1.91	1.79	1.65	1.57	1.51	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
16	4.15	2.69	*2.23*	2.01	1.88	1.73	1.65	1.58	1.54	1.51	1.49	1.48	1.47	1.45	1.43	1.42	1.42	1.41	1.40	1.39
20	4.36	2.81	*2.32*	2.08	1.95	1.79	1.70	1.63	1.59	1.56	1.54	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
30	4.75	3.02	2.49	*2.22*	2.07	1.90	1.80	1.73	1.68	1.65	1.63	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51
40	5.02	3.17	2.60	*2.32*	*2.16*	1.97	1.87	1.79	1.75	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
50	5.22	3.29	2.69	2.40	*2.23*	2.03	1.93	1.84	1.79	1.76	1.73	1.71	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60
60	5.39	3.38	2.76	2.46	*2.28*	2.08	1.97	1.89	1.83	1.79	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.63
75	5.61	3.50	2.85	2.53	*2.35*	*2.14*	2.02	1.94	1.88	1.84	1.81	1.79	1.77	1.75	1.73	1.72	1.71	1.70	1.68	1.67
100	5.86	3.64	2.96	2.63	2.44	*2.21*	2.09	2.00	1.94	1.90	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
125	6.06	3.76	3.05	2.71	2.50	*2.27*	*2.14*	2.05	1.99	1.94	1.91	1.89	1.87	1.84	1.82	1.81	1.80	1.79	1.77	1.76
150	6.23	3.85	3.12	2.77	2.56	*2.32*	*2.19*	2.09	2.03	1.98	1.95	1.92	1.90	1.87	1.85	1.84	1.83	1.82	1.80	1.79
175	6.37	3.93	3.18	2.82	2.60	*2.36*	*2.22*	*2.12*	2.06	2.01	1.98	1.95	1.93	1.90	1.88	1.87	1.85	1.84	1.83	1.82
200	6.48	3.99	3.23	2.86	2.64	*2.39*	*2.26*	*2.15*	2.08	2.04	2.00	1.98	1.96	1.93	1.90	1.89	1.88	1.87	1.85	1.84

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (10 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.29	1.63	1.41	1.29	1.23	1.15	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
2	2.87	1.97	1.68	1.53	1.45	1.35	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
3	3.23	2.18	1.84	1.67	1.57	1.46	1.39	1.35	1.32	1.29	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.20	1.20
4	3.50	2.32	1.95	1.77	1.66	1.54	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
5	3.70	2.44	2.04	1.85	1.73	1.60	1.53	1.47	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
8	4.14	2.69	2.23	2.01	1.88	1.73	1.65	1.58	1.54	1.51	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
12	4.53	2.90	2.39	2.15	2.00	1.84	1.75	1.68	1.63	1.60	1.58	1.56	1.55	1.53	1.51	1.50	1.50	1.49	1.48	1.47
16	4.80	3.06	*2.51*	2.25	2.09	1.91	1.82	1.74	1.70	1.66	1.64	1.62	1.61	1.58	1.57	1.56	1.55	1.54	1.53	1.52
20	5.02	3.17	*2.60*	2.32	2.16	1.97	1.87	1.79	1.75	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
30	5.39	3.38	2.76	*2.46*	2.28	2.08	1.97	1.88	1.83	1.79	1.77	1.75	1.73	1.71	1.69	1.67	1.67	1.66	1.64	1.63
40	5.66	3.53	2.88	*2.56*	2.37	2.16	2.04	1.95	1.89	1.85	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.69	1.68
50	5.86	3.64	2.96	*2.63*	2.44	2.21	2.09	2.00	1.94	1.90	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
60	6.04	3.74	3.03	*2.69*	*2.49*	2.26	2.14	2.04	1.98	1.94	1.90	1.88	1.86	1.83	1.81	1.80	1.79	1.78	1.76	1.75
75	6.23	3.85	3.12	2.76	*2.56*	2.32	2.19	2.09	2.02	1.98	1.95	1.92	1.90	1.88	1.85	1.84	1.83	1.82	1.80	1.79
100	6.48	3.99	3.23	2.86	*2.64*	2.39	2.26	2.15	2.08	2.04	2.00	1.98	1.96	1.93	1.90	1.89	1.88	1.87	1.85	1.84
125	6.68	4.10	3.32	2.93	*2.71*	*2.45*	2.31	2.20	2.13	2.08	2.05	2.02	2.00	1.97	1.94	1.93	1.91	1.90	1.88	1.87
150	6.84	4.19	3.38	2.99	2.76	*2.50*	2.35	2.24	2.17	2.12	2.08	2.05	2.03	2.00	1.97	1.96	1.94	1.93	1.91	1.90
175	6.97	4.27	3.44	3.04	2.80	*2.53*	2.39	2.27	2.20	2.15	2.11	2.08	2.06	2.02	2.00	1.98	1.97	1.96	1.94	1.93
200	7.07	4.34	3.49	3.08	2.84	*2.57*	2.41	2.30	2.22	2.17	2.13	2.10	2.08	2.05	2.02	2.00	1.99	1.98	1.96	1.95

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (10 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.87	1.97	1.68	1.53	1.45	1.35	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
2	3.50	2.32	1.95	1.77	1.66	1.54	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
3	3.88	2.54	2.11	1.91	1.79	1.65	1.57	1.51	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
4	4.14	2.69	2.23	2.01	1.88	1.73	1.65	1.58	1.54	1.51	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
5	4.36	*2.81*	2.32	2.08	1.95	1.79	1.70	1.64	1.59	1.56	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
8	4.80	3.06	2.51	2.25	2.09	1.91	1.82	1.74	1.70	1.66	1.64	1.62	1.61	1.58	1.57	1.56	1.55	1.54	1.53	1.52
12	5.19	3.27	2.67	2.38	2.21	2.02	1.92	1.84	1.78	1.75	1.72	1.70	1.69	1.66	1.65	1.63	1.63	1.62	1.60	1.60
16	5.45	3.42	*2.79*	2.48	2.30	2.10	1.98	1.90	1.85	1.81	1.78	1.76	1.74	1.72	1.70	1.69	1.68	1.67	1.66	1.65
20	5.66	3.53	*2.88*	2.56	2.37	2.16	2.04	1.95	1.89	1.85	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.69	1.68
30	6.04	3.74	3.03	2.69	2.49	2.26	2.14	2.04	1.98	1.94	1.90	1.88	1.86	1.83	1.81	1.80	1.79	1.78	1.76	1.75
40	6.29	3.88	3.14	*2.79*	2.58	2.34	2.20	2.10	2.04	1.99	1.96	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80
50	6.48	3.99	3.23	*2.86*	2.64	2.39	2.26	2.15	2.08	2.04	2.00	1.98	1.96	1.93	1.90	1.89	1.88	1.87	1.85	1.84
60	6.64	4.08	3.30	*2.92*	2.70	2.44	2.30	2.19	2.12	2.07	2.04	2.01	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
75	6.84	4.19	3.38	*2.99*	2.76	2.50	2.35	2.24	2.17	2.12	2.08	2.05	2.03	2.00	1.97	1.96	1.94	1.93	1.91	1.90
100	7.07	4.34	3.49	3.08	*2.84*	2.57	2.41	2.30	2.22	2.17	2.13	2.10	2.08	2.05	2.02	2.00	1.99	1.98	1.96	1.95
125	7.27	4.44	3.57	3.15	*2.91*	2.62	2.47	2.35	2.27	2.22	2.18	2.15	2.12	2.08	2.06	2.04	2.03	2.02	2.00	1.98
150	7.42	4.52	3.64	3.21	*2.96*	2.67	2.51	2.39	2.30	2.25	2.21	2.18	2.15	2.12	2.09	2.07	2.06	2.05	2.02	2.01
175	7.54	4.60	3.70	3.26	*3.00*	2.71	2.54	2.42	2.34	2.28	2.24	2.21	2.18	2.14	2.12	2.10	2.08	2.07	2.05	2.03
200	7.66	4.66	3.75	3.30	*3.04*	2.74	2.57	2.44	2.36	2.30	2.26	2.23	2.20	2.17	2.14	2.12	2.10	2.09	2.07	2.05

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (20 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.34	1.65	1.42	1.30	1.23	1.15	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
2	2.99	2.00	1.69	1.54	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
3	3.40	2.22	1.86	1.68	1.58	1.46	1.40	1.35	1.32	1.29	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.21	1.20
4	3.71	2.38	1.98	1.78	1.67	1.54	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
5	3.95	2.50	2.07	1.87	1.74	1.61	1.53	1.47	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
8	4.48	2.77	2.27	2.03	1.89	1.73	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
12	4.96	3.01	2.44	2.18	2.02	1.85	1.75	1.68	1.64	1.60	1.58	1.56	1.55	1.53	1.52	1.50	1.50	1.49	1.48	1.47
16	5.29	3.18	2.57	2.28	2.11	1.92	1.82	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
20	5.57	3.32	2.67	2.36	2.18	1.98	1.88	1.80	1.75	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
30	6.04	3.55	2.84	2.50	2.31	2.09	1.98	1.89	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
40	6.39	3.73	2.97	2.61	2.40	2.17	2.05	1.96	1.90	1.86	1.83	1.80	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.68
50	6.64	3.87	3.07	2.69	2.47	2.23	2.10	2.01	1.95	1.90	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
60	6.86	3.97	3.14	2.75	2.53	2.28	2.15	2.05	1.98	1.94	1.91	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.76	1.75
75	7.11	4.10	3.24	2.84	2.60	2.34	2.20	2.10	2.03	1.98	1.95	1.93	1.91	1.88	1.86	1.84	1.83	1.82	1.80	1.79
100	7.46	4.27	3.36	2.94	2.69	2.42	2.27	2.16	2.09	2.04	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
125	7.70	4.40	3.46	3.02	2.76	2.48	2.32	2.21	2.14	2.09	2.05	2.02	2.00	1.97	1.94	1.93	1.91	1.90	1.88	1.87
150	7.91	4.51	3.54	3.08	2.82	2.53	2.37	2.25	2.18	2.12	2.08	2.06	2.03	2.00	1.98	1.96	1.94	1.93	1.92	1.90
175	8.09	4.59	3.60	3.13	2.87	2.57	2.40	2.28	2.21	2.15	2.11	2.08	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
200	8.24	4.67	3.66	3.18	2.91	2.60	2.44	2.31	2.23	2.18	2.14	2.11	2.08	2.05	2.02	2.01	1.99	1.98	1.96	1.95

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (20 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.99	2.00	1.69	1.54	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
2	3.71	2.38	1.98	1.78	1.67	1.54	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
3	4.16	2.61	2.15	1.93	1.80	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
4	4.48	2.77	2.27	2.03	1.89	1.73	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
5	4.75	2.91	2.36	2.11	1.96	1.80	1.71	1.64	1.59	1.56	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
8	5.29	3.18	2.57	2.28	2.11	1.92	1.82	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
12	5.78	3.43	2.74	2.43	2.24	2.04	1.92	1.84	1.79	1.75	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.62	1.60	1.60
16	6.11	3.59	2.87	2.53	2.33	2.11	1.99	1.91	1.85	1.81	1.78	1.76	1.74	1.72	1.70	1.69	1.68	1.67	1.66	1.65
20	6.39	3.73	2.97	2.61	2.40	2.17	2.05	1.96	1.90	1.86	1.83	1.80	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.68
30	6.86	3.97	3.14	2.75	2.53	2.28	2.15	2.05	1.98	1.94	1.91	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.76	1.75
40	7.19	4.14	3.27	2.86	2.62	2.36	2.22	2.11	2.04	2.00	1.96	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80
50	7.46	4.27	3.36	2.94	2.69	2.42	2.27	2.16	2.09	2.04	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
60	7.66	4.38	3.44	3.00	2.75	2.47	2.31	2.20	2.13	2.08	2.04	2.01	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
75	7.91	4.51	3.54	3.08	2.82	2.53	2.37	2.25	2.18	2.12	2.08	2.06	2.03	2.00	1.98	1.96	1.94	1.93	1.92	1.90
100	8.24	4.67	3.66	3.18	2.91	2.60	2.44	2.31	2.23	2.18	2.14	2.11	2.08	2.05	2.02	2.01	1.99	1.98	1.96	1.95
125	8.48	4.79	3.75	3.26	2.98	2.66	2.49	2.36	2.28	2.22	2.18	2.15	2.12	2.09	2.06	2.04	2.03	2.02	2.00	1.98
150	8.67	4.90	3.83	3.33	3.03	2.71	2.53	2.40	2.32	2.26	2.21	2.18	2.16	2.12	2.09	2.07	2.06	2.05	2.03	2.01
175	8.83	4.98	3.89	3.38	3.08	2.75	2.57	2.43	2.35	2.29	2.24	2.21	2.19	2.15	2.12	2.10	2.08	2.07	2.05	2.03
200	8.98	5.06	3.95	3.42	3.12	2.78	2.60	2.46	2.37	2.31	2.27	2.23	2.21	2.17	2.14	2.12	2.10	2.09	2.07	2.06

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (20 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.71	2.38	1.98	1.78	1.67	1.54	1.47	1.42	1.38	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
2	4.48	2.77	2.27	2.03	1.89	1.73	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
3	4.96	3.01	2.44	2.18	2.02	1.85	1.75	1.68	1.64	1.60	1.58	1.56	1.55	1.53	1.52	1.50	1.50	1.49	1.48	1.47
4	5.29	3.18	2.57	2.28	2.11	1.92	1.82	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
5	5.57	3.32	2.67	2.36	2.18	1.98	1.88	1.80	1.75	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
8	6.11	3.59	2.87	2.53	2.33	2.11	1.99	1.91	1.85	1.81	1.78	1.76	1.74	1.72	1.70	1.69	1.68	1.67	1.66	1.65
12	6.60	3.84	3.05	2.68	2.46	2.22	2.09	2.00	1.94	1.89	1.86	1.84	1.82	1.79	1.77	1.76	1.75	1.74	1.73	1.72
16	6.93	4.00	3.17	2.78	2.55	2.30	2.16	2.06	2.00	1.95	1.92	1.89	1.88	1.85	1.83	1.81	1.80	1.79	1.77	1.76
20	7.19	4.14	3.27	2.86	2.62	2.36	2.22	2.11	2.04	2.00	1.96	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80
30	7.66	4.38	3.44	3.00	2.75	2.47	2.31	2.20	2.13	2.08	2.04	2.01	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
40	7.97	4.54	3.56	3.11	2.84	2.54	2.38	2.26	2.19	2.14	2.10	2.07	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.91
50	8.24	4.67	3.66	3.18	2.91	2.60	2.44	2.31	2.23	2.18	2.14	2.11	2.08	2.05	2.02	2.01	1.99	1.98	1.96	1.95
60	8.44	4.78	3.73	3.25	2.96	2.65	2.48	2.35	2.27	2.21	2.17	2.14	2.12	2.08	2.06	2.04	2.02	2.01	1.99	1.98
75	8.67	4.90	3.83	3.33	3.03	2.71	2.53	2.40	2.32	2.26	2.21	2.18	2.16	2.12	2.09	2.07	2.06	2.05	2.03	2.01
100	8.98	5.06	3.95	3.42	3.12	2.78	2.60	2.46	2.37	2.31	2.27	2.23	2.21	2.17	2.14	2.12	2.10	2.09	2.07	2.06
125	9.22	5.18	4.03	3.50	3.19	2.84	2.65	2.51	2.42	2.36	2.31	2.28	2.25	2.21	2.18	2.16	2.14	2.13	2.10	2.09
150	9.41	5.28	4.11	3.56	3.24	2.89	2.70	2.55	2.45	2.39	2.34	2.31	2.28	2.24	2.21	2.19	2.17	2.16	2.13	2.12
175	9.57	5.37	4.17	3.61	3.29	2.93	2.73	2.58	2.49	2.42	2.37	2.33	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
200	9.69	5.44	4.23	3.66	3.33	2.96	2.76	2.61	2.51	2.44	2.40	2.36	2.33	2.29	2.25	2.23	2.21	2.20	2.18	2.16

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (40 COCs, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.05	2.03	1.70	1.55	1.45	1.35	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.12
2	3.84	2.41	1.99	1.79	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.29	1.28	1.27	1.27	1.26
3	4.36	2.65	2.16	1.94	1.81	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
4	4.72	2.83	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.42	1.42	1.41	1.40	1.40
5	5.05	2.97	2.39	2.12	1.97	1.80	1.71	1.64	1.59	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44
8	5.70	3.27	2.61	2.30	2.12	1.93	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
12	6.30	3.54	2.79	2.45	2.26	2.04	1.93	1.84	1.79	1.75	1.72	1.70	1.69	1.66	1.65	1.64	1.63	1.62	1.60	1.59
16	6.74	3.73	2.93	2.56	2.35	2.12	2.00	1.91	1.85	1.81	1.79	1.76	1.75	1.72	1.70	1.69	1.68	1.67	1.66	1.65
20	7.07	3.90	3.04	2.64	2.42	2.18	2.05	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.68
30	7.67	4.17	3.23	2.80	2.56	2.29	2.16	2.05	1.98	1.94	1.91	1.89	1.86	1.83	1.82	1.80	1.79	1.78	1.77	1.75
40	8.11	4.36	3.37	2.91	2.65	2.37	2.22	2.11	2.05	2.00	1.96	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80
50	8.44	4.51	3.46	3.00	2.72	2.44	2.28	2.17	2.09	2.05	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
60	8.71	4.64	3.56	3.06	2.79	2.48	2.33	2.21	2.13	2.08	2.05	2.02	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
75	8.98	4.79	3.65	3.15	2.86	2.54	2.38	2.26	2.18	2.12	2.09	2.06	2.03	2.00	1.98	1.96	1.95	1.93	1.92	1.90
100	9.42	4.98	3.79	3.26	2.96	2.63	2.45	2.32	2.24	2.18	2.14	2.11	2.09	2.05	2.03	2.01	1.99	1.98	1.96	1.95
125	9.75	5.13	3.90	3.34	3.03	2.69	2.50	2.37	2.29	2.23	2.18	2.15	2.13	2.09	2.06	2.05	2.03	2.02	2.00	1.98
150	10.02	5.24	3.98	3.41	3.09	2.74	2.55	2.41	2.32	2.26	2.22	2.19	2.16	2.12	2.09	2.07	2.06	2.05	2.03	2.01
175	10.24	5.35	4.06	3.47	3.14	2.78	2.59	2.44	2.35	2.29	2.25	2.21	2.19	2.15	2.12	2.10	2.08	2.07	2.05	2.04
200	10.41	5.43	4.12	3.52	3.19	2.81	2.62	2.47	2.38	2.32	2.27	2.24	2.21	2.17	2.14	2.12	2.10	2.09	2.07	2.06

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (40 COCs, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.84	2.41	1.99	1.79	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.29	1.28	1.27	1.27	1.26
2	4.72	2.83	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.42	1.42	1.41	1.40	1.40
3	5.29	3.09	2.48	2.19	2.03	1.85	1.75	1.68	1.64	1.60	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
4	5.70	3.27	2.61	2.30	2.12	1.93	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
5	6.03	3.42	2.71	2.38	2.20	1.99	1.88	1.80	1.75	1.71	1.69	1.67	1.65	1.63	1.62	1.60	1.59	1.58	1.57	1.56
8	6.74	3.73	2.93	2.56	2.35	2.12	2.00	1.91	1.85	1.81	1.79	1.76	1.75	1.72	1.70	1.69	1.68	1.67	1.66	1.65
12	7.34	4.01	3.12	2.71	2.48	2.23	2.10	2.00	1.94	1.90	1.86	1.84	1.82	1.79	1.78	1.76	1.75	1.74	1.72	1.72
16	7.78	4.20	3.26	2.83	2.58	2.31	2.17	2.07	2.00	1.96	1.92	1.90	1.88	1.85	1.83	1.81	1.80	1.79	1.78	1.77
20	8.11	4.36	3.37	2.91	2.65	2.37	2.22	2.11	2.05	2.00	1.96	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80
30	8.71	4.64	3.56	3.06	2.79	2.48	2.33	2.21	2.13	2.08	2.05	2.02	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
40	9.09	4.83	3.69	3.17	2.89	2.57	2.39	2.27	2.19	2.14	2.10	2.07	2.05	2.01	1.99	1.97	1.96	1.94	1.93	1.91
50	9.42	4.98	3.79	3.26	2.96	2.63	2.45	2.32	2.24	2.18	2.14	2.11	2.09	2.05	2.03	2.01	1.99	1.98	1.96	1.95
60	9.70	5.10	3.88	3.32	3.02	2.67	2.50	2.36	2.28	2.22	2.18	2.15	2.12	2.08	2.06	2.04	2.02	2.01	1.99	1.98
75	10.02	5.24	3.98	3.41	3.09	2.74	2.55	2.41	2.32	2.26	2.22	2.19	2.16	2.12	2.09	2.07	2.06	2.05	2.03	2.01
100	10.41	5.43	4.12	3.52	3.19	2.81	2.62	2.47	2.38	2.32	2.27	2.24	2.21	2.17	2.14	2.12	2.10	2.09	2.07	2.06
125	10.73	5.59	4.23	3.61	3.26	2.87	2.67	2.52	2.43	2.36	2.31	2.28	2.25	2.21	2.18	2.16	2.14	2.13	2.10	2.09
150	10.95	5.70	4.31	3.68	3.32	2.93	2.72	2.56	2.46	2.39	2.35	2.31	2.28	2.24	2.21	2.19	2.17	2.16	2.13	2.12
175	11.17	5.81	4.38	3.73	3.37	2.97	2.75	2.59	2.49	2.43	2.37	2.34	2.31	2.26	2.23	2.21	2.19	2.18	2.16	2.14
200	11.39	5.89	4.43	3.78	3.41	3.00	2.78	2.62	2.52	2.45	2.40	2.36	2.33	2.29	2.25	2.23	2.21	2.20	2.18	2.16

Table 19-2. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Observations (40 COCs, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.72	2.83	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.42	1.42	1.41	1.40	1.40
2	5.70	3.27	2.61	2.30	2.12	1.93	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
3	6.30	3.54	2.79	2.45	2.26	2.04	1.93	1.84	1.79	1.75	1.72	1.70	1.69	1.66	1.65	1.64	1.63	1.62	1.60	1.59
4	6.74	3.73	2.93	2.56	2.35	2.12	2.00	1.91	1.85	1.81	1.79	1.76	1.75	1.72	1.70	1.69	1.68	1.67	1.66	1.65
5	7.07	3.90	3.04	2.64	2.42	2.18	2.05	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.68
8	7.78	4.20	3.26	2.83	2.58	2.31	2.17	2.07	2.00	1.96	1.92	1.90	1.88	1.85	1.83	1.81	1.80	1.79	1.78	1.77
12	8.38	4.49	3.45	2.98	2.72	2.42	2.27	2.16	2.09	2.04	2.00	1.97	1.95	1.92	1.90	1.88	1.87	1.86	1.84	1.83
16	8.77	4.68	3.58	3.09	2.80	2.50	2.34	2.22	2.15	2.09	2.06	2.03	2.00	1.97	1.95	1.93	1.92	1.91	1.89	1.88
20	9.09	4.83	3.69	3.17	2.89	2.57	2.39	2.27	2.19	2.14	2.10	2.07	2.05	2.01	1.99	1.97	1.96	1.94	1.93	1.91
30	9.70	5.10	3.88	3.32	3.02	2.67	2.50	2.36	2.28	2.22	2.18	2.15	2.12	2.08	2.06	2.04	2.02	2.01	1.99	1.98
40	10.08	5.29	4.01	3.43	3.11	2.76	2.57	2.42	2.33	2.27	2.23	2.20	2.17	2.13	2.10	2.08	2.07	2.06	2.04	2.02
50	10.41	5.43	4.12	3.52	3.19	2.81	2.62	2.47	2.38	2.32	2.27	2.24	2.21	2.17	2.14	2.12	2.10	2.09	2.07	2.06
60	10.68	5.57	4.20	3.59	3.24	2.87	2.66	2.51	2.42	2.35	2.31	2.27	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.08
75	10.95	5.70	4.31	3.68	3.32	2.93	2.72	2.56	2.46	2.39	2.35	2.31	2.28	2.24	2.21	2.19	2.17	2.16	2.13	2.12
100	11.39	5.89	4.43	3.78	3.41	3.00	2.78	2.62	2.52	2.45	2.40	2.36	2.33	2.29	2.25	2.23	2.21	2.20	2.18	2.16
125	11.66	6.03	4.54	3.86	3.48	3.06	2.84	2.67	2.57	2.49	2.44	2.40	2.37	2.32	2.29	2.27	2.25	2.23	2.21	2.19
150	11.94	6.14	4.61	3.93	3.54	3.11	2.88	2.71	2.60	2.53	2.47	2.43	2.40	2.35	2.32	2.30	2.28	2.26	2.24	2.22
175	12.10	6.25	4.69	3.98	3.58	3.15	2.91	2.74	2.63	2.56	2.50	2.46	2.43	2.38	2.35	2.32	2.30	2.29	2.26	2.24
200	12.27	6.30	4.75	4.04	3.63	3.19	2.95	2.77	2.66	2.58	2.52	2.48	2.45	2.40	2.37	2.34	2.32	2.31	2.28	2.26

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.06	-0.01	-0.04	-0.07	-0.08	-0.10	-0.11	-0.12	-0.13	-0.13	-0.13	-0.14	-0.14	-0.14	-0.14	-0.15	-0.15	-0.15	-0.15	-0.15
2	0.36	0.25	0.21	0.18	0.16	0.13	0.12	0.11	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07
3	0.52	0.40	0.34	0.31	0.29	0.26	0.24	0.23	0.22	0.21	0.21	0.20	0.20	0.20	0.19	0.19	0.19	0.19	0.18	0.18
4	0.64	0.50	0.44	0.40	0.37	0.34	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26	0.26
5	0.73	0.58	0.51	0.47	0.44	0.41	0.39	0.37	0.36	0.35	0.35	0.34	0.34	0.33	0.33	0.33	0.32	0.32	0.32	0.32
8	0.92	0.73	0.65	0.60	0.57	0.53	0.51	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.44	0.43	0.43
12	1.08	0.86	0.77	0.72	0.68	0.64	0.61	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.53	0.53	0.53	0.52	0.52
16	1.19	0.95	0.85	0.79	0.76	0.71	0.68	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59	0.58
20	1.28	1.02	0.91	0.85	0.81	0.76	0.73	0.71	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65	0.64	0.64	0.63	0.63
30	1.44	1.15	1.03	0.96	0.91	0.86	0.83	0.80	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.71
40	1.55	1.23	1.10	1.03	0.98	0.93	0.89	0.86	0.84	0.83	0.82	0.81	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77
50	1.63	1.30	1.16	1.09	1.04	0.98	0.94	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.83	0.82	0.82	0.81
60	1.70	1.35	1.21	1.13	1.08	1.02	0.98	0.95	0.93	0.91	0.90	0.89	0.89	0.88	0.87	0.86	0.86	0.86	0.85	0.85
75	1.78	1.42	1.27	1.19	1.13	1.07	1.03	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90	0.89	0.89
100	1.89	1.50	1.34	1.26	1.20	1.13	1.09	1.05	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.95	0.94	0.94
125	1.97	1.56	1.40	1.31	1.25	1.18	1.13	1.10	1.07	1.06	1.04	1.03	1.03	1.01	1.01	1.00	0.99	0.99	0.98	0.98
150	2.03	1.61	1.44	1.35	1.29	1.21	1.17	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01	1.01
175	*2.09*	1.66	1.48	1.39	1.32	1.25	1.20	1.16	1.14	1.12	1.11	1.10	1.09	1.07	1.06	1.06	1.05	1.05	1.04	1.03
200	2.14	1.69	1.51	1.42	1.35	1.27	1.23	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06	1.06

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.36	0.25	0.21	0.18	0.16	0.13	0.12	0.11	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07
2	0.64	0.50	0.44	0.40	0.37	0.34	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26	0.26
3	0.80	0.64	0.56	0.52	0.49	0.46	0.43	0.42	0.41	0.40	0.39	0.39	0.38	0.38	0.37	0.37	0.37	0.37	0.36	0.36
4	0.92	0.73	0.65	0.60	0.57	0.53	0.51	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.44	0.43	0.43
5	1.01	0.80	0.72	0.66	0.63	0.59	0.57	0.55	0.53	0.52	0.52	0.51	0.51	0.50	0.50	0.49	0.49	0.49	0.48	0.48
8	1.19	0.95	0.85	0.79	0.76	0.71	0.68	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59	0.58
12	1.35	1.08	0.97	0.90	0.86	0.81	0.78	0.75	0.74	0.72	0.71	0.71	0.70	0.69	0.69	0.68	0.68	0.68	0.67	0.67
16	1.46	1.17	1.04	0.98	0.93	0.87	0.84	0.82	0.80	0.78	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.74	0.73	0.73
20	1.55	1.23	1.10	1.03	0.98	0.93	0.89	0.86	0.84	0.83	0.82	0.81	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77
30	1.70	1.35	1.21	1.13	1.08	1.02	0.98	0.95	0.93	0.91	0.90	0.89	0.89	0.88	0.87	0.86	0.86	0.86	0.85	0.85
40	1.81	1.44	1.29	1.20	1.15	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90	0.90
50	1.89	1.50	1.34	1.26	1.20	1.13	1.09	1.05	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.95	0.94	0.94
60	1.95	1.55	1.39	1.30	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.98	0.97
75	2.03	1.61	1.44	1.35	1.29	1.21	1.17	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01	1.01
100	2.14	1.69	1.51	1.42	1.35	1.27	1.23	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06	1.06
125	2.21	1.75	1.57	1.47	1.40	1.32	1.27	1.23	1.20	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.11	1.10	1.09
150	2.28	1.80	1.61	1.51	1.44	1.35	1.30	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.12
175	*2.33*	1.84	1.65	1.54	1.47	1.38	1.33	1.29	1.26	1.24	1.23	1.22	1.21	1.19	1.18	1.17	1.17	1.16	1.15	1.15
200	2.37	1.88	1.68	1.57	1.50	1.41	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.20	1.20	1.19	1.19	1.18	1.17

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.64	0.50	0.44	0.40	0.37	0.34	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26	0.26
2	0.92	0.73	0.65	0.60	0.57	0.53	0.51	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.44	0.43	0.43
3	1.08	0.86	0.77	0.72	0.68	0.64	0.61	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.53	0.53	0.53	0.52	0.52
4	1.19	0.95	0.85	0.79	0.76	0.71	0.68	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59	0.58
5	1.28	1.02	0.91	0.85	0.81	0.76	0.73	0.71	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65	0.64	0.64	0.63	0.63
8	1.46	1.17	1.04	0.98	0.93	0.87	0.84	0.82	0.80	0.78	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.74	0.73	0.73
12	1.62	1.29	1.15	1.08	1.03	0.97	0.93	0.90	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.80
16	1.72	1.37	1.23	1.15	1.10	1.03	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.88	0.87	0.87	0.86	0.86
20	1.81	1.44	1.29	1.20	1.15	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90	0.90
30	1.95	1.55	1.39	1.30	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.98	0.97
40	2.06	1.63	1.46	1.36	1.30	1.23	1.18	1.14	1.12	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02	1.02
50	2.14	1.69	1.51	1.42	1.35	1.27	1.23	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06	1.06
60	2.20	1.74	1.56	1.46	1.39	1.31	1.26	1.22	1.20	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.09
75	2.28	1.80	1.61	1.51	1.44	1.35	1.30	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.12
100	2.37	1.88	1.68	1.57	1.50	1.41	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.20	1.20	1.19	1.19	1.18	1.17
125	2.45	1.94	1.73	1.62	1.54	1.45	1.40	1.36	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.23	1.23	1.22	1.21	1.20
150	2.51	1.98	1.77	1.66	1.58	1.49	1.43	1.39	1.36	1.34	1.32	1.31	1.30	1.28	1.27	1.26	1.26	1.25	1.24	1.23
175	2.56	2.02	1.81	1.69	1.61	1.52	1.46	1.42	1.39	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.26
200	2.60	2.06	1.84	1.72	1.64	1.54	1.49	1.44	1.41	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28	1.28

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (2 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.39	0.28	0.22	0.19	0.17	0.14	0.12	0.11	0.10	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.07	0.07	0.07	0.07
2	0.70	0.53	0.46	0.41	0.39	0.35	0.33	0.31	0.30	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26
3	0.88	0.67	0.59	0.54	0.50	0.47	0.44	0.42	0.41	0.40	0.40	0.39	0.39	0.38	0.38	0.37	0.37	0.37	0.36	0.36
4	1.01	0.78	0.68	0.62	0.59	0.54	0.52	0.50	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
5	1.12	0.85	0.75	0.69	0.65	0.60	0.58	0.55	0.54	0.53	0.52	0.52	0.51	0.50	0.50	0.49	0.49	0.49	0.48	0.48
8	1.33	1.02	0.89	0.82	0.78	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.60	0.60	0.60	0.59	0.59	0.59
12	1.52	1.16	1.01	0.93	0.88	0.82	0.79	0.76	0.74	0.73	0.72	0.71	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67
16	1.66	1.26	1.10	1.01	0.96	0.89	0.86	0.83	0.81	0.79	0.78	0.77	0.77	0.76	0.75	0.74	0.74	0.74	0.73	0.73
20	1.76	1.33	1.16	1.07	1.02	0.95	0.91	0.87	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77
30	1.95	1.47	1.28	1.18	1.12	1.04	1.00	0.96	0.94	0.92	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.86	0.85	0.85
40	*2.09*	1.56	1.36	1.26	1.19	1.11	1.06	1.02	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90	0.90
50	2.19	1.64	1.43	1.31	1.24	1.16	1.11	1.07	1.04	1.02	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.95	0.94
60	2.27	1.70	1.48	1.36	1.29	1.20	1.15	1.11	1.08	1.06	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.99	0.98	0.97
75	2.37	1.77	1.54	1.42	1.34	1.25	1.19	1.15	1.12	1.10	1.09	1.08	1.07	1.05	1.04	1.03	1.03	1.02	1.02	1.01
100	2.50	1.86	1.62	1.49	1.41	1.31	1.25	1.21	1.18	1.16	1.14	1.13	1.12	1.10	1.09	1.08	1.08	1.07	1.07	1.06
125	2.60	1.93	1.68	1.54	1.46	1.36	1.30	1.25	1.22	1.20	1.18	1.17	1.16	1.14	1.13	1.12	1.12	1.11	1.10	1.10
150	2.68	1.99	1.73	1.59	1.50	1.40	1.33	1.29	1.25	1.23	1.21	1.20	1.19	1.17	1.16	1.15	1.15	1.14	1.13	1.13
175	2.75	*2.04*	1.77	1.63	1.54	1.43	1.37	1.32	1.28	1.26	1.24	1.23	1.22	1.20	1.19	1.18	1.17	1.17	1.16	1.15
200	2.81	*2.08*	1.81	1.66	1.57	1.46	1.39	1.34	1.31	1.28	1.26	1.25	1.24	1.22	1.21	1.20	1.19	1.19	1.18	1.17

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.70	0.53	0.46	0.41	0.39	0.35	0.33	0.31	0.30	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26
2	1.01	0.78	0.68	0.62	0.59	0.54	0.52	0.50	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
3	1.20	0.92	0.80	0.74	0.70	0.65	0.62	0.60	0.58	0.57	0.56	0.56	0.55	0.55	0.54	0.54	0.53	0.53	0.53	0.52
4	1.33	1.02	0.89	0.82	0.78	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.60	0.60	0.60	0.59	0.59	0.59
5	1.44	1.10	0.96	0.88	0.84	0.78	0.75	0.72	0.70	0.69	0.68	0.67	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
8	1.66	1.26	1.10	1.01	0.96	0.89	0.86	0.83	0.81	0.79	0.78	0.77	0.77	0.76	0.75	0.74	0.74	0.74	0.73	0.73
12	1.85	1.39	1.22	1.12	1.06	0.99	0.95	0.91	0.89	0.88	0.86	0.86	0.85	0.84	0.83	0.82	0.82	0.82	0.81	0.81
16	1.98	1.49	1.30	1.20	1.13	1.06	1.01	0.98	0.95	0.94	0.92	0.91	0.91	0.89	0.89	0.88	0.87	0.87	0.86	0.86
20	2.09	1.56	1.36	1.26	1.19	1.11	1.06	1.02	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90	0.90
30	2.27	1.70	1.48	1.36	1.29	1.20	1.15	1.11	1.08	1.06	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.99	0.98	0.97
40	2.40	1.79	1.56	1.43	1.36	1.26	1.21	1.16	1.13	1.11	1.10	1.09	1.08	1.06	1.05	1.05	1.04	1.04	1.03	1.02
50	2.50	1.86	1.62	1.49	1.41	1.31	1.25	1.21	1.18	1.16	1.14	1.13	1.12	1.10	1.09	1.08	1.08	1.07	1.07	1.06
60	2.59	1.92	1.67	1.53	1.45	1.35	1.29	1.24	1.21	1.19	1.17	1.16	1.15	1.14	1.12	1.12	1.11	1.10	1.10	1.09
75	2.68	1.99	1.73	1.59	1.50	1.40	1.33	1.29	1.25	1.23	1.21	1.20	1.19	1.17	1.16	1.15	1.15	1.14	1.13	1.13
100	2.81	2.08	1.81	1.66	1.57	1.46	1.39	1.34	1.31	1.28	1.26	1.25	1.24	1.22	1.21	1.20	1.19	1.19	1.18	1.17
125	2.91	2.15	1.86	1.71	1.62	1.50	1.44	1.38	1.35	1.32	1.30	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21
150	2.99	2.20	1.91	1.76	1.66	1.54	1.47	1.42	1.38	1.35	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.24
175	3.05	2.25	1.95	1.79	1.69	1.57	1.50	1.44	1.41	1.38	1.36	1.35	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26
200	3.11	*2.29*	1.98	1.82	1.72	1.60	1.53	1.47	1.43	1.40	1.38	1.37	1.36	1.34	1.32	1.31	1.31	1.30	1.29	1.28

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.01	0.78	0.68	0.62	0.59	0.54	0.52	0.50	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
2	1.33	1.02	0.89	0.82	0.78	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.60	0.60	0.60	0.59	0.59	0.59
3	1.52	1.16	1.01	0.93	0.88	0.82	0.79	0.76	0.74	0.73	0.72	0.71	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67
4	1.66	1.26	1.10	1.01	0.96	0.89	0.86	0.83	0.81	0.79	0.78	0.77	0.77	0.76	0.75	0.74	0.74	0.74	0.73	0.73
5	1.76	1.33	1.16	1.07	1.02	0.95	0.91	0.87	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77
8	1.98	1.49	1.30	1.20	1.13	1.06	1.01	0.98	0.95	0.94	0.92	0.91	0.91	0.89	0.89	0.88	0.87	0.87	0.86	0.86
12	2.17	1.62	1.42	1.30	1.23	1.15	1.10	1.06	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.95	0.94	0.93
16	2.30	1.72	1.50	1.38	1.30	1.21	1.16	1.12	1.09	1.07	1.06	1.05	1.04	1.02	1.01	1.01	1.00	1.00	0.99	0.98
20	2.40	1.79	1.56	1.43	1.36	1.26	1.21	1.16	1.13	1.11	1.10	1.09	1.08	1.06	1.05	1.05	1.04	1.04	1.03	1.02
30	2.59	1.92	1.67	1.53	1.45	1.35	1.29	1.24	1.21	1.19	1.17	1.16	1.15	1.14	1.12	1.12	1.11	1.10	1.10	1.09
40	2.71	2.01	1.75	1.61	1.52	1.41	1.35	1.30	1.27	1.24	1.23	1.21	1.20	1.18	1.17	1.16	1.16	1.15	1.14	1.14
50	2.81	2.08	1.81	1.66	1.57	1.46	1.39	1.34	1.31	1.28	1.26	1.25	1.24	1.22	1.21	1.20	1.19	1.19	1.18	1.17
60	2.89	2.13	1.85	1.70	1.61	1.49	1.43	1.37	1.34	1.32	1.30	1.28	1.27	1.25	1.24	1.23	1.22	1.22	1.21	1.20
75	2.99	2.20	1.91	1.76	1.66	1.54	1.47	1.42	1.38	1.35	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.24
100	3.11	2.29	1.98	1.82	1.72	1.60	1.53	1.47	1.43	1.40	1.38	1.37	1.36	1.34	1.32	1.31	1.31	1.30	1.29	1.28
125	3.20	2.35	2.04	1.87	1.77	1.64	1.57	1.51	1.47	1.44	1.42	1.41	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
150	3.28	2.41	2.09	1.92	1.81	1.68	1.60	1.54	1.50	1.47	1.45	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
175	3.34	2.45	2.13	1.95	1.84	1.71	1.63	1.57	1.53	1.50	1.48	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36
200	3.39	2.49	2.16	1.98	1.87	1.74	1.66	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.85	0.63	0.54	0.49	0.46	0.42	0.40	0.38	0.37	0.36	0.35	0.35	0.34	0.34	0.33	0.33	0.33	0.32	0.32	0.32
2	1.21	0.89	0.77	0.70	0.66	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.50	0.50	0.50	0.49	0.49	0.49	0.48
3	1.43	1.04	0.90	0.82	0.77	0.72	0.68	0.66	0.64	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.58	0.57
4	1.59	1.15	0.99	0.91	0.85	0.79	0.75	0.72	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
5	1.71	1.24	1.06	0.97	0.91	0.85	0.81	0.78	0.76	0.74	0.73	0.72	0.72	0.71	0.70	0.70	0.69	0.69	0.68	0.68
8	1.99	1.42	1.21	1.11	1.04	0.96	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.80	0.79	0.79	0.78	0.78	0.77
12	2.24	1.58	1.34	1.22	1.15	1.06	1.01	0.97	0.95	0.93	0.91	0.90	0.90	0.88	0.88	0.87	0.86	0.86	0.85	0.85
16	2.41	1.69	1.43	1.30	1.22	1.13	1.07	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
20	2.55	1.77	1.50	1.36	1.28	1.18	1.12	1.08	1.05	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.95	0.94
30	2.80	1.93	1.63	1.48	1.38	1.27	1.21	1.16	1.13	1.11	1.09	1.08	1.07	1.06	1.04	1.04	1.03	1.03	1.02	1.01
40	2.97	*2.04*	1.72	1.56	1.46	1.34	1.27	1.22	1.19	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06
50	3.11	*2.13*	1.79	1.62	1.51	1.39	1.32	1.27	1.23	1.21	1.19	1.17	1.16	1.15	1.13	1.13	1.12	1.11	1.10	1.10
60	3.22	2.20	1.85	1.67	1.56	1.43	1.36	1.30	1.27	1.24	1.22	1.21	1.20	1.18	1.17	1.16	1.15	1.14	1.13	1.13
75	3.36	2.28	1.91	1.73	1.61	1.48	1.40	1.35	1.31	1.28	1.26	1.25	1.23	1.22	1.20	1.19	1.19	1.18	1.17	1.16
100	3.53	2.39	*2.00*	1.80	1.68	1.54	1.46	1.40	1.36	1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.23	1.23	1.22	1.21
125	3.67	2.47	*2.07*	1.86	1.74	1.59	1.51	1.44	1.40	1.37	1.35	1.33	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24
150	3.77	2.54	*2.12*	1.91	1.78	1.63	1.55	1.48	1.44	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27
175	3.86	2.60	*2.17*	1.95	1.82	1.66	1.58	1.51	1.46	1.43	1.41	1.39	1.38	1.36	1.34	1.33	1.32	1.32	1.30	1.30
200	3.94	2.65	2.21	*1.99*	1.85	1.69	1.60	1.53	1.49	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.34	1.32	1.32

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.21	0.89	0.77	0.70	0.66	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.50	0.50	0.50	0.49	0.49	0.49	0.48
2	1.59	1.15	0.99	0.91	0.85	0.79	0.75	0.72	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
3	1.82	1.31	1.12	1.02	0.96	0.89	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
4	1.99	1.42	1.21	1.11	1.04	0.96	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.80	0.79	0.79	0.78	0.78	0.77
5	2.13	1.50	1.28	1.17	1.10	1.02	0.97	0.93	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.83	0.82	0.81
8	2.41	1.69	1.43	1.30	1.22	1.13	1.07	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
12	2.66	1.84	1.56	1.41	1.33	1.22	1.16	1.12	1.09	1.07	1.05	1.04	1.03	1.01	1.00	1.00	0.99	0.99	0.98	0.97
16	2.84	1.95	1.65	1.49	1.40	1.29	1.22	1.18	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
20	2.97	2.04	1.72	1.56	1.46	1.34	1.27	1.22	1.19	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06
30	3.22	2.20	1.85	1.67	1.56	1.43	1.36	1.30	1.27	1.24	1.22	1.21	1.20	1.18	1.17	1.16	1.15	1.14	1.13	1.13
40	3.40	*2.30*	1.93	1.74	1.63	1.49	1.42	1.36	1.32	1.29	1.27	1.26	1.25	1.23	1.21	1.20	1.20	1.19	1.18	1.17
50	3.53	*2.39*	2.00	1.80	1.68	1.54	1.46	1.40	1.36	1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.23	1.23	1.22	1.21
60	3.64	2.46	2.06	1.85	1.73	1.58	1.50	1.44	1.40	1.37	1.34	1.33	1.31	1.30	1.28	1.27	1.26	1.26	1.24	1.24
75	3.77	2.54	2.12	1.91	1.78	1.63	1.55	1.48	1.44	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27
100	3.94	2.65	2.21	1.99	1.85	1.69	1.60	1.53	1.49	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.34	1.32	1.32
125	4.07	2.73	*2.27*	2.04	1.90	1.74	1.65	1.58	1.53	1.50	1.47	1.45	1.44	1.41	1.40	1.39	1.38	1.37	1.36	1.35
150	4.18	2.79	*2.33*	2.09	1.95	1.78	1.68	1.61	1.56	1.53	1.50	1.48	1.47	1.44	1.43	1.41	1.41	1.40	1.38	1.38
175	4.27	2.85	*2.37*	2.13	1.98	1.81	1.71	1.64	1.59	1.55	1.53	1.51	1.49	1.47	1.45	1.44	1.43	1.42	1.41	1.40
200	4.34	2.90	*2.41*	2.17	2.02	1.84	1.74	1.66	1.61	1.58	1.55	1.53	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.59	1.15	0.99	0.91	0.85	0.79	0.75	0.72	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
2	1.99	1.42	1.21	1.11	1.04	0.96	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.80	0.79	0.79	0.78	0.78	0.77
3	2.24	1.58	1.34	1.22	1.15	1.06	1.01	0.97	0.95	0.93	0.91	0.90	0.90	0.88	0.88	0.87	0.86	0.86	0.85	0.85
4	2.41	1.69	1.43	1.30	1.22	1.13	1.07	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
5	2.55	1.77	1.50	1.36	1.28	1.18	1.12	1.08	1.05	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.95	0.94
8	2.84	1.95	1.65	1.49	1.40	1.29	1.22	1.18	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
12	3.09	2.11	1.78	1.61	1.50	1.38	1.31	1.26	1.22	1.20	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
16	3.26	2.22	1.87	1.68	1.57	1.44	1.37	1.31	1.28	1.25	1.23	1.22	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.14
20	3.40	2.30	1.93	1.74	1.63	1.49	1.42	1.36	1.32	1.29	1.27	1.26	1.25	1.23	1.21	1.20	1.20	1.19	1.18	1.17
30	3.64	2.46	2.06	1.85	1.73	1.58	1.50	1.44	1.40	1.37	1.34	1.33	1.31	1.30	1.28	1.27	1.26	1.26	1.24	1.24
40	3.81	2.56	2.14	1.93	1.80	1.65	1.56	1.49	1.45	1.42	1.39	1.38	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28
50	3.94	*2.65*	2.21	1.99	1.85	1.69	1.60	1.53	1.49	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.34	1.32	1.32
60	4.05	2.71	2.26	2.03	1.89	1.73	1.64	1.57	1.52	1.49	1.46	1.45	1.43	1.41	1.39	1.38	1.37	1.36	1.35	1.34
75	4.18	2.79	2.33	2.09	1.95	1.78	1.68	1.61	1.56	1.53	1.50	1.48	1.47	1.44	1.43	1.41	1.41	1.40	1.38	1.38
100	4.34	2.90	2.41	2.17	2.02	1.84	1.74	1.66	1.61	1.58	1.55	1.53	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
125	4.47	2.97	2.48	2.22	2.07	1.89	1.78	1.70	1.65	1.61	1.59	1.57	1.55	1.52	1.51	1.49	1.48	1.47	1.46	1.45
150	4.57	3.04	2.53	2.27	2.11	1.92	1.82	1.74	1.68	1.64	1.62	1.59	1.58	1.55	1.53	1.52	1.51	1.50	1.49	1.48
175	4.66	3.09	*2.57*	2.31	2.14	1.96	1.85	1.76	1.71	1.67	1.64	1.62	1.60	1.57	1.56	1.54	1.53	1.52	1.51	1.50
200	4.73	3.14	*2.61*	2.34	2.17	1.98	1.87	1.79	1.73	1.69	1.66	1.64	1.62	1.60	1.58	1.56	1.55	1.54	1.53	1.52

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.24	0.91	0.78	0.71	0.67	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.51	0.50	0.50	0.49	0.49	0.49	0.48
2	1.66	1.18	1.01	0.91	0.86	0.79	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
3	1.92	1.34	1.14	1.03	0.97	0.90	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
4	2.11	1.46	1.23	1.12	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.82	0.80	0.80	0.79	0.79	0.78	0.78	0.77
5	2.27	1.55	1.31	1.18	1.11	1.02	0.97	0.93	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.83	0.82	0.81
8	2.61	1.75	1.46	1.32	1.23	1.13	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
12	2.91	1.92	1.60	1.44	1.34	1.23	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
16	3.13	*2.05*	1.70	1.52	1.42	1.30	1.23	1.18	1.15	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
20	3.30	*2.14*	1.77	1.59	1.48	1.35	1.28	1.23	1.19	1.17	1.15	1.14	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06
30	3.61	2.32	1.91	1.70	1.58	1.44	1.37	1.31	1.27	1.24	1.22	1.21	1.20	1.18	1.17	1.16	1.15	1.14	1.13	1.13
40	3.83	2.44	*2.00*	1.79	1.66	1.51	1.43	1.37	1.32	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
50	4.00	2.54	*2.08*	1.85	1.72	1.56	1.47	1.41	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.23	1.23	1.22	1.21
60	4.14	2.62	*2.14*	1.90	1.76	1.60	1.51	1.44	1.40	1.37	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
75	4.31	2.71	2.21	*1.97*	1.82	1.65	1.56	1.49	1.44	1.41	1.39	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27
100	4.52	2.84	2.31	*2.05*	1.89	1.72	1.62	1.54	1.50	1.46	1.44	1.42	1.40	1.38	1.37	1.35	1.34	1.34	1.32	1.32
125	4.69	2.93	2.38	*2.11*	1.95	1.77	1.66	1.59	1.54	1.50	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35
150	4.83	3.01	2.44	*2.16*	*2.00*	1.81	1.70	1.62	1.57	1.53	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
175	4.94	3.07	2.49	*2.21*	*2.04*	1.84	1.73	1.65	1.60	1.56	1.53	1.51	1.49	1.47	1.45	1.44	1.43	1.42	1.41	1.40
200	5.04	3.13	2.54	2.24	*2.07*	1.87	1.76	1.68	1.62	1.58	1.55	1.53	1.52	1.49	1.47	1.46	1.45	1.44	1.43	1.42

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.66	1.18	1.01	0.91	0.86	0.79	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
2	2.11	1.46	1.23	1.12	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.82	0.80	0.80	0.79	0.79	0.78	0.78	0.77
3	2.40	1.63	1.37	1.24	1.16	1.07	1.01	0.97	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85
4	2.61	1.75	1.46	1.32	1.23	1.13	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
5	2.77	1.84	1.54	1.39	1.29	1.19	1.13	1.08	1.05	1.03	1.02	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.95	0.94
8	3.13	2.05	1.70	1.52	1.42	1.30	1.23	1.18	1.15	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
12	3.44	2.22	1.83	1.64	1.52	1.39	1.32	1.26	1.23	1.20	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
16	3.66	*2.35*	1.93	1.72	1.60	1.46	1.38	1.32	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
20	3.83	2.44	2.00	1.79	1.66	1.51	1.43	1.37	1.32	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
30	4.14	2.62	2.14	1.90	1.76	1.60	1.51	1.44	1.40	1.37	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
40	4.36	2.74	2.24	1.99	1.84	1.67	1.57	1.50	1.45	1.42	1.40	1.38	1.37	1.34	1.33	1.32	1.31	1.30	1.29	1.28
50	4.52	2.84	*2.31*	2.05	1.89	1.72	1.62	1.54	1.50	1.46	1.44	1.42	1.40	1.38	1.37	1.35	1.34	1.34	1.32	1.32
60	4.66	2.92	*2.37*	2.10	1.94	1.76	1.66	1.58	1.53	1.49	1.47	1.45	1.43	1.41	1.39	1.38	1.37	1.37	1.35	1.34
75	4.83	3.01	*2.44*	2.16	2.00	1.81	1.70	1.62	1.57	1.53	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
100	5.04	3.13	2.54	2.24	2.07	1.87	1.76	1.68	1.62	1.58	1.55	1.53	1.52	1.49	1.47	1.46	1.45	1.44	1.43	1.42
125	5.20	3.22	2.61	*2.31*	2.13	1.92	1.80	1.72	1.66	1.62	1.59	1.57	1.55	1.53	1.51	1.49	1.48	1.47	1.46	1.45
150	5.34	3.30	2.67	*2.36*	2.17	1.96	1.84	1.75	1.69	1.65	1.62	1.60	1.58	1.55	1.54	1.52	1.51	1.50	1.49	1.48
175	5.45	3.36	2.71	*2.40*	2.21	1.99	1.87	1.78	1.72	1.68	1.65	1.62	1.61	1.58	1.56	1.54	1.53	1.52	1.51	1.50
200	5.54	3.42	2.76	*2.43*	2.24	2.02	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.58	1.56	1.55	1.54	1.53	1.52

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.11	1.46	1.23	1.12	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.82	0.80	0.80	0.79	0.79	0.78	0.78	0.77
2	2.61	1.75	1.46	1.32	1.23	1.13	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
3	2.91	1.92	1.60	1.44	1.34	1.23	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
4	3.13	2.05	1.70	1.52	1.42	1.30	1.23	1.18	1.15	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
5	3.30	2.14	1.77	1.59	1.48	1.35	1.28	1.23	1.19	1.17	1.15	1.14	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06
8	3.66	2.35	1.93	1.72	1.60	1.46	1.38	1.32	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	3.97	2.52	2.06	1.84	1.71	1.55	1.47	1.40	1.36	1.33	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20
16	4.19	*2.65*	2.16	1.92	1.78	1.62	1.53	1.46	1.41	1.38	1.36	1.34	1.33	1.31	1.29	1.28	1.27	1.27	1.26	1.25
20	4.36	2.74	2.24	1.99	1.84	1.67	1.57	1.50	1.45	1.42	1.40	1.38	1.37	1.34	1.33	1.32	1.31	1.30	1.29	1.28
30	4.66	2.92	2.37	2.10	1.94	1.76	1.66	1.58	1.53	1.49	1.47	1.45	1.43	1.41	1.39	1.38	1.37	1.37	1.35	1.34
40	4.88	3.04	2.46	2.18	2.01	1.82	1.71	1.63	1.58	1.54	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.39	1.39
50	5.04	3.13	2.54	2.24	2.07	1.87	1.76	1.68	1.62	1.58	1.55	1.53	1.52	1.49	1.47	1.46	1.45	1.44	1.43	1.42
60	5.17	3.21	*2.59*	2.29	2.11	1.91	1.80	1.71	1.65	1.61	1.59	1.56	1.55	1.52	1.50	1.49	1.48	1.47	1.45	1.44
75	5.34	3.30	*2.67*	2.36	2.17	1.96	1.84	1.75	1.69	1.65	1.62	1.60	1.58	1.55	1.54	1.52	1.51	1.50	1.49	1.48
100	5.54	3.42	*2.76*	2.43	2.24	2.02	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.58	1.56	1.55	1.54	1.53	1.52
125	5.70	3.51	2.83	2.50	2.30	2.07	1.94	1.85	1.78	1.74	1.71	1.68	1.66	1.63	1.61	1.60	1.58	1.58	1.56	1.55
150	5.83	3.58	2.88	2.54	2.34	2.11	1.98	1.88	1.81	1.77	1.74	1.71	1.69	1.66	1.64	1.62	1.61	1.60	1.58	1.57
175	5.94	3.64	2.93	*2.59*	2.38	2.14	2.01	1.91	1.84	1.79	1.76	1.73	1.71	1.68	1.66	1.64	1.63	1.62	1.60	1.59
200	6.03	3.70	2.97	*2.62*	2.41	2.17	2.03	1.93	1.86	1.82	1.78	1.75	1.73	1.70	1.68	1.66	1.65	1.64	1.62	1.61

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (20 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.70	1.19	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
2	2.20	1.48	1.25	1.13	1.05	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
3	2.51	1.66	1.38	1.25	1.16	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85
4	2.75	1.79	1.48	1.33	1.24	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
5	2.94	1.89	1.56	1.40	1.30	1.19	1.13	1.08	1.05	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
8	3.36	*2.11*	1.72	1.54	1.43	1.30	1.24	1.18	1.15	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
12	3.74	2.30	1.87	1.66	1.54	1.40	1.32	1.27	1.23	1.20	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
16	4.01	2.44	*1.97*	1.75	1.62	1.47	1.39	1.32	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
20	4.22	2.55	*2.05*	1.82	1.68	1.52	1.43	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
30	4.61	2.75	*2.20*	1.94	1.78	1.61	1.52	1.45	1.40	1.37	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
40	4.88	2.89	2.30	*2.03*	1.86	1.68	1.58	1.51	1.46	1.42	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
50	5.10	3.00	2.39	*2.09*	1.92	1.73	1.63	1.55	1.50	1.46	1.44	1.42	1.40	1.38	1.37	1.35	1.34	1.34	1.32	1.32
60	5.27	3.09	2.45	*2.15*	*1.97*	1.77	1.67	1.58	1.53	1.50	1.47	1.45	1.43	1.41	1.39	1.38	1.37	1.37	1.35	1.34
75	5.49	3.20	2.53	*2.22*	*2.03*	1.82	1.71	1.63	1.57	1.54	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
100	5.76	3.34	2.64	2.30	*2.11*	1.89	1.77	1.68	1.63	1.59	1.56	1.54	1.52	1.49	1.47	1.46	1.45	1.44	1.43	1.42
125	5.97	3.45	2.72	2.37	*2.17*	1.94	1.82	1.73	1.67	1.63	1.60	1.57	1.55	1.53	1.51	1.49	1.48	1.48	1.46	1.45
150	6.14	3.54	2.78	2.42	*2.22*	*1.98*	1.86	1.76	1.70	1.66	1.63	1.60	1.58	1.56	1.54	1.52	1.51	1.50	1.49	1.48
175	6.28	3.61	2.84	2.47	2.26	*2.02*	1.89	1.79	1.73	1.68	1.65	1.63	1.61	1.58	1.56	1.54	1.53	1.52	1.51	1.50
200	6.41	3.68	2.88	2.51	2.29	*2.05*	1.91	1.81	1.75	1.71	1.67	1.65	1.63	1.60	1.58	1.56	1.55	1.54	1.53	1.52

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (20 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.20	1.48	1.25	1.13	1.05	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
2	2.75	1.79	1.48	1.33	1.24	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
3	3.11	1.98	1.62	1.45	1.35	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
4	3.36	2.11	1.72	1.54	1.43	1.30	1.24	1.18	1.15	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
5	3.57	2.21	1.80	1.61	1.49	1.36	1.28	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
8	4.01	2.44	1.97	1.75	1.62	1.47	1.39	1.32	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	4.39	2.64	2.12	1.87	1.72	1.56	1.47	1.41	1.36	1.33	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20
16	4.67	2.78	2.22	1.96	1.80	1.63	1.53	1.46	1.42	1.38	1.36	1.34	1.33	1.31	1.29	1.28	1.27	1.27	1.26	1.25
20	4.88	2.89	*2.30*	2.03	1.86	1.68	1.58	1.51	1.46	1.42	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
30	5.27	3.09	*2.45*	2.15	1.97	1.77	1.67	1.58	1.53	1.50	1.47	1.45	1.43	1.41	1.39	1.38	1.37	1.37	1.35	1.34
40	5.55	3.23	2.56	2.24	2.05	1.84	1.73	1.64	1.59	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39
50	5.76	3.34	2.64	*2.30*	2.11	1.89	1.77	1.68	1.63	1.59	1.56	1.54	1.52	1.49	1.47	1.46	1.45	1.44	1.43	1.42
60	5.93	3.43	2.70	*2.36*	2.16	1.93	1.81	1.72	1.66	1.62	1.59	1.57	1.55	1.52	1.50	1.49	1.48	1.47	1.45	1.44
75	6.14	3.54	2.78	*2.42*	2.22	1.98	1.86	1.76	1.70	1.66	1.63	1.60	1.58	1.56	1.54	1.52	1.51	1.50	1.49	1.48
100	6.41	3.68	2.88	*2.51*	*2.29*	2.05	1.91	1.81	1.75	1.71	1.67	1.65	1.63	1.60	1.58	1.56	1.55	1.54	1.53	1.52
125	6.61	3.78	2.96	2.58	*2.35*	2.10	1.96	1.86	1.79	1.74	1.71	1.68	1.66	1.63	1.61	1.60	1.59	1.58	1.56	1.55
150	6.78	3.87	3.03	2.63	*2.40*	2.14	2.00	1.89	1.82	1.77	1.74	1.71	1.69	1.66	1.64	1.62	1.61	1.60	1.58	1.57
175	6.92	3.94	3.08	2.68	*2.44*	2.17	2.03	1.92	1.85	1.80	1.76	1.74	1.72	1.68	1.66	1.65	1.63	1.62	1.61	1.59
200	7.04	4.00	3.13	2.71	*2.47*	2.20	2.06	1.94	1.87	1.82	1.79	1.76	1.74	1.70	1.68	1.67	1.65	1.64	1.62	1.61

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (20 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.75	**1.79**	**1.48**	**1.33**	**1.24**	**1.14**	**1.08**	**1.04**	**1.01**	**0.99**	**0.97**	**0.96**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.91**	**0.90**
2	3.36	**2.11**	**1.72**	**1.54**	**1.43**	**1.30**	**1.24**	**1.18**	**1.15**	**1.13**	**1.11**	**1.09**	**1.08**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.03**	**1.02**
3	3.74	**2.30**	**1.87**	**1.66**	**1.54**	**1.40**	**1.32**	**1.27**	**1.23**	**1.20**	**1.18**	**1.17**	**1.16**	**1.14**	**1.13**	**1.12**	**1.11**	**1.11**	**1.10**	**1.09**
4	4.01	**2.44**	**1.97**	**1.75**	**1.62**	**1.47**	**1.39**	**1.32**	**1.28**	**1.26**	**1.24**	**1.22**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.16**	**1.15**	**1.14**
5	4.22	**2.55**	**2.05**	**1.82**	**1.68**	**1.52**	**1.43**	**1.37**	**1.33**	**1.30**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
8	4.67	2.78	**2.22**	**1.96**	**1.80**	**1.63**	**1.53**	**1.46**	**1.42**	**1.38**	**1.36**	**1.34**	**1.33**	**1.31**	**1.29**	**1.28**	**1.27**	**1.27**	**1.26**	**1.25**
12	5.06	2.98	**2.37**	**2.08**	**1.91**	**1.72**	**1.62**	**1.54**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	5.34	3.12	**2.47**	**2.17**	**1.99**	**1.79**	**1.68**	**1.60**	**1.54**	**1.51**	**1.48**	**1.46**	**1.45**	**1.42**	**1.41**	**1.39**	**1.38**	**1.38**	**1.36**	**1.35**
20	5.55	3.23	*2.56*	**2.24**	**2.05**	**1.84**	**1.73**	**1.64**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
30	5.93	3.43	*2.70*	**2.36**	**2.16**	**1.93**	**1.81**	**1.72**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.47**	**1.45**	**1.44**
40	6.20	3.57	2.81	**2.44**	**2.23**	**2.00**	**1.87**	**1.77**	**1.71**	**1.67**	**1.64**	**1.61**	**1.59**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**
50	6.41	3.68	2.88	**2.51**	**2.29**	**2.05**	**1.91**	**1.81**	**1.75**	**1.71**	**1.67**	**1.65**	**1.63**	**1.60**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
60	6.57	3.76	2.95	*2.56*	**2.34**	**2.09**	**1.95**	**1.85**	**1.78**	**1.74**	**1.70**	**1.68**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.57**	**1.55**	**1.54**
75	6.78	3.87	3.03	*2.63*	**2.40**	**2.14**	**2.00**	**1.89**	**1.82**	**1.77**	**1.74**	**1.71**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.60**	**1.58**	**1.57**
100	7.04	4.00	3.13	*2.71*	**2.47**	**2.20**	**2.06**	**1.94**	**1.87**	**1.82**	**1.79**	**1.76**	**1.74**	**1.70**	**1.68**	**1.67**	**1.65**	**1.64**	**1.62**	**1.61**
125	7.24	4.11	3.21	*2.78*	**2.53**	**2.25**	**2.10**	**1.98**	**1.91**	**1.86**	**1.82**	**1.79**	**1.77**	**1.74**	**1.71**	**1.70**	**1.68**	**1.67**	**1.65**	**1.64**
150	7.40	4.19	3.27	2.83	*2.58*	**2.29**	**2.14**	**2.02**	**1.94**	**1.89**	**1.85**	**1.82**	**1.80**	**1.76**	**1.74**	**1.72**	**1.71**	**1.70**	**1.68**	**1.67**
175	7.54	4.26	3.32	2.88	*2.62*	**2.33**	**2.17**	**2.05**	**1.97**	**1.92**	**1.88**	**1.85**	**1.82**	**1.79**	**1.76**	**1.74**	**1.73**	**1.72**	**1.70**	**1.69**
200	7.66	4.32	3.37	2.92	*2.65*	**2.36**	**2.19**	**2.07**	**1.99**	**1.94**	**1.90**	**1.87**	**1.84**	**1.81**	**1.78**	**1.76**	**1.75**	**1.74**	**1.72**	**1.70**

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (40 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.25	**1.49**	**1.25**	**1.13**	**1.05**	**0.97**	**0.92**	**0.89**	**0.86**	**0.85**	**0.83**	**0.82**	**0.82**	**0.81**	**0.80**	**0.79**	**0.79**	**0.78**	**0.78**	**0.77**
2	2.85	**1.81**	**1.49**	**1.34**	**1.25**	**1.14**	**1.08**	**1.04**	**1.01**	**0.99**	**0.97**	**0.96**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.91**	**0.90**
3	3.25	*2.01*	**1.64**	**1.46**	**1.35**	**1.24**	**1.17**	**1.12**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.01**	**1.00**	**0.99**	**0.99**	**0.98**	**0.97**
4	3.54	*2.15*	**1.74**	**1.55**	**1.44**	**1.31**	**1.24**	**1.18**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.03**	**1.02**
5	3.78	2.27	**1.82**	**1.62**	**1.50**	**1.36**	**1.29**	**1.23**	**1.19**	**1.17**	**1.15**	**1.14**	**1.13**	**1.11**	**1.10**	**1.09**	**1.08**	**1.08**	**1.07**	**1.06**
8	4.30	2.51	*2.00*	**1.76**	**1.62**	**1.47**	**1.39**	**1.33**	**1.29**	**1.26**	**1.24**	**1.22**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.16**	**1.15**	**1.14**
12	4.77	2.73	*2.16*	**1.89**	**1.74**	**1.57**	**1.48**	**1.41**	**1.36**	**1.33**	**1.31**	**1.29**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
16	5.11	2.89	2.27	*1.98*	**1.82**	**1.64**	**1.54**	**1.46**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.31**	**1.29**	**1.28**	**1.27**	**1.27**	**1.26**	**1.25**
20	5.37	3.01	2.35	*2.05*	**1.88**	**1.69**	**1.58**	**1.51**	**1.46**	**1.43**	**1.40**	**1.38**	**1.37**	**1.35**	**1.33**	**1.32**	**1.31**	**1.30**	**1.29**	**1.28**
30	5.86	3.23	2.51	*2.18*	*1.99*	**1.78**	**1.67**	**1.59**	**1.54**	**1.50**	**1.47**	**1.45**	**1.44**	**1.41**	**1.40**	**1.38**	**1.37**	**1.37**	**1.35**	**1.34**
40	6.21	3.39	2.63	2.27	*2.07*	**1.85**	**1.73**	**1.64**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
50	6.47	3.52	2.71	2.34	*2.13*	**1.90**	**1.78**	**1.69**	**1.63**	**1.59**	**1.56**	**1.54**	**1.52**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
60	6.70	3.62	2.79	2.40	*2.19*	*1.95*	**1.82**	**1.72**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.47**	**1.46**	**1.44**
75	6.95	3.75	2.87	2.48	2.25	*2.00*	**1.87**	**1.77**	**1.70**	**1.66**	**1.63**	**1.60**	**1.58**	**1.56**	**1.54**	**1.52**	**1.51**	**1.50**	**1.49**	**1.48**
100	7.30	3.91	2.99	2.57	2.33	*2.07*	**1.92**	**1.82**	**1.75**	**1.71**	**1.67**	**1.65**	**1.63**	**1.60**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
125	7.58	4.03	3.08	2.64	2.39	*2.12*	*1.97*	**1.86**	**1.79**	**1.75**	**1.71**	**1.69**	**1.67**	**1.63**	**1.61**	**1.60**	**1.59**	**1.58**	**1.56**	**1.55**
150	7.79	4.13	3.15	2.70	2.44	*2.16*	*2.01*	**1.90**	**1.83**	**1.78**	**1.74**	**1.72**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.60**	**1.58**	**1.57**
175	7.97	4.22	3.21	2.75	2.48	*2.19*	*2.04*	**1.93**	**1.85**	**1.80**	**1.77**	**1.74**	**1.72**	**1.69**	**1.66**	**1.65**	**1.63**	**1.62**	**1.61**	**1.59**
200	8.12	4.29	3.26	2.79	2.52	*2.23*	*2.07*	*1.95*	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.71**	**1.68**	**1.67**	**1.65**	**1.64**	**1.62**	**1.61**

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (40 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.85	**1.81**	**1.49**	**1.34**	**1.25**	**1.14**	**1.08**	**1.04**	**1.01**	**0.99**	**0.97**	**0.96**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.91**	**0.90**
2	3.54	**2.15**	**1.74**	**1.55**	**1.44**	**1.31**	**1.24**	**1.18**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.03**	**1.02**
3	3.98	*2.36*	**1.89**	**1.67**	**1.55**	**1.40**	**1.33**	**1.27**	**1.23**	**1.20**	**1.19**	**1.17**	**1.16**	**1.14**	**1.13**	**1.12**	**1.11**	**1.11**	**1.10**	**1.09**
4	4.30	2.51	**2.00**	**1.76**	**1.62**	**1.47**	**1.39**	**1.33**	**1.29**	**1.26**	**1.24**	**1.22**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.16**	**1.15**	**1.14**
5	4.56	2.63	**2.08**	**1.83**	**1.69**	**1.52**	**1.44**	**1.37**	**1.33**	**1.30**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
8	5.11	2.89	*2.27*	**1.98**	**1.82**	**1.64**	**1.54**	**1.46**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.31**	**1.29**	**1.28**	**1.27**	**1.27**	**1.26**	**1.25**
12	5.59	3.11	*2.42*	**2.11**	**1.93**	**1.73**	**1.62**	**1.54**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	5.94	3.27	2.54	**2.20**	**2.01**	**1.80**	**1.68**	**1.60**	**1.55**	**1.51**	**1.48**	**1.46**	**1.45**	**1.42**	**1.41**	**1.39**	**1.38**	**1.38**	**1.36**	**1.35**
20	6.21	3.39	2.63	*2.27*	**2.07**	**1.85**	**1.73**	**1.64**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
30	6.70	3.62	2.79	*2.40*	**2.19**	**1.95**	**1.82**	**1.72**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.47**	**1.46**	**1.44**
40	7.03	3.78	2.90	*2.50*	*2.27*	**2.01**	**1.88**	**1.78**	**1.71**	**1.67**	**1.64**	**1.61**	**1.59**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**
50	7.30	3.91	2.99	2.57	*2.33*	**2.07**	**1.92**	**1.82**	**1.75**	**1.71**	**1.67**	**1.65**	**1.63**	**1.60**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
60	7.52	4.01	3.06	2.63	*2.38*	**2.11**	**1.96**	**1.85**	**1.79**	**1.74**	**1.71**	**1.68**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.57**	**1.55**	**1.54**
75	7.79	4.13	3.15	2.70	*2.44*	**2.16**	**2.01**	**1.90**	**1.83**	**1.78**	**1.74**	**1.72**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.60**	**1.58**	**1.57**
100	8.12	4.29	3.26	2.79	*2.52*	**2.23**	**2.07**	**1.95**	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.71**	**1.68**	**1.67**	**1.65**	**1.64**	**1.62**	**1.61**
125	8.38	4.41	3.34	2.86	2.58	*2.28*	**2.11**	**1.99**	**1.92**	**1.86**	**1.82**	**1.80**	**1.77**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.65**	**1.64**
150	8.59	4.51	3.42	2.92	2.63	*2.32*	**2.15**	**2.03**	**1.95**	**1.89**	**1.85**	**1.82**	**1.80**	**1.77**	**1.74**	**1.72**	**1.71**	**1.70**	**1.68**	**1.67**
175	8.77	4.59	3.48	2.97	2.68	*2.36*	**2.18**	**2.06**	**1.98**	**1.92**	**1.88**	**1.85**	**1.82**	**1.79**	**1.76**	**1.74**	**1.73**	**1.72**	**1.70**	**1.69**
200	8.91	4.67	3.53	3.01	2.71	*2.39*	**2.21**	**2.08**	**2.00**	**1.94**	**1.90**	**1.87**	**1.84**	**1.81**	**1.78**	**1.76**	**1.75**	**1.74**	**1.72**	**1.70**

Table 19-3. κ-Multipliers for 1-of-4 Interwell Prediction Limits on Observations (40 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.54	**2.15**	**1.74**	**1.55**	**1.44**	**1.31**	**1.24**	**1.18**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.03**	**1.02**
2	4.30	**2.51**	**2.00**	**1.76**	**1.62**	**1.47**	**1.39**	**1.33**	**1.29**	**1.26**	**1.24**	**1.22**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.16**	**1.15**	**1.14**
3	4.77	2.73	**2.16**	**1.89**	**1.74**	**1.57**	**1.48**	**1.41**	**1.36**	**1.33**	**1.31**	**1.29**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
4	5.11	2.89	**2.27**	**1.98**	**1.82**	**1.64**	**1.54**	**1.46**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.31**	**1.29**	**1.28**	**1.27**	**1.27**	**1.26**	**1.25**
5	5.37	3.01	**2.35**	**2.05**	**1.88**	**1.69**	**1.58**	**1.51**	**1.46**	**1.43**	**1.40**	**1.38**	**1.37**	**1.35**	**1.33**	**1.32**	**1.31**	**1.30**	**1.29**	**1.28**
8	5.94	3.27	**2.54**	**2.20**	**2.01**	**1.80**	**1.68**	**1.60**	**1.55**	**1.51**	**1.48**	**1.46**	**1.45**	**1.42**	**1.41**	**1.39**	**1.38**	**1.38**	**1.36**	**1.35**
12	6.43	3.50	*2.70*	**2.33**	**2.12**	**1.89**	**1.77**	**1.68**	**1.62**	**1.58**	**1.55**	**1.53**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.42**	**1.41**
16	6.78	3.66	2.81	**2.42**	**2.20**	**1.96**	**1.83**	**1.73**	**1.67**	**1.63**	**1.60**	**1.58**	**1.56**	**1.53**	**1.51**	**1.50**	**1.49**	**1.48**	**1.46**	**1.45**
20	7.03	3.78	2.90	**2.50**	**2.27**	**2.01**	**1.88**	**1.78**	**1.71**	**1.67**	**1.64**	**1.61**	**1.59**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**
30	7.52	4.01	3.06	*2.63*	**2.38**	**2.11**	**1.96**	**1.85**	**1.79**	**1.74**	**1.71**	**1.68**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.57**	**1.55**	**1.54**
40	7.85	4.17	3.17	*2.72*	**2.46**	**2.18**	**2.02**	**1.91**	**1.84**	**1.79**	**1.75**	**1.72**	**1.70**	**1.67**	**1.65**	**1.63**	**1.62**	**1.61**	**1.59**	**1.58**
50	8.12	4.29	3.26	*2.79*	**2.52**	**2.23**	**2.07**	**1.95**	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.71**	**1.68**	**1.67**	**1.65**	**1.64**	**1.62**	**1.61**
60	8.34	4.39	3.33	2.85	*2.57*	**2.27**	**2.11**	**1.98**	**1.91**	**1.86**	**1.82**	**1.79**	**1.77**	**1.73**	**1.71**	**1.69**	**1.68**	**1.67**	**1.65**	**1.64**
75	8.59	4.51	3.42	2.92	*2.63*	**2.32**	**2.15**	**2.03**	**1.95**	**1.89**	**1.85**	**1.82**	**1.80**	**1.77**	**1.74**	**1.72**	**1.71**	**1.70**	**1.68**	**1.67**
100	8.91	4.67	3.53	3.01	*2.71*	**2.39**	**2.21**	**2.08**	**2.00**	**1.94**	**1.90**	**1.87**	**1.84**	**1.81**	**1.78**	**1.76**	**1.75**	**1.74**	**1.72**	**1.70**
125	9.16	4.79	3.61	3.08	*2.77*	**2.44**	**2.26**	**2.12**	**2.04**	**1.98**	**1.94**	**1.90**	**1.88**	**1.84**	**1.81**	**1.79**	**1.78**	**1.77**	**1.75**	**1.73**
150	9.38	4.88	3.68	3.13	*2.82*	**2.48**	**2.29**	**2.16**	**2.07**	**2.01**	**1.97**	**1.93**	**1.91**	**1.87**	**1.84**	**1.82**	**1.80**	**1.79**	**1.77**	**1.76**
175	9.53	4.96	3.74	3.18	*2.87*	**2.51**	**2.33**	**2.19**	**2.10**	**2.03**	**1.99**	**1.96**	**1.93**	**1.89**	**1.86**	**1.84**	**1.82**	**1.81**	**1.79**	**1.78**
200	9.69	5.03	3.79	3.23	2.90	*2.54*	**2.35**	**2.21**	**2.12**	**2.06**	**2.01**	**1.98**	**1.95**	**1.91**	**1.88**	**1.86**	**1.84**	**1.83**	**1.81**	**1.79**

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.71	0.59	0.53	0.50	0.48	0.45	0.44	0.43	0.42	0.41	0.41	0.41	0.40	0.40	0.40	0.40	0.39	0.39	0.39	0.39
2	1.07	0.88	0.81	0.76	0.73	0.70	0.68	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61	0.61
3	1.28	1.05	0.96	0.90	0.87	0.83	0.80	0.78	0.77	0.76	0.76	0.75	0.75	0.74	0.74	0.73	0.73	0.73	0.72	0.72
4	1.43	1.17	1.06	1.00	0.96	0.92	0.89	0.87	0.85	0.84	0.84	0.83	0.83	0.82	0.81	0.81	0.81	0.81	0.80	0.80
5	1.54	1.26	1.14	1.07	1.03	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87	0.87	0.86	0.86	0.86
8	1.78	1.44	1.30	1.23	1.18	1.12	1.09	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99	0.99	0.98	0.98	0.97	0.97
12	1.98	1.59	1.44	1.35	1.30	1.23	1.20	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.08	1.08	1.08	1.07	1.07
16	2.11	1.70	1.53	1.44	1.38	1.31	1.27	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13	1.13
20	2.22	1.78	1.60	1.51	1.44	1.37	1.33	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18	1.18
30	2.41	1.92	1.73	1.62	1.56	1.48	1.43	1.39	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.28	1.28	1.27	1.26
40	2.54	2.02	1.82	1.71	1.63	1.55	1.50	1.46	1.43	1.41	1.40	1.38	1.38	1.36	1.35	1.34	1.34	1.33	1.33	1.32
50	2.64	2.10	1.88	1.77	1.69	1.60	1.55	1.51	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.38	1.37	1.37
60	2.72	2.16	1.94	1.82	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.46	1.45	1.43	1.43	1.42	1.42	1.41	1.40
75	2.81	2.23	2.00	1.88	1.80	1.70	1.64	1.60	1.57	1.55	1.53	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44
100	2.93	2.32	2.08	1.95	1.87	1.77	1.71	1.66	1.63	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.50	1.50
125	3.03	2.39	2.15	2.01	1.93	1.82	1.76	1.71	1.67	1.65	1.63	1.62	1.61	1.59	1.58	1.57	1.56	1.56	1.54	1.54
150	3.10	2.45	2.20	2.06	1.97	1.86	1.80	1.75	1.71	1.69	1.67	1.65	1.64	1.62	1.61	1.60	1.59	1.59	1.58	1.57
175	3.16	2.50	2.24	2.10	2.01	1.90	1.83	1.78	1.74	1.72	1.70	1.68	1.67	1.65	1.64	1.63	1.62	1.62	1.61	1.60
200	3.21	2.54	2.27	2.13	2.04	1.93	1.86	1.81	1.77	1.74	1.72	1.71	1.70	1.68	1.67	1.66	1.65	1.64	1.63	1.62

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.07	0.88	0.81	0.76	0.73	0.70	0.68	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61	0.61
2	1.43	1.17	1.06	1.00	0.96	0.92	0.89	0.87	0.85	0.84	0.84	0.83	0.83	0.82	0.81	0.81	0.81	0.81	0.80	0.80
3	1.63	1.33	1.20	1.13	1.09	1.04	1.01	0.98	0.97	0.95	0.95	0.94	0.93	0.93	0.92	0.92	0.91	0.91	0.90	0.90
4	1.78	1.44	1.30	1.23	1.18	1.12	1.09	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99	0.99	0.98	0.98	0.97	0.97
5	1.89	1.52	1.38	1.30	1.25	1.18	1.15	1.12	1.10	1.09	1.08	1.07	1.06	1.05	1.05	1.04	1.04	1.03	1.03	1.02
8	2.11	1.70	1.53	1.44	1.38	1.31	1.27	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13	1.13
12	2.31	1.84	1.66	1.56	1.50	1.42	1.37	1.34	1.31	1.30	1.28	1.27	1.26	1.25	1.24	1.24	1.23	1.23	1.22	1.22
16	2.44	1.94	1.75	1.64	1.57	1.49	1.44	1.40	1.38	1.36	1.35	1.34	1.33	1.31	1.31	1.30	1.29	1.29	1.28	1.28
20	2.54	2.02	1.82	1.71	1.63	1.55	1.50	1.46	1.43	1.41	1.40	1.38	1.38	1.36	1.35	1.34	1.34	1.33	1.33	1.32
30	2.72	2.16	1.94	1.82	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.46	1.45	1.43	1.43	1.42	1.42	1.41	1.40
40	2.84	2.25	2.02	1.89	1.81	1.72	1.66	1.61	1.58	1.56	1.54	1.53	1.52	1.50	1.49	1.48	1.48	1.47	1.46	1.46
50	2.93	2.32	2.08	1.95	1.87	1.77	1.71	1.66	1.63	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.50	1.50
60	3.01	2.38	2.13	2.00	1.92	1.81	1.75	1.70	1.67	1.64	1.62	1.61	1.60	1.58	1.57	1.56	1.55	1.55	1.54	1.53
75	3.10	2.45	2.20	2.06	1.97	1.86	1.80	1.75	1.71	1.69	1.67	1.65	1.64	1.62	1.61	1.60	1.59	1.59	1.58	1.57
100	3.21	2.54	2.27	2.13	2.04	1.93	1.86	1.81	1.77	1.74	1.72	1.71	1.70	1.68	1.67	1.66	1.65	1.64	1.63	1.62
125	3.30	2.61	2.33	2.19	2.09	1.98	1.91	1.85	1.81	1.79	1.77	1.75	1.74	1.72	1.71	1.70	1.69	1.68	1.67	1.66
150	3.37	2.66	2.38	2.23	2.13	2.02	1.95	1.89	1.85	1.82	1.80	1.79	1.77	1.75	1.74	1.73	1.72	1.71	1.70	1.69
175	3.43	2.70	2.42	2.27	2.17	2.05	1.98	1.92	1.88	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.72
200	3.48	2.74	2.46	2.30	2.20	2.08	2.00	1.95	1.91	1.88	1.86	1.84	1.83	1.80	1.79	1.78	1.77	1.76	1.75	1.74

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.43	1.17	1.06	1.00	0.96	0.92	0.89	0.87	0.85	0.84	0.84	0.83	0.83	0.82	0.81	0.81	0.81	0.81	0.80	0.80
2	1.78	1.44	1.30	1.23	1.18	1.12	1.09	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99	0.99	0.98	0.98	0.97	0.97
3	1.98	1.59	1.44	1.35	1.30	1.23	1.20	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.08	1.08	1.08	1.07	1.07
4	2.11	1.70	1.53	1.44	1.38	1.31	1.27	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13	1.13
5	2.22	1.78	1.60	1.51	1.44	1.37	1.33	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18	1.18
8	2.44	1.94	1.75	1.64	1.57	1.49	1.44	1.40	1.38	1.36	1.35	1.34	1.33	1.31	1.31	1.30	1.29	1.29	1.28	1.28
12	2.62	2.08	1.87	1.76	1.68	1.59	1.54	1.50	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.38	1.37	1.36	1.36
16	2.74	2.18	1.96	1.83	1.76	1.66	1.61	1.56	1.53	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
20	2.84	2.25	2.02	1.89	1.81	1.72	1.66	1.61	1.58	1.56	1.54	1.53	1.52	1.50	1.49	1.48	1.48	1.47	1.46	1.46
30	3.01	2.38	2.13	2.00	1.92	1.81	1.75	1.70	1.67	1.64	1.62	1.61	1.60	1.58	1.57	1.56	1.55	1.55	1.54	1.53
40	3.13	2.47	2.21	2.07	1.99	1.88	1.81	1.76	1.73	1.70	1.68	1.67	1.65	1.64	1.62	1.61	1.61	1.60	1.59	1.58
50	3.21	2.54	2.27	2.13	2.04	1.93	1.86	1.81	1.77	1.74	1.72	1.71	1.70	1.68	1.67	1.66	1.65	1.64	1.63	1.62
60	3.29	2.59	2.32	2.18	2.08	1.97	1.90	1.84	1.81	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.68	1.67	1.66	1.65
75	3.37	2.66	2.38	2.23	2.13	2.02	1.95	1.89	1.85	1.82	1.80	1.79	1.77	1.75	1.74	1.73	1.72	1.71	1.70	1.69
100	3.48	2.74	2.46	2.30	2.20	2.08	2.00	1.95	1.91	1.88	1.86	1.84	1.83	1.80	1.79	1.78	1.77	1.76	1.75	1.74
125	3.57	2.81	2.51	2.35	2.25	2.12	2.05	1.99	1.95	1.92	1.90	1.88	1.87	1.84	1.83	1.82	1.81	1.80	1.79	1.78
150	3.63	2.86	2.56	2.40	2.29	2.16	2.09	2.02	1.98	1.95	1.93	1.91	1.90	1.88	1.86	1.85	1.84	1.83	1.82	1.81
175	3.69	2.90	2.60	2.43	2.32	2.19	2.12	2.05	2.01	1.98	1.96	1.94	1.93	1.90	1.89	1.87	1.86	1.86	1.84	1.83
200	3.74	2.94	2.63	2.46	2.35	2.22	2.14	2.08	2.04	2.01	1.98	1.96	1.95	1.93	1.91	1.90	1.89	1.88	1.86	1.85

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.14	0.92	0.83	0.78	0.74	0.71	0.68	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61	0.61
2	1.55	1.22	1.09	1.02	0.98	0.93	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.81	0.80	0.80
3	1.79	1.40	1.25	1.16	1.11	1.05	1.02	0.99	0.97	0.96	0.95	0.94	0.94	0.93	0.92	0.92	0.91	0.91	0.91	0.90
4	1.97	1.52	1.35	1.26	1.20	1.14	1.10	1.07	1.05	1.04	1.02	1.02	1.01	1.00	0.99	0.99	0.99	0.98	0.98	0.97
5	2.10	1.62	1.43	1.34	1.27	1.20	1.16	1.13	1.11	1.09	1.08	1.07	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.02
8	2.39	1.81	1.60	1.49	1.42	1.34	1.29	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
12	2.63	1.98	1.74	1.62	1.54	1.45	1.39	1.35	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.24	1.23	1.22	1.22
16	2.80	2.10	1.84	1.71	1.62	1.52	1.47	1.42	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.30	1.29	1.28	1.28
20	2.93	2.19	1.92	1.78	1.69	1.58	1.52	1.47	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.32
30	3.16	2.35	2.06	1.90	1.80	1.69	1.62	1.57	1.54	1.51	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.42	1.41	1.40
40	3.32	2.46	2.15	1.99	1.88	1.76	1.69	1.64	1.60	1.57	1.55	1.54	1.53	1.51	1.50	1.49	1.48	1.47	1.46	1.46
50	3.44	2.55	2.22	2.05	1.94	1.82	1.74	1.69	1.65	1.62	1.60	1.59	1.57	1.55	1.54	1.53	1.52	1.52	1.51	1.50
60	3.54	*2.62*	2.28	2.10	1.99	1.86	1.79	1.73	1.69	1.66	1.64	1.62	1.61	1.59	1.58	1.57	1.56	1.55	1.54	1.53
75	3.66	2.70	2.35	2.17	2.05	1.92	1.84	1.78	1.73	1.71	1.68	1.67	1.65	1.63	1.62	1.61	1.60	1.59	1.58	1.57
100	3.81	2.81	2.44	2.25	2.13	1.99	1.90	1.84	1.79	1.76	1.74	1.72	1.71	1.69	1.67	1.66	1.65	1.65	1.63	1.62
125	3.93	2.89	2.51	2.31	2.19	2.04	1.95	1.89	1.84	1.81	1.79	1.77	1.75	1.73	1.71	1.70	1.69	1.69	1.67	1.66
150	4.02	2.95	*2.57*	2.36	2.23	2.08	1.99	1.92	1.88	1.85	1.82	1.80	1.79	1.76	1.75	1.74	1.73	1.72	1.70	1.70
175	4.10	3.01	*2.61*	2.40	2.27	2.12	2.03	1.96	1.91	1.88	1.85	1.83	1.82	1.79	1.78	1.76	1.75	1.74	1.73	1.72
200	4.17	3.06	*2.65*	2.44	2.31	2.15	2.06	1.98	1.94	1.90	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.77	1.75	1.74

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.55	1.22	1.09	1.02	0.98	0.93	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.81	0.80	0.80
2	1.97	1.52	1.35	1.26	1.20	1.14	1.10	1.07	1.05	1.04	1.02	1.02	1.01	1.00	0.99	0.99	0.99	0.98	0.98	0.97
3	2.21	1.69	1.50	1.40	1.33	1.25	1.21	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
4	2.39	1.81	1.60	1.49	1.42	1.34	1.29	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
5	2.52	1.91	1.68	1.56	1.49	1.40	1.35	1.31	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.18	1.18
8	2.80	2.10	1.84	1.71	1.62	1.52	1.47	1.42	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.30	1.29	1.28	1.28
12	3.03	2.26	1.98	1.83	1.74	1.63	1.57	1.52	1.49	1.46	1.45	1.43	1.42	1.41	1.39	1.39	1.38	1.37	1.37	1.36
16	3.20	2.38	2.08	1.92	1.82	1.70	1.64	1.58	1.55	1.53	1.51	1.49	1.48	1.46	1.45	1.44	1.44	1.43	1.42	1.42
20	3.32	2.46	2.15	1.99	1.88	1.76	1.69	1.64	1.60	1.57	1.55	1.54	1.53	1.51	1.50	1.49	1.48	1.47	1.46	1.46
30	3.54	2.62	2.28	2.10	1.99	1.86	1.79	1.73	1.69	1.66	1.64	1.62	1.61	1.59	1.58	1.57	1.56	1.55	1.54	1.53
40	3.70	2.73	2.37	2.19	2.07	1.93	1.85	1.79	1.75	1.72	1.70	1.68	1.67	1.65	1.63	1.62	1.61	1.61	1.59	1.59
50	3.81	2.81	2.44	2.25	2.13	1.99	1.90	1.84	1.79	1.76	1.74	1.72	1.71	1.69	1.67	1.66	1.65	1.65	1.63	1.62
60	3.91	*2.87*	2.50	2.30	2.18	2.03	1.94	1.88	1.83	1.80	1.78	1.76	1.74	1.72	1.71	1.69	1.69	1.68	1.67	1.66
75	4.02	2.95	2.57	2.36	2.23	2.08	1.99	1.92	1.88	1.85	1.82	1.80	1.79	1.76	1.75	1.74	1.73	1.72	1.70	1.70
100	4.17	3.06	2.65	2.44	2.31	2.15	2.06	1.98	1.94	1.90	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.77	1.75	1.74
125	4.28	3.13	2.72	2.50	2.36	2.20	2.11	2.03	1.98	1.94	1.92	1.90	1.88	1.86	1.84	1.83	1.81	1.81	1.79	1.78
150	4.37	3.19	2.77	2.55	2.41	2.24	2.14	2.07	2.02	1.98	1.95	1.93	1.91	1.89	1.87	1.86	1.85	1.84	1.82	1.81
175	4.44	3.25	*2.82*	2.59	2.45	2.28	2.18	2.10	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.87	1.86	1.85	1.84
200	4.51	3.29	*2.85*	2.62	2.48	2.31	2.20	2.13	2.07	2.03	2.01	1.98	1.97	1.94	1.92	1.91	1.89	1.89	1.87	1.86

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.97	1.52	1.35	1.26	1.20	1.14	1.10	1.07	1.05	1.04	1.02	1.02	1.01	1.00	0.99	0.99	0.99	0.98	0.98	0.97
2	2.39	1.81	1.60	1.49	1.42	1.34	1.29	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
3	2.63	1.98	1.74	1.62	1.54	1.45	1.39	1.35	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.24	1.23	1.22	1.22
4	2.80	2.10	1.84	1.71	1.62	1.52	1.47	1.42	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.30	1.29	1.28	1.28
5	2.93	2.19	1.92	1.78	1.69	1.58	1.52	1.47	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.32
8	3.20	2.38	2.08	1.92	1.82	1.70	1.64	1.58	1.55	1.53	1.51	1.49	1.48	1.46	1.45	1.44	1.44	1.43	1.42	1.42
12	3.42	2.53	2.21	2.04	1.93	1.81	1.73	1.68	1.64	1.61	1.59	1.58	1.56	1.55	1.53	1.52	1.52	1.51	1.50	1.49
16	3.58	2.64	2.30	2.12	2.01	1.88	1.80	1.74	1.70	1.67	1.65	1.64	1.62	1.60	1.59	1.58	1.57	1.56	1.55	1.54
20	3.70	2.73	2.37	2.19	2.07	1.93	1.85	1.79	1.75	1.72	1.70	1.68	1.67	1.65	1.63	1.62	1.61	1.61	1.59	1.59
30	3.91	2.87	2.50	2.30	2.18	2.03	1.94	1.88	1.83	1.80	1.78	1.76	1.74	1.72	1.71	1.69	1.69	1.68	1.67	1.66
40	4.06	2.98	2.59	2.38	2.25	2.10	2.01	1.94	1.89	1.86	1.83	1.81	1.80	1.78	1.76	1.75	1.74	1.73	1.72	1.71
50	4.17	3.06	2.65	2.44	2.31	2.15	2.06	1.98	1.94	1.90	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.77	1.75	1.74
60	4.26	*3.12*	2.71	2.49	2.35	2.19	2.10	2.02	1.97	1.94	1.91	1.89	1.87	1.85	1.83	1.82	1.81	1.80	1.78	1.77
75	4.37	3.19	2.77	2.55	2.41	2.24	2.14	2.07	2.02	1.98	1.95	1.93	1.91	1.89	1.87	1.86	1.85	1.84	1.82	1.81
100	4.51	3.29	2.85	2.62	2.48	2.31	2.20	2.13	2.07	2.03	2.01	1.98	1.97	1.94	1.92	1.91	1.89	1.89	1.87	1.86
125	4.61	3.37	2.92	2.68	2.53	2.35	2.25	2.17	2.12	2.08	2.05	2.02	2.01	1.98	1.96	1.94	1.93	1.92	1.91	1.89
150	4.69	3.43	2.97	2.73	2.58	2.39	2.29	2.21	2.15	2.11	2.08	2.06	2.04	2.01	1.99	1.97	1.96	1.95	1.93	1.92
175	4.77	3.48	3.01	2.76	2.61	2.43	2.32	2.24	2.18	2.14	2.11	2.08	2.06	2.04	2.01	2.00	1.99	1.98	1.96	1.95
200	4.83	3.52	3.05	2.80	2.64	2.46	2.35	2.26	2.20	2.16	2.13	2.11	2.09	2.06	2.04	2.02	2.01	2.00	1.98	1.97

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.80	1.36	1.20	1.12	1.07	1.01	0.97	0.94	0.93	0.91	0.90	0.90	0.89	0.88	0.88	0.87	0.87	0.87	0.86	0.86
2	2.30	1.69	1.47	1.36	1.29	1.21	1.17	1.13	1.11	1.10	1.08	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
3	2.62	1.88	1.63	1.50	1.42	1.33	1.28	1.24	1.22	1.20	1.18	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.12
4	2.84	2.02	1.74	1.60	1.52	1.41	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
5	3.02	2.13	1.83	1.68	1.59	1.48	1.42	1.37	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.25	1.24	1.23	1.23
8	3.39	2.36	2.01	1.84	1.73	1.61	1.54	1.49	1.45	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.34	1.34	1.33	1.32
12	3.71	2.56	2.17	1.97	1.85	1.72	1.64	1.58	1.55	1.52	1.50	1.49	1.47	1.46	1.44	1.43	1.43	1.42	1.41	1.40
16	3.94	2.69	2.28	2.07	1.94	1.80	1.71	1.65	1.61	1.58	1.56	1.55	1.53	1.51	1.50	1.49	1.48	1.48	1.47	1.46
20	4.11	2.80	2.36	2.14	2.01	1.86	1.77	1.70	1.66	1.63	1.61	1.59	1.58	1.56	1.55	1.53	1.53	1.52	1.51	1.50
30	4.43	2.99	2.51	2.27	2.13	1.96	1.87	1.80	1.75	1.72	1.69	1.68	1.66	1.64	1.62	1.61	1.60	1.60	1.58	1.58
40	4.64	3.12	*2.62*	2.37	2.21	2.04	1.94	1.86	1.81	1.78	1.75	1.73	1.72	1.69	1.68	1.67	1.66	1.65	1.64	1.63
50	4.81	3.22	2.70	2.44	2.28	2.09	1.99	1.91	1.86	1.82	1.80	1.78	1.76	1.74	1.72	1.71	1.70	1.69	1.67	1.67
60	4.94	3.31	2.76	2.49	2.33	2.14	2.03	1.95	1.90	1.86	1.83	1.81	1.80	1.77	1.75	1.74	1.73	1.72	1.71	1.70
75	5.10	3.40	2.84	*2.56*	2.39	2.20	2.09	2.00	1.94	1.91	1.88	1.86	1.84	1.81	1.79	1.78	1.77	1.76	1.75	1.74
100	5.31	3.53	2.95	*2.65*	2.47	2.27	2.15	2.06	2.00	1.96	1.93	1.91	1.89	1.86	1.84	1.83	1.82	1.81	1.79	1.78
125	5.46	3.63	3.02	*2.72*	*2.54*	2.32	2.20	2.11	2.05	2.01	1.98	1.95	1.93	1.90	1.88	1.87	1.86	1.85	1.83	1.82
150	5.59	3.71	3.09	2.78	*2.59*	2.37	2.24	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.86	1.85
175	5.69	3.77	3.14	2.82	*2.63*	2.41	2.28	2.18	2.12	2.07	2.04	2.01	1.99	1.96	1.94	1.93	1.91	1.90	1.89	1.88
200	5.79	3.83	3.18	2.86	*2.67*	2.44	2.31	2.21	2.15	2.10	2.07	2.04	2.02	1.99	1.97	1.95	1.94	1.93	1.91	1.90

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.30	1.69	1.47	1.36	1.29	1.21	1.17	1.13	1.11	1.10	1.08	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
2	2.84	2.02	1.74	1.60	1.52	1.41	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
3	3.16	2.22	1.90	1.74	1.64	1.53	1.47	1.42	1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.28	1.27	1.27
4	3.39	2.36	2.01	1.84	1.73	1.61	1.54	1.49	1.45	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.34	1.34	1.33	1.32
5	3.57	2.47	2.10	1.91	1.80	1.67	1.60	1.54	1.50	1.48	1.46	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.37
8	3.94	2.69	2.28	2.07	1.94	1.80	1.71	1.65	1.61	1.58	1.56	1.55	1.53	1.51	1.50	1.49	1.48	1.48	1.47	1.46
12	4.26	*2.88*	2.43	2.20	2.06	1.90	1.81	1.75	1.70	1.67	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.55	1.54	1.53
16	4.47	3.02	2.54	2.29	2.15	1.98	1.88	1.81	1.76	1.73	1.71	1.69	1.67	1.65	1.64	1.62	1.62	1.61	1.60	1.59
20	4.64	3.12	2.62	2.37	2.21	2.04	1.94	1.86	1.81	1.78	1.75	1.73	1.72	1.69	1.68	1.67	1.66	1.65	1.64	1.63
30	4.94	3.31	2.76	2.49	2.33	2.14	2.03	1.95	1.90	1.86	1.83	1.81	1.80	1.77	1.75	1.74	1.73	1.72	1.71	1.70
40	5.15	3.43	*2.87*	2.58	2.41	2.21	2.10	2.01	1.96	1.92	1.89	1.87	1.85	1.82	1.80	1.79	1.78	1.77	1.76	1.75
50	5.31	3.53	*2.95*	2.65	2.47	2.27	2.15	2.06	2.00	1.96	1.93	1.91	1.89	1.86	1.84	1.83	1.82	1.81	1.79	1.78
60	5.43	3.61	3.01	2.71	2.53	2.31	2.19	2.10	2.04	2.00	1.97	1.94	1.93	1.90	1.88	1.86	1.85	1.84	1.82	1.81
75	5.59	3.71	3.09	2.78	2.59	2.37	2.24	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.86	1.85
100	5.79	3.83	3.18	*2.86*	2.67	2.44	2.31	2.21	2.15	2.10	2.07	2.04	2.02	1.99	1.97	1.95	1.94	1.93	1.91	1.90
125	5.94	3.92	3.26	*2.93*	2.73	2.49	2.36	2.26	2.19	2.14	2.11	2.08	2.06	2.03	2.00	1.99	1.97	1.96	1.95	1.93
150	6.05	4.00	3.32	*2.98*	2.77	2.54	2.40	2.29	2.23	2.18	2.14	2.11	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96
175	6.16	4.06	3.37	3.02	*2.82*	2.57	2.43	2.33	2.26	2.21	2.17	2.14	2.12	2.08	2.06	2.04	2.03	2.02	2.00	1.99
200	6.24	4.11	3.41	3.06	*2.85*	2.60	2.46	2.35	2.28	2.23	2.19	2.17	2.14	2.11	2.08	2.07	2.05	2.04	2.02	2.01

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.84**	**2.02**	**1.74**	**1.60**	**1.52**	**1.41**	**1.36**	**1.32**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**	**1.20**	**1.19**	**1.19**	**1.18**
2	3.39	**2.36**	**2.01**	**1.84**	**1.73**	**1.61**	**1.54**	**1.49**	**1.45**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.34**	**1.34**	**1.33**	**1.32**
3	3.71	**2.56**	**2.17**	**1.97**	**1.85**	**1.72**	**1.64**	**1.58**	**1.55**	**1.52**	**1.50**	**1.49**	**1.47**	**1.46**	**1.44**	**1.43**	**1.43**	**1.42**	**1.41**	**1.40**
4	3.94	**2.69**	**2.28**	**2.07**	**1.94**	**1.80**	**1.71**	**1.65**	**1.61**	**1.58**	**1.56**	**1.55**	**1.53**	**1.51**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
5	4.11	**2.80**	**2.36**	**2.14**	**2.01**	**1.86**	**1.77**	**1.70**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.53**	**1.53**	**1.52**	**1.51**	**1.50**
8	4.47	**3.02**	**2.54**	**2.29**	**2.15**	**1.98**	**1.88**	**1.81**	**1.76**	**1.73**	**1.71**	**1.69**	**1.67**	**1.65**	**1.64**	**1.62**	**1.62**	**1.61**	**1.60**	**1.59**
12	4.78	3.20	**2.68**	**2.42**	**2.27**	**2.08**	**1.98**	**1.90**	**1.85**	**1.82**	**1.79**	**1.77**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
16	4.99	3.33	**2.79**	**2.51**	**2.35**	**2.16**	**2.05**	**1.97**	**1.91**	**1.87**	**1.85**	**1.82**	**1.81**	**1.78**	**1.76**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
20	5.15	3.43	**2.87**	**2.58**	**2.41**	**2.21**	**2.10**	**2.01**	**1.96**	**1.92**	**1.89**	**1.87**	**1.85**	**1.82**	**1.80**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
30	5.43	3.61	**3.01**	**2.71**	**2.53**	**2.31**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.94**	**1.93**	**1.90**	**1.88**	**1.86**	**1.85**	**1.84**	**1.82**	**1.81**
40	5.63	3.73	*3.11*	**2.79**	**2.60**	**2.38**	**2.26**	**2.16**	**2.10**	**2.06**	**2.02**	**2.00**	**1.98**	**1.95**	**1.93**	**1.91**	**1.90**	**1.89**	**1.87**	**1.86**
50	5.79	3.83	*3.18*	**2.86**	**2.67**	**2.44**	**2.31**	**2.21**	**2.15**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.95**	**1.94**	**1.93**	**1.91**	**1.90**
60	5.91	3.90	3.25	**2.92**	**2.71**	**2.48**	**2.35**	**2.25**	**2.18**	**2.13**	**2.10**	**2.07**	**2.05**	**2.02**	**2.00**	**1.98**	**1.97**	**1.96**	**1.94**	**1.93**
75	6.05	4.00	3.32	**2.98**	**2.77**	**2.54**	**2.40**	**2.29**	**2.23**	**2.18**	**2.14**	**2.11**	**2.09**	**2.06**	**2.04**	**2.02**	**2.00**	**1.99**	**1.97**	**1.96**
100	6.24	4.11	3.41	**3.06**	**2.85**	**2.60**	**2.46**	**2.35**	**2.28**	**2.23**	**2.19**	**2.17**	**2.14**	**2.11**	**2.08**	**2.07**	**2.05**	**2.04**	**2.02**	**2.01**
125	6.39	4.20	3.49	*3.13*	**2.91**	**2.66**	**2.51**	**2.40**	**2.33**	**2.27**	**2.23**	**2.20**	**2.18**	**2.15**	**2.12**	**2.10**	**2.09**	**2.08**	**2.05**	**2.04**
150	6.50	4.27	3.54	*3.18*	**2.96**	**2.70**	**2.55**	**2.44**	**2.36**	**2.31**	**2.27**	**2.24**	**2.21**	**2.18**	**2.15**	**2.13**	**2.12**	**2.10**	**2.08**	**2.07**
175	6.60	4.33	3.59	*3.22*	**3.00**	**2.73**	**2.58**	**2.47**	**2.39**	**2.34**	**2.30**	**2.26**	**2.24**	**2.20**	**2.18**	**2.16**	**2.14**	**2.13**	**2.11**	**2.09**
200	6.68	4.39	3.64	*3.26*	**3.03**	**2.76**	**2.61**	**2.49**	**2.42**	**2.36**	**2.32**	**2.29**	**2.26**	**2.22**	**2.20**	**2.18**	**2.16**	**2.15**	**2.13**	**2.11**

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.40**	**1.72**	**1.49**	**1.37**	**1.30**	**1.22**	**1.17**	**1.14**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.04**	**1.03**	**1.03**
2	3.01	**2.08**	**1.77**	**1.62**	**1.53**	**1.42**	**1.36**	**1.32**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**	**1.20**	**1.19**	**1.19**	**1.18**
3	3.40	**2.29**	**1.94**	**1.76**	**1.66**	**1.54**	**1.47**	**1.42**	**1.39**	**1.37**	**1.35**	**1.34**	**1.33**	**1.31**	**1.30**	**1.29**	**1.29**	**1.28**	**1.27**	**1.27**
4	3.68	**2.44**	**2.05**	**1.86**	**1.75**	**1.62**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
5	3.89	**2.57**	**2.15**	**1.94**	**1.82**	**1.68**	**1.60**	**1.55**	**1.51**	**1.48**	**1.46**	**1.45**	**1.44**	**1.42**	**1.41**	**1.40**	**1.39**	**1.39**	**1.38**	**1.37**
8	4.36	2.82	**2.34**	**2.11**	**1.97**	**1.81**	**1.72**	**1.66**	**1.62**	**1.59**	**1.56**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
12	4.76	3.04	**2.51**	**2.25**	**2.09**	**1.92**	**1.83**	**1.75**	**1.71**	**1.67**	**1.65**	**1.63**	**1.62**	**1.60**	**1.58**	**1.57**	**1.56**	**1.56**	**1.54**	**1.54**
16	5.04	3.20	*2.62*	**2.35**	**2.18**	**2.00**	**1.90**	**1.82**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
20	5.26	3.32	2.72	**2.43**	**2.25**	**2.06**	**1.95**	**1.87**	**1.82**	**1.78**	**1.76**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**
30	5.65	3.53	2.88	*2.56*	**2.38**	**2.17**	**2.05**	**1.96**	**1.91**	**1.87**	**1.84**	**1.82**	**1.80**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**	**1.70**
40	5.92	3.69	3.00	*2.66*	**2.47**	**2.24**	**2.12**	**2.03**	**1.97**	**1.93**	**1.90**	**1.87**	**1.85**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
50	6.13	3.80	3.08	2.74	*2.53*	**2.30**	**2.17**	**2.08**	**2.01**	**1.97**	**1.94**	**1.91**	**1.90**	**1.87**	**1.85**	**1.83**	**1.82**	**1.81**	**1.80**	**1.78**
60	6.30	3.90	3.16	2.80	*2.59*	**2.35**	**2.22**	**2.12**	**2.05**	**2.01**	**1.97**	**1.95**	**1.93**	**1.90**	**1.88**	**1.86**	**1.85**	**1.84**	**1.83**	**1.81**
75	6.50	4.01	3.24	2.87	*2.66*	**2.41**	**2.27**	**2.17**	**2.10**	**2.05**	**2.02**	**1.99**	**1.97**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.86**	**1.85**
100	6.76	4.15	3.36	2.97	*2.74*	**2.48**	**2.34**	**2.23**	**2.16**	**2.11**	**2.07**	**2.05**	**2.02**	**1.99**	**1.97**	**1.95**	**1.94**	**1.93**	**1.91**	**1.90**
125	6.96	4.27	3.44	3.04	2.81	*2.54*	**2.39**	**2.28**	**2.20**	**2.15**	**2.12**	**2.09**	**2.06**	**2.03**	**2.01**	**1.99**	**1.98**	**1.97**	**1.95**	**1.93**
150	7.12	4.35	3.51	3.10	2.86	*2.58*	**2.43**	**2.32**	**2.24**	**2.19**	**2.15**	**2.12**	**2.10**	**2.06**	**2.04**	**2.02**	**2.01**	**2.00**	**1.98**	**1.96**
175	7.25	4.43	3.57	3.15	2.90	*2.62*	**2.47**	**2.35**	**2.27**	**2.22**	**2.18**	**2.15**	**2.12**	**2.09**	**2.06**	**2.05**	**2.03**	**2.02**	**2.00**	**1.99**
200	7.36	4.49	3.62	3.19	2.94	*2.66*	**2.50**	**2.38**	**2.30**	**2.24**	**2.20**	**2.17**	**2.15**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.02**	**2.01**

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.01	**2.08**	**1.77**	**1.62**	**1.53**	**1.42**	**1.36**	**1.32**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**	**1.20**	**1.19**	**1.19**	**1.18**
2	3.68	**2.44**	**2.05**	**1.86**	**1.75**	**1.62**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
3	4.08	**2.66**	**2.22**	**2.00**	**1.88**	**1.73**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.46**	**1.44**	**1.43**	**1.43**	**1.42**	**1.41**	**1.40**
4	4.36	**2.82**	**2.34**	**2.11**	**1.97**	**1.81**	**1.72**	**1.66**	**1.62**	**1.59**	**1.56**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
5	4.58	2.94	**2.43**	**2.18**	**2.04**	**1.87**	**1.78**	**1.71**	**1.67**	**1.64**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**	**1.51**	**1.50**
8	5.04	3.20	**2.62**	**2.35**	**2.18**	**2.00**	**1.90**	**1.82**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
12	5.44	3.41	**2.79**	**2.49**	**2.31**	**2.11**	**2.00**	**1.91**	**1.86**	**1.82**	**1.79**	**1.77**	**1.76**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
16	5.71	3.57	*2.91*	**2.59**	**2.40**	**2.18**	**2.07**	**1.98**	**1.92**	**1.88**	**1.85**	**1.83**	**1.81**	**1.78**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
20	5.92	3.69	3.00	**2.66**	**2.47**	**2.24**	**2.12**	**2.03**	**1.97**	**1.93**	**1.90**	**1.87**	**1.85**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
30	6.30	3.90	3.16	*2.80*	**2.59**	**2.35**	**2.22**	**2.12**	**2.05**	**2.01**	**1.97**	**1.95**	**1.93**	**1.90**	**1.88**	**1.86**	**1.85**	**1.84**	**1.83**	**1.81**
40	6.56	4.04	3.27	*2.90*	**2.67**	**2.42**	**2.29**	**2.18**	**2.11**	**2.07**	**2.03**	**2.00**	**1.98**	**1.95**	**1.93**	**1.91**	**1.90**	**1.89**	**1.87**	**1.86**
50	6.76	4.15	3.36	*2.97*	**2.74**	**2.48**	**2.34**	**2.23**	**2.16**	**2.11**	**2.07**	**2.05**	**2.02**	**1.99**	**1.97**	**1.95**	**1.94**	**1.93**	**1.91**	**1.90**
60	6.92	4.25	3.43	3.03	**2.79**	**2.53**	**2.38**	**2.27**	**2.20**	**2.14**	**2.11**	**2.08**	**2.06**	**2.02**	**2.00**	**1.98**	**1.97**	**1.96**	**1.94**	**1.93**
75	7.12	4.35	3.51	3.10	*2.86*	**2.58**	**2.43**	**2.32**	**2.24**	**2.19**	**2.15**	**2.12**	**2.10**	**2.06**	**2.04**	**2.02**	**2.01**	**2.00**	**1.98**	**1.96**
100	7.36	4.49	3.62	3.19	*2.94*	**2.66**	**2.50**	**2.38**	**2.30**	**2.24**	**2.20**	**2.17**	**2.15**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.02**	**2.01**
125	7.55	4.60	3.70	3.27	*3.01*	**2.71**	**2.55**	**2.42**	**2.34**	**2.29**	**2.24**	**2.21**	**2.19**	**2.15**	**2.12**	**2.10**	**2.09**	**2.08**	**2.06**	**2.04**
150	7.71	4.69	3.77	3.32	*3.06*	**2.76**	**2.59**	**2.46**	**2.38**	**2.32**	**2.28**	**2.25**	**2.22**	**2.18**	**2.15**	**2.13**	**2.12**	**2.11**	**2.08**	**2.07**
175	7.83	4.76	3.82	3.37	3.10	*2.80*	**2.62**	**2.49**	**2.41**	**2.35**	**2.31**	**2.27**	**2.25**	**2.21**	**2.18**	**2.16**	**2.14**	**2.13**	**2.11**	**2.09**
200	7.94	4.82	3.87	3.41	3.14	*2.83*	**2.65**	**2.52**	**2.44**	**2.38**	**2.33**	**2.30**	**2.27**	**2.23**	**2.20**	**2.18**	**2.16**	**2.15**	**2.13**	**2.11**

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.68	**2.44**	**2.05**	**1.86**	**1.75**	**1.62**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
2	4.36	**2.82**	**2.34**	**2.11**	**1.97**	**1.81**	**1.72**	**1.66**	**1.62**	**1.59**	**1.56**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
3	4.76	**3.04**	**2.51**	**2.25**	**2.09**	**1.92**	**1.83**	**1.75**	**1.71**	**1.67**	**1.65**	**1.63**	**1.62**	**1.60**	**1.58**	**1.57**	**1.56**	**1.56**	**1.54**	**1.54**
4	5.04	3.20	**2.62**	**2.35**	**2.18**	**2.00**	**1.90**	**1.82**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
5	5.26	3.32	**2.72**	**2.43**	**2.25**	**2.06**	**1.95**	**1.87**	**1.82**	**1.78**	**1.76**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**
8	5.71	3.57	**2.91**	**2.59**	**2.40**	**2.18**	**2.07**	**1.98**	**1.92**	**1.88**	**1.85**	**1.83**	**1.81**	**1.78**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
12	6.09	3.78	**3.07**	**2.72**	**2.52**	**2.29**	**2.16**	**2.07**	**2.01**	**1.96**	**1.93**	**1.91**	**1.89**	**1.86**	**1.84**	**1.83**	**1.81**	**1.80**	**1.79**	**1.78**
16	6.36	3.93	*3.18*	**2.82**	**2.61**	**2.37**	**2.23**	**2.13**	**2.07**	**2.02**	**1.99**	**1.96**	**1.94**	**1.91**	**1.89**	**1.88**	**1.86**	**1.85**	**1.84**	**1.83**
20	6.56	4.04	3.27	**2.90**	**2.67**	**2.42**	**2.29**	**2.18**	**2.11**	**2.07**	**2.03**	**2.00**	**1.98**	**1.95**	**1.93**	**1.91**	**1.90**	**1.89**	**1.87**	**1.86**
30	6.92	4.25	3.43	**3.03**	**2.79**	**2.53**	**2.38**	**2.27**	**2.20**	**2.14**	**2.11**	**2.08**	**2.06**	**2.02**	**2.00**	**1.98**	**1.97**	**1.96**	**1.94**	**1.93**
40	7.17	4.39	3.53	*3.12*	**2.88**	**2.60**	**2.45**	**2.33**	**2.25**	**2.20**	**2.16**	**2.13**	**2.11**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.97**
50	7.36	4.49	3.62	*3.19*	**2.94**	**2.66**	**2.50**	**2.38**	**2.30**	**2.24**	**2.20**	**2.17**	**2.15**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.02**	**2.01**
60	7.52	4.58	3.69	*3.25*	**2.99**	**2.70**	**2.54**	**2.41**	**2.33**	**2.28**	**2.24**	**2.21**	**2.18**	**2.14**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**
75	7.71	4.69	3.77	3.32	**3.06**	**2.76**	**2.59**	**2.46**	**2.38**	**2.32**	**2.28**	**2.25**	**2.22**	**2.18**	**2.15**	**2.13**	**2.12**	**2.11**	**2.08**	**2.07**
100	7.94	4.82	3.87	3.41	*3.14*	**2.83**	**2.65**	**2.52**	**2.44**	**2.38**	**2.33**	**2.30**	**2.27**	**2.23**	**2.20**	**2.18**	**2.16**	**2.15**	**2.13**	**2.11**
125	8.12	4.93	3.95	3.48	*3.20*	**2.88**	**2.70**	**2.57**	**2.48**	**2.42**	**2.37**	**2.34**	**2.31**	**2.27**	**2.24**	**2.22**	**2.20**	**2.19**	**2.16**	**2.15**
150	8.27	5.01	4.02	3.54	*3.25*	**2.93**	**2.74**	**2.60**	**2.51**	**2.45**	**2.40**	**2.37**	**2.34**	**2.30**	**2.27**	**2.24**	**2.23**	**2.21**	**2.19**	**2.17**
175	8.38	5.08	4.07	3.58	*3.29*	**2.96**	**2.78**	**2.64**	**2.54**	**2.48**	**2.43**	**2.39**	**2.37**	**2.32**	**2.29**	**2.27**	**2.25**	**2.24**	**2.21**	**2.20**
200	8.49	5.14	4.12	3.62	*3.33*	**2.99**	**2.81**	**2.66**	**2.57**	**2.50**	**2.46**	**2.42**	**2.39**	**2.34**	**2.31**	**2.29**	**2.27**	**2.26**	**2.23**	**2.22**

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.13	2.11	1.79	1.63	1.53	1.42	1.37	1.32	1.29	1.27	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
2	3.89	2.50	2.08	1.88	1.76	1.62	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
3	4.36	2.74	2.25	2.02	1.89	1.74	1.65	1.59	1.55	1.52	1.50	1.49	1.48	1.46	1.44	1.44	1.43	1.42	1.41	1.40
4	4.71	2.91	2.38	2.13	1.98	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
5	4.98	3.05	2.48	2.21	2.05	1.88	1.79	1.71	1.67	1.64	1.61	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
8	5.56	3.34	*2.69*	2.38	2.21	2.01	1.90	1.82	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
12	6.06	3.59	2.87	*2.53*	2.34	2.12	2.01	1.92	1.86	1.82	1.80	1.77	1.76	1.73	1.71	1.70	1.69	1.68	1.67	1.66
16	6.42	3.76	3.00	*2.64*	2.43	2.20	2.08	1.98	1.92	1.88	1.85	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
20	6.69	3.90	3.10	*2.72*	2.50	2.26	2.13	2.03	1.97	1.93	1.90	1.87	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
30	7.19	4.15	3.28	2.87	*2.63*	2.37	2.23	2.12	2.06	2.01	1.98	1.95	1.93	1.90	1.88	1.86	1.85	1.84	1.83	1.82
40	7.53	4.32	3.40	2.97	*2.73*	2.45	2.30	2.19	2.12	2.07	2.03	2.01	1.99	1.95	1.93	1.91	1.90	1.89	1.87	1.86
50	7.79	4.45	3.50	3.05	2.80	*2.51*	2.35	2.24	2.17	2.11	2.08	2.05	2.03	1.99	1.97	1.95	1.94	1.93	1.91	1.90
60	8.00	4.56	3.58	3.12	2.85	*2.56*	2.40	2.28	2.20	2.15	2.11	2.08	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
75	8.25	4.69	3.67	3.20	2.92	*2.62*	2.45	2.33	2.25	2.19	2.15	2.12	2.10	2.06	2.04	2.02	2.01	2.00	1.98	1.96
100	8.58	4.86	3.80	3.30	3.01	*2.70*	*2.52*	2.39	2.31	2.25	2.21	2.18	2.15	2.11	2.09	2.07	2.05	2.04	2.02	2.01
125	8.83	4.98	3.89	3.38	3.08	*2.75*	*2.57*	2.44	2.35	2.29	2.25	2.22	2.19	2.15	2.13	2.11	2.09	2.08	2.06	2.04
150	9.02	5.09	3.97	3.44	3.14	2.80	*2.62*	2.48	2.39	2.33	2.28	2.25	2.22	2.18	2.16	2.14	2.12	2.11	2.09	2.07
175	9.19	5.17	4.03	3.50	3.19	2.84	*2.65*	*2.51*	2.42	2.36	2.31	2.28	2.25	2.21	2.18	2.16	2.14	2.13	2.11	2.09
200	9.33	5.25	4.09	3.54	3.23	2.88	*2.68*	*2.54*	2.45	2.38	2.34	2.30	2.27	2.23	2.20	2.18	2.17	2.15	2.13	2.11

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.89	2.50	2.08	1.88	1.76	1.62	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
2	4.71	2.91	2.38	2.13	1.98	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
3	5.21	3.16	2.56	2.28	2.11	1.93	1.83	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
4	5.56	3.34	2.69	2.38	2.21	2.01	1.90	1.82	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
5	5.84	3.47	2.79	2.46	2.28	2.07	1.96	1.88	1.82	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
8	6.42	3.76	3.00	2.64	2.43	2.20	2.08	1.98	1.92	1.88	1.85	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
12	6.91	4.01	3.18	2.79	2.56	2.31	2.18	2.08	2.01	1.97	1.93	1.91	1.89	1.86	1.84	1.83	1.81	1.81	1.79	1.78
16	7.26	4.19	3.30	*2.89*	2.65	2.39	2.25	2.14	2.07	2.03	1.99	1.96	1.94	1.91	1.89	1.88	1.86	1.85	1.84	1.83
20	7.53	4.32	3.40	*2.97*	2.73	2.45	2.30	2.19	2.12	2.07	2.03	2.01	1.99	1.95	1.93	1.91	1.90	1.89	1.87	1.86
30	8.00	4.56	3.58	3.12	*2.85*	2.56	2.40	2.28	2.20	2.15	2.11	2.08	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
40	8.33	4.73	3.70	3.22	*2.94*	2.64	2.47	2.34	2.26	2.21	2.17	2.14	2.11	2.08	2.05	2.03	2.02	2.01	1.99	1.97
50	8.58	4.86	3.80	3.30	*3.01*	2.70	2.52	2.39	2.31	2.25	2.21	2.18	2.15	2.11	2.09	2.07	2.05	2.04	2.02	2.01
60	8.78	4.96	3.87	3.37	3.07	2.74	2.56	2.43	2.35	2.29	2.24	2.21	2.18	2.15	2.12	2.10	2.08	2.07	2.05	2.04
75	9.02	5.09	3.97	3.44	3.14	*2.80*	2.62	2.48	2.39	2.33	2.28	2.25	2.22	2.18	2.16	2.14	2.12	2.11	2.09	2.07
100	9.33	5.25	4.09	3.54	3.23	*2.88*	2.68	2.54	2.45	2.38	2.34	2.30	2.27	2.23	2.20	2.18	2.17	2.15	2.13	2.11
125	9.57	5.37	4.18	3.62	3.30	*2.93*	2.74	2.59	2.49	2.43	2.38	2.34	2.31	2.27	2.24	2.22	2.20	2.19	2.16	2.15
150	9.77	5.47	4.25	3.68	3.35	*2.98*	2.78	2.63	2.53	2.46	2.41	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.19	2.17
175	9.93	5.55	4.31	3.73	3.40	*3.02*	*2.81*	2.66	2.56	2.49	2.44	2.40	2.37	2.33	2.29	2.27	2.25	2.24	2.21	2.20
200	10.06	5.62	4.37	3.78	3.44	*3.05*	*2.85*	2.69	2.59	2.52	2.46	2.43	2.39	2.35	2.32	2.29	2.27	2.26	2.23	2.22

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.72	2.92	2.38	2.13	1.98	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
2	5.57	3.33	2.69	2.38	2.21	2.01	1.90	1.82	1.77	1.74	1.71	1.69	1.68	1.66	1.64	1.63	1.62	1.61	1.60	1.59
3	6.06	3.59	2.87	2.53	2.34	2.12	2.01	1.92	1.86	1.82	1.79	1.77	1.76	1.73	1.71	1.70	1.69	1.68	1.67	1.66
4	6.42	3.76	3.00	2.64	2.43	2.20	2.08	1.98	1.92	1.88	1.85	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
5	6.69	3.90	*3.10*	2.72	2.50	2.26	2.13	2.03	1.97	1.93	1.90	1.88	1.85	1.83	1.81	1.79	1.78	1.77	1.76	1.75
8	7.27	4.19	3.30	2.89	2.66	2.39	2.25	2.14	2.07	2.03	1.99	1.96	1.94	1.91	1.89	1.88	1.86	1.85	1.84	1.83
12	7.73	4.42	3.48	3.04	2.78	2.50	2.34	2.23	2.16	2.11	2.07	2.04	2.02	1.99	1.96	1.95	1.93	1.92	1.90	1.89
16	8.09	4.60	3.60	3.14	2.87	2.58	2.41	2.29	2.22	2.16	2.12	2.09	2.07	2.04	2.01	2.00	1.98	1.97	1.95	1.94
20	8.32	4.73	3.70	3.22	2.94	2.64	2.47	2.34	2.26	2.21	2.17	2.14	2.11	2.08	2.05	2.03	2.02	2.01	1.99	1.97
30	8.79	4.97	3.87	3.37	3.07	2.74	2.56	2.43	2.35	2.29	2.24	2.21	2.18	2.15	2.12	2.10	2.08	2.07	2.05	2.04
40	9.08	5.13	3.99	3.46	*3.16*	2.82	2.63	2.49	2.40	2.34	2.30	2.26	2.24	2.20	2.17	2.15	2.13	2.12	2.09	2.08
50	9.35	5.24	4.09	3.54	*3.23*	2.87	2.68	2.54	2.45	2.38	2.34	2.30	2.27	2.23	2.20	2.18	2.17	2.15	2.13	2.11
60	9.52	5.35	4.16	3.60	*3.28*	2.92	2.73	2.58	2.48	2.42	2.37	2.33	2.31	2.26	2.23	2.21	2.19	2.18	2.16	2.14
75	9.76	5.46	4.25	3.68	3.35	2.98	2.78	2.63	2.53	2.46	2.41	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.19	2.17
100	10.08	5.62	4.37	3.78	3.44	3.05	2.85	2.69	2.59	2.52	2.46	2.42	2.39	2.35	2.32	2.29	2.27	2.26	2.23	2.22
125	10.28	5.74	4.45	3.85	3.50	*3.11*	2.90	2.73	2.63	2.56	2.50	2.46	2.43	2.38	2.35	2.33	2.31	2.29	2.27	2.25
150	10.49	5.84	4.53	3.91	3.56	*3.16*	2.94	2.77	2.67	2.59	2.54	2.50	2.46	2.42	2.38	2.35	2.34	2.32	2.29	2.27
175	10.63	5.92	4.58	3.96	3.60	*3.20*	2.97	2.80	2.70	2.62	2.57	2.52	2.49	2.44	2.40	2.38	2.36	2.34	2.32	2.30
200	10.75	5.99	4.64	4.01	3.64	*3.23*	3.00	2.83	2.72	2.64	2.59	2.55	2.51	2.46	2.43	2.40	2.38	2.36	2.33	2.32

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.02	2.53	2.09	1.88	1.76	1.63	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
2	4.97	2.97	2.40	2.14	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
3	5.56	3.24	*2.59*	2.29	2.12	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
4	5.99	3.43	2.73	2.40	2.22	2.02	1.91	1.83	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
5	6.34	3.58	2.83	2.49	2.29	2.08	1.96	1.88	1.82	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
8	7.06	3.91	3.06	*2.67*	2.45	2.21	2.08	1.99	1.93	1.89	1.85	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
12	7.70	4.19	3.26	2.83	*2.59*	2.32	2.18	2.08	2.01	1.97	1.94	1.91	1.89	1.86	1.84	1.83	1.81	1.81	1.79	1.78
16	8.14	4.40	3.40	2.94	*2.68*	2.40	2.26	2.14	2.08	2.03	1.99	1.97	1.95	1.91	1.89	1.88	1.86	1.85	1.84	1.83
20	8.49	4.56	3.50	3.03	2.76	2.47	2.31	2.19	2.12	2.07	2.04	2.01	1.99	1.95	1.93	1.92	1.90	1.89	1.87	1.86
30	9.10	4.83	3.70	3.19	2.90	*2.58*	2.41	2.29	2.21	2.15	2.11	2.08	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
40	9.53	5.03	3.84	3.30	2.99	*2.66*	2.48	2.35	2.27	2.21	2.17	2.14	2.11	2.08	2.05	2.03	2.02	2.01	1.99	1.97
50	9.86	5.19	3.95	3.39	3.07	*2.72*	*2.54*	2.40	2.31	2.25	2.21	2.18	2.15	2.12	2.09	2.07	2.06	2.04	2.02	2.01
60	10.13	5.31	4.03	3.45	3.13	*2.77*	*2.58*	2.44	2.35	2.29	2.25	2.21	2.19	2.15	2.12	2.10	2.08	2.07	2.05	2.04
75	10.45	5.46	4.14	3.54	3.20	2.83	*2.64*	2.49	2.40	2.33	2.29	2.25	2.23	2.19	2.16	2.14	2.12	2.11	2.09	2.07
100	10.86	5.65	4.27	3.65	3.30	2.91	*2.71*	*2.55*	2.46	2.39	2.34	2.31	2.28	2.23	2.20	2.18	2.17	2.15	2.13	2.11
125	11.17	5.80	4.38	3.74	3.37	2.97	*2.76*	*2.60*	*2.50*	2.43	2.38	2.34	2.32	2.27	2.24	2.22	2.20	2.19	2.16	2.15
150	11.43	5.92	4.46	3.80	3.43	3.02	*2.80*	*2.64*	*2.54*	2.47	2.42	2.38	2.35	2.30	2.27	2.25	2.23	2.22	2.19	2.17
175	11.63	6.02	4.53	3.86	3.48	3.06	2.84	*2.67*	*2.57*	*2.50*	2.45	2.41	2.37	2.33	2.30	2.27	2.25	2.24	2.21	2.20
200	11.82	6.10	4.59	3.91	3.52	3.10	2.87	*2.70*	*2.60*	*2.52*	2.47	2.43	2.40	2.35	2.32	2.29	2.27	2.26	2.23	2.22

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.97	2.97	**2.40**	**2.14**	**1.99**	**1.82**	**1.73**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
2	5.99	3.43	**2.73**	**2.40**	**2.22**	**2.02**	**1.91**	**1.83**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
3	6.62	3.71	*2.92*	**2.56**	**2.35**	**2.13**	**2.01**	**1.92**	**1.86**	**1.82**	**1.80**	**1.77**	**1.76**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
4	7.06	3.91	3.06	**2.67**	**2.45**	**2.21**	**2.08**	**1.99**	**1.93**	**1.89**	**1.85**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
5	7.41	4.07	3.17	**2.76**	**2.53**	**2.27**	**2.14**	**2.04**	**1.98**	**1.93**	**1.90**	**1.88**	**1.86**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
8	8.14	4.40	3.40	*2.94*	**2.68**	**2.40**	**2.26**	**2.14**	**2.08**	**2.03**	**1.99**	**1.97**	**1.95**	**1.91**	**1.89**	**1.88**	**1.86**	**1.85**	**1.84**	**1.83**
12	8.76	4.68	3.59	3.10	*2.82*	**2.52**	**2.36**	**2.24**	**2.16**	**2.11**	**2.07**	**2.04**	**2.02**	**1.99**	**1.96**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**
16	9.20	4.88	3.73	3.21	*2.92*	**2.60**	**2.43**	**2.30**	**2.22**	**2.17**	**2.13**	**2.10**	**2.07**	**2.04**	**2.01**	**2.00**	**1.98**	**1.97**	**1.95**	**1.94**
20	9.53	5.03	3.84	3.30	*2.99*	**2.66**	**2.48**	**2.35**	**2.27**	**2.21**	**2.17**	**2.14**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.97**
30	10.13	5.31	4.03	3.45	3.13	**2.77**	**2.58**	**2.44**	**2.35**	**2.29**	**2.25**	**2.21**	**2.19**	**2.15**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**
40	10.54	5.50	4.17	3.56	3.22	*2.85*	**2.65**	**2.50**	**2.41**	**2.35**	**2.30**	**2.27**	**2.24**	**2.20**	**2.17**	**2.15**	**2.13**	**2.12**	**2.10**	**2.08**
50	10.86	5.65	4.27	3.65	3.30	*2.91*	**2.71**	**2.55**	**2.46**	**2.39**	**2.34**	**2.31**	**2.28**	**2.23**	**2.20**	**2.18**	**2.17**	**2.15**	**2.13**	**2.11**
60	11.11	5.77	4.36	3.72	3.36	*2.96*	**2.75**	**2.59**	**2.49**	**2.42**	**2.38**	**2.34**	**2.31**	**2.27**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
75	11.43	5.92	4.46	3.80	3.43	*3.02*	*2.80*	**2.64**	**2.54**	**2.47**	**2.42**	**2.38**	**2.35**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.19**	**2.17**
100	11.82	6.10	4.59	3.91	3.52	*3.10*	*2.87*	**2.70**	**2.60**	**2.52**	**2.47**	**2.43**	**2.40**	**2.35**	**2.32**	**2.29**	**2.27**	**2.26**	**2.23**	**2.22**
125	12.11	6.24	4.69	3.99	3.59	3.16	*2.92*	**2.75**	**2.64**	**2.57**	**2.51**	**2.47**	**2.44**	**2.39**	**2.35**	**2.33**	**2.31**	**2.29**	**2.27**	**2.25**
150	12.35	6.36	4.78	4.06	3.65	3.21	*2.97*	*2.79*	**2.68**	**2.60**	**2.54**	**2.50**	**2.47**	**2.42**	**2.38**	**2.36**	**2.34**	**2.32**	**2.29**	**2.28**
175	12.56	6.46	4.84	4.12	3.70	3.25	*3.00*	*2.82*	**2.71**	**2.63**	**2.57**	**2.53**	**2.49**	**2.44**	**2.41**	**2.38**	**2.36**	**2.34**	**2.32**	**2.30**
200	12.73	6.53	4.90	4.17	3.75	3.28	*3.04*	*2.85*	**2.73**	**2.65**	**2.60**	**2.55**	**2.52**	**2.46**	**2.43**	**2.40**	**2.38**	**2.36**	**2.34**	**2.32**

Table 19-4. κ-Multipliers for Modified Calif. Interwell Prediction Limits on Observations (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.99	3.43	**2.73**	**2.40**	**2.22**	**2.02**	**1.91**	**1.83**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
2	7.06	3.91	**3.06**	**2.67**	**2.45**	**2.21**	**2.08**	**1.99**	**1.93**	**1.89**	**1.85**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
3	7.70	4.19	3.26	**2.83**	**2.59**	**2.32**	**2.18**	**2.08**	**2.01**	**1.97**	**1.94**	**1.91**	**1.89**	**1.86**	**1.84**	**1.83**	**1.81**	**1.81**	**1.79**	**1.78**
4	8.14	4.40	3.40	**2.94**	**2.68**	**2.40**	**2.26**	**2.14**	**2.08**	**2.03**	**1.99**	**1.97**	**1.95**	**1.91**	**1.89**	**1.88**	**1.86**	**1.85**	**1.84**	**1.83**
5	8.49	4.56	3.50	**3.03**	**2.76**	**2.47**	**2.31**	**2.19**	**2.12**	**2.07**	**2.04**	**2.01**	**1.99**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.87**	**1.86**
8	9.20	4.88	3.73	*3.21*	**2.92**	**2.60**	**2.43**	**2.30**	**2.22**	**2.17**	**2.13**	**2.10**	**2.07**	**2.04**	**2.01**	**2.00**	**1.98**	**1.97**	**1.95**	**1.94**
12	9.80	5.16	3.93	3.37	**3.05**	**2.71**	**2.53**	**2.39**	**2.31**	**2.25**	**2.20**	**2.17**	**2.15**	**2.11**	**2.08**	**2.06**	**2.05**	**2.04**	**2.02**	**2.00**
16	10.21	5.35	4.06	3.48	*3.15*	**2.79**	**2.60**	**2.45**	**2.36**	**2.30**	**2.26**	**2.22**	**2.20**	**2.16**	**2.13**	**2.11**	**2.10**	**2.08**	**2.06**	**2.05**
20	10.54	5.50	4.17	3.56	*3.22*	**2.85**	**2.65**	**2.50**	**2.41**	**2.35**	**2.30**	**2.27**	**2.24**	**2.20**	**2.17**	**2.15**	**2.13**	**2.12**	**2.10**	**2.08**
30	11.11	5.77	4.36	3.72	3.36	**2.96**	**2.75**	**2.59**	**2.49**	**2.42**	**2.38**	**2.34**	**2.31**	**2.27**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
40	11.50	5.96	4.49	3.83	3.45	**3.04**	**2.82**	**2.65**	**2.55**	**2.48**	**2.43**	**2.39**	**2.36**	**2.31**	**2.28**	**2.26**	**2.24**	**2.23**	**2.20**	**2.18**
50	11.82	6.10	4.59	3.91	3.52	*3.10*	**2.87**	**2.70**	**2.60**	**2.52**	**2.47**	**2.43**	**2.40**	**2.35**	**2.32**	**2.29**	**2.27**	**2.26**	**2.23**	**2.22**
60	12.05	6.22	4.68	3.98	3.58	*3.15*	**2.92**	**2.74**	**2.63**	**2.56**	**2.50**	**2.46**	**2.43**	**2.38**	**2.35**	**2.32**	**2.30**	**2.29**	**2.26**	**2.24**
75	12.35	6.36	4.78	4.06	3.65	*3.21*	**2.97**	**2.79**	**2.68**	**2.60**	**2.54**	**2.50**	**2.47**	**2.42**	**2.38**	**2.36**	**2.34**	**2.32**	**2.29**	**2.28**
100	12.73	6.53	4.90	4.17	3.75	*3.28*	**3.04**	**2.85**	**2.73**	**2.65**	**2.60**	**2.55**	**2.52**	**2.46**	**2.43**	**2.40**	**2.38**	**2.36**	**2.34**	**2.32**
125	13.01	6.67	5.00	4.25	3.82	*3.34*	*3.09*	**2.90**	**2.78**	**2.70**	**2.64**	**2.59**	**2.55**	**2.50**	**2.46**	**2.43**	**2.41**	**2.40**	**2.37**	**2.35**
150	13.24	6.78	5.08	4.31	3.87	*3.39*	*3.13*	**2.94**	**2.81**	**2.73**	**2.67**	**2.62**	**2.58**	**2.53**	**2.49**	**2.46**	**2.44**	**2.42**	**2.39**	**2.37**
175	13.44	6.88	5.15	4.37	3.92	3.43	*3.17*	**2.97**	**2.84**	**2.76**	**2.70**	**2.65**	**2.61**	**2.55**	**2.51**	**2.49**	**2.46**	**2.45**	**2.42**	**2.40**
200	13.61	6.95	5.21	4.41	3.96	3.47	*3.20*	**3.00**	**2.87**	**2.78**	**2.72**	**2.67**	**2.63**	**2.58**	**2.54**	**2.51**	**2.48**	**2.47**	**2.43**	**2.41**

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D STATISTICAL TABLES

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Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.42	1.21	1.12	1.07	1.04	1.01	0.98	0.97	0.96	0.95	0.94	0.94	0.94	0.93	0.93	0.93	0.92	0.92	0.92	0.92
2	1.89	1.58	1.45	1.38	1.34	1.29	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.18	1.18	1.18	1.18	1.17	1.17
3	2.17	1.79	1.64	1.56	1.51	1.45	1.41	1.39	1.37	1.36	1.35	1.34	1.34	1.33	1.32	1.32	1.31	1.31	1.31	1.30
4	2.37	1.94	1.77	1.68	1.62	1.56	1.52	1.49	1.47	1.45	1.44	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.40	1.39
5	2.52	2.05	1.87	1.77	1.71	1.64	1.60	1.56	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.48	1.47	1.47	1.46	1.46
8	2.83	2.28	2.07	1.96	1.89	1.80	1.76	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61	1.60	1.60
12	3.09	2.48	2.24	2.12	2.04	1.94	1.89	1.85	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.73	1.72	1.71	1.71
16	3.27	2.61	2.36	2.22	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.80	1.79	1.78
20	3.40	2.71	2.45	2.31	2.22	2.11	2.05	2.00	1.97	1.94	1.93	1.91	1.90	1.89	1.88	1.87	1.86	1.86	1.85	1.84
30	3.64	2.89	2.61	2.45	2.36	2.24	2.17	2.12	2.08	2.06	2.04	2.02	2.01	1.99	1.98	1.97	1.96	1.96	1.95	1.94
40	3.81	3.02	2.72	2.55	2.45	2.33	2.26	2.20	2.16	2.13	2.11	2.10	2.09	2.07	2.05	2.04	2.04	2.03	2.02	2.01
50	3.93	3.11	2.80	2.63	2.52	2.40	2.32	2.26	2.22	2.19	2.17	2.16	2.14	2.12	2.11	2.10	2.09	2.08	2.07	2.06
60	4.03	3.19	2.87	2.69	2.58	2.45	2.37	2.31	2.27	2.24	2.22	2.20	2.19	2.17	2.15	2.14	2.13	2.13	2.11	2.11
75	4.15	3.28	2.95	2.77	2.65	2.52	2.43	2.37	2.33	2.30	2.28	2.26	2.24	2.22	2.21	2.19	2.19	2.18	2.17	2.16
100	4.31	3.40	*3.05*	2.86	2.74	2.60	2.51	2.45	2.40	2.37	2.35	2.33	2.31	2.29	2.27	2.26	2.25	2.24	2.23	2.22
125	4.42	3.48	3.13	2.93	2.81	2.66	2.57	2.51	2.46	2.43	2.40	2.38	2.37	2.34	2.32	2.31	2.30	2.29	2.28	2.27
150	4.51	3.55	3.19	2.99	2.86	2.71	2.62	2.55	2.50	2.47	2.44	2.42	2.41	2.38	2.37	2.35	2.34	2.33	2.32	2.31
175	4.59	3.61	3.24	*3.04*	2.91	2.76	2.66	2.59	2.54	2.51	2.48	2.46	2.44	2.42	2.40	2.39	2.38	2.37	2.35	2.34
200	4.66	3.66	3.28	*3.08*	2.95	2.79	2.70	2.62	2.58	2.54	2.51	2.49	2.47	2.45	2.43	2.42	2.41	2.40	2.38	2.37

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.89	1.58	1.45	1.38	1.34	1.29	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.18	1.18	1.18	1.18	1.17	1.17
2	2.37	1.94	1.77	1.68	1.62	1.56	1.52	1.49	1.47	1.45	1.44	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.40	1.39
3	2.64	2.14	1.95	1.84	1.78	1.70	1.66	1.62	1.60	1.59	1.57	1.56	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
4	2.83	2.28	2.07	1.96	1.89	1.80	1.76	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61	1.60	1.60
5	2.97	2.39	2.17	2.05	1.97	1.88	1.83	1.79	1.76	1.74	1.73	1.72	1.71	1.70	1.69	1.68	1.67	1.67	1.66	1.66
8	3.27	2.61	2.36	2.22	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.80	1.79	1.78
12	3.51	2.79	2.52	2.37	2.28	2.17	2.10	2.05	2.02	2.00	1.98	1.96	1.95	1.94	1.92	1.92	1.91	1.90	1.89	1.89
16	3.68	2.92	2.63	2.48	2.38	2.26	2.19	2.14	2.10	2.07	2.06	2.04	2.03	2.01	2.00	1.99	1.98	1.98	1.96	1.96
20	3.81	3.02	2.72	2.55	2.45	2.33	2.26	2.20	2.16	2.13	2.11	2.10	2.09	2.07	2.05	2.04	2.04	2.03	2.02	2.01
30	4.03	3.19	2.87	2.69	2.58	2.45	2.37	2.31	2.27	2.24	2.22	2.20	2.19	2.17	2.15	2.14	2.13	2.13	2.11	2.11
40	4.19	3.31	2.97	2.79	2.67	2.53	2.45	2.39	2.35	2.31	2.29	2.27	2.26	2.24	2.22	2.21	2.20	2.19	2.18	2.17
50	4.31	3.40	3.05	2.86	2.74	2.60	2.51	2.45	2.40	2.37	2.35	2.33	2.31	2.29	2.27	2.26	2.25	2.24	2.23	2.22
60	4.40	3.47	3.11	2.92	2.80	2.65	2.56	2.49	2.45	2.42	2.39	2.37	2.36	2.33	2.32	2.30	2.29	2.28	2.27	2.26
75	4.51	3.55	3.19	2.99	2.86	2.71	2.62	2.55	2.50	2.47	2.44	2.42	2.41	2.38	2.37	2.35	2.34	2.33	2.32	2.31
100	4.66	3.66	3.28	3.08	2.95	2.79	2.70	2.62	2.58	2.54	2.51	2.49	2.47	2.45	2.43	2.42	2.41	2.40	2.38	2.37
125	4.77	3.75	*3.36*	3.15	3.01	2.85	2.76	2.68	2.63	2.59	2.56	2.54	2.53	2.50	2.48	2.46	2.45	2.44	2.43	2.42
150	4.85	3.81	3.42	3.20	3.07	2.90	2.80	2.73	2.67	2.63	2.61	2.58	2.57	2.54	2.52	2.50	2.49	2.48	2.47	2.45
175	4.93	3.87	3.47	3.25	3.11	2.94	2.84	2.76	2.71	2.67	2.64	2.62	2.60	2.57	2.55	2.54	2.52	2.51	2.50	2.49
200	4.99	3.92	3.51	*3.29*	3.15	2.98	2.88	2.79	2.74	2.70	2.67	2.65	2.63	2.60	2.58	2.56	2.55	2.54	2.52	2.51

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.37	1.94	1.77	1.68	1.62	1.56	1.52	1.49	1.47	1.45	1.44	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.40	1.39
2	2.83	2.28	2.07	1.96	1.89	1.80	1.76	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61	1.60	1.60
3	3.09	2.48	2.24	2.12	2.04	1.94	1.89	1.85	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.73	1.72	1.71	1.71
4	3.27	2.61	2.36	2.22	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.80	1.79	1.78
5	3.40	2.71	2.45	2.31	2.22	2.11	2.05	2.00	1.97	1.94	1.93	1.91	1.90	1.89	1.88	1.87	1.86	1.86	1.85	1.84
8	3.68	2.92	2.63	2.48	2.38	2.26	2.19	2.14	2.10	2.07	2.06	2.04	2.03	2.01	2.00	1.99	1.98	1.98	1.96	1.96
12	3.91	3.10	2.78	2.62	2.51	2.38	2.31	2.25	2.21	2.18	2.16	2.15	2.13	2.11	2.10	2.09	2.08	2.07	2.06	2.05
16	4.07	3.22	2.89	2.71	2.60	2.47	2.39	2.33	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.15	2.14	2.13	2.12
20	4.19	3.31	2.97	2.79	2.67	2.53	2.45	2.39	2.35	2.31	2.29	2.27	2.26	2.24	2.22	2.21	2.20	2.19	2.18	2.17
30	4.40	3.47	3.11	2.92	2.80	2.65	2.56	2.49	2.45	2.42	2.39	2.37	2.36	2.33	2.32	2.30	2.29	2.28	2.27	2.26
40	4.55	3.58	3.21	3.01	2.88	2.73	2.64	2.57	2.52	2.49	2.46	2.44	2.42	2.40	2.38	2.37	2.36	2.35	2.33	2.32
50	4.66	3.66	3.28	3.08	2.95	2.79	2.70	2.62	2.58	2.54	2.51	2.49	2.47	2.45	2.43	2.42	2.41	2.40	2.38	2.37
60	4.75	3.73	3.34	3.13	3.00	2.84	2.75	2.67	2.62	2.58	2.56	2.53	2.52	2.49	2.47	2.46	2.44	2.44	2.42	2.41
75	4.85	3.81	3.42	3.20	3.07	2.90	2.80	2.73	2.67	2.63	2.61	2.58	2.57	2.54	2.52	2.50	2.49	2.48	2.47	2.45
100	4.99	3.92	3.51	3.29	3.15	2.98	2.88	2.79	2.74	2.70	2.67	2.65	2.63	2.60	2.58	2.56	2.55	2.54	2.52	2.51
125	5.09	4.00	3.58	3.35	3.21	3.03	2.93	2.85	2.79	2.75	2.72	2.70	2.68	2.65	2.63	2.61	2.60	2.59	2.57	2.56
150	5.18	4.06	3.63	3.40	3.26	3.08	2.98	2.89	2.83	2.79	2.76	2.74	2.72	2.69	2.67	2.65	2.64	2.63	2.61	2.59
175	5.24	4.11	3.68	3.45	3.30	3.12	3.01	2.93	2.87	2.83	2.80	2.77	2.75	2.72	2.70	2.68	2.67	2.66	2.64	2.62
200	5.30	4.16	3.72	3.49	3.34	3.15	3.04	2.96	2.90	2.86	2.82	2.80	2.78	2.75	2.72	2.71	2.69	2.68	2.66	2.65

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.01	1.63	1.48	1.41	1.36	1.30	1.27	1.25	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18	1.18	1.18	1.17	1.17
2	2.57	2.02	1.82	1.71	1.65	1.57	1.53	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.41	1.40	1.40	1.39
3	2.90	2.25	2.01	1.89	1.81	1.72	1.67	1.64	1.61	1.59	1.58	1.57	1.56	1.55	1.54	1.54	1.53	1.53	1.52	1.52
4	3.14	2.41	2.15	2.01	1.93	1.83	1.77	1.73	1.70	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.60
5	3.33	2.54	2.25	2.10	2.01	1.91	1.85	1.80	1.77	1.75	1.74	1.72	1.72	1.70	1.69	1.68	1.68	1.67	1.67	1.66
8	3.71	2.79	2.47	2.30	2.19	2.07	2.00	1.95	1.92	1.89	1.87	1.86	1.85	1.83	1.82	1.81	1.81	1.80	1.79	1.79
12	4.03	3.01	2.65	2.46	2.35	2.21	2.13	2.07	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.91	1.90	1.89
16	4.25	3.16	2.78	2.57	2.45	2.31	2.22	2.16	2.12	2.09	2.07	2.05	2.04	2.02	2.00	1.99	1.99	1.98	1.97	1.96
20	4.42	3.28	2.87	2.66	2.53	2.38	2.29	2.23	2.18	2.15	2.13	2.11	2.10	2.08	2.06	2.05	2.04	2.03	2.02	2.01
30	4.72	3.48	*3.04*	2.82	2.68	2.51	2.42	2.34	2.29	2.26	2.23	2.22	2.20	2.18	2.16	2.15	2.14	2.13	2.12	2.11
40	4.93	3.63	3.16	2.92	2.77	2.60	2.50	2.42	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.21	2.20	2.18	2.17
50	5.08	3.74	3.26	*3.01*	2.85	2.67	2.57	2.48	2.43	2.39	2.37	2.34	2.33	2.30	2.28	2.27	2.26	2.25	2.23	2.22
60	5.21	3.82	3.33	*3.07*	2.91	2.73	2.62	2.53	2.48	2.44	2.41	2.39	2.37	2.34	2.32	2.31	2.30	2.29	2.27	2.26
75	5.36	3.93	3.42	*3.15*	*2.99*	2.79	2.68	2.59	2.54	2.50	2.47	2.44	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.31
100	5.55	4.06	3.53	3.25	*3.08*	2.88	2.76	2.67	2.61	2.57	2.54	2.51	2.49	2.46	2.44	2.43	2.41	2.40	2.39	2.37
125	5.70	4.16	3.61	3.33	*3.15*	2.94	2.82	2.73	2.67	2.62	2.59	2.56	2.54	2.51	2.49	2.47	2.46	2.45	2.43	2.42
150	5.82	4.24	3.68	3.39	3.21	*3.00*	2.87	2.78	2.71	2.67	2.63	2.61	2.59	2.55	2.53	2.51	2.50	2.49	2.47	2.46
175	5.91	4.31	3.74	3.44	3.26	*3.04*	2.92	2.82	2.75	2.70	2.67	2.64	2.62	2.59	2.56	2.55	2.53	2.52	2.50	2.49
200	6.00	4.37	3.79	3.49	3.30	*3.08*	2.95	2.85	2.78	2.74	2.70	2.67	2.65	2.62	2.59	2.58	2.56	2.55	2.53	2.52

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.57**	**2.02**	**1.82**	**1.71**	**1.65**	**1.57**	**1.53**	**1.50**	**1.48**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**	**1.42**	**1.41**	**1.41**	**1.40**	**1.40**	**1.39**
2	3.14	**2.41**	**2.15**	**2.01**	**1.93**	**1.83**	**1.77**	**1.73**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.61**	**1.60**	**1.60**
3	3.47	**2.64**	**2.34**	**2.18**	**2.08**	**1.97**	**1.91**	**1.86**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.75**	**1.74**	**1.73**	**1.73**	**1.72**	**1.71**	**1.71**
4	3.71	**2.79**	**2.47**	**2.30**	**2.19**	**2.07**	**2.00**	**1.95**	**1.92**	**1.89**	**1.87**	**1.86**	**1.85**	**1.83**	**1.82**	**1.81**	**1.81**	**1.80**	**1.79**	**1.79**
5	3.88	**2.91**	**2.57**	**2.39**	**2.28**	**2.15**	**2.08**	**2.02**	**1.98**	**1.96**	**1.94**	**1.92**	**1.91**	**1.89**	**1.88**	**1.87**	**1.87**	**1.86**	**1.85**	**1.84**
8	4.25	**3.16**	**2.78**	**2.57**	**2.45**	**2.31**	**2.22**	**2.16**	**2.12**	**2.09**	**2.07**	**2.05**	**2.04**	**2.02**	**2.00**	**1.99**	**1.99**	**1.98**	**1.97**	**1.96**
12	4.56	3.37	**2.95**	**2.73**	**2.60**	**2.44**	**2.35**	**2.28**	**2.23**	**2.20**	**2.18**	**2.16**	**2.14**	**2.12**	**2.11**	**2.09**	**2.09**	**2.08**	**2.07**	**2.06**
16	4.77	3.52	**3.07**	**2.84**	**2.70**	**2.53**	**2.43**	**2.36**	**2.31**	**2.28**	**2.25**	**2.23**	**2.22**	**2.19**	**2.18**	**2.16**	**2.15**	**2.15**	**2.13**	**2.12**
20	4.93	3.63	**3.16**	**2.92**	**2.77**	**2.60**	**2.50**	**2.42**	**2.37**	**2.34**	**2.31**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**	**2.21**	**2.20**	**2.18**	**2.17**
30	5.21	3.82	*3.33*	**3.07**	**2.91**	**2.73**	**2.62**	**2.53**	**2.48**	**2.44**	**2.41**	**2.39**	**2.37**	**2.34**	**2.32**	**2.31**	**2.30**	**2.29**	**2.27**	**2.26**
40	5.40	3.96	3.44	**3.17**	**3.01**	**2.81**	**2.70**	**2.61**	**2.55**	**2.51**	**2.48**	**2.46**	**2.44**	**2.41**	**2.39**	**2.38**	**2.36**	**2.35**	**2.34**	**2.33**
50	5.55	4.06	3.53	**3.25**	**3.08**	**2.88**	**2.76**	**2.67**	**2.61**	**2.57**	**2.54**	**2.51**	**2.49**	**2.46**	**2.44**	**2.43**	**2.41**	**2.40**	**2.39**	**2.37**
60	5.67	4.14	3.60	*3.32*	**3.14**	**2.93**	**2.81**	**2.72**	**2.66**	**2.61**	**2.58**	**2.55**	**2.53**	**2.50**	**2.48**	**2.47**	**2.45**	**2.44**	**2.42**	**2.41**
75	5.82	4.24	3.68	*3.39*	**3.21**	**3.00**	**2.87**	**2.78**	**2.71**	**2.67**	**2.63**	**2.61**	**2.59**	**2.55**	**2.53**	**2.51**	**2.50**	**2.49**	**2.47**	**2.46**
100	6.00	4.37	3.79	3.49	*3.30*	**3.08**	**2.95**	**2.85**	**2.78**	**2.74**	**2.70**	**2.67**	**2.65**	**2.62**	**2.59**	**2.58**	**2.56**	**2.55**	**2.53**	**2.52**
125	6.13	4.47	3.87	3.56	*3.37*	**3.14**	**3.01**	**2.91**	**2.84**	**2.79**	**2.75**	**2.72**	**2.70**	**2.67**	**2.64**	**2.62**	**2.61**	**2.60**	**2.58**	**2.56**
150	6.24	4.54	3.94	3.62	*3.43*	**3.19**	**3.06**	**2.95**	**2.88**	**2.83**	**2.79**	**2.76**	**2.74**	**2.71**	**2.68**	**2.66**	**2.65**	**2.63**	**2.61**	**2.60**
175	6.34	4.61	3.99	3.67	3.47	**3.24**	**3.10**	**2.99**	**2.92**	**2.87**	**2.83**	**2.80**	**2.77**	**2.74**	**2.71**	**2.69**	**2.68**	**2.67**	**2.64**	**2.63**
200	6.42	4.66	4.04	3.72	3.51	*3.27*	**3.13**	**3.02**	**2.95**	**2.90**	**2.86**	**2.83**	**2.80**	**2.77**	**2.74**	**2.72**	**2.70**	**2.69**	**2.67**	**2.65**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**3.14**	**2.41**	**2.15**	**2.01**	**1.93**	**1.83**	**1.77**	**1.73**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.61**	**1.60**	**1.60**
2	3.71	**2.79**	**2.47**	**2.30**	**2.19**	**2.07**	**2.00**	**1.95**	**1.92**	**1.89**	**1.87**	**1.86**	**1.85**	**1.83**	**1.82**	**1.81**	**1.81**	**1.80**	**1.79**	**1.79**
3	4.03	**3.01**	**2.65**	**2.46**	**2.35**	**2.21**	**2.13**	**2.07**	**2.04**	**2.01**	**1.99**	**1.97**	**1.96**	**1.94**	**1.93**	**1.92**	**1.91**	**1.91**	**1.90**	**1.89**
4	4.25	**3.16**	**2.78**	**2.57**	**2.45**	**2.31**	**2.22**	**2.16**	**2.12**	**2.09**	**2.07**	**2.05**	**2.04**	**2.02**	**2.00**	**1.99**	**1.99**	**1.98**	**1.97**	**1.96**
5	4.42	**3.28**	**2.87**	**2.66**	**2.53**	**2.38**	**2.29**	**2.23**	**2.18**	**2.15**	**2.13**	**2.11**	**2.10**	**2.08**	**2.06**	**2.05**	**2.04**	**2.03**	**2.02**	**2.01**
8	4.77	**3.52**	**3.07**	**2.84**	**2.70**	**2.53**	**2.43**	**2.36**	**2.31**	**2.28**	**2.25**	**2.23**	**2.22**	**2.19**	**2.18**	**2.16**	**2.15**	**2.15**	**2.13**	**2.12**
12	5.06	3.72	**3.24**	**2.99**	**2.84**	**2.66**	**2.55**	**2.47**	**2.42**	**2.38**	**2.36**	**2.33**	**2.32**	**2.29**	**2.27**	**2.26**	**2.25**	**2.24**	**2.23**	**2.22**
16	5.25	3.85	**3.35**	**3.09**	**2.93**	**2.75**	**2.64**	**2.55**	**2.50**	**2.46**	**2.43**	**2.40**	**2.39**	**2.36**	**2.34**	**2.33**	**2.31**	**2.31**	**2.29**	**2.28**
20	5.40	3.96	**3.44**	**3.17**	**3.01**	**2.81**	**2.70**	**2.61**	**2.55**	**2.51**	**2.48**	**2.46**	**2.44**	**2.41**	**2.39**	**2.38**	**2.36**	**2.35**	**2.34**	**2.33**
30	5.67	4.14	*3.60*	**3.32**	**3.14**	**2.93**	**2.81**	**2.72**	**2.66**	**2.61**	**2.58**	**2.55**	**2.53**	**2.50**	**2.48**	**2.47**	**2.45**	**2.44**	**2.42**	**2.41**
40	5.86	4.27	3.71	**3.41**	**3.23**	**3.02**	**2.89**	**2.79**	**2.73**	**2.68**	**2.65**	**2.62**	**2.60**	**2.57**	**2.55**	**2.53**	**2.51**	**2.50**	**2.48**	**2.47**
50	6.00	4.37	3.79	**3.49**	**3.30**	**3.08**	**2.95**	**2.85**	**2.78**	**2.74**	**2.70**	**2.67**	**2.65**	**2.62**	**2.59**	**2.58**	**2.56**	**2.55**	**2.53**	**2.52**
60	6.10	4.45	3.86	**3.55**	**3.36**	**3.13**	**3.00**	**2.90**	**2.83**	**2.78**	**2.74**	**2.71**	**2.69**	**2.66**	**2.63**	**2.61**	**2.60**	**2.59**	**2.57**	**2.55**
75	6.24	4.54	3.94	*3.62*	**3.43**	**3.19**	**3.06**	**2.95**	**2.88**	**2.83**	**2.79**	**2.76**	**2.74**	**2.71**	**2.68**	**2.66**	**2.65**	**2.63**	**2.61**	**2.60**
100	6.42	4.66	4.04	3.72	**3.51**	**3.27**	**3.13**	**3.02**	**2.95**	**2.90**	**2.86**	**2.83**	**2.80**	**2.77**	**2.74**	**2.72**	**2.70**	**2.69**	**2.67**	**2.65**
125	6.55	4.76	4.12	3.79	**3.58**	**3.33**	**3.19**	**3.08**	**3.00**	**2.95**	**2.91**	**2.88**	**2.85**	**2.81**	**2.79**	**2.77**	**2.75**	**2.74**	**2.71**	**2.70**
150	6.65	4.83	4.18	3.84	*3.63*	**3.38**	**3.24**	**3.12**	**3.04**	**2.99**	**2.95**	**2.92**	**2.89**	**2.85**	**2.82**	**2.80**	**2.79**	**2.77**	**2.75**	**2.73**
175	6.74	4.89	4.23	3.89	*3.68*	**3.42**	**3.28**	**3.16**	**3.08**	**3.02**	**2.98**	**2.95**	**2.92**	**2.88**	**2.85**	**2.83**	**2.82**	**2.80**	**2.78**	**2.76**
200	6.82	4.94	4.28	3.93	*3.72*	**3.46**	**3.31**	**3.19**	**3.11**	**3.05**	**3.01**	**2.98**	**2.95**	**2.91**	**2.88**	**2.86**	**2.84**	**2.83**	**2.80**	**2.79**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.97	2.22	1.97	1.84	1.76	1.67	1.62	1.58	1.56	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.48	1.47	1.47	1.46
2	3.68	2.66	2.32	2.15	2.04	1.93	1.86	1.81	1.78	1.76	1.74	1.73	1.72	1.70	1.69	1.69	1.68	1.67	1.67	1.66
3	4.12	2.92	2.52	2.33	2.21	2.07	2.00	1.94	1.91	1.88	1.86	1.85	1.84	1.82	1.81	1.80	1.79	1.79	1.78	1.77
4	4.43	3.10	2.67	2.45	2.32	2.18	2.09	2.03	1.99	1.96	1.94	1.93	1.92	1.90	1.88	1.88	1.87	1.86	1.85	1.84
5	4.68	3.25	2.78	2.55	2.41	2.25	2.17	2.10	2.06	2.03	2.01	1.99	1.98	1.96	1.94	1.93	1.93	1.92	1.91	1.90
8	5.19	3.55	3.02	2.75	2.60	2.42	2.32	2.24	2.20	2.16	2.14	2.12	2.10	2.08	2.06	2.05	2.04	2.04	2.02	2.02
12	5.62	3.81	3.22	2.93	2.75	2.56	2.45	2.36	2.31	2.27	2.25	2.22	2.21	2.18	2.17	2.15	2.14	2.13	2.12	2.11
16	5.92	3.99	3.36	3.05	2.86	2.65	2.54	2.45	2.39	2.35	2.32	2.30	2.28	2.25	2.23	2.22	2.21	2.20	2.19	2.18
20	6.15	4.12	3.47	3.14	2.95	2.73	2.61	2.51	2.45	2.41	2.38	2.35	2.34	2.31	2.29	2.27	2.26	2.25	2.24	2.23
30	6.55	4.37	3.66	3.31	3.10	2.86	2.73	2.63	2.56	2.51	2.48	2.45	2.43	2.40	2.38	2.37	2.35	2.34	2.33	2.32
40	6.83	4.54	3.80	3.43	3.21	2.96	2.81	2.71	2.64	2.59	2.55	2.53	2.50	2.47	2.45	2.43	2.42	2.41	2.39	2.38
50	7.04	4.67	3.90	3.52	3.29	3.03	2.88	2.77	2.70	2.65	2.61	2.58	2.56	2.52	2.50	2.48	2.47	2.46	2.44	2.42
60	7.21	4.77	3.98	3.59	3.36	3.09	2.93	2.82	2.74	2.69	2.65	2.62	2.60	2.56	2.54	2.52	2.51	2.50	2.48	2.46
75	7.42	4.90	4.08	3.68	3.44	3.16	3.00	2.88	2.80	2.75	2.71	2.67	2.65	2.61	2.59	2.57	2.55	2.54	2.52	2.51
100	7.68	5.06	4.21	3.79	3.54	3.25	3.08	2.96	2.87	2.82	2.77	2.74	2.72	2.68	2.65	2.63	2.61	2.60	2.58	2.57
125	7.88	5.18	4.31	3.88	3.62	3.31	3.15	3.02	2.93	2.87	2.83	2.79	2.77	2.73	2.70	2.68	2.66	2.65	2.63	2.61
150	8.04	5.28	4.39	3.95	3.68	3.37	3.20	3.06	2.98	2.92	2.87	2.84	2.81	2.77	2.74	2.72	2.70	2.69	2.66	2.65
175	8.17	5.36	4.45	4.00	3.73	3.42	3.24	3.10	3.01	2.95	2.91	2.87	2.84	2.80	2.77	2.75	2.73	2.72	2.69	2.68
200	8.28	5.43	4.51	4.05	3.78	3.46	3.28	3.14	3.05	2.98	2.94	2.90	2.87	2.83	2.80	2.78	2.76	2.74	2.72	2.70

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.68	2.66	2.32	2.15	2.04	1.93	1.86	1.81	1.78	1.76	1.74	1.73	1.72	1.70	1.69	1.69	1.68	1.67	1.67	1.66
2	4.43	3.10	2.67	2.45	2.32	2.18	2.09	2.03	1.99	1.96	1.94	1.93	1.92	1.90	1.88	1.88	1.87	1.86	1.85	1.84
3	4.88	3.37	2.87	2.63	2.48	2.32	2.23	2.16	2.11	2.08	2.06	2.04	2.03	2.01	1.99	1.98	1.97	1.97	1.95	1.95
4	5.19	3.55	3.02	2.75	2.60	2.42	2.32	2.24	2.20	2.16	2.14	2.12	2.10	2.08	2.06	2.05	2.04	2.04	2.02	2.02
5	5.42	3.69	3.13	2.85	2.68	2.49	2.39	2.31	2.26	2.22	2.20	2.18	2.16	2.14	2.12	2.11	2.10	2.09	2.08	2.07
8	5.92	3.99	3.36	3.05	2.86	2.65	2.54	2.45	2.39	2.35	2.32	2.30	2.28	2.25	2.23	2.22	2.21	2.20	2.19	2.18
12	6.33	4.23	3.56	3.22	3.02	2.79	2.66	2.56	2.50	2.46	2.42	2.40	2.38	2.35	2.33	2.32	2.30	2.29	2.28	2.27
16	6.61	4.41	3.69	3.34	3.13	2.88	2.75	2.64	2.58	2.53	2.50	2.47	2.45	2.42	2.40	2.38	2.37	2.36	2.34	2.33
20	6.83	4.54	3.80	3.43	3.21	2.96	2.81	2.71	2.64	2.59	2.55	2.53	2.50	2.47	2.45	2.43	2.42	2.41	2.39	2.38
30	7.21	4.77	3.98	3.59	3.36	3.09	2.93	2.82	2.74	2.69	2.65	2.62	2.60	2.56	2.54	2.52	2.51	2.50	2.48	2.46
40	7.48	4.94	4.11	3.70	3.46	3.18	3.02	2.90	2.82	2.76	2.72	2.69	2.67	2.63	2.60	2.58	2.57	2.56	2.53	2.52
50	7.68	5.06	4.21	3.79	3.54	3.25	3.08	2.96	2.87	2.82	2.77	2.74	2.72	2.68	2.65	2.63	2.61	2.60	2.58	2.57
60	7.84	5.16	4.29	3.86	3.60	3.30	3.13	3.00	2.92	2.86	2.82	2.78	2.76	2.72	2.69	2.67	2.65	2.64	2.62	2.60
75	8.04	5.28	4.39	3.95	3.68	3.37	3.20	3.06	2.98	2.92	2.87	2.84	2.81	2.77	2.74	2.72	2.70	2.69	2.66	2.65
100	8.28	5.43	4.51	4.05	3.78	3.46	3.28	3.14	3.05	2.98	2.94	2.90	2.87	2.83	2.80	2.78	2.76	2.74	2.72	2.70
125	8.47	5.55	4.61	4.14	3.85	3.52	3.34	3.20	3.10	3.04	2.99	2.95	2.92	2.88	2.84	2.82	2.80	2.79	2.76	2.74
150	8.62	5.64	4.68	4.20	3.91	3.58	3.39	3.24	3.15	3.08	3.03	2.99	2.96	2.91	2.88	2.86	2.84	2.82	2.80	2.78
175	8.75	5.72	4.74	4.26	3.96	3.62	3.43	3.28	3.18	3.12	3.06	3.02	2.99	2.95	2.91	2.89	2.87	2.85	2.83	2.81
200	8.85	5.79	4.80	4.31	4.01	3.66	3.47	3.31	3.22	3.15	3.09	3.05	3.02	2.97	2.94	2.91	2.90	2.88	2.85	2.83

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.43	3.10	2.67	2.45	2.32	2.18	2.09	2.03	1.99	1.96	1.94	1.93	1.92	1.90	1.88	1.88	1.87	1.86	1.85	1.84
2	5.19	3.55	3.02	2.75	2.60	2.42	2.32	2.24	2.20	2.16	2.14	2.12	2.10	2.08	2.06	2.05	2.04	2.04	2.02	2.02
3	5.62	3.81	3.22	2.93	2.75	2.56	2.45	2.36	2.31	2.27	2.25	2.22	2.21	2.18	2.17	2.15	2.14	2.13	2.12	2.11
4	5.92	3.99	3.36	3.05	2.86	2.65	2.54	2.45	2.39	2.35	2.32	2.30	2.28	2.25	2.23	2.22	2.21	2.20	2.19	2.18
5	6.15	4.12	3.47	3.14	2.95	2.73	2.61	2.51	2.45	2.41	2.38	2.35	2.34	2.31	2.29	2.27	2.26	2.25	2.24	2.23
8	6.61	4.41	3.69	3.34	3.13	2.88	2.75	2.64	2.58	2.53	2.50	2.47	2.45	2.42	2.40	2.38	2.37	2.36	2.34	2.33
12	7.00	4.65	3.88	3.50	3.28	3.01	2.87	2.76	2.69	2.63	2.60	2.57	2.55	2.51	2.49	2.47	2.46	2.45	2.43	2.42
16	7.27	4.81	4.01	3.62	3.38	3.11	2.95	2.84	2.76	2.71	2.67	2.64	2.61	2.58	2.55	2.53	2.52	2.51	2.49	2.48
20	7.48	4.94	4.11	3.70	3.46	3.18	3.02	2.90	2.82	2.76	2.72	2.69	2.67	2.63	2.60	2.58	2.57	2.56	2.53	2.52
30	7.84	5.16	4.29	3.86	3.60	3.30	3.13	3.00	2.92	2.86	2.82	2.78	2.76	2.72	2.69	2.67	2.65	2.64	2.62	2.60
40	8.09	5.32	4.42	3.97	3.70	3.39	3.21	3.08	2.99	2.93	2.89	2.85	2.82	2.78	2.75	2.73	2.71	2.70	2.67	2.66
50	8.28	5.43	4.51	4.05	3.78	3.46	3.28	3.14	3.05	2.98	2.94	2.90	2.87	2.83	2.80	2.78	2.76	2.74	2.72	2.70
60	8.43	5.53	4.59	4.12	3.84	3.51	3.33	3.19	3.09	3.03	2.98	2.94	2.91	2.87	2.84	2.81	2.79	2.78	2.75	2.74
75	8.62	5.64	4.68	4.20	3.91	3.58	3.39	3.24	3.15	3.08	3.03	2.99	2.96	2.91	2.88	2.86	2.84	2.82	2.80	2.78
100	8.85	5.79	4.80	4.31	4.01	3.66	3.47	3.31	3.22	3.15	3.09	3.05	3.02	2.97	2.94	2.91	2.90	2.88	2.85	2.83
125	9.03	5.90	4.89	4.38	4.08	3.73	3.53	3.37	3.27	3.20	3.14	3.10	3.07	3.02	2.99	2.96	2.94	2.92	2.89	2.87
150	9.18	5.99	4.96	4.45	4.14	3.78	3.57	3.42	3.31	3.24	3.18	3.14	3.11	3.06	3.02	2.99	2.97	2.96	2.93	2.91
175	9.29	6.07	5.02	4.50	4.19	3.82	3.62	3.45	3.35	3.27	3.22	3.17	3.14	3.09	3.05	3.02	3.00	2.99	2.96	2.94
200	9.40	6.13	5.07	4.55	4.23	3.86	3.65	3.49	3.38	3.30	3.25	3.20	3.17	3.12	3.08	3.05	3.03	3.01	2.98	2.96

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.86	2.71	2.34	2.16	2.06	1.93	1.87	1.82	1.78	1.76	1.74	1.73	1.72	1.71	1.69	1.69	1.68	1.68	1.67	1.66
2	4.75	3.20	2.71	2.48	2.34	2.19	2.10	2.04	2.00	1.97	1.95	1.93	1.92	1.90	1.89	1.88	1.87	1.86	1.85	1.84
3	5.29	3.49	2.93	2.66	2.51	2.33	2.24	2.16	2.12	2.08	2.06	2.04	2.03	2.01	1.99	1.98	1.97	1.97	1.95	1.95
4	5.68	3.69	3.09	2.80	2.62	2.43	2.33	2.25	2.20	2.17	2.14	2.12	2.10	2.08	2.07	2.05	2.04	2.04	2.02	2.02
5	5.99	3.86	3.21	2.90	2.72	2.51	2.40	2.32	2.26	2.23	2.20	2.18	2.16	2.14	2.12	2.11	2.10	2.09	2.08	2.07
8	6.62	4.20	3.46	3.11	2.91	2.68	2.55	2.46	2.40	2.35	2.32	2.30	2.28	2.26	2.24	2.22	2.21	2.20	2.19	2.18
12	7.16	4.49	3.68	3.29	3.07	2.82	2.68	2.57	2.51	2.46	2.43	2.40	2.38	2.35	2.33	2.32	2.31	2.30	2.28	2.27
16	7.54	4.69	3.83	3.42	3.18	2.91	2.77	2.66	2.59	2.54	2.50	2.48	2.45	2.42	2.40	2.38	2.37	2.36	2.34	2.33
20	7.83	4.85	3.95	3.52	3.27	2.99	2.84	2.72	2.65	2.60	2.56	2.53	2.51	2.47	2.45	2.43	2.42	2.41	2.39	2.38
30	8.33	5.13	4.17	3.70	3.43	3.13	2.96	2.84	2.76	2.70	2.66	2.63	2.60	2.57	2.54	2.52	2.51	2.50	2.48	2.46
40	8.69	5.33	4.31	3.83	3.54	3.22	3.05	2.92	2.83	2.77	2.73	2.70	2.67	2.63	2.61	2.59	2.57	2.56	2.54	2.52
50	8.95	5.48	4.43	3.92	3.63	3.30	3.12	2.98	2.89	2.83	2.78	2.75	2.72	2.68	2.65	2.63	2.62	2.60	2.58	2.57
60	9.17	5.60	4.52	4.00	3.70	3.36	3.16	3.03	2.94	2.81	2.83	2.79	2.76	2.72	2.69	2.67	2.66	2.64	2.62	2.60
75	9.43	5.74	4.63	4.10	3.78	3.43	3.24	3.09	2.99	2.93	2.88	2.84	2.81	2.77	2.74	2.72	2.70	2.69	2.66	2.65
100	9.76	5.93	4.77	4.22	3.89	3.52	3.32	3.17	3.07	3.00	2.95	2.91	2.88	2.83	2.80	2.78	2.76	2.75	2.72	2.70
125	10.01	6.07	4.88	4.31	3.97	3.59	3.39	3.23	3.12	3.05	3.00	2.96	2.93	2.88	2.85	2.82	2.81	2.79	2.76	2.75
150	10.21	6.18	4.97	4.38	4.04	3.65	3.44	3.27	3.17	3.10	3.04	3.00	2.97	2.92	2.89	2.86	2.84	2.83	2.80	2.78
175	10.37	6.28	5.04	4.45	4.10	3.70	3.48	3.32	3.21	3.13	3.08	3.04	3.00	2.95	2.92	2.89	2.87	2.86	2.83	2.81
200	10.52	6.36	5.10	4.50	4.15	3.74	3.52	3.35	3.24	3.16	3.11	3.07	3.03	2.98	2.95	2.92	2.90	2.88	2.85	2.83

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.75	**3.20**	**2.71**	**2.48**	**2.34**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
2	5.68	3.69	**3.09**	**2.80**	**2.62**	**2.43**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**	**2.04**	**2.02**	**2.02**
3	6.23	3.99	*3.31*	**2.98**	**2.79**	**2.58**	**2.46**	**2.37**	**2.32**	**2.28**	**2.25**	**2.23**	**2.21**	**2.18**	**2.17**	**2.15**	**2.14**	**2.14**	**2.12**	**2.11**
4	6.62	4.20	3.46	**3.11**	**2.91**	**2.68**	**2.55**	**2.46**	**2.40**	**2.35**	**2.32**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
5	6.92	4.36	3.58	**3.21**	**3.00**	**2.75**	**2.62**	**2.52**	**2.46**	**2.41**	**2.38**	**2.36**	**2.34**	**2.31**	**2.29**	**2.27**	**2.26**	**2.25**	**2.24**	**2.23**
8	7.54	4.69	3.83	3.42	**3.18**	**2.91**	**2.77**	**2.66**	**2.59**	**2.54**	**2.50**	**2.48**	**2.45**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
12	8.06	4.98	4.05	3.60	*3.34*	**3.05**	**2.89**	**2.77**	**2.70**	**2.64**	**2.60**	**2.57**	**2.55**	**2.52**	**2.49**	**2.47**	**2.46**	**2.45**	**2.43**	**2.42**
16	8.42	5.18	4.20	3.73	*3.46*	**3.15**	**2.98**	**2.85**	**2.77**	**2.72**	**2.68**	**2.64**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
20	8.69	5.33	4.31	3.83	3.54	**3.22**	**3.05**	**2.92**	**2.83**	**2.77**	**2.73**	**2.70**	**2.67**	**2.63**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
30	9.17	5.60	4.52	4.00	3.70	*3.36*	**3.16**	**3.03**	**2.94**	**2.81**	**2.83**	**2.79**	**2.76**	**2.72**	**2.69**	**2.67**	**2.66**	**2.64**	**2.62**	**2.60**
40	9.50	5.78	4.66	4.12	3.81	*3.45*	**3.25**	**3.11**	**3.01**	**2.94**	**2.90**	**2.86**	**2.83**	**2.79**	**2.76**	**2.73**	**2.71**	**2.70**	**2.68**	**2.66**
50	9.76	5.93	4.77	4.22	3.89	*3.52*	*3.32*	**3.17**	**3.07**	**3.00**	**2.95**	**2.91**	**2.88**	**2.83**	**2.80**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**
60	9.96	6.04	4.86	4.29	3.96	3.58	*3.37*	**3.22**	**3.11**	**3.04**	**2.99**	**2.95**	**2.92**	**2.87**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
75	10.21	6.18	4.97	4.38	4.04	3.65	*3.44*	*3.27*	**3.17**	**3.10**	**3.04**	**3.00**	**2.97**	**2.92**	**2.89**	**2.86**	**2.84**	**2.83**	**2.80**	**2.78**
100	10.52	6.36	5.10	4.50	4.15	3.74	*3.52*	*3.35*	**3.24**	**3.16**	**3.11**	**3.07**	**3.03**	**2.98**	**2.95**	**2.92**	**2.90**	**2.88**	**2.85**	**2.83**
125	10.76	6.49	5.21	4.59	4.23	3.81	3.58	*3.41*	*3.30*	**3.22**	**3.16**	**3.12**	**3.08**	**3.03**	**2.99**	**2.96**	**2.94**	**2.93**	**2.90**	**2.88**
150	10.95	6.60	5.29	4.66	4.29	3.87	3.64	*3.46*	*3.34*	*3.26*	**3.20**	**3.16**	**3.12**	**3.07**	**3.03**	**3.00**	**2.98**	**2.96**	**2.93**	**2.91**
175	11.10	6.69	5.36	4.72	4.35	3.92	3.68	*3.50*	*3.38*	*3.30*	**3.24**	**3.19**	**3.15**	**3.10**	**3.06**	**3.03**	**3.01**	**2.99**	**2.96**	**2.94**
200	11.24	6.77	5.42	4.77	4.39	3.96	3.72	*3.53*	*3.41*	*3.33*	*3.27*	**3.22**	**3.18**	**3.12**	**3.09**	**3.06**	**3.03**	**3.02**	**2.98**	**2.96**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.68	3.69	**3.09**	**2.80**	**2.62**	**2.43**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**	**2.04**	**2.02**	**2.02**
2	6.62	4.20	**3.46**	**3.11**	**2.91**	**2.68**	**2.55**	**2.46**	**2.40**	**2.35**	**2.32**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
3	7.16	4.49	3.68	**3.29**	**3.07**	**2.82**	**2.68**	**2.57**	**2.51**	**2.46**	**2.43**	**2.40**	**2.38**	**2.35**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
4	7.54	4.69	3.83	**3.42**	**3.18**	**2.91**	**2.77**	**2.66**	**2.59**	**2.54**	**2.50**	**2.48**	**2.45**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
5	7.83	4.85	3.95	**3.52**	**3.27**	**2.99**	**2.84**	**2.72**	**2.65**	**2.60**	**2.56**	**2.53**	**2.51**	**2.47**	**2.45**	**2.43**	**2.42**	**2.41**	**2.39**	**2.38**
8	8.42	5.18	4.20	3.73	**3.46**	**3.15**	**2.98**	**2.85**	**2.77**	**2.72**	**2.68**	**2.64**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
12	8.91	5.45	4.41	3.91	*3.61*	**3.28**	**3.10**	**2.97**	**2.88**	**2.82**	**2.77**	**2.74**	**2.71**	**2.67**	**2.65**	**2.62**	**2.61**	**2.60**	**2.57**	**2.56**
16	9.25	5.64	4.55	4.03	*3.72*	**3.38**	**3.19**	**3.05**	**2.95**	**2.89**	**2.84**	**2.81**	**2.78**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
20	9.50	5.78	4.66	4.12	3.81	**3.45**	**3.25**	**3.11**	**3.01**	**2.94**	**2.90**	**2.86**	**2.83**	**2.79**	**2.76**	**2.73**	**2.71**	**2.70**	**2.68**	**2.66**
30	9.96	6.04	4.86	4.29	3.96	*3.58*	**3.37**	**3.22**	**3.11**	**3.04**	**2.99**	**2.95**	**2.92**	**2.87**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
40	10.28	6.22	5.00	4.41	4.06	*3.67*	**3.46**	**3.29**	**3.19**	**3.11**	**3.06**	**3.02**	**2.98**	**2.93**	**2.90**	**2.87**	**2.85**	**2.84**	**2.81**	**2.79**
50	10.52	6.36	5.10	4.50	4.15	*3.74*	**3.52**	**3.35**	**3.24**	**3.16**	**3.11**	**3.07**	**3.03**	**2.98**	**2.95**	**2.92**	**2.90**	**2.88**	**2.85**	**2.83**
60	10.71	6.47	5.19	4.57	4.21	*3.80*	**3.57**	**3.40**	**3.29**	**3.21**	**3.15**	**3.11**	**3.07**	**3.02**	**2.98**	**2.96**	**2.93**	**2.92**	**2.89**	**2.87**
75	10.95	6.60	5.29	4.66	4.29	3.87	*3.64*	**3.46**	**3.34**	**3.26**	**3.20**	**3.16**	**3.12**	**3.07**	**3.03**	**3.00**	**2.98**	**2.96**	**2.93**	**2.91**
100	11.24	6.77	5.42	4.77	4.39	3.96	*3.72*	**3.53**	**3.41**	**3.33**	**3.27**	**3.22**	**3.18**	**3.12**	**3.09**	**3.06**	**3.03**	**3.02**	**2.98**	**2.96**
125	11.47	6.90	5.52	4.86	4.47	4.02	*3.78*	*3.59*	**3.46**	**3.38**	**3.32**	**3.27**	**3.23**	**3.17**	**3.13**	**3.10**	**3.08**	**3.06**	**3.02**	**3.00**
150	11.65	7.00	5.60	4.93	4.53	4.08	*3.83*	*3.63*	**3.51**	**3.42**	**3.36**	**3.31**	**3.27**	**3.21**	**3.17**	**3.13**	**3.11**	**3.09**	**3.06**	**3.03**
175	11.80	7.09	5.67	4.99	4.58	4.12	*3.87*	*3.67*	**3.54**	**3.46**	**3.39**	**3.34**	**3.30**	**3.24**	**3.20**	**3.16**	**3.14**	**3.12**	**3.09**	**3.06**
200	11.93	7.16	5.73	5.04	4.63	4.16	3.90	*3.71*	*3.58*	**3.49**	**3.42**	**3.37**	**3.33**	**3.26**	**3.22**	**3.19**	**3.16**	**3.14**	**3.11**	**3.09**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.97	3.25	**2.74**	**2.49**	**2.35**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
2	6.07	3.79	3.13	**2.82**	**2.64**	**2.44**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
3	6.75	4.12	3.36	*3.01*	**2.81**	**2.59**	**2.47**	**2.38**	**2.32**	**2.28**	**2.25**	**2.23**	**2.21**	**2.19**	**2.17**	**2.15**	**2.14**	**2.14**	**2.12**	**2.11**
4	7.24	4.35	3.53	*3.15*	**2.93**	**2.69**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
5	7.62	4.54	3.66	3.26	*3.02*	**2.77**	**2.63**	**2.53**	**2.46**	**2.42**	**2.38**	**2.36**	**2.34**	**2.31**	**2.29**	**2.28**	**2.26**	**2.25**	**2.24**	**2.23**
8	8.41	4.92	3.94	3.48	3.22	**2.93**	**2.78**	**2.66**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
12	9.09	5.26	4.18	3.68	3.39	*3.08*	**2.91**	**2.78**	**2.70**	**2.65**	**2.61**	**2.58**	**2.55**	**2.52**	**2.49**	**2.47**	**2.46**	**2.45**	**2.43**	**2.42**
16	9.57	5.49	4.34	3.81	3.51	*3.18*	*3.00*	**2.86**	**2.78**	**2.72**	**2.68**	**2.65**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
20	9.93	5.67	4.47	3.92	3.60	3.25	*3.07*	**2.93**	**2.84**	**2.78**	**2.73**	**2.70**	**2.67**	**2.63**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
30	10.57	5.99	4.71	4.11	3.77	3.39	*3.19*	*3.04*	*2.95*	**2.88**	**2.83**	**2.80**	**2.77**	**2.72**	**2.70**	**2.67**	**2.66**	**2.64**	**2.62**	**2.60**
40	11.01	6.22	4.87	4.25	3.89	3.49	3.28	*3.12*	*3.02*	*2.95*	**2.90**	**2.86**	**2.83**	**2.79**	**2.76**	**2.73**	**2.72**	**2.70**	**2.68**	**2.66**
50	11.35	6.39	5.00	4.35	3.98	3.57	3.35	*3.18*	*3.08*	*3.01*	*2.95*	**2.89**	**2.88**	**2.84**	**2.80**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**
60	11.62	6.53	5.10	4.43	4.05	3.63	3.40	*3.23*	*3.13*	*3.05*	*3.00*	*2.96*	**2.92**	**2.88**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
75	11.95	6.69	5.22	4.54	4.14	3.71	3.47	3.30	*3.18*	*3.11*	*3.05*	*3.01*	*2.97*	**2.92**	**2.89**	**2.86**	**2.84**	**2.83**	**2.80**	**2.78**
100	12.36	6.91	5.38	4.67	4.26	3.80	3.56	3.37	*3.26*	*3.18*	*3.12*	*3.07*	*3.04*	*2.98*	*2.95*	**2.92**	**2.90**	**2.88**	**2.85**	**2.84**
125	12.68	7.07	5.50	4.77	4.34	3.88	3.62	3.43	3.31	*3.23*	*3.17*	*3.12*	*3.09*	*3.03*	*2.99*	*2.97*	*2.94*	**2.93**	**2.90**	**2.88**
150	12.93	7.20	5.59	4.85	4.42	3.94	3.68	3.48	3.36	*3.27*	*3.21*	*3.16*	*3.13*	*3.07*	*3.03*	*3.00*	*2.98*	*2.96*	**2.93**	**2.91**
175	13.14	7.31	5.67	4.91	4.48	3.99	3.72	3.52	3.40	3.31	*3.25*	*3.20*	*3.16*	*3.10*	*3.06*	*3.03*	*3.01*	*2.99*	*2.96*	*2.94*
200	13.32	7.40	5.74	4.97	4.53	4.03	3.76	3.56	3.43	3.34	*3.28*	*3.23*	*3.19*	*3.13*	*3.09*	*3.06*	*3.04*	*3.02*	*2.98*	*2.96*

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.07	3.79	**3.13**	**2.82**	**2.64**	**2.44**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
2	7.24	4.35	3.53	**3.15**	**2.93**	**2.69**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
3	7.93	4.69	3.77	*3.34*	**3.10**	**2.83**	**2.69**	**2.58**	**2.51**	**2.47**	**2.43**	**2.41**	**2.39**	**2.36**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
4	8.41	4.92	3.94	3.48	**3.22**	**2.93**	**2.78**	**2.66**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
5	8.79	5.11	4.07	3.59	*3.31*	**3.01**	**2.85**	**2.73**	**2.65**	**2.60**	**2.56**	**2.53**	**2.51**	**2.48**	**2.45**	**2.43**	**2.42**	**2.41**	**2.39**	**2.38**
8	9.57	5.49	4.34	3.81	3.51	**3.18**	**3.00**	**2.86**	**2.78**	**2.72**	**2.68**	**2.65**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
12	10.22	5.82	4.58	4.01	3.68	*3.32*	**3.12**	**2.98**	**2.89**	**2.82**	**2.78**	**2.74**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.60**	**2.57**	**2.56**
16	10.67	6.04	4.74	4.14	3.80	*3.42*	**3.21**	**3.06**	**2.96**	**2.90**	**2.85**	**2.81**	**2.78**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
20	11.01	6.22	4.87	4.25	3.89	*3.49*	*3.28*	**3.12**	**3.02**	**2.95**	**2.90**	**2.86**	**2.83**	**2.79**	**2.76**	**2.73**	**2.72**	**2.70**	**2.68**	**2.66**
30	11.62	6.53	5.10	4.43	4.05	3.63	*3.40*	**3.23**	**3.13**	**3.05**	**3.00**	**2.96**	**2.92**	**2.88**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
40	12.04	6.74	5.25	4.57	4.17	3.73	*3.49*	*3.31*	**3.20**	**3.12**	**3.07**	**3.02**	**2.99**	**2.94**	**2.90**	**2.88**	**2.86**	**2.84**	**2.81**	**2.79**
50	12.36	6.91	5.38	4.67	4.26	3.80	*3.56*	*3.37*	*3.26*	**3.18**	**3.12**	**3.07**	**3.04**	**2.98**	**2.95**	**2.92**	**2.90**	**2.88**	**2.85**	**2.84**
60	12.62	7.04	5.47	4.75	4.33	3.86	3.61	*3.42*	*3.30*	**3.22**	**3.16**	**3.11**	**3.08**	**3.02**	**2.99**	**2.96**	**2.94**	**2.92**	**2.89**	**2.87**
75	12.93	7.20	5.59	4.85	4.42	3.94	3.68	*3.48*	*3.36*	*3.27*	**3.21**	**3.16**	**3.13**	**3.07**	**3.03**	**3.00**	**2.98**	**2.96**	**2.93**	**2.91**
100	13.32	7.40	5.74	4.97	4.53	4.03	3.76	*3.56*	*3.43*	*3.34*	*3.28*	**3.23**	**3.19**	**3.13**	**3.09**	**3.06**	**3.04**	**3.02**	**2.98**	**2.96**
125	13.62	7.56	5.86	5.07	4.61	4.10	3.83	3.62	*3.49*	*3.39*	*3.33*	*3.28*	**3.24**	**3.18**	**3.13**	**3.10**	**3.08**	**3.06**	**3.03**	**3.00**
150	13.86	7.68	5.95	5.15	4.68	4.16	3.88	3.67	*3.53*	*3.44*	*3.37*	*3.32*	*3.27*	**3.21**	**3.17**	**3.14**	**3.11**	**3.09**	**3.06**	**3.04**
175	14.06	7.79	6.03	5.21	4.74	4.21	3.93	3.71	*3.57*	*3.47*	*3.40*	*3.35*	*3.31*	**3.24**	**3.20**	**3.17**	**3.14**	**3.12**	**3.09**	**3.06**
200	14.23	7.88	6.10	5.27	4.79	4.26	3.96	3.74	*3.60*	*3.50*	*3.43*	*3.38*	*3.33*	*3.27*	**3.23**	**3.19**	**3.17**	**3.15**	**3.11**	**3.09**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	7.24	4.35	**3.53**	**3.15**	**2.93**	**2.69**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
2	8.41	4.92	3.94	**3.48**	**3.22**	**2.93**	**2.78**	**2.66**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
3	9.09	5.26	4.18	*3.68*	**3.39**	**3.08**	**2.91**	**2.78**	**2.70**	**2.65**	**2.61**	**2.58**	**2.55**	**2.52**	**2.49**	**2.47**	**2.46**	**2.45**	**2.43**	**2.42**
4	9.57	5.49	4.34	3.81	**3.51**	**3.18**	**3.00**	**2.86**	**2.78**	**2.72**	**2.68**	**2.65**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
5	9.93	5.67	4.47	3.92	*3.60*	**3.25**	**3.07**	**2.93**	**2.84**	**2.78**	**2.73**	**2.70**	**2.67**	**2.63**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
8	10.67	6.04	4.74	4.14	3.80	**3.42**	**3.21**	**3.06**	**2.96**	**2.90**	**2.85**	**2.81**	**2.78**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
12	11.29	6.36	4.97	4.33	3.96	**3.56**	**3.34**	**3.17**	**3.07**	**3.00**	**2.94**	**2.91**	**2.87**	**2.83**	**2.80**	**2.77**	**2.75**	**2.74**	**2.71**	**2.70**
16	11.72	6.57	5.13	4.46	4.08	*3.65*	**3.42**	**3.25**	**3.14**	**3.07**	**3.01**	**2.97**	**2.94**	**2.89**	**2.86**	**2.83**	**2.81**	**2.80**	**2.77**	**2.75**
20	12.04	6.74	5.25	4.57	4.17	*3.73*	**3.49**	**3.31**	**3.20**	**3.12**	**3.07**	**3.02**	**2.99**	**2.94**	**2.90**	**2.88**	**2.86**	**2.84**	**2.81**	**2.79**
30	12.62	7.04	5.47	4.75	4.33	3.86	*3.61*	**3.42**	**3.30**	**3.22**	**3.16**	**3.11**	**3.08**	**3.02**	**2.99**	**2.96**	**2.94**	**2.92**	**2.89**	**2.87**
40	13.02	7.24	5.63	4.87	4.44	3.96	*3.70*	**3.50**	**3.38**	**3.29**	**3.23**	**3.18**	**3.14**	**3.08**	**3.04**	**3.01**	**2.99**	**2.97**	**2.94**	**2.92**
50	13.32	7.40	5.74	4.97	4.53	4.03	*3.76*	**3.56**	**3.43**	**3.34**	**3.28**	**3.23**	**3.19**	**3.13**	**3.09**	**3.06**	**3.04**	**3.02**	**2.98**	**2.96**
60	13.56	7.53	5.84	5.05	4.60	4.09	*3.82*	*3.61*	**3.48**	**3.39**	**3.32**	**3.27**	**3.23**	**3.17**	**3.13**	**3.09**	**3.07**	**3.05**	**3.02**	**3.00**
75	13.86	7.68	5.95	5.15	4.68	4.16	*3.88*	*3.67*	**3.53**	**3.44**	**3.37**	**3.32**	**3.27**	**3.21**	**3.17**	**3.14**	**3.11**	**3.09**	**3.06**	**3.04**
100	14.23	7.88	6.10	5.27	4.79	4.26	3.96	*3.74*	*3.60*	**3.50**	**3.43**	**3.38**	**3.33**	**3.27**	**3.23**	**3.19**	**3.17**	**3.15**	**3.11**	**3.09**
125	14.52	8.03	6.21	5.36	4.87	4.33	4.03	*3.80*	*3.66*	**3.56**	**3.48**	**3.43**	**3.38**	**3.32**	**3.27**	**3.23**	**3.21**	**3.19**	**3.15**	**3.13**
150	14.74	8.15	6.30	5.44	4.94	4.38	4.08	*3.85*	*3.70*	*3.60*	**3.52**	**3.46**	**3.42**	**3.35**	**3.30**	**3.27**	**3.24**	**3.22**	**3.18**	**3.16**
175	14.94	8.25	6.37	5.50	5.00	4.43	4.12	*3.89*	*3.74*	*3.63*	**3.56**	**3.50**	**3.45**	**3.38**	**3.33**	**3.30**	**3.27**	**3.25**	**3.21**	**3.18**
200	15.10	8.33	6.44	5.55	5.04	4.47	4.16	*3.92*	*3.77*	*3.66*	*3.59*	**3.53**	**3.48**	**3.41**	**3.36**	**3.32**	**3.30**	**3.27**	**3.23**	**3.21**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.34	3.85	3.15	**2.83**	**2.65**	**2.45**	**2.34**	**2.26**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
2	7.72	4.46	3.57	3.17	**2.94**	**2.70**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
3	8.57	4.83	3.83	3.37	*3.12*	**2.84**	**2.69**	**2.58**	**2.52**	**2.47**	**2.43**	**2.41**	**2.39**	**2.36**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
4	9.18	5.10	4.01	3.52	3.24	**2.94**	**2.79**	**2.67**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
5	9.66	5.30	4.15	3.63	3.34	*3.02*	**2.86**	**2.73**	**2.66**	**2.60**	**2.56**	**2.53**	**2.51**	**2.48**	**2.45**	**2.43**	**2.42**	**2.41**	**2.39**	**2.38**
8	10.66	5.75	4.45	3.87	3.55	*3.19*	*3.01*	**2.87**	**2.78**	**2.72**	**2.68**	**2.65**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
12	11.51	6.13	4.71	4.08	3.72	3.34	*3.14*	*2.99*	**2.89**	**2.83**	**2.78**	**2.75**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.60**	**2.57**	**2.56**
16	12.11	6.39	4.90	4.22	3.85	3.44	*3.23*	*3.07*	*2.97*	**2.90**	**2.85**	**2.81**	**2.78**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
20	12.56	6.60	5.04	4.34	3.95	3.52	3.30	*3.13*	*3.03*	*2.96*	**2.90**	**2.87**	**2.83**	**2.79**	**2.76**	**2.73**	**2.72**	**2.70**	**2.68**	**2.66**
30	13.37	6.97	5.29	4.54	4.12	3.66	3.42	*3.24*	*3.13*	*3.06*	*3.00*	*2.96*	**2.93**	**2.88**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
40	13.93	7.22	5.47	4.69	4.25	3.77	3.51	3.33	*3.21*	*3.13*	*3.07*	*3.03*	*2.99*	*2.94*	**2.90**	**2.88**	**2.86**	**2.84**	**2.81**	**2.79**
50	14.35	7.42	5.61	4.80	4.34	3.85	3.58	3.32	*3.27*	*3.18*	*3.12*	*3.08*	*3.04*	*2.99*	*2.95*	**2.92**	**2.90**	**2.88**	**2.86**	**2.84**
60	14.70	7.58	5.72	4.89	4.42	3.91	3.64	3.44	3.31	*3.23*	*3.16*	*3.12*	*3.08*	*3.03*	*2.99*	*2.96*	*2.94*	**2.92**	**2.89**	**2.87**
75	15.11	7.77	5.86	5.00	4.51	3.99	3.71	3.50	3.37	*3.28*	*3.22*	*3.17*	*3.13*	*3.07*	*3.03*	*3.00*	*2.98*	*2.96*	**2.93**	**2.91**
100	15.63	8.02	6.03	5.14	4.64	4.09	3.80	3.58	3.44	3.35	*3.28*	*3.23*	*3.19*	*3.13*	*3.09*	*3.06*	*3.04*	*3.02*	*2.99*	*2.96*
125	16.02	8.20	6.16	5.25	4.73	4.16	3.86	3.64	3.50	3.40	3.33	*3.28*	*3.24*	*3.18*	*3.14*	*3.10*	*3.08*	*3.06*	*3.03*	*3.00*
150	16.34	8.35	6.27	5.33	4.80	4.23	3.92	3.69	3.55	3.45	3.38	3.32	*3.28*	*3.22*	*3.17*	*3.14*	*3.11*	*3.09*	*3.06*	*3.04*
175	16.61	8.48	6.36	5.41	4.87	4.28	3.97	3.73	3.59	3.48	3.41	3.36	*3.31*	*3.25*	*3.20*	*3.17*	*3.14*	*3.12*	*3.09*	*3.06*
200	16.84	8.59	6.44	5.47	4.92	4.33	4.01	3.77	3.62	3.52	3.44	3.38	3.34	*3.27*	*3.23*	*3.19*	*3.17*	*3.15*	*3.11*	*3.09*

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	7.72	4.46	3.57	**3.17**	**2.94**	**2.70**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
2	9.18	5.10	4.01	3.52	**3.24**	**2.94**	**2.79**	**2.67**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
3	10.05	5.48	4.27	3.72	*3.42*	**3.09**	**2.92**	**2.79**	**2.71**	**2.65**	**2.61**	**2.58**	**2.55**	**2.52**	**2.49**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
4	10.66	5.75	4.45	3.87	3.55	**3.19**	**3.01**	**2.87**	**2.78**	**2.72**	**2.68**	**2.65**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
5	11.13	5.95	4.59	3.99	3.64	*3.27*	**3.08**	**2.93**	**2.84**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
8	12.11	6.39	4.90	4.22	3.85	*3.44*	**3.23**	**3.07**	**2.97**	**2.90**	**2.85**	**2.81**	**2.78**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
12	12.93	6.76	5.15	4.43	4.02	3.59	*3.35*	**3.18**	**3.08**	**3.00**	**2.95**	**2.91**	**2.88**	**2.83**	**2.80**	**2.77**	**2.75**	**2.74**	**2.71**	**2.70**
16	13.50	7.03	5.33	4.58	4.15	3.69	*3.44*	*3.26*	**3.15**	**3.07**	**3.02**	**2.97**	**2.94**	**2.89**	**2.86**	**2.83**	**2.81**	**2.80**	**2.77**	**2.75**
20	13.93	7.22	5.47	4.69	4.25	3.77	*3.51*	*3.33*	**3.21**	**3.13**	**3.07**	**3.03**	**2.99**	**2.94**	**2.90**	**2.88**	**2.86**	**2.84**	**2.81**	**2.79**
30	14.70	7.58	5.72	4.89	4.42	3.91	3.64	*3.44*	*3.31*	**3.23**	**3.16**	**3.12**	**3.08**	**3.03**	**2.99**	**2.96**	**2.94**	**2.92**	**2.89**	**2.87**
40	15.23	7.83	5.90	5.03	4.54	4.01	3.73	*3.52*	*3.39*	*3.30*	**3.23**	**3.18**	**3.14**	**3.09**	**3.05**	**3.02**	**2.99**	**2.98**	**2.94**	**2.92**
50	15.63	8.02	6.03	5.14	4.64	4.09	3.80	*3.58*	*3.44*	*3.35*	*3.28*	**3.23**	**3.19**	**3.13**	**3.09**	**3.06**	**3.04**	**3.02**	**2.99**	**2.96**
60	15.95	8.17	6.14	5.23	4.71	4.15	3.85	3.63	*3.49*	*3.39*	*3.32*	*3.27*	**3.23**	**3.17**	**3.13**	**3.10**	**3.07**	**3.05**	**3.02**	**3.00**
75	16.34	8.35	6.27	5.33	4.80	4.23	3.92	3.69	*3.55*	*3.45*	*3.38*	*3.32*	*3.28*	**3.22**	**3.17**	**3.14**	**3.11**	**3.09**	**3.06**	**3.04**
100	16.84	8.59	6.44	5.47	4.92	4.33	4.01	3.77	*3.62*	*3.52*	*3.44*	*3.38*	*3.34*	*3.27*	**3.23**	**3.19**	**3.17**	**3.15**	**3.11**	**3.09**
125	17.21	8.77	6.56	5.57	5.01	4.40	4.07	3.83	3.67	*3.57*	*3.49*	*3.43*	*3.39*	*3.32*	*3.27*	**3.24**	**3.21**	**3.19**	**3.15**	**3.13**
150	17.52	8.91	6.67	5.66	5.09	4.46	4.13	3.88	3.72	*3.61*	*3.53*	*3.47*	*3.43*	*3.36*	*3.31*	*3.27*	**3.24**	**3.22**	**3.18**	**3.16**
175	17.77	9.03	6.75	5.73	5.15	4.51	4.17	3.92	3.76	*3.65*	*3.57*	*3.51*	*3.46*	*3.39*	*3.34*	*3.30*	*3.27*	**3.25**	**3.21**	**3.18**
200	17.99	9.14	6.83	5.79	5.20	4.56	4.21	3.95	3.79	3.68	*3.60*	*3.53*	*3.49*	*3.41*	*3.36*	*3.33*	*3.30*	*3.27*	**3.23**	**3.21**

Table 19-5. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 2 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	9.18	5.10	4.01	**3.52**	**3.24**	**2.94**	**2.79**	**2.67**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
2	10.66	5.75	4.45	3.87	**3.55**	**3.19**	**3.01**	**2.87**	**2.78**	**2.72**	**2.68**	**2.65**	**2.62**	**2.58**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
3	11.51	6.13	4.71	4.08	*3.72*	**3.34**	**3.14**	**2.99**	**2.89**	**2.83**	**2.78**	**2.75**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.60**	**2.57**	**2.56**
4	12.11	6.39	4.90	4.22	3.85	**3.44**	**3.23**	**3.07**	**2.97**	**2.90**	**2.85**	**2.81**	**2.78**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
5	12.56	6.60	5.04	4.34	3.95	**3.52**	**3.30**	**3.13**	**3.03**	**2.96**	**2.90**	**2.87**	**2.83**	**2.79**	**2.76**	**2.73**	**2.72**	**2.70**	**2.68**	**2.66**
8	13.50	7.03	5.33	4.58	4.15	*3.69*	**3.44**	**3.26**	**3.15**	**3.07**	**3.02**	**2.97**	**2.94**	**2.89**	**2.86**	**2.83**	**2.81**	**2.80**	**2.77**	**2.75**
12	14.28	7.39	5.59	4.78	4.32	*3.83*	**3.57**	**3.38**	**3.26**	**3.17**	**3.11**	**3.07**	**3.03**	**2.98**	**2.94**	**2.91**	**2.89**	**2.88**	**2.85**	**2.83**
16	14.82	7.64	5.76	4.92	4.45	3.93	*3.66*	**3.46**	**3.33**	**3.24**	**3.18**	**3.13**	**3.09**	**3.04**	**3.00**	**2.97**	**2.95**	**2.93**	**2.90**	**2.87**
20	15.23	7.83	5.90	5.03	4.54	4.01	*3.73*	**3.52**	**3.39**	**3.30**	**3.23**	**3.18**	**3.14**	**3.09**	**3.05**	**3.02**	**2.99**	**2.98**	**2.94**	**2.92**
30	15.95	8.17	6.14	5.23	4.71	4.15	*3.85*	*3.63*	**3.49**	**3.39**	**3.32**	**3.27**	**3.23**	**3.17**	**3.13**	**3.10**	**3.07**	**3.05**	**3.02**	**3.00**
40	16.46	8.41	6.31	5.36	4.83	4.25	3.94	*3.71*	**3.56**	**3.46**	**3.39**	**3.34**	**3.29**	**3.23**	**3.18**	**3.15**	**3.13**	**3.11**	**3.07**	**3.05**
50	16.84	8.59	6.44	5.47	4.92	4.33	4.01	*3.77*	*3.62*	**3.52**	**3.44**	**3.38**	**3.34**	**3.27**	**3.23**	**3.19**	**3.17**	**3.15**	**3.11**	**3.09**
60	17.15	8.73	6.54	5.55	5.00	4.39	4.06	*3.82*	*3.66*	**3.56**	**3.48**	**3.42**	**3.38**	**3.31**	**3.26**	**3.23**	**3.20**	**3.18**	**3.14**	**3.12**
75	17.52	8.91	6.67	5.66	5.09	4.46	4.13	*3.88*	*3.72*	*3.61*	**3.53**	**3.47**	**3.43**	**3.36**	**3.31**	**3.27**	**3.24**	**3.22**	**3.18**	**3.16**
100	17.99	9.14	6.83	5.79	5.20	4.56	4.21	3.95	*3.79*	*3.68*	*3.60*	**3.53**	**3.49**	**3.41**	**3.36**	**3.33**	**3.30**	**3.27**	**3.23**	**3.21**
125	18.34	9.31	6.95	5.89	5.29	4.63	4.28	4.01	*3.85*	*3.73*	*3.65*	*3.58*	**3.53**	**3.46**	**3.41**	**3.37**	**3.34**	**3.31**	**3.27**	**3.25**
150	18.63	9.44	7.05	5.97	5.36	4.69	4.33	4.06	*3.89*	*3.77*	*3.69*	*3.62*	**3.57**	**3.49**	**3.44**	**3.40**	**3.37**	**3.35**	**3.30**	**3.28**
175	18.88	9.56	7.13	6.04	5.42	4.74	4.38	4.10	*3.93*	*3.81*	*3.72*	*3.65*	*3.60*	**3.52**	**3.47**	**3.43**	**3.40**	**3.37**	**3.33**	**3.30**
200	19.08	9.66	7.21	6.10	5.47	4.79	4.42	4.14	*3.96*	*3.84*	*3.75*	*3.68*	*3.63*	**3.55**	**3.49**	**3.45**	**3.42**	**3.40**	**3.35**	**3.33**

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.69	0.56	0.50	0.47	0.45	0.42	0.40	0.39	0.38	0.37	0.37	0.37	0.36	0.36	0.36	0.35	0.35	0.35	0.35	0.35
2	1.02	0.83	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.57	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.54	0.54
3	1.21	0.98	0.89	0.83	0.80	0.76	0.73	0.71	0.70	0.69	0.68	0.68	0.67	0.66	0.66	0.66	0.65	0.65	0.65	0.64
4	1.34	1.09	0.98	0.92	0.88	0.84	0.81	0.79	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.72	0.71
5	1.44	1.17	1.05	0.99	0.95	0.90	0.87	0.84	0.83	0.82	0.81	0.80	0.80	0.79	0.78	0.78	0.78	0.77	0.77	0.77
8	1.65	1.33	1.20	1.12	1.08	1.02	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.90	0.89	0.88	0.88	0.88	0.87	0.87
12	1.83	1.47	1.32	1.24	1.19	1.12	1.08	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96	0.95
16	1.95	1.56	1.40	1.32	1.26	1.19	1.15	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01	1.01
20	2.05	1.63	1.47	1.38	1.32	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.07	1.06	1.05
30	2.21	1.76	1.58	1.48	1.42	1.34	1.29	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.13	1.13
40	2.33	1.85	1.66	1.55	1.49	1.40	1.35	1.31	1.29	1.27	1.26	1.24	1.24	1.22	1.21	1.21	1.20	1.20	1.19	1.18
50	2.41	1.92	1.72	1.61	1.54	1.45	1.40	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25	1.25	1.24	1.24	1.23	1.22
60	2.48	1.97	1.77	1.65	1.58	1.49	1.44	1.40	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25
75	2.57	2.03	1.82	1.71	1.63	1.54	1.48	1.44	1.41	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.29
100	2.68	2.12	1.90	1.77	1.70	1.60	1.54	1.50	1.47	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.36	1.35	1.34
125	2.76	2.18	1.95	1.83	1.74	1.65	1.59	1.54	1.51	1.48	1.47	1.45	1.44	1.43	1.41	1.41	1.40	1.39	1.38	1.38
150	2.82	2.23	2.00	1.87	1.78	1.68	1.62	1.57	1.54	1.52	1.50	1.49	1.47	1.46	1.44	1.44	1.43	1.42	1.41	1.40
175	2.88	2.27	2.03	1.90	1.82	1.71	1.65	1.60	1.57	1.54	1.53	1.51	1.50	1.48	1.47	1.46	1.45	1.45	1.44	1.43
200	2.92	2.31	2.06	1.93	1.85	1.74	1.68	1.63	1.59	1.57	1.55	1.53	1.52	1.51	1.49	1.48	1.47	1.47	1.46	1.45

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.02	0.83	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.57	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.54	0.54
2	1.34	1.09	0.98	0.92	0.88	0.84	0.81	0.79	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.72	0.71
3	1.52	1.23	1.11	1.04	1.00	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.81
4	1.65	1.33	1.20	1.12	1.08	1.02	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.90	0.89	0.88	0.88	0.88	0.87	0.87
5	1.75	1.41	1.26	1.19	1.14	1.08	1.04	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.93	0.92	0.92
8	1.95	1.56	1.40	1.32	1.26	1.19	1.15	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01	1.01
12	2.12	1.69	1.52	1.42	1.36	1.29	1.24	1.21	1.18	1.17	1.15	1.14	1.14	1.12	1.12	1.11	1.10	1.10	1.09	1.09
16	2.24	1.78	1.60	1.50	1.43	1.35	1.31	1.27	1.24	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.15	1.14
20	2.33	1.85	1.66	1.55	1.49	1.40	1.35	1.31	1.29	1.27	1.26	1.24	1.24	1.22	1.21	1.21	1.20	1.20	1.19	1.18
30	2.48	1.97	1.77	1.65	1.58	1.49	1.44	1.40	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25
40	2.59	2.05	1.84	1.72	1.65	1.55	1.50	1.45	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.32	1.31	1.30
50	2.68	2.12	1.90	1.77	1.70	1.60	1.54	1.50	1.47	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.36	1.35	1.34
60	2.74	2.17	1.94	1.82	1.74	1.64	1.58	1.53	1.50	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
75	2.82	2.23	2.00	1.87	1.78	1.68	1.62	1.57	1.54	1.52	1.50	1.49	1.47	1.46	1.44	1.44	1.43	1.42	1.41	1.40
100	2.92	2.31	2.06	1.93	1.85	1.74	1.68	1.63	1.59	1.57	1.55	1.53	1.52	1.51	1.49	1.48	1.47	1.47	1.46	1.45
125	3.00	2.37	2.12	1.98	1.89	1.78	1.72	1.67	1.63	1.61	1.59	1.57	1.56	1.54	1.53	1.52	1.51	1.50	1.49	1.48
150	3.06	2.41	2.16	2.02	1.93	1.82	1.75	1.70	1.66	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51
175	3.11	2.45	2.19	2.05	1.96	1.85	1.78	1.73	1.69	1.66	1.64	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
200	3.16	2.49	2.23	2.08	1.99	1.87	1.81	1.75	1.71	1.69	1.67	1.65	1.64	1.62	1.60	1.59	1.58	1.58	1.56	1.56

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.34	1.09	0.98	0.92	0.88	0.84	0.81	0.79	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.72	0.71
2	1.65	1.33	1.20	1.12	1.08	1.02	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.90	0.89	0.88	0.88	0.88	0.87	0.87
3	1.83	1.47	1.32	1.24	1.19	1.12	1.08	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96	0.95
4	1.95	1.56	1.40	1.32	1.26	1.19	1.15	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01	1.01
5	2.05	1.63	1.47	1.38	1.32	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.07	1.06	1.05
8	2.24	1.78	1.60	1.50	1.43	1.35	1.31	1.27	1.24	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.15	1.14
12	2.40	1.90	1.71	1.60	1.53	1.44	1.39	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.23	1.22	1.21
16	2.51	1.99	1.78	1.67	1.60	1.51	1.45	1.41	1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.28	1.27	1.26
20	2.59	2.05	1.84	1.72	1.65	1.55	1.50	1.45	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.32	1.31	1.30
30	2.74	2.17	1.94	1.82	1.74	1.64	1.58	1.53	1.50	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
40	2.85	2.25	2.01	1.88	1.80	1.70	1.64	1.59	1.55	1.53	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.41
50	2.92	2.31	2.06	1.93	1.85	1.74	1.68	1.63	1.59	1.57	1.55	1.53	1.52	1.51	1.49	1.48	1.47	1.47	1.46	1.45
60	2.99	2.36	2.11	1.97	1.88	1.78	1.71	1.66	1.63	1.60	1.58	1.57	1.55	1.54	1.52	1.51	1.50	1.50	1.49	1.48
75	3.06	2.41	2.16	2.02	1.93	1.82	1.75	1.70	1.66	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51
100	3.16	2.49	2.23	2.08	1.99	1.87	1.81	1.75	1.71	1.69	1.67	1.65	1.64	1.62	1.60	1.59	1.58	1.58	1.56	1.56
125	3.23	2.54	2.28	2.13	2.03	1.92	1.85	1.79	1.75	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60	1.59
150	3.29	2.59	2.32	2.17	2.07	1.95	1.88	1.82	1.78	1.75	1.73	1.72	1.70	1.68	1.67	1.65	1.64	1.64	1.62	1.62
175	3.34	2.63	2.35	2.20	2.10	1.98	1.91	1.85	1.81	1.78	1.76	1.74	1.73	1.70	1.69	1.68	1.67	1.66	1.65	1.64
200	3.38	2.66	2.38	2.22	2.12	2.00	1.93	1.87	1.83	1.80	1.78	1.76	1.75	1.72	1.71	1.70	1.69	1.68	1.67	1.66

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (2 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.10	0.87	0.77	0.72	0.69	0.65	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.54
2	1.47	1.14	1.02	0.95	0.90	0.85	0.82	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.72
3	1.69	1.30	1.15	1.07	1.02	0.96	0.93	0.90	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.82	0.81	0.81
4	1.85	1.42	1.25	1.16	1.10	1.04	1.00	0.97	0.95	0.93	0.92	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
5	1.97	1.50	1.32	1.23	1.17	1.10	1.05	1.02	1.00	0.99	0.97	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
8	2.22	1.68	1.48	1.37	1.30	1.22	1.17	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01
12	2.44	1.83	1.60	1.48	1.41	1.32	1.26	1.22	1.20	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.11	1.10	1.10	1.09
16	2.59	1.94	1.69	1.56	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.14
20	2.71	2.02	1.76	1.63	1.54	1.44	1.38	1.33	1.30	1.28	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.18
30	2.91	2.16	1.88	1.74	1.64	1.53	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.25
40	3.05	2.26	1.97	1.81	1.72	1.60	1.53	1.48	1.44	1.42	1.40	1.38	1.37	1.36	1.34	1.33	1.33	1.32	1.31	1.30
50	3.16	2.34	2.03	1.87	1.77	1.65	1.58	1.52	1.49	1.46	1.44	1.43	1.41	1.40	1.38	1.37	1.37	1.36	1.35	1.34
60	3.25	2.40	2.08	1.92	1.81	1.69	1.62	1.56	1.52	1.49	1.47	1.46	1.45	1.43	1.41	1.40	1.40	1.39	1.38	1.37
75	3.35	2.47	2.15	1.98	1.87	1.74	1.66	1.60	1.56	1.54	1.51	1.50	1.49	1.47	1.45	1.44	1.43	1.43	1.41	1.41
100	3.49	2.57	2.23	2.05	1.93	1.80	1.72	1.66	1.62	1.59	1.57	1.55	1.54	1.51	1.50	1.49	1.48	1.47	1.46	1.45
125	3.59	2.64	2.29	2.10	1.99	1.85	1.77	1.70	1.66	1.63	1.61	1.59	1.57	1.55	1.54	1.53	1.52	1.51	1.50	1.49
150	3.67	*2.69*	2.34	2.15	2.03	1.89	1.80	1.74	1.69	1.66	1.64	1.62	1.60	1.58	1.57	1.55	1.55	1.54	1.52	1.52
175	3.74	*2.74*	2.38	2.18	2.06	1.92	1.83	1.76	1.72	1.69	1.66	1.64	1.63	1.61	1.59	1.58	1.57	1.56	1.55	1.54
200	3.80	2.78	2.41	2.22	2.09	1.94	1.86	1.79	1.74	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.47	1.14	1.02	0.95	0.90	0.85	0.82	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.72
2	1.85	1.42	1.25	1.16	1.10	1.04	1.00	0.97	0.95	0.93	0.92	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
3	2.07	1.57	1.38	1.28	1.22	1.14	1.10	1.07	1.04	1.03	1.01	1.01	1.00	0.99	0.98	0.97	0.97	0.97	0.96	0.95
4	2.22	1.68	1.48	1.37	1.30	1.22	1.17	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01
5	2.34	1.76	1.55	1.43	1.36	1.27	1.22	1.18	1.16	1.14	1.12	1.11	1.11	1.09	1.08	1.08	1.07	1.07	1.06	1.06
8	2.59	1.94	1.69	1.56	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	2.80	2.08	1.82	1.68	1.59	1.48	1.42	1.37	1.34	1.32	1.30	1.29	1.28	1.26	1.25	1.24	1.24	1.23	1.22	1.22
16	2.94	2.18	1.90	1.75	1.66	1.55	1.48	1.43	1.40	1.37	1.36	1.34	1.33	1.32	1.30	1.29	1.29	1.28	1.27	1.27
20	3.05	2.26	1.97	1.81	1.72	1.60	1.53	1.48	1.44	1.42	1.40	1.38	1.37	1.36	1.34	1.33	1.33	1.32	1.31	1.30
30	3.25	2.40	2.08	1.92	1.81	1.69	1.62	1.56	1.52	1.49	1.47	1.46	1.45	1.43	1.41	1.40	1.40	1.39	1.38	1.37
40	3.38	2.49	2.17	1.99	1.88	1.75	1.68	1.62	1.58	1.55	1.53	1.51	1.50	1.48	1.46	1.45	1.44	1.44	1.43	1.42
50	3.49	2.57	2.23	2.05	1.93	1.80	1.72	1.66	1.62	1.59	1.57	1.55	1.54	1.51	1.50	1.49	1.48	1.47	1.46	1.45
60	3.57	2.62	2.28	2.09	1.98	1.84	1.76	1.69	1.65	1.62	1.60	1.58	1.57	1.55	1.53	1.52	1.51	1.50	1.49	1.48
75	3.67	2.69	2.34	2.15	2.03	1.89	1.80	1.74	1.69	1.66	1.64	1.62	1.60	1.58	1.57	1.55	1.55	1.54	1.52	1.52
100	3.80	2.78	2.41	2.22	2.09	1.94	1.86	1.79	1.74	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
125	3.90	2.85	2.47	2.27	2.14	1.99	1.90	1.83	1.78	1.75	1.72	1.70	1.69	1.66	1.65	1.63	1.62	1.62	1.60	1.59
150	3.97	*2.91*	2.52	2.31	2.18	2.03	1.94	1.86	1.82	1.78	1.75	1.73	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62
175	4.04	2.95	2.56	2.35	2.22	2.06	1.96	1.89	1.84	1.81	1.78	1.76	1.74	1.72	1.70	1.69	1.68	1.67	1.65	1.64
200	4.09	2.99	2.59	2.38	2.24	2.08	1.99	1.92	1.87	1.83	1.80	1.78	1.76	1.74	1.72	1.71	1.70	1.69	1.67	1.66

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.85	1.42	1.25	1.16	1.10	1.04	1.00	0.97	0.95	0.93	0.92	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
2	2.22	1.68	1.48	1.37	1.30	1.22	1.17	1.13	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01
3	2.44	1.83	1.60	1.48	1.41	1.32	1.26	1.22	1.20	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.11	1.10	1.10	1.09
4	2.59	1.94	1.69	1.56	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.14
5	2.71	2.02	1.76	1.63	1.54	1.44	1.38	1.33	1.30	1.28	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.18
8	2.94	2.18	1.90	1.75	1.66	1.55	1.48	1.43	1.40	1.37	1.36	1.34	1.33	1.32	1.30	1.29	1.29	1.28	1.27	1.27
12	3.14	2.32	2.02	1.86	1.76	1.64	1.57	1.51	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.33
16	3.28	2.42	2.10	1.93	1.83	1.70	1.63	1.57	1.53	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.41	1.40	1.39	1.38
20	3.38	2.49	2.17	1.99	1.88	1.75	1.68	1.62	1.58	1.55	1.53	1.51	1.50	1.48	1.46	1.45	1.44	1.44	1.43	1.42
30	3.57	2.62	2.28	2.09	1.98	1.84	1.76	1.69	1.65	1.62	1.60	1.58	1.57	1.55	1.53	1.52	1.51	1.50	1.49	1.48
40	3.70	2.71	2.35	2.16	2.04	1.90	1.81	1.75	1.70	1.67	1.65	1.63	1.61	1.59	1.58	1.56	1.56	1.55	1.53	1.53
50	3.80	2.78	2.41	2.22	2.09	1.94	1.86	1.79	1.74	1.71	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.58	1.57	1.56
60	3.88	2.84	2.46	2.26	2.13	1.98	1.89	1.82	1.78	1.74	1.72	1.70	1.68	1.66	1.64	1.63	1.62	1.61	1.60	1.59
75	3.97	2.91	2.52	2.31	2.18	2.03	1.94	1.86	1.82	1.78	1.75	1.73	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62
100	4.09	2.99	2.59	2.38	2.24	2.08	1.99	1.92	1.87	1.83	1.80	1.78	1.76	1.74	1.72	1.71	1.70	1.69	1.67	1.66
125	4.19	3.06	2.65	2.43	2.29	2.13	2.03	1.95	1.90	1.87	1.84	1.82	1.80	1.77	1.75	1.74	1.73	1.72	1.70	1.69
150	4.26	*3.11*	2.69	2.47	2.33	2.16	2.06	1.99	1.93	1.90	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
175	4.32	3.15	2.73	2.50	2.36	2.19	2.09	2.01	1.96	1.92	1.89	1.87	1.85	1.82	1.80	1.79	1.78	1.77	1.75	1.74
200	4.38	3.19	2.76	2.53	2.39	2.22	2.12	2.04	1.98	1.94	1.91	1.89	1.87	1.84	1.82	1.81	1.80	1.79	1.77	1.76

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (5 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.72	1.28	1.12	1.04	0.98	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77
2	2.19	1.58	1.37	1.26	1.19	1.11	1.06	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
3	2.47	1.76	1.51	1.39	1.31	1.22	1.17	1.13	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.01	1.00
4	2.67	1.89	1.61	1.48	1.39	1.29	1.24	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
5	2.83	1.98	1.69	1.54	1.45	1.35	1.29	1.24	1.21	1.19	1.18	1.16	1.15	1.14	1.13	1.12	1.12	1.11	1.10	1.10
8	3.17	2.19	1.86	1.69	1.59	1.47	1.40	1.35	1.31	1.29	1.27	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.18
12	3.45	2.37	2.00	1.81	1.70	1.57	1.49	1.43	1.40	1.37	1.35	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
16	3.65	2.49	2.10	1.90	1.78	1.64	1.56	1.50	1.46	1.43	1.41	1.39	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
20	3.81	2.58	2.17	1.96	1.84	1.69	1.61	1.54	1.50	1.47	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
30	4.08	*2.75*	2.31	2.08	1.94	1.78	1.69	1.63	1.58	1.55	1.53	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.41
40	4.27	2.87	2.40	2.16	2.02	1.85	1.76	1.68	1.64	1.60	1.58	1.56	1.54	1.52	1.50	1.49	1.48	1.48	1.46	1.45
50	4.42	2.96	2.47	2.23	2.08	1.90	1.80	1.73	1.68	1.64	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.50	1.49
60	4.54	3.03	2.53	2.28	2.12	1.94	1.84	1.76	1.71	1.68	1.65	1.63	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
75	4.68	3.12	2.60	2.34	2.18	1.99	1.89	1.81	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.59	1.58	1.58	1.56	1.55
100	4.86	3.23	*2.69*	2.42	2.25	2.06	1.95	1.86	1.81	1.77	1.74	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
125	5.00	3.32	*2.76*	2.48	2.31	2.11	1.99	1.91	1.85	1.81	1.78	1.75	1.74	1.71	1.69	1.67	1.66	1.65	1.64	1.63
150	5.11	3.39	*2.82*	2.53	2.35	2.15	2.03	1.94	1.88	1.84	1.81	1.79	1.77	1.74	1.72	1.70	1.69	1.68	1.66	1.65
175	5.20	3.44	2.86	2.57	2.39	2.18	2.06	1.97	1.91	1.87	1.84	1.81	1.79	1.76	1.74	1.73	1.71	1.70	1.69	1.67
200	5.28	3.49	2.90	*2.60*	2.42	2.21	2.09	2.00	1.93	1.89	1.86	1.83	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.70

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (5 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.19	1.58	1.37	1.26	1.19	1.11	1.06	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
2	2.67	1.89	1.61	1.48	1.39	1.29	1.24	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
3	2.96	2.06	1.76	1.60	1.50	1.39	1.33	1.28	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.13
4	3.17	2.19	1.86	1.69	1.59	1.47	1.40	1.35	1.31	1.29	1.27	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.18
5	3.32	2.29	1.93	1.76	1.65	1.52	1.45	1.40	1.36	1.33	1.32	1.30	1.29	1.27	1.26	1.25	1.25	1.24	1.23	1.22
8	3.65	2.49	2.10	1.90	1.78	1.64	1.56	1.50	1.46	1.43	1.41	1.39	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
12	3.93	2.66	2.23	2.02	1.88	1.73	1.65	1.58	1.54	1.51	1.48	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37
16	4.13	2.78	2.33	2.10	1.96	1.80	1.71	1.64	1.59	1.56	1.54	1.52	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42
20	4.27	*2.87*	2.40	2.16	2.02	1.85	1.76	1.68	1.64	1.60	1.58	1.56	1.54	1.52	1.50	1.49	1.48	1.48	1.46	1.45
30	4.54	3.03	2.53	2.28	2.12	1.94	1.84	1.76	1.71	1.68	1.65	1.63	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
40	4.72	3.15	2.62	2.36	2.20	2.01	1.90	1.82	1.77	1.73	1.70	1.68	1.66	1.64	1.62	1.60	1.59	1.59	1.57	1.56
50	4.86	3.23	2.69	2.42	2.25	2.06	1.95	1.86	1.81	1.77	1.74	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
60	4.97	3.30	2.75	2.47	2.30	2.10	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.70	1.68	1.67	1.66	1.65	1.63	1.62
75	5.11	3.39	2.82	2.53	2.35	2.15	2.03	1.94	1.88	1.84	1.81	1.79	1.77	1.74	1.72	1.70	1.69	1.68	1.66	1.65
100	5.28	3.49	*2.90*	2.60	2.42	2.21	2.09	2.00	1.93	1.89	1.86	1.83	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.70
125	5.41	3.58	*2.97*	2.66	2.48	2.26	2.13	2.04	1.97	1.93	1.89	1.87	1.85	1.82	1.80	1.78	1.77	1.76	1.74	1.73
150	5.52	3.64	3.02	2.71	2.52	2.30	2.17	2.07	2.01	1.96	1.92	1.90	1.88	1.85	1.82	1.81	1.79	1.78	1.76	1.75
175	5.61	3.70	3.07	2.75	2.56	2.33	2.20	2.10	2.03	1.99	1.95	1.92	1.90	1.87	1.85	1.83	1.82	1.81	1.79	1.77
200	5.68	3.75	3.11	2.78	2.59	2.36	2.23	2.12	2.06	2.01	1.97	1.94	1.92	1.89	1.87	1.85	1.84	1.82	1.80	1.79

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.67	1.89	1.61	1.48	1.39	1.29	1.24	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
2	3.17	2.19	1.86	1.69	1.59	1.47	1.40	1.35	1.31	1.29	1.27	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.18
3	3.45	2.37	2.00	1.81	1.70	1.57	1.49	1.43	1.40	1.37	1.35	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
4	3.65	2.49	2.10	1.90	1.78	1.64	1.56	1.50	1.46	1.43	1.41	1.39	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
5	3.81	2.58	2.17	1.96	1.84	1.69	1.61	1.54	1.50	1.47	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
8	4.13	2.78	2.33	2.10	1.96	1.80	1.71	1.64	1.59	1.56	1.54	1.52	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42
12	4.39	2.94	2.46	2.21	2.07	1.89	1.80	1.72	1.67	1.64	1.61	1.59	1.58	1.55	1.54	1.52	1.51	1.51	1.49	1.48
16	4.58	3.06	2.55	2.30	2.14	1.96	1.86	1.78	1.73	1.69	1.66	1.64	1.62	1.60	1.58	1.57	1.56	1.55	1.54	1.53
20	4.72	3.15	2.62	2.36	2.20	2.01	1.90	1.82	1.77	1.73	1.70	1.68	1.66	1.64	1.62	1.60	1.59	1.59	1.57	1.56
30	4.97	3.30	2.75	2.47	2.30	2.10	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.70	1.68	1.67	1.66	1.65	1.63	1.62
40	5.15	3.41	2.84	2.55	2.37	2.16	2.04	1.95	1.89	1.85	1.82	1.80	1.78	1.75	1.73	1.71	1.70	1.69	1.67	1.66
50	5.28	3.49	2.90	2.60	2.42	2.21	2.09	2.00	1.93	1.89	1.86	1.83	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.70
60	5.39	3.56	2.96	2.65	2.47	2.25	2.13	2.03	1.97	1.92	1.89	1.86	1.84	1.81	1.79	1.77	1.76	1.75	1.73	1.72
75	5.52	3.64	3.02	2.71	2.52	2.30	2.17	2.07	2.01	1.96	1.92	1.90	1.88	1.85	1.82	1.81	1.79	1.78	1.76	1.75
100	5.68	3.75	*3.11*	2.78	2.59	2.36	2.23	2.12	2.06	2.01	1.97	1.94	1.92	1.89	1.87	1.85	1.84	1.82	1.80	1.79
125	5.81	3.82	*3.17*	2.84	2.64	2.40	2.27	2.16	2.09	2.04	2.01	1.98	1.96	1.92	1.90	1.88	1.87	1.86	1.84	1.82
150	5.91	3.89	*3.22*	2.88	2.68	2.44	2.30	2.20	2.12	2.07	2.04	2.01	1.99	1.95	1.93	1.91	1.89	1.88	1.86	1.85
175	5.99	3.94	3.26	2.92	2.71	2.47	2.33	2.22	2.15	2.10	2.06	2.03	2.01	1.97	1.95	1.93	1.91	1.90	1.88	1.87
200	6.07	3.99	3.30	2.96	2.74	2.50	2.36	2.25	2.17	2.12	2.08	2.05	2.03	1.99	1.97	1.95	1.93	1.92	1.90	1.89

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.29	1.62	1.39	1.27	1.20	1.11	1.07	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
2	2.86	1.94	1.64	1.49	1.40	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
3	3.21	2.14	1.79	1.62	1.52	1.40	1.34	1.29	1.26	1.23	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.13
4	3.46	2.28	1.90	1.72	1.60	1.48	1.41	1.35	1.32	1.29	1.27	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
5	3.66	2.39	1.99	1.79	1.67	1.53	1.46	1.40	1.36	1.34	1.32	1.30	1.29	1.28	1.26	1.25	1.25	1.24	1.23	1.22
8	4.07	2.62	2.16	1.94	1.80	1.65	1.57	1.50	1.46	1.43	1.41	1.39	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
12	4.43	2.82	2.31	2.06	1.92	1.75	1.66	1.59	1.54	1.51	1.49	1.47	1.46	1.43	1.42	1.41	1.40	1.39	1.38	1.37
16	4.68	2.96	2.42	2.15	2.00	1.82	1.72	1.65	1.60	1.57	1.54	1.52	1.51	1.48	1.47	1.46	1.45	1.44	1.43	1.42
20	4.87	3.06	2.50	2.22	2.06	1.87	1.77	1.69	1.64	1.61	1.58	1.56	1.55	1.52	1.51	1.49	1.49	1.48	1.46	1.46
30	5.22	3.26	*2.65*	2.35	2.17	1.97	1.86	1.78	1.72	1.68	1.66	1.63	1.62	1.59	1.57	1.56	1.55	1.54	1.53	1.52
40	5.46	3.39	*2.75*	2.44	2.25	2.04	1.92	1.83	1.78	1.74	1.71	1.68	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
50	5.64	3.49	2.83	2.50	2.31	2.09	1.97	1.88	1.82	1.78	1.75	1.72	1.70	1.68	1.66	1.64	1.63	1.62	1.61	1.60
60	5.79	3.58	2.89	2.56	2.36	2.14	2.01	1.91	1.85	1.81	1.78	1.75	1.73	1.71	1.69	1.67	1.66	1.65	1.63	1.62
75	5.97	3.68	2.97	*2.63*	2.42	2.19	2.06	1.96	1.89	1.85	1.82	1.79	1.77	1.74	1.72	1.70	1.69	1.68	1.67	1.65
100	6.19	3.81	3.07	*2.71*	2.50	2.25	2.12	2.01	1.95	1.90	1.87	1.84	1.82	1.79	1.76	1.75	1.74	1.73	1.71	1.70
125	6.37	3.90	3.15	*2.78*	2.56	2.30	2.16	2.06	1.99	1.94	1.90	1.88	1.85	1.82	1.80	1.78	1.77	1.76	1.74	1.73
150	6.51	3.98	3.21	*2.83*	*2.60*	2.35	2.20	2.09	2.02	1.97	1.93	1.91	1.88	1.85	1.83	1.81	1.80	1.78	1.77	1.75
175	6.62	4.05	3.26	2.87	*2.64*	2.38	2.23	2.12	2.05	2.00	1.96	1.93	1.91	1.87	1.85	1.83	1.82	1.81	1.79	1.77
200	6.72	4.11	3.30	2.91	*2.68*	2.41	2.26	2.15	2.07	2.02	1.98	1.95	1.93	1.89	1.87	1.85	1.84	1.83	1.81	1.79

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.86	**1.94**	**1.64**	**1.49**	**1.40**	**1.30**	**1.24**	**1.20**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.07**	**1.06**	**1.06**
2	3.46	**2.28**	**1.90**	**1.72**	**1.60**	**1.48**	**1.41**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
3	3.82	**2.48**	**2.05**	**1.84**	**1.72**	**1.58**	**1.50**	**1.44**	**1.40**	**1.37**	**1.35**	**1.34**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**	**1.26**
4	4.07	**2.62**	**2.16**	**1.94**	**1.80**	**1.65**	**1.57**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
5	4.27	**2.73**	**2.24**	**2.01**	**1.87**	**1.71**	**1.62**	**1.55**	**1.51**	**1.48**	**1.45**	**1.44**	**1.42**	**1.40**	**1.39**	**1.38**	**1.37**	**1.36**	**1.35**	**1.34**
8	4.68	2.96	**2.42**	**2.15**	**2.00**	**1.82**	**1.72**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.48**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
12	5.03	3.15	**2.57**	**2.28**	**2.11**	**1.92**	**1.81**	**1.73**	**1.68**	**1.64**	**1.62**	**1.59**	**1.58**	**1.55**	**1.54**	**1.52**	**1.51**	**1.51**	**1.49**	**1.48**
16	5.27	3.29	**2.67**	**2.37**	**2.19**	**1.99**	**1.87**	**1.79**	**1.73**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.58**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
20	5.46	3.39	**2.75**	**2.44**	**2.25**	**2.04**	**1.92**	**1.83**	**1.78**	**1.74**	**1.71**	**1.68**	**1.67**	**1.64**	**1.62**	**1.61**	**1.60**	**1.59**	**1.57**	**1.56**
30	5.79	3.58	*2.89*	**2.56**	**2.36**	**2.14**	**2.01**	**1.91**	**1.85**	**1.81**	**1.78**	**1.75**	**1.73**	**1.71**	**1.69**	**1.67**	**1.66**	**1.65**	**1.63**	**1.62**
40	6.02	3.71	*2.99*	**2.65**	**2.44**	**2.20**	**2.07**	**1.97**	**1.91**	**1.86**	**1.83**	**1.80**	**1.78**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.66**
50	6.19	3.81	3.07	**2.71**	**2.50**	**2.25**	**2.12**	**2.01**	**1.95**	**1.90**	**1.87**	**1.84**	**1.82**	**1.79**	**1.76**	**1.75**	**1.74**	**1.73**	**1.71**	**1.70**
60	6.34	3.89	3.13	**2.76**	**2.54**	**2.30**	**2.16**	**2.05**	**1.98**	**1.93**	**1.90**	**1.87**	**1.85**	**1.82**	**1.79**	**1.78**	**1.76**	**1.75**	**1.73**	**1.72**
75	6.51	3.98	3.21	*2.83*	**2.60**	**2.35**	**2.20**	**2.09**	**2.02**	**1.97**	**1.93**	**1.91**	**1.88**	**1.85**	**1.83**	**1.81**	**1.80**	**1.78**	**1.77**	**1.75**
100	6.72	4.11	3.30	*2.91*	**2.68**	**2.41**	**2.26**	**2.15**	**2.07**	**2.02**	**1.98**	**1.95**	**1.93**	**1.89**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.79**
125	6.89	4.20	3.38	*2.97*	**2.73**	**2.46**	**2.31**	**2.19**	**2.11**	**2.06**	**2.02**	**1.99**	**1.96**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.84**	**1.82**
150	7.02	4.28	3.43	*3.02*	**2.78**	**2.50**	**2.34**	**2.22**	**2.14**	**2.09**	**2.05**	**2.02**	**1.99**	**1.96**	**1.93**	**1.91**	**1.90**	**1.88**	**1.86**	**1.85**
175	7.13	4.34	3.48	*3.07*	**2.82**	**2.53**	**2.37**	**2.25**	**2.17**	**2.12**	**2.07**	**2.04**	**2.02**	**1.98**	**1.95**	**1.93**	**1.92**	**1.90**	**1.88**	**1.87**
200	7.23	4.40	3.53	3.10	*2.85*	**2.56**	**2.40**	**2.28**	**2.19**	**2.14**	**2.10**	**2.06**	**2.04**	**2.00**	**1.97**	**1.95**	**1.94**	**1.92**	**1.90**	**1.89**

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.46	**2.28**	**1.90**	**1.72**	**1.60**	**1.48**	**1.41**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
2	4.07	**2.62**	**2.16**	**1.94**	**1.80**	**1.65**	**1.57**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
3	4.43	**2.82**	**2.31**	**2.06**	**1.92**	**1.75**	**1.66**	**1.59**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**	**1.38**	**1.37**
4	4.68	**2.96**	**2.42**	**2.15**	**2.00**	**1.82**	**1.72**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.48**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
5	4.87	**3.06**	**2.50**	**2.22**	**2.06**	**1.87**	**1.77**	**1.69**	**1.64**	**1.61**	**1.58**	**1.56**	**1.55**	**1.52**	**1.51**	**1.49**	**1.49**	**1.48**	**1.46**	**1.46**
8	5.27	3.29	**2.67**	**2.37**	**2.19**	**1.99**	**1.87**	**1.79**	**1.73**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.58**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
12	5.61	3.48	**2.81**	**2.49**	**2.30**	**2.08**	**1.96**	**1.87**	**1.81**	**1.77**	**1.74**	**1.72**	**1.70**	**1.67**	**1.65**	**1.64**	**1.62**	**1.62**	**1.60**	**1.59**
16	5.84	3.61	**2.92**	**2.58**	**2.38**	**2.15**	**2.02**	**1.93**	**1.86**	**1.82**	**1.79**	**1.76**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.64**	**1.63**
20	6.02	3.71	**2.99**	**2.65**	**2.44**	**2.20**	**2.07**	**1.97**	**1.91**	**1.86**	**1.83**	**1.80**	**1.78**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.66**
30	6.34	3.89	*3.13*	**2.76**	**2.54**	**2.30**	**2.16**	**2.05**	**1.98**	**1.93**	**1.90**	**1.87**	**1.85**	**1.82**	**1.79**	**1.78**	**1.76**	**1.75**	**1.73**	**1.72**
40	6.56	4.01	3.23	**2.85**	**2.62**	**2.36**	**2.22**	**2.10**	**2.03**	**1.98**	**1.94**	**1.92**	**1.89**	**1.86**	**1.84**	**1.82**	**1.81**	**1.79**	**1.77**	**1.76**
50	6.72	4.11	3.30	**2.91**	**2.68**	**2.41**	**2.26**	**2.15**	**2.07**	**2.02**	**1.98**	**1.95**	**1.93**	**1.89**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.79**
60	6.86	4.18	3.36	**2.96**	**2.72**	**2.45**	**2.30**	**2.18**	**2.10**	**2.05**	**2.01**	**1.98**	**1.96**	**1.92**	**1.90**	**1.88**	**1.86**	**1.85**	**1.83**	**1.82**
75	7.02	4.28	3.43	**3.02**	**2.78**	**2.50**	**2.34**	**2.22**	**2.14**	**2.09**	**2.05**	**2.02**	**1.99**	**1.96**	**1.93**	**1.91**	**1.90**	**1.88**	**1.86**	**1.85**
100	7.23	4.40	3.53	*3.10*	**2.85**	**2.56**	**2.40**	**2.28**	**2.19**	**2.14**	**2.10**	**2.06**	**2.04**	**2.00**	**1.97**	**1.95**	**1.94**	**1.92**	**1.90**	**1.89**
125	7.39	4.49	3.60	*3.16*	**2.91**	**2.61**	**2.44**	**2.32**	**2.23**	**2.17**	**2.13**	**2.10**	**2.07**	**2.03**	**2.00**	**1.98**	**1.97**	**1.95**	**1.93**	**1.92**
150	7.52	4.56	3.65	*3.21*	**2.95**	**2.65**	**2.48**	**2.35**	**2.26**	**2.20**	**2.16**	**2.13**	**2.10**	**2.06**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.93**
175	7.62	4.62	3.70	*3.25*	**2.99**	**2.68**	**2.51**	**2.38**	**2.29**	**2.23**	**2.18**	**2.15**	**2.12**	**2.08**	**2.05**	**2.03**	**2.01**	**2.00**	**1.98**	**1.96**
200	7.71	4.67	3.74	*3.29*	**3.02**	**2.71**	**2.54**	**2.40**	**2.31**	**2.25**	**2.21**	**2.17**	**2.14**	**2.10**	**2.07**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (20 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.99	**1.98**	**1.66**	**1.50**	**1.41**	**1.30**	**1.24**	**1.20**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.07**	**1.06**	**1.06**
2	3.68	**2.34**	**1.93**	**1.73**	**1.61**	**1.48**	**1.41**	**1.35**	**1.32**	**1.29**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
3	4.12	**2.56**	**2.09**	**1.87**	**1.73**	**1.59**	**1.50**	**1.44**	**1.40**	**1.38**	**1.36**	**1.34**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**	**1.26**
4	4.43	*2.71*	**2.20**	**1.96**	**1.82**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
5	4.68	2.84	**2.29**	**2.04**	**1.88**	**1.72**	**1.62**	**1.55**	**1.51**	**1.48**	**1.45**	**1.44**	**1.42**	**1.40**	**1.39**	**1.38**	**1.37**	**1.36**	**1.35**	**1.34**
8	5.20	3.10	**2.48**	**2.19**	**2.02**	**1.83**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
12	5.65	3.32	*2.65*	**2.33**	**2.14**	**1.93**	**1.82**	**1.74**	**1.68**	**1.65**	**1.62**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.52**	**1.51**	**1.49**	**1.48**
16	5.96	3.48	*2.76*	**2.42**	**2.23**	**2.01**	**1.89**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
20	6.20	3.60	2.85	**2.50**	**2.29**	**2.06**	**1.94**	**1.84**	**1.78**	**1.74**	**1.71**	**1.69**	**1.67**	**1.64**	**1.62**	**1.61**	**1.60**	**1.59**	**1.57**	**1.56**
30	6.63	3.82	3.01	*2.63*	**2.41**	**2.16**	**2.02**	**1.92**	**1.86**	**1.81**	**1.78**	**1.76**	**1.74**	**1.71**	**1.69**	**1.67**	**1.66**	**1.65**	**1.63**	**1.62**
40	6.94	3.98	3.12	*2.72*	**2.49**	**2.23**	**2.09**	**1.98**	**1.91**	**1.87**	**1.83**	**1.80**	**1.78**	**1.75**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.66**
50	7.17	4.09	3.21	*2.80*	**2.55**	**2.28**	**2.14**	**2.03**	**1.95**	**1.91**	**1.87**	**1.84**	**1.82**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.71**	**1.70**
60	7.35	4.19	3.28	*2.85*	*2.61*	**2.33**	**2.18**	**2.06**	**1.99**	**1.94**	**1.90**	**1.87**	**1.85**	**1.82**	**1.79**	**1.78**	**1.76**	**1.75**	**1.73**	**1.72**
75	7.58	4.31	3.37	2.92	*2.67*	**2.38**	**2.22**	**2.11**	**2.03**	**1.98**	**1.94**	**1.91**	**1.89**	**1.85**	**1.83**	**1.81**	**1.80**	**1.79**	**1.77**	**1.75**
100	7.86	4.45	3.48	3.02	*2.75*	**2.45**	**2.28**	**2.16**	**2.08**	**2.03**	**1.99**	**1.96**	**1.93**	**1.90**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.79**
125	8.08	4.56	3.56	3.09	*2.81*	**2.50**	**2.33**	**2.20**	**2.12**	**2.07**	**2.02**	**1.99**	**1.97**	**1.93**	**1.90**	**1.89**	**1.87**	**1.86**	**1.84**	**1.82**
150	8.26	4.66	3.63	3.14	*2.86*	**2.54**	**2.37**	**2.24**	**2.16**	**2.10**	**2.06**	**2.02**	**2.00**	**1.96**	**1.93**	**1.91**	**1.90**	**1.88**	**1.86**	**1.85**
175	8.40	4.73	3.68	3.19	2.90	*2.58*	**2.40**	**2.27**	**2.18**	**2.12**	**2.08**	**2.05**	**2.02**	**1.98**	**1.95**	**1.93**	**1.92**	**1.91**	**1.88**	**1.87**
200	8.53	4.80	3.73	3.23	2.94	*2.61*	**2.43**	**2.29**	**2.21**	**2.15**	**2.10**	**2.07**	**2.04**	**2.00**	**1.97**	**1.95**	**1.94**	**1.92**	**1.90**	**1.89**

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (20 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.68	**2.34**	**1.93**	**1.73**	**1.61**	**1.48**	**1.41**	**1.35**	**1.32**	**1.29**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
2	4.43	**2.71**	**2.20**	**1.96**	**1.82**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
3	4.88	2.94	**2.37**	**2.10**	**1.94**	**1.76**	**1.66**	**1.59**	**1.55**	**1.51**	**1.49**	**1.47**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.39**	**1.38**	**1.37**
4	5.20	3.10	**2.48**	**2.19**	**2.02**	**1.83**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
5	5.44	3.22	**2.57**	**2.27**	**2.09**	**1.89**	**1.78**	**1.70**	**1.65**	**1.61**	**1.58**	**1.56**	**1.55**	**1.52**	**1.51**	**1.50**	**1.49**	**1.48**	**1.46**	**1.46**
8	5.96	3.48	**2.76**	**2.42**	**2.23**	**2.01**	**1.89**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
12	6.40	3.70	*2.92*	**2.56**	**2.34**	**2.11**	**1.98**	**1.88**	**1.82**	**1.77**	**1.74**	**1.72**	**1.70**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.60**	**1.59**
16	6.70	3.86	3.04	**2.65**	**2.43**	**2.18**	**2.04**	**1.94**	**1.87**	**1.83**	**1.79**	**1.77**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.64**	**1.63**
20	6.94	3.98	3.12	**2.72**	**2.49**	**2.23**	**2.09**	**1.98**	**1.91**	**1.87**	**1.83**	**1.80**	**1.78**	**1.75**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.66**
30	7.35	4.19	3.28	*2.85*	**2.61**	**2.33**	**2.18**	**2.06**	**1.99**	**1.94**	**1.90**	**1.87**	**1.85**	**1.82**	**1.79**	**1.78**	**1.76**	**1.75**	**1.73**	**1.72**
40	7.64	4.34	3.39	*2.95*	**2.69**	**2.40**	**2.24**	**2.12**	**2.04**	**1.99**	**1.95**	**1.92**	**1.90**	**1.86**	**1.84**	**1.82**	**1.81**	**1.80**	**1.78**	**1.76**
50	7.86	4.45	3.48	*3.02*	**2.75**	**2.45**	**2.28**	**2.16**	**2.08**	**2.03**	**1.99**	**1.96**	**1.93**	**1.90**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.79**
60	8.04	4.54	3.54	3.07	**2.80**	**2.49**	**2.32**	**2.20**	**2.12**	**2.06**	**2.02**	**1.99**	**1.96**	**1.92**	**1.90**	**1.88**	**1.86**	**1.85**	**1.83**	**1.82**
75	8.26	4.66	3.63	3.14	*2.86*	**2.54**	**2.37**	**2.24**	**2.16**	**2.10**	**2.06**	**2.02**	**2.00**	**1.96**	**1.93**	**1.91**	**1.90**	**1.88**	**1.86**	**1.85**
100	8.53	4.80	3.73	3.23	*2.94*	**2.61**	**2.43**	**2.29**	**2.21**	**2.15**	**2.10**	**2.07**	**2.04**	**2.00**	**1.97**	**1.95**	**1.94**	**1.92**	**1.90**	**1.89**
125	8.74	4.91	3.81	3.30	*3.00*	**2.66**	**2.48**	**2.34**	**2.25**	**2.18**	**2.14**	**2.10**	**2.08**	**2.04**	**2.01**	**1.98**	**1.97**	**1.96**	**1.93**	**1.92**
150	8.91	4.99	3.88	3.35	*3.05*	**2.70**	**2.51**	**2.37**	**2.28**	**2.22**	**2.17**	**2.13**	**2.10**	**2.06**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.94**
175	9.05	5.07	3.93	3.40	*3.09*	**2.74**	**2.55**	**2.40**	**2.31**	**2.24**	**2.19**	**2.16**	**2.13**	**2.08**	**2.05**	**2.03**	**2.02**	**2.00**	**1.98**	**1.96**
200	9.17	5.13	3.98	3.44	3.12	**2.77**	**2.57**	**2.43**	**2.33**	**2.26**	**2.22**	**2.18**	**2.15**	**2.10**	**2.07**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.43	2.71	2.20	1.96	1.82	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
2	5.20	3.10	2.48	2.19	2.02	1.83	1.73	1.65	1.60	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
3	5.65	3.32	2.65	2.33	2.14	1.93	1.82	1.74	1.68	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
4	5.96	3.48	2.76	2.42	2.23	2.01	1.89	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
5	6.20	3.60	2.85	2.50	2.29	2.06	1.94	1.84	1.78	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
8	6.70	3.86	3.04	2.65	2.43	2.18	2.04	1.94	1.87	1.83	1.79	1.77	1.75	1.72	1.70	1.68	1.67	1.66	1.64	1.63
12	7.12	4.07	*3.19*	2.78	2.54	2.27	2.13	2.02	1.95	1.90	1.86	1.84	1.81	1.78	1.76	1.74	1.73	1.72	1.70	1.69
16	7.42	4.22	3.31	2.87	2.62	2.34	2.19	2.07	2.00	1.95	1.91	1.88	1.86	1.83	1.80	1.79	1.77	1.76	1.74	1.73
20	7.64	4.34	3.39	2.95	2.69	2.40	2.24	2.12	2.04	1.99	1.95	1.92	1.90	1.86	1.84	1.82	1.81	1.80	1.78	1.76
30	8.04	4.54	3.54	3.07	2.80	2.49	2.32	2.20	2.12	2.06	2.02	1.99	1.96	1.92	1.90	1.88	1.86	1.85	1.83	1.82
40	8.32	4.69	3.65	*3.16*	2.88	2.56	2.38	2.25	2.17	2.11	2.07	2.03	2.01	1.97	1.94	1.92	1.91	1.89	1.87	1.86
50	8.53	4.80	3.73	*3.23*	2.94	2.61	2.43	2.29	2.21	2.15	2.10	2.07	2.04	2.00	1.97	1.95	1.94	1.92	1.90	1.89
60	8.70	4.89	3.80	*3.29*	2.99	2.65	2.47	2.33	2.24	2.18	2.13	2.10	2.07	2.03	2.00	1.98	1.96	1.95	1.93	1.91
75	8.91	4.99	3.88	3.35	3.05	2.70	2.51	2.37	2.28	2.22	2.17	2.13	2.10	2.06	2.03	2.01	1.99	1.98	1.96	1.94
100	9.17	5.13	3.98	3.44	*3.12*	2.77	2.57	2.43	2.33	2.26	2.22	2.18	2.15	2.10	2.07	2.05	2.03	2.02	1.99	1.98
125	9.37	5.23	4.06	3.50	*3.18*	2.82	2.62	2.47	2.37	2.30	2.25	2.21	2.18	2.14	2.11	2.08	2.06	2.05	2.02	2.01
150	9.53	5.32	4.12	3.56	*3.23*	2.86	2.66	2.50	2.40	2.33	2.28	2.24	2.21	2.16	2.13	2.11	2.09	2.07	2.05	2.03
175	9.66	5.39	4.17	3.60	*3.27*	2.89	2.69	2.53	2.43	2.36	2.30	2.26	2.23	2.19	2.15	2.13	2.11	2.09	2.07	2.05
200	9.78	5.45	4.22	3.64	*3.30*	2.92	2.71	2.55	2.45	2.38	2.33	2.28	2.25	2.20	2.17	2.15	2.13	2.11	2.08	2.07

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.84	2.38	1.94	1.74	1.62	1.49	1.41	1.36	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
2	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
3	5.25	3.02	2.40	2.12	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
4	5.64	3.20	2.53	2.22	2.04	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
5	5.95	3.34	*2.62*	2.29	2.10	1.90	1.79	1.70	1.65	1.61	1.59	1.56	1.55	1.52	1.51	1.50	1.49	1.48	1.46	1.46
8	6.60	3.64	2.83	2.46	2.25	2.02	1.89	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
12	7.16	3.89	3.00	*2.60*	2.37	2.12	1.98	1.88	1.82	1.78	1.74	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
16	7.56	4.07	3.13	*2.70*	2.46	2.19	2.05	1.94	1.87	1.83	1.79	1.77	1.75	1.72	1.70	1.68	1.67	1.66	1.64	1.63
20	7.86	4.21	3.23	*2.78*	2.53	2.25	2.10	1.99	1.92	1.87	1.83	1.81	1.79	1.75	1.73	1.72	1.70	1.69	1.68	1.66
30	8.41	4.46	3.40	2.92	*2.65*	2.35	2.19	2.07	1.99	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.75	1.74	1.72
40	8.79	4.64	3.53	3.02	*2.73*	2.42	2.25	2.13	2.05	1.99	1.95	1.92	1.90	1.86	1.84	1.82	1.81	1.80	1.78	1.76
50	9.08	4.77	3.62	3.10	*2.80*	2.48	2.30	2.17	2.09	2.03	1.99	1.96	1.93	1.90	1.87	1.85	1.84	1.83	1.81	1.79
60	9.31	4.88	3.70	3.16	*2.86*	2.52	2.34	2.21	2.12	2.06	2.02	1.99	1.96	1.93	1.90	1.88	1.87	1.85	1.83	1.82
75	9.59	5.01	3.79	3.24	2.92	2.58	2.39	2.25	2.16	2.10	2.06	2.03	2.00	1.96	1.93	1.91	1.90	1.89	1.86	1.85
100	9.95	5.18	3.91	3.34	3.01	*2.65*	2.45	2.31	2.22	2.15	2.11	2.07	2.04	2.00	1.98	1.95	1.94	1.93	1.90	1.89
125	10.23	5.31	4.00	3.41	3.07	*2.70*	2.50	2.35	2.26	2.19	2.14	2.11	2.08	2.04	2.01	1.99	1.97	1.96	1.93	1.92
150	10.45	5.42	4.08	3.47	3.13	*2.74*	2.54	2.39	2.29	2.22	2.17	2.14	2.11	2.06	2.03	2.01	1.99	1.98	1.96	1.94
175	10.63	5.50	4.14	3.52	3.17	*2.78*	*2.57*	2.42	2.32	2.25	2.20	2.16	2.13	2.09	2.06	2.03	2.02	2.00	1.98	1.96
200	10.79	5.58	4.20	3.57	3.21	*2.81*	*2.60*	2.44	2.34	2.27	2.22	2.18	2.15	2.11	2.08	2.05	2.03	2.02	1.99	1.98

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.71	**2.78**	**2.23**	**1.98**	**1.83**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
2	5.64	3.20	**2.53**	**2.22**	**2.04**	**1.84**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
3	6.20	3.45	**2.70**	**2.36**	**2.16**	**1.94**	**1.83**	**1.74**	**1.68**	**1.65**	**1.62**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.52**	**1.51**	**1.49**	**1.48**
4	6.60	3.64	**2.83**	**2.46**	**2.25**	**2.02**	**1.89**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
5	6.91	3.78	*2.92*	**2.54**	**2.32**	**2.07**	**1.94**	**1.85**	**1.78**	**1.74**	**1.71**	**1.69**	**1.67**	**1.64**	**1.62**	**1.61**	**1.60**	**1.59**	**1.57**	**1.56**
8	7.56	4.07	3.13	**2.70**	**2.46**	**2.19**	**2.05**	**1.94**	**1.87**	**1.83**	**1.79**	**1.77**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.64**	**1.63**
12	8.11	4.32	3.31	*2.84*	**2.58**	**2.29**	**2.14**	**2.02**	**1.95**	**1.90**	**1.87**	**1.84**	**1.82**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.70**	**1.69**
16	8.49	4.50	3.43	*2.94*	**2.67**	**2.36**	**2.20**	**2.08**	**2.01**	**1.95**	**1.91**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.76**	**1.74**	**1.73**
20	8.79	4.64	3.53	*3.02*	**2.73**	**2.42**	**2.25**	**2.13**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.86**	**1.84**	**1.82**	**1.81**	**1.80**	**1.78**	**1.76**
30	9.31	4.88	3.70	3.16	*2.86*	**2.52**	**2.34**	**2.21**	**2.12**	**2.06**	**2.02**	**1.99**	**1.96**	**1.93**	**1.90**	**1.88**	**1.87**	**1.85**	**1.83**	**1.82**
40	9.68	5.05	3.82	3.26	*2.94*	**2.59**	**2.40**	**2.26**	**2.17**	**2.11**	**2.07**	**2.04**	**2.01**	**1.97**	**1.94**	**1.92**	**1.91**	**1.89**	**1.87**	**1.86**
50	9.95	5.18	3.91	3.34	*3.01*	**2.65**	**2.45**	**2.31**	**2.22**	**2.15**	**2.11**	**2.07**	**2.04**	**2.00**	**1.98**	**1.95**	**1.94**	**1.93**	**1.90**	**1.89**
60	10.18	5.29	3.99	3.40	*3.06*	**2.69**	**2.49**	**2.34**	**2.25**	**2.18**	**2.14**	**2.10**	**2.07**	**2.03**	**2.00**	**1.98**	**1.96**	**1.95**	**1.93**	**1.91**
75	10.45	5.42	4.08	3.47	3.13	**2.74**	**2.54**	**2.39**	**2.29**	**2.22**	**2.17**	**2.14**	**2.11**	**2.06**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.94**
100	10.79	5.58	4.20	3.57	3.21	**2.81**	**2.60**	**2.44**	**2.34**	**2.27**	**2.22**	**2.18**	**2.15**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**
125	11.05	5.70	4.28	3.64	3.27	*2.87*	**2.65**	**2.48**	**2.38**	**2.31**	**2.26**	**2.22**	**2.19**	**2.14**	**2.11**	**2.08**	**2.06**	**2.05**	**2.02**	**2.01**
150	11.27	5.81	4.36	3.70	3.32	*2.91*	**2.69**	**2.52**	**2.41**	**2.34**	**2.29**	**2.25**	**2.21**	**2.17**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.03**
175	11.44	5.89	4.42	3.75	3.37	*2.95*	**2.72**	**2.55**	**2.44**	**2.37**	**2.31**	**2.27**	**2.24**	**2.19**	**2.15**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**
200	11.60	5.96	4.47	3.79	3.41	*2.98*	**2.75**	**2.57**	**2.46**	**2.39**	**2.33**	**2.29**	**2.26**	**2.21**	**2.17**	**2.15**	**2.13**	**2.11**	**2.08**	**2.07**

Table 19-6. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 2 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.64	3.20	**2.53**	**2.22**	**2.04**	**1.84**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
2	6.60	3.64	**2.83**	**2.46**	**2.25**	**2.02**	**1.89**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
3	7.16	3.89	**3.00**	**2.60**	**2.37**	**2.12**	**1.98**	**1.88**	**1.82**	**1.78**	**1.74**	**1.72**	**1.70**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.60**	**1.59**
4	7.56	4.07	*3.13*	**2.70**	**2.46**	**2.19**	**2.05**	**1.94**	**1.87**	**1.83**	**1.79**	**1.77**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.64**	**1.63**
5	7.86	4.21	3.23	**2.78**	**2.53**	**2.25**	**2.10**	**1.99**	**1.92**	**1.87**	**1.83**	**1.81**	**1.79**	**1.75**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.66**
8	8.49	4.50	3.43	**2.94**	**2.67**	**2.36**	**2.20**	**2.08**	**2.01**	**1.95**	**1.91**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.76**	**1.74**	**1.73**
12	9.02	4.75	3.60	*3.09*	**2.79**	**2.47**	**2.29**	**2.16**	**2.08**	**2.02**	**1.98**	**1.95**	**1.93**	**1.89**	**1.87**	**1.85**	**1.83**	**1.82**	**1.79**	**1.79**
16	9.39	4.92	3.73	*3.18*	**2.88**	**2.54**	**2.35**	**2.22**	**2.13**	**2.08**	**2.03**	**2.00**	**1.97**	**1.94**	**1.91**	**1.89**	**1.87**	**1.86**	**1.84**	**1.83**
20	9.68	5.05	3.82	*3.26*	**2.94**	**2.59**	**2.40**	**2.26**	**2.17**	**2.11**	**2.07**	**2.04**	**2.01**	**1.97**	**1.94**	**1.92**	**1.91**	**1.89**	**1.87**	**1.86**
30	10.18	5.29	3.99	3.40	**3.06**	**2.69**	**2.49**	**2.34**	**2.25**	**2.18**	**2.14**	**2.10**	**2.07**	**2.03**	**2.00**	**1.98**	**1.96**	**1.95**	**1.93**	**1.91**
40	10.53	5.45	4.11	3.49	*3.14*	**2.76**	**2.55**	**2.40**	**2.30**	**2.23**	**2.18**	**2.15**	**2.12**	**2.07**	**2.04**	**2.02**	**2.00**	**1.99**	**1.97**	**1.95**
50	10.79	5.58	4.20	3.57	*3.21*	**2.81**	**2.60**	**2.44**	**2.34**	**2.27**	**2.22**	**2.18**	**2.15**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**
60	11.01	5.68	4.27	3.63	*3.26*	**2.86**	**2.64**	**2.48**	**2.37**	**2.30**	**2.25**	**2.21**	**2.18**	**2.13**	**2.10**	**2.08**	**2.06**	**2.04**	**2.02**	**2.00**
75	11.27	5.81	4.36	3.70	*3.32*	**2.91**	**2.69**	**2.52**	**2.41**	**2.34**	**2.29**	**2.25**	**2.21**	**2.17**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.03**
100	11.60	5.96	4.47	3.79	3.41	**2.98**	**2.75**	**2.57**	**2.46**	**2.39**	**2.33**	**2.29**	**2.26**	**2.21**	**2.17**	**2.15**	**2.13**	**2.11**	**2.08**	**2.07**
125	11.85	6.08	4.56	3.86	3.47	**3.03**	**2.79**	**2.62**	**2.50**	**2.43**	**2.37**	**2.32**	**2.29**	**2.24**	**2.20**	**2.18**	**2.16**	**2.14**	**2.11**	**2.09**
150	12.05	6.18	4.63	3.92	3.52	*3.07*	**2.83**	**2.65**	**2.54**	**2.46**	**2.40**	**2.35**	**2.32**	**2.27**	**2.23**	**2.20**	**2.18**	**2.17**	**2.14**	**2.12**
175	12.22	6.26	4.68	3.97	3.56	*3.11*	**2.86**	**2.68**	**2.56**	**2.48**	**2.42**	**2.38**	**2.34**	**2.29**	**2.25**	**2.22**	**2.20**	**2.19**	**2.16**	**2.14**
200	12.37	6.33	4.74	4.01	3.60	*3.14*	**2.89**	**2.70**	**2.59**	**2.50**	**2.44**	**2.40**	**2.36**	**2.31**	**2.27**	**2.24**	**2.22**	**2.20**	**2.17**	**2.15**

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.37	0.27	0.22	0.19	0.17	0.14	0.13	0.11	0.11	0.10	0.10	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.07	0.07
2	0.64	0.50	0.43	0.39	0.36	0.33	0.31	0.30	0.29	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.25	0.25	0.24	0.24
3	0.79	0.62	0.54	0.50	0.47	0.43	0.41	0.39	0.38	0.37	0.37	0.36	0.36	0.35	0.35	0.34	0.34	0.34	0.33	0.33
4	0.90	0.71	0.62	0.57	0.54	0.50	0.48	0.46	0.45	0.44	0.43	0.42	0.42	0.41	0.41	0.40	0.40	0.40	0.39	0.39
5	0.98	0.77	0.68	0.63	0.60	0.55	0.53	0.51	0.49	0.48	0.48	0.47	0.47	0.46	0.45	0.45	0.45	0.44	0.44	0.44
8	1.15	0.91	0.80	0.75	0.71	0.66	0.63	0.61	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.54	0.53	0.53	0.53
12	1.29	1.02	0.90	0.84	0.80	0.74	0.71	0.69	0.67	0.66	0.65	0.64	0.63	0.63	0.62	0.61	0.61	0.61	0.60	0.60
16	1.38	1.10	0.97	0.90	0.86	0.80	0.77	0.74	0.72	0.71	0.70	0.69	0.69	0.68	0.67	0.66	0.66	0.66	0.65	0.65
20	1.46	1.15	1.03	0.95	0.91	0.85	0.81	0.78	0.76	0.75	0.74	0.73	0.73	0.72	0.71	0.70	0.70	0.69	0.69	0.68
30	1.59	1.26	1.12	1.04	0.99	0.93	0.89	0.86	0.84	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75
40	1.68	1.33	1.18	1.10	1.05	0.98	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.81	0.81	0.80	0.79
50	1.75	1.38	1.23	1.15	1.09	1.02	0.98	0.95	0.92	0.91	0.89	0.88	0.88	0.86	0.86	0.85	0.84	0.84	0.83	0.83
60	1.81	1.43	1.27	1.18	1.13	1.06	1.01	0.98	0.95	0.94	0.92	0.91	0.91	0.89	0.88	0.88	0.87	0.87	0.86	0.85
75	1.87	1.48	1.32	1.23	1.17	1.10	1.05	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.89	0.89
100	1.96	1.55	1.38	1.28	1.22	1.15	1.10	1.06	1.04	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.93
125	2.02	1.60	1.42	1.33	1.26	1.18	1.14	1.10	1.07	1.05	1.04	1.03	1.02	1.00	0.99	0.98	0.98	0.97	0.97	0.96
150	2.08	1.64	1.46	1.36	1.30	1.21	1.17	1.13	1.10	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.98
175	2.12	1.67	1.49	1.39	1.32	1.24	1.19	1.15	1.12	1.10	1.09	1.08	1.07	1.05	1.04	1.03	1.03	1.02	1.01	1.00
200	2.16	1.70	1.52	1.41	1.35	1.26	1.21	1.17	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.64	0.50	0.43	0.39	0.36	0.33	0.31	0.30	0.29	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.25	0.25	0.24	0.24
2	0.90	0.71	0.62	0.57	0.54	0.50	0.48	0.46	0.45	0.44	0.43	0.42	0.42	0.41	0.41	0.40	0.40	0.40	0.39	0.39
3	1.04	0.83	0.73	0.68	0.64	0.59	0.57	0.55	0.53	0.52	0.51	0.51	0.50	0.49	0.49	0.49	0.48	0.48	0.48	0.47
4	1.15	0.91	0.80	0.75	0.71	0.66	0.63	0.61	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.54	0.53	0.53	0.53
5	1.22	0.97	0.86	0.80	0.76	0.71	0.68	0.65	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57	0.57
8	1.38	1.10	0.97	0.90	0.86	0.80	0.77	0.74	0.72	0.71	0.70	0.69	0.69	0.68	0.67	0.66	0.66	0.66	0.65	0.65
12	1.52	1.20	1.07	0.99	0.94	0.88	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.71
16	1.61	1.27	1.13	1.05	1.00	0.94	0.90	0.87	0.85	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.77	0.77	0.76	0.76
20	1.68	1.33	1.18	1.10	1.05	0.98	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.81	0.81	0.80	0.79
30	1.81	1.43	1.27	1.18	1.13	1.06	1.01	0.98	0.95	0.94	0.92	0.91	0.91	0.89	0.88	0.88	0.87	0.87	0.86	0.85
40	1.89	1.50	1.33	1.24	1.18	1.11	1.06	1.03	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90
50	1.96	1.55	1.38	1.28	1.22	1.15	1.10	1.06	1.04	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.93
60	2.01	1.59	1.42	1.32	1.26	1.18	1.13	1.09	1.06	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.97	0.97	0.96	0.95
75	2.08	1.64	1.46	1.36	1.30	1.21	1.17	1.13	1.10	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.98
100	2.16	1.70	1.52	1.41	1.35	1.26	1.21	1.17	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
125	2.22	1.75	1.56	1.45	1.38	1.30	1.25	1.20	1.18	1.15	1.14	1.13	1.12	1.10	1.09	1.08	1.07	1.07	1.06	1.05
150	2.27	1.79	1.60	1.49	1.42	1.33	1.28	1.23	1.20	1.18	1.16	1.15	1.14	1.13	1.11	1.11	1.10	1.09	1.08	1.08
175	2.31	1.82	1.62	1.51	1.44	1.35	1.30	1.25	1.22	1.20	1.19	1.17	1.16	1.15	1.13	1.13	1.12	1.11	1.10	1.10
200	2.35	1.85	1.65	1.54	1.46	1.37	1.32	1.27	1.24	1.22	1.20	1.19	1.18	1.16	1.15	1.14	1.14	1.13	1.12	1.11

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.90	0.71	0.62	0.57	0.54	0.50	0.48	0.46	0.45	0.44	0.43	0.42	0.42	0.41	0.41	0.40	0.40	0.40	0.39	0.39
2	1.15	0.91	0.80	0.75	0.71	0.66	0.63	0.61	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.54	0.53	0.53	0.53
3	1.29	1.02	0.90	0.84	0.80	0.74	0.71	0.69	0.67	0.66	0.65	0.64	0.63	0.63	0.62	0.61	0.61	0.61	0.60	0.60
4	1.38	1.10	0.97	0.90	0.86	0.80	0.77	0.74	0.72	0.71	0.70	0.69	0.69	0.68	0.67	0.66	0.66	0.66	0.65	0.65
5	1.46	1.15	1.03	0.95	0.91	0.85	0.81	0.78	0.76	0.75	0.74	0.73	0.73	0.72	0.71	0.70	0.70	0.69	0.69	0.68
8	1.61	1.27	1.13	1.05	1.00	0.94	0.90	0.87	0.85	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.77	0.77	0.76	0.76
12	1.74	1.37	1.22	1.14	1.08	1.01	0.97	0.94	0.92	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.82
16	1.83	1.44	1.29	1.20	1.14	1.07	1.02	0.99	0.96	0.95	0.93	0.92	0.92	0.90	0.89	0.89	0.88	0.88	0.87	0.86
20	1.89	1.50	1.33	1.24	1.18	1.11	1.06	1.03	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90
30	2.01	1.59	1.42	1.32	1.26	1.18	1.13	1.09	1.06	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.97	0.97	0.96	0.95
40	2.09	1.65	1.47	1.37	1.31	1.23	1.18	1.14	1.11	1.09	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.01	1.00	0.99
50	2.16	1.70	1.52	1.41	1.35	1.26	1.21	1.17	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
60	2.21	1.74	1.55	1.45	1.38	1.29	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.09	1.08	1.08	1.07	1.06	1.05	1.05
75	2.27	1.79	1.60	1.49	1.42	1.33	1.28	1.23	1.20	1.18	1.16	1.15	1.14	1.13	1.11	1.11	1.10	1.09	1.08	1.08
100	2.35	1.85	1.65	1.54	1.46	1.37	1.32	1.27	1.24	1.22	1.20	1.19	1.18	1.16	1.15	1.14	1.14	1.13	1.12	1.11
125	2.41	1.90	1.69	1.58	1.50	1.41	1.35	1.31	1.28	1.25	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
150	2.46	1.93	1.72	1.61	1.53	1.44	1.38	1.33	1.30	1.28	1.26	1.25	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16
175	2.50	1.96	1.75	1.63	1.56	1.46	1.40	1.36	1.32	1.30	1.28	1.27	1.26	1.24	1.22	1.21	1.21	1.20	1.19	1.18
200	2.53	1.99	1.78	1.66	1.58	1.48	1.42	1.37	1.34	1.32	1.30	1.28	1.27	1.25	1.24	1.23	1.22	1.22	1.21	1.20

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.71	0.53	0.45	0.41	0.38	0.34	0.32	0.30	0.29	0.28	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.25	0.25	0.24
2	1.00	0.76	0.66	0.60	0.56	0.51	0.49	0.47	0.45	0.44	0.43	0.43	0.42	0.41	0.41	0.41	0.40	0.40	0.40	0.39
3	1.17	0.89	0.77	0.70	0.66	0.61	0.58	0.55	0.54	0.53	0.52	0.51	0.51	0.50	0.49	0.49	0.48	0.48	0.48	0.47
4	1.30	0.98	0.85	0.78	0.73	0.68	0.64	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.54	0.53	0.53
5	1.39	1.05	0.91	0.83	0.78	0.72	0.69	0.66	0.64	0.63	0.62	0.61	0.60	0.60	0.59	0.58	0.58	0.58	0.57	0.57
8	1.59	1.19	1.04	0.95	0.89	0.83	0.79	0.75	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65
12	1.76	1.31	1.14	1.04	0.98	0.91	0.87	0.83	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72
16	1.88	1.40	1.21	1.11	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76
20	1.97	1.46	1.27	1.16	1.09	1.01	0.96	0.92	0.90	0.88	0.87	0.86	0.85	0.83	0.83	0.82	0.81	0.81	0.80	0.80
30	2.13	1.58	1.37	1.25	1.18	1.09	1.04	1.00	0.97	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.88	0.87	0.86	0.86
40	2.24	1.66	1.44	1.32	1.24	1.15	1.09	1.05	1.02	1.00	0.98	0.97	0.96	0.94	0.93	0.92	0.92	0.91	0.90	0.90
50	2.33	1.72	1.49	1.36	1.28	1.19	1.13	1.08	1.05	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93
60	2.40	1.77	1.53	1.40	1.32	1.22	1.16	1.11	1.08	1.06	1.04	1.03	1.02	1.00	0.99	0.98	0.98	0.97	0.96	0.96
75	2.48	1.83	1.58	1.45	1.36	1.26	1.20	1.15	1.12	1.10	1.08	1.06	1.05	1.04	1.02	1.02	1.01	1.00	0.99	0.99
100	2.59	1.90	1.65	1.51	1.42	1.31	1.25	1.20	1.16	1.14	1.12	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.03
125	2.67	1.96	1.70	1.55	1.46	1.35	1.29	1.23	1.20	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
150	2.73	2.01	1.74	1.59	1.50	1.38	1.32	1.26	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.10	1.09	1.08
175	2.79	2.05	1.77	1.62	1.52	1.41	1.34	1.29	1.25	1.22	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.12	1.11	1.10
200	2.83	2.08	1.80	1.65	1.55	1.43	1.36	1.31	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.14	1.12	1.12

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.00	0.76	0.66	0.60	0.56	0.51	0.49	0.47	0.45	0.44	0.43	0.43	0.42	0.41	0.41	0.41	0.40	0.40	0.40	0.39
2	1.30	0.98	0.85	0.78	0.73	0.68	0.64	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.54	0.53	0.53
3	1.47	1.10	0.96	0.88	0.83	0.76	0.73	0.70	0.68	0.66	0.65	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60
4	1.59	1.19	1.04	0.95	0.89	0.83	0.79	0.75	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65
5	1.68	1.26	1.09	1.00	0.94	0.87	0.83	0.80	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
8	1.88	1.40	1.21	1.11	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76
12	2.04	1.52	1.31	1.20	1.13	1.05	1.00	0.96	0.93	0.91	0.90	0.89	0.88	0.86	0.85	0.85	0.84	0.84	0.83	0.82
16	2.15	1.60	1.38	1.27	1.19	1.10	1.05	1.01	0.98	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
20	2.24	1.66	1.44	1.32	1.24	1.15	1.09	1.05	1.02	1.00	0.98	0.97	0.96	0.94	0.93	0.92	0.92	0.91	0.90	0.90
30	2.40	1.77	1.53	1.40	1.32	1.22	1.16	1.11	1.08	1.06	1.04	1.03	1.02	1.00	0.99	0.98	0.98	0.97	0.96	0.96
40	2.50	1.85	1.60	1.46	1.38	1.27	1.21	1.16	1.13	1.11	1.09	1.07	1.06	1.05	1.03	1.02	1.02	1.01	1.00	1.00
50	2.59	1.90	1.65	1.51	1.42	1.31	1.25	1.20	1.16	1.14	1.12	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.03
60	2.65	1.95	1.69	1.54	1.45	1.34	1.28	1.23	1.19	1.17	1.15	1.13	1.12	1.10	1.09	1.08	1.07	1.07	1.06	1.05
75	2.73	2.01	1.74	1.59	1.50	1.38	1.32	1.26	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.10	1.09	1.08
100	2.83	2.08	1.80	1.65	1.55	1.43	1.36	1.31	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.14	1.12	1.12
125	2.91	2.14	1.85	1.69	1.59	1.47	1.40	1.34	1.30	1.27	1.25	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.14
150	2.97	2.18	1.88	1.72	1.62	1.50	1.43	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.19	1.17	1.17
175	3.03	2.22	1.92	1.75	1.65	1.53	1.45	1.39	1.35	1.32	1.30	1.28	1.27	1.25	1.23	1.22	1.21	1.21	1.19	1.19
200	3.07	2.25	1.94	1.78	1.67	1.55	1.47	1.41	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.30	0.98	0.85	0.78	0.73	0.68	0.64	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.54	0.53	0.53
2	1.59	1.19	1.04	0.95	0.89	0.83	0.79	0.75	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65
3	1.76	1.31	1.14	1.04	0.98	0.91	0.87	0.83	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72
4	1.88	1.40	1.21	1.11	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76
5	1.97	1.46	1.27	1.16	1.09	1.01	0.96	0.92	0.90	0.88	0.87	0.86	0.85	0.83	0.83	0.82	0.81	0.81	0.80	0.80
8	2.15	1.60	1.38	1.27	1.19	1.10	1.05	1.01	0.98	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
12	2.31	1.71	1.48	1.35	1.28	1.18	1.12	1.08	1.05	1.03	1.01	1.00	0.99	0.97	0.96	0.95	0.95	0.94	0.93	0.92
16	2.42	1.79	1.55	1.42	1.33	1.23	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.01	1.00	0.99	0.99	0.98	0.97	0.96
20	2.50	1.85	1.60	1.46	1.38	1.27	1.21	1.16	1.13	1.11	1.09	1.07	1.06	1.05	1.03	1.02	1.02	1.01	1.00	1.00
30	2.65	1.95	1.69	1.54	1.45	1.34	1.28	1.23	1.19	1.17	1.15	1.13	1.12	1.10	1.09	1.08	1.07	1.07	1.06	1.05
40	2.75	2.02	1.75	1.60	1.51	1.39	1.33	1.27	1.24	1.21	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.09	1.09
50	2.83	2.08	1.80	1.65	1.55	1.43	1.36	1.31	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.14	1.12	1.12
60	2.90	2.13	1.84	1.68	1.58	1.46	1.39	1.33	1.30	1.27	1.25	1.23	1.22	1.20	1.18	1.17	1.17	1.16	1.15	1.14
75	2.97	2.18	1.88	1.72	1.62	1.50	1.43	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.19	1.17	1.17
100	3.07	2.25	1.94	1.78	1.67	1.55	1.47	1.41	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
125	3.14	2.30	1.99	1.82	1.71	1.58	1.51	1.44	1.40	1.37	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.25	1.24	1.23
150	3.20	2.35	2.03	1.85	1.74	1.61	1.53	1.47	1.43	1.40	1.37	1.36	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.25
175	3.25	2.38	2.06	1.88	1.77	1.64	1.56	1.49	1.45	1.42	1.39	1.38	1.36	1.34	1.32	1.31	1.30	1.29	1.28	1.27
200	3.30	2.41	2.08	1.91	1.79	1.66	1.58	1.51	1.47	1.44	1.41	1.39	1.38	1.36	1.34	1.33	1.32	1.31	1.29	1.28

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (5 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.20	0.87	0.74	0.67	0.63	0.58	0.54	0.52	0.50	0.49	0.48	0.48	0.47	0.46	0.46	0.45	0.45	0.45	0.44	0.44
2	1.56	1.11	0.95	0.86	0.80	0.74	0.70	0.67	0.65	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
3	1.78	1.26	1.07	0.97	0.90	0.83	0.78	0.75	0.73	0.71	0.70	0.69	0.68	0.67	0.66	0.66	0.65	0.65	0.64	0.64
4	1.93	1.36	1.15	1.04	0.97	0.89	0.84	0.81	0.78	0.76	0.75	0.74	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
5	2.05	1.44	1.21	1.10	1.02	0.94	0.89	0.85	0.82	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.74	0.73	0.72
8	2.31	1.60	1.35	1.21	1.13	1.04	0.98	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
12	2.53	1.74	1.46	1.31	1.22	1.12	1.06	1.01	0.98	0.96	0.94	0.93	0.92	0.90	0.89	0.88	0.88	0.87	0.86	0.86
16	2.69	1.84	1.54	1.38	1.29	1.18	1.11	1.06	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90
20	2.81	1.91	1.60	1.44	1.34	1.22	1.15	1.10	1.07	1.04	1.02	1.01	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93
30	3.02	2.05	1.71	1.53	1.43	1.30	1.23	1.17	1.13	1.11	1.09	1.07	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99
40	3.17	2.14	1.78	1.60	1.49	1.35	1.28	1.22	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.05	1.03	1.03
50	3.29	2.21	1.84	1.65	1.53	1.40	1.32	1.26	1.21	1.19	1.16	1.15	1.13	1.12	1.10	1.09	1.08	1.08	1.06	1.06
60	3.38	2.27	1.89	1.69	1.57	1.43	1.35	1.29	1.24	1.21	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08
75	3.49	2.34	1.95	1.74	1.62	1.47	1.39	1.32	1.28	1.25	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12	1.11
100	3.63	2.43	2.02	1.81	1.68	1.53	1.44	1.37	1.32	1.29	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.17	1.15	1.15
125	3.74	2.50	2.07	1.86	1.72	1.57	1.47	1.40	1.36	1.32	1.30	1.28	1.26	1.24	1.23	1.21	1.20	1.20	1.18	1.17
150	3.83	2.55	2.12	1.90	1.76	1.60	1.51	1.43	1.38	1.35	1.32	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.20	1.20
175	3.90	2.60	2.16	1.93	1.79	1.63	1.53	1.46	1.41	1.37	1.35	1.33	1.31	1.29	1.27	1.26	1.25	1.24	1.22	1.21
200	3.97	2.64	2.19	1.96	1.82	1.65	1.55	1.48	1.43	1.39	1.37	1.34	1.33	1.30	1.29	1.27	1.26	1.25	1.24	1.23

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (5 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.56	1.11	0.95	0.86	0.80	0.74	0.70	0.67	0.65	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
2	1.93	1.36	1.15	1.04	0.97	0.89	0.84	0.81	0.78	0.76	0.75	0.74	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
3	2.15	1.50	1.26	1.14	1.07	0.98	0.92	0.88	0.86	0.84	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.76	0.75
4	2.31	1.60	1.35	1.21	1.13	1.04	0.98	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
5	2.43	1.68	1.41	1.27	1.18	1.08	1.02	0.98	0.95	0.93	0.91	0.90	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
8	2.69	1.84	1.54	1.38	1.29	1.18	1.11	1.06	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90
12	2.91	1.97	1.65	1.48	1.38	1.26	1.19	1.13	1.10	1.07	1.05	1.04	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.96
16	3.06	2.07	1.72	1.55	1.44	1.31	1.24	1.18	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.01	1.00	1.00
20	3.17	2.14	1.78	1.60	1.49	1.35	1.28	1.22	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.05	1.03	1.03
30	3.38	2.27	1.89	1.69	1.57	1.43	1.35	1.29	1.24	1.21	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08
40	3.52	2.36	1.96	1.76	1.63	1.48	1.40	1.33	1.29	1.26	1.23	1.22	1.20	1.18	1.17	1.15	1.15	1.14	1.13	1.12
50	3.63	2.43	2.02	1.81	1.68	1.53	1.44	1.37	1.32	1.29	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.17	1.15	1.15
60	3.72	2.49	2.06	1.85	1.72	1.56	1.47	1.40	1.35	1.32	1.29	1.27	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17
75	3.83	2.55	2.12	1.90	1.76	1.60	1.51	1.43	1.38	1.35	1.32	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.20	1.20
100	3.97	*2.64*	2.19	1.96	1.82	1.65	1.55	1.48	1.43	1.39	1.37	1.34	1.33	1.30	1.29	1.27	1.26	1.25	1.24	1.23
125	4.07	*2.71*	2.24	2.01	1.86	1.69	1.59	1.51	1.46	1.42	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.27	1.26
150	4.16	*2.76*	2.29	2.04	1.90	1.72	1.62	1.54	1.49	1.45	1.42	1.40	1.38	1.36	1.34	1.32	1.31	1.30	1.29	1.28
175	4.23	2.80	2.32	2.08	1.93	1.75	1.64	1.56	1.51	1.47	1.44	1.42	1.40	1.38	1.36	1.34	1.33	1.32	1.31	1.30
200	4.29	2.84	2.35	2.10	1.95	1.77	1.67	1.58	1.53	1.49	1.46	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.93	1.36	1.15	1.04	0.97	0.89	0.84	0.81	0.78	0.76	0.75	0.74	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
2	2.31	1.60	1.35	1.21	1.13	1.04	0.98	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
3	2.53	1.74	1.46	1.31	1.22	1.12	1.06	1.01	0.98	0.96	0.94	0.93	0.92	0.90	0.89	0.88	0.88	0.87	0.86	0.86
4	2.69	1.84	1.54	1.38	1.29	1.18	1.11	1.06	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90
5	2.81	1.91	1.60	1.44	1.34	1.22	1.15	1.10	1.07	1.04	1.02	1.01	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93
8	3.06	2.07	1.72	1.55	1.44	1.31	1.24	1.18	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.01	1.00	1.00
12	3.27	2.20	1.83	1.64	1.53	1.39	1.31	1.25	1.21	1.18	1.16	1.14	1.13	1.11	1.10	1.08	1.08	1.07	1.06	1.05
16	3.41	2.29	1.91	1.71	1.59	1.44	1.36	1.30	1.25	1.22	1.20	1.18	1.17	1.15	1.14	1.12	1.12	1.11	1.10	1.09
20	3.52	2.36	1.96	1.76	1.63	1.48	1.40	1.33	1.29	1.26	1.23	1.22	1.20	1.18	1.17	1.15	1.15	1.14	1.13	1.12
30	3.72	2.49	2.06	1.85	1.72	1.56	1.47	1.40	1.35	1.32	1.29	1.27	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17
40	3.86	2.57	2.14	1.91	1.77	1.61	1.52	1.44	1.39	1.36	1.33	1.31	1.30	1.27	1.26	1.24	1.23	1.23	1.21	1.20
50	3.97	2.64	2.19	1.96	1.82	1.65	1.55	1.48	1.43	1.39	1.37	1.34	1.33	1.30	1.29	1.27	1.26	1.25	1.24	1.23
60	4.05	2.69	2.23	2.00	1.85	1.68	1.58	1.51	1.45	1.42	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.26	1.25
75	4.16	2.76	2.29	2.04	1.90	1.72	1.62	1.54	1.49	1.45	1.42	1.40	1.38	1.36	1.34	1.32	1.31	1.30	1.29	1.28
100	4.29	2.84	2.35	2.10	1.95	1.77	1.67	1.58	1.53	1.49	1.46	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31
125	4.39	*2.91*	2.41	2.15	1.99	1.81	1.70	1.62	1.56	1.52	1.49	1.47	1.45	1.42	1.40	1.39	1.37	1.37	1.35	1.34
150	4.47	2.96	2.45	2.19	2.03	1.84	1.73	1.64	1.59	1.55	1.52	1.49	1.47	1.44	1.42	1.41	1.40	1.39	1.37	1.36
175	4.54	3.00	2.48	2.22	2.06	1.86	1.75	1.67	1.61	1.57	1.54	1.51	1.49	1.46	1.44	1.43	1.41	1.41	1.39	1.38
200	4.60	3.04	2.51	2.24	2.08	1.89	1.77	1.69	1.63	1.58	1.55	1.53	1.51	1.48	1.46	1.44	1.43	1.42	1.40	1.39

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.64	1.14	0.96	0.87	0.81	0.74	0.70	0.67	0.65	0.63	0.62	0.62	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
2	2.07	1.40	1.17	1.05	0.98	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
3	2.34	1.56	1.29	1.16	1.08	0.98	0.93	0.89	0.86	0.84	0.83	0.81	0.81	0.79	0.78	0.78	0.77	0.77	0.76	0.75
4	2.53	1.67	1.38	1.24	1.15	1.04	0.99	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
5	2.68	1.76	1.45	1.29	1.20	1.09	1.03	0.98	0.95	0.93	0.91	0.90	0.89	0.88	0.86	0.86	0.85	0.85	0.84	0.83
8	3.00	1.94	1.59	1.42	1.31	1.19	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
12	3.27	2.09	1.71	1.52	1.40	1.27	1.20	1.14	1.10	1.07	1.05	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.96	0.96
16	3.47	2.20	1.80	1.59	1.47	1.33	1.25	1.19	1.15	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.02	1.00	1.00
20	3.62	2.29	1.86	1.65	1.52	1.37	1.29	1.23	1.19	1.16	1.13	1.12	1.10	1.09	1.07	1.06	1.05	1.05	1.04	1.03
30	3.88	2.44	1.98	1.75	1.61	1.45	1.36	1.30	1.25	1.22	1.20	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08
40	4.07	*2.55*	2.06	1.82	1.68	1.51	1.42	1.34	1.30	1.26	1.24	1.22	1.21	1.18	1.17	1.16	1.15	1.14	1.13	1.12
50	4.21	2.63	2.13	1.88	1.73	1.55	1.46	1.38	1.33	1.30	1.27	1.25	1.24	1.21	1.20	1.19	1.18	1.17	1.16	1.15
60	4.33	2.70	2.18	1.92	1.77	1.59	1.49	1.41	1.36	1.32	1.30	1.28	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17
75	4.47	2.78	2.24	1.97	1.81	1.63	1.53	1.45	1.39	1.36	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
100	4.65	2.88	2.32	2.04	1.88	1.68	1.58	1.49	1.44	1.40	1.37	1.35	1.33	1.31	1.29	1.28	1.26	1.26	1.24	1.23
125	4.78	2.96	2.38	2.10	1.92	1.73	1.61	1.53	1.47	1.43	1.40	1.38	1.36	1.34	1.32	1.30	1.29	1.28	1.27	1.26
150	4.90	3.02	*2.43*	2.14	1.96	1.76	1.65	1.56	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
175	4.99	3.07	*2.47*	2.17	1.99	1.79	1.67	1.58	1.52	1.48	1.45	1.43	1.41	1.38	1.36	1.35	1.33	1.32	1.31	1.30
200	5.07	3.12	*2.51*	2.20	2.02	1.81	1.69	1.60	1.54	1.50	1.47	1.44	1.43	1.40	1.38	1.36	1.35	1.34	1.32	1.31

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.07	1.40	1.17	1.05	0.98	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
2	2.53	1.67	1.38	1.24	1.15	1.04	0.99	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
3	2.80	1.83	1.50	1.34	1.24	1.13	1.06	1.01	0.98	0.96	0.94	0.93	0.92	0.90	0.89	0.88	0.88	0.87	0.86	0.86
4	3.00	1.94	1.59	1.42	1.31	1.19	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
5	3.15	2.02	1.66	1.47	1.36	1.23	1.16	1.11	1.07	1.04	1.03	1.01	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93
8	3.47	2.20	1.80	1.59	1.47	1.33	1.25	1.19	1.15	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.02	1.00	1.00
12	3.74	2.36	1.91	1.70	1.56	1.41	1.32	1.26	1.21	1.18	1.16	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
16	3.93	2.46	2.00	1.77	1.63	1.47	1.38	1.31	1.26	1.23	1.21	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09
20	4.07	2.55	2.06	1.82	1.68	1.51	1.42	1.34	1.30	1.26	1.24	1.22	1.21	1.18	1.17	1.16	1.15	1.14	1.13	1.12
30	4.33	*2.70*	2.18	1.92	1.77	1.59	1.49	1.41	1.36	1.32	1.30	1.28	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17
40	4.51	2.80	2.26	1.99	1.83	1.64	1.54	1.46	1.40	1.37	1.34	1.32	1.30	1.28	1.26	1.25	1.24	1.23	1.21	1.20
50	4.65	2.88	2.32	2.04	1.88	1.68	1.58	1.49	1.44	1.40	1.37	1.35	1.33	1.31	1.29	1.28	1.26	1.26	1.24	1.23
60	4.76	2.94	2.37	2.09	1.92	1.72	1.61	1.52	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
75	4.90	3.02	2.43	2.14	1.96	1.76	1.65	1.56	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
100	5.07	3.12	2.51	2.20	2.02	1.81	1.69	1.60	1.54	1.50	1.47	1.44	1.43	1.40	1.38	1.36	1.35	1.34	1.32	1.31
125	5.20	3.19	2.57	2.26	2.07	1.85	1.73	1.64	1.58	1.53	1.50	1.47	1.45	1.43	1.40	1.39	1.38	1.37	1.35	1.34
150	5.30	3.26	*2.61*	2.30	2.11	1.89	1.76	1.66	1.60	1.56	1.52	1.50	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
175	5.39	3.31	*2.65*	2.33	2.14	1.91	1.79	1.69	1.62	1.58	1.55	1.52	1.50	1.47	1.45	1.43	1.42	1.41	1.39	1.38
200	5.47	3.35	*2.69*	2.36	2.16	1.94	1.81	1.71	1.64	1.60	1.56	1.54	1.52	1.48	1.46	1.45	1.43	1.42	1.40	1.39

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.53	1.67	1.38	1.24	1.15	1.04	0.99	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
2	3.00	1.94	1.59	1.42	1.31	1.19	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
3	3.27	2.09	1.71	1.52	1.40	1.27	1.20	1.14	1.10	1.07	1.05	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.96	0.96
4	3.47	2.20	1.80	1.59	1.47	1.33	1.25	1.19	1.15	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.02	1.00	1.00
5	3.62	2.29	1.86	1.65	1.52	1.37	1.29	1.23	1.19	1.16	1.13	1.12	1.10	1.09	1.07	1.06	1.05	1.05	1.04	1.03
8	3.93	2.46	2.00	1.77	1.63	1.47	1.38	1.31	1.26	1.23	1.21	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09
12	4.19	2.61	2.11	1.87	1.72	1.54	1.45	1.37	1.32	1.29	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14
16	4.37	2.72	2.20	1.94	1.78	1.60	1.50	1.42	1.37	1.33	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
20	4.51	2.80	2.26	1.99	1.83	1.64	1.54	1.46	1.40	1.37	1.34	1.32	1.30	1.28	1.26	1.25	1.24	1.23	1.21	1.20
30	4.76	2.94	2.37	2.09	1.92	1.72	1.61	1.52	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
40	4.93	3.04	2.45	2.15	1.98	1.77	1.66	1.57	1.51	1.47	1.44	1.41	1.40	1.37	1.35	1.33	1.32	1.31	1.30	1.29
50	5.07	3.12	2.51	2.20	2.02	1.81	1.69	1.60	1.54	1.50	1.47	1.44	1.43	1.40	1.38	1.36	1.35	1.34	1.32	1.31
60	5.17	3.18	2.55	2.25	2.06	1.85	1.72	1.63	1.57	1.53	1.49	1.47	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.33
75	5.30	3.26	2.61	2.30	2.11	1.89	1.76	1.66	1.60	1.56	1.52	1.50	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
100	5.47	3.35	2.69	2.36	2.16	1.94	1.81	1.71	1.64	1.60	1.56	1.54	1.52	1.48	1.46	1.45	1.43	1.42	1.40	1.39
125	5.60	3.42	2.74	2.41	2.21	1.98	1.84	1.74	1.68	1.63	1.59	1.57	1.54	1.51	1.49	1.47	1.46	1.45	1.43	1.42
150	5.70	3.48	2.79	2.45	2.25	2.01	1.87	1.77	1.70	1.65	1.62	1.59	1.57	1.53	1.51	1.49	1.48	1.47	1.45	1.44
175	5.78	3.53	2.83	2.48	2.28	2.04	1.90	1.79	1.72	1.67	1.64	1.61	1.59	1.55	1.53	1.51	1.50	1.49	1.47	1.45
200	5.86	3.57	*2.86*	2.51	2.30	2.06	1.92	1.81	1.74	1.69	1.66	1.63	1.60	1.57	1.55	1.53	1.51	1.50	1.48	1.47

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (20 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.16	1.43	1.19	1.06	0.99	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
2	2.69	1.72	1.40	1.25	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
3	3.02	1.89	1.53	1.36	1.25	1.14	1.07	1.02	0.98	0.96	0.94	0.93	0.92	0.90	0.89	0.89	0.88	0.87	0.86	0.86
4	3.26	2.01	1.62	1.44	1.32	1.20	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
5	3.45	2.11	1.70	1.50	1.38	1.24	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93
8	3.84	2.31	1.85	1.62	1.49	1.34	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.00	1.00
12	4.19	*2.49*	1.98	1.73	1.59	1.42	1.33	1.26	1.22	1.19	1.16	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
16	4.43	*2.61*	2.07	1.81	1.65	1.48	1.38	1.31	1.26	1.23	1.21	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09
20	4.62	2.71	2.14	1.87	1.71	1.53	1.43	1.35	1.30	1.27	1.24	1.22	1.21	1.18	1.17	1.16	1.15	1.14	1.13	1.12
30	4.95	2.88	2.27	1.97	1.80	1.61	1.50	1.42	1.36	1.33	1.30	1.28	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17
40	5.19	3.00	2.36	2.05	1.87	1.66	1.55	1.47	1.41	1.37	1.34	1.32	1.30	1.28	1.26	1.25	1.24	1.23	1.21	1.20
50	5.37	3.10	*2.43*	2.11	1.92	1.71	1.59	1.50	1.44	1.40	1.38	1.35	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23
60	5.51	3.17	*2.48*	2.16	1.96	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
75	5.69	3.26	*2.55*	2.21	2.01	1.79	1.66	1.57	1.51	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
100	5.91	3.38	*2.64*	2.29	2.08	1.84	1.71	1.61	1.55	1.51	1.47	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.32	1.31
125	6.08	3.47	2.71	2.34	2.13	1.89	1.75	1.65	1.58	1.54	1.50	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
150	6.22	3.54	2.76	2.39	2.17	1.92	1.78	1.68	1.61	1.56	1.53	1.50	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
175	6.34	3.60	2.81	*2.43*	2.20	1.95	1.81	1.70	1.63	1.59	1.55	1.52	1.50	1.47	1.45	1.43	1.42	1.41	1.39	1.38
200	6.44	3.66	2.85	*2.46*	2.23	1.98	1.83	1.72	1.65	1.61	1.57	1.54	1.52	1.49	1.46	1.45	1.43	1.42	1.41	1.39

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (20 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.69	1.72	1.40	1.25	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
2	3.26	2.01	1.62	1.44	1.32	1.20	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
3	3.60	2.19	1.75	1.55	1.42	1.28	1.20	1.14	1.10	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.96	0.96
4	3.84	2.31	1.85	1.62	1.49	1.34	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.00	1.00
5	4.03	2.41	1.92	1.68	1.54	1.39	1.30	1.23	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
8	4.43	2.61	2.07	1.81	1.65	1.48	1.38	1.31	1.26	1.23	1.21	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09
12	4.77	2.79	2.20	1.92	1.75	1.56	1.46	1.38	1.33	1.29	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14
16	5.00	2.91	2.29	1.99	1.82	1.62	1.51	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
20	5.19	3.00	2.36	2.05	1.87	1.66	1.55	1.47	1.41	1.37	1.34	1.32	1.30	1.28	1.26	1.25	1.24	1.23	1.21	1.20
30	5.51	3.17	2.48	2.16	1.96	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
40	5.74	3.29	2.57	2.23	2.03	1.80	1.67	1.58	1.52	1.47	1.44	1.42	1.40	1.37	1.35	1.34	1.32	1.31	1.30	1.29
50	5.91	3.38	*2.64*	2.29	2.08	1.84	1.71	1.61	1.55	1.51	1.47	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.32	1.31
60	6.05	3.45	*2.69*	2.33	2.12	1.88	1.74	1.64	1.58	1.53	1.50	1.47	1.45	1.42	1.40	1.39	1.37	1.36	1.35	1.33
75	6.22	3.54	*2.76*	2.39	2.17	1.92	1.78	1.68	1.61	1.56	1.53	1.50	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
100	6.44	3.66	2.85	2.46	2.23	1.98	1.83	1.72	1.65	1.61	1.57	1.54	1.52	1.49	1.46	1.45	1.43	1.42	1.41	1.39
125	6.61	3.74	2.91	2.51	2.28	2.02	1.87	1.76	1.69	1.64	1.60	1.57	1.55	1.51	1.49	1.47	1.46	1.45	1.43	1.42
150	6.74	3.81	2.96	2.56	2.32	2.05	1.90	1.79	1.71	1.66	1.62	1.59	1.57	1.54	1.51	1.50	1.48	1.47	1.45	1.44
175	6.85	3.87	3.01	2.60	2.35	2.08	1.93	1.81	1.74	1.68	1.64	1.62	1.59	1.56	1.53	1.51	1.50	1.49	1.47	1.45
200	6.95	3.92	3.04	*2.63*	2.38	2.10	1.95	1.83	1.76	1.70	1.66	1.63	1.61	1.57	1.55	1.53	1.51	1.50	1.48	1.47

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (20 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.26	2.01	1.62	1.44	1.32	1.20	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
2	3.84	2.31	1.85	1.62	1.49	1.34	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.00	1.00
3	4.19	2.49	1.98	1.73	1.59	1.42	1.33	1.26	1.22	1.19	1.16	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
4	4.43	2.61	2.07	1.81	1.65	1.48	1.38	1.31	1.26	1.23	1.21	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09
5	4.62	2.71	2.14	1.87	1.71	1.53	1.43	1.35	1.30	1.27	1.24	1.22	1.21	1.18	1.17	1.16	1.15	1.14	1.13	1.12
8	5.00	*2.91*	2.29	1.99	1.82	1.62	1.51	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
12	5.33	3.08	2.42	2.10	1.91	1.70	1.58	1.50	1.44	1.40	1.37	1.35	1.33	1.30	1.28	1.27	1.26	1.25	1.24	1.23
16	5.56	3.20	2.50	2.17	1.98	1.76	1.63	1.54	1.48	1.44	1.41	1.39	1.37	1.34	1.32	1.31	1.30	1.29	1.27	1.26
20	5.74	3.29	2.57	2.23	2.03	1.80	1.67	1.58	1.52	1.47	1.44	1.42	1.40	1.37	1.35	1.34	1.32	1.31	1.30	1.29
30	6.05	3.45	2.69	2.33	2.12	1.88	1.74	1.64	1.58	1.53	1.50	1.47	1.45	1.42	1.40	1.39	1.37	1.36	1.35	1.33
40	6.27	3.57	2.78	2.40	2.18	1.93	1.79	1.69	1.62	1.57	1.54	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
50	6.44	3.66	*2.85*	2.46	2.23	1.98	1.83	1.72	1.65	1.61	1.57	1.54	1.52	1.49	1.46	1.45	1.43	1.42	1.41	1.39
60	6.57	3.73	*2.90*	2.50	2.27	2.01	1.86	1.75	1.68	1.63	1.59	1.57	1.54	1.51	1.49	1.47	1.46	1.44	1.43	1.41
75	6.74	3.81	*2.96*	2.56	2.32	2.05	1.90	1.79	1.71	1.66	1.62	1.59	1.57	1.54	1.51	1.50	1.48	1.47	1.45	1.44
100	6.95	3.92	3.04	2.63	2.38	2.10	1.95	1.83	1.76	1.70	1.66	1.63	1.61	1.57	1.55	1.53	1.51	1.50	1.48	1.47
125	7.11	4.01	3.11	2.68	2.43	2.15	1.99	1.87	1.79	1.73	1.69	1.66	1.64	1.60	1.57	1.55	1.54	1.53	1.51	1.49
150	7.24	4.07	3.16	2.72	2.47	2.18	2.02	1.89	1.81	1.76	1.72	1.68	1.66	1.62	1.60	1.58	1.56	1.55	1.53	1.51
175	7.34	4.13	3.20	2.76	2.50	2.21	2.04	1.92	1.84	1.78	1.74	1.70	1.68	1.64	1.61	1.59	1.58	1.57	1.54	1.53
200	7.44	4.18	3.24	2.79	2.53	2.23	2.06	1.94	1.85	1.80	1.75	1.72	1.70	1.66	1.63	1.61	1.59	1.58	1.56	1.54

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (40 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.81	1.74	1.42	1.26	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
2	3.46	2.06	1.65	1.45	1.33	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
3	3.87	2.25	1.78	1.56	1.43	1.28	1.20	1.14	1.10	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.97	0.96
4	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
5	4.40	*2.50*	1.96	1.70	1.56	1.39	1.30	1.23	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
8	4.90	2.73	2.12	1.84	1.67	1.49	1.39	1.31	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
12	5.33	2.93	2.26	1.95	1.77	1.57	1.47	1.38	1.33	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14
16	5.63	3.07	2.36	2.03	1.84	1.63	1.52	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
20	5.86	3.18	*2.44*	2.09	1.90	1.68	1.56	1.47	1.41	1.37	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.21	1.20
30	6.29	3.38	*2.58*	2.21	2.00	1.76	1.63	1.54	1.48	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
40	6.58	3.51	2.68	2.29	2.07	1.82	1.69	1.59	1.52	1.48	1.44	1.42	1.40	1.37	1.35	1.34	1.32	1.32	1.30	1.29
50	6.81	3.62	2.75	2.35	2.12	1.86	1.73	1.62	1.56	1.51	1.48	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.31
60	6.99	3.71	2.81	2.40	2.16	1.90	1.76	1.65	1.58	1.54	1.50	1.48	1.45	1.42	1.40	1.39	1.37	1.36	1.35	1.34
75	7.21	3.81	2.89	*2.46*	2.22	1.95	1.80	1.69	1.62	1.57	1.53	1.51	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
100	7.49	3.95	2.98	*2.54*	2.29	2.00	1.85	1.73	1.66	1.61	1.57	1.54	1.52	1.49	1.47	1.45	1.43	1.42	1.41	1.39
125	7.71	4.05	3.06	*2.60*	2.34	2.05	1.89	1.77	1.69	1.64	1.60	1.57	1.55	1.52	1.49	1.47	1.46	1.45	1.43	1.42
150	7.89	4.13	3.12	*2.65*	2.38	2.08	1.92	1.80	1.72	1.67	1.63	1.60	1.57	1.54	1.51	1.50	1.48	1.47	1.45	1.44
175	8.03	4.20	3.17	*2.69*	*2.42*	2.11	1.95	1.82	1.74	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.50	1.49	1.47	1.45
200	8.16	4.26	3.21	2.73	*2.45*	2.14	1.97	1.84	1.76	1.71	1.67	1.64	1.61	1.57	1.55	1.53	1.52	1.50	1.48	1.47

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.46	2.06	1.65	1.45	1.33	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
2	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
3	4.59	2.59	2.02	1.76	1.60	1.43	1.34	1.27	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
4	4.90	*2.73*	2.12	1.84	1.67	1.49	1.39	1.31	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
5	5.13	2.84	2.20	1.90	1.73	1.54	1.43	1.35	1.30	1.27	1.24	1.22	1.21	1.18	1.17	1.16	1.15	1.14	1.13	1.12
8	5.63	3.07	2.36	2.03	1.84	1.63	1.52	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
12	6.06	3.27	2.50	2.15	1.94	1.72	1.59	1.50	1.44	1.40	1.37	1.35	1.33	1.30	1.29	1.27	1.26	1.25	1.24	1.23
16	6.35	3.41	2.60	2.23	2.01	1.77	1.65	1.55	1.49	1.44	1.41	1.39	1.37	1.34	1.32	1.31	1.30	1.29	1.27	1.26
20	6.58	3.51	*2.68*	2.29	2.07	1.82	1.69	1.59	1.52	1.48	1.44	1.42	1.40	1.37	1.35	1.34	1.32	1.32	1.30	1.29
30	6.99	3.71	*2.81*	2.40	2.16	1.90	1.76	1.65	1.58	1.54	1.50	1.48	1.45	1.42	1.40	1.39	1.37	1.36	1.35	1.34
40	7.28	3.84	2.91	2.48	2.23	1.96	1.81	1.70	1.63	1.58	1.54	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
50	7.49	3.95	2.98	2.54	2.29	2.00	1.85	1.73	1.66	1.61	1.57	1.54	1.52	1.49	1.47	1.45	1.43	1.42	1.41	1.39
60	7.67	4.03	3.04	2.59	2.33	2.04	1.88	1.76	1.69	1.64	1.60	1.57	1.55	1.51	1.49	1.47	1.46	1.45	1.43	1.41
75	7.89	4.13	3.12	*2.65*	2.38	2.08	1.92	1.80	1.72	1.67	1.63	1.60	1.57	1.54	1.51	1.50	1.48	1.47	1.45	1.44
100	8.16	4.26	3.21	*2.73*	2.45	2.14	1.97	1.84	1.76	1.71	1.67	1.64	1.61	1.57	1.55	1.53	1.52	1.50	1.48	1.47
125	8.37	4.36	3.28	*2.79*	2.50	2.18	2.01	1.88	1.80	1.74	1.70	1.66	1.64	1.60	1.58	1.56	1.54	1.53	1.51	1.49
150	8.53	4.44	3.34	*2.83*	2.54	2.22	2.04	1.91	1.82	1.76	1.72	1.69	1.66	1.62	1.60	1.58	1.56	1.55	1.53	1.51
175	8.68	4.51	3.39	*2.87*	2.58	2.25	2.07	1.93	1.85	1.79	1.74	1.71	1.68	1.64	1.61	1.59	1.58	1.57	1.54	1.53
200	8.80	4.57	3.43	2.91	*2.61*	2.27	2.09	1.95	1.87	1.80	1.76	1.73	1.70	1.66	1.63	1.61	1.59	1.58	1.56	1.54

Table 19-7. κ-Multipliers for 1-of-3 Interwell Prediction Limits on Means of Order 2 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
2	4.90	2.73	2.12	1.84	1.67	1.49	1.39	1.31	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
3	5.33	*2.93*	2.26	1.95	1.77	1.57	1.47	1.38	1.33	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14
4	5.63	3.07	2.36	2.03	1.84	1.63	1.52	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
5	5.86	3.18	2.44	2.09	1.90	1.68	1.56	1.47	1.41	1.37	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.21	1.20
8	6.35	3.41	2.60	2.23	2.01	1.77	1.65	1.55	1.49	1.44	1.41	1.39	1.37	1.34	1.32	1.31	1.30	1.29	1.27	1.26
12	6.77	3.60	2.74	2.34	2.11	1.86	1.72	1.61	1.55	1.50	1.47	1.44	1.42	1.40	1.37	1.36	1.35	1.34	1.32	1.31
16	7.06	3.74	*2.83*	2.42	2.18	1.91	1.77	1.66	1.59	1.55	1.51	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
20	7.28	3.84	*2.91*	2.48	2.23	1.96	1.81	1.70	1.63	1.58	1.54	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
30	7.67	4.03	3.04	2.59	2.33	2.04	1.88	1.76	1.69	1.64	1.60	1.57	1.55	1.51	1.49	1.47	1.46	1.45	1.43	1.41
40	7.95	4.16	3.14	2.67	2.40	2.10	1.93	1.81	1.73	1.68	1.64	1.61	1.58	1.55	1.52	1.50	1.49	1.48	1.46	1.45
50	8.16	4.26	3.21	2.73	2.45	2.14	1.97	1.84	1.76	1.71	1.67	1.64	1.61	1.57	1.55	1.53	1.52	1.50	1.48	1.47
60	8.33	4.34	3.27	2.77	2.49	2.18	2.00	1.87	1.79	1.73	1.69	1.66	1.63	1.60	1.57	1.55	1.54	1.52	1.50	1.49
75	8.53	4.44	3.34	*2.83*	2.54	2.22	2.04	1.91	1.82	1.76	1.72	1.69	1.66	1.62	1.60	1.58	1.56	1.55	1.53	1.51
100	8.80	4.57	3.43	*2.91*	2.61	2.27	2.09	1.95	1.87	1.80	1.76	1.73	1.70	1.66	1.63	1.61	1.59	1.58	1.56	1.54
125	9.00	4.67	3.50	*2.97*	2.66	2.32	2.13	1.99	1.90	1.84	1.79	1.75	1.73	1.69	1.66	1.64	1.62	1.61	1.58	1.57
150	9.16	4.74	3.56	*3.01*	2.70	2.35	2.16	2.02	1.92	1.86	1.81	1.78	1.75	1.71	1.68	1.66	1.64	1.63	1.60	1.59
175	9.29	4.81	3.60	*3.05*	2.73	2.38	2.19	2.04	1.95	1.88	1.83	1.80	1.77	1.73	1.69	1.67	1.66	1.64	1.62	1.60
200	9.41	4.86	3.64	*3.08*	2.76	2.40	2.21	2.06	1.97	1.90	1.85	1.81	1.78	1.74	1.71	1.69	1.67	1.66	1.63	1.62

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.25	1.04	0.96	0.91	0.88	0.84	0.82	0.81	0.79	0.79	0.78	0.78	0.77	0.77	0.76	0.76	0.76	0.76	0.75	0.75
2	1.65	1.35	1.24	1.17	1.13	1.08	1.05	1.03	1.01	1.00	1.00	0.99	0.99	0.98	0.97	0.97	0.97	0.96	0.96	0.96
3	1.87	1.53	1.39	1.32	1.27	1.21	1.18	1.15	1.13	1.12	1.11	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07	1.07
4	2.03	1.65	1.50	1.42	1.36	1.30	1.26	1.23	1.22	1.20	1.19	1.18	1.18	1.17	1.16	1.16	1.15	1.15	1.14	1.14
5	2.16	1.74	1.58	1.49	1.43	1.37	1.33	1.30	1.28	1.26	1.25	1.24	1.24	1.23	1.22	1.21	1.21	1.21	1.20	1.20
8	2.41	1.93	1.75	1.65	1.58	1.50	1.46	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31
12	2.62	2.09	1.89	1.77	1.70	1.62	1.57	1.53	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40	1.40
16	2.76	2.20	1.98	1.86	1.79	1.70	1.64	1.60	1.57	1.55	1.54	1.53	1.52	1.50	1.49	1.48	1.48	1.47	1.47	1.46
20	2.87	2.28	2.05	1.93	1.85	1.76	1.70	1.65	1.63	1.60	1.59	1.58	1.57	1.55	1.54	1.53	1.53	1.52	1.51	1.51
30	3.06	2.43	2.18	2.05	1.96	1.86	1.80	1.75	1.72	1.70	1.68	1.67	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.59
40	3.19	2.53	2.27	2.13	2.04	1.93	1.87	1.82	1.78	1.76	1.74	1.73	1.72	1.70	1.69	1.68	1.67	1.66	1.65	1.65
50	3.29	2.61	2.34	2.19	2.10	1.99	1.92	1.87	1.83	1.81	1.79	1.77	1.76	1.74	1.73	1.72	1.71	1.71	1.70	1.69
60	3.38	2.67	2.39	2.24	2.15	2.03	1.96	1.91	1.87	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.72
75	3.47	2.74	2.46	2.30	2.21	2.09	2.02	1.96	1.92	1.89	1.87	1.86	1.84	1.83	1.81	1.80	1.79	1.79	1.77	1.77
100	3.59	2.83	2.54	2.38	2.28	2.15	2.08	2.02	1.98	1.95	1.93	1.92	1.90	1.88	1.87	1.86	1.85	1.84	1.83	1.82
125	3.69	2.90	2.60	2.44	2.33	2.21	2.13	2.07	2.03	2.00	1.98	1.96	1.94	1.92	1.91	1.90	1.89	1.88	1.87	1.86
150	3.76	2.96	2.65	2.48	2.38	2.25	2.17	2.11	2.07	2.04	2.01	1.99	1.98	1.96	1.94	1.93	1.92	1.91	1.90	1.89
175	3.82	3.01	2.69	2.52	2.41	2.28	2.20	2.14	2.10	2.07	2.04	2.02	2.01	1.99	1.97	1.96	1.95	1.94	1.93	1.92
200	3.88	3.05	2.73	2.56	2.45	2.31	2.23	2.17	2.12	2.09	2.07	2.05	2.03	2.01	1.99	1.98	1.97	1.96	1.95	1.94

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.65	1.35	1.24	1.17	1.13	1.08	1.05	1.03	1.01	1.00	1.00	0.99	0.99	0.98	0.97	0.97	0.97	0.96	0.96	0.96
2	2.03	1.65	1.50	1.42	1.36	1.30	1.26	1.23	1.22	1.20	1.19	1.18	1.18	1.17	1.16	1.16	1.15	1.15	1.14	1.14
3	2.25	1.82	1.65	1.55	1.49	1.42	1.38	1.35	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.25	1.24	1.24
4	2.41	1.93	1.75	1.65	1.58	1.50	1.46	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31
5	2.52	2.02	1.82	1.72	1.65	1.57	1.52	1.48	1.46	1.44	1.43	1.42	1.41	1.39	1.39	1.38	1.37	1.37	1.36	1.36
8	2.76	2.20	1.98	1.86	1.79	1.70	1.64	1.60	1.57	1.55	1.54	1.53	1.52	1.50	1.49	1.48	1.48	1.47	1.47	1.46
12	2.96	2.35	2.11	1.98	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.62	1.61	1.59	1.58	1.57	1.57	1.56	1.55	1.54
16	3.09	2.45	2.20	2.07	1.98	1.88	1.82	1.77	1.73	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.62	1.62	1.61	1.60
20	3.19	2.53	2.27	2.13	2.04	1.93	1.87	1.82	1.78	1.76	1.74	1.73	1.72	1.70	1.69	1.68	1.67	1.66	1.65	1.65
30	3.38	2.67	2.39	2.24	2.15	2.03	1.96	1.91	1.87	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.72
40	3.50	2.76	2.48	2.32	2.22	2.10	2.03	1.97	1.94	1.91	1.89	1.87	1.86	1.84	1.82	1.81	1.80	1.80	1.79	1.78
50	3.59	2.83	2.54	2.38	2.28	2.15	2.08	2.02	1.98	1.95	1.93	1.92	1.90	1.88	1.87	1.86	1.85	1.84	1.83	1.82
60	3.67	2.89	2.59	2.43	2.32	2.20	2.12	2.06	2.02	1.99	1.97	1.95	1.94	1.92	1.90	1.89	1.88	1.87	1.86	1.85
75	3.76	2.96	2.65	2.48	2.38	2.25	2.17	2.11	2.07	2.04	2.01	1.99	1.98	1.96	1.94	1.93	1.92	1.91	1.90	1.89
100	3.88	3.05	2.73	2.56	2.45	2.31	2.23	2.17	2.12	2.09	2.07	2.05	2.03	2.01	1.99	1.98	1.97	1.96	1.95	1.94
125	3.96	3.12	2.79	2.61	2.50	2.36	2.28	2.21	2.17	2.13	2.11	2.09	2.08	2.05	2.03	2.02	2.01	2.00	1.99	1.98
150	4.03	3.17	2.84	2.66	2.54	2.40	2.32	2.25	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.04	2.03	2.02	2.01
175	4.09	3.22	2.88	2.69	2.58	2.43	2.35	2.28	2.23	2.20	2.17	2.15	2.14	2.11	2.09	2.08	2.07	2.06	2.05	2.03
200	4.14	3.25	2.91	2.73	2.61	2.46	2.37	2.31	2.26	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.09	2.08	2.07	2.06

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.03	1.65	1.50	1.42	1.36	1.30	1.26	1.23	1.22	1.20	1.19	1.18	1.18	1.17	1.16	1.16	1.15	1.15	1.14	1.14
2	2.41	1.93	1.75	1.65	1.58	1.50	1.46	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31
3	2.62	2.09	1.89	1.77	1.70	1.62	1.57	1.53	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40	1.40
4	2.76	2.20	1.98	1.86	1.79	1.70	1.64	1.60	1.57	1.55	1.54	1.53	1.52	1.50	1.49	1.48	1.48	1.47	1.47	1.46
5	2.87	2.28	2.05	1.93	1.85	1.76	1.70	1.65	1.63	1.60	1.59	1.58	1.57	1.55	1.54	1.53	1.53	1.52	1.51	1.51
8	3.09	2.45	2.20	2.07	1.98	1.88	1.82	1.77	1.73	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.62	1.62	1.61	1.60
12	3.28	2.59	2.33	2.18	2.09	1.98	1.91	1.86	1.82	1.80	1.78	1.77	1.75	1.74	1.72	1.71	1.71	1.70	1.69	1.68
16	3.40	2.69	2.41	2.26	2.16	2.05	1.98	1.92	1.89	1.86	1.84	1.83	1.81	1.79	1.78	1.77	1.76	1.76	1.74	1.74
20	3.50	2.76	2.48	2.32	2.22	2.10	2.03	1.97	1.94	1.91	1.89	1.87	1.86	1.84	1.82	1.81	1.80	1.80	1.79	1.78
30	3.67	2.89	2.59	2.43	2.32	2.20	2.12	2.06	2.02	1.99	1.97	1.95	1.94	1.92	1.90	1.89	1.88	1.87	1.86	1.85
40	3.79	2.98	2.67	2.50	2.39	2.26	2.18	2.12	2.08	2.05	2.02	2.01	1.99	1.97	1.95	1.94	1.93	1.92	1.91	1.90
50	3.88	3.05	2.73	2.56	2.45	2.31	2.23	2.17	2.12	2.09	2.07	2.05	2.03	2.01	1.99	1.98	1.97	1.96	1.95	1.94
60	3.95	3.10	2.78	2.60	2.49	2.35	2.27	2.20	2.16	2.13	2.10	2.08	2.07	2.04	2.03	2.01	2.00	2.00	1.98	1.97
75	4.03	3.17	2.84	2.66	2.54	2.40	2.32	2.25	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.04	2.03	2.02	2.01
100	4.14	3.25	2.91	2.73	2.61	2.46	2.37	2.31	2.26	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.09	2.08	2.07	2.06
125	4.23	3.32	2.97	2.78	2.66	2.51	2.42	2.35	2.30	2.27	2.24	2.22	2.20	2.17	2.16	2.14	2.13	2.12	2.10	2.09
150	4.29	3.37	3.01	2.82	2.70	2.55	2.46	2.38	2.33	2.30	2.27	2.25	2.23	2.21	2.19	2.17	2.16	2.15	2.13	2.12
175	4.35	3.41	3.05	2.86	2.73	2.58	2.49	2.41	2.36	2.33	2.30	2.28	2.26	2.23	2.21	2.20	2.19	2.18	2.16	2.15
200	4.40	3.45	3.08	2.89	2.76	2.60	2.51	2.44	2.39	2.35	2.32	2.30	2.28	2.25	2.24	2.22	2.21	2.20	2.18	2.17

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.78	1.41	1.27	1.20	1.15	1.09	1.06	1.04	1.02	1.01	1.00	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96
2	2.24	1.74	1.55	1.45	1.39	1.32	1.28	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.15	1.15	1.14
3	2.52	1.93	1.71	1.60	1.53	1.44	1.40	1.36	1.33	1.32	1.30	1.29	1.29	1.28	1.27	1.26	1.26	1.25	1.25	1.24
4	2.71	2.06	1.82	1.70	1.62	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
5	2.86	2.16	1.91	1.78	1.69	1.60	1.54	1.50	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.38	1.37	1.36	1.36
8	3.17	2.38	2.09	1.94	1.84	1.73	1.67	1.62	1.59	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.48	1.47	1.46
12	3.43	2.55	2.24	2.07	1.97	1.84	1.77	1.72	1.68	1.66	1.64	1.63	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.55
16	3.60	2.68	2.34	2.16	2.05	1.92	1.85	1.79	1.75	1.73	1.71	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.60
20	3.74	2.77	2.42	2.23	2.12	1.98	1.91	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.69	1.68	1.67	1.67	1.66	1.65
30	3.98	2.94	2.56	2.36	2.24	2.09	2.01	1.94	1.90	1.87	1.84	1.83	1.81	1.79	1.77	1.76	1.75	1.75	1.74	1.73
40	4.15	3.05	2.65	2.45	2.32	2.16	2.08	2.01	1.96	1.93	1.90	1.89	1.87	1.85	1.83	1.82	1.81	1.80	1.79	1.78
50	4.27	3.14	2.73	2.51	2.38	2.22	2.13	2.06	2.01	1.98	1.95	1.93	1.92	1.89	1.87	1.86	1.85	1.84	1.83	1.82
60	4.37	3.21	2.79	2.57	2.43	2.27	2.17	2.10	2.05	2.01	1.99	1.97	1.95	1.93	1.91	1.90	1.89	1.88	1.86	1.85
75	4.49	3.29	2.86	2.63	2.49	2.32	2.22	2.15	2.10	2.06	2.03	2.01	2.00	1.97	1.95	1.94	1.93	1.92	1.90	1.89
100	4.65	3.40	2.95	2.71	2.57	2.39	2.29	2.21	2.16	2.12	2.09	2.07	2.05	2.02	2.01	1.99	1.98	1.97	1.96	1.94
125	4.76	3.48	3.02	2.78	2.63	2.45	2.34	2.26	2.20	2.16	2.13	2.11	2.09	2.07	2.05	2.03	2.02	2.01	1.99	1.98
150	4.86	3.55	*3.07*	2.83	2.67	2.49	2.38	2.30	2.24	2.20	2.17	2.15	2.13	2.10	2.08	2.06	2.05	2.04	2.03	2.01
175	4.94	3.60	*3.12*	2.87	2.71	2.52	2.42	2.33	2.27	2.23	2.20	2.18	2.16	2.13	2.11	2.09	2.08	2.07	2.05	2.04
200	5.00	3.65	*3.16*	2.91	2.75	2.56	2.44	2.36	2.30	2.26	2.23	2.20	2.18	2.15	2.13	2.11	2.10	2.09	2.07	2.06

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.24	1.74	1.55	1.45	1.39	1.32	1.28	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.15	1.15	1.14
2	2.71	2.06	1.82	1.70	1.62	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
3	2.98	2.25	1.98	1.84	1.75	1.65	1.59	1.54	1.51	1.49	1.48	1.47	1.46	1.44	1.43	1.42	1.42	1.41	1.41	1.40
4	3.17	2.38	2.09	1.94	1.84	1.73	1.67	1.62	1.59	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.48	1.47	1.46
5	3.31	2.47	2.17	2.01	1.91	1.79	1.73	1.67	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.51
8	3.60	2.68	2.34	2.16	2.05	1.92	1.85	1.79	1.75	1.73	1.71	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.60
12	3.85	2.84	2.48	2.29	2.17	2.03	1.95	1.89	1.85	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.70	1.69	1.68
16	4.02	2.96	2.58	2.38	2.25	2.11	2.02	1.96	1.91	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.77	1.76	1.75	1.74
20	4.15	3.05	2.65	2.45	2.32	2.16	2.08	2.01	1.96	1.93	1.90	1.89	1.87	1.85	1.83	1.82	1.81	1.80	1.79	1.78
30	4.37	3.21	2.79	2.57	2.43	2.27	2.17	2.10	2.05	2.01	1.99	1.97	1.95	1.93	1.91	1.90	1.89	1.88	1.86	1.85
40	4.53	3.32	2.88	2.65	2.51	2.34	2.24	2.16	2.11	2.07	2.05	2.02	2.01	1.98	1.96	1.95	1.94	1.93	1.92	1.91
50	4.65	3.40	2.95	2.71	2.57	2.39	2.29	2.21	2.16	2.12	2.09	2.07	2.05	2.02	2.01	1.99	1.98	1.97	1.96	1.94
60	4.74	3.47	3.01	2.77	2.62	2.44	2.33	2.25	2.19	2.16	2.13	2.10	2.09	2.06	2.04	2.02	2.01	2.00	1.99	1.98
75	4.86	3.55	3.07	2.83	2.67	2.49	2.38	2.30	2.24	2.20	2.17	2.15	2.13	2.10	2.08	2.06	2.05	2.04	2.03	2.01
100	5.00	3.65	3.16	2.91	2.75	2.56	2.44	2.36	2.30	2.26	2.23	2.20	2.18	2.15	2.13	2.11	2.10	2.09	2.07	2.06
125	5.11	3.73	3.23	2.97	2.80	2.61	2.49	2.40	2.34	2.30	2.27	2.24	2.22	2.19	2.17	2.15	2.14	2.13	2.11	2.10
150	5.20	3.79	*3.28*	3.01	2.85	2.65	2.53	2.44	2.38	2.33	2.30	2.28	2.25	2.22	2.20	2.18	2.17	2.16	2.14	2.13
175	5.28	3.84	*3.32*	3.05	2.88	2.68	2.56	2.47	2.41	2.36	2.33	2.30	2.28	2.25	2.23	2.21	2.20	2.19	2.17	2.15
200	5.34	3.89	3.36	3.09	2.92	2.71	2.59	2.50	2.43	2.39	2.35	2.33	2.31	2.27	2.25	2.23	2.22	2.21	2.19	2.17

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.71	2.06	1.82	1.70	1.62	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
2	3.17	2.38	2.09	1.94	1.84	1.73	1.67	1.62	1.59	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.48	1.47	1.46
3	3.43	2.55	2.24	2.07	1.97	1.84	1.77	1.72	1.68	1.66	1.64	1.63	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.55
4	3.60	2.68	2.34	2.16	2.05	1.92	1.85	1.79	1.75	1.73	1.71	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.60
5	3.74	2.77	2.42	2.23	2.12	1.98	1.91	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.69	1.68	1.67	1.67	1.66	1.65
8	4.02	2.96	2.58	2.38	2.25	2.11	2.02	1.96	1.91	1.88	1.86	1.84	1.82	1.80	1.79	1.78	1.77	1.76	1.75	1.74
12	4.25	3.12	2.71	2.50	2.37	2.21	2.12	2.05	2.00	1.97	1.94	1.92	1.91	1.88	1.87	1.85	1.84	1.84	1.82	1.81
16	4.41	3.23	2.81	2.59	2.45	2.28	2.19	2.11	2.06	2.03	2.00	1.98	1.96	1.94	1.92	1.91	1.90	1.89	1.88	1.87
20	4.53	3.32	2.88	2.65	2.51	2.34	2.24	2.16	2.11	2.07	2.05	2.02	2.01	1.98	1.96	1.95	1.94	1.93	1.92	1.91
30	4.74	3.47	3.01	2.77	2.62	2.44	2.33	2.25	2.19	2.16	2.13	2.10	2.09	2.06	2.04	2.02	2.01	2.00	1.99	1.98
40	4.89	3.57	3.09	2.84	2.69	2.50	2.40	2.31	2.25	2.21	2.18	2.16	2.14	2.11	2.09	2.07	2.06	2.05	2.04	2.02
50	5.00	3.65	3.16	2.91	2.75	2.56	2.44	2.36	2.30	2.26	2.23	2.20	2.18	2.15	2.13	2.11	2.10	2.09	2.07	2.06
60	5.09	3.71	3.22	2.95	2.79	2.60	2.48	2.39	2.33	2.29	2.26	2.23	2.21	2.18	2.16	2.15	2.13	2.12	2.10	2.09
75	5.20	3.79	3.28	3.01	2.85	2.65	2.53	2.44	2.38	2.33	2.30	2.28	2.25	2.22	2.20	2.18	2.17	2.16	2.14	2.13
100	5.34	3.89	3.36	3.09	2.92	2.71	2.59	2.50	2.43	2.39	2.35	2.33	2.31	2.27	2.25	2.23	2.22	2.21	2.19	2.17
125	5.44	3.96	3.43	3.15	2.97	2.76	2.64	2.54	2.48	2.43	2.40	2.37	2.35	2.31	2.29	2.27	2.26	2.24	2.22	2.21
150	5.53	4.02	*3.48*	3.19	3.01	2.80	2.68	2.58	2.51	2.46	2.43	2.40	2.38	2.34	2.32	2.30	2.29	2.27	2.25	2.24
175	5.60	4.07	*3.52*	3.23	3.05	2.83	2.71	2.61	2.54	2.49	2.46	2.43	2.40	2.37	2.34	2.32	2.31	2.30	2.28	2.26
200	5.66	4.11	3.56	3.26	3.08	2.86	2.74	2.63	2.57	2.52	2.48	2.45	2.43	2.39	2.37	2.35	2.33	2.32	2.30	2.28

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.62	1.92	1.68	1.56	1.49	1.40	1.35	1.31	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.21	1.20	1.20
2	3.22	2.29	1.98	1.82	1.73	1.62	1.55	1.51	1.48	1.45	1.44	1.43	1.42	1.40	1.39	1.38	1.38	1.37	1.37	1.36
3	3.58	2.51	2.15	1.97	1.86	1.74	1.67	1.61	1.58	1.56	1.54	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
4	3.83	2.66	2.27	2.08	1.96	1.82	1.75	1.69	1.65	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
5	4.03	2.78	2.36	2.16	2.03	1.89	1.81	1.75	1.71	1.68	1.66	1.64	1.63	1.61	1.60	1.59	1.58	1.57	1.56	1.56
8	4.44	3.03	2.56	2.32	2.18	2.02	1.93	1.86	1.82	1.79	1.76	1.75	1.73	1.71	1.70	1.69	1.68	1.67	1.66	1.65
12	4.79	3.23	2.72	2.47	2.31	2.14	2.04	1.96	1.91	1.88	1.85	1.83	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73
16	5.03	3.38	2.84	2.57	2.40	2.22	2.11	2.03	1.98	1.94	1.92	1.89	1.88	1.85	1.84	1.82	1.81	1.81	1.79	1.78
20	5.21	3.49	2.93	2.64	2.47	2.28	2.17	2.08	2.03	1.99	1.96	1.94	1.92	1.90	1.88	1.87	1.86	1.85	1.83	1.82
30	5.53	3.69	*3.08*	2.78	2.60	2.39	2.27	2.18	2.12	2.08	2.05	2.02	2.01	1.98	1.96	1.94	1.93	1.92	1.91	1.90
40	5.76	3.83	3.19	2.88	2.69	2.46	2.34	2.24	2.18	2.14	2.11	2.08	2.06	2.03	2.01	2.00	1.98	1.98	1.96	1.95
50	5.93	3.93	3.28	2.95	2.75	2.52	2.39	2.30	2.23	2.19	2.15	2.13	2.11	2.07	2.05	2.04	2.02	2.01	2.00	1.99
60	6.07	4.02	3.35	*3.01*	2.81	2.57	2.44	2.34	2.27	2.22	2.19	2.16	2.14	2.11	2.09	2.07	2.06	2.05	2.03	2.02
75	6.23	4.12	3.43	*3.08*	2.87	2.63	2.49	2.39	2.32	2.27	2.23	2.20	2.18	2.15	2.13	2.11	2.10	2.09	2.07	2.05
100	6.44	4.25	3.53	*3.17*	2.95	2.70	2.56	2.45	2.38	2.33	2.29	2.26	2.24	2.20	2.18	2.16	2.15	2.13	2.11	2.10
125	6.60	4.35	3.61	3.24	*3.02*	2.76	2.61	2.50	2.42	2.37	2.33	2.30	2.28	2.24	2.22	2.20	2.18	2.17	2.15	2.14
150	6.72	4.42	3.67	3.30	*3.07*	2.80	2.65	2.54	2.46	2.41	2.37	2.34	2.31	2.28	2.25	2.23	2.21	2.20	2.18	2.17
175	6.83	4.49	3.73	3.34	*3.11*	2.84	2.69	2.57	2.49	2.44	2.40	2.37	2.34	2.30	2.28	2.26	2.24	2.23	2.21	2.19
200	6.92	4.55	3.77	3.38	*3.15*	2.87	2.72	2.60	2.52	2.46	2.42	2.39	2.36	2.33	2.30	2.28	2.26	2.25	2.23	2.21

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.22	2.29	1.98	1.82	1.73	1.62	1.55	1.51	1.48	1.45	1.44	1.43	1.42	1.40	1.39	1.38	1.38	1.37	1.37	1.36
2	3.83	2.66	2.27	2.08	1.96	1.82	1.75	1.69	1.65	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
3	4.19	2.88	2.44	2.22	2.09	1.94	1.86	1.79	1.75	1.72	1.70	1.68	1.67	1.65	1.64	1.63	1.62	1.61	1.60	1.59
4	4.44	3.03	2.56	2.32	2.18	2.02	1.93	1.86	1.82	1.79	1.76	1.75	1.73	1.71	1.70	1.69	1.68	1.67	1.66	1.65
5	4.63	3.14	2.65	2.40	2.26	2.09	1.99	1.92	1.87	1.84	1.81	1.79	1.78	1.76	1.74	1.73	1.72	1.72	1.70	1.69
8	5.03	3.38	2.84	2.57	2.40	2.22	2.11	2.03	1.98	1.94	1.92	1.89	1.88	1.85	1.84	1.82	1.81	1.81	1.79	1.78
12	5.36	3.58	3.00	2.71	2.53	2.33	2.21	2.13	2.07	2.03	2.00	1.98	1.96	1.93	1.92	1.90	1.89	1.88	1.87	1.86
16	5.59	3.72	3.11	2.80	2.62	2.41	2.29	2.19	2.13	2.09	2.06	2.04	2.02	1.99	1.97	1.96	1.94	1.93	1.92	1.91
20	5.76	3.83	3.19	2.88	2.69	2.46	2.34	2.24	2.18	2.14	2.11	2.08	2.06	2.03	2.01	2.00	1.98	1.98	1.96	1.95
30	6.07	4.02	3.35	3.01	2.81	2.57	2.44	2.34	2.27	2.22	2.19	2.16	2.14	2.11	2.09	2.07	2.06	2.05	2.03	2.02
40	6.28	4.15	3.45	3.10	2.89	2.65	2.51	2.40	2.33	2.28	2.25	2.22	2.20	2.16	2.14	2.12	2.11	2.10	2.08	2.06
50	6.44	4.25	3.53	3.17	2.95	2.70	2.56	2.45	2.38	2.33	2.29	2.26	2.24	2.20	2.18	2.16	2.15	2.13	2.11	2.10
60	6.57	4.33	3.59	*3.23*	3.01	2.75	2.60	2.49	2.42	2.36	2.32	2.29	2.27	2.24	2.21	2.19	2.17	2.17	2.14	2.13
75	6.72	4.42	3.67	*3.30*	3.07	2.80	2.65	2.54	2.46	2.41	2.37	2.34	2.31	2.28	2.25	2.23	2.21	2.20	2.18	2.17
100	6.92	4.55	3.77	*3.38*	3.15	2.87	2.72	2.60	2.52	2.46	2.42	2.39	2.36	2.33	2.30	2.28	2.26	2.25	2.23	2.21
125	7.07	4.64	3.85	3.45	3.21	2.93	2.77	2.65	2.56	2.51	2.46	2.43	2.40	2.36	2.33	2.32	2.30	2.29	2.26	2.25
150	7.19	4.72	3.91	3.50	*3.26*	2.97	2.81	2.68	2.60	2.54	2.50	2.46	2.44	2.40	2.37	2.35	2.33	2.32	2.29	2.28
175	7.29	4.78	3.96	3.55	*3.30*	3.01	2.84	2.72	2.63	2.57	2.53	2.49	2.46	2.42	2.39	2.37	2.35	2.34	2.32	2.30
200	7.34	4.83	4.00	3.59	*3.34*	3.04	2.87	2.74	2.66	2.60	2.55	2.52	2.49	2.45	2.42	2.39	2.38	2.36	2.34	2.32

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.83	**2.66**	**2.27**	**2.08**	**1.96**	**1.82**	**1.75**	**1.69**	**1.65**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
2	4.44	**3.03**	**2.56**	**2.32**	**2.18**	**2.02**	**1.93**	**1.86**	**1.82**	**1.79**	**1.76**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
3	4.79	**3.23**	**2.72**	**2.47**	**2.31**	**2.14**	**2.04**	**1.96**	**1.91**	**1.88**	**1.85**	**1.83**	**1.82**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.74**	**1.73**
4	5.03	**3.38**	**2.84**	**2.57**	**2.40**	**2.22**	**2.11**	**2.03**	**1.98**	**1.94**	**1.92**	**1.89**	**1.88**	**1.85**	**1.84**	**1.82**	**1.81**	**1.81**	**1.79**	**1.78**
5	5.21	3.49	**2.93**	**2.64**	**2.47**	**2.28**	**2.17**	**2.08**	**2.03**	**1.99**	**1.96**	**1.94**	**1.92**	**1.90**	**1.88**	**1.87**	**1.86**	**1.85**	**1.83**	**1.82**
8	5.59	3.72	**3.11**	**2.80**	**2.62**	**2.41**	**2.29**	**2.19**	**2.13**	**2.09**	**2.06**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.94**	**1.93**	**1.92**	**1.91**
12	5.90	3.91	**3.26**	**2.94**	**2.74**	**2.51**	**2.38**	**2.29**	**2.22**	**2.18**	**2.14**	**2.12**	**2.10**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.98**
16	6.11	4.05	**3.37**	**3.03**	**2.83**	**2.59**	**2.45**	**2.35**	**2.28**	**2.24**	**2.20**	**2.17**	**2.15**	**2.12**	**2.10**	**2.08**	**2.07**	**2.06**	**2.04**	**2.03**
20	6.28	4.15	**3.45**	**3.10**	**2.89**	**2.65**	**2.51**	**2.40**	**2.33**	**2.28**	**2.25**	**2.22**	**2.20**	**2.16**	**2.14**	**2.12**	**2.11**	**2.10**	**2.08**	**2.06**
30	6.57	4.33	3.59	**3.23**	**3.01**	**2.75**	**2.60**	**2.49**	**2.42**	**2.36**	**2.32**	**2.29**	**2.27**	**2.24**	**2.21**	**2.19**	**2.17**	**2.17**	**2.14**	**2.13**
40	6.77	4.45	3.70	**3.32**	**3.09**	**2.82**	**2.67**	**2.55**	**2.47**	**2.42**	**2.38**	**2.35**	**2.32**	**2.29**	**2.26**	**2.24**	**2.23**	**2.21**	**2.19**	**2.18**
50	6.92	4.55	3.77	**3.38**	**3.15**	**2.87**	**2.72**	**2.60**	**2.52**	**2.46**	**2.42**	**2.39**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.25**	**2.23**	**2.21**
60	7.04	4.62	3.83	**3.44**	**3.20**	**2.92**	**2.76**	**2.64**	**2.56**	**2.50**	**2.46**	**2.42**	**2.40**	**2.36**	**2.33**	**2.31**	**2.29**	**2.28**	**2.26**	**2.24**
75	7.19	4.72	3.91	*3.50*	**3.26**	**2.97**	**2.81**	**2.68**	**2.60**	**2.54**	**2.50**	**2.46**	**2.44**	**2.40**	**2.37**	**2.35**	**2.33**	**2.32**	**2.29**	**2.28**
100	7.34	4.83	4.00	*3.59*	**3.34**	**3.04**	**2.87**	**2.74**	**2.66**	**2.60**	**2.55**	**2.52**	**2.49**	**2.45**	**2.42**	**2.39**	**2.38**	**2.36**	**2.34**	**2.32**
125	7.52	4.92	4.08	3.65	**3.39**	**3.09**	**2.92**	**2.79**	**2.70**	**2.64**	**2.59**	**2.56**	**2.53**	**2.48**	**2.45**	**2.43**	**2.41**	**2.40**	**2.37**	**2.35**
150	7.64	5.00	4.13	3.70	**3.44**	**3.14**	**2.96**	**2.83**	**2.74**	**2.67**	**2.62**	**2.59**	**2.56**	**2.51**	**2.48**	**2.46**	**2.44**	**2.43**	**2.40**	**2.38**
175	7.73	5.06	4.18	3.75	*3.48*	**3.17**	**2.99**	**2.86**	**2.77**	**2.70**	**2.65**	**2.61**	**2.58**	**2.54**	**2.51**	**2.48**	**2.46**	**2.45**	**2.42**	**2.40**
200	7.82	5.11	4.23	3.78	*3.52*	**3.20**	**3.02**	**2.88**	**2.79**	**2.73**	**2.68**	**2.64**	**2.61**	**2.56**	**2.53**	**2.50**	**2.49**	**2.47**	**2.44**	**2.42**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.41	**2.35**	**2.01**	**1.84**	**1.74**	**1.62**	**1.56**	**1.51**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**	**1.39**	**1.38**	**1.37**	**1.37**	**1.36**
2	4.15	**2.76**	**2.32**	**2.10**	**1.98**	**1.83**	**1.75**	**1.69**	**1.65**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
3	4.60	**3.00**	**2.50**	**2.26**	**2.12**	**1.95**	**1.86**	**1.80**	**1.75**	**1.72**	**1.70**	**1.69**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
4	4.92	3.17	**2.63**	**2.37**	**2.21**	**2.04**	**1.94**	**1.87**	**1.82**	**1.79**	**1.77**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
5	5.16	3.30	**2.73**	**2.45**	**2.29**	**2.10**	**2.00**	**1.93**	**1.88**	**1.84**	**1.82**	**1.80**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.72**	**1.70**	**1.69**
8	5.68	3.58	**2.94**	**2.63**	**2.45**	**2.24**	**2.13**	**2.04**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
12	6.11	3.82	*3.12*	**2.78**	**2.58**	**2.36**	**2.23**	**2.14**	**2.08**	**2.04**	**2.01**	**1.98**	**1.96**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**
16	6.41	3.99	3.24	**2.89**	**2.68**	**2.44**	**2.31**	**2.21**	**2.14**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
20	6.64	4.11	3.34	**2.97**	**2.75**	**2.50**	**2.36**	**2.26**	**2.19**	**2.15**	**2.11**	**2.09**	**2.07**	**2.04**	**2.01**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
30	7.05	4.34	3.51	*3.11*	**2.88**	**2.61**	**2.47**	**2.35**	**2.28**	**2.23**	**2.20**	**2.17**	**2.15**	**2.11**	**2.09**	**2.07**	**2.06**	**2.05**	**2.03**	**2.02**
40	7.33	4.50	3.63	*3.22*	**2.97**	**2.69**	**2.54**	**2.42**	**2.34**	**2.29**	**2.25**	**2.22**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.10**	**2.08**	**2.07**
50	7.55	4.62	3.73	3.30	*3.04*	**2.75**	**2.59**	**2.47**	**2.39**	**2.34**	**2.30**	**2.27**	**2.24**	**2.21**	**2.18**	**2.16**	**2.15**	**2.14**	**2.12**	**2.10**
60	7.72	4.71	3.80	3.36	*3.10*	**2.80**	**2.63**	**2.50**	**2.43**	**2.38**	**2.33**	**2.30**	**2.28**	**2.24**	**2.21**	**2.19**	**2.18**	**2.17**	**2.15**	**2.13**
75	7.93	4.83	3.89	3.44	*3.17*	**2.86**	**2.69**	**2.56**	**2.48**	**2.42**	**2.38**	**2.34**	**2.32**	**2.28**	**2.25**	**2.23**	**2.22**	**2.20**	**2.18**	**2.17**
100	8.19	4.98	4.00	3.53	3.25	**2.94**	**2.76**	**2.63**	**2.54**	**2.48**	**2.43**	**2.40**	**2.37**	**2.33**	**2.30**	**2.28**	**2.27**	**2.25**	**2.23**	**2.21**
125	8.39	5.09	4.09	3.61	3.32	*2.99*	**2.81**	**2.67**	**2.58**	**2.52**	**2.48**	**2.44**	**2.41**	**2.37**	**2.34**	**2.32**	**2.30**	**2.29**	**2.27**	**2.25**
150	8.55	5.18	4.16	3.67	3.38	*3.04*	**2.86**	**2.71**	**2.62**	**2.56**	**2.51**	**2.47**	**2.45**	**2.40**	**2.37**	**2.35**	**2.33**	**2.32**	**2.29**	**2.28**
175	8.68	5.26	4.22	3.72	3.42	*3.08*	**2.89**	**2.75**	**2.65**	**2.59**	**2.54**	**2.50**	**2.47**	**2.43**	**2.40**	**2.38**	**2.36**	**2.34**	**2.32**	**2.30**
200	8.80	5.32	4.27	3.76	3.46	*3.12*	**2.92**	**2.78**	**2.68**	**2.61**	**2.57**	**2.53**	**2.50**	**2.45**	**2.42**	**2.40**	**2.38**	**2.36**	**2.34**	**2.26**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.15	**2.76**	**2.32**	**2.10**	**1.98**	**1.83**	**1.75**	**1.69**	**1.65**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
2	4.92	**3.17**	**2.63**	**2.37**	**2.21**	**2.04**	**1.94**	**1.87**	**1.82**	**1.79**	**1.77**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
3	5.36	3.41	**2.81**	**2.52**	**2.35**	**2.16**	**2.05**	**1.97**	**1.92**	**1.88**	**1.86**	**1.84**	**1.82**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.74**	**1.73**
4	5.68	3.58	**2.94**	**2.63**	**2.45**	**2.24**	**2.13**	**2.04**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
5	5.92	3.71	**3.04**	**2.71**	**2.52**	**2.30**	**2.18**	**2.09**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.85**	**1.83**	**1.82**
8	6.41	3.99	*3.24*	**2.89**	**2.68**	**2.44**	**2.31**	**2.21**	**2.14**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
12	6.83	4.22	3.42	**3.03**	**2.81**	**2.55**	**2.41**	**2.30**	**2.23**	**2.19**	**2.15**	**2.12**	**2.10**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.98**
16	7.11	4.38	3.54	**3.14**	**2.90**	**2.63**	**2.48**	**2.37**	**2.30**	**2.25**	**2.21**	**2.18**	**2.16**	**2.12**	**2.10**	**2.08**	**2.07**	**2.06**	**2.04**	**2.03**
20	7.33	4.50	3.63	*3.22*	**2.97**	**2.69**	**2.54**	**2.42**	**2.34**	**2.29**	**2.25**	**2.22**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.10**	**2.08**	**2.07**
30	7.72	4.71	3.80	*3.36*	**3.10**	**2.80**	**2.63**	**2.50**	**2.43**	**2.38**	**2.33**	**2.30**	**2.28**	**2.24**	**2.21**	**2.19**	**2.18**	**2.17**	**2.15**	**2.13**
40	7.99	4.87	3.92	3.46	**3.19**	**2.88**	**2.71**	**2.58**	**2.49**	**2.43**	**2.39**	**2.36**	**2.33**	**2.29**	**2.26**	**2.24**	**2.23**	**2.22**	**2.19**	**2.18**
50	8.19	4.98	4.00	3.53	*3.25*	**2.94**	**2.76**	**2.63**	**2.54**	**2.48**	**2.43**	**2.40**	**2.37**	**2.33**	**2.30**	**2.28**	**2.27**	**2.25**	**2.23**	**2.21**
60	8.35	5.07	4.08	3.59	*3.31*	**2.98**	**2.80**	**2.67**	**2.58**	**2.51**	**2.47**	**2.43**	**2.41**	**2.36**	**2.33**	**2.31**	**2.30**	**2.28**	**2.26**	**2.24**
75	8.55	5.18	4.16	3.67	*3.38*	**3.04**	**2.86**	**2.71**	**2.62**	**2.56**	**2.51**	**2.47**	**2.45**	**2.40**	**2.37**	**2.35**	**2.33**	**2.32**	**2.29**	**2.28**
100	8.80	5.32	4.27	3.76	3.46	**3.12**	**2.92**	**2.78**	**2.68**	**2.61**	**2.57**	**2.53**	**2.50**	**2.45**	**2.42**	**2.40**	**2.38**	**2.36**	**2.34**	**2.26**
125	8.99	5.43	4.36	3.83	3.53	**3.17**	**2.98**	**2.82**	**2.73**	**2.66**	**2.61**	**2.57**	**2.54**	**2.49**	**2.46**	**2.43**	**2.42**	**2.40**	**2.37**	**2.36**
150	9.14	5.52	4.42	3.89	3.58	*3.22*	**3.02**	**2.86**	**2.76**	**2.69**	**2.64**	**2.60**	**2.57**	**2.52**	**2.49**	**2.46**	**2.44**	**2.43**	**2.40**	**2.38**
175	9.27	5.60	4.48	3.94	3.62	*3.26*	**3.05**	**2.90**	**2.79**	**2.72**	**2.67**	**2.63**	**2.60**	**2.55**	**2.51**	**2.49**	**2.47**	**2.45**	**2.42**	**2.41**
200	9.38	5.66	4.53	3.98	3.66	*3.29*	**3.08**	**2.92**	**2.82**	**2.75**	**2.69**	**2.65**	**2.62**	**2.57**	**2.54**	**2.51**	**2.49**	**2.47**	**2.45**	**2.43**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.92	**3.17**	**2.63**	**2.37**	**2.21**	**2.04**	**1.94**	**1.87**	**1.82**	**1.79**	**1.77**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
2	5.68	3.58	**2.94**	**2.63**	**2.45**	**2.24**	**2.13**	**2.04**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
3	6.11	3.82	**3.12**	**2.78**	**2.58**	**2.36**	**2.23**	**2.14**	**2.08**	**2.04**	**2.01**	**1.98**	**1.96**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**
4	6.41	3.99	**3.24**	**2.89**	**2.68**	**2.44**	**2.31**	**2.21**	**2.14**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
5	6.64	4.11	**3.34**	**2.97**	**2.75**	**2.50**	**2.36**	**2.26**	**2.19**	**2.15**	**2.11**	**2.09**	**2.07**	**2.04**	**2.01**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
8	7.11	4.38	*3.54*	**3.14**	**2.90**	**2.63**	**2.48**	**2.37**	**2.30**	**2.25**	**2.21**	**2.18**	**2.16**	**2.12**	**2.10**	**2.08**	**2.07**	**2.06**	**2.04**	**2.03**
12	7.51	4.60	3.71	**3.28**	**3.03**	**2.74**	**2.58**	**2.46**	**2.38**	**2.33**	**2.29**	**2.26**	**2.24**	**2.20**	**2.17**	**2.16**	**2.14**	**2.13**	**2.11**	**2.10**
16	7.78	4.75	3.83	**3.38**	**3.12**	**2.82**	**2.65**	**2.53**	**2.44**	**2.39**	**2.35**	**2.31**	**2.29**	**2.25**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
20	7.99	4.87	3.92	**3.46**	**3.19**	**2.88**	**2.71**	**2.58**	**2.49**	**2.43**	**2.39**	**2.36**	**2.33**	**2.29**	**2.26**	**2.24**	**2.23**	**2.22**	**2.19**	**2.18**
30	8.35	5.07	4.08	*3.59*	**3.31**	**2.98**	**2.80**	**2.67**	**2.58**	**2.51**	**2.47**	**2.43**	**2.41**	**2.36**	**2.33**	**2.31**	**2.30**	**2.28**	**2.26**	**2.24**
40	8.60	5.22	4.19	3.69	**3.39**	**3.06**	**2.87**	**2.73**	**2.64**	**2.57**	**2.52**	**2.49**	**2.46**	**2.41**	**2.38**	**2.36**	**2.34**	**2.33**	**2.30**	**2.29**
50	8.80	5.32	4.27	3.76	**3.46**	**3.12**	**2.92**	**2.78**	**2.68**	**2.61**	**2.57**	**2.53**	**2.50**	**2.45**	**2.42**	**2.40**	**2.38**	**2.36**	**2.34**	**2.26**
60	8.95	5.42	4.34	3.82	*3.51*	**3.16**	**2.97**	**2.82**	**2.72**	**2.65**	**2.60**	**2.56**	**2.53**	**2.48**	**2.45**	**2.43**	**2.41**	**2.39**	**2.37**	**2.35**
75	9.14	5.52	4.42	3.89	*3.58*	**3.22**	**3.02**	**2.86**	**2.76**	**2.69**	**2.64**	**2.60**	**2.57**	**2.52**	**2.49**	**2.46**	**2.44**	**2.43**	**2.40**	**2.38**
100	9.38	5.66	4.53	3.98	*3.66*	**3.29**	**3.08**	**2.92**	**2.82**	**2.75**	**2.69**	**2.65**	**2.62**	**2.57**	**2.54**	**2.51**	**2.49**	**2.47**	**2.45**	**2.43**
125	9.56	5.76	4.61	4.05	3.72	**3.34**	**3.13**	**2.97**	**2.86**	**2.79**	**2.73**	**2.69**	**2.66**	**2.61**	**2.57**	**2.54**	**2.52**	**2.51**	**2.48**	**2.46**
150	9.70	5.85	4.68	4.11	3.77	**3.39**	**3.17**	**3.01**	**2.90**	**2.82**	**2.77**	**2.72**	**2.69**	**2.64**	**2.60**	**2.57**	**2.55**	**2.54**	**2.51**	**2.49**
175	9.82	5.91	4.73	4.16	3.82	**3.43**	**3.21**	**3.04**	**2.93**	**2.85**	**2.80**	**2.75**	**2.72**	**2.66**	**2.63**	**2.60**	**2.58**	**2.56**	**2.53**	**2.51**
200	9.93	5.97	4.77	4.20	3.85	**3.46**	**3.24**	**3.07**	**2.96**	**2.88**	**2.82**	**2.77**	**2.74**	**2.68**	**2.65**	**2.62**	**2.60**	**2.58**	**2.55**	**2.53**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.38	**2.82**	**2.34**	**2.12**	**1.99**	**1.84**	**1.76**	**1.70**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
2	5.31	3.27	**2.67**	**2.39**	**2.23**	**2.05**	**1.95**	**1.87**	**1.83**	**1.79**	**1.77**	**1.75**	**1.74**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
3	5.86	3.54	**2.87**	**2.55**	**2.37**	**2.17**	**2.06**	**1.97**	**1.92**	**1.89**	**1.86**	**1.84**	**1.82**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.74**	**1.73**
4	6.26	3.74	**3.01**	**2.67**	**2.47**	**2.25**	**2.13**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
5	6.57	3.89	*3.12*	**2.76**	**2.55**	**2.32**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.85**	**1.83**	**1.82**
8	7.22	4.21	3.35	**2.95**	**2.71**	**2.46**	**2.32**	**2.21**	**2.15**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
12	7.76	4.48	3.54	*3.11*	**2.85**	**2.57**	**2.42**	**2.31**	**2.24**	**2.19**	**2.15**	**2.13**	**2.10**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.98**
16	8.14	4.67	3.68	3.22	**2.95**	**2.66**	**2.50**	**2.38**	**2.30**	**2.25**	**2.21**	**2.18**	**2.16**	**2.13**	**2.10**	**2.08**	**2.07**	**2.06**	**2.04**	**2.03**
20	8.43	4.81	3.78	3.31	*3.03*	**2.72**	**2.56**	**2.43**	**2.35**	**2.30**	**2.26**	**2.23**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.10**	**2.08**	**2.07**
30	8.94	5.07	3.97	3.46	*3.17*	**2.84**	**2.66**	**2.53**	**2.44**	**2.38**	**2.34**	**2.31**	**2.28**	**2.24**	**2.22**	**2.20**	**2.18**	**2.17**	**2.15**	**2.13**
40	9.30	5.25	4.11	3.57	3.26	**2.92**	**2.73**	**2.59**	**2.50**	**2.44**	**2.40**	**2.36**	**2.33**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**	**2.17**	**2.18**
50	9.57	5.39	4.21	3.66	3.34	*2.98*	**2.79**	**2.64**	**2.55**	**2.49**	**2.44**	**2.40**	**2.38**	**2.33**	**2.31**	**2.28**	**2.27**	**2.25**	**2.23**	**2.21**
60	9.79	5.50	4.29	3.72	3.40	*3.03*	**2.83**	**2.68**	**2.59**	**2.52**	**2.48**	**2.44**	**2.41**	**2.37**	**2.34**	**2.31**	**2.30**	**2.28**	**2.26**	**2.24**
75	10.05	5.64	4.39	3.81	3.47	*3.09*	**2.89**	**2.73**	**2.64**	**2.57**	**2.52**	**2.48**	**2.45**	**2.41**	**2.37**	**2.35**	**2.33**	**2.32**	**2.29**	**2.26**
100	10.38	5.81	4.52	3.91	3.56	*3.17*	**2.96**	**2.80**	**2.70**	**2.63**	**2.57**	**2.53**	**2.50**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**
125	10.63	5.94	4.61	3.99	3.63	*3.23*	*3.01*	**2.85**	**2.74**	**2.67**	**2.62**	**2.57**	**2.54**	**2.49**	**2.46**	**2.44**	**2.42**	**2.40**	**2.37**	**2.36**
150	10.83	6.04	4.69	4.06	3.69	*3.28*	*3.06*	**2.89**	**2.78**	**2.71**	**2.65**	**2.61**	**2.58**	**2.53**	**2.49**	**2.47**	**2.45**	**2.43**	**2.40**	**2.38**
175	11.00	6.13	4.76	4.11	3.74	3.32	*3.10*	**2.92**	**2.81**	**2.74**	**2.68**	**2.64**	**2.60**	**2.55**	**2.52**	**2.49**	**2.47**	**2.45**	**2.43**	**2.41**
200	11.15	6.20	4.81	4.16	3.78	3.36	*3.13*	**2.95**	**2.84**	**2.76**	**2.70**	**2.66**	**2.63**	**2.57**	**2.54**	**2.51**	**2.49**	**2.48**	**2.45**	**2.43**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.31	3.27	**2.67**	**2.39**	**2.23**	**2.05**	**1.95**	**1.87**	**1.83**	**1.79**	**1.77**	**1.75**	**1.74**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
2	6.26	3.74	**3.01**	**2.67**	**2.47**	**2.25**	**2.13**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
3	6.82	4.01	**3.21**	**2.83**	**2.61**	**2.37**	**2.24**	**2.14**	**2.08**	**2.04**	**2.01**	**1.98**	**1.97**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**
4	7.22	4.21	3.35	**2.95**	**2.71**	**2.46**	**2.32**	**2.21**	**2.15**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
5	7.52	4.36	3.45	**3.03**	**2.79**	**2.52**	**2.38**	**2.27**	**2.20**	**2.15**	**2.12**	**2.09**	**2.07**	**2.04**	**2.02**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
8	8.14	4.67	3.68	*3.22*	**2.95**	**2.66**	**2.50**	**2.38**	**2.30**	**2.25**	**2.21**	**2.18**	**2.16**	**2.13**	**2.10**	**2.08**	**2.07**	**2.06**	**2.04**	**2.03**
12	8.66	4.93	3.87	*3.38*	**3.09**	**2.77**	**2.60**	**2.47**	**2.39**	**2.34**	**2.29**	**2.26**	**2.24**	**2.20**	**2.18**	**2.16**	**2.14**	**2.13**	**2.11**	**2.10**
16	9.02	5.11	4.00	3.49	**3.19**	**2.86**	**2.68**	**2.54**	**2.45**	**2.40**	**2.35**	**2.32**	**2.29**	**2.25**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
20	9.30	5.25	4.11	3.57	*3.26*	**2.92**	**2.73**	**2.59**	**2.50**	**2.44**	**2.40**	**2.36**	**2.33**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**	**2.17**	**2.18**
30	9.79	5.50	4.29	3.72	*3.40*	**3.03**	**2.83**	**2.68**	**2.59**	**2.52**	**2.48**	**2.44**	**2.41**	**2.37**	**2.34**	**2.31**	**2.30**	**2.28**	**2.26**	**2.24**
40	10.12	5.68	4.42	3.83	3.49	**3.11**	**2.90**	**2.75**	**2.65**	**2.58**	**2.53**	**2.49**	**2.46**	**2.42**	**2.39**	**2.36**	**2.34**	**2.33**	**2.31**	**2.29**
50	10.38	5.81	4.52	3.91	3.56	**3.17**	**2.96**	**2.80**	**2.70**	**2.63**	**2.57**	**2.53**	**2.50**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**
60	10.58	5.91	4.59	3.98	3.62	*3.22*	**3.00**	**2.84**	**2.73**	**2.66**	**2.61**	**2.57**	**2.54**	**2.49**	**2.45**	**2.43**	**2.41**	**2.40**	**2.37**	**2.35**
75	10.83	6.04	4.69	4.06	3.69	*3.28*	**3.06**	**2.89**	**2.78**	**2.71**	**2.65**	**2.61**	**2.58**	**2.53**	**2.49**	**2.47**	**2.45**	**2.43**	**2.40**	**2.38**
100	11.15	6.20	4.81	4.16	3.78	*3.36*	**3.13**	**2.95**	**2.84**	**2.76**	**2.70**	**2.66**	**2.63**	**2.57**	**2.54**	**2.51**	**2.49**	**2.48**	**2.45**	**2.43**
125	11.38	6.33	4.90	4.24	3.85	*3.42*	**3.18**	**3.00**	**2.89**	**2.81**	**2.75**	**2.70**	**2.67**	**2.61**	**2.58**	**2.55**	**2.53**	**2.51**	**2.48**	**2.46**
150	11.58	6.43	4.98	4.30	3.91	*3.47*	*3.22*	**3.04**	**2.92**	**2.84**	**2.78**	**2.73**	**2.70**	**2.64**	**2.60**	**2.58**	**2.56**	**2.54**	**2.51**	**2.49**
175	11.74	6.52	5.04	4.35	3.95	*3.51*	*3.26*	**3.07**	**2.95**	**2.87**	**2.81**	**2.76**	**2.72**	**2.67**	**2.63**	**2.60**	**2.58**	**2.56**	**2.53**	**2.51**
200	11.88	6.59	5.10	4.40	3.99	3.54	*3.29*	**3.10**	**2.98**	**2.90**	**2.83**	**2.78**	**2.75**	**2.69**	**2.65**	**2.62**	**2.60**	**2.58**	**2.55**	**2.53**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.26	3.74	**3.01**	**2.67**	**2.47**	**2.25**	**2.13**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
2	7.22	4.21	**3.35**	**2.95**	**2.71**	**2.46**	**2.32**	**2.21**	**2.15**	**2.10**	**2.07**	**2.04**	**2.02**	**1.99**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
3	7.76	4.48	*3.54*	**3.11**	**2.85**	**2.57**	**2.42**	**2.31**	**2.24**	**2.19**	**2.15**	**2.13**	**2.10**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.98**
4	8.14	4.67	3.68	**3.22**	**2.95**	**2.66**	**2.50**	**2.38**	**2.30**	**2.25**	**2.21**	**2.18**	**2.16**	**2.13**	**2.10**	**2.08**	**2.07**	**2.06**	**2.04**	**2.03**
5	8.43	4.81	3.78	**3.31**	**3.03**	**2.72**	**2.56**	**2.43**	**2.35**	**2.30**	**2.26**	**2.23**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.10**	**2.08**	**2.07**
8	9.02	5.11	4.00	*3.49*	**3.19**	**2.86**	**2.68**	**2.54**	**2.45**	**2.40**	**2.35**	**2.32**	**2.29**	**2.25**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
12	9.52	5.37	4.19	3.64	**3.32**	**2.97**	**2.78**	**2.63**	**2.54**	**2.48**	**2.43**	**2.40**	**2.37**	**2.31**	**2.30**	**2.28**	**2.26**	**2.25**	**2.22**	**2.21**
16	9.86	5.54	4.32	3.75	**3.42**	**3.05**	**2.85**	**2.70**	**2.60**	**2.54**	**2.49**	**2.45**	**2.42**	**2.38**	**2.35**	**2.33**	**2.31**	**2.29**	**2.27**	**2.25**
20	10.12	5.68	4.42	3.83	*3.49*	**3.11**	**2.90**	**2.75**	**2.65**	**2.58**	**2.53**	**2.49**	**2.46**	**2.42**	**2.39**	**2.36**	**2.34**	**2.33**	**2.31**	**2.29**
30	10.58	5.91	4.59	3.98	*3.62*	**3.22**	**3.00**	**2.84**	**2.73**	**2.66**	**2.61**	**2.57**	**2.54**	**2.49**	**2.45**	**2.43**	**2.41**	**2.40**	**2.37**	**2.35**
40	10.90	6.08	4.72	4.08	3.71	**3.30**	**3.07**	**2.90**	**2.79**	**2.72**	**2.66**	**2.62**	**2.59**	**2.54**	**2.50**	**2.48**	**2.46**	**2.44**	**2.41**	**2.39**
50	11.15	6.20	4.81	4.16	3.78	**3.36**	**3.13**	**2.95**	**2.84**	**2.76**	**2.70**	**2.66**	**2.63**	**2.57**	**2.54**	**2.51**	**2.49**	**2.48**	**2.45**	**2.43**
60	11.34	6.31	4.89	4.22	3.84	**3.41**	**3.17**	**2.99**	**2.88**	**2.80**	**2.74**	**2.69**	**2.66**	**2.61**	**2.57**	**2.54**	**2.52**	**2.50**	**2.47**	**2.45**
75	11.58	6.43	4.98	4.30	3.91	*3.47*	**3.22**	**3.04**	**2.92**	**2.84**	**2.78**	**2.73**	**2.70**	**2.64**	**2.60**	**2.58**	**2.56**	**2.54**	**2.51**	**2.49**
100	11.88	6.59	5.10	4.40	3.99	*3.54*	**3.29**	**3.10**	**2.98**	**2.90**	**2.83**	**2.78**	**2.75**	**2.69**	**2.65**	**2.62**	**2.60**	**2.58**	**2.55**	**2.53**
125	12.10	6.71	5.19	4.48	4.06	*3.60*	**3.34**	**3.15**	**3.02**	**2.94**	**2.87**	**2.82**	**2.79**	**2.73**	**2.69**	**2.66**	**2.63**	**2.62**	**2.58**	**2.56**
150	12.29	6.80	5.26	4.54	4.12	*3.64*	**3.39**	**3.19**	**3.06**	**2.97**	**2.91**	**2.86**	**2.82**	**2.76**	**2.72**	**2.69**	**2.66**	**2.64**	**2.61**	**2.59**
175	12.44	6.89	5.32	4.59	4.16	*3.68*	**3.42**	**3.22**	**3.09**	**3.00**	**2.93**	**2.88**	**2.84**	**2.78**	**2.74**	**2.71**	**2.68**	**2.67**	**2.63**	**2.61**
200	12.57	6.95	5.37	4.63	4.20	*3.69*	**3.45**	**3.25**	**3.12**	**3.03**	**2.96**	**2.91**	**2.87**	**2.80**	**2.76**	**2.73**	**2.70**	**2.69**	**2.65**	**2.63**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.59	3.33	**2.70**	**2.41**	**2.24**	**2.05**	**1.95**	**1.88**	**1.83**	**1.79**	**1.77**	**1.75**	**1.74**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
2	6.75	3.85	*3.05*	**2.69**	**2.49**	**2.26**	**2.14**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
3	7.45	4.16	3.27	**2.86**	**2.63**	**2.38**	**2.25**	**2.15**	**2.09**	**2.04**	**2.01**	**1.99**	**1.97**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**
4	7.95	4.38	3.42	**2.98**	**2.74**	**2.47**	**2.32**	**2.22**	**2.15**	**2.10**	**2.07**	**2.05**	**2.02**	**2.00**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
5	8.33	4.55	3.54	*3.08*	**2.82**	**2.53**	**2.38**	**2.27**	**2.20**	**2.15**	**2.12**	**2.09**	**2.07**	**2.04**	**2.02**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
8	9.15	4.91	3.78	3.28	**2.99**	**2.67**	**2.51**	**2.38**	**2.31**	**2.25**	**2.21**	**2.19**	**2.16**	**2.13**	**2.10**	**2.09**	**2.07**	**2.06**	**2.04**	**2.03**
12	9.83	5.22	4.00	3.45	**3.13**	**2.79**	**2.61**	**2.48**	**2.40**	**2.34**	**2.30**	**2.27**	**2.24**	**2.20**	**2.18**	**2.16**	**2.14**	**2.13**	**2.11**	**2.10**
16	10.31	5.44	4.15	3.57	*3.24*	**2.88**	**2.69**	**2.55**	**2.46**	**2.40**	**2.35**	**2.32**	**2.29**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
20	10.67	5.60	4.26	3.66	3.32	**2.95**	**2.75**	**2.60**	**2.51**	**2.45**	**2.40**	**2.36**	**2.34**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.19**	**2.18**
30	11.32	5.90	4.47	3.83	3.46	*3.06*	**2.85**	**2.70**	**2.60**	**2.53**	**2.48**	**2.44**	**2.41**	**2.37**	**2.34**	**2.32**	**2.30**	**2.28**	**2.26**	**2.20**
40	11.77	6.11	4.62	3.95	3.56	*3.15*	**2.93**	**2.76**	**2.66**	**2.59**	**2.54**	**2.50**	**2.46**	**2.42**	**2.39**	**2.33**	**2.35**	**2.33**	**2.31**	**2.29**
50	12.10	6.27	4.73	4.04	3.64	*3.20*	*2.98*	**2.81**	**2.71**	**2.63**	**2.58**	**2.54**	**2.51**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**
60	12.38	6.40	4.82	4.11	3.71	*3.27*	*3.03*	**2.85**	**2.74**	**2.67**	**2.61**	**2.57**	**2.54**	**2.49**	**2.46**	**2.43**	**2.41**	**2.40**	**2.37**	**2.34**
75	12.71	6.55	4.93	4.20	3.78	3.33	*3.09*	**2.91**	**2.79**	**2.71**	**2.66**	**2.61**	**2.58**	**2.53**	**2.49**	**2.47**	**2.44**	**2.43**	**2.40**	**2.38**
100	13.12	6.75	5.07	4.31	3.88	3.41	*3.16*	*2.97*	**2.85**	**2.77**	**2.71**	**2.67**	**2.63**	**2.58**	**2.54**	**2.51**	**2.49**	**2.48**	**2.45**	**2.43**
125	13.44	6.90	5.18	4.40	3.96	3.47	*3.22*	*3.02*	**2.90**	**2.81**	**2.75**	**2.71**	**2.67**	**2.62**	**2.58**	**2.55**	**2.53**	**2.51**	**2.48**	**2.46**
150	13.70	7.02	5.26	4.47	4.02	3.53	*3.26*	*3.06*	**2.94**	**2.85**	**2.79**	**2.74**	**2.70**	**2.65**	**2.61**	**2.58**	**2.56**	**2.52**	**2.51**	**2.49**
175	13.91	7.12	5.31	4.53	4.07	3.57	*3.30*	*3.10*	*2.97*	**2.88**	**2.82**	**2.77**	**2.73**	**2.67**	**2.63**	**2.60**	**2.58**	**2.56**	**2.53**	**2.51**
200	14.09	7.21	5.40	4.58	4.12	3.61	3.33	*3.13*	*3.00*	**2.91**	**2.84**	**2.79**	**2.75**	**2.69**	**2.65**	**2.62**	**2.60**	**2.58**	**2.54**	**2.53**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.75	3.85	**3.05**	**2.69**	**2.49**	**2.26**	**2.14**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
2	7.95	4.38	3.42	**2.98**	**2.74**	**2.47**	**2.32**	**2.22**	**2.15**	**2.10**	**2.07**	**2.05**	**2.02**	**2.00**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
3	8.65	4.69	3.63	**3.15**	**2.88**	**2.59**	**2.43**	**2.32**	**2.24**	**2.19**	**2.16**	**2.13**	**2.11**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.98**
4	9.15	4.91	3.78	*3.28*	**2.99**	**2.67**	**2.51**	**2.38**	**2.31**	**2.25**	**2.21**	**2.19**	**2.16**	**2.13**	**2.10**	**2.09**	**2.07**	**2.06**	**2.04**	**2.03**
5	9.52	5.08	3.90	*3.37*	**3.07**	**2.74**	**2.57**	**2.44**	**2.36**	**2.30**	**2.26**	**2.23**	**2.20**	**2.17**	**2.14**	**2.13**	**2.11**	**2.10**	**2.08**	**2.07**
8	10.31	5.44	4.15	3.57	*3.24*	**2.88**	**2.69**	**2.55**	**2.46**	**2.40**	**2.35**	**2.32**	**2.29**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
12	10.96	5.74	4.36	3.74	*3.38*	**3.00**	**2.79**	**2.64**	**2.55**	**2.48**	**2.44**	**2.40**	**2.37**	**2.33**	**2.30**	**2.28**	**2.26**	**2.25**	**2.22**	**2.21**
16	11.42	5.95	4.51	3.85	3.49	**3.08**	**2.87**	**2.71**	**2.61**	**2.54**	**2.49**	**2.45**	**2.42**	**2.38**	**2.35**	**2.33**	**2.31**	**2.30**	**2.26**	**2.25**
20	11.77	6.11	4.62	3.95	3.56	**3.15**	**2.93**	**2.76**	**2.66**	**2.59**	**2.54**	**2.50**	**2.46**	**2.42**	**2.39**	**2.33**	**2.35**	**2.33**	**2.31**	**2.29**
30	12.38	6.40	4.82	4.11	3.71	*3.27*	**3.03**	**2.85**	**2.74**	**2.67**	**2.61**	**2.57**	**2.54**	**2.49**	**2.46**	**2.43**	**2.41**	**2.40**	**2.37**	**2.34**
40	12.80	6.60	4.96	4.22	3.81	*3.35*	**3.10**	**2.92**	**2.81**	**2.73**	**2.67**	**2.62**	**2.59**	**2.54**	**2.50**	**2.48**	**2.46**	**2.44**	**2.41**	**2.39**
50	13.12	6.75	5.07	4.31	3.88	*3.41*	**3.16**	**2.97**	**2.85**	**2.77**	**2.71**	**2.67**	**2.63**	**2.58**	**2.54**	**2.51**	**2.49**	**2.48**	**2.45**	**2.43**
60	13.38	6.87	5.16	4.38	3.94	*3.46*	*3.20*	**3.01**	**2.89**	**2.81**	**2.75**	**2.70**	**2.66**	**2.61**	**2.57**	**2.54**	**2.52**	**2.50**	**2.47**	**2.45**
75	13.70	7.02	5.26	4.47	4.02	3.53	*3.26*	**3.06**	**2.94**	**2.85**	**2.79**	**2.74**	**2.70**	**2.65**	**2.61**	**2.58**	**2.56**	**2.52**	**2.51**	**2.46**
100	14.09	7.21	5.40	4.58	4.12	3.61	*3.33*	**3.13**	**3.00**	**2.91**	**2.84**	**2.79**	**2.75**	**2.69**	**2.65**	**2.62**	**2.60**	**2.58**	**2.54**	**2.47**
125	14.39	7.35	5.50	4.66	4.19	3.67	*3.39*	**3.18**	**3.04**	**2.95**	**2.88**	**2.83**	**2.79**	**2.73**	**2.69**	**2.66**	**2.64**	**2.62**	**2.58**	**2.56**
150	14.64	7.47	5.58	4.73	4.25	3.72	*3.43*	*3.22*	**3.08**	**2.99**	**2.92**	**2.86**	**2.82**	**2.76**	**2.72**	**2.69**	**2.66**	**2.64**	**2.57**	**2.59**
175	14.84	7.56	5.65	4.79	4.30	3.76	*3.47*	*3.25*	**3.11**	**3.01**	**2.94**	**2.89**	**2.85**	**2.77**	**2.74**	**2.71**	**2.69**	**2.67**	**2.63**	**2.61**
200	15.01	7.65	5.71	4.84	4.34	3.80	*3.50*	*3.28*	**3.14**	**3.04**	**2.97**	**2.91**	**2.87**	**2.81**	**2.76**	**2.73**	**2.71**	**2.69**	**2.65**	**2.60**

Table 19-8. κ-Multipliers for 1-of-1 Interwell Prediction Limits on Means of Order 3 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	7.95	4.38	**3.42**	**2.98**	**2.74**	**2.47**	**2.32**	**2.22**	**2.15**	**2.10**	**2.07**	**2.05**	**2.02**	**2.00**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
2	9.15	4.91	3.78	**3.28**	**2.99**	**2.67**	**2.51**	**2.38**	**2.31**	**2.25**	**2.21**	**2.19**	**2.16**	**2.13**	**2.10**	**2.09**	**2.07**	**2.06**	**2.04**	**2.03**
3	9.83	5.22	4.00	**3.45**	**3.13**	**2.79**	**2.61**	**2.48**	**2.40**	**2.34**	**2.30**	**2.27**	**2.24**	**2.20**	**2.18**	**2.16**	**2.14**	**2.13**	**2.11**	**2.10**
4	10.31	5.44	4.15	*3.57*	**3.24**	**2.88**	**2.69**	**2.55**	**2.46**	**2.40**	**2.35**	**2.32**	**2.29**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
5	10.67	5.60	4.26	3.66	**3.32**	**2.95**	**2.75**	**2.60**	**2.51**	**2.45**	**2.40**	**2.36**	**2.34**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.19**	**2.18**
8	11.42	5.95	4.51	3.85	*3.49*	**3.08**	**2.87**	**2.71**	**2.61**	**2.54**	**2.49**	**2.45**	**2.42**	**2.38**	**2.35**	**2.33**	**2.31**	**2.30**	**2.26**	**2.20**
12	12.04	6.24	4.69	4.02	*3.63*	**3.20**	**2.97**	**2.80**	**2.69**	**2.62**	**2.57**	**2.53**	**2.50**	**2.45**	**2.42**	**2.39**	**2.38**	**2.36**	**2.33**	**2.32**
16	12.48	6.44	4.85	4.14	3.73	**3.28**	**3.05**	**2.87**	**2.76**	**2.68**	**2.63**	**2.58**	**2.55**	**2.50**	**2.47**	**2.44**	**2.42**	**2.41**	**2.38**	**2.32**
20	12.80	6.60	4.96	4.22	3.81	**3.35**	**3.10**	**2.92**	**2.81**	**2.73**	**2.67**	**2.62**	**2.59**	**2.54**	**2.50**	**2.48**	**2.46**	**2.44**	**2.41**	**2.39**
30	13.38	6.87	5.16	4.38	3.94	*3.46*	**3.20**	**3.01**	**2.89**	**2.81**	**2.75**	**2.70**	**2.66**	**2.61**	**2.57**	**2.54**	**2.52**	**2.50**	**2.47**	**2.46**
40	13.79	7.06	5.29	4.50	4.04	*3.54*	**3.28**	**3.08**	**2.95**	**2.86**	**2.80**	**2.75**	**2.71**	**2.66**	**2.62**	**2.58**	**2.57**	**2.55**	**2.52**	**2.50**
50	14.09	7.21	5.40	4.58	4.12	*3.61*	**3.33**	**3.13**	**3.00**	**2.91**	**2.84**	**2.79**	**2.75**	**2.69**	**2.65**	**2.62**	**2.60**	**2.58**	**2.54**	**2.53**
60	14.34	7.32	5.48	4.65	4.18	*3.66*	**3.38**	**3.17**	**3.03**	**2.94**	**2.87**	**2.82**	**2.78**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.58**	**2.56**
75	14.64	7.47	5.58	4.73	4.25	*3.72*	**3.43**	**3.22**	**3.08**	**2.99**	**2.92**	**2.86**	**2.82**	**2.76**	**2.72**	**2.69**	**2.66**	**2.64**	**2.61**	**2.59**
100	15.01	7.65	5.71	4.84	4.34	3.80	*3.50*	**3.28**	**3.14**	**3.04**	**2.97**	**2.91**	**2.87**	**2.81**	**2.76**	**2.73**	**2.71**	**2.69**	**2.65**	**2.63**
125	15.30	7.78	5.81	4.92	4.41	3.86	*3.55*	**3.33**	**3.18**	**3.08**	**3.01**	**2.95**	**2.91**	**2.84**	**2.80**	**2.77**	**2.74**	**2.72**	**2.68**	**2.66**
150	15.48	7.90	5.89	4.99	4.47	3.90	*3.60*	**3.37**	**3.22**	**3.12**	**3.04**	**2.99**	**2.94**	**2.87**	**2.83**	**2.79**	**2.77**	**2.75**	**2.71**	**2.68**
175	15.73	7.99	5.96	5.04	4.52	3.95	*3.63*	**3.40**	**3.25**	**3.15**	**3.07**	**3.01**	**2.97**	**2.90**	**2.85**	**2.82**	**2.79**	**2.77**	**2.73**	**2.71**
200	15.89	8.07	6.02	5.09	4.56	3.98	*3.67*	**3.43**	**3.28**	**3.17**	**3.09**	**3.04**	**2.99**	**2.92**	**2.87**	**2.84**	**2.81**	**2.79**	**2.75**	**2.73**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.67	0.53	0.46	0.42	0.40	0.37	0.35	0.34	0.33	0.32	0.31	0.31	0.31	0.30	0.30	0.29	0.29	0.29	0.29	0.29
2	0.95	0.75	0.67	0.62	0.59	0.55	0.53	0.51	0.50	0.49	0.48	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.45	0.45
3	1.11	0.88	0.79	0.73	0.69	0.65	0.62	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.55	0.54	0.54	0.54	0.53	0.53
4	1.22	0.97	0.86	0.80	0.77	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
5	1.30	1.04	0.92	0.86	0.82	0.77	0.74	0.71	0.70	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63	0.63
8	1.47	1.17	1.05	0.97	0.93	0.87	0.84	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.71
12	1.62	1.28	1.15	1.07	1.02	0.95	0.92	0.89	0.87	0.85	0.84	0.83	0.83	0.82	0.81	0.80	0.80	0.79	0.79	0.78
16	1.72	1.36	1.21	1.13	1.08	1.01	0.97	0.94	0.92	0.90	0.89	0.88	0.88	0.86	0.86	0.85	0.85	0.84	0.84	0.83
20	1.79	1.42	1.27	1.18	1.13	1.06	1.02	0.98	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
30	1.92	1.52	1.36	1.27	1.21	1.13	1.09	1.05	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.93
40	2.02	1.59	1.42	1.33	1.26	1.19	1.14	1.10	1.08	1.06	1.05	1.03	1.03	1.01	1.00	1.00	0.99	0.98	0.98	0.97
50	2.09	1.65	1.47	1.37	1.31	1.23	1.18	1.14	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.00
60	2.14	1.69	1.51	1.41	1.34	1.26	1.21	1.17	1.14	1.12	1.11	1.10	1.09	1.07	1.06	1.06	1.05	1.04	1.04	1.03
75	2.21	1.74	1.56	1.45	1.38	1.30	1.25	1.21	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.06
100	2.29	1.81	1.62	1.51	1.44	1.35	1.30	1.25	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.13	1.12	1.12	1.11	1.10
125	2.36	1.86	1.66	1.55	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
150	2.41	1.90	1.70	1.58	1.51	1.42	1.36	1.32	1.29	1.26	1.25	1.23	1.22	1.21	1.19	1.18	1.18	1.17	1.16	1.15
175	2.45	1.93	1.73	1.61	1.53	1.44	1.38	1.34	1.31	1.29	1.27	1.25	1.24	1.23	1.21	1.20	1.20	1.19	1.18	1.17
200	2.49	1.96	1.75	1.63	1.56	1.46	1.41	1.36	1.33	1.30	1.29	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.20	1.19

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.95	0.75	0.67	0.62	0.59	0.55	0.53	0.51	0.50	0.49	0.48	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.45	0.45
2	1.22	0.97	0.86	0.80	0.77	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
3	1.37	1.09	0.97	0.91	0.86	0.81	0.78	0.75	0.73	0.72	0.71	0.70	0.70	0.69	0.68	0.68	0.67	0.67	0.67	0.66
4	1.47	1.17	1.05	0.97	0.93	0.87	0.84	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.71
5	1.55	1.23	1.10	1.03	0.98	0.92	0.88	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76	0.75
8	1.72	1.36	1.21	1.13	1.08	1.01	0.97	0.94	0.92	0.90	0.89	0.88	0.88	0.86	0.86	0.85	0.85	0.84	0.84	0.83
12	1.85	1.47	1.31	1.22	1.16	1.09	1.05	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90	0.89
16	1.94	1.54	1.37	1.28	1.22	1.15	1.10	1.06	1.04	1.02	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.94	0.94
20	2.02	1.59	1.42	1.33	1.26	1.19	1.14	1.10	1.08	1.06	1.05	1.03	1.03	1.01	1.00	1.00	0.99	0.98	0.98	0.97
30	2.14	1.69	1.51	1.41	1.34	1.26	1.21	1.17	1.14	1.12	1.11	1.10	1.09	1.07	1.06	1.06	1.05	1.04	1.04	1.03
40	2.23	1.76	1.57	1.46	1.40	1.31	1.26	1.22	1.19	1.17	1.15	1.14	1.13	1.12	1.10	1.10	1.09	1.08	1.08	1.07
50	2.29	1.81	1.62	1.51	1.44	1.35	1.30	1.25	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.13	1.12	1.12	1.11	1.10
60	2.35	1.85	1.65	1.54	1.47	1.38	1.33	1.28	1.25	1.23	1.21	1.20	1.19	1.17	1.16	1.15	1.15	1.14	1.13	1.12
75	2.41	1.90	1.70	1.58	1.51	1.42	1.36	1.32	1.29	1.26	1.25	1.23	1.22	1.21	1.19	1.18	1.18	1.17	1.16	1.15
100	2.49	1.96	1.75	1.63	1.56	1.46	1.41	1.36	1.33	1.30	1.29	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.20	1.19
125	2.55	2.01	1.79	1.67	1.60	1.50	1.44	1.39	1.36	1.34	1.32	1.30	1.29	1.27	1.26	1.25	1.24	1.24	1.23	1.22
150	2.60	2.05	1.83	1.71	1.63	1.53	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24
175	2.64	2.08	1.86	1.73	1.65	1.55	1.49	1.44	1.41	1.38	1.36	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.27	1.26
200	2.68	2.11	1.88	1.75	1.67	1.57	1.51	1.46	1.43	1.40	1.38	1.37	1.36	1.34	1.32	1.31	1.30	1.30	1.29	1.28

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.22	0.97	0.86	0.80	0.77	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
2	1.47	1.17	1.05	0.97	0.93	0.87	0.84	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.71
3	1.62	1.28	1.15	1.07	1.02	0.95	0.92	0.89	0.87	0.85	0.84	0.83	0.83	0.82	0.81	0.80	0.80	0.79	0.79	0.78
4	1.72	1.36	1.21	1.13	1.08	1.01	0.97	0.94	0.92	0.90	0.89	0.88	0.88	0.86	0.86	0.85	0.85	0.84	0.84	0.83
5	1.79	1.42	1.27	1.18	1.13	1.06	1.02	0.98	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
8	1.94	1.54	1.37	1.28	1.22	1.15	1.10	1.06	1.04	1.02	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.94	0.94
12	2.07	1.64	1.46	1.36	1.30	1.22	1.17	1.13	1.11	1.09	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.00	1.00
16	2.16	1.71	1.52	1.42	1.35	1.27	1.22	1.18	1.15	1.13	1.12	1.11	1.10	1.08	1.07	1.06	1.06	1.05	1.04	1.04
20	2.23	1.76	1.57	1.46	1.40	1.31	1.26	1.22	1.19	1.17	1.15	1.14	1.13	1.12	1.10	1.10	1.09	1.08	1.08	1.07
30	2.35	1.85	1.65	1.54	1.47	1.38	1.33	1.28	1.25	1.23	1.21	1.20	1.19	1.17	1.16	1.15	1.15	1.14	1.13	1.12
40	2.43	1.91	1.71	1.59	1.52	1.43	1.37	1.33	1.29	1.27	1.26	1.24	1.23	1.21	1.20	1.19	1.19	1.18	1.17	1.16
50	2.49	1.96	1.75	1.63	1.56	1.46	1.41	1.36	1.33	1.30	1.29	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.20	1.19
60	2.54	2.00	1.79	1.67	1.59	1.49	1.43	1.39	1.35	1.33	1.31	1.30	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.21
75	2.60	2.05	1.83	1.71	1.63	1.53	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24
100	2.68	2.11	1.88	1.75	1.67	1.57	1.51	1.46	1.43	1.40	1.38	1.37	1.36	1.34	1.32	1.31	1.30	1.30	1.29	1.28
125	2.74	2.15	1.92	1.79	1.71	1.61	1.54	1.49	1.46	1.43	1.41	1.40	1.38	1.37	1.35	1.34	1.33	1.33	1.31	1.30
150	2.79	2.19	1.95	1.82	1.74	1.63	1.57	1.52	1.48	1.46	1.44	1.42	1.41	1.39	1.37	1.36	1.35	1.35	1.33	1.33
175	2.82	2.22	1.98	1.85	1.76	1.66	1.59	1.54	1.50	1.48	1.46	1.44	1.43	1.41	1.39	1.38	1.37	1.37	1.35	1.34
200	2.86	2.25	2.00	1.87	1.78	1.68	1.61	1.56	1.52	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.04	0.80	0.70	0.64	0.61	0.56	0.54	0.52	0.50	0.49	0.49	0.48	0.47	0.47	0.46	0.46	0.46	0.45	0.45	0.45
2	1.36	1.04	0.91	0.84	0.79	0.73	0.70	0.67	0.66	0.64	0.64	0.63	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
3	1.55	1.17	1.02	0.94	0.89	0.83	0.79	0.76	0.74	0.73	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.66
4	1.68	1.27	1.11	1.02	0.96	0.89	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
5	1.78	1.34	1.17	1.07	1.01	0.94	0.90	0.87	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.76	0.75
8	1.99	1.49	1.29	1.19	1.12	1.04	0.99	0.96	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.84	0.83
12	2.17	1.61	1.40	1.29	1.21	1.12	1.07	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90
16	2.29	1.70	1.47	1.35	1.28	1.18	1.13	1.08	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.95	0.95	0.94
20	2.38	1.76	1.53	1.40	1.32	1.23	1.17	1.12	1.09	1.07	1.06	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
30	2.54	1.88	1.63	1.49	1.41	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.10	1.08	1.07	1.06	1.05	1.05	1.04	1.03
40	2.66	1.96	1.70	1.56	1.47	1.36	1.29	1.24	1.21	1.18	1.17	1.15	1.14	1.12	1.11	1.10	1.09	1.09	1.08	1.07
50	2.74	2.02	1.75	1.60	1.51	1.40	1.33	1.28	1.25	1.22	1.20	1.19	1.17	1.16	1.14	1.13	1.13	1.12	1.11	1.10
60	2.81	2.07	1.79	1.64	1.55	1.43	1.36	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.13	1.13
75	2.90	2.13	1.84	1.69	1.59	1.47	1.40	1.35	1.31	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.18	1.18	1.16	1.16
100	3.00	2.21	1.91	1.75	1.65	1.52	1.45	1.39	1.35	1.33	1.30	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.19
125	3.08	2.26	1.96	1.79	1.69	1.56	1.49	1.43	1.39	1.36	1.34	1.32	1.31	1.29	1.27	1.26	1.25	1.24	1.23	1.22
150	3.15	2.31	2.00	1.83	1.72	1.59	1.52	1.46	1.41	1.39	1.36	1.35	1.33	1.31	1.29	1.28	1.27	1.27	1.25	1.25
175	3.20	2.35	2.03	1.86	1.75	1.62	1.54	1.48	1.44	1.41	1.38	1.37	1.35	1.33	1.32	1.30	1.29	1.29	1.27	1.26
200	3.25	2.38	2.06	1.88	1.77	1.64	1.56	1.50	1.46	1.43	1.40	1.39	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.36	1.04	0.91	0.84	0.79	0.73	0.70	0.67	0.66	0.64	0.64	0.63	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
2	1.68	1.27	1.11	1.02	0.96	0.89	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
3	1.86	1.40	1.22	1.12	1.06	0.98	0.94	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.81	0.80	0.80	0.79	0.78
4	1.99	1.49	1.29	1.19	1.12	1.04	0.99	0.96	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.84	0.83
5	2.09	1.56	1.35	1.24	1.17	1.09	1.04	1.00	0.97	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
8	2.29	1.70	1.47	1.35	1.28	1.18	1.13	1.08	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.95	0.95	0.94
12	2.45	1.82	1.57	1.44	1.36	1.26	1.20	1.16	1.12	1.10	1.09	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.00
16	2.57	1.90	1.64	1.51	1.42	1.32	1.25	1.21	1.17	1.15	1.13	1.12	1.11	1.09	1.08	1.07	1.06	1.06	1.05	1.04
20	2.66	1.96	1.70	1.56	1.47	1.36	1.29	1.24	1.21	1.18	1.17	1.15	1.14	1.12	1.11	1.10	1.09	1.09	1.08	1.07
30	2.81	2.07	1.79	1.64	1.55	1.43	1.36	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.13	1.13
40	2.92	2.15	1.86	1.70	1.60	1.48	1.41	1.36	1.32	1.29	1.27	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16
50	3.00	2.21	1.91	1.75	1.65	1.52	1.45	1.39	1.35	1.33	1.30	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.19
60	3.07	2.25	1.95	1.78	1.68	1.55	1.48	1.42	1.38	1.35	1.33	1.31	1.30	1.28	1.27	1.25	1.25	1.24	1.23	1.22
75	3.15	2.31	2.00	1.83	1.72	1.59	1.52	1.46	1.41	1.39	1.36	1.35	1.33	1.31	1.29	1.28	1.27	1.27	1.25	1.25
100	3.25	2.38	2.06	1.88	1.77	1.64	1.56	1.50	1.46	1.43	1.40	1.39	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
125	3.33	2.44	2.10	1.93	1.81	1.68	1.60	1.53	1.49	1.46	1.43	1.42	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31
150	3.39	2.48	2.14	1.96	1.85	1.71	1.63	1.56	1.52	1.48	1.46	1.44	1.43	1.40	1.39	1.37	1.36	1.35	1.34	1.33
175	3.44	2.52	2.17	1.99	1.87	1.73	1.65	1.58	1.54	1.51	1.48	1.46	1.45	1.42	1.40	1.39	1.38	1.37	1.36	1.35
200	3.49	2.55	2.20	2.01	1.90	1.75	1.67	1.60	1.56	1.52	1.50	1.48	1.46	1.44	1.42	1.41	1.40	1.39	1.37	1.36

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.68	1.27	1.11	1.02	0.96	0.89	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
2	1.99	1.49	1.29	1.19	1.12	1.04	0.99	0.96	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.84	0.83
3	2.17	1.61	1.40	1.29	1.21	1.12	1.07	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90
4	2.29	1.70	1.47	1.35	1.28	1.18	1.13	1.08	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.95	0.95	0.94
5	2.38	1.76	1.53	1.40	1.32	1.23	1.17	1.12	1.09	1.07	1.06	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
8	2.57	1.90	1.64	1.51	1.42	1.32	1.25	1.21	1.17	1.15	1.13	1.12	1.11	1.09	1.08	1.07	1.06	1.06	1.05	1.04
12	2.73	2.01	1.74	1.60	1.50	1.39	1.33	1.27	1.24	1.21	1.19	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.10
16	2.84	2.09	1.81	1.66	1.56	1.44	1.38	1.32	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.14
20	2.92	2.15	1.86	1.70	1.60	1.48	1.41	1.36	1.32	1.29	1.27	1.26	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16
30	3.07	2.25	1.95	1.78	1.68	1.55	1.48	1.42	1.38	1.35	1.33	1.31	1.30	1.28	1.27	1.25	1.25	1.24	1.23	1.22
40	3.17	2.33	2.01	1.84	1.73	1.60	1.53	1.47	1.42	1.39	1.37	1.35	1.34	1.32	1.30	1.29	1.28	1.28	1.26	1.25
50	3.25	2.38	2.06	1.88	1.77	1.64	1.56	1.50	1.46	1.43	1.40	1.39	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
60	3.31	2.43	2.10	1.92	1.81	1.67	1.59	1.53	1.48	1.45	1.43	1.41	1.40	1.37	1.36	1.34	1.33	1.33	1.31	1.30
75	3.39	2.48	2.14	1.96	1.85	1.71	1.63	1.56	1.52	1.48	1.46	1.44	1.43	1.40	1.39	1.37	1.36	1.35	1.34	1.33
100	3.49	2.55	2.20	2.01	1.90	1.75	1.67	1.60	1.56	1.52	1.50	1.48	1.46	1.44	1.42	1.41	1.40	1.39	1.37	1.36
125	3.56	2.60	2.24	2.06	1.94	1.79	1.70	1.63	1.59	1.55	1.53	1.51	1.49	1.47	1.45	1.44	1.42	1.42	1.40	1.39
150	3.62	2.64	2.28	2.09	1.97	1.82	1.73	1.66	1.61	1.58	1.55	1.53	1.52	1.49	1.47	1.46	1.45	1.44	1.42	1.41
175	3.67	2.68	2.31	2.12	1.99	1.84	1.75	1.68	1.63	1.60	1.57	1.55	1.53	1.51	1.49	1.48	1.47	1.46	1.44	1.43
200	3.71	2.71	2.34	2.14	2.01	1.86	1.77	1.70	1.65	1.62	1.59	1.57	1.55	1.53	1.51	1.49	1.48	1.47	1.46	1.44

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (5 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.62	1.17	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.70	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
2	2.02	1.43	1.22	1.11	1.04	0.96	0.91	0.87	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
3	2.26	1.58	1.34	1.22	1.14	1.05	1.00	0.95	0.93	0.91	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.84	0.83	0.82
4	2.43	1.69	1.43	1.29	1.21	1.11	1.05	1.01	0.98	0.96	0.95	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
5	2.57	1.77	1.50	1.35	1.26	1.16	1.10	1.05	1.02	1.00	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90
8	2.84	1.94	1.63	1.47	1.37	1.26	1.19	1.14	1.11	1.08	1.06	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
12	3.07	2.09	1.75	1.57	1.47	1.34	1.27	1.21	1.18	1.15	1.13	1.12	1.10	1.09	1.07	1.06	1.06	1.05	1.04	1.03
16	3.24	2.19	1.83	1.65	1.53	1.40	1.32	1.26	1.22	1.20	1.18	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
20	3.36	2.27	1.89	1.70	1.58	1.44	1.36	1.30	1.26	1.23	1.21	1.19	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.10
30	3.58	2.40	2.00	1.80	1.67	1.52	1.44	1.37	1.33	1.30	1.27	1.26	1.24	1.22	1.21	1.19	1.19	1.18	1.17	1.16
40	3.73	2.50	2.08	1.86	1.73	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.28	1.26	1.25	1.23	1.22	1.22	1.20	1.20
50	3.85	2.57	2.14	1.92	1.78	1.62	1.53	1.46	1.41	1.38	1.35	1.33	1.32	1.29	1.28	1.26	1.25	1.25	1.23	1.22
60	3.94	2.63	2.19	1.96	1.82	1.65	1.56	1.49	1.44	1.40	1.38	1.36	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.25
75	4.06	2.70	2.24	2.01	1.86	1.70	1.60	1.52	1.47	1.44	1.41	1.39	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
100	4.20	*2.79*	2.32	2.07	1.92	1.75	1.65	1.57	1.52	1.48	1.45	1.43	1.41	1.39	1.37	1.36	1.34	1.34	1.32	1.31
125	4.31	*2.86*	2.37	2.12	1.97	1.79	1.68	1.60	1.55	1.51	1.48	1.46	1.44	1.42	1.40	1.38	1.37	1.36	1.35	1.34
150	4.40	2.91	2.42	2.16	2.01	1.82	1.72	1.63	1.58	1.54	1.51	1.49	1.47	1.44	1.42	1.41	1.40	1.39	1.37	1.36
175	4.47	2.96	2.45	2.19	2.04	1.85	1.74	1.66	1.60	1.56	1.53	1.51	1.49	1.46	1.44	1.43	1.41	1.41	1.39	1.38
200	4.53	3.00	2.49	2.22	2.06	1.87	1.76	1.68	1.62	1.58	1.55	1.52	1.51	1.48	1.46	1.44	1.43	1.42	1.40	1.39

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (5 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.02	1.43	1.22	1.11	1.04	0.96	0.91	0.87	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
2	2.43	1.69	1.43	1.29	1.21	1.11	1.05	1.01	0.98	0.96	0.95	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
3	2.67	1.84	1.55	1.40	1.31	1.20	1.13	1.09	1.05	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93
4	2.84	1.94	1.63	1.47	1.37	1.26	1.19	1.14	1.11	1.08	1.06	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
5	2.97	2.02	1.70	1.53	1.42	1.30	1.23	1.18	1.14	1.12	1.10	1.09	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.01
8	3.24	2.19	1.83	1.65	1.53	1.40	1.32	1.26	1.22	1.20	1.18	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
12	3.46	2.33	1.94	1.74	1.62	1.48	1.40	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.17	1.16	1.16	1.15	1.14	1.13
16	3.61	2.43	2.02	1.81	1.68	1.53	1.45	1.38	1.34	1.31	1.28	1.27	1.25	1.23	1.21	1.20	1.19	1.19	1.18	1.17
20	3.73	2.50	2.08	1.86	1.73	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.28	1.26	1.25	1.23	1.22	1.22	1.20	1.20
30	3.94	2.63	2.19	1.96	1.82	1.65	1.56	1.49	1.44	1.40	1.38	1.36	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.25
40	4.09	2.72	2.26	2.02	1.88	1.71	1.61	1.53	1.48	1.45	1.42	1.40	1.38	1.36	1.34	1.33	1.32	1.31	1.29	1.28
50	4.20	2.79	2.32	2.07	1.92	1.75	1.65	1.57	1.52	1.48	1.45	1.43	1.41	1.39	1.37	1.36	1.34	1.34	1.32	1.31
60	4.29	2.85	2.36	2.11	1.96	1.78	1.68	1.60	1.54	1.51	1.48	1.46	1.44	1.41	1.39	1.38	1.37	1.36	1.34	1.33
75	4.40	*2.91*	2.42	2.16	2.01	1.82	1.72	1.63	1.58	1.54	1.51	1.49	1.47	1.44	1.42	1.41	1.40	1.39	1.37	1.36
100	4.53	3.00	2.49	2.22	2.06	1.87	1.76	1.68	1.62	1.58	1.55	1.52	1.51	1.48	1.46	1.44	1.43	1.42	1.40	1.39
125	4.64	3.07	2.54	2.27	2.11	1.91	1.80	1.71	1.65	1.61	1.58	1.55	1.54	1.51	1.49	1.47	1.46	1.45	1.43	1.42
150	4.72	3.12	2.58	2.31	2.14	1.94	1.83	1.74	1.68	1.64	1.60	1.58	1.56	1.53	1.51	1.49	1.48	1.47	1.45	1.44
175	4.79	3.16	2.62	2.34	2.17	1.97	1.85	1.76	1.70	1.66	1.62	1.60	1.58	1.55	1.53	1.51	1.50	1.49	1.47	1.46
200	4.85	3.20	2.65	2.37	2.19	1.99	1.87	1.78	1.72	1.68	1.64	1.62	1.60	1.57	1.54	1.53	1.51	1.50	1.49	1.47

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.43	1.69	1.43	1.29	1.21	1.11	1.05	1.01	0.98	0.96	0.95	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
2	2.84	1.94	1.63	1.47	1.37	1.26	1.19	1.14	1.11	1.08	1.06	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
3	3.07	2.09	1.75	1.57	1.47	1.34	1.27	1.21	1.18	1.15	1.13	1.12	1.10	1.09	1.07	1.06	1.06	1.05	1.04	1.03
4	3.24	2.19	1.83	1.65	1.53	1.40	1.32	1.26	1.22	1.20	1.18	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
5	3.36	2.27	1.89	1.70	1.58	1.44	1.36	1.30	1.26	1.23	1.21	1.19	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.10
8	3.61	2.43	2.02	1.81	1.68	1.53	1.45	1.38	1.34	1.31	1.28	1.27	1.25	1.23	1.21	1.20	1.19	1.19	1.18	1.17
12	3.83	2.56	2.13	1.91	1.77	1.61	1.52	1.45	1.40	1.37	1.34	1.33	1.31	1.29	1.27	1.26	1.25	1.24	1.23	1.22
16	3.98	2.65	2.20	1.97	1.83	1.67	1.57	1.50	1.45	1.41	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
20	4.09	2.72	2.26	2.02	1.88	1.71	1.61	1.53	1.48	1.45	1.42	1.40	1.38	1.36	1.34	1.33	1.32	1.31	1.29	1.28
30	4.29	2.85	2.36	2.11	1.96	1.78	1.68	1.60	1.54	1.51	1.48	1.46	1.44	1.41	1.39	1.38	1.37	1.36	1.34	1.33
40	4.43	2.93	2.43	2.18	2.02	1.83	1.73	1.64	1.59	1.55	1.52	1.49	1.48	1.45	1.43	1.41	1.40	1.39	1.38	1.37
50	4.53	3.00	2.49	2.22	2.06	1.87	1.76	1.68	1.62	1.58	1.55	1.52	1.51	1.48	1.46	1.44	1.43	1.42	1.40	1.39
60	4.62	3.05	2.53	2.26	2.10	1.90	1.79	1.70	1.65	1.60	1.57	1.55	1.53	1.50	1.48	1.47	1.45	1.44	1.43	1.41
75	4.72	*3.12*	2.58	2.31	2.14	1.94	1.83	1.74	1.68	1.64	1.60	1.58	1.56	1.53	1.51	1.49	1.48	1.47	1.45	1.44
100	4.85	3.20	2.65	2.37	2.19	1.99	1.87	1.78	1.72	1.68	1.64	1.62	1.60	1.57	1.54	1.53	1.51	1.50	1.49	1.47
125	4.95	3.26	2.70	2.41	2.24	2.03	1.91	1.81	1.75	1.71	1.67	1.65	1.62	1.59	1.57	1.55	1.54	1.53	1.51	1.50
150	5.03	3.31	2.74	2.45	2.27	2.06	1.94	1.84	1.78	1.73	1.70	1.67	1.65	1.62	1.59	1.58	1.56	1.55	1.53	1.52
175	5.10	3.36	2.78	2.48	2.30	2.08	1.96	1.86	1.80	1.75	1.72	1.69	1.67	1.63	1.61	1.59	1.58	1.57	1.55	1.53
200	5.16	3.39	2.81	2.51	2.32	2.11	1.98	1.88	1.82	1.77	1.73	1.71	1.68	1.65	1.63	1.61	1.59	1.58	1.56	1.55

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.15	1.48	1.24	1.12	1.05	0.96	0.91	0.88	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
2	2.64	1.76	1.46	1.31	1.22	1.12	1.06	1.01	0.98	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.89	0.88	0.87	0.87
3	2.94	1.93	1.59	1.43	1.32	1.21	1.14	1.09	1.06	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93
4	3.15	2.04	1.68	1.50	1.40	1.27	1.20	1.14	1.11	1.08	1.07	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
5	3.32	2.14	1.75	1.56	1.45	1.32	1.24	1.19	1.15	1.12	1.10	1.09	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.01
8	3.66	2.33	1.90	1.69	1.56	1.42	1.33	1.27	1.23	1.20	1.18	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
12	3.95	2.49	2.03	1.80	1.66	1.50	1.41	1.34	1.30	1.27	1.24	1.23	1.21	1.19	1.18	1.16	1.16	1.15	1.14	1.13
16	4.15	2.61	2.12	1.87	1.73	1.56	1.46	1.39	1.35	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
20	4.31	2.69	2.18	1.93	1.78	1.60	1.51	1.43	1.38	1.35	1.32	1.30	1.29	1.26	1.25	1.24	1.23	1.22	1.21	1.20
30	4.58	*2.85*	2.30	2.03	1.87	1.68	1.58	1.50	1.45	1.41	1.38	1.36	1.35	1.32	1.30	1.29	1.28	1.27	1.26	1.25
40	4.77	2.96	2.39	2.11	1.94	1.74	1.63	1.55	1.49	1.45	1.43	1.40	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
50	4.92	3.04	2.45	2.16	1.99	1.78	1.67	1.58	1.53	1.49	1.46	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31
60	5.04	3.11	2.50	2.21	2.03	1.82	1.70	1.61	1.56	1.51	1.48	1.46	1.44	1.41	1.40	1.38	1.37	1.36	1.34	1.33
75	5.18	3.19	2.57	2.26	2.08	1.86	1.74	1.65	1.59	1.55	1.52	1.49	1.47	1.44	1.42	1.41	1.40	1.39	1.37	1.36
100	5.36	3.29	2.65	2.33	2.14	1.92	1.79	1.70	1.63	1.59	1.56	1.53	1.51	1.48	1.46	1.45	1.43	1.42	1.41	1.39
125	5.50	3.37	*2.71*	2.38	2.18	1.96	1.83	1.73	1.67	1.62	1.59	1.56	1.54	1.51	1.49	1.47	1.46	1.45	1.43	1.42
150	5.61	3.43	*2.76*	2.42	2.22	1.99	1.86	1.76	1.69	1.65	1.61	1.59	1.57	1.53	1.51	1.50	1.48	1.47	1.45	1.44
175	5.70	3.49	*2.80*	2.46	2.26	2.02	1.89	1.78	1.72	1.67	1.64	1.61	1.59	1.55	1.53	1.51	1.50	1.49	1.47	1.46
200	5.78	3.53	*2.83*	2.49	2.28	2.05	1.91	1.81	1.74	1.69	1.65	1.63	1.60	1.57	1.55	1.53	1.52	1.51	1.49	1.47

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.64**	**1.76**	**1.46**	**1.31**	**1.22**	**1.12**	**1.06**	**1.01**	**0.98**	**0.96**	**0.95**	**0.94**	**0.93**	**0.91**	**0.90**	**0.89**	**0.89**	**0.88**	**0.87**	**0.87**
2	3.15	**2.04**	**1.68**	**1.50**	**1.40**	**1.27**	**1.20**	**1.14**	**1.11**	**1.08**	**1.07**	**1.05**	**1.04**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.97**
3	3.45	**2.21**	**1.81**	**1.61**	**1.49**	**1.36**	**1.28**	**1.22**	**1.18**	**1.15**	**1.13**	**1.12**	**1.11**	**1.09**	**1.07**	**1.06**	**1.06**	**1.05**	**1.04**	**1.03**
4	3.66	**2.33**	**1.90**	**1.69**	**1.56**	**1.42**	**1.33**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
5	3.82	**2.42**	**1.97**	**1.75**	**1.62**	**1.46**	**1.38**	**1.31**	**1.27**	**1.24**	**1.21**	**1.20**	**1.18**	**1.16**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**
8	4.15	**2.61**	**2.12**	**1.87**	**1.73**	**1.56**	**1.46**	**1.39**	**1.35**	**1.31**	**1.29**	**1.27**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
12	4.43	**2.76**	**2.24**	**1.98**	**1.82**	**1.64**	**1.54**	**1.46**	**1.41**	**1.38**	**1.35**	**1.33**	**1.31**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
16	4.63	**2.87**	**2.32**	**2.05**	**1.89**	**1.70**	**1.59**	**1.51**	**1.46**	**1.42**	**1.39**	**1.37**	**1.35**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
20	4.77	*2.96*	**2.39**	**2.11**	**1.94**	**1.74**	**1.63**	**1.55**	**1.49**	**1.45**	**1.43**	**1.40**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.29**	**1.28**
30	5.04	3.11	**2.50**	**2.21**	**2.03**	**1.82**	**1.70**	**1.61**	**1.56**	**1.51**	**1.48**	**1.46**	**1.44**	**1.41**	**1.40**	**1.38**	**1.37**	**1.36**	**1.34**	**1.33**
40	5.22	3.21	**2.58**	**2.28**	**2.09**	**1.87**	**1.75**	**1.66**	**1.60**	**1.56**	**1.53**	**1.50**	**1.48**	**1.45**	**1.43**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
50	5.36	3.29	**2.65**	**2.33**	**2.14**	**1.92**	**1.79**	**1.70**	**1.63**	**1.59**	**1.56**	**1.53**	**1.51**	**1.48**	**1.46**	**1.45**	**1.43**	**1.42**	**1.41**	**1.39**
60	5.47	3.36	**2.70**	**2.37**	**2.18**	**1.95**	**1.82**	**1.73**	**1.66**	**1.62**	**1.58**	**1.56**	**1.54**	**1.51**	**1.48**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
75	5.61	3.43	**2.76**	**2.42**	**2.22**	**1.99**	**1.86**	**1.76**	**1.69**	**1.65**	**1.61**	**1.59**	**1.57**	**1.53**	**1.51**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**
100	5.78	3.53	**2.83**	**2.49**	**2.28**	**2.05**	**1.91**	**1.81**	**1.74**	**1.69**	**1.65**	**1.63**	**1.60**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
125	5.91	3.61	*2.89*	**2.54**	**2.33**	**2.09**	**1.95**	**1.84**	**1.77**	**1.72**	**1.68**	**1.66**	**1.63**	**1.60**	**1.57**	**1.56**	**1.54**	**1.53**	**1.51**	**1.50**
150	6.02	3.67	*2.94*	**2.58**	**2.37**	**2.12**	**1.98**	**1.87**	**1.80**	**1.75**	**1.71**	**1.68**	**1.66**	**1.62**	**1.60**	**1.58**	**1.56**	**1.55**	**1.53**	**1.52**
175	6.10	3.72	*2.98*	**2.62**	**2.40**	**2.15**	**2.00**	**1.89**	**1.82**	**1.77**	**1.73**	**1.70**	**1.68**	**1.64**	**1.62**	**1.60**	**1.58**	**1.57**	**1.55**	**1.54**
200	6.18	3.76	*3.01*	**2.65**	**2.42**	**2.17**	**2.02**	**1.91**	**1.84**	**1.79**	**1.75**	**1.72**	**1.69**	**1.66**	**1.63**	**1.61**	**1.60**	**1.59**	**1.56**	**1.55**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.15	**2.04**	**1.68**	**1.50**	**1.40**	**1.27**	**1.20**	**1.14**	**1.11**	**1.08**	**1.07**	**1.05**	**1.04**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.97**
2	3.66	**2.33**	**1.90**	**1.69**	**1.56**	**1.42**	**1.33**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
3	3.95	**2.49**	**2.03**	**1.80**	**1.66**	**1.50**	**1.41**	**1.34**	**1.30**	**1.27**	**1.24**	**1.23**	**1.21**	**1.19**	**1.18**	**1.16**	**1.16**	**1.15**	**1.14**	**1.13**
4	4.15	**2.61**	**2.12**	**1.87**	**1.73**	**1.56**	**1.46**	**1.39**	**1.35**	**1.31**	**1.29**	**1.27**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
5	4.31	**2.69**	**2.18**	**1.93**	**1.78**	**1.60**	**1.51**	**1.43**	**1.38**	**1.35**	**1.32**	**1.30**	**1.29**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
8	4.63	**2.87**	**2.32**	**2.05**	**1.89**	**1.70**	**1.59**	**1.51**	**1.46**	**1.42**	**1.39**	**1.37**	**1.35**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
12	4.89	**3.03**	**2.44**	**2.15**	**1.98**	**1.78**	**1.66**	**1.58**	**1.52**	**1.48**	**1.45**	**1.43**	**1.41**	**1.38**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	5.08	3.13	**2.52**	**2.22**	**2.04**	**1.83**	**1.72**	**1.62**	**1.57**	**1.52**	**1.49**	**1.47**	**1.45**	**1.42**	**1.40**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
20	5.22	3.21	**2.58**	**2.28**	**2.09**	**1.87**	**1.75**	**1.66**	**1.60**	**1.56**	**1.53**	**1.50**	**1.48**	**1.45**	**1.43**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
30	5.47	3.36	**2.70**	**2.37**	**2.18**	**1.95**	**1.82**	**1.73**	**1.66**	**1.62**	**1.58**	**1.56**	**1.54**	**1.51**	**1.48**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
40	5.65	3.46	**2.77**	**2.44**	**2.24**	**2.00**	**1.87**	**1.77**	**1.70**	**1.66**	**1.62**	**1.60**	**1.57**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.46**	**1.45**
50	5.78	3.53	**2.83**	**2.49**	**2.28**	**2.05**	**1.91**	**1.81**	**1.74**	**1.69**	**1.65**	**1.63**	**1.60**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
60	5.89	3.60	**2.88**	**2.53**	**2.32**	**2.08**	**1.94**	**1.83**	**1.76**	**1.71**	**1.68**	**1.65**	**1.63**	**1.59**	**1.57**	**1.55**	**1.54**	**1.53**	**1.51**	**1.49**
75	6.02	3.67	**2.94**	**2.58**	**2.37**	**2.12**	**1.98**	**1.87**	**1.80**	**1.75**	**1.71**	**1.68**	**1.66**	**1.62**	**1.60**	**1.58**	**1.56**	**1.55**	**1.53**	**1.52**
100	6.18	3.76	**3.01**	**2.65**	**2.42**	**2.17**	**2.02**	**1.91**	**1.84**	**1.79**	**1.75**	**1.72**	**1.69**	**1.66**	**1.63**	**1.61**	**1.60**	**1.59**	**1.56**	**1.55**
125	6.31	3.84	*3.07*	**2.69**	**2.47**	**2.21**	**2.06**	**1.95**	**1.87**	**1.82**	**1.78**	**1.75**	**1.72**	**1.68**	**1.66**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**
150	6.41	3.90	*3.12*	**2.73**	**2.51**	**2.24**	**2.09**	**1.97**	**1.90**	**1.84**	**1.80**	**1.77**	**1.74**	**1.71**	**1.68**	**1.66**	**1.64**	**1.63**	**1.61**	**1.59**
175	6.49	3.94	*3.16*	**2.77**	**2.54**	**2.27**	**2.11**	**2.00**	**1.92**	**1.86**	**1.82**	**1.79**	**1.76**	**1.72**	**1.70**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**
200	6.56	3.99	*3.19*	**2.80**	**2.56**	**2.29**	**2.13**	**2.01**	**1.94**	**1.88**	**1.84**	**1.81**	**1.78**	**1.74**	**1.71**	**1.69**	**1.68**	**1.66**	**1.64**	**1.63**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (20 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.80	**1.80**	**1.48**	**1.33**	**1.23**	**1.12**	**1.06**	**1.02**	**0.99**	**0.96**	**0.95**	**0.94**	**0.93**	**0.91**	**0.90**	**0.89**	**0.89**	**0.88**	**0.88**	**0.87**
2	3.41	**2.12**	**1.72**	**1.52**	**1.41**	**1.28**	**1.20**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.97**
3	3.78	**2.30**	**1.85**	**1.64**	**1.51**	**1.36**	**1.28**	**1.22**	**1.18**	**1.16**	**1.13**	**1.12**	**1.11**	**1.09**	**1.07**	**1.07**	**1.06**	**1.05**	**1.04**	**1.03**
4	4.04	**2.44**	**1.95**	**1.72**	**1.58**	**1.43**	**1.34**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
5	4.24	**2.54**	**2.03**	**1.78**	**1.64**	**1.47**	**1.38**	**1.31**	**1.27**	**1.24**	**1.22**	**1.20**	**1.19**	**1.17**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**
8	4.67	*2.76*	**2.19**	**1.91**	**1.75**	**1.57**	**1.47**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
12	5.04	2.94	**2.32**	**2.03**	**1.85**	**1.66**	**1.55**	**1.47**	**1.42**	**1.38**	**1.35**	**1.33**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
16	5.29	3.07	**2.42**	**2.11**	**1.92**	**1.72**	**1.60**	**1.52**	**1.46**	**1.42**	**1.40**	**1.37**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
20	5.48	3.17	**2.49**	**2.17**	**1.98**	**1.76**	**1.65**	**1.56**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.29**	**1.28**
30	5.83	3.35	**2.62**	**2.28**	**2.08**	**1.85**	**1.72**	**1.62**	**1.56**	**1.52**	**1.49**	**1.46**	**1.45**	**1.42**	**1.40**	**1.38**	**1.37**	**1.36**	**1.34**	**1.33**
40	6.07	3.47	*2.72*	**2.36**	**2.14**	**1.90**	**1.77**	**1.67**	**1.61**	**1.56**	**1.53**	**1.50**	**1.48**	**1.46**	**1.43**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
50	6.25	3.57	*2.79*	**2.41**	**2.20**	**1.95**	**1.81**	**1.71**	**1.64**	**1.60**	**1.56**	**1.54**	**1.52**	**1.48**	**1.46**	**1.45**	**1.43**	**1.42**	**1.41**	**1.39**
60	6.40	3.65	*2.84*	**2.46**	**2.24**	**1.99**	**1.85**	**1.74**	**1.67**	**1.62**	**1.59**	**1.56**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
75	6.58	3.74	*2.91*	**2.52**	**2.29**	**2.03**	**1.89**	**1.78**	**1.71**	**1.66**	**1.62**	**1.59**	**1.57**	**1.54**	**1.51**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**
100	6.81	3.86	3.00	**2.59**	**2.36**	**2.09**	**1.94**	**1.82**	**1.75**	**1.70**	**1.66**	**1.63**	**1.61**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
125	6.98	3.95	3.07	**2.65**	**2.41**	**2.13**	**1.98**	**1.86**	**1.78**	**1.73**	**1.69**	**1.66**	**1.64**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.51**	**1.50**
150	7.12	4.02	3.12	*2.70*	**2.45**	**2.16**	**2.01**	**1.89**	**1.81**	**1.76**	**1.72**	**1.69**	**1.66**	**1.62**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**
175	7.24	4.08	3.17	*2.74*	**2.48**	**2.19**	**2.03**	**1.91**	**1.83**	**1.78**	**1.74**	**1.71**	**1.68**	**1.64**	**1.62**	**1.60**	**1.58**	**1.57**	**1.55**	**1.54**
200	7.34	4.13	3.21	*2.77*	**2.51**	**2.22**	**2.06**	**1.93**	**1.85**	**1.80**	**1.75**	**1.72**	**1.70**	**1.66**	**1.63**	**1.61**	**1.60**	**1.59**	**1.57**	**1.55**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (20 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.41	**2.12**	**1.72**	**1.52**	**1.41**	**1.28**	**1.20**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.97**
2	4.04	**2.44**	**1.95**	**1.72**	**1.58**	**1.43**	**1.34**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
3	4.41	**2.62**	**2.09**	**1.83**	**1.68**	**1.51**	**1.42**	**1.35**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**
4	4.67	**2.76**	**2.19**	**1.91**	**1.75**	**1.57**	**1.47**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
5	4.87	**2.86**	**2.26**	**1.98**	**1.81**	**1.62**	**1.52**	**1.44**	**1.39**	**1.35**	**1.32**	**1.30**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
8	5.29	3.07	**2.42**	**2.11**	**1.92**	**1.72**	**1.60**	**1.52**	**1.46**	**1.42**	**1.40**	**1.37**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
12	5.64	3.25	**2.55**	**2.22**	**2.02**	**1.80**	**1.68**	**1.59**	**1.53**	**1.49**	**1.46**	**1.43**	**1.41**	**1.39**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	5.89	3.38	**2.64**	**2.30**	**2.09**	**1.86**	**1.73**	**1.64**	**1.57**	**1.53**	**1.50**	**1.47**	**1.45**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
20	6.07	3.47	**2.72**	**2.36**	**2.14**	**1.90**	**1.77**	**1.67**	**1.61**	**1.56**	**1.53**	**1.50**	**1.48**	**1.46**	**1.43**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
30	6.40	3.65	**2.84**	**2.46**	**2.24**	**1.99**	**1.85**	**1.74**	**1.67**	**1.62**	**1.59**	**1.56**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
40	6.63	3.77	*2.93*	**2.54**	**2.30**	**2.04**	**1.90**	**1.79**	**1.72**	**1.67**	**1.63**	**1.60**	**1.58**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.46**	**1.45**
50	6.81	3.86	*3.00*	**2.59**	**2.36**	**2.09**	**1.94**	**1.82**	**1.75**	**1.70**	**1.66**	**1.63**	**1.61**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
60	6.95	3.93	3.06	**2.64**	**2.40**	**2.12**	**1.97**	**1.85**	**1.78**	**1.72**	**1.69**	**1.66**	**1.63**	**1.60**	**1.57**	**1.55**	**1.54**	**1.53**	**1.51**	**1.49**
75	7.12	4.02	3.12	**2.70**	**2.45**	**2.16**	**2.01**	**1.89**	**1.81**	**1.76**	**1.72**	**1.69**	**1.66**	**1.62**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**
100	7.34	4.13	3.21	**2.77**	**2.51**	**2.22**	**2.06**	**1.93**	**1.85**	**1.80**	**1.75**	**1.72**	**1.70**	**1.66**	**1.63**	**1.61**	**1.60**	**1.59**	**1.57**	**1.55**
125	7.50	4.22	3.27	**2.82**	**2.56**	**2.26**	**2.09**	**1.97**	**1.89**	**1.83**	**1.79**	**1.75**	**1.73**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.59**	**1.58**
150	7.64	4.29	3.33	*2.87*	**2.60**	**2.29**	**2.13**	**2.00**	**1.91**	**1.85**	**1.81**	**1.78**	**1.75**	**1.71**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**	**1.60**
175	7.75	4.35	3.37	*2.90*	**2.63**	**2.32**	**2.15**	**2.02**	**1.93**	**1.87**	**1.83**	**1.80**	**1.77**	**1.73**	**1.70**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**
200	7.84	4.40	3.41	*2.94*	**2.66**	**2.35**	**2.17**	**2.04**	**1.95**	**1.89**	**1.85**	**1.81**	**1.79**	**1.75**	**1.72**	**1.69**	**1.68**	**1.67**	**1.64**	**1.63**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (20 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.04	**2.44**	**1.95**	**1.72**	**1.58**	**1.43**	**1.34**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
2	4.67	**2.76**	**2.19**	**1.91**	**1.75**	**1.57**	**1.47**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
3	5.04	**2.94**	**2.32**	**2.03**	**1.85**	**1.66**	**1.55**	**1.47**	**1.42**	**1.38**	**1.35**	**1.33**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
4	5.29	**3.07**	**2.42**	**2.11**	**1.92**	**1.72**	**1.60**	**1.52**	**1.46**	**1.42**	**1.40**	**1.37**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
5	5.48	3.17	**2.49**	**2.17**	**1.98**	**1.76**	**1.65**	**1.56**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.29**	**1.28**
8	5.89	3.38	**2.64**	**2.30**	**2.09**	**1.86**	**1.73**	**1.64**	**1.57**	**1.53**	**1.50**	**1.47**	**1.45**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
12	6.22	3.55	**2.77**	**2.40**	**2.19**	**1.94**	**1.81**	**1.70**	**1.64**	**1.59**	**1.56**	**1.53**	**1.51**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**
16	6.46	3.67	**2.86**	**2.48**	**2.25**	**2.00**	**1.86**	**1.75**	**1.68**	**1.63**	**1.60**	**1.57**	**1.55**	**1.52**	**1.49**	**1.48**	**1.46**	**1.45**	**1.44**	**1.42**
20	6.63	3.77	**2.93**	**2.54**	**2.30**	**2.04**	**1.90**	**1.79**	**1.72**	**1.67**	**1.63**	**1.60**	**1.58**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.46**	**1.45**
30	6.95	3.93	*3.06*	**2.64**	**2.40**	**2.12**	**1.97**	**1.85**	**1.78**	**1.72**	**1.69**	**1.66**	**1.63**	**1.60**	**1.57**	**1.55**	**1.54**	**1.53**	**1.51**	**1.49**
40	7.17	4.05	*3.14*	**2.71**	**2.46**	**2.18**	**2.02**	**1.90**	**1.82**	**1.76**	**1.72**	**1.69**	**1.67**	**1.63**	**1.61**	**1.59**	**1.57**	**1.56**	**1.54**	**1.53**
50	7.34	4.13	*3.21*	**2.77**	**2.51**	**2.22**	**2.06**	**1.93**	**1.85**	**1.80**	**1.75**	**1.72**	**1.70**	**1.66**	**1.63**	**1.61**	**1.60**	**1.59**	**1.57**	**1.55**
60	7.47	4.20	3.26	**2.81**	**2.55**	**2.25**	**2.09**	**1.96**	**1.88**	**1.82**	**1.78**	**1.75**	**1.72**	**1.68**	**1.66**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**
75	7.64	4.29	3.33	**2.87**	**2.60**	**2.29**	**2.13**	**2.00**	**1.91**	**1.85**	**1.81**	**1.78**	**1.75**	**1.71**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**	**1.60**
100	7.84	4.40	3.41	**2.94**	**2.66**	**2.35**	**2.17**	**2.04**	**1.95**	**1.89**	**1.85**	**1.81**	**1.79**	**1.75**	**1.72**	**1.69**	**1.68**	**1.67**	**1.64**	**1.63**
125	8.00	4.48	3.47	**2.99**	**2.71**	**2.39**	**2.21**	**2.07**	**1.99**	**1.92**	**1.88**	**1.84**	**1.81**	**1.77**	**1.74**	**1.72**	**1.70**	**1.69**	**1.67**	**1.65**
150	8.13	4.55	3.52	**3.03**	**2.75**	**2.42**	**2.24**	**2.10**	**2.01**	**1.95**	**1.90**	**1.87**	**1.84**	**1.79**	**1.76**	**1.74**	**1.72**	**1.71**	**1.68**	**1.67**
175	8.23	4.61	3.56	*3.07*	**2.78**	**2.45**	**2.27**	**2.13**	**2.03**	**1.97**	**1.92**	**1.88**	**1.86**	**1.81**	**1.78**	**1.76**	**1.74**	**1.73**	**1.70**	**1.68**
200	8.33	4.65	3.60	*3.10*	**2.81**	**2.47**	**2.29**	**2.15**	**2.05**	**1.99**	**1.94**	**1.90**	**1.87**	**1.83**	**1.80**	**1.77**	**1.76**	**1.74**	**1.72**	**1.70**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (40 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.60	**2.16**	**1.74**	**1.53**	**1.42**	**1.28**	**1.21**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.98**
2	4.35	**2.51**	**1.98**	**1.74**	**1.59**	**1.43**	**1.34**	**1.28**	**1.23**	**1.20**	**1.18**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
3	4.81	**2.72**	**2.13**	**1.86**	**1.70**	**1.52**	**1.42**	**1.35**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**
4	5.14	2.87	**2.24**	**1.94**	**1.77**	**1.58**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
5	5.40	2.99	**2.32**	**2.01**	**1.83**	**1.63**	**1.52**	**1.44**	**1.39**	**1.35**	**1.33**	**1.31**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
8	5.94	3.24	**2.49**	**2.15**	**1.95**	**1.73**	**1.61**	**1.52**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
12	6.39	3.45	**2.64**	**2.27**	**2.05**	**1.82**	**1.69**	**1.59**	**1.53**	**1.49**	**1.46**	**1.43**	**1.42**	**1.39**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	6.71	3.60	*2.74*	**2.35**	**2.13**	**1.88**	**1.74**	**1.64**	**1.58**	**1.53**	**1.50**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
20	6.96	3.71	*2.82*	**2.42**	**2.18**	**1.92**	**1.78**	**1.68**	**1.61**	**1.57**	**1.53**	**1.51**	**1.49**	**1.46**	**1.43**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
30	7.39	3.91	2.97	**2.53**	**2.29**	**2.01**	**1.86**	**1.75**	**1.68**	**1.63**	**1.59**	**1.56**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
40	7.70	4.06	3.07	**2.62**	**2.36**	**2.07**	**1.91**	**1.80**	**1.72**	**1.67**	**1.63**	**1.60**	**1.58**	**1.55**	**1.52**	**1.51**	**1.49**	**1.48**	**1.46**	**1.45**
50	7.93	4.16	3.15	*2.68*	**2.41**	**2.12**	**1.95**	**1.83**	**1.76**	**1.70**	**1.66**	**1.63**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
60	8.11	4.25	3.21	*2.73*	**2.46**	**2.15**	**1.99**	**1.86**	**1.78**	**1.73**	**1.69**	**1.66**	**1.63**	**1.60**	**1.57**	**1.55**	**1.54**	**1.53**	**1.51**	**1.50**
75	8.34	4.36	3.29	*2.79*	**2.51**	**2.20**	**2.03**	**1.90**	**1.82**	**1.76**	**1.72**	**1.69**	**1.66**	**1.63**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**
100	8.62	4.49	3.38	*2.87*	**2.58**	**2.26**	**2.08**	**1.95**	**1.86**	**1.80**	**1.76**	**1.73**	**1.70**	**1.66**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**
125	8.84	4.60	3.46	*2.93*	**2.63**	**2.30**	**2.12**	**1.98**	**1.90**	**1.83**	**1.79**	**1.76**	**1.73**	**1.69**	**1.66**	**1.64**	**1.63**	**1.61**	**1.59**	**1.58**
150	9.01	4.68	3.52	2.98	*2.68*	**2.34**	**2.15**	**2.01**	**1.92**	**1.86**	**1.82**	**1.78**	**1.75**	**1.71**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**	**1.60**
175	9.16	4.75	3.57	3.03	*2.71*	**2.37**	**2.18**	**2.04**	**1.95**	**1.88**	**1.84**	**1.80**	**1.77**	**1.73**	**1.70**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**
200	9.29	4.81	3.61	3.06	*2.75*	**2.39**	**2.20**	**2.06**	**1.97**	**1.90**	**1.86**	**1.82**	**1.79**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.64**	**1.62**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.35	**2.51**	**1.98**	**1.74**	**1.59**	**1.43**	**1.34**	**1.28**	**1.23**	**1.20**	**1.18**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
2	5.14	**2.87**	**2.24**	**1.94**	**1.77**	**1.58**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
3	5.61	3.09	**2.39**	**2.06**	**1.88**	**1.67**	**1.56**	**1.47**	**1.42**	**1.38**	**1.35**	**1.33**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
4	5.94	3.24	**2.49**	**2.15**	**1.95**	**1.73**	**1.61**	**1.52**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
5	6.19	3.35	**2.57**	**2.21**	**2.01**	**1.78**	**1.65**	**1.56**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.29**	**1.28**
8	6.71	3.60	**2.74**	**2.35**	**2.13**	**1.88**	**1.74**	**1.64**	**1.58**	**1.53**	**1.50**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
12	7.15	3.80	*2.89*	**2.47**	**2.23**	**1.96**	**1.82**	**1.71**	**1.64**	**1.59**	**1.56**	**1.53**	**1.51**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**
16	7.46	3.95	*2.99*	**2.55**	**2.30**	**2.02**	**1.87**	**1.76**	**1.69**	**1.64**	**1.60**	**1.57**	**1.55**	**1.52**	**1.49**	**1.48**	**1.46**	**1.45**	**1.44**	**1.42**
20	7.70	4.06	3.07	**2.62**	**2.36**	**2.07**	**1.91**	**1.80**	**1.72**	**1.67**	**1.63**	**1.60**	**1.58**	**1.55**	**1.52**	**1.51**	**1.49**	**1.48**	**1.46**	**1.45**
30	8.11	4.25	3.21	**2.73**	**2.46**	**2.15**	**1.99**	**1.86**	**1.78**	**1.73**	**1.69**	**1.66**	**1.63**	**1.60**	**1.57**	**1.55**	**1.54**	**1.53**	**1.51**	**1.50**
40	8.40	4.39	3.31	**2.81**	**2.53**	**2.21**	**2.04**	**1.91**	**1.83**	**1.77**	**1.73**	**1.70**	**1.67**	**1.63**	**1.61**	**1.59**	**1.57**	**1.56**	**1.54**	**1.53**
50	8.62	4.49	3.38	*2.87*	**2.58**	**2.26**	**2.08**	**1.95**	**1.86**	**1.80**	**1.76**	**1.73**	**1.70**	**1.66**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**
60	8.80	4.58	3.44	*2.92*	**2.62**	**2.29**	**2.11**	**1.98**	**1.89**	**1.83**	**1.78**	**1.75**	**1.72**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**
75	9.01	4.68	3.52	*2.98*	**2.68**	**2.34**	**2.15**	**2.01**	**1.92**	**1.86**	**1.82**	**1.78**	**1.75**	**1.71**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**	**1.60**
100	9.29	4.81	3.61	*3.06*	**2.75**	**2.39**	**2.20**	**2.06**	**1.97**	**1.90**	**1.86**	**1.82**	**1.79**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.64**	**1.62**
125	9.50	4.91	3.68	3.12	**2.80**	**2.44**	**2.24**	**2.09**	**2.00**	**1.93**	**1.89**	**1.85**	**1.82**	**1.77**	**1.74**	**1.72**	**1.70**	**1.69**	**1.67**	**1.65**
150	9.67	4.99	3.74	3.17	**2.84**	**2.47**	**2.27**	**2.12**	**2.03**	**1.96**	**1.91**	**1.87**	**1.84**	**1.80**	**1.77**	**1.74**	**1.72**	**1.71**	**1.69**	**1.67**
175	9.81	5.06	3.79	3.21	*2.87*	**2.50**	**2.30**	**2.15**	**2.05**	**1.98**	**1.93**	**1.89**	**1.86**	**1.82**	**1.78**	**1.76**	**1.74**	**1.73**	**1.70**	**1.69**
200	9.93	5.12	3.83	3.24	*2.90*	**2.53**	**2.32**	**2.17**	**2.07**	**2.00**	**1.95**	**1.91**	**1.88**	**1.83**	**1.80**	**1.77**	**1.76**	**1.74**	**1.72**	**1.70**

Table 19-9. κ-Multipliers for 1-of-2 Interwell Prediction Limits on Means of Order 3 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.14	**2.87**	**2.24**	**1.94**	**1.77**	**1.58**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
2	5.94	3.24	**2.49**	**2.15**	**1.95**	**1.73**	**1.61**	**1.52**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
3	6.39	3.45	**2.64**	**2.27**	**2.05**	**1.82**	**1.69**	**1.59**	**1.53**	**1.49**	**1.46**	**1.43**	**1.42**	**1.39**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
4	6.71	3.60	**2.74**	**2.35**	**2.13**	**1.88**	**1.74**	**1.64**	**1.58**	**1.53**	**1.50**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
5	6.96	3.71	**2.82**	**2.42**	**2.18**	**1.92**	**1.78**	**1.68**	**1.61**	**1.57**	**1.53**	**1.51**	**1.49**	**1.46**	**1.43**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
8	7.46	3.95	**2.99**	**2.55**	**2.30**	**2.02**	**1.87**	**1.76**	**1.69**	**1.64**	**1.60**	**1.57**	**1.55**	**1.52**	**1.49**	**1.48**	**1.46**	**1.45**	**1.44**	**1.42**
12	7.88	4.14	*3.13*	**2.67**	**2.40**	**2.11**	**1.95**	**1.83**	**1.75**	**1.70**	**1.66**	**1.63**	**1.60**	**1.57**	**1.55**	**1.53**	**1.51**	**1.50**	**1.48**	**1.47**
16	8.18	4.28	3.23	**2.75**	**2.47**	**2.17**	**2.00**	**1.87**	**1.79**	**1.74**	**1.70**	**1.67**	**1.64**	**1.61**	**1.58**	**1.56**	**1.55**	**1.54**	**1.52**	**1.50**
20	8.40	4.39	3.31	**2.81**	**2.53**	**2.21**	**2.04**	**1.91**	**1.83**	**1.77**	**1.73**	**1.70**	**1.67**	**1.63**	**1.61**	**1.59**	**1.57**	**1.56**	**1.54**	**1.53**
30	8.80	4.58	3.44	**2.92**	**2.62**	**2.29**	**2.11**	**1.98**	**1.89**	**1.83**	**1.78**	**1.75**	**1.72**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**
40	9.08	4.71	3.54	**3.00**	**2.69**	**2.35**	**2.16**	**2.02**	**1.93**	**1.87**	**1.82**	**1.79**	**1.76**	**1.72**	**1.69**	**1.67**	**1.65**	**1.64**	**1.62**	**1.60**
50	9.29	4.81	3.61	*3.06*	**2.75**	**2.39**	**2.20**	**2.06**	**1.97**	**1.90**	**1.86**	**1.82**	**1.79**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.64**	**1.62**
60	9.46	4.89	3.67	*3.11*	**2.79**	**2.43**	**2.23**	**2.09**	**1.99**	**1.93**	**1.88**	**1.84**	**1.81**	**1.77**	**1.74**	**1.72**	**1.70**	**1.69**	**1.66**	**1.65**
75	9.67	4.99	3.74	*3.17*	**2.84**	**2.47**	**2.27**	**2.12**	**2.03**	**1.96**	**1.91**	**1.87**	**1.84**	**1.80**	**1.77**	**1.74**	**1.72**	**1.71**	**1.69**	**1.67**
100	9.93	5.12	3.83	*3.24*	**2.90**	**2.53**	**2.32**	**2.17**	**2.07**	**2.00**	**1.95**	**1.91**	**1.88**	**1.83**	**1.80**	**1.77**	**1.76**	**1.74**	**1.72**	**1.70**
125	10.12	5.21	3.90	3.30	**2.95**	**2.57**	**2.36**	**2.20**	**2.10**	**2.03**	**1.98**	**1.94**	**1.90**	**1.86**	**1.82**	**1.80**	**1.78**	**1.77**	**1.74**	**1.72**
150	10.29	5.29	3.96	3.35	**3.00**	**2.61**	**2.39**	**2.23**	**2.13**	**2.05**	**2.00**	**1.96**	**1.93**	**1.88**	**1.85**	**1.82**	**1.80**	**1.79**	**1.76**	**1.74**
175	10.42	5.36	4.00	3.39	**3.03**	**2.63**	**2.42**	**2.25**	**2.15**	**2.07**	**2.02**	**1.98**	**1.95**	**1.90**	**1.86**	**1.84**	**1.82**	**1.80**	**1.77**	**1.76**
200	10.54	5.41	4.04	3.42	*3.06*	**2.66**	**2.44**	**2.27**	**2.17**	**2.09**	**2.04**	**2.00**	**1.96**	**1.91**	**1.88**	**1.85**	**1.83**	**1.82**	**1.79**	**1.77**

D STATISTICAL TABLES

D.3 TABLES FROM CHAPTER 19: INTRAWELL PREDICTION LIMITS FOR FUTURE OBSERVATIONS

TABLE 19-10 *κ*-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations.....D-118

TABLE 19-11 *κ*-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations.....D-127

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Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.78	0.67	0.61	0.58	0.57	0.54	0.53	0.52	0.51	0.51	0.50	0.50	0.50	0.50	0.49	0.49	0.49	0.49	0.49	0.49
2	1.27	1.05	0.97	0.92	0.89	0.85	0.83	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77	0.77	0.77	0.76	0.76
3	1.59	1.28	1.17	1.10	1.06	1.02	0.99	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92	0.91	0.91	0.91	0.91
4	1.82	1.45	1.31	1.23	1.19	1.13	1.10	1.08	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.00
5	2.02	1.58	1.41	1.33	1.28	1.22	1.19	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.09	1.09	1.08	1.08	1.08
8	2.47	1.86	1.65	1.54	1.47	1.40	1.36	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.24	1.24	1.23	1.23	1.22
12	2.90	2.11	1.85	1.72	1.64	1.55	1.50	1.46	1.43	1.42	1.40	1.39	1.39	1.38	1.37	1.36	1.36	1.35	1.35	1.34
16	3.24	2.30	1.99	1.84	1.75	1.65	1.60	1.55	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
20	3.52	2.45	2.11	1.94	1.84	1.73	1.67	1.62	1.59	1.57	1.56	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
30	4.09	2.73	2.32	2.12	2.01	1.88	1.80	1.75	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.59
40	4.54	2.94	*2.47*	2.25	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
50	4.91	3.11	2.59	2.35	2.21	2.06	1.97	1.91	1.87	1.84	1.82	1.80	1.79	1.77	1.76	1.75	1.74	1.74	1.73	1.72
60	5.24	3.26	2.70	*2.43*	2.29	2.12	2.03	1.96	1.92	1.89	1.87	1.85	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.76
75	5.67	3.44	2.82	*2.54*	2.38	2.20	2.10	2.03	1.98	1.95	1.93	1.91	1.90	1.88	1.86	1.85	1.84	1.84	1.83	1.82
100	6.26	3.68	2.99	2.67	*2.49*	2.30	2.19	2.11	2.07	2.03	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
125	6.76	3.88	3.12	2.78	*2.59*	2.37	2.26	2.18	2.13	2.09	2.07	2.05	2.03	2.01	1.99	1.98	1.97	1.96	1.95	1.94
150	7.20	4.05	3.23	2.87	2.66	*2.44*	2.32	2.23	2.18	2.14	2.11	2.09	2.07	2.05	2.03	2.02	2.01	2.00	1.99	1.98
175	7.59	4.19	3.33	2.94	2.72	*2.49*	2.37	2.28	2.22	2.18	2.15	2.13	2.11	2.09	2.07	2.06	2.05	2.04	2.02	2.01
200	7.95	4.32	3.41	3.01	2.78	*2.54*	*2.41*	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.21	1.03	0.95	0.90	0.88	0.84	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77	0.77	0.77	0.77	0.76	0.76	0.76
2	1.76	1.42	1.29	1.22	1.18	1.13	1.10	1.08	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.02	1.01	1.01	1.01	1.00
3	2.11	1.66	1.49	1.40	1.35	1.28	1.25	1.22	1.20	1.19	1.18	1.17	1.17	1.16	1.15	1.15	1.14	1.14	1.14	1.13
4	2.39	1.83	1.63	1.53	1.47	1.39	1.35	1.32	1.30	1.29	1.27	1.27	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
5	2.62	1.97	1.74	1.63	1.56	1.48	1.43	1.40	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
8	3.15	2.27	1.98	1.83	1.75	1.65	1.59	1.55	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
12	3.67	2.55	2.19	2.01	1.91	1.79	1.73	1.68	1.65	1.63	1.61	1.60	1.59	1.57	1.56	1.56	1.55	1.55	1.54	1.53
16	4.08	2.75	2.34	2.14	2.03	1.90	1.82	1.77	1.74	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.63	1.62	1.61	1.61
20	4.42	2.91	2.46	2.24	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
30	5.11	3.23	2.68	2.43	2.28	2.12	2.03	1.96	1.92	1.89	1.87	1.85	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.76
40	5.65	3.46	*2.85*	2.56	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
50	6.11	3.65	2.98	2.67	2.49	2.30	2.19	2.11	2.06	2.03	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
60	6.50	3.81	3.08	2.75	2.56	2.36	2.25	2.17	2.12	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.94	1.93
75	7.02	4.02	3.22	*2.86*	2.66	2.44	2.32	2.23	2.18	2.14	2.11	2.09	2.07	2.05	2.03	2.02	2.01	2.00	1.99	1.98
100	7.75	4.29	3.40	3.00	*2.78*	2.54	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
125	8.36	4.51	3.54	3.11	*2.87*	2.62	2.48	2.38	2.32	2.27	2.24	2.22	2.20	2.17	2.15	2.14	2.13	2.12	2.10	2.09
150	8.91	4.70	3.66	3.20	*2.95*	2.68	2.54	2.43	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
175	9.38	4.86	3.76	3.28	3.02	2.73	2.59	2.48	2.41	2.36	2.33	2.30	2.28	2.25	2.23	2.22	2.20	2.19	2.18	2.17
200	9.81	5.01	3.85	3.35	3.07	*2.78*	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.65	1.37	1.26	1.20	1.16	1.12	1.09	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.00	1.00
2	2.27	1.78	1.61	1.51	1.45	1.39	1.35	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
3	2.68	2.04	1.81	1.69	1.62	1.54	1.49	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.34	1.34
4	3.00	2.22	1.96	1.82	1.74	1.64	1.59	1.55	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
5	3.27	2.37	2.07	1.92	1.83	1.72	1.66	1.62	1.59	1.57	1.56	1.54	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
8	3.90	2.70	2.32	2.13	2.02	1.89	1.82	1.77	1.73	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.63	1.62	1.61	1.61
12	4.52	3.00	2.54	2.31	2.18	2.03	1.95	1.89	1.85	1.83	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.73	1.72	1.71
16	5.01	3.23	2.70	2.45	2.30	2.14	2.05	1.98	1.94	1.91	1.89	1.87	1.86	1.84	1.82	1.81	1.80	1.80	1.79	1.78
20	5.42	3.41	2.83	2.55	2.39	2.21	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
30	6.24	3.76	3.06	2.74	2.56	2.36	2.25	2.17	2.11	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.94	1.93
40	6.89	4.02	3.24	2.88	2.68	2.46	2.34	2.25	2.19	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
50	7.45	4.23	3.38	2.99	2.77	2.53	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
60	7.92	4.41	3.49	3.08	2.85	2.60	2.47	2.37	2.31	2.26	2.23	2.21	2.19	2.16	2.14	2.13	2.12	2.11	2.09	2.08
75	8.55	4.64	3.64	*3.19*	2.94	2.68	2.54	2.43	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
100	9.43	4.95	3.83	3.34	3.07	2.78	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19
125	10.17	5.20	3.98	3.46	*3.17*	2.86	2.70	2.58	2.50	2.45	2.42	2.39	2.37	2.33	2.31	2.29	2.28	2.27	2.25	2.24
150	10.82	5.41	4.11	3.55	*3.25*	2.92	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.37	2.35	2.33	2.32	2.31	2.29	2.28
175	11.40	5.59	4.22	3.64	*3.32*	2.98	2.80	2.67	2.59	2.54	2.50	2.47	2.44	2.41	2.39	2.37	2.35	2.34	2.32	2.31
200	11.92	5.76	4.32	3.71	3.37	3.02	2.84	2.71	2.63	2.57	2.53	2.50	2.48	2.44	2.42	2.40	2.38	2.37	2.35	2.34

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (2 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.27	1.05	0.97	0.92	0.89	0.85	0.83	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77	0.77	0.77	0.76	0.76
2	1.82	1.45	1.31	1.23	1.19	1.13	1.10	1.08	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.00
3	2.19	1.68	1.50	1.41	1.35	1.29	1.25	1.22	1.20	1.19	1.18	1.17	1.17	1.16	1.15	1.15	1.15	1.14	1.14	1.13
4	2.47	1.86	1.65	1.54	1.47	1.40	1.36	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.24	1.24	1.23	1.23	1.22
5	2.70	1.99	1.76	1.64	1.56	1.48	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
8	3.24	2.30	1.99	1.84	1.75	1.65	1.60	1.55	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
12	3.77	2.57	2.20	2.02	1.92	1.80	1.73	1.68	1.65	1.63	1.61	1.60	1.59	1.57	1.56	1.56	1.55	1.55	1.54	1.53
16	4.19	2.78	2.35	2.15	2.03	1.90	1.83	1.77	1.74	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.63	1.62	1.61	1.61
20	4.54	2.94	*2.47*	2.25	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
30	5.24	3.26	2.70	*2.43*	2.29	2.12	2.03	1.96	1.92	1.89	1.87	1.85	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.76
40	5.80	3.49	2.86	*2.57*	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
50	6.26	3.68	2.99	2.67	*2.49*	2.30	2.19	2.11	2.07	2.03	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
60	6.67	3.84	3.10	2.76	*2.57*	2.36	2.25	2.17	2.12	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.93	1.92
75	7.20	4.05	3.23	2.87	2.66	*2.44*	2.32	2.23	2.18	2.14	2.11	2.09	2.08	2.05	2.03	2.02	2.01	2.00	1.99	1.98
100	7.95	4.32	3.41	3.01	2.78	*2.54*	*2.41*	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
125	8.57	4.55	3.55	3.12	2.88	*2.62*	*2.48*	2.38	2.32	2.27	2.24	2.22	2.20	2.17	2.15	2.14	2.13	2.12	2.10	2.09
150	9.12	4.74	3.67	3.21	2.95	2.68	*2.54*	*2.43*	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
175	9.61	4.90	3.77	3.29	3.02	2.73	*2.59*	*2.48*	*2.41*	2.36	2.33	2.30	2.28	2.25	2.23	2.22	2.20	2.19	2.18	2.17
200	10.06	5.04	3.86	3.35	3.08	2.78	*2.63*	*2.51*	*2.44*	*2.40*	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (2 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.76	1.42	1.29	1.22	1.18	1.13	1.10	1.08	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.02	1.01	1.01	1.01	1.00
2	2.39	1.83	1.63	1.53	1.47	1.39	1.35	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
3	2.82	2.08	1.83	1.71	1.63	1.54	1.49	1.46	1.43	1.42	1.40	1.39	1.39	1.37	1.37	1.36	1.36	1.35	1.35	1.34
4	3.15	2.27	1.98	1.83	1.75	1.65	1.59	1.55	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
5	3.43	2.42	2.09	1.93	1.84	1.73	1.67	1.62	1.59	1.57	1.56	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
8	4.08	2.75	2.34	2.14	2.03	1.90	1.82	1.77	1.74	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.63	1.62	1.61	1.61
12	4.72	3.05	2.56	2.33	2.19	2.04	1.96	1.89	1.86	1.83	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.73	1.72	1.71
16	5.22	3.28	2.72	2.46	2.31	2.14	2.05	1.98	1.94	1.91	1.89	1.87	1.86	1.84	1.82	1.81	1.81	1.80	1.79	1.78
20	5.65	3.46	*2.85*	2.56	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
30	6.50	3.81	3.08	2.75	2.57	2.36	2.25	2.17	2.12	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.93	1.92
40	7.18	4.08	3.26	*2.89*	2.68	2.46	2.34	2.25	2.20	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
50	7.75	4.29	3.40	3.00	*2.78*	2.54	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
60	8.25	4.47	3.51	3.09	*2.86*	2.60	2.47	2.37	2.31	2.26	2.23	2.21	2.19	2.16	2.14	2.13	2.12	2.11	2.09	2.08
75	8.91	4.70	3.66	3.20	*2.95*	2.68	2.54	2.43	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
100	9.81	5.01	3.85	3.35	3.07	*2.78*	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19
125	10.59	5.26	4.00	3.47	3.17	*2.86*	2.70	2.58	2.50	2.45	2.42	2.39	2.37	2.33	2.31	2.29	2.28	2.27	2.25	2.24
150	11.25	5.47	4.13	3.56	3.25	*2.92*	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.38	2.35	2.33	2.32	2.31	2.29	2.28
175	11.86	5.66	4.24	3.65	3.32	*2.98*	*2.80*	2.67	2.59	2.54	2.50	2.47	2.45	2.41	2.39	2.37	2.35	2.34	2.32	2.31
200	12.40	5.82	4.34	3.72	3.38	3.03	*2.84*	2.71	2.63	2.57	2.53	2.50	2.48	2.44	2.41	2.40	2.38	2.37	2.35	2.34

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (2 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.27	1.78	1.61	1.51	1.45	1.39	1.35	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	1.23	1.22
2	3.00	2.22	1.96	1.82	1.74	1.64	1.59	1.55	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.43	1.42
3	3.51	2.50	2.17	2.00	1.90	1.79	1.73	1.68	1.65	1.63	1.61	1.60	1.59	1.57	1.56	1.56	1.55	1.55	1.54	1.53
4	3.90	2.70	2.32	2.13	2.02	1.89	1.82	1.77	1.73	1.71	1.69	1.68	1.67	1.65	1.64	1.63	1.63	1.62	1.61	1.61
5	4.23	2.87	2.44	2.23	2.11	1.97	1.89	1.84	1.80	1.77	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
8	5.01	3.23	2.70	2.45	2.30	2.14	2.05	1.98	1.94	1.91	1.89	1.87	1.86	1.84	1.82	1.81	1.80	1.80	1.79	1.78
12	5.78	3.57	2.93	2.64	2.47	2.28	2.18	2.10	2.05	2.02	1.99	1.98	1.96	1.94	1.92	1.91	1.90	1.90	1.89	1.88
16	6.38	3.82	3.10	2.77	2.59	2.38	2.27	2.19	2.13	2.10	2.07	2.05	2.03	2.01	1.99	1.98	1.97	1.97	1.95	1.94
20	6.89	4.02	3.24	2.88	2.68	2.46	2.34	2.25	2.19	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
30	7.92	4.41	3.49	3.08	2.85	2.60	2.47	2.37	2.31	2.26	2.23	2.21	2.19	2.16	2.14	2.13	2.12	2.11	2.09	2.08
40	8.74	4.71	3.68	*3.23*	2.97	2.70	2.56	2.45	2.38	2.34	2.30	2.28	2.26	2.23	2.21	2.19	2.18	2.17	2.16	2.15
50	9.43	4.95	3.83	3.34	3.07	2.78	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19
60	10.03	5.15	3.95	3.44	*3.15*	2.84	2.68	2.57	2.49	2.44	2.41	2.38	2.36	2.32	2.30	2.29	2.27	2.26	2.24	2.23
75	10.82	5.41	4.11	3.55	*3.25*	2.92	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.37	2.35	2.33	2.32	2.31	2.29	2.28
100	11.92	5.76	4.32	3.71	3.38	3.02	2.84	2.71	2.63	2.57	2.53	2.50	2.48	2.44	2.41	2.40	2.38	2.37	2.35	2.34
125	12.85	6.04	4.48	3.83	3.48	3.10	2.91	2.77	2.69	2.63	2.58	2.55	2.53	2.49	2.46	2.44	2.43	2.42	2.40	2.38
150	13.67	6.28	4.62	3.93	3.56	*3.17*	2.97	2.82	2.73	2.67	2.63	2.60	2.57	2.53	2.50	2.48	2.47	2.45	2.43	2.42
175	14.39	6.49	4.74	4.02	3.63	*3.23*	3.02	2.87	2.77	2.71	2.67	2.63	2.60	2.56	2.54	2.51	2.50	2.49	2.46	2.45
200	15.04	6.68	4.85	4.10	3.69	*3.27*	3.06	2.91	2.81	2.74	2.70	2.66	2.63	2.59	2.56	2.54	2.53	2.51	2.49	2.48

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.02	1.58	1.41	1.33	1.28	1.22	1.19	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.09	1.09	1.08	1.08	1.08
2	2.70	1.99	1.76	1.64	1.56	1.48	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
3	3.16	2.25	1.96	1.81	1.73	1.63	1.57	1.53	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.42	1.41	1.40
4	3.52	2.45	2.11	1.94	1.84	1.73	1.67	1.62	1.59	1.57	1.56	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
5	3.83	2.60	2.22	2.04	1.93	1.81	1.74	1.69	1.66	1.64	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.56	1.55	1.54
8	4.54	2.94	2.47	2.25	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
12	5.24	3.26	2.69	2.43	2.29	2.12	2.03	1.96	1.92	1.89	1.87	1.85	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.76
16	5.80	3.49	2.86	2.57	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
20	6.26	3.68	2.99	2.67	2.49	2.30	2.19	2.11	2.07	2.03	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
30	7.20	4.05	3.23	2.87	2.66	2.44	2.32	2.23	2.18	2.14	2.11	2.09	2.08	2.05	2.03	2.02	2.01	2.00	1.99	1.98
40	7.95	4.32	3.41	3.01	2.78	2.54	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
50	8.57	4.55	3.55	3.12	2.87	2.62	2.48	2.38	2.32	2.28	2.24	2.22	2.20	2.17	2.15	2.14	2.13	2.12	2.10	2.09
60	9.12	4.74	3.67	3.21	2.95	2.68	2.54	2.43	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
75	9.84	4.97	3.82	3.32	3.05	2.76	2.61	2.50	2.43	2.38	2.35	2.32	2.30	2.27	2.25	2.23	2.22	2.21	2.19	2.18
100	10.84	5.30	4.02	3.47	3.17	2.86	2.70	2.58	2.50	2.45	2.42	2.39	2.37	2.33	2.31	2.29	2.28	2.27	2.25	2.24
125	11.67	5.57	4.17	3.59	3.27	2.94	2.77	2.64	2.56	2.51	2.47	2.44	2.42	2.38	2.36	2.34	2.33	2.32	2.30	2.29
150	12.45	5.79	4.30	3.69	3.36	3.00	2.83	2.69	2.61	2.56	2.51	2.48	2.46	2.43	2.40	2.38	2.37	2.36	2.34	2.33
175	13.09	5.98	4.42	3.77	3.42	3.06	2.87	2.73	2.65	2.59	2.55	2.52	2.50	2.46	2.44	2.42	2.40	2.39	2.37	2.36
200	13.67	6.15	4.52	3.85	3.49	3.11	2.91	2.77	2.69	2.63	2.58	2.55	2.53	2.49	2.46	2.44	2.43	2.42	2.40	2.38

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.62	1.97	1.74	1.63	1.56	1.48	1.43	1.40	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
2	3.43	2.42	2.09	1.93	1.84	1.73	1.67	1.62	1.59	1.57	1.56	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
3	3.98	2.70	2.31	2.11	2.00	1.87	1.80	1.75	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.59
4	4.42	2.91	2.46	2.24	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
5	4.79	3.08	2.58	2.35	2.21	2.05	1.97	1.91	1.87	1.84	1.82	1.80	1.79	1.77	1.76	1.75	1.74	1.74	1.73	1.72
8	5.65	3.46	2.85	2.56	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
12	6.50	3.81	3.08	2.75	2.57	2.36	2.25	2.17	2.12	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.93	1.93
16	7.18	4.08	3.26	2.89	2.68	2.46	2.34	2.25	2.20	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
20	7.75	4.29	3.40	3.00	2.78	2.54	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
30	8.90	4.70	3.66	3.20	2.95	2.68	2.54	2.43	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
40	9.81	5.01	3.85	3.35	3.07	2.78	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.25	2.23	2.22	2.21	2.19
50	10.58	5.26	4.00	3.47	3.17	2.86	2.70	2.58	2.50	2.45	2.42	2.39	2.37	2.33	2.31	2.29	2.28	2.27	2.25	2.24
60	11.25	5.47	4.13	3.56	3.25	2.92	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.38	2.35	2.33	2.32	2.31	2.29	2.28
75	12.13	5.74	4.29	3.68	3.35	3.00	2.82	2.69	2.61	2.56	2.51	2.48	2.46	2.43	2.40	2.38	2.37	2.36	2.34	2.33
100	13.38	6.12	4.50	3.84	3.48	3.11	2.91	2.77	2.69	2.63	2.58	2.55	2.53	2.49	2.46	2.44	2.43	2.42	2.40	2.38
125	14.40	6.41	4.68	3.97	3.58	3.19	2.98	2.84	2.75	2.68	2.64	2.60	2.58	2.54	2.51	2.49	2.47	2.46	2.44	2.43
150	15.33	6.67	4.82	4.06	3.67	3.25	3.04	2.89	2.79	2.73	2.68	2.65	2.62	2.58	2.55	2.53	2.51	2.50	2.48	2.46
175	16.11	6.88	4.94	4.16	3.74	3.31	3.09	2.93	2.83	2.76	2.72	2.68	2.66	2.61	2.58	2.56	2.55	2.53	2.50	2.48
200	16.89	7.08	5.05	4.24	3.80	3.36	3.13	2.97	2.87	2.80	2.75	2.71	2.69	2.64	2.61	2.59	2.56	2.54	2.50	2.48

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.27	2.37	2.07	1.92	1.83	1.72	1.66	1.62	1.59	1.57	1.56	1.54	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
2	4.23	2.87	2.44	2.23	2.11	1.97	1.89	1.84	1.80	1.77	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
3	4.89	3.18	2.66	2.42	2.27	2.11	2.03	1.96	1.92	1.89	1.87	1.85	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.76
4	5.42	3.41	2.83	2.55	2.39	2.21	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
5	5.86	3.60	2.96	2.66	2.48	2.29	2.19	2.11	2.06	2.03	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
8	6.89	4.02	3.24	2.88	2.68	2.46	2.34	2.25	2.19	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
12	7.93	4.41	3.49	3.08	2.85	2.60	2.47	2.37	2.31	2.26	2.23	2.21	2.19	2.16	2.14	2.13	2.12	2.11	2.09	2.08
16	8.74	4.71	3.68	3.23	2.97	2.70	2.56	2.45	2.38	2.34	2.30	2.28	2.26	2.23	2.21	2.19	2.18	2.17	2.16	2.15
20	9.43	4.95	3.83	3.34	3.07	2.78	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19
30	10.82	5.41	4.11	3.55	3.25	2.92	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.38	2.35	2.33	2.32	2.31	2.29	2.28
40	11.93	5.76	4.32	3.71	3.38	3.02	2.84	2.71	2.63	2.57	2.53	2.50	2.48	2.44	2.41	2.40	2.38	2.37	2.35	2.34
50	12.85	6.04	4.48	3.83	3.48	3.10	2.91	2.77	2.69	2.63	2.58	2.55	2.53	2.49	2.46	2.44	2.43	2.42	2.40	2.38
60	13.67	6.28	4.62	3.93	3.56	3.17	2.97	2.82	2.73	2.67	2.63	2.59	2.57	2.53	2.50	2.48	2.47	2.46	2.43	2.42
75	14.75	6.59	4.80	4.06	3.66	3.25	3.04	2.89	2.79	2.73	2.68	2.65	2.62	2.58	2.55	2.53	2.51	2.50	2.48	2.46
100	16.21	6.99	5.03	4.22	3.80	3.36	3.13	2.97	2.87	2.80	2.75	2.71	2.68	2.64	2.61	2.59	2.57	2.55	2.51	2.48
125	17.48	7.34	5.21	4.36	3.91	3.44	3.20	3.03	2.92	2.85	2.80	2.76	2.73	2.69	2.65	2.62	2.62	2.60	2.58	2.56
150	18.55	7.62	5.37	4.47	3.99	3.50	3.26	3.08	2.97	2.90	2.84	2.81	2.77	2.73	2.69	2.67	2.66	2.64	2.62	2.60
175	19.53	7.86	5.51	4.57	4.06	3.56	3.31	3.12	3.02	2.94	2.88	2.84	2.81	2.76	2.73	2.70	2.69	2.67	2.64	2.63
200	20.51	8.11	5.62	4.65	4.14	3.61	3.35	3.16	3.05	2.97	2.92	2.87	2.84	2.79	2.76	2.73	2.71	2.69	2.67	2.65

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.70	1.99	1.76	1.64	1.56	1.48	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.29	1.29
2	3.52	2.45	2.11	1.94	1.84	1.73	1.67	1.62	1.59	1.57	1.56	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
3	4.09	2.73	2.32	2.12	2.01	1.88	1.80	1.75	1.72	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.59
4	4.54	2.94	2.47	2.25	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
5	4.91	3.11	2.59	2.35	2.21	2.06	1.97	1.91	1.87	1.84	1.82	1.80	1.79	1.77	1.76	1.75	1.74	1.74	1.73	1.72
8	5.79	3.49	2.86	2.57	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.92	1.89	1.88	1.87	1.86	1.85	1.84	1.83
12	6.67	3.84	3.10	2.76	2.57	2.36	2.25	2.17	2.12	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.93	1.92
16	7.36	4.11	3.27	2.90	2.69	2.46	2.34	2.25	2.20	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
20	7.95	4.32	3.41	3.01	2.78	2.54	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
30	9.13	4.73	3.67	3.21	2.95	2.68	2.54	2.43	2.37	2.32	2.29	2.26	2.24	2.22	2.20	2.18	2.17	2.16	2.14	2.13
40	10.05	5.05	3.86	3.35	3.08	2.78	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19
50	10.84	5.30	4.01	3.47	3.18	2.86	2.70	2.58	2.50	2.45	2.42	2.39	2.37	2.33	2.31	2.29	2.28	2.27	2.25	2.24
60	11.54	5.51	4.15	3.57	3.26	2.93	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.37	2.35	2.33	2.32	2.31	2.29	2.28
75	12.42	5.79	4.31	3.68	3.35	3.00	2.82	2.69	2.61	2.56	2.52	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
100	13.71	6.15	4.51	3.85	3.49	3.11	2.92	2.77	2.69	2.63	2.59	2.55	2.53	2.49	2.46	2.44	2.43	2.42	2.40	2.38
125	14.77	6.45	4.69	3.97	3.59	3.19	2.98	2.83	2.75	2.68	2.64	2.60	2.58	2.54	2.51	2.49	2.48	2.46	2.44	2.43
150	15.70	6.71	4.83	4.07	3.67	3.25	3.04	2.89	2.79	2.73	2.68	2.64	2.62	2.58	2.55	2.53	2.51	2.50	2.48	2.46
175	16.52	6.91	4.95	4.16	3.74	3.31	3.09	2.93	2.83	2.77	2.72	2.68	2.65	2.61	2.59	2.56	2.54	2.52	2.51	2.49
200	17.34	7.12	5.07	4.25	3.81	3.35	3.13	2.97	2.87	2.80	2.75	2.71	2.68	2.64	2.61	2.59	2.57	2.54	2.53	2.52

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.43	2.42	2.09	1.93	1.84	1.73	1.67	1.62	1.59	1.57	1.56	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.49	1.48
2	4.42	2.91	2.46	2.24	2.12	1.98	1.90	1.84	1.80	1.78	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
3	5.11	3.23	2.68	2.43	2.28	2.12	2.03	1.96	1.92	1.89	1.87	1.85	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.76
4	5.65	3.46	2.85	2.56	2.40	2.22	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
5	6.11	3.65	2.98	2.67	2.49	2.30	2.19	2.11	2.06	2.03	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.91	1.89	1.89
8	7.18	4.08	3.26	2.89	2.68	2.46	2.34	2.25	2.20	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
12	8.25	4.47	3.51	3.09	2.86	2.60	2.47	2.37	2.31	2.26	2.23	2.21	2.19	2.16	2.14	2.13	2.12	2.11	2.09	2.08
16	9.10	4.77	3.70	3.24	2.98	2.70	2.56	2.45	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.19	2.18	2.17	2.16	2.15
20	9.81	5.01	3.85	3.35	3.07	2.78	2.63	2.51	2.44	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.23	2.22	2.21	2.19
30	11.25	5.47	4.13	3.56	3.25	2.92	2.75	2.63	2.55	2.50	2.46	2.43	2.41	2.37	2.35	2.33	2.32	2.31	2.29	2.28
40	12.39	5.82	4.34	3.72	3.38	3.03	2.84	2.71	2.63	2.57	2.53	2.50	2.48	2.44	2.42	2.40	2.38	2.37	2.35	2.34
50	13.36	6.11	4.50	3.84	3.48	3.11	2.91	2.77	2.69	2.63	2.59	2.55	2.53	2.49	2.46	2.44	2.43	2.42	2.40	2.38
60	14.24	6.36	4.64	3.94	3.56	3.17	2.97	2.82	2.74	2.67	2.63	2.59	2.57	2.53	2.50	2.48	2.47	2.46	2.43	2.42
75	15.35	6.67	4.82	4.06	3.67	3.25	3.04	2.89	2.79	2.73	2.68	2.65	2.62	2.58	2.55	2.53	2.51	2.50	2.48	2.46
100	16.88	7.08	5.05	4.23	3.80	3.36	3.13	2.97	2.87	2.80	2.75	2.71	2.68	2.64	2.61	2.59	2.56	2.54	2.50	2.52
125	18.16	7.41	5.24	4.37	3.91	3.44	3.20	3.03	2.93	2.85	2.80	2.76	2.73	2.68	2.65	2.62	2.60	2.60	2.58	2.56
150	19.34	7.71	5.39	4.48	4.00	3.51	3.26	3.08	2.97	2.90	2.85	2.81	2.78	2.72	2.69	2.67	2.66	2.64	2.61	2.60
175	20.39	7.97	5.54	4.57	4.07	3.57	3.31	3.13	3.02	2.94	2.89	2.84	2.81	2.76	2.72	2.70	2.68	2.67	2.64	2.63
200	21.33	8.20	5.65	4.66	4.13	3.62	3.35	3.16	3.05	2.97	2.92	2.87	2.84	2.79	2.75	2.73	2.71	2.70	2.67	2.65

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.23	2.87	2.44	2.23	2.11	1.97	1.89	1.84	1.80	1.77	1.76	1.74	1.73	1.71	1.70	1.69	1.69	1.68	1.67	1.66
2	5.42	3.41	2.83	2.55	2.39	2.21	2.12	2.05	2.00	1.97	1.95	1.93	1.91	1.89	1.88	1.87	1.86	1.85	1.84	1.83
3	6.24	3.76	3.06	2.74	2.56	2.36	2.25	2.17	2.11	2.08	2.05	2.03	2.02	2.00	1.98	1.97	1.96	1.95	1.94	1.93
4	6.89	4.02	3.24	2.88	2.68	2.46	2.34	2.25	2.19	2.16	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.02	2.00	1.99
5	7.45	4.23	3.38	2.99	2.77	2.53	2.41	2.32	2.26	2.22	2.19	2.16	2.15	2.12	2.10	2.09	2.08	2.07	2.05	2.04
8	8.74	4.71	3.68	3.23	2.97	2.70	2.56	2.45	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.19	2.18	2.17	2.16	2.15
12	10.03	5.15	3.95	3.44	3.15	2.84	2.68	2.57	2.49	2.44	2.41	2.38	2.36	2.32	2.30	2.29	2.27	2.26	2.24	2.23
16	11.06	5.49	4.16	3.59	3.28	2.95	2.77	2.65	2.57	2.52	2.48	2.45	2.42	2.39	2.37	2.35	2.33	2.32	2.30	2.29
20	11.92	5.76	4.32	3.71	3.37	3.02	2.84	2.71	2.63	2.57	2.53	2.50	2.48	2.44	2.42	2.40	2.38	2.37	2.35	2.34
30	13.67	6.28	4.62	3.93	3.56	3.17	2.97	2.82	2.73	2.67	2.63	2.59	2.57	2.53	2.50	2.48	2.47	2.46	2.43	2.42
40	15.06	6.68	4.85	4.10	3.69	3.27	3.06	2.90	2.81	2.74	2.70	2.66	2.63	2.59	2.56	2.54	2.53	2.51	2.49	2.48
50	16.23	7.00	5.03	4.23	3.80	3.35	3.13	2.97	2.87	2.80	2.75	2.71	2.68	2.64	2.61	2.59	2.57	2.55	2.51	2.48
60	17.23	7.27	5.18	4.34	3.89	3.42	3.19	3.02	2.92	2.84	2.79	2.75	2.72	2.68	2.64	2.61	2.61	2.57	2.57	2.55
75	18.57	7.62	5.37	4.47	3.99	3.50	3.26	3.08	2.97	2.90	2.85	2.81	2.77	2.72	2.70	2.67	2.65	2.64	2.61	2.60
100	20.51	8.09	5.62	4.64	4.13	3.61	3.35	3.16	3.05	2.97	2.92	2.87	2.84	2.79	2.75	2.73	2.71	2.70	2.67	2.65
125	22.03	8.47	5.83	4.79	4.25	3.70	3.42	3.23	3.11	3.03	2.97	2.93	2.89	2.83	2.80	2.78	2.75	2.74	2.71	2.69
150	23.44	8.79	5.99	4.91	4.34	3.76	3.48	3.28	3.16	3.07	3.01	2.96	2.93	2.87	2.84	2.81	2.79	2.78	2.75	2.72
175	24.61	9.08	6.15	5.01	4.41	3.82	3.53	3.33	3.19	3.11	3.05	3.00	2.96	2.91	2.87	2.84	2.82	2.81	2.77	2.75
200	25.78	9.32	6.27	5.10	4.48	3.88	3.57	3.36	3.23	3.14	3.08	3.02	2.99	2.93	2.89	2.87	2.84	2.82	2.78	2.75

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.52	**2.45**	**2.11**	**1.94**	**1.84**	**1.73**	**1.67**	**1.62**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.52**	**1.51**	**1.51**	**1.50**	**1.50**	**1.49**	**1.48**
2	4.54	2.94	*2.47*	**2.25**	**2.12**	**1.98**	**1.90**	**1.84**	**1.80**	**1.78**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
3	5.24	3.26	2.70	*2.43*	**2.28**	**2.12**	**2.03**	**1.96**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.82**	**1.81**	**1.80**	**1.79**	**1.78**	**1.77**	**1.76**
4	5.79	3.49	2.86	*2.57*	**2.40**	**2.22**	**2.12**	**2.05**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
5	6.26	3.68	2.99	2.67	*2.49*	**2.30**	**2.19**	**2.11**	**2.07**	**2.03**	**2.01**	**1.99**	**1.97**	**1.95**	**1.93**	**1.92**	**1.91**	**1.91**	**1.89**	**1.89**
8	7.36	4.11	3.27	2.90	2.69	*2.46*	**2.34**	**2.25**	**2.20**	**2.16**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
12	8.45	4.50	3.52	3.10	2.86	*2.60*	*2.47*	**2.37**	**2.31**	**2.26**	**2.23**	**2.21**	**2.19**	**2.16**	**2.14**	**2.13**	**2.12**	**2.11**	**2.09**	**2.08**
16	9.32	4.80	3.71	3.24	2.98	2.70	2.56	2.45	2.38	**2.34**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
20	10.05	5.04	3.86	3.36	3.08	2.78	2.63	2.51	2.44	*2.40*	**2.36**	**2.33**	**2.31**	**2.28**	**2.26**	**2.25**	**2.23**	**2.22**	**2.21**	**2.19**
30	11.53	5.51	4.14	3.57	3.25	2.92	2.75	*2.63*	*2.55*	*2.50*	*2.46*	*2.43*	*2.41*	**2.38**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**	**2.28**
40	12.71	5.86	4.35	3.72	3.38	3.03	2.84	2.71	*2.63*	*2.57*	*2.53*	*2.50*	*2.48*	*2.44*	*2.42*	*2.40*	*2.38*	**2.37**	**2.35**	**2.34**
50	13.69	6.15	4.52	3.85	3.48	3.11	2.91	2.77	2.69	*2.63*	*2.58*	*2.55*	*2.53*	*2.49*	*2.46*	*2.44*	*2.43*	*2.42*	*2.40*	*2.38*
60	14.57	6.39	4.66	3.95	3.57	3.17	2.97	2.82	2.73	2.67	*2.63*	*2.60*	*2.57*	*2.53*	*2.50*	*2.48*	*2.47*	*2.45*	*2.43*	*2.42*
75	15.70	6.70	4.83	4.07	3.67	3.25	3.04	2.89	2.79	2.73	2.68	*2.65*	*2.62*	*2.58*	*2.55*	*2.53*	*2.51*	*2.50*	*2.48*	*2.46*
100	17.27	7.12	5.06	4.24	3.80	3.36	3.13	2.97	2.87	2.80	2.75	2.71	*2.68*	*2.64*	*2.60*	*2.58*	*2.55*	*2.54*	*2.50*	*2.48*
125	18.59	7.46	5.24	4.37	3.91	3.44	3.20	3.03	2.93	2.86	2.80	2.76	2.73	*2.69*	*2.65*	*2.62*	*2.60*	*2.58*	*2.58*	*2.56*
150	19.77	7.75	5.40	4.48	4.00	3.51	3.26	3.08	2.97	2.90	2.85	2.81	2.77	2.72	*2.69*	*2.66*	*2.64*	*2.64*	*2.61*	*2.60*
175	20.86	8.01	5.54	4.58	4.07	3.56	3.31	3.13	3.02	2.94	2.88	2.84	2.81	2.76	2.72	2.70	*2.68*	*2.67*	*2.64*	*2.63*
200	21.80	8.24	5.65	4.66	4.14	3.61	3.35	3.16	3.05	2.97	2.92	2.87	2.84	2.79	2.76	2.73	2.71	*2.70*	*2.67*	*2.65*

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.42	2.91	**2.46**	**2.24**	**2.12**	**1.97**	**1.90**	**1.84**	**1.80**	**1.78**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
2	5.65	3.46	*2.85*	**2.56**	**2.40**	**2.22**	**2.12**	**2.05**	**2.00**	**1.97**	**1.95**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
3	6.50	3.81	3.08	**2.75**	**2.57**	**2.36**	**2.25**	**2.17**	**2.12**	**2.08**	**2.05**	**2.03**	**2.02**	**2.00**	**1.98**	**1.97**	**1.96**	**1.95**	**1.93**	**1.92**
4	7.18	4.08	3.26	*2.89*	**2.68**	**2.46**	**2.34**	**2.25**	**2.20**	**2.16**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
5	7.75	4.29	3.40	3.00	*2.78*	**2.54**	**2.41**	**2.32**	**2.26**	**2.22**	**2.19**	**2.16**	**2.15**	**2.12**	**2.10**	**2.09**	**2.08**	**2.07**	**2.05**	**2.04**
8	9.10	4.77	3.70	3.24	2.98	**2.70**	**2.56**	**2.45**	**2.38**	**2.34**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
12	10.44	5.21	3.97	3.44	3.15	*2.84*	**2.68**	**2.57**	**2.49**	**2.44**	**2.41**	**2.38**	**2.36**	**2.32**	**2.30**	**2.29**	**2.27**	**2.26**	**2.24**	**2.23**
16	11.50	5.55	4.18	3.60	3.28	*2.95*	*2.77*	**2.65**	**2.57**	**2.52**	**2.48**	**2.45**	**2.42**	**2.39**	**2.37**	**2.35**	**2.33**	**2.32**	**2.30**	**2.29**
20	12.40	5.83	4.34	3.72	3.38	3.03	*2.84*	**2.71**	**2.63**	**2.57**	**2.53**	**2.50**	**2.48**	**2.44**	**2.42**	**2.40**	**2.38**	**2.37**	**2.35**	**2.34**
30	14.22	6.35	4.64	3.94	3.56	3.17	*2.97*	*2.82*	**2.73**	**2.67**	**2.63**	**2.60**	**2.57**	**2.53**	**2.50**	**2.48**	**2.47**	**2.46**	**2.43**	**2.42**
40	15.66	6.75	4.87	4.11	3.70	3.28	3.06	*2.91*	*2.81*	**2.74**	**2.70**	**2.66**	**2.63**	**2.59**	**2.56**	**2.54**	**2.53**	**2.51**	**2.49**	**2.48**
50	16.88	7.08	5.05	4.23	3.80	3.36	3.13	*2.97*	*2.87*	*2.80*	*2.75*	**2.71**	**2.68**	**2.64**	**2.61**	**2.59**	**2.56**	**2.54**	**2.50**	**2.48**
60	17.93	7.35	5.20	4.34	3.89	3.43	3.19	*3.02*	*2.92*	*2.84*	*2.79*	*2.75*	**2.72**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.54**	**2.55**
75	19.34	7.71	5.39	4.48	4.00	3.51	3.26	3.08	*2.97*	*2.90*	*2.85*	*2.81*	*2.77*	**2.72**	**2.69**	**2.67**	**2.65**	**2.64**	**2.61**	**2.60**
100	21.25	8.18	5.64	4.65	4.14	3.61	3.35	3.16	*3.05*	*2.97*	*2.92*	*2.87*	*2.84*	*2.79*	*2.75*	**2.73**	**2.71**	**2.70**	**2.67**	**2.65**
125	22.97	8.55	5.85	4.79	4.25	3.70	3.42	3.23	3.11	*3.03*	*2.97*	*2.92*	*2.89*	*2.83*	*2.80*	*2.77*	*2.75*	**2.74**	**2.71**	**2.69**
150	24.38	8.91	6.02	4.91	4.34	3.77	3.48	3.28	3.15	*3.07*	*3.01*	*2.97*	*2.93*	*2.87*	*2.84*	*2.81*	*2.79*	*2.77*	**2.74**	**2.73**
175	25.62	9.18	6.17	5.01	4.42	3.83	3.53	3.33	3.20	*3.11*	*3.05*	*3.01*	*2.96*	*2.91*	*2.87*	*2.84*	*2.81*	*2.80*	*2.77*	**2.76**
200	26.88	9.45	6.29	5.10	4.49	3.88	3.57	3.36	3.23	3.14	*3.08*	*3.04*	*2.99*	*2.93*	*2.89*	*2.86*	*2.84*	*2.83*	*2.80*	*2.78*

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.42	3.41	**2.83**	**2.55**	**2.39**	**2.21**	**2.12**	**2.05**	**2.00**	**1.97**	**1.95**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
2	6.89	4.02	3.24	**2.88**	**2.68**	**2.46**	**2.34**	**2.25**	**2.19**	**2.16**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
3	7.92	4.41	3.49	**3.08**	**2.85**	**2.60**	**2.47**	**2.37**	**2.31**	**2.26**	**2.23**	**2.21**	**2.19**	**2.16**	**2.14**	**2.13**	**2.12**	**2.11**	**2.09**	**2.08**
4	8.74	4.71	3.68	*3.23*	**2.97**	**2.70**	**2.56**	**2.45**	**2.38**	**2.34**	**2.30**	**2.28**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.17**	**2.16**	**2.15**
5	9.43	4.95	3.83	3.34	**3.07**	**2.78**	**2.63**	**2.51**	**2.44**	**2.40**	**2.36**	**2.33**	**2.31**	**2.28**	**2.26**	**2.24**	**2.23**	**2.22**	**2.21**	**2.19**
8	11.06	5.49	4.16	3.59	*3.28*	**2.94**	**2.77**	**2.65**	**2.57**	**2.52**	**2.48**	**2.45**	**2.42**	**2.39**	**2.37**	**2.35**	**2.33**	**2.32**	**2.30**	**2.29**
12	12.68	5.99	4.45	3.81	3.46	**3.09**	**2.90**	**2.76**	**2.68**	**2.62**	**2.57**	**2.54**	**2.52**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.37**
16	13.96	6.37	4.67	3.97	3.59	*3.19*	**2.99**	**2.84**	**2.75**	**2.69**	**2.64**	**2.61**	**2.58**	**2.54**	**2.52**	**2.50**	**2.48**	**2.47**	**2.45**	**2.43**
20	15.06	6.68	4.85	4.10	3.69	*3.27*	**3.06**	**2.90**	**2.81**	**2.74**	**2.70**	**2.66**	**2.63**	**2.59**	**2.56**	**2.54**	**2.53**	**2.51**	**2.49**	**2.48**
30	17.25	7.27	5.18	4.33	3.89	3.42	*3.19*	**3.02**	**2.91**	**2.84**	**2.79**	**2.75**	**2.72**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.57**	**2.55**
40	18.98	7.72	5.42	4.51	4.02	3.53	*3.28*	**3.10**	**2.99**	**2.91**	**2.86**	**2.82**	**2.79**	**2.74**	**2.71**	**2.68**	**2.67**	**2.65**	**2.63**	**2.61**
50	20.47	8.10	5.62	4.64	4.13	3.61	*3.35*	*3.16*	**3.05**	**2.97**	**2.92**	**2.87**	**2.84**	**2.79**	**2.76**	**2.73**	**2.71**	**2.70**	**2.67**	**2.65**
60	21.76	8.40	5.79	4.76	4.22	3.68	*3.41*	*3.22*	**3.10**	**3.02**	**2.96**	**2.92**	**2.88**	**2.83**	**2.79**	**2.77**	**2.75**	**2.73**	**2.70**	**2.69**
75	23.44	8.81	6.00	4.90	4.34	3.76	3.48	*3.28*	*3.16*	**3.07**	**3.01**	**2.96**	**2.92**	**2.87**	**2.84**	**2.81**	**2.79**	**2.77**	**2.75**	**2.73**
100	25.78	9.34	6.27	5.09	4.48	3.88	3.57	*3.36*	*3.23*	*3.14*	**3.08**	**3.02**	**2.99**	**2.93**	**2.90**	**2.87**	**2.84**	**2.82**	**2.78**	**2.76**
125	27.81	9.77	6.48	5.24	4.60	3.96	3.65	*3.42*	*3.29*	*3.20*	*3.12*	**3.08**	**3.04**	**2.98**	**2.94**	**2.91**	**2.88**	**2.86**	**2.82**	**2.80**
150	29.53	10.16	6.68	5.37	4.70	4.03	3.71	*3.48*	*3.34*	*3.24*	*3.17*	**3.12**	**3.08**	**3.02**	**2.97**	**2.94**	**2.92**	**2.90**	**2.87**	**2.85**
175	31.09	10.47	6.84	5.47	4.79	4.09	3.75	3.52	*3.38*	*3.28*	*3.20*	*3.15*	**3.11**	**3.05**	**3.01**	**2.98**	**2.95**	**2.93**	**2.90**	**2.88**
200	32.50	10.78	6.97	5.57	4.85	4.15	3.80	3.55	*3.41*	*3.32*	*3.25*	*3.18*	*3.14*	**3.08**	**3.04**	**3.00**	**2.98**	**2.96**	**2.92**	**2.90**

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.54	2.94	*2.47*	**2.25**	**2.12**	**1.98**	**1.90**	**1.84**	**1.80**	**1.78**	**1.76**	**1.74**	**1.73**	**1.71**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.66**
2	5.79	3.49	2.86	*2.57*	**2.40**	**2.22**	**2.12**	**2.05**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
3	6.67	3.84	3.10	2.76	*2.57*	**2.36**	**2.25**	**2.17**	**2.12**	**2.08**	**2.05**	**2.03**	**2.02**	**2.00**	**1.98**	**1.97**	**1.96**	**1.95**	**1.93**	**1.92**
4	7.36	4.11	3.27	2.90	2.69	*2.46*	**2.34**	**2.25**	**2.20**	**2.16**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
5	7.95	4.32	3.41	3.01	2.78	*2.54*	*2.41*	**2.32**	**2.26**	**2.22**	**2.19**	**2.16**	**2.15**	**2.12**	**2.10**	**2.09**	**2.08**	**2.07**	**2.05**	**2.04**
8	9.32	4.80	3.71	3.24	2.98	2.70	*2.56*	*2.45*	*2.38*	**2.34**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
12	10.70	5.25	3.99	3.45	3.16	2.85	2.68	*2.57*	*2.49*	*2.44*	*2.41*	**2.38**	**2.36**	**2.32**	**2.30**	**2.29**	**2.27**	**2.26**	**2.24**	**2.23**
16	11.79	5.59	4.19	3.60	3.28	2.95	2.77	*2.65*	*2.57*	*2.52*	*2.48*	*2.45*	*2.42*	*2.39*	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**
20	12.71	5.86	4.35	3.72	3.38	3.03	2.84	2.71	*2.63*	*2.57*	*2.53*	*2.50*	*2.48*	*2.44*	*2.41*	*2.40*	*2.38*	**2.37**	**2.35**	**2.34**
30	14.56	6.39	4.66	3.95	3.57	3.17	2.97	2.82	2.73	2.67	*2.63*	*2.59*	*2.57*	*2.53*	*2.50*	*2.48*	*2.47*	*2.46*	*2.43*	*2.42*
40	16.04	6.80	4.88	4.11	3.70	3.28	3.06	2.91	2.81	2.74	2.70	*2.66*	*2.63*	*2.59*	*2.56*	*2.54*	*2.53*	*2.51*	*2.49*	*2.46*
50	17.29	7.12	5.06	4.24	3.81	3.36	3.13	2.97	2.87	2.80	2.75	2.71	*2.68*	*2.64*	*2.60*	*2.58*	*2.56*	*2.54*	*2.50*	*2.48*
60	18.37	7.40	5.21	4.35	3.89	3.42	3.19	3.02	2.92	2.85	2.79	2.75	2.72	*2.68*	*2.64*	*2.61*	*2.59*	*2.57*	*2.54*	*2.56*
75	19.80	7.76	5.41	4.48	4.00	3.51	3.26	3.08	2.97	2.90	2.85	2.81	2.77	2.73	*2.69*	*2.66*	*2.64*	*2.64*	*2.61*	*2.60*
100	21.80	8.23	5.66	4.66	4.14	3.61	3.35	3.16	3.05	2.97	2.92	2.87	2.84	2.79	2.76	2.73	2.71	*2.70*	*2.67*	*2.65*
125	23.47	8.61	5.86	4.80	4.25	3.70	3.42	3.23	3.11	3.03	2.97	2.93	2.89	2.84	2.81	2.77	2.76	2.74	2.71	*2.69*
150	24.96	8.94	6.03	4.92	4.35	3.77	3.48	3.28	3.16	3.07	3.01	2.97	2.93	2.88	2.85	2.80	2.79	2.77	2.75	2.72
175	26.28	9.23	6.17	5.02	4.42	3.83	3.53	3.32	3.20	3.11	3.05	3.00	2.97	2.92	2.89	2.84	2.82	2.80	2.77	2.76
200	27.42	9.49	6.31	5.11	4.49	3.88	3.58	3.36	3.23	3.14	3.08	3.04	3.00	2.95	2.92	2.86	2.84	2.82	2.79	2.78

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.65	3.46	*2.85*	**2.56**	**2.40**	**2.22**	**2.12**	**2.05**	**2.00**	**1.97**	**1.95**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.83**
2	7.18	4.08	3.26	*2.89*	**2.68**	**2.46**	**2.34**	**2.25**	**2.20**	**2.16**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
3	8.25	4.47	3.51	3.09	*2.86*	**2.60**	**2.47**	**2.37**	**2.31**	**2.26**	**2.23**	**2.21**	**2.19**	**2.16**	**2.14**	**2.13**	**2.12**	**2.11**	**2.09**	**2.08**
4	9.10	4.77	3.70	3.24	2.98	**2.70**	**2.56**	**2.45**	**2.38**	**2.34**	**2.31**	**2.28**	**2.26**	**2.23**	**2.21**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**
5	9.81	5.01	3.85	3.35	3.07	*2.78*	**2.63**	**2.51**	**2.44**	**2.40**	**2.36**	**2.33**	**2.31**	**2.28**	**2.26**	**2.25**	**2.23**	**2.22**	**2.21**	**2.19**
8	11.51	5.55	4.18	3.60	3.28	*2.95*	*2.77*	**2.65**	**2.57**	**2.52**	**2.48**	**2.45**	**2.42**	**2.39**	**2.37**	**2.35**	**2.33**	**2.32**	**2.30**	**2.29**
12	13.19	6.06	4.47	3.82	3.46	3.09	*2.90*	*2.76*	**2.68**	**2.62**	**2.57**	**2.54**	**2.52**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.37**
16	14.53	6.44	4.69	3.98	3.59	3.19	*2.99*	*2.84*	*2.75*	**2.69**	**2.64**	**2.61**	**2.58**	**2.54**	**2.52**	**2.50**	**2.48**	**2.47**	**2.45**	**2.43**
20	15.66	6.75	4.87	4.10	3.70	3.28	3.06	2.91	2.81	***2.74***	***2.70***	***2.66***	***2.63***	***2.59***	***2.56***	***2.54***	***2.53***	***2.51***	***2.49***	***2.48***
30	17.94	7.35	5.20	4.34	3.89	3.42	3.19	3.02	2.91	2.84	2.79	2.75	***2.72***	***2.68***	***2.64***	***2.61***	***2.59***	***2.60***	***2.57***	***2.55***
40	19.75	7.81	5.45	4.52	4.03	3.53	3.28	3.10	*2.99*	*2.92*	*2.86*	*2.82*	*2.79*	**2.74**	**2.70**	**2.68**	**2.67**	**2.65**	**2.63**	**2.61**
50	21.27	8.18	5.64	4.65	4.14	3.61	3.35	3.16	*3.05*	*2.97*	*2.92*	*2.87*	*2.84*	*2.79*	*2.75*	**2.73**	**2.71**	**2.70**	**2.67**	**2.65**
60	22.63	8.49	5.81	4.77	4.23	3.68	3.41	3.22	*3.10*	*3.02*	*2.96*	*2.92*	*2.88*	*2.83*	*2.79*	*2.77*	**2.75**	**2.73**	**2.70**	**2.69**
75	24.39	8.90	6.02	4.91	4.34	3.77	3.48	3.28	3.16	*3.07*	*3.01*	*2.97*	*2.93*	*2.87*	*2.84*	*2.81*	*2.79*	*2.77*	**2.75**	**2.73**
100	26.81	9.44	6.30	5.10	4.49	3.88	3.57	3.36	3.23	3.14	*3.08*	*3.04*	*2.99*	*2.93*	*2.89*	*2.86*	*2.84*	*2.82*	*2.78*	*2.75*
125	28.92	9.89	6.51	5.25	4.60	3.96	3.65	3.42	3.29	3.20	3.14	*3.09*	*3.04*	*2.98*	*2.94*	*2.91*	*2.88*	*2.86*	*2.82*	*2.80*
150	30.76	10.26	6.70	5.37	4.70	4.03	3.71	3.48	3.34	3.24	3.18	*3.13*	*3.10*	*3.02*	*2.97*	*2.94*	*2.92*	*2.90*	*2.86*	*2.83*
175	32.34	10.59	6.87	5.48	4.78	4.09	3.76	3.52	3.38	3.28	3.22	3.17	*3.13*	*3.05*	*3.01*	*2.97*	*2.95*	*2.93*	*2.89*	*2.87*
200	33.75	10.88	7.01	5.58	4.86	4.15	3.80	3.56	3.41	3.32	3.25	3.20	3.16	*3.08*	*3.03*	*3.00*	*2.98*	*2.96*	*2.92*	*2.90*

Table 19-10. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Observations (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.89	4.02	3.24	**2.88**	**2.68**	**2.46**	**2.34**	**2.25**	**2.19**	**2.16**	**2.13**	**2.11**	**2.09**	**2.07**	**2.05**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
2	8.74	4.71	3.68	*3.23*	**2.97**	**2.70**	**2.56**	**2.45**	**2.38**	**2.34**	**2.30**	**2.28**	**2.26**	**2.23**	**2.21**	**2.19**	**2.18**	**2.17**	**2.16**	**2.15**
3	10.03	5.15	3.95	3.44	*3.15*	**2.84**	**2.68**	**2.57**	**2.49**	**2.44**	**2.41**	**2.38**	**2.36**	**2.32**	**2.30**	**2.29**	**2.27**	**2.26**	**2.24**	**2.23**
4	11.06	5.49	4.16	3.59	*3.28*	**2.94**	**2.77**	**2.65**	**2.57**	**2.52**	**2.48**	**2.45**	**2.42**	**2.39**	**2.37**	**2.35**	**2.33**	**2.32**	**2.30**	**2.29**
5	11.92	5.76	4.32	3.71	3.38	**3.02**	**2.84**	**2.71**	**2.63**	**2.57**	**2.53**	**2.50**	**2.48**	**2.44**	**2.41**	**2.40**	**2.38**	**2.37**	**2.35**	**2.34**
8	13.97	6.37	4.67	3.97	3.59	*3.19*	**2.99**	**2.84**	**2.75**	**2.69**	**2.64**	**2.61**	**2.58**	**2.54**	**2.52**	**2.50**	**2.48**	**2.47**	**2.45**	**2.43**
12	16.00	6.94	5.00	4.20	3.78	*3.34*	*3.12*	**2.96**	**2.86**	**2.79**	**2.74**	**2.70**	**2.67**	**2.63**	**2.60**	**2.58**	**2.56**	**2.55**	**2.51**	**2.51**
16	17.62	7.37	5.23	4.37	3.92	3.45	*3.21*	**3.04**	**2.93**	**2.86**	**2.81**	**2.77**	**2.74**	**2.69**	**2.66**	**2.63**	**2.62**	**2.61**	**2.58**	**2.57**
20	19.00	7.72	5.42	4.51	4.02	3.53	*3.28*	**3.10**	**2.99**	**2.91**	**2.86**	**2.82**	**2.79**	**2.74**	**2.71**	**2.68**	**2.67**	**2.65**	**2.63**	**2.61**
30	21.75	8.40	5.79	4.76	4.22	3.68	*3.41*	*3.22*	**3.10**	**3.02**	**2.96**	**2.92**	**2.88**	**2.83**	**2.79**	**2.77**	**2.75**	**2.73**	**2.70**	**2.69**
40	23.95	8.92	6.05	4.94	4.37	3.79	3.50	*3.30*	*3.17*	**3.09**	**3.03**	**2.98**	**2.94**	**2.89**	**2.85**	**2.82**	**2.80**	**2.79**	**2.76**	**2.74**
50	25.80	9.34	6.27	5.09	4.49	3.88	3.57	*3.36*	*3.23*	*3.14*	**3.08**	**3.02**	**2.99**	**2.93**	**2.89**	**2.87**	**2.84**	**2.82**	**2.78**	**2.76**
60	27.42	9.70	6.45	5.21	4.58	3.95	3.63	*3.41*	*3.28*	*3.19*	*3.12*	**3.07**	**3.03**	**2.97**	**2.93**	**2.90**	**2.87**	**2.85**	**2.82**	**2.79**
75	29.53	10.15	6.67	5.37	4.70	4.03	3.71	*3.48*	*3.34*	*3.25*	*3.17*	*3.12*	**3.08**	**3.02**	**2.98**	**2.94**	**2.92**	**2.90**	**2.87**	**2.85**
100	32.52	10.77	6.98	5.56	4.85	4.14	3.80	3.56	*3.41*	*3.32*	*3.25*	*3.18*	*3.14*	**3.08**	**3.03**	**3.00**	**2.98**	**2.96**	**2.93**	**2.90**
125	35.07	11.27	7.22	5.72	4.98	4.24	3.87	3.62	*3.47*	*3.37*	*3.30*	*3.23*	*3.19*	*3.13*	**3.08**	**3.05**	**3.02**	**3.00**	**2.96**	**2.94**
150	37.27	11.69	7.43	5.86	5.08	4.31	3.93	3.67	3.52	*3.41*	*3.34*	*3.27*	*3.23*	*3.16*	*3.12*	**3.09**	**3.05**	**3.03**	**2.99**	**2.96**
175	39.20	12.08	7.60	5.98	5.16	4.37	3.98	3.72	3.56	*3.45*	*3.38*	*3.32*	*3.26*	*3.20*	*3.15*	*3.12*	**3.08**	**3.06**	**3.03**	**3.00**
200	40.96	12.39	7.76	6.08	5.24	4.42	4.03	3.76	3.59	*3.48*	*3.41*	*3.35*	*3.31*	*3.22*	*3.18*	*3.15*	**3.11**	**3.09**	**3.05**	**3.03**

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.33	0.25	0.21	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10
2	0.71	0.57	0.50	0.46	0.44	0.41	0.39	0.38	0.37	0.37	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34
3	0.95	0.75	0.67	0.62	0.59	0.55	0.53	0.52	0.51	0.50	0.49	0.49	0.48	0.48	0.47	0.47	0.47	0.47	0.46	0.46
4	1.13	0.88	0.78	0.73	0.69	0.65	0.63	0.61	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55
5	1.27	0.98	0.87	0.81	0.77	0.72	0.70	0.68	0.66	0.65	0.65	0.64	0.64	0.63	0.62	0.62	0.62	0.62	0.61	0.61
8	1.60	1.20	1.05	0.98	0.93	0.87	0.84	0.82	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74
12	1.91	1.39	1.21	1.12	1.06	1.00	0.96	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85	0.85	0.84	0.84
16	2.15	1.54	1.33	1.22	1.16	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91
20	2.35	1.65	1.42	1.30	1.23	1.15	1.10	1.07	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
30	2.75	1.86	1.58	1.44	1.36	1.27	1.21	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.05
40	3.06	2.02	1.70	1.55	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
50	3.32	2.15	1.80	1.63	1.53	1.41	1.35	1.30	1.27	1.25	1.24	1.23	1.22	1.20	1.19	1.19	1.18	1.17	1.17	1.16
60	3.55	*2.26*	1.87	1.69	1.59	1.47	1.40	1.35	1.32	1.30	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.21	1.20
75	3.84	2.39	1.97	1.77	1.66	1.53	1.46	1.40	1.37	1.35	1.33	1.32	1.31	1.29	1.28	1.27	1.27	1.26	1.25	1.25
100	4.26	2.57	2.10	1.88	1.75	1.61	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
125	4.60	2.72	*2.20*	1.96	1.82	1.67	1.59	1.53	1.49	1.46	1.44	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.35
150	4.90	2.84	*2.28*	2.03	1.88	1.72	1.64	1.57	1.53	1.50	1.48	1.47	1.45	1.44	1.42	1.41	1.41	1.40	1.39	1.38
175	5.17	2.95	2.35	2.09	1.93	1.77	1.68	1.61	1.57	1.54	1.52	1.50	1.49	1.47	1.45	1.44	1.44	1.43	1.42	1.41
200	5.42	3.04	2.42	2.14	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.67	0.54	0.49	0.45	0.43	0.41	0.39	0.38	0.37	0.36	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34
2	1.08	0.86	0.77	0.72	0.69	0.65	0.62	0.61	0.59	0.59	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
3	1.35	1.05	0.93	0.87	0.83	0.78	0.75	0.73	0.71	0.70	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.67	0.66	0.66
4	1.55	1.18	1.04	0.97	0.92	0.87	0.84	0.81	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74
5	1.71	1.29	1.13	1.05	1.00	0.94	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
8	2.09	1.52	1.32	1.22	1.15	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91
12	2.46	1.73	1.48	1.36	1.29	1.20	1.15	1.12	1.09	1.07	1.06	1.05	1.05	1.03	1.03	1.02	1.02	1.01	1.01	1.00
16	2.75	1.88	1.60	1.46	1.38	1.28	1.23	1.19	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
20	2.99	2.00	1.69	1.54	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
30	3.47	2.24	1.87	1.69	1.58	1.46	1.40	1.35	1.32	1.30	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.21	1.20
40	3.84	2.41	1.99	1.79	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
50	4.16	*2.55*	2.09	1.87	1.75	1.61	1.53	1.47	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
60	4.44	2.67	2.17	1.94	1.81	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
75	4.80	2.82	2.28	2.03	1.88	1.72	1.64	1.57	1.53	1.50	1.48	1.47	1.45	1.44	1.42	1.41	1.41	1.40	1.39	1.38
100	5.30	3.02	2.41	2.13	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
125	5.73	3.19	*2.52*	2.22	2.05	1.87	1.77	1.69	1.65	1.61	1.59	1.57	1.56	1.54	1.52	1.51	1.50	1.50	1.49	1.48
150	6.10	3.32	*2.61*	2.29	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
175	6.43	3.44	2.69	2.35	2.16	1.96	1.85	1.77	1.72	1.69	1.66	1.64	1.63	1.60	1.59	1.58	1.57	1.56	1.55	1.54
200	6.73	3.55	2.75	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.01	0.82	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
2	1.47	1.15	1.02	0.96	0.91	0.86	0.83	0.81	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75	0.74	0.74	0.74	0.73
3	1.77	1.34	1.19	1.10	1.05	0.99	0.95	0.93	0.91	0.89	0.89	0.88	0.87	0.86	0.86	0.85	0.85	0.85	0.84	0.84
4	2.00	1.49	1.30	1.21	1.15	1.08	1.04	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91
5	2.19	1.60	1.39	1.29	1.22	1.14	1.10	1.06	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
8	2.64	1.85	1.59	1.45	1.37	1.28	1.23	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
12	3.07	2.08	1.76	1.60	1.50	1.40	1.34	1.29	1.26	1.24	1.23	1.22	1.21	1.19	1.18	1.18	1.17	1.17	1.16	1.15
16	3.42	2.24	1.88	1.70	1.60	1.48	1.41	1.36	1.33	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.23	1.22	1.21
20	3.70	2.38	1.98	1.78	1.67	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
30	4.28	2.64	2.16	1.93	1.80	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
40	4.74	*2.83*	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
50	5.12	2.99	2.40	2.13	1.97	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
60	5.45	3.12	2.49	2.20	2.03	1.85	1.76	1.68	1.64	1.61	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
75	5.89	3.29	2.60	2.28	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
100	6.50	3.51	2.74	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
125	7.02	3.70	*2.86*	2.49	2.28	2.06	1.94	1.85	1.80	1.76	1.73	1.71	1.70	1.67	1.66	1.64	1.63	1.63	1.61	1.60
150	7.46	3.85	2.95	2.56	2.34	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
175	7.87	3.98	3.04	2.63	2.40	2.15	2.03	1.93	1.87	1.83	1.80	1.78	1.76	1.74	1.72	1.71	1.69	1.69	1.67	1.66
200	8.23	4.10	3.11	2.68	2.44	2.19	2.06	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.69

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.71	0.57	0.50	0.46	0.44	0.41	0.39	0.38	0.37	0.37	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34
2	1.13	0.88	0.78	0.73	0.69	0.65	0.63	0.61	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55
3	1.39	1.07	0.94	0.87	0.83	0.78	0.75	0.73	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.67	0.66	0.66
4	1.60	1.20	1.05	0.98	0.93	0.87	0.84	0.82	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74
5	1.76	1.31	1.14	1.06	1.00	0.94	0.91	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
8	2.15	1.54	1.33	1.22	1.16	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91
12	2.52	1.74	1.49	1.37	1.29	1.20	1.15	1.12	1.09	1.08	1.06	1.05	1.05	1.03	1.03	1.02	1.02	1.01	1.01	1.00
16	2.81	1.90	1.61	1.47	1.38	1.29	1.23	1.19	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
20	3.06	2.02	1.70	1.55	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
30	3.55	*2.26*	1.87	1.69	1.59	1.47	1.40	1.35	1.32	1.30	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.21	1.20
40	3.93	2.43	2.00	1.80	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
50	4.26	2.57	2.10	1.88	1.75	1.61	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
60	4.54	2.69	*2.18*	1.95	1.81	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
75	4.90	2.84	*2.28*	2.03	1.88	1.72	1.64	1.57	1.53	1.50	1.48	1.47	1.45	1.44	1.42	1.41	1.41	1.40	1.39	1.38
100	5.42	3.04	2.42	2.14	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
125	5.85	3.20	2.53	*2.22*	2.05	1.87	1.77	1.69	1.65	1.61	1.59	1.57	1.56	1.54	1.52	1.51	1.50	1.50	1.49	1.48
150	6.23	3.34	2.61	*2.29*	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
175	6.56	3.46	2.69	*2.35*	2.17	1.96	1.85	1.77	1.72	1.69	1.66	1.64	1.63	1.60	1.59	1.58	1.57	1.56	1.55	1.54
200	6.87	3.57	2.76	2.41	*2.21*	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (2 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.08	0.86	0.77	0.72	0.69	0.65	0.62	0.61	0.59	0.59	0.58	0.57	0.57	0.56	0.56	0.56	0.55	0.55	0.55	0.55
2	1.55	1.18	1.04	0.97	0.92	0.87	0.84	0.81	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.74
3	1.85	1.38	1.20	1.11	1.06	0.99	0.96	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85	0.85	0.84	0.84
4	2.09	1.52	1.32	1.22	1.15	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91
5	2.29	1.63	1.41	1.30	1.23	1.15	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
8	2.75	1.88	1.60	1.46	1.38	1.28	1.23	1.19	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
12	3.19	2.11	1.77	1.61	1.51	1.40	1.34	1.29	1.26	1.24	1.23	1.22	1.21	1.19	1.18	1.18	1.17	1.17	1.16	1.15
16	3.55	2.28	1.89	1.71	1.60	1.48	1.42	1.37	1.33	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.23	1.22	1.21
20	3.84	2.41	1.99	1.79	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
30	4.44	2.67	2.17	1.94	1.81	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
40	4.91	2.87	2.31	2.05	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
50	5.30	3.02	2.41	2.13	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
60	5.65	3.15	*2.50*	2.20	2.04	1.85	1.76	1.68	1.64	1.61	1.58	1.57	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
75	6.10	3.32	*2.61*	2.29	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
100	6.73	3.55	2.75	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
125	7.27	3.73	2.87	*2.49*	2.29	2.06	1.94	1.85	1.80	1.76	1.73	1.71	1.70	1.67	1.66	1.64	1.63	1.63	1.61	1.60
150	7.73	3.89	2.97	*2.57*	2.35	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
175	8.14	4.02	3.05	*2.63*	2.40	2.16	2.03	1.93	1.87	1.83	1.80	1.78	1.76	1.74	1.72	1.70	1.69	1.69	1.67	1.66
200	8.53	4.15	3.12	*2.69*	2.45	2.19	2.06	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.68

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (2 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.47	1.15	1.02	0.96	0.91	0.86	0.83	0.81	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75	0.74	0.74	0.74	0.73
2	2.00	1.49	1.30	1.21	1.15	1.08	1.04	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91
3	2.36	1.70	1.47	1.35	1.28	1.20	1.15	1.11	1.09	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.00
4	2.64	1.85	1.59	1.45	1.37	1.28	1.23	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
5	2.87	1.97	1.68	1.53	1.45	1.34	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
8	3.42	2.24	1.88	1.70	1.60	1.48	1.41	1.36	1.33	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.23	1.22	1.21
12	3.95	2.49	2.06	1.85	1.73	1.59	1.52	1.46	1.43	1.40	1.38	1.37	1.36	1.34	1.33	1.32	1.32	1.31	1.30	1.29
16	4.38	2.68	2.19	1.96	1.82	1.68	1.59	1.53	1.49	1.47	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.37	1.36	1.35
20	4.74	*2.83*	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
30	5.45	3.12	2.49	2.20	2.03	1.85	1.76	1.68	1.64	1.61	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
40	6.02	3.34	2.63	2.31	2.13	1.93	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
50	6.50	3.51	2.74	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
60	6.92	3.66	*2.84*	2.47	2.27	2.05	1.93	1.85	1.79	1.75	1.73	1.71	1.69	1.67	1.65	1.64	1.63	1.62	1.60	1.60
75	7.46	3.85	2.95	2.56	2.34	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
100	8.23	4.10	3.11	2.68	2.44	2.19	2.06	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.69
125	8.88	4.31	3.23	*2.78*	2.52	2.26	2.12	2.01	1.95	1.91	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
150	9.43	4.48	3.34	*2.85*	2.59	2.31	2.16	2.06	1.99	1.94	1.91	1.89	1.87	1.84	1.82	1.80	1.79	1.78	1.76	1.75
175	9.95	4.64	3.43	*2.92*	2.64	2.35	2.20	2.09	2.02	1.98	1.94	1.92	1.90	1.86	1.84	1.83	1.82	1.81	1.79	1.78
200	10.40	4.78	3.51	*2.98*	2.69	2.39	2.23	2.12	2.05	2.00	1.97	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.27	0.98	0.87	0.81	0.77	0.72	0.70	0.68	0.66	0.65	0.65	0.64	0.64	0.63	0.62	0.62	0.62	0.62	0.61	0.61
2	1.76	1.31	1.14	1.06	1.00	0.94	0.91	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
3	2.09	1.50	1.30	1.20	1.14	1.06	1.02	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90	0.90	0.89
4	2.35	1.65	1.42	1.30	1.23	1.15	1.10	1.07	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
5	2.56	1.77	1.51	1.38	1.30	1.21	1.16	1.13	1.10	1.09	1.07	1.06	1.06	1.04	1.04	1.03	1.03	1.02	1.01	1.01
8	3.06	2.02	1.70	1.55	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
12	3.55	*2.26*	1.87	1.69	1.59	1.47	1.40	1.35	1.32	1.30	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.21	1.20
16	3.93	2.43	2.00	1.80	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
20	4.26	2.57	2.10	1.88	1.75	1.61	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
30	4.90	2.84	*2.28*	2.03	1.88	1.72	1.64	1.57	1.53	1.50	1.48	1.47	1.45	1.44	1.42	1.41	1.41	1.40	1.39	1.38
40	5.42	3.04	2.42	2.14	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
50	5.85	3.20	2.53	*2.22*	2.05	1.87	1.77	1.69	1.65	1.61	1.59	1.57	1.56	1.54	1.52	1.51	1.50	1.50	1.49	1.48
60	6.23	3.34	2.61	*2.29*	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
75	6.72	3.52	2.73	2.38	*2.19*	1.98	1.87	1.79	1.74	1.70	1.68	1.66	1.64	1.62	1.60	1.59	1.58	1.57	1.56	1.55
100	7.41	3.76	2.88	2.50	*2.29*	2.06	1.94	1.85	1.80	1.76	1.74	1.71	1.70	1.67	1.66	1.64	1.63	1.63	1.61	1.60
125	8.00	3.95	3.00	2.59	*2.36*	2.12	2.00	1.91	1.85	1.81	1.78	1.76	1.74	1.72	1.70	1.68	1.67	1.67	1.65	1.64
150	8.50	4.11	3.09	2.66	2.43	*2.18*	2.04	1.95	1.89	1.85	1.82	1.79	1.78	1.75	1.73	1.72	1.71	1.70	1.68	1.67
175	8.96	4.25	3.18	2.72	2.48	*2.22*	2.08	1.98	1.92	1.88	1.85	1.82	1.81	1.78	1.76	1.74	1.74	1.72	1.71	1.70
200	9.38	4.38	3.25	2.78	2.53	*2.26*	2.12	2.01	1.95	1.91	1.88	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.71	1.29	1.13	1.05	1.00	0.94	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
2	2.29	1.63	1.41	1.30	1.23	1.15	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
3	2.68	1.85	1.57	1.44	1.36	1.27	1.21	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.05
4	2.99	2.00	1.69	1.54	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
5	3.24	2.13	1.79	1.62	1.52	1.41	1.35	1.30	1.27	1.25	1.24	1.23	1.22	1.20	1.19	1.18	1.18	1.17	1.17	1.16
8	3.84	2.41	1.99	1.79	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
12	4.44	2.67	2.17	1.94	1.81	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
16	4.91	2.87	2.31	2.05	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
20	5.30	3.02	2.41	2.13	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
30	6.10	3.32	*2.61*	2.29	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
40	6.73	3.55	2.75	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
50	7.27	3.73	2.87	*2.49*	2.29	2.06	1.94	1.85	1.80	1.76	1.73	1.71	1.70	1.67	1.66	1.64	1.63	1.63	1.61	1.60
60	7.73	3.89	2.97	*2.57*	2.35	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
75	8.33	4.09	3.09	*2.66*	2.42	2.18	2.05	1.95	1.89	1.85	1.82	1.79	1.78	1.75	1.73	1.72	1.71	1.70	1.68	1.67
100	9.20	4.35	3.25	2.78	*2.53*	2.26	2.12	2.01	1.95	1.91	1.88	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
125	9.90	4.57	3.38	2.88	*2.61*	2.32	2.17	2.07	2.00	1.95	1.92	1.89	1.87	1.84	1.82	1.81	1.80	1.79	1.77	1.76
150	10.55	4.75	3.49	2.96	*2.67*	2.37	2.22	2.11	2.04	1.99	1.96	1.93	1.91	1.88	1.86	1.84	1.83	1.82	1.80	1.79
175	11.07	4.91	3.57	3.02	*2.72*	2.42	2.26	2.14	2.07	2.02	1.98	1.96	1.94	1.90	1.88	1.87	1.85	1.85	1.83	1.82
200	11.60	5.05	3.65	3.08	2.78	*2.45*	2.29	2.17	2.10	2.05	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.19	1.60	1.39	1.29	1.22	1.14	1.10	1.06	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
2	2.87	1.97	1.68	1.53	1.45	1.34	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
3	3.34	2.21	1.85	1.68	1.58	1.46	1.40	1.35	1.32	1.29	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.20	1.20
4	3.70	2.38	1.98	1.78	1.67	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
5	4.01	2.52	2.08	1.87	1.74	1.61	1.53	1.47	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
8	4.74	*2.83*	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
12	5.45	3.12	2.49	2.20	2.03	1.85	1.76	1.68	1.64	1.61	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
16	6.02	3.34	2.63	2.31	2.13	1.93	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
20	6.50	3.51	2.74	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
30	7.46	3.85	2.95	2.56	2.34	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
40	8.23	4.10	3.11	2.68	2.44	2.19	2.06	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.69
50	8.88	4.31	3.23	*2.78*	2.52	2.26	2.12	2.01	1.95	1.91	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
60	9.43	4.48	3.34	*2.85*	2.59	2.31	2.16	2.06	1.99	1.94	1.91	1.89	1.87	1.84	1.82	1.80	1.79	1.78	1.76	1.75
75	10.18	4.71	3.47	*2.95*	2.67	2.37	2.22	2.11	2.04	1.99	1.95	1.93	1.91	1.88	1.86	1.84	1.83	1.82	1.80	1.79
100	11.22	5.01	3.64	3.08	*2.77*	2.45	2.29	2.17	2.10	2.05	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
125	12.07	5.25	3.78	3.18	*2.86*	2.52	2.35	2.22	2.15	2.09	2.05	2.03	2.00	1.97	1.95	1.93	1.92	1.90	1.89	1.87
150	12.83	5.46	3.90	3.26	*2.92*	2.57	2.40	2.26	2.18	2.13	2.09	2.06	2.04	2.00	1.98	1.96	1.95	1.94	1.92	1.90
175	13.54	5.64	4.00	3.33	*2.98*	2.62	2.43	2.30	2.22	2.16	2.12	2.09	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
200	14.18	5.80	4.09	3.40	*3.03*	2.66	2.47	2.33	2.24	2.19	2.15	2.11	2.09	2.05	2.03	2.01	1.99	1.98	1.96	1.95

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.76	1.31	1.14	1.06	1.00	0.94	0.91	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79
2	2.35	1.65	1.42	1.30	1.23	1.15	1.10	1.07	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
3	2.75	1.86	1.58	1.44	1.36	1.27	1.21	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.05
4	3.06	2.02	1.70	1.55	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
5	3.32	2.15	1.80	1.63	1.53	1.41	1.35	1.30	1.27	1.25	1.24	1.23	1.22	1.20	1.19	1.19	1.18	1.17	1.17	1.16
8	3.93	2.43	2.00	1.80	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.29	1.28	1.27	1.26	1.26
12	4.53	2.69	*2.18*	1.95	1.81	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
16	5.02	2.88	*2.31*	2.05	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
20	5.42	3.04	2.42	2.14	1.98	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
30	6.23	3.34	2.62	*2.29*	2.11	1.92	1.81	1.74	1.69	1.65	1.63	1.61	1.60	1.57	1.56	1.55	1.54	1.53	1.52	1.51
40	6.87	3.57	2.76	2.41	*2.21*	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
50	7.42	3.75	2.87	2.50	*2.29*	2.06	1.94	1.86	1.80	1.76	1.73	1.71	1.70	1.67	1.66	1.64	1.63	1.63	1.61	1.60
60	7.89	3.91	2.97	2.57	*2.35*	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.64
75	8.50	4.11	3.09	2.66	2.43	*2.18*	2.04	1.95	1.89	1.85	1.82	1.79	1.78	1.75	1.73	1.72	1.71	1.70	1.68	1.67
100	9.38	4.37	3.25	2.78	2.53	*2.26*	2.12	2.01	1.95	1.91	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
125	10.11	4.59	3.38	2.88	2.61	*2.32*	*2.17*	2.07	2.00	1.95	1.92	1.89	1.87	1.84	1.82	1.81	1.80	1.79	1.77	1.76
150	10.74	4.79	3.49	2.95	2.67	*2.37*	*2.22*	2.11	2.04	1.99	1.95	1.93	1.91	1.88	1.86	1.84	1.83	1.82	1.80	1.79
175	11.33	4.93	3.59	3.03	2.73	2.42	*2.26*	*2.14*	2.07	2.02	1.98	1.96	1.93	1.90	1.88	1.87	1.86	1.84	1.83	1.82
200	11.82	5.08	3.66	3.09	2.78	2.45	*2.29*	*2.17*	2.10	2.04	2.01	1.98	1.96	1.93	1.90	1.89	1.88	1.87	1.85	1.84

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.29	1.63	1.41	1.30	1.23	1.15	1.10	1.07	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
2	2.99	2.00	1.69	1.54	1.45	1.35	1.29	1.25	1.22	1.20	1.19	1.17	1.17	1.15	1.14	1.14	1.13	1.13	1.12	1.11
3	3.47	2.24	1.87	1.69	1.58	1.46	1.40	1.35	1.32	1.30	1.28	1.27	1.26	1.24	1.23	1.22	1.22	1.21	1.21	1.20
4	3.84	2.41	1.99	1.79	1.68	1.55	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
5	4.16	*2.55*	2.09	1.87	1.75	1.61	1.53	1.47	1.44	1.41	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.32	1.31	1.30
8	4.91	2.87	2.31	2.05	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
12	5.65	3.16	*2.50*	2.20	2.04	1.85	1.76	1.68	1.64	1.61	1.58	1.57	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
16	6.24	3.37	*2.64*	2.32	2.13	1.94	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
20	6.73	3.55	2.75	2.40	2.21	2.00	1.89	1.80	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.59	1.59	1.57	1.56
30	7.73	3.89	2.97	*2.57*	2.35	2.11	1.99	1.90	1.84	1.80	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.64	1.63
40	8.52	4.14	3.12	*2.69*	2.45	2.19	2.06	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.70	1.68
50	9.18	4.35	3.25	2.78	*2.53*	2.26	2.12	2.01	1.95	1.91	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.73	1.72
60	9.77	4.53	3.35	2.86	*2.59*	2.31	2.16	2.06	1.99	1.94	1.91	1.89	1.87	1.84	1.82	1.80	1.79	1.78	1.76	1.75
75	10.55	4.75	3.48	2.95	*2.67*	2.37	2.22	2.11	2.04	1.99	1.95	1.93	1.91	1.88	1.86	1.84	1.83	1.82	1.80	1.79
100	11.62	5.05	3.66	3.08	2.78	*2.46*	2.29	2.17	2.10	2.05	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
125	12.50	5.30	3.80	3.19	2.86	*2.52*	2.35	2.22	2.15	2.09	2.05	2.03	2.00	1.97	1.95	1.93	1.92	1.90	1.89	1.87
150	13.28	5.52	3.91	3.27	2.93	*2.58*	2.39	2.26	2.19	2.13	2.09	2.06	2.04	2.00	1.98	1.96	1.95	1.93	1.92	1.90
175	13.96	5.69	4.00	3.34	2.98	*2.62*	2.44	2.30	2.22	2.16	2.12	2.09	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
200	14.65	5.86	4.10	3.41	3.04	*2.66*	*2.47*	2.33	2.25	2.19	2.14	2.11	2.09	2.05	2.03	2.01	2.00	1.98	1.96	1.95

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.87	1.97	1.68	1.53	1.45	1.34	1.29	1.25	1.22	1.20	1.18	1.17	1.16	1.15	1.14	1.14	1.13	1.13	1.12	1.11
2	3.70	2.38	1.98	1.78	1.67	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.32	1.30	1.29	1.28	1.28	1.27	1.26	1.26
3	4.28	2.64	2.16	1.93	1.80	1.66	1.58	1.52	1.48	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.36	1.36	1.35	1.34
4	4.74	*2.83*	2.29	2.04	1.90	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39
5	5.12	2.99	2.40	2.13	1.97	1.80	1.71	1.64	1.60	1.57	1.54	1.53	1.51	1.49	1.48	1.47	1.46	1.46	1.44	1.44
8	6.02	3.34	2.63	2.31	2.13	1.93	1.83	1.75	1.70	1.67	1.64	1.62	1.61	1.59	1.57	1.56	1.55	1.54	1.53	1.52
12	6.92	3.66	*2.84*	2.47	2.27	2.05	1.93	1.85	1.79	1.75	1.73	1.71	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60
16	7.63	3.90	2.99	2.59	2.37	2.13	2.00	1.91	1.85	1.81	1.78	1.76	1.75	1.72	1.70	1.69	1.68	1.67	1.66	1.65
20	8.23	4.10	3.11	2.68	2.44	2.19	2.06	1.96	1.90	1.86	1.83	1.81	1.79	1.76	1.74	1.73	1.72	1.71	1.69	1.68
30	9.44	4.49	3.34	*2.85*	2.59	2.31	2.16	2.06	1.99	1.94	1.91	1.89	1.87	1.84	1.82	1.80	1.79	1.78	1.76	1.75
40	10.40	4.77	3.51	*2.98*	2.69	2.39	2.23	2.12	2.05	2.00	1.97	1.94	1.92	1.89	1.87	1.85	1.84	1.83	1.81	1.80
50	11.21	5.00	3.64	3.08	*2.77*	2.46	2.29	2.17	2.10	2.05	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
60	11.91	5.21	3.75	3.16	*2.84*	2.51	2.34	2.21	2.14	2.08	2.05	2.02	1.99	1.96	1.94	1.92	1.91	1.90	1.88	1.87
75	12.84	5.46	3.89	3.26	*2.92*	2.57	2.39	2.26	2.18	2.13	2.09	2.06	2.04	2.00	1.98	1.96	1.95	1.93	1.92	1.90
100	14.16	5.80	4.09	3.40	*3.03*	2.66	2.47	2.33	2.24	2.19	2.15	2.11	2.09	2.05	2.03	2.01	1.99	1.98	1.96	1.95
125	15.23	6.08	4.24	3.50	3.12	2.72	2.52	2.38	2.29	2.23	2.19	2.15	2.13	2.09	2.06	2.04	2.03	2.02	2.00	1.98
150	16.21	6.32	4.37	3.59	3.19	*2.78*	2.57	2.42	2.33	2.26	2.22	2.19	2.16	2.12	2.09	2.07	2.06	2.05	2.02	2.01
175	17.09	6.52	4.47	3.67	3.25	*2.82*	2.61	2.45	2.36	2.29	2.25	2.22	2.19	2.15	2.12	2.10	2.08	2.07	2.05	2.03
200	17.77	6.69	4.57	3.74	3.31	*2.86*	2.64	2.48	2.39	2.33	2.28	2.24	2.21	2.17	2.14	2.12	2.11	2.09	2.07	2.05

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.35	**1.65**	**1.42**	**1.30**	**1.23**	**1.15**	**1.10**	**1.07**	**1.05**	**1.03**	**1.02**	**1.01**	**1.00**	**0.99**	**0.98**	**0.98**	**0.97**	**0.97**	**0.96**	**0.96**
2	3.06	**2.02**	**1.70**	**1.55**	**1.45**	**1.35**	**1.29**	**1.25**	**1.22**	**1.20**	**1.19**	**1.17**	**1.17**	**1.15**	**1.14**	**1.14**	**1.13**	**1.13**	**1.12**	**1.11**
3	3.55	*2.26*	**1.87**	**1.69**	**1.59**	**1.47**	**1.40**	**1.35**	**1.32**	**1.30**	**1.28**	**1.27**	**1.26**	**1.24**	**1.23**	**1.22**	**1.22**	**1.21**	**1.21**	**1.20**
4	3.93	2.43	**2.00**	**1.80**	**1.68**	**1.55**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.30**	**1.29**	**1.29**	**1.28**	**1.27**	**1.26**	**1.26**
5	4.25	2.57	**2.10**	**1.88**	**1.75**	**1.61**	**1.53**	**1.48**	**1.44**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.32**	**1.31**	**1.30**
8	5.02	2.88	*2.31*	**2.05**	**1.90**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
12	5.77	3.17	2.51	*2.21*	**2.04**	**1.86**	**1.76**	**1.68**	**1.64**	**1.61**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**	**1.48**	**1.47**
16	6.37	3.39	2.65	*2.32*	*2.14*	**1.94**	**1.83**	**1.75**	**1.70**	**1.67**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
20	6.87	3.57	2.76	2.41	*2.21*	**2.00**	**1.89**	**1.80**	**1.75**	**1.72**	**1.69**	**1.67**	**1.65**	**1.63**	**1.61**	**1.60**	**1.59**	**1.59**	**1.57**	**1.56**
30	7.89	3.91	2.97	2.57	*2.35*	**2.11**	**1.99**	**1.90**	**1.84**	**1.80**	**1.77**	**1.75**	**1.73**	**1.71**	**1.69**	**1.68**	**1.67**	**1.66**	**1.64**	**1.64**
40	8.69	4.17	3.13	2.69	2.45	*2.19*	**2.06**	**1.96**	**1.90**	**1.86**	**1.83**	**1.81**	**1.79**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.69**	**1.68**
50	9.38	4.38	3.25	2.78	2.53	*2.26*	**2.12**	**2.01**	**1.95**	**1.91**	**1.87**	**1.85**	**1.83**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.73**	**1.72**
60	9.97	4.55	3.36	2.86	2.59	*2.31*	*2.16*	**2.06**	**1.99**	**1.94**	**1.91**	**1.89**	**1.87**	**1.84**	**1.82**	**1.80**	**1.79**	**1.78**	**1.76**	**1.75**
75	10.74	4.78	3.49	2.96	2.67	*2.37*	*2.22*	**2.11**	**2.04**	**1.99**	**1.95**	**1.93**	**1.91**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.80**	**1.79**
100	11.84	5.08	3.66	3.09	2.78	2.46	*2.29*	*2.17*	**2.10**	**2.05**	**2.01**	**1.98**	**1.96**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.85**	**1.84**
125	12.74	5.32	3.80	3.19	2.86	2.52	*2.35*	*2.22*	*2.15*	**2.09**	**2.05**	**2.02**	**2.00**	**1.97**	**1.95**	**1.93**	**1.91**	**1.90**	**1.89**	**1.87**
150	13.57	5.54	3.92	3.27	2.93	2.58	*2.39*	*2.26*	*2.18*	*2.13*	**2.09**	**2.06**	**2.04**	**2.00**	**1.98**	**1.96**	**1.95**	**1.93**	**1.92**	**1.90**
175	14.26	5.71	4.02	3.34	2.98	2.62	2.43	*2.30*	*2.22*	*2.16*	*2.12*	**2.09**	**2.06**	**2.03**	**2.00**	**1.98**	**1.97**	**1.96**	**1.94**	**1.93**
200	14.94	5.88	4.11	3.41	3.04	2.66	2.47	*2.33*	*2.24*	*2.19*	*2.15*	*2.11*	**2.09**	**2.05**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.95**

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.99	**2.00**	**1.69**	**1.54**	**1.45**	**1.35**	**1.29**	**1.25**	**1.22**	**1.20**	**1.19**	**1.17**	**1.17**	**1.15**	**1.14**	**1.14**	**1.13**	**1.13**	**1.12**	**1.11**
2	3.84	**2.41**	**1.99**	**1.79**	**1.68**	**1.55**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.30**	**1.29**	**1.28**	**1.28**	**1.27**	**1.26**	**1.26**
3	4.44	2.67	**2.17**	**1.94**	**1.81**	**1.66**	**1.58**	**1.52**	**1.48**	**1.45**	**1.43**	**1.42**	**1.41**	**1.39**	**1.38**	**1.37**	**1.36**	**1.36**	**1.35**	**1.34**
4	4.91	2.87	**2.31**	**2.05**	**1.90**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
5	5.30	3.02	**2.41**	**2.13**	**1.98**	**1.80**	**1.71**	**1.64**	**1.60**	**1.57**	**1.54**	**1.53**	**1.51**	**1.49**	**1.48**	**1.47**	**1.46**	**1.46**	**1.44**	**1.44**
8	6.24	3.37	*2.64*	**2.32**	**2.13**	**1.94**	**1.83**	**1.75**	**1.70**	**1.67**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
12	7.16	3.70	2.85	*2.48*	**2.27**	**2.05**	**1.93**	**1.85**	**1.79**	**1.75**	**1.73**	**1.71**	**1.69**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**
16	7.90	3.94	3.00	*2.59*	**2.37**	**2.13**	**2.00**	**1.91**	**1.85**	**1.81**	**1.79**	**1.76**	**1.75**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
20	8.52	4.14	3.12	*2.69*	*2.45*	**2.19**	**2.06**	**1.96**	**1.90**	**1.86**	**1.83**	**1.81**	**1.79**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.69**	**1.68**
30	9.77	4.53	3.35	2.86	*2.59*	**2.31**	**2.16**	**2.06**	**1.99**	**1.94**	**1.91**	**1.89**	**1.87**	**1.84**	**1.82**	**1.80**	**1.79**	**1.78**	**1.76**	**1.75**
40	10.77	4.82	3.52	2.98	*2.69*	**2.39**	**2.24**	**2.12**	**2.05**	**2.00**	**1.97**	**1.94**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.80**
50	11.60	5.05	3.66	3.08	2.78	*2.46*	**2.29**	**2.17**	**2.10**	**2.05**	**2.01**	**1.98**	**1.96**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.85**	**1.84**
60	12.33	5.26	3.77	3.16	2.84	*2.51*	**2.34**	**2.21**	**2.14**	**2.08**	**2.05**	**2.02**	**2.00**	**1.96**	**1.94**	**1.92**	**1.91**	**1.90**	**1.88**	**1.87**
75	13.28	5.51	3.91	3.27	2.93	*2.57*	**2.39**	**2.26**	**2.18**	**2.13**	**2.09**	**2.06**	**2.04**	**2.00**	**1.98**	**1.96**	**1.95**	**1.93**	**1.92**	**1.90**
100	14.65	5.85	4.10	3.40	3.04	*2.66*	*2.47*	**2.33**	**2.24**	**2.19**	**2.14**	**2.11**	**2.09**	**2.05**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.95**
125	15.77	6.14	4.25	3.51	3.12	*2.73*	*2.52*	**2.38**	**2.29**	**2.23**	**2.19**	**2.15**	**2.13**	**2.09**	**2.06**	**2.04**	**2.03**	**2.02**	**1.99**	**1.98**
150	16.80	6.37	4.38	3.60	3.19	2.78	*2.57*	**2.42**	**2.33**	**2.27**	**2.22**	**2.19**	**2.16**	**2.12**	**2.09**	**2.07**	**2.06**	**2.05**	**2.03**	**2.01**
175	17.68	6.58	4.49	3.67	3.25	2.83	*2.61*	*2.46*	**2.36**	**2.30**	**2.25**	**2.22**	**2.19**	**2.15**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.03**
200	18.46	6.76	4.58	3.74	3.31	2.86	*2.64*	*2.48*	**2.39**	**2.32**	**2.28**	**2.24**	**2.21**	**2.17**	**2.14**	**2.12**	**2.10**	**2.09**	**2.07**	**2.05**

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.70	**2.38**	**1.98**	**1.78**	**1.67**	**1.54**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.30**	**1.29**	**1.28**	**1.28**	**1.27**	**1.26**	**1.26**
2	4.74	*2.83*	**2.29**	**2.04**	**1.90**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
3	5.45	3.12	**2.49**	**2.20**	**2.03**	**1.85**	**1.76**	**1.68**	**1.64**	**1.61**	**1.58**	**1.56**	**1.55**	**1.53**	**1.52**	**1.50**	**1.50**	**1.49**	**1.48**	**1.47**
4	6.02	3.34	**2.63**	**2.31**	**2.13**	**1.93**	**1.83**	**1.75**	**1.70**	**1.67**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
5	6.50	3.51	**2.74**	**2.40**	**2.21**	**2.00**	**1.89**	**1.80**	**1.75**	**1.72**	**1.69**	**1.67**	**1.65**	**1.63**	**1.61**	**1.60**	**1.59**	**1.59**	**1.57**	**1.56**
8	7.63	3.91	2.99	**2.59**	**2.37**	**2.13**	**2.00**	**1.91**	**1.85**	**1.81**	**1.78**	**1.76**	**1.75**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
12	8.76	4.27	3.21	**2.76**	**2.51**	**2.24**	**2.11**	**2.00**	**1.94**	**1.90**	**1.87**	**1.84**	**1.82**	**1.80**	**1.78**	**1.76**	**1.75**	**1.74**	**1.73**	**1.72**
16	9.65	4.55	3.38	*2.88*	**2.61**	**2.33**	**2.18**	**2.07**	**2.00**	**1.96**	**1.92**	**1.90**	**1.88**	**1.85**	**1.83**	**1.81**	**1.80**	**1.79**	**1.78**	**1.76**
20	10.40	4.77	3.51	*2.98*	**2.69**	**2.39**	**2.23**	**2.12**	**2.05**	**2.00**	**1.97**	**1.94**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.80**
30	11.93	5.21	3.76	3.16	*2.84*	**2.51**	**2.34**	**2.21**	**2.14**	**2.08**	**2.05**	**2.02**	**1.99**	**1.96**	**1.94**	**1.92**	**1.91**	**1.90**	**1.88**	**1.87**
40	13.13	5.54	3.94	3.29	*2.95*	**2.59**	**2.41**	**2.28**	**2.20**	**2.14**	**2.10**	**2.07**	**2.05**	**2.01**	**1.99**	**1.97**	**1.96**	**1.95**	**1.93**	**1.91**
50	14.16	5.80	4.09	3.40	*3.03*	**2.66**	**2.47**	**2.33**	**2.24**	**2.19**	**2.14**	**2.11**	**2.09**	**2.05**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.95**
60	15.04	6.03	4.21	3.49	3.10	**2.71**	**2.51**	**2.37**	**2.28**	**2.22**	**2.18**	**2.15**	**2.12**	**2.08**	**2.06**	**2.04**	**2.02**	**2.01**	**1.99**	**1.98**
75	16.21	6.32	4.36	3.59	3.19	*2.78*	**2.57**	**2.42**	**2.33**	**2.27**	**2.22**	**2.19**	**2.16**	**2.12**	**2.09**	**2.07**	**2.06**	**2.04**	**2.02**	**2.00**
100	17.87	6.70	4.57	3.74	3.30	*2.86*	**2.64**	**2.49**	**2.39**	**2.32**	**2.28**	**2.24**	**2.21**	**2.17**	**2.14**	**2.12**	**2.10**	**2.09**	**2.06**	**2.04**
125	19.24	7.03	4.74	3.85	3.39	*2.93*	**2.70**	**2.54**	**2.44**	**2.37**	**2.32**	**2.28**	**2.25**	**2.21**	**2.18**	**2.16**	**2.14**	**2.13**	**2.10**	**2.09**
150	20.41	7.30	4.87	3.94	3.47	*2.98*	**2.75**	**2.58**	**2.47**	**2.40**	**2.35**	**2.31**	**2.28**	**2.24**	**2.21**	**2.19**	**2.17**	**2.15**	**2.13**	**2.12**
175	21.48	7.53	4.99	4.03	3.53	*3.03*	*2.79*	**2.61**	**2.51**	**2.43**	**2.38**	**2.34**	**2.31**	**2.27**	**2.24**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
200	22.46	7.74	5.09	4.10	3.58	*3.08*	*2.82*	**2.64**	**2.53**	**2.46**	**2.40**	**2.37**	**2.33**	**2.29**	**2.26**	**2.23**	**2.22**	**2.20**	**2.18**	**2.16**

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.06	**2.02**	**1.70**	**1.55**	**1.45**	**1.35**	**1.29**	**1.25**	**1.22**	**1.20**	**1.19**	**1.17**	**1.17**	**1.15**	**1.14**	**1.14**	**1.13**	**1.13**	**1.12**	**1.11**
2	3.93	2.43	**2.00**	**1.80**	**1.68**	**1.55**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.30**	**1.29**	**1.29**	**1.28**	**1.27**	**1.26**	**1.26**
3	4.54	2.69	*2.18*	**1.95**	**1.81**	**1.66**	**1.58**	**1.52**	**1.48**	**1.45**	**1.43**	**1.42**	**1.41**	**1.39**	**1.38**	**1.37**	**1.36**	**1.36**	**1.35**	**1.34**
4	5.02	2.88	*2.31*	**2.05**	**1.90**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
5	5.42	3.04	2.42	**2.14**	**1.98**	**1.80**	**1.71**	**1.64**	**1.60**	**1.57**	**1.54**	**1.53**	**1.51**	**1.49**	**1.48**	**1.47**	**1.46**	**1.46**	**1.44**	**1.44**
8	6.37	3.39	2.65	*2.32*	*2.14*	**1.94**	**1.83**	**1.75**	**1.70**	**1.67**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
12	7.31	3.72	2.86	2.48	*2.27*	**2.05**	**1.93**	**1.85**	**1.79**	**1.75**	**1.73**	**1.71**	**1.69**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**
16	8.06	3.97	3.01	2.60	*2.37*	*2.13*	**2.01**	**1.91**	**1.85**	**1.81**	**1.79**	**1.76**	**1.75**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
20	8.69	4.17	3.13	2.69	2.45	*2.19*	**2.06**	**1.96**	**1.90**	**1.86**	**1.83**	**1.81**	**1.79**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.69**	**1.68**
30	9.97	4.55	3.36	2.86	2.59	*2.31*	*2.16*	**2.06**	**1.99**	**1.94**	**1.91**	**1.89**	**1.87**	**1.84**	**1.82**	**1.80**	**1.79**	**1.78**	**1.76**	**1.75**
40	10.98	4.84	3.53	2.99	2.70	*2.39*	*2.24*	*2.12*	**2.05**	**2.00**	**1.97**	**1.94**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.80**
50	11.84	5.08	3.66	3.09	2.78	2.46	*2.29*	*2.17*	**2.10**	**2.05**	**2.01**	**1.98**	**1.96**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.85**	**1.84**
60	12.58	5.28	3.77	3.17	2.84	2.51	*2.34*	*2.21*	*2.14*	**2.08**	**2.05**	**2.02**	**1.99**	**1.96**	**1.94**	**1.92**	**1.91**	**1.90**	**1.88**	**1.87**
75	13.55	5.54	3.92	3.27	2.93	2.57	*2.39*	*2.26*	*2.18*	*2.13*	**2.09**	**2.06**	**2.04**	**2.00**	**1.98**	**1.96**	**1.95**	**1.93**	**1.92**	**1.90**
100	14.92	5.88	4.11	3.41	3.04	2.66	2.47	*2.33*	*2.24*	*2.19*	*2.14*	*2.11*	**2.09**	**2.05**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.94**
125	16.09	6.17	4.26	3.52	3.12	2.72	2.52	*2.38*	*2.29*	*2.23*	*2.19*	*2.15*	*2.13*	**2.09**	**2.06**	**2.04**	**2.03**	**2.02**	**1.99**	**1.98**
150	17.11	6.41	4.38	3.60	3.19	2.78	2.57	*2.42*	*2.33*	*2.27*	*2.22*	*2.19*	*2.16*	*2.12*	**2.09**	**2.07**	**2.06**	**2.05**	**2.03**	**2.01**
175	17.97	6.60	4.49	3.67	3.25	2.82	2.61	2.46	*2.36*	*2.29*	*2.25*	*2.22*	*2.19*	*2.15*	*2.12*	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**
200	18.75	6.80	4.59	3.75	3.30	2.86	2.65	2.49	*2.39*	*2.32*	*2.28*	*2.24*	*2.21*	*2.17*	*2.14*	*2.12*	**2.10**	**2.09**	**2.09**	**2.06**

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.84	**2.41**	**1.99**	**1.79**	**1.68**	**1.55**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.30**	**1.29**	**1.28**	**1.28**	**1.27**	**1.26**	**1.26**
2	4.91	2.87	**2.31**	**2.05**	**1.90**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
3	5.65	3.16	*2.50*	**2.20**	**2.04**	**1.85**	**1.76**	**1.68**	**1.64**	**1.61**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**	**1.48**	**1.47**
4	6.24	3.37	*2.64*	**2.32**	**2.13**	**1.94**	**1.83**	**1.75**	**1.70**	**1.67**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
5	6.73	3.55	2.75	**2.40**	**2.21**	**2.00**	**1.89**	**1.80**	**1.75**	**1.72**	**1.69**	**1.67**	**1.65**	**1.63**	**1.61**	**1.60**	**1.59**	**1.59**	**1.57**	**1.56**
8	7.90	3.94	3.00	*2.59*	**2.37**	**2.13**	**2.00**	**1.91**	**1.85**	**1.81**	**1.79**	**1.76**	**1.75**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
12	9.06	4.31	3.22	2.76	*2.51*	**2.25**	**2.11**	**2.01**	**1.94**	**1.90**	**1.87**	**1.84**	**1.82**	**1.80**	**1.78**	**1.76**	**1.75**	**1.74**	**1.73**	**1.72**
16	9.98	4.59	3.39	2.89	*2.61*	**2.33**	**2.18**	**2.07**	**2.00**	**1.96**	**1.92**	**1.90**	**1.88**	**1.85**	**1.83**	**1.81**	**1.80**	**1.79**	**1.78**	**1.76**
20	10.76	4.82	3.52	2.98	*2.69*	**2.39**	**2.24**	**2.12**	**2.05**	**2.00**	**1.97**	**1.94**	**1.92**	**1.89**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.80**
30	12.33	5.25	3.77	3.17	2.84	*2.51*	**2.34**	**2.21**	**2.14**	**2.08**	**2.05**	**2.02**	**1.99**	**1.96**	**1.94**	**1.92**	**1.91**	**1.90**	**1.88**	**1.87**
40	13.59	5.59	3.95	3.30	2.95	*2.59*	**2.41**	**2.28**	**2.20**	**2.14**	**2.10**	**2.07**	**2.05**	**2.01**	**1.99**	**1.97**	**1.96**	**1.95**	**1.93**	**1.91**
50	14.65	5.85	4.10	3.40	3.03	*2.66*	*2.47*	**2.33**	**2.24**	**2.19**	**2.14**	**2.11**	**2.09**	**2.05**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.95**
60	15.55	6.08	4.22	3.49	3.11	*2.71*	*2.51*	**2.37**	**2.28**	**2.22**	**2.18**	**2.15**	**2.12**	**2.08**	**2.06**	**2.04**	**2.02**	**2.01**	**1.99**	**1.97**
75	16.80	6.37	4.38	3.60	3.19	2.78	*2.57*	**2.42**	**2.33**	**2.27**	**2.22**	**2.19**	**2.16**	**2.12**	**2.09**	**2.07**	**2.06**	**2.04**	**2.02**	**2.00**
100	18.44	6.76	4.58	3.74	3.31	2.87	*2.64*	*2.49*	**2.39**	**2.32**	**2.28**	**2.24**	**2.21**	**2.17**	**2.14**	**2.12**	**2.10**	**2.09**	**2.06**	**2.04**
125	19.84	7.09	4.75	3.86	3.40	2.93	*2.70*	*2.53*	**2.44**	**2.37**	**2.32**	**2.28**	**2.25**	**2.21**	**2.18**	**2.16**	**2.14**	**2.12**	**2.10**	**2.08**
150	21.09	7.34	4.88	3.95	3.47	2.99	*2.74*	*2.58*	*2.47*	**2.40**	**2.35**	**2.31**	**2.29**	**2.24**	**2.21**	**2.19**	**2.17**	**2.16**	**2.13**	**2.12**
175	22.19	7.58	5.00	4.03	3.54	3.04	*2.79*	*2.61*	*2.50*	**2.43**	**2.38**	**2.34**	**2.31**	**2.27**	**2.24**	**2.21**	**2.19**	**2.18**	**2.16**	**2.14**
200	23.28	7.81	5.12	4.10	3.59	3.08	2.82	*2.64*	*2.53*	*2.46*	**2.41**	**2.36**	**2.33**	**2.29**	**2.26**	**2.24**	**2.22**	**2.20**	**2.18**	**2.16**

Table 19-11. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Observations (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.74	*2.83*	**2.29**	**2.04**	**1.90**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
2	6.02	3.34	**2.63**	**2.31**	**2.13**	**1.93**	**1.83**	**1.75**	**1.70**	**1.67**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
3	6.92	3.66	*2.84*	**2.47**	**2.27**	**2.05**	**1.93**	**1.85**	**1.79**	**1.75**	**1.73**	**1.71**	**1.69**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**
4	7.63	3.91	2.99	**2.59**	**2.37**	**2.13**	**2.00**	**1.91**	**1.85**	**1.81**	**1.78**	**1.76**	**1.75**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
5	8.23	4.10	3.11	**2.68**	**2.44**	**2.19**	**2.06**	**1.96**	**1.90**	**1.86**	**1.83**	**1.81**	**1.79**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.69**	**1.68**
8	9.65	4.55	3.38	*2.88*	**2.61**	**2.33**	**2.18**	**2.07**	**2.00**	**1.96**	**1.92**	**1.90**	**1.88**	**1.85**	**1.83**	**1.81**	**1.80**	**1.79**	**1.78**	**1.76**
12	11.06	4.96	3.62	3.06	**2.76**	**2.44**	**2.28**	**2.16**	**2.09**	**2.04**	**2.00**	**1.98**	**1.95**	**1.92**	**1.90**	**1.88**	**1.87**	**1.86**	**1.84**	**1.83**
16	12.19	5.28	3.80	3.19	*2.86*	**2.53**	**2.35**	**2.23**	**2.15**	**2.10**	**2.06**	**2.03**	**2.01**	**1.97**	**1.95**	**1.93**	**1.92**	**1.91**	**1.89**	**1.88**
20	13.13	5.53	3.94	3.29	*2.95*	**2.59**	**2.41**	**2.28**	**2.20**	**2.14**	**2.10**	**2.07**	**2.05**	**2.01**	**1.99**	**1.97**	**1.96**	**1.95**	**1.93**	**1.91**
30	15.04	6.03	4.21	3.48	3.10	**2.71**	**2.51**	**2.37**	**2.28**	**2.22**	**2.18**	**2.15**	**2.12**	**2.08**	**2.06**	**2.04**	**2.02**	**2.01**	**1.99**	**1.98**
40	16.56	6.40	4.41	3.63	3.22	*2.80*	**2.59**	**2.44**	**2.34**	**2.28**	**2.23**	**2.20**	**2.17**	**2.13**	**2.10**	**2.08**	**2.07**	**2.05**	**2.03**	**2.01**
50	17.85	6.71	4.57	3.74	3.30	*2.86*	**2.64**	**2.49**	**2.39**	**2.32**	**2.28**	**2.24**	**2.21**	**2.17**	**2.14**	**2.12**	**2.10**	**2.09**	**2.06**	**2.04**
60	18.98	6.96	4.70	3.83	3.38	*2.92*	**2.69**	**2.53**	**2.43**	**2.36**	**2.31**	**2.27**	**2.24**	**2.20**	**2.17**	**2.15**	**2.13**	**2.12**	**2.10**	**2.08**
75	20.47	7.29	4.87	3.95	3.47	*2.99*	**2.75**	**2.58**	**2.47**	**2.40**	**2.35**	**2.31**	**2.28**	**2.24**	**2.21**	**2.19**	**2.17**	**2.16**	**2.13**	**2.12**
100	22.50	7.73	5.10	4.10	3.58	*3.08*	*2.82*	**2.64**	**2.53**	**2.46**	**2.40**	**2.37**	**2.33**	**2.29**	**2.26**	**2.23**	**2.22**	**2.20**	**2.18**	**2.16**
125	24.22	8.11	5.27	4.22	3.68	3.14	*2.88*	**2.69**	**2.58**	**2.50**	**2.45**	**2.41**	**2.37**	**2.33**	**2.29**	**2.27**	**2.25**	**2.24**	**2.21**	**2.19**
150	25.78	8.40	5.43	4.32	3.76	3.20	*2.92*	**2.73**	**2.62**	**2.54**	**2.48**	**2.44**	**2.40**	**2.36**	**2.32**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**
175	27.19	8.67	5.55	4.40	3.83	3.25	*2.97*	*2.77*	**2.65**	**2.57**	**2.51**	**2.47**	**2.43**	**2.38**	**2.35**	**2.32**	**2.31**	**2.29**	**2.26**	**2.24**
200	28.44	8.91	5.66	4.47	3.89	3.29	*3.00*	*2.80*	**2.68**	**2.59**	**2.53**	**2.49**	**2.46**	**2.41**	**2.37**	**2.35**	**2.33**	**2.31**	**2.28**	**2.26**

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.06	-0.01	-0.04	-0.07	-0.08	-0.10	-0.11	-0.12	-0.13	-0.13	-0.13	-0.14	-0.14	-0.14	-0.14	-0.15	-0.15	-0.15	-0.15	-0.15
2	0.39	0.28	0.22	0.19	0.17	0.14	0.12	0.11	0.10	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.07	0.07	0.07	0.07
3	0.59	0.44	0.37	0.33	0.30	0.27	0.25	0.23	0.22	0.22	0.21	0.21	0.20	0.20	0.20	0.19	0.19	0.19	0.19	0.18
4	0.74	0.55	0.47	0.42	0.39	0.36	0.33	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26
5	0.85	0.63	0.54	0.49	0.46	0.42	0.40	0.38	0.37	0.36	0.35	0.35	0.34	0.34	0.33	0.33	0.33	0.32	0.32	0.32
8	1.11	0.82	0.70	0.64	0.60	0.55	0.52	0.50	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
12	1.36	0.98	0.84	0.76	0.72	0.66	0.63	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.53	0.53	0.53	0.52
16	1.54	1.10	0.94	0.85	0.80	0.74	0.70	0.67	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59
20	1.70	1.19	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
30	2.01	1.37	1.15	1.04	0.98	0.90	0.86	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
40	2.25	1.49	1.25	1.13	1.06	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
50	2.45	1.60	1.33	1.20	1.12	1.03	0.98	0.94	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.82	0.81
60	2.62	1.68	1.39	1.25	1.17	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.85	0.85
75	2.85	1.79	1.47	1.32	1.23	1.13	1.07	1.02	1.00	0.98	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.90	0.89
100	3.16	1.94	1.58	1.41	1.31	1.20	1.13	1.09	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
125	3.42	*2.05*	1.66	1.48	1.37	1.25	1.18	1.13	1.10	1.08	1.06	1.05	1.04	1.02	1.01	1.01	1.00	1.00	0.99	0.98
150	3.65	*2.15*	1.73	1.54	1.42	1.29	1.22	1.17	1.14	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01
175	3.85	2.24	1.79	1.58	1.46	1.33	1.26	1.20	1.17	1.14	1.13	1.11	1.10	1.09	1.07	1.07	1.06	1.05	1.04	1.04
200	4.04	2.31	1.84	1.63	1.50	1.36	1.29	1.23	1.20	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.36	0.25	0.21	0.18	0.16	0.13	0.12	0.11	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07
2	0.70	0.53	0.46	0.41	0.39	0.35	0.33	0.31	0.30	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26
3	0.91	0.69	0.60	0.54	0.51	0.47	0.44	0.43	0.41	0.40	0.40	0.39	0.39	0.38	0.38	0.37	0.37	0.37	0.36	0.36
4	1.07	0.80	0.69	0.63	0.60	0.55	0.52	0.50	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
5	1.21	0.89	0.77	0.70	0.66	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.50	0.50	0.50	0.49	0.49	0.49	0.48
8	1.50	1.08	0.93	0.85	0.80	0.74	0.70	0.67	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59
12	1.79	1.26	1.07	0.97	0.91	0.84	0.80	0.77	0.75	0.73	0.72	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67
16	2.01	1.38	1.17	1.06	0.99	0.91	0.87	0.84	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73
20	2.20	1.48	1.24	1.13	1.05	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
30	2.56	1.67	1.39	1.25	1.17	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85
40	2.85	1.81	1.49	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
50	3.10	1.92	1.57	1.41	1.31	1.19	1.13	1.09	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
60	3.31	2.02	1.64	1.46	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
75	3.58	2.14	1.73	1.53	1.42	1.29	1.22	1.17	1.14	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01
100	3.96	*2.30*	1.84	1.62	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
125	4.28	*2.43*	1.93	1.70	1.56	1.42	1.34	1.28	1.24	1.21	1.19	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.10
150	4.57	2.54	2.00	1.75	1.61	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
175	4.82	2.63	2.06	1.80	1.66	1.50	1.41	1.35	1.30	1.28	1.25	1.24	1.23	1.21	1.19	1.18	1.18	1.17	1.16	1.15
200	5.04	2.72	2.12	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.64	0.50	0.44	0.40	0.37	0.34	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26	0.26
2	1.01	0.78	0.68	0.62	0.59	0.54	0.52	0.50	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
3	1.25	0.94	0.82	0.75	0.71	0.65	0.62	0.60	0.58	0.57	0.57	0.56	0.55	0.55	0.54	0.54	0.53	0.53	0.53	0.52
4	1.44	1.06	0.92	0.84	0.79	0.73	0.70	0.67	0.65	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59
5	1.59	1.15	0.99	0.91	0.85	0.79	0.75	0.72	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
8	1.93	1.36	1.16	1.05	0.99	0.91	0.87	0.83	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73
12	2.27	1.54	1.30	1.17	1.10	1.01	0.96	0.93	0.90	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.81	0.81
16	2.53	1.68	1.40	1.26	1.18	1.08	1.03	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.88	0.87	0.87	0.86
20	2.75	1.79	1.48	1.33	1.24	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
30	3.20	2.00	1.63	1.46	1.35	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
40	3.54	2.15	1.74	1.55	1.43	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
50	3.83	2.27	1.83	1.62	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
60	4.09	2.38	1.90	1.68	1.55	1.40	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
75	4.42	2.51	1.99	1.75	1.61	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
100	4.89	2.69	2.11	1.84	1.69	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
125	5.27	2.84	2.20	1.92	1.76	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.24	1.23	1.22	1.21
150	5.62	2.96	2.28	1.98	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
175	5.92	3.06	2.35	2.03	1.86	1.66	1.56	1.48	1.44	1.40	1.38	1.36	1.35	1.32	1.31	1.30	1.29	1.28	1.27	1.26
200	6.19	3.16	2.41	2.08	1.89	1.70	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.39	0.28	0.22	0.19	0.17	0.14	0.12	0.11	0.10	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.07	0.07	0.07	0.07
2	0.74	0.55	0.47	0.42	0.39	0.36	0.33	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26
3	0.95	0.70	0.61	0.55	0.51	0.47	0.45	0.43	0.41	0.41	0.40	0.39	0.39	0.38	0.38	0.37	0.37	0.37	0.37	0.36
4	1.11	0.82	0.70	0.64	0.60	0.55	0.52	0.50	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
5	1.24	0.91	0.78	0.71	0.67	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.51	0.50	0.50	0.49	0.49	0.49	0.48
8	1.54	1.10	0.94	0.85	0.80	0.74	0.70	0.67	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59
12	1.83	1.27	1.07	0.97	0.91	0.84	0.80	0.77	0.75	0.74	0.72	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67
16	*2.06*	1.39	1.17	1.06	0.99	0.92	0.87	0.84	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73
20	2.25	1.49	1.25	1.13	1.06	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
30	2.62	1.68	1.39	1.25	1.17	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.85	0.85
40	2.91	1.82	1.50	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
50	3.16	1.94	1.58	1.41	1.31	1.20	1.13	1.09	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
60	3.37	*2.03*	1.65	1.47	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
75	3.65	*2.15*	1.73	1.54	1.42	1.29	1.22	1.17	1.14	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01
100	4.04	2.31	1.84	1.63	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
125	4.37	2.44	1.93	1.70	1.56	1.42	1.34	1.28	1.24	1.21	1.19	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.11	1.10
150	4.65	2.55	*2.01*	1.76	1.62	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
175	4.91	2.65	*2.07*	1.81	1.66	1.50	1.41	1.35	1.30	1.28	1.25	1.24	1.23	1.21	1.19	1.18	1.18	1.17	1.16	1.15
200	5.14	2.73	*2.12*	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.70	0.53	0.46	0.41	0.39	0.35	0.33	0.31	0.30	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26
2	1.07	0.80	0.69	0.63	0.60	0.55	0.52	0.50	0.49	0.48	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
3	1.32	0.97	0.83	0.76	0.71	0.66	0.63	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.53	0.53	0.53	0.52
4	1.50	1.08	0.93	0.85	0.80	0.74	0.70	0.67	0.66	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59
5	1.66	1.18	1.01	0.92	0.86	0.79	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
8	2.01	1.38	1.17	1.06	0.99	0.91	0.87	0.84	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73
12	2.36	1.57	1.31	1.18	1.10	1.01	0.96	0.93	0.90	0.88	0.87	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.81	0.81
16	2.63	1.70	1.41	1.27	1.18	1.09	1.03	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.88	0.87	0.87	0.86
20	2.85	1.81	1.49	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
30	3.31	2.02	1.64	1.46	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
40	3.66	2.17	1.75	1.55	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
50	3.96	*2.30*	1.84	1.62	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
60	4.22	*2.41*	1.91	1.68	1.55	1.41	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
75	4.56	2.54	2.00	1.75	1.61	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
100	5.04	2.72	2.12	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
125	5.45	2.86	2.21	1.92	1.76	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.24	1.23	1.22	1.21
150	5.79	2.99	*2.29*	1.98	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
175	6.11	3.09	*2.36*	2.04	1.86	1.66	1.56	1.48	1.44	1.40	1.38	1.36	1.35	1.32	1.31	1.30	1.29	1.28	1.27	1.26
200	6.39	3.19	*2.42*	2.08	1.90	1.70	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.01	0.78	0.68	0.62	0.59	0.54	0.52	0.50	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43
2	1.44	1.06	0.92	0.84	0.79	0.73	0.70	0.67	0.65	0.64	0.63	0.63	0.62	0.61	0.61	0.60	0.60	0.59	0.59	0.59
3	1.72	1.23	1.06	0.96	0.91	0.84	0.80	0.77	0.75	0.73	0.72	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67
4	1.93	1.36	1.16	1.05	0.99	0.91	0.87	0.83	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73
5	2.11	1.46	1.23	1.12	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.82	0.80	0.80	0.79	0.79	0.78	0.78	0.77
8	2.53	1.68	1.40	1.26	1.18	1.08	1.03	0.99	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.88	0.87	0.87	0.86
12	2.95	1.88	1.55	1.39	1.29	1.18	1.12	1.08	1.05	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93
16	3.27	2.03	1.66	1.48	1.37	1.25	1.19	1.14	1.11	1.08	1.07	1.05	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.98
20	3.54	2.15	1.74	1.55	1.43	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
30	4.09	2.38	1.90	1.68	1.55	1.40	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
40	4.52	2.55	2.02	1.77	1.63	1.47	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
50	4.89	2.69	2.11	1.84	1.69	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
60	5.20	2.81	2.19	1.90	1.75	1.57	1.48	1.41	1.36	1.33	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20
75	5.62	2.96	2.28	1.98	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
100	6.20	3.16	2.41	2.08	1.89	1.69	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
125	6.69	3.32	2.51	2.16	1.96	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.33	1.32
150	7.11	3.46	*2.59*	2.22	2.01	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
175	7.49	3.58	*2.67*	2.28	2.06	1.83	1.71	1.62	1.57	1.53	1.50	1.48	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
200	7.84	3.69	*2.73*	2.32	2.10	1.86	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.85	0.63	0.54	0.49	0.46	0.42	0.40	0.38	0.37	0.36	0.35	0.35	0.34	0.34	0.33	0.33	0.33	0.32	0.32	0.32
2	1.24	0.91	0.78	0.71	0.67	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.51	0.50	0.50	0.49	0.49	0.49	0.48
3	1.50	1.07	0.91	0.83	0.78	0.72	0.69	0.66	0.64	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.58	0.57
4	1.70	1.19	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
5	1.86	1.29	1.09	0.99	0.92	0.85	0.81	0.78	0.76	0.74	0.73	0.73	0.72	0.71	0.70	0.70	0.69	0.69	0.68	0.68
8	2.25	1.49	1.25	1.13	1.06	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
12	2.62	1.68	1.39	1.25	1.17	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.85	0.85
16	2.91	1.82	1.50	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
20	3.16	1.94	1.58	1.41	1.31	1.20	1.13	1.09	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
30	3.65	*2.15*	1.73	1.54	1.42	1.29	1.22	1.17	1.14	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01
40	4.04	2.31	1.84	1.63	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
50	4.37	2.44	1.93	1.70	1.56	1.42	1.34	1.28	1.24	1.21	1.19	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.11	1.10
60	4.65	2.55	*2.01*	1.76	1.62	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
75	5.02	2.69	*2.10*	1.83	1.68	1.51	1.43	1.36	1.32	1.29	1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16
100	5.55	2.88	2.22	1.92	1.76	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.24	1.23	1.22	1.21
125	5.99	3.03	2.31	*2.00*	1.83	1.64	1.54	1.46	1.42	1.38	1.36	1.34	1.33	1.31	1.29	1.28	1.27	1.26	1.25	1.25
150	6.37	3.16	2.40	*2.06*	1.88	1.68	1.58	1.50	1.45	1.42	1.39	1.37	1.36	1.34	1.32	1.31	1.30	1.29	1.28	1.27
175	6.71	3.27	2.46	*2.12*	1.92	1.72	1.61	1.53	1.48	1.44	1.42	1.40	1.38	1.36	1.35	1.33	1.32	1.32	1.30	1.30
200	7.03	3.37	2.52	*2.16*	*1.96*	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.33	1.32

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.21	0.89	0.77	0.70	0.66	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.50	0.50	0.50	0.49	0.49	0.49	0.48
2	1.66	1.18	1.01	0.92	0.86	0.79	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
3	1.96	1.35	1.14	1.04	0.97	0.90	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
4	2.20	1.48	1.24	1.13	1.05	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
5	2.39	1.58	1.32	1.19	1.11	1.02	0.97	0.94	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.82	0.81
8	2.85	1.81	1.49	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
12	3.31	2.02	1.64	1.46	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
16	3.66	2.17	1.75	1.55	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
20	3.96	*2.30*	1.84	1.62	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
30	4.56	2.54	2.00	1.75	1.61	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
40	5.04	2.72	2.12	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
50	5.45	2.86	2.21	1.92	1.76	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.24	1.23	1.22	1.21
60	5.79	2.99	*2.29*	1.98	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
75	6.25	3.14	*2.39*	2.06	1.88	1.68	1.58	1.50	1.45	1.42	1.39	1.37	1.36	1.34	1.32	1.31	1.30	1.29	1.28	1.27
100	6.90	3.35	2.52	2.16	1.96	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.32	1.32
125	7.44	3.52	2.62	2.24	2.03	1.80	1.69	1.60	1.55	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35
150	7.91	3.67	2.71	*2.30*	2.08	1.85	1.72	1.64	1.58	1.54	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
175	8.33	3.79	2.78	*2.36*	2.13	1.89	1.76	1.67	1.61	1.57	1.54	1.52	1.50	1.47	1.45	1.44	1.43	1.42	1.41	1.40
200	8.73	3.90	2.85	*2.41*	2.17	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.59	1.15	0.99	0.91	0.85	0.79	0.75	0.72	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
2	2.11	1.46	1.23	1.12	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.82	0.80	0.80	0.79	0.79	0.78	0.78	0.77
3	2.47	1.65	1.38	1.24	1.16	1.07	1.01	0.97	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85
4	2.75	1.79	1.48	1.33	1.24	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
5	2.99	1.90	1.56	1.40	1.30	1.19	1.13	1.08	1.05	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
8	3.54	2.15	1.74	1.55	1.43	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
12	4.09	2.38	1.90	1.68	1.55	1.40	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
16	4.52	2.55	2.02	1.77	1.63	1.47	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
20	4.89	2.69	2.11	1.84	1.69	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
30	5.62	2.96	2.28	1.98	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
40	6.20	3.16	2.41	2.08	1.89	1.69	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
50	6.69	3.32	2.51	2.16	1.96	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.33	1.32
60	7.11	3.46	*2.59*	2.22	2.01	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
75	7.68	3.64	*2.70*	2.30	2.08	1.85	1.72	1.64	1.58	1.54	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
100	8.45	3.87	2.84	2.41	2.17	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42
125	9.11	4.06	2.95	2.49	2.24	1.97	1.83	1.73	1.67	1.63	1.60	1.57	1.56	1.53	1.51	1.50	1.48	1.48	1.46	1.45
150	9.70	4.23	3.05	*2.56*	2.29	2.02	1.88	1.77	1.71	1.66	1.63	1.60	1.59	1.56	1.54	1.52	1.51	1.50	1.49	1.48
175	10.20	4.37	3.13	*2.61*	2.34	2.05	1.91	1.80	1.73	1.69	1.66	1.63	1.61	1.58	1.56	1.55	1.53	1.53	1.51	1.50
200	10.66	4.50	3.20	*2.67*	2.38	2.09	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.54	1.53	1.52

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.24	0.91	0.78	0.71	0.67	0.61	0.58	0.56	0.54	0.53	0.52	0.52	0.51	0.51	0.50	0.50	0.49	0.49	0.49	0.48
2	1.70	1.19	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
3	2.01	1.37	1.15	1.04	0.98	0.90	0.86	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
4	2.25	1.49	1.25	1.13	1.06	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
5	2.45	1.60	1.33	1.20	1.12	1.03	0.98	0.94	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.82	0.81
8	2.91	1.82	1.50	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
12	3.37	*2.03*	1.65	1.47	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
16	3.74	2.19	1.76	1.56	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
20	4.04	2.31	1.84	1.63	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
30	4.65	2.55	*2.00*	1.76	1.62	1.46	1.38	1.31	1.27	1.25	1.23	1.21	1.20	1.18	1.17	1.16	1.15	1.15	1.14	1.13
40	5.14	2.73	*2.12*	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
50	5.55	2.88	2.22	1.92	1.76	1.58	1.49	1.42	1.37	1.34	1.32	1.30	1.29	1.27	1.25	1.24	1.24	1.23	1.22	1.21
60	5.90	3.00	2.30	*1.99*	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
75	6.37	3.16	2.39	*2.06*	1.88	1.68	1.58	1.50	1.45	1.42	1.39	1.37	1.36	1.34	1.32	1.31	1.30	1.29	1.28	1.27
100	7.03	3.37	2.52	*2.16*	*1.96*	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.32	1.32
125	7.59	3.54	2.63	2.24	*2.03*	1.80	1.69	1.60	1.55	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35
150	8.06	3.68	2.71	2.31	*2.08*	1.85	1.72	1.64	1.58	1.54	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.38	1.38
175	8.50	3.81	2.79	2.36	*2.13*	1.89	1.76	1.67	1.61	1.57	1.54	1.52	1.50	1.47	1.45	1.44	1.43	1.42	1.41	1.40
200	8.88	3.93	2.86	2.41	*2.17*	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (10 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.66	1.18	1.01	0.92	0.86	0.79	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
2	2.20	1.48	1.24	1.13	1.05	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
3	2.56	1.67	1.39	1.25	1.17	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85
4	2.85	1.81	1.49	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
5	3.10	1.92	1.57	1.41	1.31	1.19	1.13	1.09	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
8	3.66	2.17	1.75	1.55	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
12	4.22	*2.41*	1.91	1.68	1.55	1.41	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
16	4.67	2.58	2.03	1.78	1.63	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
20	5.04	2.72	2.12	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
30	5.79	2.99	*2.29*	1.98	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
40	6.39	3.19	*2.42*	2.08	1.90	1.70	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
50	6.90	3.35	2.52	2.16	1.96	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.32	1.32
60	7.34	3.49	2.60	2.22	2.02	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
75	7.91	3.67	2.71	*2.30*	2.08	1.85	1.72	1.64	1.58	1.54	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
100	8.72	3.90	2.85	*2.41*	2.17	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42
125	9.40	4.10	2.96	*2.49*	2.24	1.97	1.83	1.74	1.67	1.63	1.60	1.57	1.56	1.53	1.51	1.50	1.48	1.48	1.46	1.45
150	9.99	4.26	3.05	2.56	*2.30*	2.02	1.88	1.77	1.71	1.66	1.63	1.60	1.59	1.56	1.54	1.52	1.51	1.50	1.49	1.48
175	10.55	4.41	3.13	2.62	*2.34*	2.05	1.91	1.80	1.73	1.69	1.66	1.63	1.61	1.58	1.56	1.55	1.53	1.53	1.51	1.50
200	11.02	4.54	3.21	2.67	*2.39*	2.09	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.55	1.53	1.52

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (10 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.11	1.46	1.23	1.12	1.05	0.97	0.92	0.88	0.86	0.84	0.83	0.82	0.82	0.80	0.80	0.79	0.79	0.78	0.78	0.77
2	2.75	1.79	1.48	1.33	1.24	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
3	3.20	2.00	1.63	1.46	1.35	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
4	3.54	2.15	1.74	1.55	1.43	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
5	3.83	2.27	1.83	1.62	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
8	4.52	2.55	2.02	1.77	1.63	1.47	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	5.20	2.81	2.19	1.90	1.75	1.57	1.48	1.41	1.36	1.33	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20
16	5.74	3.00	2.31	2.00	1.83	1.64	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.31	1.29	1.28	1.28	1.27	1.26	1.25
20	6.20	3.16	2.41	2.08	1.89	1.69	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
30	7.11	3.46	*2.59*	2.22	2.01	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
40	7.84	3.69	*2.73*	2.32	2.10	1.86	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39
50	8.45	3.87	2.84	2.41	2.17	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42
60	8.99	4.03	2.93	2.47	2.22	1.96	1.83	1.73	1.66	1.62	1.59	1.57	1.55	1.52	1.50	1.49	1.48	1.47	1.45	1.44
75	9.70	4.23	3.05	*2.56*	2.29	2.02	1.87	1.77	1.71	1.66	1.63	1.60	1.59	1.56	1.54	1.52	1.51	1.50	1.49	1.48
100	10.66	4.50	3.20	*2.67*	2.38	2.09	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.54	1.53	1.52
125	11.51	4.72	3.32	*2.75*	2.45	2.14	1.98	1.87	1.80	1.75	1.71	1.69	1.67	1.64	1.61	1.60	1.59	1.58	1.56	1.55
150	12.25	4.91	3.42	2.83	2.51	2.19	2.03	1.90	1.83	1.78	1.74	1.72	1.70	1.66	1.64	1.62	1.61	1.60	1.58	1.57
175	12.89	5.07	3.51	2.89	*2.56*	2.23	2.06	1.94	1.86	1.81	1.77	1.74	1.72	1.69	1.66	1.65	1.64	1.62	1.61	1.59
200	13.48	5.21	3.59	2.94	*2.61*	2.26	2.09	1.96	1.88	1.83	1.79	1.76	1.74	1.71	1.68	1.67	1.65	1.64	1.62	1.61

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.70	1.19	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.69	0.68	0.68	0.67	0.66	0.65	0.65	0.65	0.64	0.64	0.63
2	2.25	1.49	1.25	1.13	1.06	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
3	2.62	1.68	1.39	1.25	1.17	1.07	1.02	0.98	0.95	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.87	0.86	0.85	0.85
4	2.91	1.82	1.50	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
5	3.16	1.94	1.58	1.41	1.31	1.20	1.13	1.09	1.06	1.03	1.02	1.01	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.94
8	3.74	2.19	1.76	1.56	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
12	4.31	2.42	1.91	1.68	1.55	1.41	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
16	4.76	2.59	2.03	1.78	1.63	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
20	5.14	2.73	2.12	1.85	1.70	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
30	5.90	3.00	2.30	1.99	1.81	1.63	1.53	1.45	1.41	1.38	1.35	1.33	1.32	1.30	1.28	1.27	1.26	1.26	1.25	1.24
40	6.51	3.20	2.42	2.08	1.90	1.70	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
50	7.02	3.37	2.52	2.16	1.96	1.75	1.64	1.56	1.50	1.47	1.44	1.42	1.41	1.38	1.37	1.35	1.35	1.34	1.32	1.32
60	7.47	3.51	2.61	2.23	2.02	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
75	8.06	3.68	2.71	2.31	2.08	1.85	1.72	1.64	1.58	1.54	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.37
100	8.88	3.93	2.85	2.41	2.17	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42
125	9.58	4.12	2.97	2.49	2.24	1.97	1.83	1.74	1.67	1.63	1.60	1.57	1.56	1.53	1.51	1.50	1.48	1.48	1.46	1.45
150	10.20	4.28	3.06	2.56	2.30	2.02	1.88	1.77	1.71	1.66	1.63	1.60	1.59	1.56	1.54	1.52	1.51	1.50	1.49	1.48
175	10.72	4.42	3.14	2.62	2.34	2.06	1.91	1.80	1.73	1.69	1.66	1.63	1.61	1.58	1.56	1.55	1.53	1.53	1.51	1.50
200	11.19	4.56	3.21	2.67	2.39	2.09	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.55	1.53	1.52

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.20	1.48	1.24	1.13	1.05	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
2	2.85	1.81	1.49	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
3	3.31	2.02	1.64	1.46	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
4	3.66	2.17	1.75	1.55	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
5	3.96	2.30	1.84	1.62	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
8	4.67	2.58	2.03	1.78	1.63	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	5.37	2.84	2.19	1.91	1.75	1.57	1.48	1.41	1.36	1.33	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20
16	5.93	3.03	2.32	2.01	1.83	1.64	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.31	1.29	1.28	1.28	1.27	1.26	1.25
20	6.39	3.19	2.42	2.08	1.90	1.70	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
30	7.34	3.49	2.60	2.22	2.02	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
40	8.09	3.72	2.74	2.33	2.10	1.86	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39
50	8.72	3.91	2.85	2.41	2.17	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42
60	9.27	4.06	2.94	2.48	2.23	1.96	1.83	1.73	1.67	1.62	1.59	1.57	1.55	1.52	1.50	1.49	1.48	1.47	1.45	1.44
75	9.99	4.26	3.05	2.56	2.29	2.02	1.88	1.77	1.71	1.66	1.63	1.60	1.59	1.56	1.54	1.52	1.51	1.50	1.49	1.48
100	11.02	4.53	3.21	2.67	2.39	2.09	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.54	1.53	1.52
125	11.87	4.75	3.33	2.76	2.46	2.14	1.98	1.87	1.80	1.75	1.71	1.69	1.67	1.64	1.61	1.60	1.59	1.58	1.56	1.55
150	12.60	4.94	3.43	2.83	2.52	2.19	2.03	1.90	1.83	1.78	1.74	1.72	1.70	1.66	1.64	1.62	1.61	1.60	1.58	1.57
175	13.30	5.10	3.52	2.89	2.56	2.23	2.06	1.94	1.86	1.81	1.77	1.74	1.72	1.69	1.66	1.65	1.63	1.62	1.61	1.59
200	13.89	5.24	3.60	2.95	2.61	2.26	2.09	1.96	1.88	1.83	1.79	1.76	1.74	1.71	1.68	1.67	1.65	1.64	1.62	1.61

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.75	1.79	1.48	1.33	1.24	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
2	3.54	2.15	1.74	1.55	1.43	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02
3	4.09	2.38	1.90	1.68	1.55	1.40	1.33	1.27	1.23	1.20	1.19	1.17	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09
4	4.52	2.55	2.02	1.77	1.63	1.47	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
5	4.88	2.69	2.11	1.84	1.69	1.53	1.44	1.37	1.33	1.30	1.28	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17
8	5.74	3.00	2.31	2.00	1.83	1.64	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.31	1.29	1.28	1.28	1.27	1.26	1.25
12	6.59	3.29	2.49	2.14	1.95	1.74	1.63	1.55	1.50	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31
16	7.27	3.51	*2.62*	2.24	2.03	1.81	1.69	1.60	1.55	1.51	1.48	1.46	1.45	1.42	1.41	1.39	1.38	1.38	1.36	1.35
20	7.84	3.69	*2.73*	2.32	2.10	1.86	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39
30	8.99	4.03	2.93	2.47	2.22	1.96	1.83	1.73	1.66	1.62	1.59	1.57	1.55	1.52	1.50	1.49	1.48	1.47	1.45	1.44
40	9.90	4.29	3.08	*2.58*	2.31	2.03	1.89	1.78	1.72	1.67	1.64	1.62	1.60	1.57	1.55	1.53	1.52	1.51	1.50	1.49
50	10.68	4.50	3.20	*2.67*	2.38	2.09	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.54	1.53	1.52
60	11.35	4.68	3.30	*2.74*	2.44	2.13	1.98	1.86	1.79	1.74	1.71	1.68	1.66	1.63	1.61	1.59	1.58	1.57	1.55	1.54
75	12.23	4.90	3.42	2.83	2.51	2.19	2.02	1.91	1.83	1.78	1.74	1.72	1.70	1.66	1.64	1.63	1.61	1.60	1.58	1.57
100	13.48	5.21	3.59	2.94	*2.61*	2.26	2.09	1.96	1.88	1.83	1.79	1.76	1.74	1.71	1.68	1.67	1.65	1.64	1.62	1.61
125	14.53	5.46	3.72	3.04	*2.68*	2.32	2.14	2.00	1.92	1.87	1.83	1.80	1.77	1.74	1.72	1.70	1.68	1.67	1.65	1.64
150	15.41	5.67	3.83	3.11	*2.74*	2.37	2.18	2.04	1.96	1.90	1.86	1.83	1.80	1.77	1.74	1.72	1.71	1.70	1.68	1.67
175	16.23	5.86	3.93	3.18	*2.79*	2.41	2.21	2.07	1.98	1.92	1.88	1.85	1.83	1.79	1.76	1.74	1.73	1.72	1.70	1.69
200	16.99	6.02	4.01	3.24	*2.84*	2.44	2.24	2.09	2.01	1.95	1.90	1.87	1.85	1.81	1.78	1.76	1.75	1.74	1.72	1.70

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.25	1.49	1.25	1.13	1.06	0.97	0.92	0.89	0.86	0.85	0.83	0.82	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.77
2	2.91	1.82	1.50	1.34	1.25	1.14	1.08	1.04	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.91	0.90
3	3.37	*2.03*	1.65	1.47	1.36	1.24	1.17	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.99	0.98	0.97
4	3.74	2.19	1.76	1.56	1.44	1.31	1.24	1.18	1.15	1.13	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.04	1.03	1.02
5	4.04	2.31	1.84	1.63	1.50	1.36	1.29	1.23	1.19	1.17	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.08	1.07	1.06
8	4.76	2.59	*2.03*	1.78	1.63	1.48	1.39	1.33	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	5.47	2.85	*2.20*	1.91	1.75	1.57	1.48	1.41	1.37	1.33	1.31	1.29	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.20
16	6.04	3.05	2.32	*2.01*	1.83	1.64	1.54	1.47	1.42	1.39	1.36	1.34	1.33	1.31	1.29	1.28	1.28	1.27	1.26	1.25
20	6.51	3.20	2.42	*2.08*	1.90	1.70	1.59	1.51	1.46	1.43	1.40	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28
30	7.47	3.51	2.61	2.23	*2.02*	1.79	1.68	1.59	1.54	1.50	1.47	1.45	1.44	1.41	1.40	1.38	1.37	1.37	1.35	1.34
40	8.23	3.74	2.74	2.33	*2.10*	1.86	1.74	1.65	1.59	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.39	1.38
50	8.88	3.93	2.85	2.41	*2.17*	1.92	1.79	1.69	1.63	1.59	1.56	1.54	1.52	1.49	1.48	1.46	1.45	1.44	1.43	1.42
60	9.43	4.08	2.94	2.48	*2.23*	*1.96*	1.83	1.73	1.66	1.62	1.59	1.57	1.55	1.52	1.50	1.49	1.48	1.47	1.45	1.44
75	10.20	4.28	3.06	2.56	2.30	*2.02*	1.88	1.77	1.71	1.66	1.63	1.60	1.59	1.56	1.54	1.52	1.51	1.50	1.49	1.48
100	11.19	4.56	3.21	2.67	2.39	*2.09*	1.94	1.83	1.76	1.71	1.68	1.65	1.63	1.60	1.58	1.57	1.55	1.55	1.53	1.52
125	12.07	4.78	3.33	2.76	2.46	*2.15*	*1.98*	1.87	1.80	1.75	1.71	1.69	1.67	1.64	1.61	1.60	1.59	1.58	1.56	1.55
150	12.89	4.95	3.44	2.83	2.52	*2.19*	*2.03*	1.90	1.83	1.78	1.74	1.72	1.70	1.66	1.64	1.63	1.61	1.60	1.59	1.57
175	13.48	5.13	3.52	2.90	2.56	*2.23*	*2.06*	1.93	1.86	1.81	1.77	1.74	1.72	1.69	1.66	1.65	1.63	1.62	1.60	1.59
200	14.18	5.27	3.60	2.94	2.61	*2.26*	*2.09*	*1.96*	1.88	1.83	1.79	1.77	1.74	1.71	1.68	1.67	1.65	1.64	1.62	1.61

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.85	**1.81**	**1.49**	**1.34**	**1.25**	**1.14**	**1.08**	**1.04**	**1.01**	**0.99**	**0.97**	**0.96**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.91**	**0.90**
2	3.66	**2.17**	**1.75**	**1.55**	**1.44**	**1.31**	**1.24**	**1.18**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.03**	**1.02**
3	4.22	*2.41*	**1.91**	**1.68**	**1.55**	**1.41**	**1.33**	**1.27**	**1.23**	**1.20**	**1.19**	**1.17**	**1.16**	**1.14**	**1.13**	**1.12**	**1.11**	**1.11**	**1.10**	**1.09**
4	4.67	2.58	**2.03**	**1.78**	**1.63**	**1.48**	**1.39**	**1.33**	**1.29**	**1.26**	**1.24**	**1.22**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.16**	**1.15**	**1.14**
5	5.04	2.72	**2.12**	**1.85**	**1.70**	**1.53**	**1.44**	**1.37**	**1.33**	**1.30**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
8	5.93	3.03	*2.32*	**2.01**	**1.83**	**1.64**	**1.54**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.31**	**1.29**	**1.28**	**1.28**	**1.27**	**1.26**	**1.25**
12	6.80	3.32	2.50	**2.15**	**1.95**	**1.74**	**1.63**	**1.55**	**1.50**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	7.50	3.54	2.63	**2.25**	**2.04**	**1.81**	**1.69**	**1.60**	**1.55**	**1.51**	**1.48**	**1.46**	**1.45**	**1.42**	**1.41**	**1.39**	**1.38**	**1.38**	**1.36**	**1.35**
20	8.09	3.72	2.74	*2.33*	**2.10**	**1.86**	**1.74**	**1.65**	**1.59**	**1.55**	**1.52**	**1.50**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**	**1.39**
30	9.27	4.06	2.94	*2.48*	**2.23**	**1.96**	**1.83**	**1.73**	**1.67**	**1.62**	**1.59**	**1.57**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.47**	**1.45**	**1.44**
40	10.21	4.32	3.09	2.59	*2.31*	**2.03**	**1.89**	**1.78**	**1.72**	**1.67**	**1.64**	**1.61**	**1.60**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**
50	11.02	4.53	3.21	2.67	*2.39*	**2.09**	**1.94**	**1.83**	**1.76**	**1.71**	**1.68**	**1.65**	**1.63**	**1.60**	**1.58**	**1.57**	**1.55**	**1.54**	**1.53**	**1.52**
60	11.72	4.72	3.31	2.74	*2.44*	**2.14**	**1.98**	**1.86**	**1.79**	**1.74**	**1.71**	**1.68**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.57**	**1.55**	**1.54**
75	12.60	4.94	3.43	2.83	*2.52*	**2.19**	**2.03**	**1.90**	**1.83**	**1.78**	**1.74**	**1.72**	**1.70**	**1.66**	**1.64**	**1.62**	**1.61**	**1.60**	**1.58**	**1.57**
100	13.89	5.24	3.60	2.95	2.61	*2.26*	**2.09**	**1.96**	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.71**	**1.68**	**1.67**	**1.65**	**1.64**	**1.62**	**1.61**
125	15.00	5.51	3.73	3.04	2.68	*2.32*	**2.14**	**2.00**	**1.92**	**1.87**	**1.83**	**1.80**	**1.77**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.65**	**1.64**
150	15.94	5.71	3.84	3.12	2.75	*2.37*	**2.18**	**2.04**	**1.96**	**1.90**	**1.86**	**1.83**	**1.80**	**1.77**	**1.74**	**1.72**	**1.71**	**1.70**	**1.68**	**1.67**
175	16.76	5.89	3.94	3.18	2.80	*2.40*	**2.21**	**2.07**	**1.98**	**1.93**	**1.88**	**1.85**	**1.83**	**1.79**	**1.77**	**1.75**	**1.73**	**1.72**	**1.70**	**1.69**
200	17.58	6.06	4.01	3.24	2.84	*2.44*	**2.24**	**2.09**	**2.01**	**1.95**	**1.90**	**1.87**	**1.85**	**1.81**	**1.78**	**1.77**	**1.75**	**1.74**	**1.72**	**1.70**

Table 19-12. κ-Multipliers for 1-of-4 Intrawell Prediction Limits on Observations (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.54	**2.15**	**1.74**	**1.55**	**1.43**	**1.31**	**1.24**	**1.18**	**1.15**	**1.13**	**1.11**	**1.10**	**1.08**	**1.07**	**1.06**	**1.05**	**1.04**	**1.04**	**1.03**	**1.02**
2	4.52	**2.55**	**2.02**	**1.77**	**1.63**	**1.47**	**1.39**	**1.33**	**1.29**	**1.26**	**1.24**	**1.22**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.16**	**1.15**	**1.14**
3	5.20	2.81	**2.19**	**1.90**	**1.75**	**1.57**	**1.48**	**1.41**	**1.36**	**1.33**	**1.31**	**1.29**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
4	5.74	3.00	**2.31**	**2.00**	**1.83**	**1.64**	**1.54**	**1.47**	**1.42**	**1.39**	**1.36**	**1.34**	**1.33**	**1.31**	**1.29**	**1.28**	**1.28**	**1.27**	**1.26**	**1.25**
5	6.20	3.16	**2.41**	**2.08**	**1.89**	**1.69**	**1.59**	**1.51**	**1.46**	**1.43**	**1.40**	**1.38**	**1.37**	**1.35**	**1.33**	**1.32**	**1.31**	**1.30**	**1.29**	**1.28**
8	7.27	3.51	*2.62*	**2.24**	**2.03**	**1.81**	**1.69**	**1.60**	**1.55**	**1.51**	**1.48**	**1.46**	**1.45**	**1.42**	**1.41**	**1.39**	**1.38**	**1.38**	**1.36**	**1.35**
12	8.34	3.84	2.82	**2.39**	**2.16**	**1.91**	**1.78**	**1.68**	**1.62**	**1.58**	**1.55**	**1.53**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.42**	**1.41**
16	9.18	4.09	2.96	**2.50**	**2.24**	**1.98**	**1.84**	**1.74**	**1.68**	**1.63**	**1.60**	**1.58**	**1.56**	**1.53**	**1.51**	**1.50**	**1.49**	**1.48**	**1.46**	**1.45**
20	9.90	4.29	3.08	*2.58*	**2.31**	**2.03**	**1.89**	**1.78**	**1.72**	**1.67**	**1.64**	**1.62**	**1.60**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**
30	11.35	4.68	3.30	*2.74*	**2.44**	**2.13**	**1.98**	**1.86**	**1.79**	**1.74**	**1.71**	**1.68**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.57**	**1.55**	**1.54**
40	12.51	4.97	3.46	2.85	**2.53**	**2.21**	**2.04**	**1.92**	**1.84**	**1.79**	**1.76**	**1.73**	**1.71**	**1.67**	**1.65**	**1.63**	**1.62**	**1.61**	**1.59**	**1.58**
50	13.48	5.21	3.59	2.94	*2.61*	**2.26**	**2.09**	**1.96**	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.71**	**1.68**	**1.67**	**1.65**	**1.64**	**1.62**	**1.61**
60	14.33	5.41	3.69	3.02	*2.67*	**2.31**	**2.13**	**2.00**	**1.92**	**1.86**	**1.82**	**1.79**	**1.77**	**1.73**	**1.71**	**1.69**	**1.68**	**1.67**	**1.65**	**1.63**
75	15.41	5.67	3.83	3.11	*2.74*	**2.37**	**2.18**	**2.04**	**1.96**	**1.90**	**1.86**	**1.83**	**1.80**	**1.77**	**1.74**	**1.72**	**1.71**	**1.70**	**1.68**	**1.66**
100	16.99	6.02	4.01	3.24	*2.84*	**2.44**	**2.24**	**2.09**	**2.01**	**1.95**	**1.90**	**1.87**	**1.85**	**1.81**	**1.78**	**1.76**	**1.75**	**1.74**	**1.72**	**1.70**
125	18.28	6.30	4.15	3.34	2.92	**2.50**	**2.29**	**2.14**	**2.05**	**1.98**	**1.94**	**1.91**	**1.88**	**1.84**	**1.81**	**1.79**	**1.78**	**1.77**	**1.74**	**1.73**
150	19.45	6.56	4.28	3.42	2.98	*2.55*	**2.33**	**2.18**	**2.08**	**2.01**	**1.97**	**1.93**	**1.91**	**1.87**	**1.84**	**1.82**	**1.81**	**1.79**	**1.77**	**1.76**
175	20.51	6.77	4.38	3.49	3.03	*2.59*	**2.36**	**2.20**	**2.11**	**2.04**	**1.99**	**1.96**	**1.93**	**1.89**	**1.86**	**1.84**	**1.83**	**1.81**	**1.79**	**1.78**
200	21.33	6.94	4.47	3.54	3.08	*2.62*	**2.39**	**2.23**	**2.13**	**2.07**	**2.01**	**1.98**	**1.95**	**1.91**	**1.88**	**1.86**	**1.85**	**1.83**	**1.81**	**1.79**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.71	0.59	0.53	0.50	0.48	0.45	0.44	0.43	0.42	0.41	0.41	0.41	0.40	0.40	0.40	0.40	0.39	0.39	0.39	0.39
2	1.14	0.92	0.83	0.78	0.74	0.71	0.68	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61	0.61
3	1.42	1.11	0.99	0.93	0.89	0.84	0.81	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73	0.73	0.73	0.72
4	1.63	1.25	1.11	1.04	0.99	0.94	0.90	0.88	0.86	0.85	0.84	0.84	0.83	0.82	0.82	0.81	0.81	0.81	0.80	0.80
5	1.80	1.36	1.20	1.12	1.07	1.01	0.97	0.94	0.93	0.91	0.90	0.90	0.89	0.88	0.88	0.87	0.87	0.87	0.86	0.86
8	2.19	1.60	1.40	1.29	1.23	1.15	1.11	1.08	1.05	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.99	0.98	0.98	0.97
12	2.58	1.82	1.57	1.44	1.36	1.27	1.22	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
16	2.88	1.98	1.69	1.54	1.46	1.36	1.30	1.26	1.23	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
20	3.13	2.11	1.78	1.63	1.53	1.42	1.37	1.32	1.29	1.27	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
30	3.63	2.35	1.96	1.78	1.67	1.54	1.47	1.42	1.39	1.37	1.35	1.34	1.33	1.31	1.30	1.29	1.29	1.28	1.27	1.27
40	4.02	2.53	2.09	1.88	1.76	1.63	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
50	4.36	2.68	2.20	1.97	1.84	1.69	1.61	1.55	1.51	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
60	4.65	2.80	2.28	2.04	1.90	1.74	1.66	1.59	1.55	1.53	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40
75	5.02	2.96	2.39	2.12	1.97	1.81	1.72	1.65	1.60	1.58	1.55	1.54	1.52	1.50	1.49	1.48	1.47	1.47	1.46	1.45
100	5.55	3.17	2.53	2.24	2.07	1.89	1.79	1.72	1.67	1.64	1.62	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
125	5.99	3.34	*2.64*	2.32	2.15	1.95	1.85	1.77	1.72	1.69	1.66	1.64	1.63	1.61	1.59	1.58	1.57	1.56	1.55	1.54
150	6.38	3.48	2.73	2.40	2.21	2.00	1.89	1.81	1.76	1.73	1.70	1.68	1.67	1.64	1.63	1.61	1.61	1.60	1.58	1.58
175	6.72	3.60	2.81	2.46	2.26	2.05	1.93	1.85	1.80	1.76	1.73	1.71	1.70	1.67	1.66	1.64	1.63	1.63	1.61	1.60
200	7.04	3.72	2.88	2.51	2.31	2.09	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.07	0.88	0.81	0.76	0.73	0.70	0.68	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61	0.61
2	1.55	1.22	1.09	1.02	0.98	0.93	0.90	0.88	0.86	0.85	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.81	0.80	0.80
3	1.86	1.42	1.26	1.18	1.12	1.06	1.02	0.99	0.97	0.96	0.95	0.94	0.94	0.93	0.92	0.92	0.91	0.91	0.91	0.90
4	2.10	1.57	1.38	1.28	1.22	1.15	1.11	1.07	1.05	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.99	0.98	0.98	0.97
5	2.30	1.69	1.47	1.36	1.29	1.21	1.17	1.13	1.11	1.10	1.08	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
8	2.77	1.95	1.67	1.54	1.45	1.36	1.30	1.26	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
12	3.23	2.18	1.85	1.69	1.59	1.47	1.41	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25	1.24	1.24	1.23	1.22	1.22
16	3.59	2.36	1.98	1.79	1.68	1.56	1.49	1.44	1.40	1.38	1.36	1.35	1.34	1.32	1.31	1.31	1.30	1.29	1.29	1.28
20	3.89	2.50	2.08	1.88	1.76	1.62	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
30	4.49	2.77	2.27	2.03	1.89	1.74	1.66	1.59	1.55	1.53	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40
40	4.97	2.97	2.40	2.14	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
50	5.37	3.13	2.51	2.23	2.07	1.89	1.79	1.72	1.67	1.64	1.61	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
60	5.72	3.27	2.60	2.30	2.13	1.94	1.84	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
75	6.17	3.44	2.72	2.39	2.20	2.00	1.89	1.81	1.76	1.73	1.70	1.68	1.67	1.64	1.63	1.61	1.61	1.60	1.58	1.58
100	6.81	3.68	*2.87*	2.51	2.30	2.08	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
125	7.35	3.87	2.99	2.60	2.38	2.15	2.02	1.93	1.87	1.83	1.81	1.78	1.77	1.74	1.72	1.71	1.70	1.69	1.68	1.67
150	7.82	4.03	3.09	2.68	2.45	2.20	2.07	1.97	1.91	1.87	1.84	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.70
175	8.25	4.17	3.17	2.74	2.50	2.24	2.11	2.01	1.95	1.91	1.87	1.85	1.83	1.80	1.78	1.77	1.76	1.75	1.74	1.72
200	8.63	4.29	3.25	2.80	2.55	2.28	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.43	1.17	1.06	1.00	0.96	0.92	0.89	0.87	0.85	0.84	0.84	0.83	0.83	0.82	0.81	0.81	0.81	0.81	0.80	0.80
2	1.97	1.52	1.35	1.26	1.20	1.14	1.10	1.07	1.05	1.04	1.02	1.02	1.01	1.00	0.99	0.99	0.99	0.98	0.98	0.97
3	2.33	1.74	1.52	1.41	1.34	1.26	1.22	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.08	1.07	1.07
4	2.61	1.90	1.65	1.52	1.44	1.35	1.30	1.26	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
5	2.84	2.02	1.74	1.60	1.52	1.41	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
8	3.39	2.31	1.95	1.78	1.67	1.55	1.49	1.44	1.40	1.38	1.36	1.35	1.34	1.32	1.31	1.31	1.30	1.29	1.28	1.28
12	3.93	2.56	2.14	1.93	1.81	1.67	1.59	1.54	1.50	1.47	1.45	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
16	4.35	2.76	2.27	2.04	1.91	1.75	1.67	1.61	1.57	1.54	1.52	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.42
20	4.71	2.91	2.38	2.13	1.98	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
30	5.42	3.21	2.58	2.29	2.12	1.93	1.83	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
40	5.99	3.43	2.73	2.40	2.22	2.02	1.91	1.83	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
50	6.47	3.61	2.84	2.50	2.30	2.08	1.96	1.88	1.82	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
60	6.89	3.77	2.94	2.57	2.36	2.13	2.01	1.92	1.86	1.83	1.80	1.78	1.76	1.73	1.72	1.70	1.69	1.68	1.67	1.66
75	7.43	3.96	3.06	2.66	2.44	2.20	2.07	1.97	1.91	1.87	1.84	1.82	1.80	1.77	1.76	1.74	1.73	1.72	1.71	1.70
100	8.20	4.22	3.23	2.79	2.54	2.28	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
125	8.84	4.43	3.35	2.88	2.62	2.35	2.20	2.09	2.02	1.98	1.95	1.92	1.90	1.87	1.85	1.83	1.82	1.81	1.80	1.79
150	9.40	4.62	3.46	2.96	2.69	2.40	2.25	2.13	2.06	2.02	1.98	1.95	1.93	1.90	1.88	1.87	1.85	1.84	1.83	1.82
175	9.90	4.77	3.55	3.03	2.75	2.44	2.29	2.17	2.10	2.05	2.01	1.98	1.96	1.93	1.91	1.89	1.88	1.87	1.85	1.84
200	10.36	4.91	3.64	*3.09*	2.80	2.48	2.32	2.20	2.13	2.07	2.04	2.01	1.99	1.96	1.93	1.92	1.90	1.89	1.88	1.86

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (2 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.14	0.92	0.83	0.78	0.74	0.71	0.68	0.66	0.65	0.64	0.64	0.63	0.63	0.62	0.62	0.62	0.61	0.61	0.61	0.61
2	1.63	1.25	1.11	1.04	0.99	0.94	0.90	0.88	0.86	0.85	0.84	0.84	0.83	0.82	0.82	0.81	0.81	0.81	0.80	0.80
3	1.95	1.46	1.28	1.19	1.13	1.06	1.03	1.00	0.98	0.96	0.95	0.95	0.94	0.93	0.92	0.92	0.92	0.91	0.91	0.90
4	2.19	1.60	1.40	1.29	1.23	1.15	1.11	1.08	1.05	1.04	1.03	1.02	1.01	1.00	1.00	0.99	0.99	0.98	0.98	0.97
5	2.40	1.72	1.49	1.37	1.30	1.22	1.17	1.14	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
8	2.88	1.98	1.69	1.54	1.46	1.36	1.30	1.26	1.23	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.14	1.13
12	3.35	2.22	1.86	1.69	1.59	1.48	1.41	1.37	1.34	1.31	1.30	1.29	1.28	1.26	1.25	1.24	1.24	1.23	1.23	1.22
16	3.72	2.39	1.99	1.80	1.69	1.56	1.49	1.44	1.41	1.38	1.36	1.35	1.34	1.32	1.31	1.31	1.30	1.29	1.29	1.28
20	4.02	2.53	2.09	1.88	1.76	1.63	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
30	4.65	2.80	2.28	2.04	1.90	1.74	1.66	1.59	1.55	1.53	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40
40	5.14	3.00	2.42	2.15	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
50	5.55	3.17	2.53	2.24	2.07	1.89	1.79	1.72	1.67	1.64	1.62	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
60	5.91	3.31	*2.62*	2.31	2.13	1.94	1.84	1.76	1.71	1.68	1.65	1.64	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
75	6.38	3.48	2.73	2.40	2.21	2.00	1.89	1.81	1.76	1.73	1.70	1.68	1.67	1.64	1.63	1.61	1.61	1.60	1.58	1.58
100	7.04	3.72	2.88	2.51	2.31	2.09	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
125	7.60	3.91	3.00	*2.61*	2.39	2.15	2.03	1.93	1.88	1.83	1.81	1.78	1.77	1.74	1.72	1.71	1.70	1.69	1.68	1.67
150	8.09	4.07	3.10	*2.68*	2.45	2.20	2.07	1.98	1.92	1.87	1.84	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.70
175	8.51	4.21	3.19	2.75	2.50	2.25	2.11	2.01	1.95	1.91	1.87	1.85	1.83	1.80	1.79	1.77	1.76	1.75	1.74	1.72
200	8.91	4.34	3.26	2.81	*2.55*	2.29	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**1.55**	**1.22**	**1.09**	**1.02**	**0.98**	**0.93**	**0.90**	**0.88**	**0.86**	**0.85**	**0.84**	**0.83**	**0.83**	**0.82**	**0.82**	**0.81**	**0.81**	**0.81**	**0.80**	**0.80**
2	**2.10**	**1.57**	**1.38**	**1.28**	**1.22**	**1.15**	**1.11**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.99**	**0.98**	**0.98**	**0.97**
3	**2.48**	**1.79**	**1.55**	**1.43**	**1.36**	**1.27**	**1.22**	**1.18**	**1.16**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.09**	**1.08**	**1.08**	**1.07**	**1.07**
4	**2.77**	**1.95**	**1.67**	**1.54**	**1.45**	**1.36**	**1.30**	**1.26**	**1.23**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.15**	**1.14**	**1.14**	**1.13**
5	3.01	**2.08**	**1.77**	**1.62**	**1.53**	**1.42**	**1.36**	**1.32**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**	**1.20**	**1.19**	**1.19**	**1.18**
8	3.59	**2.36**	**1.98**	**1.79**	**1.68**	**1.56**	**1.49**	**1.44**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.32**	**1.31**	**1.31**	**1.30**	**1.29**	**1.29**	**1.28**
12	4.15	**2.62**	**2.16**	**1.95**	**1.82**	**1.68**	**1.60**	**1.54**	**1.50**	**1.47**	**1.46**	**1.44**	**1.43**	**1.41**	**1.40**	**1.39**	**1.38**	**1.38**	**1.37**	**1.36**
16	4.59	**2.81**	**2.30**	**2.06**	**1.91**	**1.76**	**1.67**	**1.61**	**1.57**	**1.54**	**1.52**	**1.50**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**	**1.42**
20	4.97	2.97	**2.40**	**2.14**	**1.99**	**1.82**	**1.73**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
30	5.72	3.27	**2.60**	**2.30**	**2.13**	**1.94**	**1.84**	**1.76**	**1.71**	**1.68**	**1.65**	**1.63**	**1.62**	**1.60**	**1.58**	**1.57**	**1.56**	**1.56**	**1.54**	**1.54**
40	6.31	3.50	**2.75**	**2.42**	**2.23**	**2.02**	**1.91**	**1.83**	**1.78**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
50	6.81	3.68	*2.87*	**2.51**	**2.30**	**2.08**	**1.97**	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**
60	7.25	3.83	*2.97*	**2.58**	**2.37**	**2.14**	**2.01**	**1.92**	**1.87**	**1.83**	**1.80**	**1.78**	**1.76**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
75	7.82	4.03	3.09	**2.68**	**2.45**	**2.20**	**2.07**	**1.97**	**1.91**	**1.87**	**1.84**	**1.82**	**1.80**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.70**
100	8.63	4.29	3.25	**2.80**	**2.55**	**2.28**	**2.14**	**2.04**	**1.98**	**1.93**	**1.90**	**1.88**	**1.86**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
125	9.30	4.51	3.38	*2.89*	**2.63**	**2.35**	**2.20**	**2.09**	**2.03**	**1.98**	**1.95**	**1.92**	**1.90**	**1.87**	**1.85**	**1.83**	**1.82**	**1.81**	**1.80**	**1.79**
150	9.89	4.69	3.49	*2.98*	**2.70**	**2.40**	**2.25**	**2.14**	**2.06**	**2.02**	**1.98**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.85**	**1.84**	**1.83**	**1.82**
175	10.43	4.85	3.58	3.04	**2.75**	**2.45**	**2.29**	**2.17**	**2.10**	**2.05**	**2.01**	**1.98**	**1.96**	**1.93**	**1.91**	**1.89**	**1.88**	**1.87**	**1.85**	**1.84**
200	10.90	5.00	3.66	3.11	*2.80*	**2.48**	**2.32**	**2.20**	**2.13**	**2.07**	**2.04**	**2.01**	**1.99**	**1.96**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.86**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**1.97**	**1.52**	**1.35**	**1.26**	**1.20**	**1.14**	**1.10**	**1.07**	**1.05**	**1.04**	**1.02**	**1.02**	**1.01**	**1.00**	**0.99**	**0.99**	**0.99**	**0.98**	**0.98**	**0.97**
2	**2.61**	**1.90**	**1.65**	**1.52**	**1.44**	**1.35**	**1.30**	**1.26**	**1.23**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.15**	**1.14**	**1.14**	**1.13**
3	3.04	**2.13**	**1.82**	**1.67**	**1.58**	**1.47**	**1.41**	**1.36**	**1.33**	**1.31**	**1.30**	**1.28**	**1.27**	**1.26**	**1.25**	**1.24**	**1.24**	**1.23**	**1.22**	**1.22**
4	3.39	**2.31**	**1.95**	**1.78**	**1.67**	**1.55**	**1.49**	**1.44**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.32**	**1.31**	**1.31**	**1.30**	**1.29**	**1.28**	**1.28**
5	3.68	**2.45**	**2.05**	**1.86**	**1.75**	**1.62**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
8	4.35	**2.76**	**2.27**	**2.04**	**1.91**	**1.75**	**1.67**	**1.61**	**1.57**	**1.54**	**1.52**	**1.50**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**	**1.42**
12	5.02	**3.04**	**2.47**	**2.20**	**2.04**	**1.87**	**1.78**	**1.70**	**1.66**	**1.63**	**1.61**	**1.59**	**1.57**	**1.55**	**1.54**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**
16	5.55	3.26	**2.61**	**2.31**	**2.14**	**1.95**	**1.85**	**1.77**	**1.72**	**1.69**	**1.67**	**1.65**	**1.63**	**1.61**	**1.60**	**1.58**	**1.58**	**1.57**	**1.56**	**1.55**
20	5.99	3.43	**2.73**	**2.40**	**2.22**	**2.02**	**1.91**	**1.83**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
30	6.89	3.77	**2.94**	**2.57**	**2.36**	**2.13**	**2.01**	**1.92**	**1.86**	**1.83**	**1.80**	**1.78**	**1.76**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
40	7.60	4.02	*3.10*	**2.69**	**2.46**	**2.22**	**2.09**	**1.99**	**1.93**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
50	8.20	4.22	3.23	**2.79**	**2.54**	**2.28**	**2.14**	**2.04**	**1.98**	**1.93**	**1.90**	**1.88**	**1.86**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
60	8.72	4.40	3.33	**2.87**	**2.61**	**2.33**	**2.19**	**2.08**	**2.02**	**1.97**	**1.94**	**1.91**	**1.89**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
75	9.40	4.62	3.46	**2.96**	**2.69**	**2.40**	**2.25**	**2.13**	**2.06**	**2.02**	**1.98**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.85**	**1.84**	**1.83**	**1.82**
100	10.36	4.91	3.64	*3.09*	**2.80**	**2.48**	**2.32**	**2.20**	**2.13**	**2.07**	**2.04**	**2.01**	**1.99**	**1.96**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.86**
125	11.18	5.16	3.78	*3.20*	**2.88**	**2.55**	**2.38**	**2.25**	**2.17**	**2.12**	**2.08**	**2.05**	**2.03**	**1.99**	**1.97**	**1.95**	**1.94**	**1.93**	**1.91**	**1.90**
150	11.88	5.36	3.89	*3.28*	**2.95**	**2.60**	**2.42**	**2.29**	**2.21**	**2.16**	**2.12**	**2.09**	**2.06**	**2.03**	**2.00**	**1.98**	**1.97**	**1.96**	**1.94**	**1.93**
175	12.51	5.54	4.00	3.35	**3.01**	**2.65**	**2.46**	**2.33**	**2.24**	**2.19**	**2.15**	**2.11**	**2.09**	**2.05**	**2.03**	**2.01**	**2.00**	**1.99**	**1.97**	**1.95**
200	13.10	5.70	4.08	3.42	**3.06**	**2.69**	**2.50**	**2.36**	**2.27**	**2.21**	**2.17**	**2.14**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.97**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.80	1.36	1.20	1.12	1.07	1.01	0.97	0.94	0.93	0.91	0.90	0.90	0.89	0.88	0.88	0.87	0.87	0.87	0.86	0.86
2	2.40	1.72	1.49	1.37	1.30	1.22	1.17	1.14	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
3	2.81	1.94	1.66	1.52	1.44	1.34	1.29	1.25	1.22	1.20	1.19	1.18	1.17	1.15	1.15	1.14	1.13	1.13	1.12	1.12
4	3.13	2.11	1.78	1.63	1.53	1.42	1.37	1.32	1.29	1.27	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
5	3.40	2.24	1.88	1.71	1.61	1.49	1.43	1.38	1.35	1.32	1.31	1.30	1.29	1.27	1.26	1.25	1.25	1.24	1.23	1.23
8	4.02	2.53	2.09	1.88	1.76	1.63	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
12	4.65	2.80	2.28	2.04	1.90	1.74	1.66	1.59	1.55	1.53	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40
16	5.14	3.00	2.42	2.15	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
20	5.55	3.17	2.53	2.24	2.07	1.89	1.79	1.72	1.67	1.64	1.62	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
30	6.38	3.48	2.73	2.40	2.21	2.00	1.89	1.81	1.76	1.73	1.70	1.68	1.67	1.64	1.63	1.61	1.61	1.60	1.59	1.58
40	7.04	3.72	2.88	2.51	2.31	2.09	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
50	7.60	3.91	3.00	*2.60*	2.39	2.15	2.03	1.93	1.88	1.83	1.81	1.78	1.77	1.74	1.72	1.71	1.70	1.69	1.68	1.67
60	8.09	4.07	3.10	*2.68*	2.45	2.20	2.07	1.98	1.91	1.87	1.84	1.82	1.80	1.77	1.76	1.74	1.73	1.72	1.71	1.70
75	8.71	4.28	3.23	2.78	*2.53*	2.27	2.13	2.03	1.96	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74
100	9.61	4.55	3.39	2.90	*2.63*	2.35	2.20	2.09	2.03	1.98	1.95	1.92	1.90	1.87	1.85	1.83	1.82	1.81	1.80	1.78
125	10.35	4.78	3.53	3.00	*2.71*	2.41	2.26	2.14	2.07	2.02	1.99	1.96	1.94	1.91	1.89	1.87	1.86	1.85	1.83	1.82
150	11.02	4.96	3.63	3.08	2.78	2.47	2.30	2.19	2.11	2.06	2.03	2.00	1.98	1.94	1.92	1.90	1.89	1.88	1.87	1.85
175	11.60	5.14	3.73	3.15	2.84	*2.51*	2.34	2.22	2.14	2.09	2.06	2.03	2.00	1.97	1.95	1.93	1.92	1.91	1.89	1.88
200	12.11	5.27	3.82	3.21	2.89	*2.55*	2.38	2.25	2.17	2.12	2.08	2.05	2.03	1.99	1.97	1.95	1.94	1.93	1.91	1.90

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.30	1.69	1.47	1.36	1.29	1.21	1.17	1.13	1.11	1.10	1.08	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
2	3.01	2.08	1.77	1.62	1.53	1.42	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
3	3.50	2.32	1.95	1.77	1.66	1.54	1.47	1.42	1.39	1.37	1.35	1.34	1.33	1.31	1.30	1.29	1.29	1.28	1.27	1.27
4	3.89	2.50	2.08	1.88	1.76	1.62	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
5	4.21	2.65	2.18	1.96	1.83	1.69	1.61	1.55	1.51	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
8	4.97	2.97	2.40	2.14	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
12	5.72	3.27	2.60	2.30	2.13	1.94	1.84	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
16	6.31	3.49	2.75	2.42	2.23	2.02	1.91	1.83	1.78	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
20	6.81	3.68	*2.87*	2.51	2.30	2.08	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
30	7.82	4.03	3.09	2.68	2.45	2.20	2.07	1.97	1.91	1.87	1.84	1.82	1.80	1.77	1.76	1.74	1.73	1.72	1.71	1.70
40	8.62	4.29	3.25	2.80	2.55	2.28	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
50	9.30	4.51	3.38	*2.90*	2.63	2.35	2.20	2.09	2.03	1.98	1.95	1.92	1.90	1.87	1.85	1.83	1.82	1.81	1.80	1.79
60	9.88	4.69	3.49	*2.98*	2.70	2.40	2.25	2.14	2.06	2.02	1.98	1.95	1.93	1.90	1.88	1.87	1.85	1.84	1.83	1.82
75	10.66	4.92	3.62	3.08	2.78	2.47	2.30	2.19	2.11	2.06	2.03	2.00	1.98	1.94	1.92	1.90	1.89	1.88	1.86	1.85
100	11.76	5.23	3.80	3.21	*2.89*	2.55	2.38	2.25	2.17	2.12	2.08	2.05	2.03	1.99	1.97	1.95	1.94	1.93	1.91	1.90
125	12.66	5.49	3.95	3.31	*2.97*	2.62	2.43	2.30	2.22	2.17	2.12	2.09	2.07	2.03	2.01	1.99	1.98	1.97	1.95	1.93
150	13.44	5.70	4.06	3.40	*3.04*	2.67	2.48	2.34	2.26	2.20	2.16	2.13	2.10	2.07	2.04	2.02	2.01	2.00	1.98	1.96
175	14.14	5.90	4.17	3.47	3.10	2.71	2.52	2.38	2.29	2.23	2.19	2.16	2.13	2.09	2.07	2.05	2.03	2.02	2.00	1.99
200	14.84	6.05	4.26	3.54	3.15	2.75	2.55	2.41	2.32	2.26	2.21	2.18	2.16	2.12	2.09	2.07	2.06	2.04	2.02	2.01

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.84	2.02	1.74	1.60	1.52	1.41	1.36	1.32	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
2	3.68	2.45	2.05	1.86	1.75	1.62	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
3	4.25	2.71	2.24	2.02	1.88	1.74	1.65	1.59	1.55	1.52	1.50	1.49	1.48	1.46	1.44	1.44	1.43	1.42	1.41	1.40
4	4.71	2.91	2.38	2.13	1.98	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
5	5.09	3.07	2.49	2.22	2.06	1.88	1.79	1.71	1.67	1.64	1.61	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
8	5.99	3.43	2.73	2.40	2.22	2.02	1.91	1.83	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
12	6.89	3.77	2.94	2.57	2.36	2.13	2.01	1.92	1.86	1.83	1.80	1.78	1.76	1.73	1.71	1.70	1.69	1.68	1.67	1.66
16	7.60	4.02	*3.10*	2.69	2.46	2.22	2.09	1.99	1.93	1.89	1.86	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
20	8.20	4.22	3.23	2.79	2.54	2.28	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
30	9.40	4.62	3.46	2.97	2.69	2.40	2.25	2.13	2.06	2.02	1.98	1.95	1.93	1.90	1.88	1.87	1.85	1.84	1.83	1.82
40	10.36	4.91	3.64	*3.09*	2.80	2.48	2.32	2.20	2.13	2.07	2.04	2.01	1.99	1.96	1.93	1.92	1.90	1.89	1.88	1.86
50	11.17	5.16	3.78	*3.20*	2.88	2.55	2.38	2.25	2.17	2.12	2.08	2.05	2.03	2.00	1.97	1.95	1.94	1.93	1.91	1.90
60	11.88	5.36	3.89	*3.28*	2.95	2.60	2.42	2.29	2.21	2.16	2.12	2.09	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
75	12.81	5.62	4.04	3.39	3.03	2.67	2.48	2.34	2.26	2.20	2.16	2.13	2.10	2.07	2.04	2.02	2.01	2.00	1.98	1.96
100	14.10	5.98	4.23	3.53	*3.15*	2.75	2.55	2.41	2.32	2.26	2.21	2.18	2.16	2.12	2.09	2.07	2.06	2.04	2.02	2.00
125	15.20	6.26	4.39	3.63	*3.23*	2.82	2.61	2.46	2.37	2.30	2.26	2.22	2.19	2.16	2.13	2.11	2.09	2.08	2.05	2.04
150	16.17	6.50	4.52	3.73	*3.31*	2.88	2.66	2.50	2.40	2.34	2.29	2.26	2.23	2.19	2.16	2.14	2.12	2.11	2.08	2.07
175	17.03	6.72	4.63	3.81	3.37	2.92	2.70	2.54	2.44	2.37	2.32	2.29	2.26	2.21	2.18	2.16	2.15	2.13	2.11	2.09
200	17.81	6.91	4.74	3.88	3.43	2.96	2.73	2.57	2.47	2.40	2.35	2.31	2.28	2.24	2.21	2.18	2.17	2.15	2.13	2.11

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.40	1.72	1.49	1.37	1.30	1.22	1.17	1.14	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.03	1.03
2	3.13	2.11	1.78	1.63	1.53	1.42	1.37	1.32	1.29	1.27	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
3	3.63	2.35	1.96	1.78	1.67	1.54	1.47	1.42	1.39	1.37	1.35	1.34	1.33	1.31	1.30	1.29	1.29	1.28	1.27	1.27
4	4.02	2.53	2.09	1.88	1.76	1.63	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
5	4.36	2.68	2.20	1.97	1.84	1.69	1.61	1.55	1.51	1.48	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
8	5.14	3.00	2.42	2.15	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.49	1.48	1.47	1.46
12	5.91	3.31	*2.62*	2.31	2.13	1.94	1.84	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
16	6.52	3.53	2.76	2.42	2.23	2.02	1.91	1.83	1.78	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
20	7.04	3.72	2.88	2.51	2.31	2.09	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
30	8.08	4.07	3.10	*2.68*	2.45	2.20	2.07	1.97	1.91	1.87	1.84	1.82	1.80	1.78	1.76	1.74	1.73	1.72	1.71	1.70
40	8.91	4.33	3.26	2.80	*2.55*	2.29	2.15	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
50	9.62	4.55	3.39	2.90	*2.63*	2.35	2.20	2.09	2.02	1.98	1.95	1.92	1.90	1.87	1.85	1.83	1.82	1.81	1.80	1.79
60	10.21	4.74	3.50	2.98	*2.70*	2.40	2.25	2.13	2.06	2.02	1.98	1.95	1.93	1.90	1.88	1.87	1.85	1.84	1.83	1.82
75	11.04	4.97	3.64	3.08	2.78	2.47	2.30	2.19	2.11	2.06	2.03	2.00	1.97	1.94	1.92	1.90	1.89	1.88	1.86	1.85
100	12.11	5.27	3.82	3.21	2.89	*2.55*	2.38	2.25	2.17	2.12	2.08	2.05	2.03	2.00	1.97	1.95	1.94	1.93	1.91	1.90
125	13.09	5.54	3.96	3.32	2.97	*2.62*	2.44	2.30	2.22	2.17	2.12	2.09	2.07	2.04	2.01	1.99	1.98	1.97	1.95	1.93
150	13.87	5.76	4.08	3.41	3.04	*2.67*	2.48	2.34	2.26	2.20	2.16	2.13	2.11	2.07	2.04	2.02	2.01	2.00	1.98	1.96
175	14.65	5.96	4.17	3.48	3.10	*2.72*	*2.52*	2.38	2.29	2.23	2.19	2.15	2.13	2.09	2.07	2.05	2.03	2.02	2.00	1.99
200	15.23	6.10	4.27	3.54	3.15	*2.76*	*2.55*	2.41	2.32	2.26	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.04	2.02	2.01

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.01	**2.08**	**1.77**	**1.62**	**1.53**	**1.42**	**1.36**	**1.32**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**	**1.20**	**1.19**	**1.19**	**1.18**
2	3.89	**2.50**	**2.08**	**1.88**	**1.76**	**1.62**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
3	4.49	**2.77**	**2.27**	**2.03**	**1.89**	**1.74**	**1.66**	**1.59**	**1.55**	**1.53**	**1.50**	**1.49**	**1.48**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**	**1.41**	**1.40**
4	4.97	2.97	**2.40**	**2.14**	**1.99**	**1.82**	**1.73**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
5	5.37	3.13	**2.51**	**2.23**	**2.07**	**1.89**	**1.79**	**1.72**	**1.67**	**1.64**	**1.61**	**1.60**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**	**1.51**	**1.50**
8	6.31	3.50	**2.75**	**2.42**	**2.23**	**2.02**	**1.91**	**1.83**	**1.78**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
12	7.25	3.83	*2.97*	**2.58**	**2.37**	**2.14**	**2.01**	**1.92**	**1.87**	**1.83**	**1.80**	**1.78**	**1.76**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
16	8.00	4.09	3.12	**2.70**	**2.47**	**2.22**	**2.09**	**1.99**	**1.93**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
20	8.63	4.29	3.25	**2.80**	**2.55**	**2.28**	**2.14**	**2.04**	**1.98**	**1.93**	**1.90**	**1.88**	**1.86**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
30	9.89	4.69	3.49	*2.98*	**2.70**	**2.40**	**2.25**	**2.13**	**2.06**	**2.02**	**1.98**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.85**	**1.84**	**1.83**	**1.82**
40	10.89	4.99	3.66	3.10	*2.80*	**2.48**	**2.32**	**2.20**	**2.13**	**2.08**	**2.04**	**2.01**	**1.99**	**1.96**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.86**
50	11.77	5.24	3.80	3.21	*2.89*	**2.55**	**2.38**	**2.25**	**2.17**	**2.12**	**2.08**	**2.05**	**2.03**	**2.00**	**1.97**	**1.95**	**1.94**	**1.93**	**1.91**	**1.90**
60	12.50	5.44	3.92	3.29	*2.95*	**2.60**	**2.42**	**2.29**	**2.21**	**2.16**	**2.12**	**2.09**	**2.06**	**2.03**	**2.00**	**1.99**	**1.97**	**1.96**	**1.94**	**1.93**
75	13.48	5.71	4.06	3.40	*3.04*	**2.67**	**2.48**	**2.34**	**2.26**	**2.20**	**2.16**	**2.13**	**2.10**	**2.07**	**2.04**	**2.02**	**2.01**	**2.00**	**1.98**	**1.96**
100	14.84	6.05	4.26	3.54	3.15	**2.76**	**2.55**	**2.41**	**2.32**	**2.26**	**2.22**	**2.18**	**2.15**	**2.12**	**2.09**	**2.07**	**2.06**	**2.04**	**2.02**	**2.00**
125	16.02	6.35	4.42	3.65	3.24	*2.82*	**2.61**	**2.46**	**2.37**	**2.30**	**2.26**	**2.22**	**2.20**	**2.15**	**2.13**	**2.11**	**2.09**	**2.08**	**2.05**	**2.04**
150	16.99	6.59	4.54	3.74	3.31	*2.88*	**2.66**	**2.50**	**2.40**	**2.34**	**2.29**	**2.26**	**2.23**	**2.19**	**2.16**	**2.14**	**2.12**	**2.11**	**2.09**	**2.07**
175	17.97	6.84	4.66	3.81	3.37	*2.93*	**2.70**	**2.54**	**2.44**	**2.37**	**2.32**	**2.28**	**2.26**	**2.22**	**2.19**	**2.16**	**2.15**	**2.13**	**2.11**	**2.09**
200	18.75	7.03	4.76	3.88	3.43	*2.97*	**2.73**	**2.56**	**2.47**	**2.39**	**2.34**	**2.31**	**2.28**	**2.23**	**2.20**	**2.19**	**2.17**	**2.15**	**2.13**	**2.11**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.68	**2.45**	**2.05**	**1.86**	**1.75**	**1.62**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
2	4.71	**2.91**	**2.38**	**2.13**	**1.98**	**1.82**	**1.73**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
3	5.42	3.21	**2.58**	**2.29**	**2.12**	**1.93**	**1.83**	**1.76**	**1.71**	**1.68**	**1.65**	**1.63**	**1.62**	**1.60**	**1.58**	**1.57**	**1.56**	**1.56**	**1.54**	**1.54**
4	5.99	3.43	**2.73**	**2.40**	**2.22**	**2.02**	**1.91**	**1.83**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
5	6.47	3.61	**2.84**	**2.50**	**2.30**	**2.08**	**1.96**	**1.88**	**1.82**	**1.79**	**1.76**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**
8	7.60	4.02	*3.10*	**2.69**	**2.46**	**2.22**	**2.09**	**1.99**	**1.93**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
12	8.72	4.40	3.33	**2.87**	**2.61**	**2.33**	**2.19**	**2.08**	**2.02**	**1.97**	**1.94**	**1.91**	**1.89**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
16	9.61	4.68	3.50	**2.99**	**2.71**	**2.42**	**2.26**	**2.15**	**2.08**	**2.03**	**1.99**	**1.97**	**1.95**	**1.91**	**1.89**	**1.88**	**1.87**	**1.86**	**1.84**	**1.83**
20	10.36	4.91	3.64	*3.09*	**2.80**	**2.48**	**2.32**	**2.20**	**2.13**	**2.07**	**2.04**	**2.01**	**1.99**	**1.96**	**1.93**	**1.92**	**1.90**	**1.89**	**1.87**	**1.86**
30	11.89	5.36	3.89	*3.28*	**2.95**	**2.60**	**2.42**	**2.29**	**2.21**	**2.16**	**2.12**	**2.09**	**2.06**	**2.03**	**2.00**	**1.99**	**1.97**	**1.96**	**1.94**	**1.93**
40	13.09	5.70	4.08	3.42	**3.06**	**2.69**	**2.50**	**2.36**	**2.27**	**2.21**	**2.17**	**2.14**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.97**
50	14.11	5.97	4.24	3.52	*3.15*	**2.75**	**2.55**	**2.41**	**2.32**	**2.26**	**2.21**	**2.18**	**2.15**	**2.12**	**2.09**	**2.07**	**2.06**	**2.04**	**2.02**	**2.00**
60	14.99	6.20	4.36	3.61	*3.22*	**2.81**	**2.60**	**2.45**	**2.36**	**2.29**	**2.25**	**2.22**	**2.19**	**2.15**	**2.12**	**2.10**	**2.08**	**2.07**	**2.04**	**2.04**
75	16.16	6.49	4.52	3.73	*3.31*	**2.87**	**2.66**	**2.50**	**2.40**	**2.34**	**2.29**	**2.26**	**2.23**	**2.19**	**2.16**	**2.14**	**2.12**	**2.11**	**2.09**	**2.07**
100	17.77	6.91	4.74	3.88	3.42	**2.97**	**2.73**	**2.57**	**2.47**	**2.40**	**2.35**	**2.31**	**2.28**	**2.24**	**2.21**	**2.19**	**2.17**	**2.15**	**2.13**	**2.11**
125	19.14	7.23	4.91	3.99	3.52	**3.03**	**2.79**	**2.62**	**2.51**	**2.44**	**2.39**	**2.35**	**2.32**	**2.27**	**2.24**	**2.22**	**2.20**	**2.19**	**2.16**	**2.15**
150	20.31	7.52	5.05	4.09	3.59	*3.09*	**2.84**	**2.66**	**2.55**	**2.48**	**2.42**	**2.38**	**2.35**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.19**	**2.17**
175	21.48	7.76	5.18	4.17	3.66	*3.14*	**2.88**	**2.69**	**2.58**	**2.50**	**2.45**	**2.41**	**2.38**	**2.33**	**2.29**	**2.27**	**2.26**	**2.24**	**2.22**	**2.20**
200	22.46	7.96	5.27	4.25	3.71	*3.17*	**2.92**	**2.72**	**2.61**	**2.53**	**2.48**	**2.44**	**2.40**	**2.35**	**2.32**	**2.29**	**2.28**	**2.26**	**2.23**	**2.22**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.13	2.11	1.78	1.63	1.53	1.42	1.37	1.32	1.29	1.27	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18
2	4.02	2.53	2.09	1.88	1.76	1.63	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
3	4.65	2.80	2.28	2.04	1.90	1.74	1.66	1.59	1.55	1.53	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40
4	5.14	3.00	2.42	2.15	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.49	1.48	1.47	1.46
5	5.55	3.17	2.53	2.24	2.07	1.89	1.79	1.72	1.67	1.64	1.62	1.60	1.58	1.56	1.55	1.54	1.53	1.52	1.51	1.50
8	6.52	3.53	2.76	2.42	2.23	2.02	1.91	1.83	1.78	1.74	1.71	1.69	1.68	1.66	1.64	1.63	1.62	1.61	1.60	1.59
12	7.49	3.87	2.98	*2.59*	2.37	2.14	2.01	1.92	1.87	1.83	1.80	1.78	1.76	1.73	1.72	1.70	1.69	1.68	1.67	1.66
16	8.26	4.13	3.14	*2.71*	2.47	2.22	2.09	1.99	1.93	1.89	1.86	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
20	8.91	4.34	3.26	2.81	*2.55*	2.29	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
30	10.22	4.74	3.50	2.98	*2.70*	2.40	2.25	2.14	2.06	2.02	1.98	1.95	1.93	1.90	1.88	1.87	1.85	1.84	1.83	1.82
40	11.25	5.04	3.68	3.11	2.81	2.49	2.32	2.20	2.13	2.07	2.04	2.01	1.99	1.96	1.93	1.92	1.90	1.89	1.88	1.86
50	12.13	5.29	3.82	3.21	2.89	*2.55*	2.38	2.25	2.17	2.12	2.08	2.05	2.03	2.00	1.97	1.95	1.94	1.93	1.91	1.90
60	12.89	5.49	3.93	3.30	2.96	*2.60*	2.42	2.29	2.21	2.16	2.12	2.09	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
75	13.89	5.76	4.08	3.41	3.04	*2.67*	2.48	2.34	2.26	2.20	2.16	2.13	2.10	2.07	2.04	2.02	2.01	2.00	1.98	1.96
100	15.35	6.12	4.28	3.54	3.16	*2.76*	*2.56*	2.41	2.32	2.26	2.22	2.18	2.16	2.12	2.09	2.07	2.05	2.04	2.02	2.00
125	16.52	6.42	4.42	3.65	3.24	2.83	*2.61*	2.46	2.37	2.30	2.26	2.22	2.20	2.15	2.13	2.11	2.09	2.08	2.05	2.03
150	17.58	6.65	4.57	3.75	3.31	2.88	*2.66*	*2.50*	2.40	2.34	2.29	2.26	2.23	2.19	2.16	2.14	2.12	2.11	2.08	2.07
175	18.52	6.86	4.69	3.82	3.38	2.93	*2.70*	*2.53*	2.44	2.37	2.32	2.29	2.26	2.21	2.18	2.16	2.15	2.13	2.11	2.09
200	19.22	7.09	4.78	3.90	3.43	2.97	*2.73*	*2.56*	2.47	2.40	2.34	2.31	2.28	2.23	2.20	2.18	2.17	2.15	2.13	2.12

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.89	2.50	2.08	1.88	1.76	1.62	1.55	1.49	1.46	1.43	1.41	1.40	1.39	1.37	1.36	1.35	1.35	1.34	1.33	1.32
2	4.97	2.97	2.40	2.14	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
3	5.72	3.27	2.60	2.30	2.13	1.94	1.84	1.76	1.71	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.56	1.54	1.54
4	6.31	3.50	2.75	2.42	2.23	2.02	1.91	1.83	1.78	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
5	6.81	3.68	*2.87*	2.51	2.30	2.08	1.97	1.88	1.83	1.79	1.76	1.74	1.72	1.70	1.68	1.67	1.66	1.65	1.64	1.63
8	8.00	4.09	3.12	2.70	2.47	2.22	2.09	1.99	1.93	1.89	1.86	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
12	9.18	4.47	3.35	*2.88*	2.61	2.34	2.19	2.08	2.02	1.97	1.94	1.91	1.89	1.86	1.84	1.83	1.82	1.81	1.79	1.78
16	10.11	4.76	3.52	*3.00*	2.72	2.42	2.26	2.15	2.08	2.03	1.99	1.97	1.95	1.92	1.89	1.88	1.86	1.86	1.84	1.83
20	10.90	4.99	3.66	3.10	*2.80*	2.48	2.32	2.20	2.13	2.07	2.04	2.01	1.99	1.96	1.93	1.92	1.90	1.89	1.88	1.86
30	12.48	5.44	3.92	3.29	*2.96*	2.60	2.42	2.29	2.21	2.16	2.12	2.09	2.06	2.03	2.00	1.98	1.97	1.96	1.94	1.93
40	13.77	5.79	4.11	3.43	3.07	2.69	2.50	2.36	2.27	2.21	2.17	2.14	2.11	2.08	2.05	2.03	2.02	2.01	1.99	1.97
50	14.82	6.06	4.26	3.54	3.15	2.76	2.55	2.41	2.32	2.26	2.22	2.18	2.16	2.12	2.09	2.07	2.06	2.04	2.02	2.00
60	15.76	6.30	4.39	3.63	3.22	*2.81*	2.60	2.45	2.36	2.30	2.25	2.22	2.19	2.15	2.12	2.10	2.08	2.07	2.04	2.04
75	16.99	6.59	4.55	3.74	3.31	*2.88*	2.66	2.50	2.41	2.34	2.29	2.26	2.23	2.19	2.16	2.14	2.12	2.11	2.09	2.07
100	18.75	7.00	4.76	3.88	3.43	*2.97*	2.73	2.57	2.46	2.40	2.35	2.31	2.28	2.24	2.20	2.18	2.17	2.15	2.13	2.11
125	20.16	7.32	4.94	4.00	3.52	*3.03*	*2.79*	2.62	2.51	2.44	2.39	2.35	2.32	2.27	2.24	2.22	2.20	2.19	2.16	2.15
150	21.33	7.62	5.07	4.10	3.60	*3.09*	*2.84*	2.66	2.55	2.48	2.42	2.38	2.35	2.31	2.27	2.25	2.23	2.22	2.19	2.18
175	22.50	7.85	5.20	4.19	3.66	3.13	*2.88*	2.70	2.58	2.50	2.45	2.41	2.38	2.33	2.30	2.27	2.26	2.24	2.22	2.20
200	23.44	8.09	5.30	4.25	3.72	3.18	*2.92*	2.72	2.61	2.53	2.48	2.43	2.40	2.35	2.32	2.29	2.28	2.26	2.23	2.22

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.71	**2.91**	**2.38**	**2.13**	**1.98**	**1.82**	**1.73**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.46**
2	5.99	3.43	**2.73**	**2.40**	**2.22**	**2.02**	**1.91**	**1.83**	**1.77**	**1.74**	**1.71**	**1.69**	**1.68**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
3	6.89	3.77	**2.94**	**2.57**	**2.36**	**2.13**	**2.01**	**1.92**	**1.86**	**1.83**	**1.80**	**1.78**	**1.76**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**
4	7.60	4.02	*3.10*	**2.69**	**2.46**	**2.22**	**2.09**	**1.99**	**1.93**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.72**	**1.71**
5	8.20	4.22	3.23	**2.79**	**2.54**	**2.28**	**2.14**	**2.04**	**1.98**	**1.93**	**1.90**	**1.88**	**1.86**	**1.83**	**1.81**	**1.79**	**1.78**	**1.77**	**1.76**	**1.75**
8	9.61	4.68	3.50	**2.99**	**2.71**	**2.42**	**2.26**	**2.15**	**2.08**	**2.03**	**1.99**	**1.97**	**1.95**	**1.91**	**1.89**	**1.88**	**1.86**	**1.86**	**1.84**	**1.83**
12	11.02	5.11	3.75	*3.18*	**2.86**	**2.54**	**2.37**	**2.24**	**2.16**	**2.11**	**2.07**	**2.04**	**2.02**	**1.99**	**1.96**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**
16	12.14	5.43	3.93	3.31	**2.97**	**2.62**	**2.44**	**2.31**	**2.23**	**2.17**	**2.13**	**2.10**	**2.07**	**2.04**	**2.01**	**2.00**	**1.98**	**1.97**	**1.95**	**1.94**
20	13.08	5.70	4.08	3.42	**3.06**	**2.69**	**2.50**	**2.36**	**2.27**	**2.21**	**2.17**	**2.14**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.97**
30	15.00	6.20	4.36	3.61	*3.22*	**2.81**	**2.60**	**2.45**	**2.36**	**2.30**	**2.25**	**2.21**	**2.19**	**2.15**	**2.12**	**2.10**	**2.08**	**2.07**	**2.04**	**2.04**
40	16.52	6.59	4.57	3.76	*3.33*	**2.90**	**2.68**	**2.52**	**2.42**	**2.35**	**2.30**	**2.27**	**2.24**	**2.20**	**2.17**	**2.15**	**2.13**	**2.12**	**2.10**	**2.08**
50	17.81	6.90	4.73	3.87	3.42	**2.96**	**2.73**	**2.57**	**2.46**	**2.40**	**2.35**	**2.31**	**2.28**	**2.24**	**2.21**	**2.18**	**2.17**	**2.15**	**2.13**	**2.11**
60	18.93	7.18	4.87	3.97	3.50	**3.02**	**2.78**	**2.61**	**2.50**	**2.43**	**2.38**	**2.34**	**2.31**	**2.27**	**2.24**	**2.21**	**2.20**	**2.18**	**2.16**	**2.14**
75	20.39	7.50	5.05	4.09	3.59	*3.09*	**2.84**	**2.66**	**2.55**	**2.48**	**2.42**	**2.38**	**2.35**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.19**	**2.18**
100	22.50	7.97	5.27	4.25	3.71	*3.18*	**2.92**	**2.72**	**2.61**	**2.53**	**2.48**	**2.44**	**2.40**	**2.35**	**2.32**	**2.29**	**2.27**	**2.26**	**2.23**	**2.22**
125	24.14	8.35	5.46	4.37	3.81	*3.25*	**2.97**	**2.78**	**2.66**	**2.57**	**2.52**	**2.48**	**2.44**	**2.39**	**2.35**	**2.33**	**2.31**	**2.30**	**2.27**	**2.25**
150	25.78	8.67	5.62	4.48	3.90	*3.31*	**3.02**	**2.82**	**2.70**	**2.61**	**2.55**	**2.50**	**2.47**	**2.42**	**2.38**	**2.36**	**2.34**	**2.32**	**2.29**	**2.27**
175	27.19	8.96	5.77	4.57	3.96	*3.35*	**3.06**	**2.85**	**2.72**	**2.64**	**2.58**	**2.53**	**2.50**	**2.45**	**2.41**	**2.38**	**2.36**	**2.34**	**2.32**	**2.30**
200	28.12	9.20	5.86	4.64	4.01	*3.40*	*3.10*	**2.89**	**2.75**	**2.67**	**2.60**	**2.56**	**2.52**	**2.47**	**2.43**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.02	**2.53**	**2.09**	**1.88**	**1.76**	**1.63**	**1.55**	**1.49**	**1.46**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.35**	**1.34**	**1.33**	**1.32**
2	5.14	3.00	**2.42**	**2.15**	**1.99**	**1.82**	**1.73**	**1.66**	**1.62**	**1.59**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**	**1.49**	**1.48**	**1.47**	**1.46**
3	5.91	3.31	*2.62*	**2.31**	**2.13**	**1.94**	**1.84**	**1.76**	**1.71**	**1.68**	**1.65**	**1.64**	**1.62**	**1.60**	**1.58**	**1.57**	**1.56**	**1.56**	**1.54**	**1.54**
4	6.52	3.53	2.76	**2.42**	**2.23**	**2.02**	**1.91**	**1.83**	**1.78**	**1.74**	**1.71**	**1.69**	**1.68**	**1.66**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
5	7.04	3.72	2.88	**2.51**	**2.31**	**2.09**	**1.97**	**1.88**	**1.83**	**1.79**	**1.76**	**1.74**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**
8	8.26	4.13	3.14	*2.71*	**2.47**	**2.22**	**2.09**	**1.99**	**1.93**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.75**	**1.74**	**1.74**	**1.72**	**1.71**
12	9.48	4.51	3.37	2.88	*2.62*	**2.34**	**2.19**	**2.08**	**2.02**	**1.97**	**1.94**	**1.91**	**1.89**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
16	10.44	4.80	3.54	3.01	*2.72*	**2.42**	**2.26**	**2.15**	**2.08**	**2.03**	**1.99**	**1.97**	**1.95**	**1.92**	**1.89**	**1.88**	**1.86**	**1.86**	**1.84**	**1.83**
20	11.25	5.04	3.68	3.11	2.81	**2.49**	**2.32**	**2.20**	**2.13**	**2.07**	**2.04**	**2.01**	**1.99**	**1.96**	**1.93**	**1.92**	**1.90**	**1.89**	**1.87**	**1.86**
30	12.90	5.49	3.93	3.30	2.96	*2.60*	**2.42**	**2.29**	**2.21**	**2.16**	**2.12**	**2.09**	**2.06**	**2.03**	**2.00**	**1.99**	**1.97**	**1.96**	**1.94**	**1.93**
40	14.20	5.84	4.12	3.43	3.07	*2.69*	**2.50**	**2.36**	**2.27**	**2.21**	**2.17**	**2.14**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.97**
50	15.31	6.12	4.27	3.54	3.15	*2.76*	*2.55*	**2.41**	**2.32**	**2.26**	**2.21**	**2.18**	**2.16**	**2.12**	**2.09**	**2.07**	**2.05**	**2.04**	**2.02**	**2.00**
60	16.27	6.36	4.40	3.63	3.23	2.81	*2.60*	**2.45**	**2.36**	**2.29**	**2.25**	**2.21**	**2.19**	**2.15**	**2.12**	**2.10**	**2.08**	**2.07**	**2.04**	**2.02**
75	17.54	6.66	4.57	3.74	3.32	2.88	*2.66*	*2.50*	**2.40**	**2.34**	**2.29**	**2.26**	**2.23**	**2.19**	**2.16**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**
100	19.30	7.07	4.78	3.89	3.43	2.97	*2.73*	*2.57*	**2.46**	**2.40**	**2.35**	**2.31**	**2.28**	**2.24**	**2.21**	**2.18**	**2.17**	**2.15**	**2.13**	**2.11**
125	20.78	7.40	4.95	4.01	3.53	3.03	*2.79*	*2.62*	*2.51*	**2.44**	**2.39**	**2.35**	**2.32**	**2.27**	**2.24**	**2.22**	**2.20**	**2.19**	**2.16**	**2.15**
150	22.11	7.70	5.09	4.11	3.60	3.09	2.84	*2.66*	*2.55*	**2.48**	**2.42**	**2.38**	**2.35**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.19**	**2.18**
175	23.28	7.93	5.21	4.19	3.66	3.14	2.88	*2.70*	*2.58*	*2.50*	**2.45**	**2.41**	**2.38**	**2.33**	**2.30**	**2.28**	**2.26**	**2.25**	**2.21**	**2.20**
200	24.38	8.16	5.32	4.26	3.72	3.18	2.92	*2.72*	*2.61*	*2.53*	**2.48**	**2.43**	**2.40**	**2.35**	**2.32**	**2.30**	**2.28**	**2.27**	**2.23**	**2.22**

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.97	2.97	2.40	2.14	1.99	1.82	1.73	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.50	1.49	1.48	1.48	1.47	1.46
2	6.31	3.49	2.75	2.42	2.23	2.02	1.91	1.83	1.78	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
3	7.25	3.83	*2.97*	2.58	2.37	2.14	2.01	1.92	1.87	1.83	1.80	1.78	1.76	1.73	1.72	1.70	1.69	1.68	1.67	1.66
4	8.00	4.09	3.12	2.70	2.47	2.22	2.09	1.99	1.93	1.89	1.86	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
5	8.63	4.29	3.25	2.80	2.55	2.28	2.14	2.04	1.98	1.93	1.90	1.88	1.86	1.83	1.81	1.79	1.78	1.77	1.76	1.75
8	10.11	4.76	3.53	*3.00*	2.72	2.42	2.26	2.15	2.08	2.03	1.99	1.97	1.95	1.91	1.89	1.88	1.86	1.86	1.84	1.83
12	11.59	5.19	3.78	3.19	*2.87*	2.54	2.37	2.24	2.16	2.11	2.07	2.04	2.02	1.99	1.96	1.95	1.93	1.92	1.90	1.89
16	12.76	5.52	3.96	3.32	*2.98*	2.62	2.44	2.31	2.23	2.17	2.13	2.10	2.07	2.04	2.01	2.00	1.98	1.97	1.95	1.94
20	13.76	5.79	4.11	3.43	3.07	2.69	2.50	2.36	2.27	2.21	2.17	2.14	2.11	2.08	2.05	2.03	2.02	2.01	1.99	1.97
30	15.76	6.30	4.39	3.63	3.22	*2.81*	2.60	2.45	2.36	2.29	2.25	2.21	2.19	2.15	2.12	2.10	2.08	2.07	2.04	2.02
40	17.34	6.69	4.59	3.77	3.34	*2.90*	2.68	2.52	2.42	2.35	2.30	2.27	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.08
50	18.71	7.01	4.76	3.89	3.43	*2.97*	2.73	2.57	2.46	2.40	2.35	2.31	2.28	2.24	2.21	2.18	2.17	2.15	2.13	2.11
60	19.88	7.28	4.90	3.98	3.51	*3.02*	2.78	2.61	2.50	2.43	2.38	2.34	2.31	2.27	2.24	2.21	2.20	2.18	2.16	2.14
75	21.41	7.62	5.08	4.10	3.60	*3.09*	*2.84*	2.66	2.55	2.48	2.42	2.38	2.35	2.30	2.27	2.25	2.23	2.22	2.19	2.18
100	23.59	8.09	5.31	4.26	3.72	3.18	*2.92*	2.72	2.61	2.53	2.48	2.43	2.40	2.35	2.32	2.29	2.28	2.26	2.23	2.22
125	25.47	8.48	5.49	4.38	3.82	3.25	*2.97*	2.77	2.66	2.57	2.52	2.47	2.44	2.39	2.36	2.33	2.31	2.29	2.27	2.25
150	27.03	8.79	5.64	4.49	3.90	3.31	*3.02*	*2.82*	2.70	2.61	2.55	2.50	2.47	2.42	2.39	2.36	2.34	2.32	2.29	2.28
175	28.44	9.06	5.78	4.57	3.96	3.36	*3.06*	*2.85*	2.72	2.64	2.58	2.53	2.50	2.45	2.41	2.39	2.36	2.34	2.32	2.30
200	29.69	9.30	5.90	4.65	4.02	3.40	*3.10*	*2.88*	2.75	2.67	2.60	2.56	2.52	2.47	2.43	2.41	2.39	2.36	2.34	2.32

Table 19-13. κ-Multipliers for Modified Calif. Intrawell Prediction Limits on Observations (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.99	3.43	2.73	2.40	2.22	2.02	1.91	1.83	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	1.60	1.59
2	7.60	4.02	*3.10*	2.69	2.46	2.22	2.09	1.99	1.93	1.89	1.86	1.83	1.81	1.79	1.77	1.75	1.74	1.73	1.72	1.71
3	8.72	4.40	3.33	2.87	2.61	2.33	2.19	2.08	2.02	1.97	1.94	1.91	1.89	1.86	1.84	1.83	1.82	1.81	1.79	1.78
4	9.61	4.68	3.50	2.99	2.71	2.42	2.26	2.15	2.08	2.03	1.99	1.97	1.95	1.91	1.89	1.88	1.86	1.86	1.84	1.83
5	10.36	4.91	3.64	*3.09*	2.80	2.48	2.32	2.20	2.13	2.07	2.04	2.01	1.99	1.96	1.93	1.92	1.90	1.89	1.87	1.86
8	12.14	5.43	3.93	3.31	2.97	2.62	2.44	2.31	2.23	2.17	2.13	2.10	2.07	2.04	2.01	2.00	1.98	1.97	1.95	1.94
12	13.91	5.92	4.21	3.51	*3.13*	2.74	2.54	2.40	2.31	2.25	2.21	2.17	2.15	2.11	2.08	2.06	2.05	2.04	2.02	2.00
16	15.32	6.29	4.41	3.65	*3.24*	2.83	2.62	2.47	2.37	2.31	2.26	2.23	2.20	2.16	2.13	2.11	2.09	2.08	2.06	2.05
20	16.50	6.59	4.57	3.76	*3.33*	2.90	2.67	2.52	2.42	2.35	2.30	2.27	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.08
30	18.91	7.17	4.87	3.97	3.50	3.02	2.78	2.61	2.50	2.43	2.38	2.34	2.31	2.27	2.24	2.21	2.20	2.18	2.16	2.14
40	20.82	7.61	5.10	4.12	3.62	*3.11*	2.85	2.67	2.56	2.49	2.43	2.39	2.36	2.32	2.28	2.26	2.24	2.23	2.20	2.18
50	22.42	7.97	5.28	4.25	3.72	*3.18*	2.91	2.72	2.61	2.53	2.48	2.43	2.40	2.35	2.32	2.29	2.28	2.26	2.23	2.22
60	23.83	8.28	5.43	4.35	3.79	*3.24*	2.96	2.77	2.65	2.57	2.51	2.47	2.43	2.38	2.35	2.32	2.30	2.29	2.26	2.24
75	25.70	8.67	5.62	4.47	3.89	*3.31*	3.02	2.82	2.69	2.61	2.55	2.51	2.47	2.42	2.38	2.36	2.34	2.32	2.29	2.27
100	28.28	9.18	5.88	4.64	4.02	*3.40*	*3.10*	2.88	2.75	2.67	2.60	2.56	2.52	2.47	2.43	2.40	2.38	2.36	2.33	2.31
125	30.47	9.61	6.07	4.78	4.12	3.47	*3.15*	2.93	2.80	2.71	2.64	2.60	2.56	2.50	2.47	2.44	2.41	2.40	2.37	2.34
150	32.34	10.00	6.25	4.88	4.20	3.54	*3.20*	2.97	2.84	2.74	2.68	2.63	2.59	2.53	2.50	2.47	2.44	2.42	2.39	2.37
175	34.06	10.31	6.41	4.98	4.28	3.58	*3.25*	3.01	2.87	2.77	2.71	2.66	2.62	2.56	2.52	2.49	2.47	2.45	2.42	2.40
200	35.62	10.59	6.52	5.06	4.34	3.62	*3.28*	3.04	2.90	2.80	2.73	2.68	2.64	2.58	2.54	2.51	2.49	2.47	2.44	2.41

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D STATISTICAL TABLES

D.3 TABLES FROM CHAPTER 19: INTRAWELL PREDICTION LIMITS FOR FUTURE MEANS

TABLE 19-14 κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Mean Order 2.....D-156

TABLE 19-15 κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Mean Order 2..... D-165

TABLE 19-16 κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Mean Order 2..... D-174

TABLE 19-17 κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Mean Order 3..... D-183

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Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (1 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.42	1.20	1.12	1.07	1.04	1.01	0.98	0.97	0.96	0.95	0.94	0.94	0.94	0.93	0.93	0.93	0.92	0.92	0.92	0.92
2	2.01	1.63	1.48	1.41	1.36	1.30	1.27	1.25	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18	1.18	1.18	1.17	1.17
3	2.41	1.88	1.70	1.60	1.54	1.47	1.43	1.40	1.38	1.37	1.35	1.35	1.34	1.33	1.32	1.32	1.32	1.31	1.31	1.30
4	2.71	2.07	1.85	1.73	1.66	1.58	1.54	1.50	1.48	1.46	1.45	1.44	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39
5	2.97	2.22	1.97	1.84	1.76	1.67	1.62	1.58	1.56	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.48	1.47	1.47	1.46
8	3.55	2.55	2.22	2.06	1.96	1.85	1.79	1.74	1.71	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.60	1.60
12	4.13	2.85	2.45	2.25	2.13	2.00	1.93	1.88	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.73	1.72	1.71
16	4.59	3.07	2.61	2.39	2.26	2.11	2.03	1.97	1.93	1.90	1.88	1.87	1.86	1.84	1.83	1.82	1.81	1.80	1.79	1.79
20	4.97	3.25	2.74	2.49	2.35	2.19	2.10	2.04	2.00	1.97	1.95	1.93	1.92	1.90	1.89	1.88	1.87	1.86	1.85	1.84
30	5.74	3.59	2.98	2.69	2.52	2.34	2.24	2.17	2.12	2.09	2.06	2.04	2.03	2.01	1.99	1.98	1.97	1.97	1.95	1.95
40	6.34	3.85	3.15	2.83	2.65	2.45	2.34	2.26	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.05	2.04	2.02	2.02
50	6.85	4.06	3.29	2.94	2.75	2.53	2.41	2.32	2.27	2.23	2.20	2.18	2.16	2.14	2.12	2.11	2.10	2.09	2.08	2.07
60	7.30	4.23	3.41	*3.04*	2.83	2.59	2.47	2.38	2.32	2.28	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.14	2.12	2.11
75	7.88	4.46	3.56	*3.15*	2.92	2.68	2.54	2.45	2.38	2.34	2.31	2.29	2.27	2.24	2.22	2.21	2.20	2.19	2.17	2.16
100	8.69	4.76	3.75	3.30	*3.05*	2.78	2.64	2.53	2.47	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.26	2.26	2.24	2.23
125	9.38	5.00	3.90	3.42	*3.15*	2.86	2.71	2.60	2.53	2.48	2.44	2.42	2.40	2.37	2.34	2.33	2.32	2.31	2.29	2.28
150	9.98	5.21	4.03	3.52	3.23	2.93	2.77	2.65	2.58	2.53	2.49	2.46	2.44	2.41	2.39	2.37	2.36	2.35	2.33	2.32
175	10.51	5.39	4.14	3.60	3.30	*2.99*	2.82	2.70	2.62	2.57	2.53	2.50	2.48	2.45	2.42	2.41	2.39	2.38	2.36	2.35
200	11.00	5.54	4.24	3.67	3.36	*3.04*	2.86	2.74	2.66	2.60	2.56	2.53	2.51	2.48	2.45	2.44	2.42	2.41	2.39	2.38

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (1 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.89	1.58	1.45	1.38	1.34	1.29	1.26	1.24	1.23	1.22	1.21	1.20	1.20	1.19	1.18	1.18	1.18	1.18	1.17	1.17
2	2.57	2.02	1.82	1.71	1.65	1.57	1.53	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.41	1.40	1.40	1.39
3	3.03	2.29	2.04	1.91	1.83	1.73	1.68	1.64	1.61	1.60	1.58	1.57	1.56	1.55	1.54	1.54	1.53	1.53	1.52	1.52
4	3.39	2.50	2.19	2.04	1.95	1.84	1.78	1.74	1.71	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.60	1.60
5	3.68	2.66	2.32	2.15	2.04	1.93	1.86	1.81	1.78	1.76	1.74	1.73	1.72	1.70	1.69	1.69	1.68	1.67	1.67	1.66
8	4.38	3.02	2.58	2.37	2.25	2.10	2.03	1.97	1.93	1.90	1.88	1.87	1.86	1.84	1.83	1.82	1.81	1.80	1.79	1.79
12	5.07	3.35	2.82	2.57	2.42	2.25	2.16	2.09	2.05	2.02	2.00	1.98	1.97	1.95	1.93	1.92	1.92	1.91	1.90	1.89
16	5.61	3.59	2.99	2.71	2.54	2.36	2.26	2.18	2.14	2.10	2.08	2.06	2.05	2.02	2.01	2.00	1.99	1.98	1.97	1.96
20	6.07	3.79	3.13	2.82	2.64	2.44	2.33	2.25	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.05	2.04	2.02	2.02
30	6.99	4.17	3.39	3.02	2.82	2.59	2.47	2.38	2.32	2.28	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.14	2.12	2.11
40	7.72	4.46	3.57	3.17	2.94	2.70	2.56	2.46	2.40	2.36	2.33	2.30	2.28	2.26	2.24	2.22	2.21	2.20	2.19	2.18
50	8.33	4.69	3.72	*3.29*	3.04	2.78	2.64	2.53	2.46	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.26	2.26	2.24	2.23
60	8.87	4.89	3.85	*3.39*	3.13	2.84	2.70	2.59	2.52	2.47	2.43	2.41	2.39	2.36	2.33	2.32	2.31	2.30	2.28	2.27
75	9.57	5.14	4.01	3.51	3.23	2.93	2.77	2.65	2.58	2.53	2.49	2.46	2.44	2.41	2.39	2.37	2.36	2.35	2.33	2.32
100	10.55	5.47	4.21	3.66	*3.36*	3.03	2.86	2.74	2.66	2.60	2.56	2.53	2.51	2.48	2.45	2.44	2.42	2.41	2.39	2.38
125	11.38	5.75	4.38	3.79	*3.46*	3.12	2.93	2.80	2.72	2.66	2.62	2.59	2.57	2.53	2.50	2.48	2.47	2.46	2.44	2.43
150	12.10	5.98	4.52	3.89	3.55	3.18	2.99	2.85	2.77	2.71	2.67	2.63	2.61	2.57	2.54	2.53	2.51	2.50	2.48	2.46
175	12.74	6.18	4.64	3.98	3.62	3.24	3.04	2.90	2.81	2.75	2.70	2.67	2.64	2.61	2.58	2.56	2.54	2.53	2.51	2.50
200	13.33	6.36	4.74	4.06	3.68	*3.29*	3.09	2.94	2.85	2.78	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.54	2.52

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.37	1.94	1.77	1.68	1.62	1.56	1.52	1.49	1.47	1.45	1.44	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.40	1.39
2	3.14	2.41	2.15	2.01	1.93	1.83	1.77	1.73	1.70	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.60
3	3.67	2.71	2.37	2.21	2.10	1.98	1.92	1.87	1.83	1.81	1.79	1.78	1.77	1.75	1.74	1.74	1.73	1.72	1.72	1.71
4	4.09	2.93	2.54	2.34	2.23	2.09	2.02	1.96	1.92	1.90	1.88	1.86	1.85	1.84	1.82	1.82	1.81	1.80	1.79	1.79
5	4.43	3.10	2.67	2.45	2.32	2.18	2.09	2.03	1.99	1.96	1.94	1.93	1.92	1.90	1.88	1.88	1.87	1.86	1.85	1.84
8	5.25	3.50	2.95	2.68	2.53	2.35	2.25	2.18	2.13	2.10	2.08	2.06	2.05	2.02	2.01	2.00	1.99	1.98	1.97	1.96
12	6.06	3.86	3.20	2.89	2.70	2.50	2.39	2.31	2.25	2.22	2.19	2.17	2.15	2.13	2.11	2.10	2.09	2.08	2.07	2.06
16	6.70	4.13	3.39	3.03	2.83	2.61	2.48	2.39	2.34	2.30	2.27	2.24	2.23	2.20	2.18	2.17	2.16	2.15	2.14	2.13
20	7.24	4.35	3.53	3.15	2.93	2.69	2.56	2.46	2.40	2.36	2.33	2.30	2.28	2.26	2.24	2.22	2.21	2.20	2.19	2.18
30	8.32	4.78	3.81	3.36	3.11	2.84	2.69	2.58	2.51	2.47	2.43	2.41	2.39	2.36	2.33	2.32	2.31	2.30	2.28	2.27
40	9.18	5.10	4.01	3.52	3.24	2.94	2.79	2.67	2.59	2.54	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
50	9.90	5.35	4.17	*3.64*	3.35	3.03	2.86	2.73	2.66	2.60	2.56	2.53	2.51	2.48	2.45	2.43	2.42	2.41	2.39	2.38
60	10.54	5.57	4.30	3.74	3.43	3.09	2.92	2.79	2.71	2.65	2.61	2.58	2.55	2.52	2.49	2.48	2.46	2.45	2.43	2.42
75	11.36	5.85	4.47	3.87	3.54	3.18	2.99	2.85	2.77	2.71	2.67	2.63	2.61	2.57	2.54	2.53	2.51	2.50	2.48	2.46
100	12.52	6.23	4.70	4.04	*3.67*	3.29	3.08	2.94	2.85	2.78	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.54	2.52
125	13.50	6.53	4.88	4.17	3.78	3.37	3.16	3.00	2.91	2.84	2.79	2.76	2.73	2.69	2.66	2.64	2.62	2.61	2.58	2.57
150	14.35	6.79	5.03	4.28	3.87	3.44	3.22	3.06	2.96	2.89	2.84	2.80	2.77	2.73	2.70	2.67	2.66	2.65	2.62	2.60
175	15.11	7.02	5.16	4.37	3.94	3.50	3.27	3.10	3.00	2.93	2.87	2.84	2.81	2.76	2.73	2.71	2.69	2.68	2.65	2.63
200	15.80	7.22	5.27	4.45	4.01	3.55	3.31	3.14	3.03	2.96	2.91	2.87	2.84	2.79	2.76	2.74	2.72	2.70	2.68	2.66

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (2 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.01	1.63	1.48	1.41	1.36	1.30	1.27	1.25	1.23	1.22	1.21	1.20	1.20	1.19	1.19	1.18	1.18	1.18	1.17	1.17
2	2.71	2.07	1.85	1.73	1.66	1.58	1.54	1.50	1.48	1.46	1.45	1.44	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39
3	3.18	2.35	2.06	1.92	1.84	1.74	1.69	1.64	1.62	1.60	1.58	1.57	1.57	1.55	1.54	1.54	1.53	1.53	1.52	1.52
4	3.55	2.55	2.22	2.06	1.96	1.85	1.79	1.74	1.71	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.60	1.60
5	3.86	2.71	2.34	2.16	2.06	1.93	1.87	1.82	1.78	1.76	1.74	1.73	1.72	1.71	1.69	1.69	1.68	1.68	1.67	1.66
8	4.59	3.07	2.61	2.39	2.26	2.11	2.03	1.97	1.93	1.90	1.88	1.87	1.86	1.84	1.83	1.82	1.81	1.80	1.79	1.79
12	5.30	3.40	2.84	2.58	2.43	2.26	2.17	2.10	2.05	2.02	2.00	1.98	1.97	1.95	1.93	1.92	1.92	1.91	1.90	1.89
16	5.87	3.65	*3.02*	2.72	2.55	2.36	2.26	2.19	2.14	2.11	2.08	2.06	2.05	2.03	2.01	2.00	1.99	1.98	1.97	1.96
20	6.34	3.85	3.15	2.83	2.65	2.45	2.34	2.26	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.05	2.04	2.02	2.02
30	7.30	4.23	3.41	*3.04*	2.83	2.59	2.47	2.38	2.32	2.28	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.14	2.12	2.11
40	8.05	4.52	3.60	3.18	2.95	2.70	2.56	2.47	2.40	2.36	2.33	2.30	2.29	2.26	2.24	2.22	2.21	2.20	2.19	2.18
50	8.69	4.76	3.75	3.30	*3.05*	2.78	2.64	2.53	2.47	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.26	2.26	2.24	2.23
60	9.25	4.95	3.87	3.40	*3.13*	2.85	2.70	2.59	2.52	2.47	2.43	2.41	2.39	2.36	2.33	2.32	2.31	2.30	2.28	2.27
75	9.98	5.21	4.03	3.52	3.23	2.93	2.77	2.65	2.58	2.53	2.49	2.46	2.44	2.41	2.39	2.37	2.36	2.35	2.33	2.32
100	11.00	5.54	4.24	3.67	3.36	*3.04*	2.86	2.74	2.66	2.60	2.56	2.53	2.51	2.48	2.45	2.44	2.42	2.41	2.39	2.38
125	11.86	5.82	4.40	3.80	3.47	*3.12*	2.94	2.80	2.72	2.66	2.62	2.59	2.57	2.53	2.50	2.48	2.47	2.46	2.44	2.43
150	12.61	6.05	4.54	3.90	3.55	*3.19*	*2.99*	2.86	2.77	2.71	2.67	2.63	2.61	2.57	2.55	2.53	2.51	2.50	2.48	2.46
175	13.28	6.26	4.66	3.99	3.63	3.24	*3.04*	2.90	2.81	2.75	2.70	2.67	2.65	2.61	2.58	2.56	2.54	2.53	2.51	2.50
200	13.89	6.44	4.77	4.07	3.69	3.29	*3.09*	2.94	2.85	2.78	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.54	2.51

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.57**	**2.02**	**1.82**	**1.71**	**1.65**	**1.57**	**1.53**	**1.50**	**1.48**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**	**1.42**	**1.41**	**1.41**	**1.40**	**1.40**	**1.39**
2	3.39	**2.50**	**2.19**	**2.04**	**1.95**	**1.84**	**1.78**	**1.74**	**1.71**	**1.69**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**	**1.62**	**1.62**	**1.61**	**1.60**	**1.60**
3	3.94	**2.80**	**2.42**	**2.23**	**2.12**	**2.00**	**1.93**	**1.87**	**1.84**	**1.81**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.74**	**1.73**	**1.73**	**1.72**	**1.71**
4	4.38	**3.02**	**2.58**	**2.37**	**2.25**	**2.10**	**2.03**	**1.97**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.80**	**1.79**	**1.79**
5	4.75	**3.20**	**2.71**	**2.48**	**2.34**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
8	5.61	3.59	**2.99**	**2.71**	**2.54**	**2.36**	**2.26**	**2.18**	**2.14**	**2.10**	**2.08**	**2.06**	**2.05**	**2.02**	**2.01**	**2.00**	**1.99**	**1.98**	**1.97**	**1.96**
12	6.47	3.96	**3.24**	**2.91**	**2.72**	**2.51**	**2.39**	**2.31**	**2.26**	**2.22**	**2.19**	**2.17**	**2.15**	**2.13**	**2.11**	**2.10**	**2.09**	**2.08**	**2.07**	**2.06**
16	7.15	4.24	3.43	**3.06**	**2.85**	**2.61**	**2.49**	**2.40**	**2.34**	**2.30**	**2.27**	**2.25**	**2.23**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**	**2.14**	**2.13**
20	7.72	4.46	3.57	**3.17**	**2.94**	**2.70**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
30	8.87	4.89	3.85	*3.39*	**3.13**	**2.84**	**2.70**	**2.59**	**2.52**	**2.47**	**2.43**	**2.41**	**2.39**	**2.36**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
40	9.78	5.21	4.05	3.54	**3.26**	**2.95**	**2.79**	**2.67**	**2.60**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
50	10.55	5.47	4.21	3.66	*3.36*	**3.03**	**2.86**	**2.74**	**2.66**	**2.60**	**2.56**	**2.53**	**2.51**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.38**
60	11.22	5.70	4.35	3.76	*3.44*	**3.10**	**2.92**	**2.79**	**2.71**	**2.65**	**2.61**	**2.58**	**2.56**	**2.52**	**2.49**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
75	12.10	5.98	4.52	3.89	3.55	**3.18**	**2.99**	**2.85**	**2.77**	**2.71**	**2.67**	**2.63**	**2.61**	**2.57**	**2.54**	**2.53**	**2.51**	**2.50**	**2.48**	**2.46**
100	13.33	6.36	4.74	4.06	3.68	*3.29*	**3.09**	**2.94**	**2.85**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
125	14.36	6.67	4.92	4.19	3.79	*3.37*	**3.16**	**3.00**	**2.91**	**2.84**	**2.79**	**2.76**	**2.73**	**2.69**	**2.66**	**2.64**	**2.62**	**2.61**	**2.58**	**2.57**
150	15.25	6.93	5.07	4.30	3.88	*3.44*	**3.22**	**3.06**	**2.96**	**2.89**	**2.84**	**2.80**	**2.77**	**2.73**	**2.70**	**2.68**	**2.66**	**2.65**	**2.62**	**2.60**
175	16.05	7.16	5.20	4.39	3.96	*3.50*	**3.27**	**3.10**	**3.00**	**2.93**	**2.87**	**2.84**	**2.81**	**2.76**	**2.73**	**2.71**	**2.69**	**2.68**	**2.65**	**2.63**
200	16.80	7.37	5.31	4.47	4.02	3.55	*3.31*	**3.14**	**3.03**	**2.96**	**2.91**	**2.87**	**2.84**	**2.79**	**2.76**	**2.74**	**2.72**	**2.70**	**2.68**	**2.66**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**3.14**	**2.41**	**2.15**	**2.01**	**1.93**	**1.83**	**1.77**	**1.73**	**1.70**	**1.68**	**1.67**	**1.66**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.61**	**1.60**	**1.60**
2	4.09	**2.93**	**2.54**	**2.34**	**2.23**	**2.09**	**2.02**	**1.96**	**1.92**	**1.90**	**1.88**	**1.86**	**1.85**	**1.84**	**1.82**	**1.82**	**1.81**	**1.80**	**1.79**	**1.79**
3	4.74	**3.25**	**2.78**	**2.54**	**2.40**	**2.24**	**2.16**	**2.09**	**2.05**	**2.02**	**2.00**	**1.98**	**1.97**	**1.95**	**1.93**	**1.92**	**1.92**	**1.91**	**1.90**	**1.89**
4	5.25	**3.50**	**2.95**	**2.68**	**2.53**	**2.35**	**2.25**	**2.18**	**2.13**	**2.10**	**2.08**	**2.06**	**2.05**	**2.02**	**2.01**	**2.00**	**1.99**	**1.98**	**1.97**	**1.96**
5	5.68	3.69	**3.09**	**2.80**	**2.62**	**2.43**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**	**2.04**	**2.02**	**2.02**
8	6.70	4.13	**3.39**	**3.03**	**2.83**	**2.61**	**2.48**	**2.39**	**2.34**	**2.30**	**2.27**	**2.24**	**2.23**	**2.20**	**2.18**	**2.17**	**2.16**	**2.15**	**2.14**	**2.13**
12	7.71	4.54	3.65	**3.25**	**3.01**	**2.76**	**2.62**	**2.52**	**2.45**	**2.41**	**2.37**	**2.35**	**2.33**	**2.30**	**2.28**	**2.27**	**2.25**	**2.25**	**2.23**	**2.22**
16	8.51	4.85	3.85	**3.40**	**3.14**	**2.86**	**2.71**	**2.60**	**2.53**	**2.48**	**2.45**	**2.42**	**2.40**	**2.37**	**2.35**	**2.33**	**2.32**	**2.31**	**2.29**	**2.28**
20	9.18	5.10	4.01	**3.52**	**3.24**	**2.94**	**2.79**	**2.67**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
30	10.54	5.57	4.30	3.74	**3.43**	**3.09**	**2.92**	**2.79**	**2.71**	**2.65**	**2.61**	**2.58**	**2.55**	**2.52**	**2.49**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
40	11.61	5.93	4.52	3.91	**3.57**	**3.20**	**3.01**	**2.87**	**2.79**	**2.73**	**2.68**	**2.65**	**2.62**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
50	12.52	6.23	4.70	4.04	*3.67*	**3.29**	**3.08**	**2.94**	**2.85**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
60	13.32	6.48	4.84	4.14	3.76	**3.35**	**3.14**	**2.99**	**2.90**	**2.83**	**2.78**	**2.75**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.60**	**2.57**	**2.56**
75	14.35	6.79	5.03	4.28	3.87	**3.44**	**3.22**	**3.06**	**2.96**	**2.89**	**2.84**	**2.80**	**2.77**	**2.73**	**2.70**	**2.67**	**2.66**	**2.65**	**2.62**	**2.60**
100	15.80	7.22	5.27	4.45	4.01	**3.55**	**3.31**	**3.14**	**3.03**	**2.96**	**2.91**	**2.87**	**2.84**	**2.79**	**2.76**	**2.74**	**2.72**	**2.70**	**2.68**	**2.66**
125	17.02	7.57	5.46	4.59	4.12	*3.63*	**3.38**	**3.20**	**3.09**	**3.02**	**2.96**	**2.92**	**2.89**	**2.84**	**2.81**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**
150	18.10	7.87	5.63	4.71	4.22	*3.70*	**3.44**	**3.26**	**3.14**	**3.06**	**3.01**	**2.96**	**2.93**	**2.88**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
175	19.02	8.12	5.77	4.81	4.29	*3.76*	**3.49**	**3.30**	**3.18**	**3.10**	**3.04**	**3.00**	**2.96**	**2.91**	**2.88**	**2.85**	**2.83**	**2.82**	**2.79**	**2.77**
200	19.90	8.35	5.89	4.89	4.36	*3.82*	**3.54**	**3.34**	**3.22**	**3.13**	**3.07**	**3.03**	**2.99**	**2.94**	**2.91**	**2.88**	**2.85**	**2.84**	**2.81**	**2.79**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.97**	**2.22**	**1.97**	**1.84**	**1.76**	**1.67**	**1.62**	**1.58**	**1.56**	**1.54**	**1.53**	**1.52**	**1.51**	**1.50**	**1.49**	**1.48**	**1.48**	**1.47**	**1.47**	**1.46**
2	3.86	**2.71**	**2.34**	**2.16**	**2.06**	**1.93**	**1.87**	**1.82**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.69**	**1.69**	**1.68**	**1.68**	**1.67**	**1.66**
3	4.48	**3.02**	**2.57**	**2.36**	**2.23**	**2.09**	**2.01**	**1.95**	**1.91**	**1.88**	**1.86**	**1.85**	**1.84**	**1.82**	**1.81**	**1.80**	**1.79**	**1.79**	**1.78**	**1.77**
4	4.97	3.25	**2.74**	**2.49**	**2.35**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
5	5.38	3.44	**2.87**	**2.60**	**2.45**	**2.27**	**2.18**	**2.11**	**2.07**	**2.03**	**2.01**	**1.99**	**1.98**	**1.96**	**1.95**	**1.94**	**1.93**	**1.92**	**1.91**	**1.90**
8	6.34	3.85	3.15	**2.83**	**2.65**	**2.45**	**2.34**	**2.26**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
12	7.30	4.23	3.41	*3.04*	**2.83**	**2.59**	**2.47**	**2.38**	**2.32**	**2.28**	**2.25**	**2.23**	**2.21**	**2.19**	**2.17**	**2.15**	**2.14**	**2.14**	**2.12**	**2.11**
16	8.05	4.52	3.60	3.18	**2.95**	**2.70**	**2.56**	**2.47**	**2.40**	**2.36**	**2.33**	**2.30**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
20	8.69	4.76	3.75	3.30	*3.05*	**2.78**	**2.64**	**2.53**	**2.47**	**2.42**	**2.39**	**2.36**	**2.34**	**2.31**	**2.29**	**2.28**	**2.26**	**2.26**	**2.24**	**2.23**
30	9.98	5.21	4.03	3.52	3.23	**2.93**	**2.77**	**2.65**	**2.58**	**2.53**	**2.49**	**2.46**	**2.44**	**2.41**	**2.39**	**2.37**	**2.36**	**2.35**	**2.33**	**2.32**
40	10.99	5.54	4.24	3.67	3.36	*3.04*	**2.86**	**2.74**	**2.66**	**2.60**	**2.56**	**2.54**	**2.51**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.38**
50	11.86	5.82	4.40	3.80	3.47	*3.12*	**2.94**	**2.80**	**2.72**	**2.66**	**2.62**	**2.59**	**2.57**	**2.53**	**2.50**	**2.48**	**2.47**	**2.46**	**2.44**	**2.43**
60	12.61	6.05	4.54	3.90	3.55	*3.19*	**2.99**	**2.86**	**2.77**	**2.71**	**2.67**	**2.63**	**2.61**	**2.57**	**2.55**	**2.53**	**2.51**	**2.50**	**2.48**	**2.46**
75	13.59	6.35	4.71	4.03	3.66	3.27	*3.07*	**2.92**	**2.83**	**2.77**	**2.72**	**2.69**	**2.66**	**2.62**	**2.59**	**2.57**	**2.56**	**2.55**	**2.52**	**2.51**
100	14.97	6.75	4.94	4.20	3.80	3.38	*3.16*	*3.00*	**2.91**	**2.84**	**2.79**	**2.76**	**2.73**	**2.69**	**2.66**	**2.64**	**2.62**	**2.60**	**2.58**	**2.57**
125	16.14	7.08	5.13	4.33	3.90	3.46	*3.23*	*3.07*	*2.97*	**2.90**	**2.85**	**2.81**	**2.78**	**2.74**	**2.71**	**2.68**	**2.67**	**2.65**	**2.62**	**2.61**
150	17.14	7.35	5.28	4.44	4.00	3.53	3.29	*3.12*	*3.01*	*2.95*	**2.89**	**2.85**	**2.82**	**2.78**	**2.75**	**2.72**	**2.70**	**2.69**	**2.66**	**2.65**
175	18.05	7.59	5.41	4.53	4.06	3.59	3.34	*3.16*	*3.05*	*2.98*	**2.92**	**2.89**	**2.86**	**2.81**	**2.78**	**2.75**	**2.74**	**2.72**	**2.70**	**2.68**
200	18.87	7.81	5.53	4.61	4.13	3.64	3.39	*3.20*	*3.09*	*3.02*	*2.96*	**2.92**	**2.89**	**2.84**	**2.81**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.68	**2.66**	**2.32**	**2.15**	**2.04**	**1.93**	**1.86**	**1.81**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.70**	**1.69**	**1.69**	**1.68**	**1.67**	**1.67**	**1.66**
2	4.75	**3.20**	**2.71**	**2.48**	**2.34**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
3	5.49	3.54	**2.95**	**2.68**	**2.52**	**2.34**	**2.24**	**2.16**	**2.12**	**2.09**	**2.06**	**2.04**	**2.03**	**2.01**	**1.99**	**1.98**	**1.97**	**1.97**	**1.95**	**1.95**
4	6.07	3.79	**3.13**	**2.82**	**2.64**	**2.44**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
5	6.56	4.00	**3.27**	**2.93**	**2.74**	**2.52**	**2.41**	**2.32**	**2.27**	**2.23**	**2.20**	**2.18**	**2.16**	**2.14**	**2.12**	**2.11**	**2.10**	**2.09**	**2.08**	**2.07**
8	7.72	4.46	3.57	**3.17**	**2.94**	**2.70**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
12	8.87	4.89	3.85	*3.39*	**3.13**	**2.84**	**2.70**	**2.59**	**2.52**	**2.47**	**2.43**	**2.41**	**2.39**	**2.36**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
16	9.78	5.21	4.05	3.54	**3.26**	**2.95**	**2.79**	**2.67**	**2.60**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
20	10.55	5.47	4.21	3.66	*3.36*	**3.03**	**2.86**	**2.74**	**2.66**	**2.60**	**2.56**	**2.53**	**2.51**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.38**
30	12.10	5.98	4.52	3.89	3.55	**3.18**	**2.99**	**2.85**	**2.77**	**2.71**	**2.67**	**2.63**	**2.61**	**2.57**	**2.54**	**2.53**	**2.51**	**2.50**	**2.48**	**2.46**
40	13.33	6.36	4.74	4.06	3.68	*3.29*	**3.09**	**2.94**	**2.85**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
50	14.36	6.67	4.92	4.19	3.79	*3.37*	**3.16**	**3.00**	**2.91**	**2.84**	**2.79**	**2.76**	**2.73**	**2.69**	**2.66**	**2.64**	**2.62**	**2.61**	**2.58**	**2.57**
60	15.26	6.93	5.07	4.30	3.88	*3.44*	**3.22**	**3.06**	**2.96**	**2.89**	**2.84**	**2.80**	**2.77**	**2.73**	**2.70**	**2.68**	**2.66**	**2.65**	**2.62**	**2.60**
75	16.44	7.27	5.26	4.43	3.99	3.53	*3.29*	**3.12**	**3.02**	**2.94**	**2.89**	**2.85**	**2.82**	**2.78**	**2.75**	**2.72**	**2.70**	**2.68**	**2.66**	**2.65**
100	18.11	7.72	5.51	4.61	4.13	3.64	*3.39*	**3.20**	**3.09**	**3.02**	**2.96**	**2.92**	**2.89**	**2.84**	**2.81**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**
125	19.51	8.09	5.71	4.75	4.24	3.72	*3.46*	*3.27*	**3.15**	**3.07**	**3.01**	**2.97**	**2.94**	**2.89**	**2.85**	**2.83**	**2.81**	**2.79**	**2.77**	**2.75**
150	20.74	8.41	5.88	4.87	4.34	3.79	*3.52*	*3.32*	**3.20**	**3.12**	**3.06**	**3.01**	**2.98**	**2.93**	**2.89**	**2.87**	**2.85**	**2.83**	**2.80**	**2.78**
175	21.80	8.67	6.03	4.97	4.42	3.85	3.57	*3.36*	**3.24**	**3.15**	**3.09**	**3.05**	**3.01**	**2.96**	**2.92**	**2.90**	**2.88**	**2.86**	**2.83**	**2.81**
200	22.85	8.91	6.15	5.06	4.49	3.91	3.61	*3.40*	*3.28*	**3.19**	**3.13**	**3.08**	**3.04**	**2.99**	**2.95**	**2.92**	**2.90**	**2.89**	**2.86**	**2.84**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.43	**3.10**	**2.67**	**2.45**	**2.32**	**2.18**	**2.09**	**2.03**	**1.99**	**1.96**	**1.94**	**1.93**	**1.92**	**1.90**	**1.88**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
2	5.68	3.69	**3.09**	**2.80**	**2.62**	**2.43**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.12**	**2.10**	**2.08**	**2.07**	**2.05**	**2.04**	**2.04**	**2.02**	**2.02**
3	6.55	4.07	**3.34**	**3.00**	**2.80**	**2.58**	**2.46**	**2.37**	**2.32**	**2.28**	**2.25**	**2.23**	**2.21**	**2.19**	**2.17**	**2.15**	**2.14**	**2.14**	**2.12**	**2.11**
4	7.24	4.35	**3.53**	**3.15**	**2.93**	**2.69**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
5	7.81	4.58	3.68	**3.27**	**3.03**	**2.77**	**2.63**	**2.53**	**2.46**	**2.42**	**2.39**	**2.36**	**2.34**	**2.31**	**2.29**	**2.28**	**2.26**	**2.25**	**2.24**	**2.23**
8	9.18	5.10	4.01	**3.52**	**3.24**	**2.94**	**2.79**	**2.67**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
12	10.54	5.57	4.30	3.74	**3.43**	**3.09**	**2.92**	**2.79**	**2.71**	**2.65**	**2.61**	**2.58**	**2.55**	**2.52**	**2.49**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
16	11.61	5.93	4.52	3.91	**3.57**	**3.20**	**3.01**	**2.87**	**2.79**	**2.73**	**2.68**	**2.65**	**2.62**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
20	12.52	6.23	4.70	4.04	*3.67*	**3.29**	**3.08**	**2.94**	**2.85**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
30	14.35	6.79	5.03	4.28	3.87	**3.44**	**3.22**	**3.06**	**2.96**	**2.89**	**2.84**	**2.80**	**2.77**	**2.73**	**2.70**	**2.67**	**2.66**	**2.65**	**2.62**	**2.60**
40	15.81	7.22	5.27	4.45	4.01	**3.55**	**3.31**	**3.14**	**3.03**	**2.96**	**2.91**	**2.87**	**2.84**	**2.79**	**2.76**	**2.74**	**2.72**	**2.70**	**2.68**	**2.66**
50	17.02	7.57	5.46	4.59	4.12	*3.63*	**3.38**	**3.20**	**3.09**	**3.02**	**2.96**	**2.92**	**2.89**	**2.84**	**2.81**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**
60	18.09	7.87	5.63	4.71	4.22	*3.70*	**3.44**	**3.26**	**3.14**	**3.06**	**3.01**	**2.96**	**2.93**	**2.88**	**2.84**	**2.82**	**2.80**	**2.78**	**2.76**	**2.74**
75	19.48	8.24	5.83	4.85	4.33	*3.79*	**3.52**	**3.32**	**3.20**	**3.12**	**3.06**	**3.01**	**2.98**	**2.93**	**2.89**	**2.87**	**2.85**	**2.83**	**2.80**	**2.78**
100	21.45	8.75	6.10	5.04	4.48	3.90	*3.61*	**3.40**	**3.28**	**3.19**	**3.13**	**3.08**	**3.04**	**2.99**	**2.95**	**2.93**	**2.90**	**2.89**	**2.86**	**2.84**
125	23.09	9.15	6.32	5.19	4.60	3.99	*3.68*	**3.47**	**3.34**	**3.25**	**3.18**	**3.13**	**3.09**	**3.04**	**3.00**	**2.97**	**2.95**	**2.93**	**2.90**	**2.88**
150	24.55	9.49	6.50	5.32	4.69	4.06	*3.74*	**3.52**	**3.39**	**3.29**	**3.22**	**3.18**	**3.13**	**3.07**	**3.03**	**3.00**	**2.98**	**2.96**	**2.93**	**2.91**
175	25.84	9.79	6.67	5.43	4.78	4.12	*3.80*	**3.57**	**3.43**	**3.33**	**3.26**	**3.21**	**3.17**	**3.11**	**3.07**	**3.04**	**3.01**	**2.99**	**2.96**	**2.94**
200	27.07	10.03	6.80	5.52	4.85	4.18	*3.84*	*3.61*	**3.46**	**3.36**	**3.29**	**3.24**	**3.20**	**3.13**	**3.09**	**3.06**	**3.04**	**3.02**	**2.99**	**2.96**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.86	**2.71**	**2.34**	**2.16**	**2.06**	**1.93**	**1.87**	**1.82**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.71**	**1.69**	**1.69**	**1.68**	**1.68**	**1.67**	**1.66**
2	4.97	3.25	**2.74**	**2.49**	**2.35**	**2.19**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**
3	5.74	3.59	**2.98**	**2.69**	**2.52**	**2.34**	**2.24**	**2.17**	**2.12**	**2.09**	**2.06**	**2.04**	**2.03**	**2.01**	**1.99**	**1.98**	**1.97**	**1.97**	**1.95**	**1.95**
4	6.34	3.85	3.15	**2.83**	**2.65**	**2.45**	**2.34**	**2.26**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
5	6.85	4.06	3.29	**2.94**	**2.75**	**2.53**	**2.41**	**2.32**	**2.27**	**2.23**	**2.20**	**2.18**	**2.16**	**2.14**	**2.12**	**2.11**	**2.10**	**2.09**	**2.08**	**2.07**
8	8.05	4.52	3.60	3.18	**2.95**	**2.70**	**2.56**	**2.47**	**2.40**	**2.36**	**2.33**	**2.30**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
12	9.25	4.95	3.87	3.40	*3.13*	**2.85**	**2.70**	**2.59**	**2.52**	**2.47**	**2.43**	**2.41**	**2.39**	**2.36**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
16	10.20	5.28	4.08	3.55	3.26	**2.95**	**2.79**	**2.67**	**2.60**	**2.55**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
20	11.00	5.54	4.24	3.67	3.36	*3.04*	**2.86**	**2.74**	**2.66**	**2.60**	**2.56**	**2.53**	**2.51**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.38**
30	12.61	6.05	4.54	3.90	3.55	*3.19*	*2.99*	**2.86**	**2.77**	**2.71**	**2.67**	**2.63**	**2.61**	**2.57**	**2.55**	**2.53**	**2.51**	**2.50**	**2.48**	**2.46**
40	13.89	6.44	4.77	4.07	3.69	3.29	*3.09*	**2.94**	**2.85**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
50	14.96	6.75	4.94	4.20	3.80	3.38	*3.16*	*3.00*	**2.91**	**2.84**	**2.79**	**2.76**	**2.73**	**2.69**	**2.66**	**2.64**	**2.62**	**2.60**	**2.58**	**2.57**
60	15.90	7.01	5.09	4.31	3.88	3.44	*3.22*	*3.06*	*2.96*	**2.89**	**2.84**	**2.80**	**2.77**	**2.73**	**2.69**	**2.67**	**2.66**	**2.64**	**2.62**	**2.60**
75	17.15	7.35	5.28	4.44	3.99	3.53	3.29	*3.12*	*3.02*	*2.95*	**2.89**	**2.85**	**2.82**	**2.78**	**2.75**	**2.72**	**2.70**	**2.69**	**2.66**	**2.65**
100	18.91	7.82	5.53	4.61	4.13	3.64	3.39	*3.20*	*3.09*	*3.02*	*2.96*	**2.92**	**2.89**	**2.84**	**2.81**	**2.78**	**2.76**	**2.75**	**2.72**	**2.70**
125	20.31	8.19	5.74	4.76	4.25	3.73	3.46	*3.27*	*3.15*	*3.07*	*3.02*	*2.97*	**2.94**	**2.89**	**2.85**	**2.83**	**2.81**	**2.79**	**2.77**	**2.75**
150	21.64	8.51	5.91	4.88	4.35	3.80	3.52	3.32	*3.20*	*3.12*	*3.06*	*3.01*	*2.98*	**2.93**	**2.89**	**2.87**	**2.85**	**2.83**	**2.80**	**2.78**
175	22.81	8.78	6.05	4.98	4.42	3.86	3.57	3.37	*3.24*	*3.16*	*3.10*	*3.05*	*3.01*	*2.96*	**2.91**	**2.90**	**2.88**	**2.86**	**2.83**	**2.81**
200	23.83	9.03	6.18	5.07	4.49	3.91	3.61	3.41	3.28	*3.19*	*3.13*	*3.08*	*3.04*	*2.99*	*2.95*	**2.92**	**2.90**	**2.89**	**2.86**	**2.84**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.75	3.20	2.71	2.48	2.34	2.19	2.10	2.04	2.00	1.97	1.95	1.93	1.92	1.90	1.89	1.88	1.87	1.86	1.85	1.84
2	6.07	3.79	3.13	2.82	2.64	2.44	2.33	2.25	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.05	2.04	2.02	2.02
3	6.99	4.17	3.39	3.02	2.82	2.59	2.47	2.38	2.32	2.28	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.14	2.12	2.11
4	7.72	4.46	3.57	3.17	2.94	2.70	2.56	2.46	2.40	2.36	2.33	2.30	2.28	2.26	2.24	2.22	2.21	2.20	2.19	2.18
5	8.33	4.69	3.72	*3.29*	3.04	2.78	2.64	2.53	2.46	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.26	2.26	2.24	2.23
8	9.78	5.21	4.05	3.54	3.26	2.95	2.79	2.67	2.60	2.54	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
12	11.22	5.70	4.35	3.76	*3.44*	3.10	2.92	2.79	2.71	2.65	2.61	2.58	2.56	2.52	2.49	2.48	2.46	2.45	2.43	2.42
16	12.36	6.06	4.57	3.93	3.58	3.21	3.01	2.87	2.79	2.73	2.68	2.65	2.62	2.59	2.56	2.54	2.52	2.51	2.49	2.48
20	13.33	6.36	4.74	4.06	3.68	*3.29*	3.09	2.94	2.85	2.78	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.54	2.52
30	15.25	6.93	5.07	4.30	3.88	*3.44*	3.22	3.06	2.96	2.89	2.84	2.80	2.77	2.73	2.70	2.68	2.66	2.65	2.62	2.60
40	16.80	7.37	5.31	4.47	4.02	3.55	*3.31*	3.14	3.03	2.96	2.91	2.87	2.84	2.79	2.76	2.74	2.72	2.70	2.68	2.66
50	18.09	7.72	5.51	4.61	4.13	3.64	*3.39*	3.20	3.09	3.02	2.96	2.92	2.89	2.84	2.81	2.78	2.76	2.75	2.72	2.70
60	19.22	8.03	5.67	4.73	4.22	3.71	*3.44*	*3.26*	3.14	3.06	3.00	2.96	2.93	2.88	2.85	2.82	2.80	2.78	2.76	2.74
75	20.70	8.40	5.88	4.87	4.34	3.79	*3.52*	*3.32*	3.20	3.12	3.06	3.01	2.98	2.93	2.89	2.87	2.85	2.83	2.80	2.78
100	22.81	8.91	6.15	5.06	4.49	3.91	3.61	*3.40*	*3.28*	3.19	3.12	3.08	3.04	2.99	2.95	2.92	2.90	2.89	2.86	2.84
125	24.61	9.30	6.38	5.21	4.61	3.99	3.69	*3.47*	*3.34*	3.25	3.17	3.13	3.09	3.04	3.00	2.97	2.95	2.93	2.90	2.88
150	26.09	9.65	6.56	5.34	4.71	4.07	3.75	*3.52*	*3.39*	*3.30*	3.22	3.17	3.13	3.07	3.03	3.00	2.98	2.96	2.93	2.91
175	27.50	9.96	6.72	5.45	4.79	4.13	3.80	*3.57*	*3.43*	*3.33*	3.26	3.21	3.17	3.11	3.07	3.04	3.01	2.99	2.96	2.94
200	28.75	10.23	6.86	5.55	4.86	4.18	3.84	3.61	*3.46*	*3.35*	*3.29*	3.24	3.20	3.13	3.09	3.06	3.04	3.02	2.99	2.96

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.68	3.69	3.09	2.80	2.62	2.43	2.33	2.25	2.20	2.17	2.14	2.12	2.10	2.08	2.07	2.05	2.04	2.04	2.02	2.02
2	7.24	4.35	3.53	3.15	2.93	2.69	2.56	2.46	2.40	2.36	2.33	2.30	2.28	2.26	2.24	2.22	2.21	2.20	2.19	2.18
3	8.32	4.78	3.81	3.36	3.11	2.84	2.69	2.58	2.51	2.47	2.43	2.41	2.39	2.36	2.33	2.32	2.31	2.30	2.28	2.27
4	9.18	5.10	4.01	3.52	3.24	2.94	2.79	2.67	2.59	2.54	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
5	9.90	5.35	4.17	*3.64*	3.35	3.03	2.86	2.73	2.66	2.60	2.56	2.53	2.51	2.48	2.45	2.43	2.42	2.41	2.39	2.38
8	11.61	5.93	4.52	3.91	3.57	3.20	3.01	2.87	2.79	2.73	2.68	2.65	2.62	2.59	2.56	2.54	2.52	2.51	2.49	2.48
12	13.32	6.48	4.84	4.14	3.76	3.35	3.14	2.99	2.90	2.83	2.78	2.75	2.72	2.68	2.65	2.63	2.61	2.60	2.57	2.56
16	14.67	6.89	5.08	4.32	3.90	3.46	3.24	3.07	2.97	2.90	2.85	2.81	2.79	2.74	2.71	2.69	2.67	2.66	2.63	2.62
20	15.80	7.22	5.27	4.45	4.01	3.55	3.31	3.14	3.03	2.96	2.91	2.87	2.84	2.79	2.76	2.74	2.72	2.70	2.68	2.66
30	18.10	7.87	5.63	4.71	4.22	*3.70*	3.44	3.26	3.14	3.06	3.01	2.96	2.93	2.88	2.84	2.82	2.80	2.78	2.76	2.74
40	19.90	8.35	5.89	4.89	4.36	*3.82*	3.54	3.34	3.22	3.13	3.07	3.03	2.99	2.94	2.91	2.88	2.86	2.84	2.81	2.79
50	21.45	8.75	6.10	5.04	4.48	3.90	*3.61*	3.40	3.28	3.19	3.13	3.08	3.04	2.99	2.95	2.92	2.90	2.89	2.86	2.84
60	22.81	9.08	6.28	5.16	4.58	3.97	*3.67*	3.46	3.33	3.23	3.17	3.12	3.08	3.03	2.99	2.96	2.94	2.92	2.89	2.87
75	24.53	9.49	6.50	5.32	4.70	4.06	*3.75*	3.52	3.39	3.29	3.22	3.17	3.13	3.07	3.03	3.00	2.98	2.96	2.93	2.91
100	27.03	10.04	6.81	5.52	4.85	4.18	*3.84*	*3.61*	3.46	3.36	3.29	3.24	3.20	3.13	3.09	3.06	3.04	3.02	2.99	2.96
125	29.14	10.51	7.03	5.68	4.98	4.27	3.92	*3.67*	3.52	3.42	3.34	3.29	3.24	3.18	3.14	3.11	3.08	3.06	3.03	3.00
150	30.94	10.90	7.21	5.81	5.08	4.35	3.97	*3.73*	3.57	3.46	3.38	3.33	3.28	3.22	3.17	3.14	3.12	3.10	3.06	3.04
175	32.66	11.25	7.36	5.93	5.17	4.40	4.03	*3.77*	*3.60*	3.50	3.42	3.36	3.32	3.25	3.20	3.17	3.14	3.12	3.09	3.06
200	34.06	11.56	7.46	6.04	5.24	4.46	4.07	*3.81*	*3.64*	3.53	3.45	3.39	3.35	3.28	3.23	3.20	3.17	3.15	3.11	3.09

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.97	3.25	2.74	2.49	2.35	2.19	2.10	2.04	2.00	1.97	1.95	1.93	1.92	1.90	1.89	1.88	1.87	1.86	1.85	1.84
2	6.34	3.85	3.15	2.83	2.65	2.45	2.34	2.26	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.05	2.04	2.02	2.02
3	7.30	4.23	3.41	3.04	2.82	2.59	2.47	2.38	2.32	2.28	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.14	2.12	2.11
4	8.05	4.52	3.60	3.18	2.95	2.70	2.56	2.47	2.40	2.36	2.33	2.30	2.29	2.26	2.24	2.22	2.21	2.20	2.19	2.18
5	8.69	4.76	3.75	3.30	3.05	2.78	2.64	2.53	2.47	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.26	2.26	2.24	2.23
8	10.20	5.28	4.08	3.55	3.26	2.95	2.79	2.67	2.60	2.55	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
12	11.69	5.77	4.37	3.78	3.45	3.10	2.92	2.79	2.71	2.65	2.61	2.58	2.56	2.52	2.49	2.48	2.46	2.45	2.43	2.42
16	12.89	6.14	4.59	3.94	3.58	3.21	3.02	2.87	2.79	2.73	2.68	2.65	2.62	2.59	2.56	2.54	2.52	2.51	2.49	2.48
20	13.89	6.44	4.77	4.07	3.69	3.29	3.09	2.94	2.85	2.78	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.54	2.52
30	15.92	7.02	5.10	4.31	3.88	3.45	3.22	3.06	2.96	2.89	2.84	2.80	2.77	2.73	2.70	2.67	2.66	2.64	2.62	2.60
40	17.53	7.46	5.34	4.48	4.03	3.56	3.31	3.14	3.03	2.96	2.91	2.87	2.84	2.79	2.76	2.74	2.72	2.70	2.68	2.66
50	18.90	7.81	5.54	4.62	4.14	3.64	3.39	3.20	3.09	3.02	2.96	2.92	2.89	2.84	2.81	2.78	2.76	2.75	2.72	2.70
60	20.12	8.12	5.70	4.74	4.23	3.71	3.45	3.26	3.14	3.06	3.01	2.96	2.93	2.88	2.84	2.82	2.80	2.78	2.76	2.74
75	21.68	8.50	5.91	4.88	4.35	3.80	3.52	3.32	3.20	3.12	3.06	3.02	2.98	2.93	2.89	2.87	2.84	2.83	2.80	2.78
100	23.83	9.03	6.18	5.08	4.49	3.91	3.61	3.41	3.28	3.19	3.12	3.08	3.04	2.99	2.95	2.92	2.90	2.89	2.86	2.84
125	25.59	9.47	6.40	5.22	4.61	4.00	3.69	3.47	3.33	3.25	3.18	3.13	3.09	3.04	3.00	2.97	2.95	2.93	2.90	2.88
150	27.34	9.81	6.59	5.35	4.71	4.06	3.75	3.52	3.38	3.29	3.22	3.17	3.13	3.08	3.03	3.00	2.98	2.97	2.93	2.91
175	28.71	10.16	6.74	5.46	4.80	4.13	3.80	3.56	3.42	3.33	3.26	3.21	3.17	3.11	3.06	3.03	3.02	2.99	2.96	2.94
200	30.08	10.45	6.88	5.54	4.86	4.19	3.85	3.60	3.45	3.36	3.29	3.23	3.20	3.14	3.09	3.06	3.04	3.02	2.99	2.96

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.07	3.79	3.13	2.82	2.64	2.44	2.33	2.25	2.20	2.17	2.14	2.12	2.11	2.08	2.07	2.05	2.05	2.04	2.02	2.02
2	7.72	4.46	3.57	3.17	2.94	2.70	2.56	2.46	2.40	2.36	2.33	2.30	2.29	2.26	2.24	2.22	2.21	2.20	2.19	2.18
3	8.87	4.89	3.85	3.39	3.13	2.84	2.70	2.59	2.52	2.47	2.43	2.41	2.39	2.36	2.33	2.32	2.31	2.30	2.28	2.27
4	9.78	5.21	4.05	3.54	3.26	2.95	2.79	2.67	2.60	2.54	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
5	10.55	5.47	4.21	3.66	3.36	3.03	2.86	2.74	2.66	2.60	2.56	2.53	2.51	2.48	2.45	2.44	2.42	2.41	2.39	2.38
8	12.37	6.06	4.57	3.93	3.58	3.21	3.01	2.87	2.79	2.73	2.68	2.65	2.62	2.59	2.56	2.54	2.52	2.51	2.49	2.48
12	14.17	6.61	4.89	4.16	3.77	3.36	3.15	2.99	2.90	2.83	2.78	2.75	2.72	2.68	2.65	2.63	2.61	2.60	2.57	2.56
16	15.61	7.03	5.12	4.34	3.91	3.47	3.24	3.08	2.97	2.90	2.85	2.82	2.79	2.74	2.71	2.69	2.67	2.66	2.63	2.62
20	16.82	7.37	5.32	4.47	4.02	3.55	3.31	3.14	3.03	2.96	2.91	2.87	2.84	2.79	2.76	2.74	2.72	2.70	2.68	2.66
30	19.29	8.02	5.68	4.73	4.22	3.71	3.45	3.26	3.14	3.06	3.01	2.96	2.93	2.88	2.85	2.82	2.80	2.78	2.76	2.74
40	21.24	8.52	5.94	4.91	4.37	3.82	3.54	3.34	3.22	3.13	3.07	3.03	2.99	2.94	2.91	2.88	2.86	2.84	2.81	2.78
50	22.85	8.92	6.15	5.06	4.49	3.91	3.61	3.41	3.28	3.19	3.13	3.08	3.04	2.99	2.95	2.92	2.90	2.89	2.86	2.84
60	24.32	9.27	6.34	5.19	4.58	3.98	3.67	3.46	3.32	3.23	3.17	3.12	3.08	3.03	2.99	2.96	2.94	2.92	2.89	2.87
75	26.17	9.72	6.56	5.33	4.71	4.06	3.75	3.52	3.38	3.29	3.22	3.17	3.13	3.07	3.03	3.01	2.98	2.96	2.93	2.91
100	28.91	10.30	6.86	5.54	4.86	4.18	3.85	3.61	3.46	3.36	3.29	3.23	3.20	3.13	3.09	3.06	3.04	3.01	2.99	2.96
125	31.05	10.79	7.10	5.70	4.98	4.27	3.92	3.67	3.52	3.42	3.34	3.29	3.24	3.18	3.14	3.11	3.08	3.06	3.03	3.00
150	33.01	11.18	7.30	5.83	5.09	4.35	3.98	3.72	3.56	3.46	3.39	3.33	3.28	3.22	3.17	3.14	3.12	3.10	3.06	3.04
175	34.77	11.52	7.47	5.96	5.18	4.41	4.03	3.77	3.61	3.50	3.42	3.36	3.31	3.25	3.20	3.17	3.14	3.12	3.09	3.06
200	36.33	11.87	7.62	6.05	5.25	4.47	4.08	3.81	3.64	3.53	3.45	3.39	3.34	3.28	3.23	3.20	3.17	3.15	3.11	3.09

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	7.24	4.35	**3.53**	**3.15**	**2.93**	**2.69**	**2.56**	**2.46**	**2.40**	**2.36**	**2.33**	**2.30**	**2.28**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
2	9.18	5.10	4.01	**3.52**	**3.24**	**2.94**	**2.79**	**2.67**	**2.59**	**2.54**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
3	0.54	5.57	4.30	3.74	**3.43**	**3.09**	**2.92**	**2.79**	**2.71**	**2.65**	**2.61**	**2.58**	**2.55**	**2.52**	**2.49**	**2.48**	**2.46**	**2.45**	**2.43**	**2.42**
4	11.61	5.93	4.52	3.91	**3.57**	**3.20**	**3.01**	**2.87**	**2.79**	**2.73**	**2.68**	**2.65**	**2.62**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
5	12.52	6.23	4.70	4.04	*3.67*	**3.29**	**3.08**	**2.94**	**2.85**	**2.78**	**2.74**	**2.70**	**2.68**	**2.64**	**2.61**	**2.59**	**2.57**	**2.56**	**2.54**	**2.52**
8	14.67	6.89	5.08	4.32	3.90	**3.46**	**3.24**	**3.07**	**2.97**	**2.90**	**2.85**	**2.81**	**2.79**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
12	16.81	7.50	5.43	4.57	4.10	*3.62*	**3.37**	**3.19**	**3.08**	**3.01**	**2.95**	**2.91**	**2.88**	**2.83**	**2.80**	**2.77**	**2.75**	**2.74**	**2.71**	**2.70**
16	18.52	7.97	5.69	4.75	4.25	*3.73*	**3.47**	**3.27**	**3.16**	**3.08**	**3.02**	**2.98**	**2.94**	**2.89**	**2.86**	**2.83**	**2.81**	**2.80**	**2.77**	**2.75**
20	19.95	8.35	5.89	4.89	4.36	*3.82*	**3.54**	**3.34**	**3.22**	**3.13**	**3.07**	**3.03**	**2.99**	**2.94**	**2.91**	**2.88**	**2.86**	**2.84**	**2.81**	**2.79**
30	22.85	9.09	6.28	5.16	4.57	3.98	*3.67*	**3.46**	**3.32**	**3.23**	**3.17**	**3.12**	**3.08**	**3.03**	**2.99**	**2.96**	**2.94**	**2.92**	**2.89**	**2.87**
40	25.20	9.64	6.57	5.36	4.73	4.09	*3.77*	**3.54**	**3.40**	**3.31**	**3.24**	**3.19**	**3.15**	**3.09**	**3.05**	**3.02**	**3.00**	**2.98**	**2.94**	**2.92**
50	27.15	10.11	6.80	5.52	4.85	4.18	*3.84*	*3.60*	**3.46**	**3.36**	**3.29**	**3.24**	**3.20**	**3.13**	**3.09**	**3.06**	**3.04**	**3.02**	**2.99**	**2.96**
60	28.81	10.47	6.99	5.65	4.96	4.25	3.91	*3.66*	**3.51**	**3.41**	**3.33**	**3.28**	**3.23**	**3.17**	**3.13**	**3.10**	**3.07**	**3.05**	**3.02**	**3.00**
75	31.05	10.99	7.25	5.81	5.08	4.35	3.98	*3.72*	**3.56**	**3.46**	**3.38**	**3.33**	**3.28**	**3.22**	**3.17**	**3.14**	**3.11**	**3.10**	**3.06**	**3.04**
100	34.18	11.62	7.57	6.03	5.25	4.46	4.08	*3.81*	*3.64*	**3.53**	**3.45**	**3.39**	**3.34**	**3.28**	**3.23**	**3.20**	**3.17**	**3.15**	**3.11**	**3.09**
125	36.72	12.21	7.84	6.20	5.37	4.55	4.15	*3.87*	*3.70*	*3.58*	**3.50**	**3.44**	**3.39**	**3.32**	**3.27**	**3.24**	**3.21**	**3.19**	**3.15**	**3.13**
150	39.06	12.65	8.06	6.35	5.48	4.63	4.21	*3.92*	*3.75*	*3.63*	**3.55**	**3.48**	**3.43**	**3.36**	**3.31**	**3.27**	**3.25**	**3.22**	**3.19**	**3.16**
175	41.41	13.09	8.25	6.47	5.57	4.70	4.27	3.97	*3.78*	*3.67*	*3.58*	**3.52**	**3.47**	**3.39**	**3.34**	**3.30**	**3.27**	**3.25**	**3.21**	**3.19**
200	42.97	13.38	8.40	6.59	5.66	4.76	4.32	4.00	*3.82*	*3.70*	*3.61*	**3.55**	**3.49**	**3.42**	**3.37**	**3.33**	**3.30**	**3.28**	**3.24**	**3.21**

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.34	3.85	3.15	**2.83**	**2.65**	**2.45**	**2.34**	**2.26**	**2.20**	**2.17**	**2.14**	**2.12**	**2.11**	**2.08**	**2.07**	**2.05**	**2.05**	**2.04**	**2.02**	**2.02**
2	8.05	4.52	3.60	3.18	**2.95**	**2.70**	**2.56**	**2.47**	**2.40**	**2.36**	**2.33**	**2.30**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.20**	**2.19**	**2.18**
3	9.25	4.95	3.87	3.40	*3.13*	**2.85**	**2.70**	**2.59**	**2.52**	**2.47**	**2.43**	**2.41**	**2.39**	**2.36**	**2.33**	**2.32**	**2.31**	**2.30**	**2.28**	**2.27**
4	10.20	5.28	4.08	3.55	3.26	**2.95**	**2.79**	**2.67**	**2.60**	**2.55**	**2.51**	**2.48**	**2.46**	**2.42**	**2.40**	**2.38**	**2.37**	**2.36**	**2.34**	**2.33**
5	11.00	5.55	4.24	3.67	3.36	*3.04*	**2.86**	**2.74**	**2.66**	**2.60**	**2.56**	**2.54**	**2.51**	**2.48**	**2.45**	**2.44**	**2.42**	**2.41**	**2.39**	**2.38**
8	12.89	6.14	4.59	3.94	3.58	*3.21*	*3.02*	**2.87**	**2.79**	**2.73**	**2.68**	**2.65**	**2.62**	**2.59**	**2.56**	**2.54**	**2.52**	**2.51**	**2.49**	**2.48**
12	14.77	6.70	4.91	4.17	3.78	3.36	*3.15*	*2.99*	**2.90**	**2.83**	**2.78**	**2.75**	**2.72**	**2.68**	**2.65**	**2.63**	**2.61**	**2.60**	**2.57**	**2.56**
16	16.27	7.12	5.15	4.35	3.92	3.47	*3.24*	*3.08*	*2.97*	**2.90**	**2.85**	**2.82**	**2.79**	**2.74**	**2.71**	**2.69**	**2.67**	**2.66**	**2.63**	**2.62**
20	17.53	7.46	5.34	4.48	4.03	3.55	3.31	*3.14*	*3.03*	*2.96*	**2.91**	**2.87**	**2.84**	**2.79**	**2.76**	**2.74**	**2.72**	**2.70**	**2.68**	**2.66**
30	20.08	8.12	5.70	4.74	4.23	3.71	3.45	*3.26*	*3.14*	*3.06*	*3.01*	*2.96*	**2.93**	**2.88**	**2.85**	**2.82**	**2.80**	**2.79**	**2.76**	**2.74**
40	22.12	8.62	5.97	4.92	4.38	3.82	3.54	3.34	*3.22*	*3.13*	*3.07*	*3.03*	*2.99*	*2.94*	**2.90**	**2.88**	**2.86**	**2.84**	**2.81**	**2.79**
50	23.82	9.03	6.18	5.07	4.50	3.91	3.61	3.40	*3.28*	*3.19*	*3.13*	*3.08*	*3.04*	*2.99*	*2.95*	**2.92**	**2.90**	**2.89**	**2.86**	**2.84**
60	25.31	9.38	6.36	5.20	4.59	3.98	3.67	3.46	3.33	*3.23*	*3.17*	*3.12*	*3.08*	*3.03*	*2.99*	*2.96*	*2.94*	**2.92**	**2.89**	**2.87**
75	27.30	9.81	6.58	5.35	4.71	4.07	3.75	3.52	3.38	*3.29*	*3.22*	*3.17*	*3.13*	*3.07*	*3.03*	*3.00*	*2.98*	*2.96*	*2.93*	**2.91**
100	30.06	10.42	6.88	5.55	4.87	4.18	3.85	3.61	3.46	3.36	*3.29*	*3.24*	*3.20*	*3.13*	*3.09*	*3.06*	*3.04*	*3.02*	*2.99*	*2.96*
125	32.34	10.90	7.13	5.71	4.99	4.27	3.92	3.67	3.52	3.42	3.34	*3.29*	*3.24*	*3.18*	*3.14*	*3.11*	*3.08*	*3.06*	*3.03*	*3.00*
150	34.45	11.31	7.32	5.84	5.10	4.35	3.98	3.72	3.57	3.46	3.38	3.33	*3.28*	*3.22*	*3.17*	*3.14*	*3.12*	*3.10*	*3.06*	*3.04*
175	36.21	11.69	7.50	5.96	5.19	4.41	4.04	3.77	3.61	3.50	3.42	3.36	*3.32*	*3.25*	*3.20*	*3.17*	*3.15*	*3.12*	*3.09*	*3.06*
200	37.85	12.01	7.66	6.06	5.26	4.47	4.08	3.81	3.64	3.53	3.45	3.39	3.35	3.28	3.23	3.20	3.17	3.15	3.11	3.09

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	7.72	4.46	3.57	3.17	2.94	2.70	2.56	2.46	2.40	2.36	2.33	2.30	2.29	2.26	2.24	2.22	2.21	2.20	2.19	2.18
2	9.78	5.21	4.05	3.54	3.26	2.95	2.79	2.67	2.60	2.54	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
3	11.22	5.70	4.35	3.76	3.44	3.10	2.92	2.79	2.71	2.65	2.61	2.58	2.56	2.52	2.49	2.48	2.46	2.45	2.43	2.42
4	12.36	6.06	4.57	3.93	3.58	3.21	3.01	2.87	2.79	2.73	2.68	2.65	2.62	2.59	2.56	2.54	2.52	2.51	2.49	2.48
5	13.33	6.36	4.74	4.06	3.68	3.29	3.09	2.94	2.85	2.78	2.74	2.70	2.68	2.64	2.61	2.59	2.57	2.56	2.54	2.52
8	15.61	7.03	5.13	4.34	3.91	3.47	3.24	3.08	2.97	2.90	2.85	2.82	2.79	2.74	2.71	2.69	2.67	2.66	2.63	2.62
12	17.89	7.66	5.47	4.59	4.11	3.62	3.37	3.19	3.08	3.01	2.95	2.91	2.88	2.83	2.80	2.77	2.75	2.74	2.71	2.70
16	19.70	8.13	5.73	4.77	4.26	3.73	3.47	3.28	3.16	3.08	3.02	2.98	2.94	2.89	2.86	2.83	2.81	2.80	2.77	2.75
20	21.23	8.52	5.94	4.91	4.37	3.82	3.54	3.34	3.22	3.13	3.07	3.03	2.99	2.94	2.90	2.88	2.86	2.84	2.81	2.78
30	24.32	9.27	6.33	5.18	4.59	3.98	3.67	3.46	3.32	3.23	3.17	3.12	3.08	3.03	2.99	2.96	2.94	2.92	2.88	2.87
40	26.78	9.83	6.62	5.38	4.74	4.09	3.77	3.54	3.40	3.31	3.24	3.19	3.15	3.09	3.05	3.02	2.99	2.98	2.94	2.92
50	28.83	10.30	6.86	5.54	4.86	4.18	3.85	3.61	3.46	3.36	3.29	3.24	3.20	3.13	3.09	3.06	3.04	3.01	2.99	2.96
60	30.64	10.69	7.05	5.67	4.96	4.26	3.91	3.66	3.51	3.41	3.33	3.28	3.23	3.17	3.13	3.10	3.07	3.05	3.02	3.00
75	33.05	11.19	7.30	5.83	5.09	4.35	3.98	3.72	3.57	3.46	3.38	3.33	3.28	3.22	3.17	3.14	3.12	3.10	3.06	3.04
100	36.33	11.87	7.62	6.05	5.26	4.46	4.08	3.81	3.64	3.53	3.45	3.39	3.35	3.28	3.23	3.20	3.17	3.15	3.11	3.09
125	39.14	12.42	7.90	6.23	5.38	4.56	4.15	3.87	3.70	3.59	3.50	3.44	3.39	3.32	3.27	3.24	3.21	3.19	3.15	3.13
150	41.60	12.89	8.12	6.36	5.49	4.64	4.22	3.93	3.75	3.63	3.54	3.48	3.43	3.36	3.31	3.27	3.25	3.22	3.19	3.16
175	43.83	13.30	8.31	6.49	5.59	4.70	4.27	3.97	3.79	3.67	3.58	3.52	3.46	3.39	3.34	3.30	3.27	3.25	3.21	3.19
200	45.70	13.65	8.47	6.60	5.67	4.76	4.31	4.01	3.82	3.70	3.61	3.54	3.49	3.42	3.37	3.33	3.30	3.28	3.24	3.21

Table 19-14. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 2 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	9.18	5.10	4.01	3.52	3.24	2.94	2.79	2.67	2.59	2.54	2.51	2.48	2.46	2.42	2.40	2.38	2.37	2.36	2.34	2.33
2	11.61	5.93	4.52	3.91	3.57	3.20	3.01	2.87	2.79	2.73	2.68	2.65	2.62	2.59	2.56	2.54	2.52	2.51	2.49	2.48
3	13.32	6.48	4.84	4.14	3.76	3.35	3.14	2.99	2.89	2.83	2.78	2.75	2.72	2.68	2.65	2.63	2.61	2.60	2.57	2.56
4	14.67	6.89	5.08	4.32	3.90	3.46	3.24	3.07	2.97	2.90	2.85	2.81	2.79	2.74	2.71	2.69	2.67	2.66	2.63	2.62
5	15.81	7.22	5.27	4.45	4.01	3.55	3.31	3.14	3.03	2.96	2.91	2.87	2.84	2.79	2.76	2.74	2.72	2.70	2.68	2.66
8	18.52	7.97	5.69	4.75	4.25	3.73	3.47	3.27	3.16	3.08	3.02	2.98	2.94	2.89	2.86	2.83	2.81	2.80	2.77	2.75
12	21.21	8.68	6.07	5.01	4.46	3.89	3.60	3.39	3.26	3.18	3.12	3.07	3.03	2.98	2.94	2.92	2.89	2.88	2.85	2.83
16	23.36	9.21	6.35	5.21	4.61	4.00	3.69	3.48	3.34	3.25	3.19	3.14	3.10	3.04	3.00	2.97	2.95	2.93	2.90	2.88
20	25.17	9.65	6.57	5.36	4.73	4.09	3.77	3.54	3.40	3.31	3.24	3.19	3.15	3.09	3.05	3.02	2.99	2.98	2.94	2.92
30	28.81	10.48	7.00	5.65	4.95	4.25	3.90	3.66	3.51	3.41	3.33	3.28	3.24	3.17	3.13	3.10	3.07	3.04	3.02	3.00
40	31.73	11.13	7.32	5.86	5.12	4.37	4.00	3.74	3.58	3.48	3.40	3.34	3.30	3.23	3.19	3.15	3.12	3.11	3.07	3.05
50	34.16	11.65	7.57	6.03	5.24	4.46	4.08	3.81	3.64	3.53	3.45	3.39	3.35	3.28	3.23	3.20	3.17	3.15	3.11	3.09
60	36.33	12.08	7.79	6.17	5.35	4.54	4.14	3.86	3.69	3.58	3.49	3.43	3.38	3.32	3.27	3.23	3.20	3.18	3.15	3.12
75	39.14	12.64	8.05	6.35	5.48	4.63	4.22	3.92	3.75	3.63	3.54	3.48	3.43	3.36	3.31	3.27	3.25	3.22	3.19	3.16
100	43.12	13.42	8.41	6.58	5.66	4.75	4.31	4.01	3.82	3.70	3.61	3.54	3.49	3.42	3.37	3.33	3.30	3.28	3.24	3.21
125	46.41	14.03	8.70	6.76	5.79	4.85	4.39	4.07	3.88	3.75	3.66	3.59	3.54	3.46	3.41	3.37	3.34	3.32	3.28	3.25
150	49.34	14.56	8.94	6.91	5.91	4.93	4.46	4.13	3.93	3.80	3.70	3.63	3.58	3.50	3.44	3.40	3.37	3.35	3.31	3.28
175	52.03	15.03	9.14	7.05	6.01	5.00	4.51	4.17	3.97	3.83	3.74	3.67	3.61	3.53	3.47	3.43	3.40	3.38	3.33	3.30
200	54.38	15.44	9.33	7.16	6.09	5.05	4.56	4.21	4.01	3.87	3.77	3.70	3.64	3.56	3.50	3.46	3.43	3.40	3.36	3.33

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.69	0.56	0.50	0.47	0.45	0.42	0.40	0.39	0.38	0.37	0.37	0.37	0.36	0.36	0.36	0.35	0.35	0.35	0.35	0.35
2	1.10	0.87	0.77	0.72	0.69	0.65	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.54
3	1.36	1.05	0.93	0.86	0.82	0.77	0.74	0.72	0.70	0.69	0.69	0.68	0.68	0.67	0.66	0.66	0.66	0.65	0.65	0.65
4	1.56	1.18	1.04	0.96	0.91	0.86	0.82	0.80	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.72
5	1.72	1.28	1.12	1.04	0.98	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77
8	2.10	1.51	1.30	1.19	1.13	1.05	1.01	0.98	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.88	0.87
12	2.46	1.71	1.46	1.33	1.25	1.16	1.11	1.08	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
16	2.75	1.86	1.57	1.43	1.34	1.24	1.19	1.14	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01
20	2.99	1.98	1.66	1.50	1.41	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
30	3.46	2.21	1.82	1.64	1.53	1.41	1.34	1.29	1.26	1.23	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.13
40	3.84	2.38	1.94	1.74	1.62	1.49	1.41	1.36	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
50	4.16	2.51	2.04	1.82	1.69	1.54	1.46	1.40	1.37	1.34	1.32	1.31	1.29	1.28	1.26	1.25	1.25	1.24	1.23	1.23
60	4.43	2.63	2.12	1.88	1.74	1.59	1.51	1.45	1.41	1.38	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
75	4.79	2.77	2.22	1.96	1.81	1.65	1.56	1.49	1.45	1.42	1.40	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.30
100	5.29	2.97	2.35	2.06	1.90	1.72	1.63	1.56	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
125	5.71	3.13	2.45	2.14	1.97	1.78	1.68	1.60	1.56	1.52	1.50	1.48	1.47	1.44	1.43	1.42	1.41	1.40	1.39	1.38
150	6.08	3.26	2.54	2.21	2.03	1.83	1.72	1.64	1.59	1.56	1.53	1.51	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.41
175	6.42	3.38	2.61	2.27	2.08	1.87	1.76	1.68	1.62	1.59	1.56	1.54	1.53	1.50	1.49	1.47	1.46	1.46	1.44	1.43
200	6.71	3.48	*2.68*	2.32	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.02	0.83	0.75	0.70	0.67	0.64	0.62	0.60	0.59	0.58	0.57	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.54	0.54
2	1.47	1.14	1.02	0.95	0.90	0.85	0.82	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.72
3	1.77	1.33	1.17	1.09	1.03	0.97	0.93	0.90	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.82	0.82	0.82	0.81	0.81
4	2.00	1.47	1.28	1.18	1.12	1.05	1.01	0.97	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
5	2.19	1.58	1.37	1.26	1.19	1.11	1.06	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
8	2.63	1.82	1.55	1.42	1.33	1.24	1.18	1.14	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01
12	3.06	2.04	1.72	1.56	1.46	1.35	1.29	1.24	1.21	1.19	1.17	1.16	1.15	1.13	1.12	1.12	1.11	1.11	1.10	1.09
16	3.40	2.21	1.84	1.65	1.55	1.42	1.36	1.30	1.27	1.25	1.23	1.22	1.21	1.19	1.18	1.17	1.17	1.16	1.15	1.14
20	3.68	2.34	1.93	1.73	1.61	1.48	1.41	1.35	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
30	4.26	2.59	2.10	1.87	1.74	1.59	1.51	1.44	1.40	1.38	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
40	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
50	5.09	*2.93*	2.33	2.06	1.90	1.72	1.63	1.56	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
60	5.42	3.06	2.42	2.12	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
75	5.85	3.22	2.52	2.20	2.02	1.83	1.72	1.64	1.59	1.56	1.53	1.51	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.41
100	6.46	3.44	2.66	2.31	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46
125	6.97	3.62	2.77	2.40	2.19	1.96	1.84	1.75	1.69	1.66	1.63	1.61	1.59	1.56	1.55	1.53	1.52	1.51	1.50	1.49
150	7.41	3.77	*2.86*	2.47	2.25	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.62	1.59	1.58	1.56	1.55	1.54	1.53	1.52
175	7.81	3.90	*2.94*	2.53	2.30	2.05	1.92	1.82	1.76	1.72	1.69	1.67	1.65	1.62	1.60	1.59	1.58	1.57	1.55	1.54
200	8.17	4.02	3.01	2.58	2.34	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.34	1.09	0.98	0.92	0.88	0.84	0.81	0.79	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.72	0.71
2	1.85	1.42	1.25	1.16	1.10	1.04	1.00	0.97	0.95	0.93	0.92	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
3	2.19	1.62	1.41	1.30	1.23	1.15	1.11	1.07	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.97	0.97	0.97	0.96	0.96
4	2.45	1.77	1.52	1.40	1.32	1.23	1.18	1.14	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.03	1.02	1.01
5	2.67	1.89	1.61	1.48	1.39	1.29	1.24	1.19	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
8	3.19	2.15	1.81	1.64	1.54	1.42	1.35	1.30	1.27	1.25	1.23	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
12	3.69	2.39	1.98	1.78	1.66	1.52	1.45	1.39	1.36	1.33	1.31	1.30	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.22
16	4.09	2.57	2.11	1.88	1.75	1.60	1.52	1.46	1.42	1.39	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.29	1.28	1.27
20	4.43	2.71	2.20	1.96	1.82	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
30	5.10	2.99	2.39	2.11	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
40	5.64	3.20	2.53	2.22	2.04	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
50	6.09	3.37	2.63	2.30	2.11	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.52	1.51	1.50	1.49	1.48	1.46	1.46
60	6.48	3.51	2.72	2.37	2.17	1.95	1.83	1.74	1.69	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
75	7.00	3.69	2.84	2.45	2.24	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.62	1.59	1.58	1.56	1.55	1.54	1.53	1.52
100	7.71	3.94	2.99	2.57	2.33	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
125	8.32	4.14	*3.11*	2.66	2.41	2.14	2.00	1.89	1.83	1.78	1.75	1.73	1.71	1.68	1.66	1.64	1.63	1.62	1.61	1.60
150	8.85	4.31	*3.21*	2.73	2.47	2.19	2.04	1.93	1.86	1.82	1.78	1.76	1.74	1.71	1.69	1.67	1.66	1.65	1.63	1.62
175	9.33	4.45	3.29	2.80	2.52	2.23	2.08	1.96	1.89	1.85	1.81	1.79	1.76	1.73	1.71	1.70	1.68	1.67	1.66	1.65
200	9.76	4.58	3.37	2.85	2.57	2.26	2.11	1.99	1.92	1.87	1.84	1.81	1.79	1.75	1.73	1.72	1.70	1.69	1.68	1.67

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.10	0.87	0.77	0.72	0.69	0.65	0.62	0.60	0.59	0.58	0.58	0.57	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.54
2	1.56	1.18	1.04	0.96	0.91	0.86	0.82	0.80	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.72
3	1.86	1.37	1.19	1.10	1.04	0.97	0.93	0.90	0.88	0.87	0.86	0.85	0.85	0.84	0.83	0.82	0.82	0.82	0.81	0.81
4	2.10	1.51	1.30	1.19	1.13	1.05	1.01	0.98	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.88	0.87
5	2.29	1.62	1.39	1.27	1.20	1.11	1.07	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
8	2.75	1.86	1.57	1.43	1.34	1.24	1.19	1.14	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01
12	3.19	2.08	1.73	1.56	1.46	1.35	1.29	1.24	1.21	1.19	1.17	1.16	1.15	1.13	1.12	1.12	1.11	1.11	1.10	1.09
16	3.54	2.24	1.85	1.66	1.55	1.43	1.36	1.30	1.27	1.25	1.23	1.22	1.21	1.19	1.18	1.17	1.17	1.16	1.15	1.15
20	3.84	2.38	1.94	1.74	1.62	1.49	1.41	1.36	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
30	4.43	2.63	2.12	1.88	1.74	1.59	1.51	1.45	1.41	1.38	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
40	4.90	2.82	2.25	1.98	1.83	1.67	1.58	1.51	1.46	1.44	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
50	5.29	2.97	2.35	2.06	1.90	1.72	1.63	1.56	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
60	5.64	3.10	2.43	2.13	1.96	1.77	1.67	1.60	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
75	6.09	3.26	2.54	2.21	2.03	1.83	1.72	1.64	1.59	1.56	1.53	1.51	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.41
100	6.71	3.48	*2.68*	2.32	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46
125	7.24	3.66	*2.79*	2.40	2.19	1.96	1.84	1.75	1.69	1.66	1.63	1.61	1.59	1.56	1.55	1.53	1.52	1.51	1.50	1.49
150	7.71	3.82	2.88	2.48	2.25	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.62	1.59	1.58	1.56	1.55	1.54	1.53	1.52
175	8.12	3.95	2.96	2.53	2.30	2.05	1.92	1.82	1.76	1.72	1.69	1.67	1.65	1.62	1.60	1.59	1.58	1.57	1.55	1.54
200	8.50	4.06	3.03	2.59	2.34	2.09	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.47	1.14	1.02	0.95	0.90	0.85	0.82	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.73	0.73	0.73	0.72	0.72	0.72
2	2.00	1.47	1.28	1.18	1.12	1.05	1.01	0.97	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
3	2.35	1.68	1.44	1.32	1.25	1.16	1.11	1.07	1.05	1.03	1.02	1.01	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.96
4	2.63	1.82	1.55	1.42	1.33	1.24	1.18	1.14	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.01
5	2.86	1.94	1.64	1.49	1.40	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
8	3.40	2.21	1.84	1.65	1.55	1.42	1.36	1.30	1.27	1.25	1.23	1.22	1.21	1.19	1.18	1.17	1.17	1.16	1.15	1.14
12	3.93	2.45	2.01	1.79	1.67	1.53	1.45	1.39	1.36	1.33	1.31	1.30	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.22
16	4.35	2.63	2.13	1.90	1.76	1.61	1.52	1.46	1.42	1.39	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.29	1.28	1.27
20	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
30	5.42	3.06	2.42	2.12	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
40	5.98	3.27	2.55	2.23	2.04	1.84	1.74	1.66	1.61	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
50	6.46	3.44	2.66	2.31	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46
60	6.87	3.59	2.75	2.38	2.17	1.95	1.83	1.74	1.69	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
75	7.41	3.77	*2.86*	2.47	2.25	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.62	1.59	1.58	1.56	1.55	1.54	1.53	1.52
100	8.17	4.02	3.01	2.58	2.34	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
125	8.82	4.22	3.13	2.67	2.42	2.14	2.00	1.90	1.83	1.79	1.75	1.73	1.71	1.68	1.66	1.64	1.63	1.62	1.61	1.60
150	9.38	4.39	3.23	2.74	2.48	2.19	2.04	1.93	1.86	1.82	1.78	1.76	1.74	1.71	1.69	1.67	1.66	1.65	1.63	1.62
175	9.87	4.53	3.32	2.81	2.53	2.23	2.08	1.96	1.90	1.85	1.81	1.79	1.76	1.73	1.71	1.70	1.68	1.67	1.66	1.65
200	10.34	4.67	3.39	*2.86*	2.57	2.27	2.11	1.99	1.92	1.87	1.83	1.81	1.79	1.76	1.73	1.72	1.70	1.69	1.68	1.66

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.85	1.42	1.25	1.16	1.10	1.04	1.00	0.97	0.95	0.93	0.92	0.92	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87
2	2.45	1.77	1.52	1.40	1.32	1.23	1.18	1.14	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01
3	2.86	1.99	1.69	1.54	1.45	1.34	1.28	1.23	1.20	1.18	1.17	1.16	1.15	1.13	1.12	1.12	1.11	1.10	1.10	1.09
4	3.19	2.15	1.81	1.64	1.54	1.42	1.35	1.30	1.27	1.25	1.23	1.22	1.21	1.19	1.18	1.17	1.16	1.16	1.15	1.14
5	3.46	2.28	1.90	1.72	1.60	1.48	1.41	1.35	1.32	1.29	1.27	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
8	4.09	2.57	2.11	1.88	1.75	1.60	1.52	1.46	1.42	1.39	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.29	1.28	1.27
12	4.72	2.84	2.29	2.03	1.88	1.71	1.61	1.54	1.50	1.47	1.45	1.43	1.42	1.40	1.38	1.37	1.36	1.36	1.34	1.34
16	5.22	3.04	2.42	2.13	1.97	1.78	1.68	1.61	1.56	1.53	1.50	1.48	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38
20	5.64	3.20	2.53	2.22	2.04	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
30	6.48	3.51	2.72	2.37	2.17	1.95	1.83	1.74	1.69	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
40	7.15	3.75	2.87	2.48	2.26	2.02	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
50	7.72	3.94	2.99	2.57	2.33	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
60	8.21	4.10	*3.08*	2.64	2.39	2.13	1.99	1.89	1.82	1.78	1.74	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
75	8.85	4.31	*3.21*	2.73	2.47	2.19	2.04	1.93	1.86	1.82	1.78	1.76	1.74	1.71	1.69	1.67	1.66	1.65	1.63	1.62
100	9.76	4.58	3.37	2.85	2.57	2.27	2.11	1.99	1.92	1.87	1.84	1.81	1.79	1.76	1.73	1.72	1.70	1.69	1.68	1.66
125	10.52	4.81	3.50	2.94	2.64	2.32	2.16	2.04	1.96	1.91	1.87	1.85	1.82	1.79	1.77	1.75	1.74	1.73	1.71	1.70
150	11.19	5.00	3.61	3.02	2.71	2.37	2.20	2.08	2.00	1.94	1.91	1.88	1.85	1.82	1.80	1.78	1.77	1.75	1.74	1.72
175	11.78	5.16	3.70	*3.09*	2.76	2.42	2.24	2.11	2.03	1.97	1.93	1.90	1.88	1.84	1.82	1.80	1.79	1.78	1.76	1.74
200	12.30	5.32	3.78	*3.15*	2.81	2.45	2.27	2.14	2.05	2.00	1.96	1.92	1.90	1.86	1.84	1.82	1.81	1.80	1.78	1.76

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.72	1.28	1.12	1.04	0.98	0.92	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77
2	2.29	1.62	1.39	1.27	1.20	1.11	1.07	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
3	2.68	1.82	1.55	1.41	1.32	1.23	1.17	1.13	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.02	1.01	1.01	1.00
4	2.99	1.98	1.66	1.50	1.41	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
5	3.24	2.10	1.75	1.58	1.48	1.36	1.30	1.25	1.22	1.20	1.18	1.17	1.16	1.14	1.13	1.12	1.12	1.11	1.10	1.10
8	3.84	2.38	1.94	1.74	1.62	1.49	1.41	1.36	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
12	4.43	2.63	2.12	1.88	1.74	1.59	1.51	1.45	1.41	1.38	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
16	4.90	2.82	2.25	1.98	1.83	1.67	1.58	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.33	1.31	1.31
20	5.29	2.97	2.35	2.06	1.90	1.72	1.63	1.56	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
30	6.08	3.26	2.54	2.21	2.03	1.83	1.72	1.64	1.59	1.56	1.53	1.51	1.50	1.48	1.46	1.45	1.44	1.43	1.42	1.41
40	6.71	3.48	*2.68*	2.32	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46
50	7.25	3.66	*2.79*	2.40	2.19	1.96	1.84	1.75	1.69	1.66	1.63	1.61	1.59	1.56	1.55	1.53	1.52	1.51	1.50	1.49
60	7.71	3.81	2.88	2.48	2.25	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.62	1.59	1.58	1.56	1.55	1.54	1.53	1.52
75	8.30	4.00	3.00	2.56	2.32	2.07	1.93	1.84	1.77	1.73	1.70	1.68	1.66	1.63	1.61	1.60	1.59	1.58	1.56	1.55
100	9.16	4.27	3.15	*2.68*	2.42	2.14	2.00	1.90	1.83	1.78	1.75	1.73	1.71	1.68	1.66	1.64	1.63	1.62	1.61	1.60
125	9.88	4.47	3.27	*2.77*	2.49	2.20	2.05	1.94	1.87	1.83	1.79	1.77	1.75	1.72	1.69	1.68	1.67	1.66	1.64	1.63
150	10.51	4.66	3.37	*2.84*	2.55	2.25	2.09	1.98	1.91	1.86	1.82	1.80	1.77	1.75	1.72	1.71	1.69	1.68	1.67	1.66
175	11.05	4.80	3.47	2.91	*2.61*	2.29	2.13	2.01	1.94	1.89	1.85	1.82	1.80	1.77	1.75	1.73	1.72	1.71	1.69	1.68
200	11.56	4.96	3.54	2.96	*2.65*	2.33	2.16	2.04	1.96	1.91	1.88	1.85	1.82	1.79	1.77	1.75	1.74	1.73	1.71	1.70

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.19	1.58	1.37	1.26	1.19	1.11	1.06	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.93	0.93	0.92	0.92
2	2.86	1.94	1.64	1.49	1.40	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
3	3.32	2.17	1.81	1.63	1.53	1.41	1.34	1.29	1.26	1.23	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.15	1.14	1.13
4	3.68	2.34	1.93	1.73	1.61	1.48	1.41	1.35	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
5	3.99	2.48	2.02	1.81	1.68	1.54	1.46	1.40	1.37	1.34	1.32	1.31	1.29	1.28	1.26	1.25	1.25	1.24	1.23	1.23
8	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
12	5.42	3.06	2.42	2.12	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
16	5.98	3.27	2.55	2.23	2.04	1.84	1.74	1.66	1.61	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
20	6.46	3.44	2.66	2.31	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46
30	7.41	3.77	*2.86*	2.47	2.25	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.62	1.59	1.58	1.56	1.55	1.54	1.53	1.52
40	8.17	4.02	3.01	2.58	2.34	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
50	8.82	4.22	3.13	2.67	2.41	2.14	2.00	1.90	1.83	1.78	1.75	1.73	1.71	1.68	1.66	1.64	1.63	1.62	1.61	1.60
60	9.38	4.39	3.23	2.74	2.48	2.19	2.04	1.93	1.87	1.82	1.78	1.76	1.74	1.71	1.69	1.67	1.66	1.65	1.63	1.62
75	10.12	4.60	3.36	*2.84*	2.55	2.25	2.09	1.98	1.91	1.86	1.82	1.80	1.78	1.74	1.72	1.71	1.69	1.68	1.67	1.66
100	11.13	4.89	3.53	*2.96*	2.65	2.33	2.16	2.04	1.96	1.91	1.88	1.85	1.82	1.79	1.77	1.75	1.74	1.73	1.71	1.70
125	11.99	5.14	3.66	*3.05*	2.73	2.39	2.21	2.08	2.01	1.95	1.91	1.88	1.86	1.83	1.80	1.78	1.77	1.76	1.74	1.73
150	12.73	5.33	3.77	3.13	2.79	2.44	2.25	2.12	2.04	1.98	1.94	1.91	1.89	1.86	1.83	1.81	1.80	1.79	1.77	1.75
175	13.44	5.51	3.87	3.20	*2.85*	2.48	2.29	2.15	2.07	2.01	1.97	1.94	1.91	1.88	1.85	1.84	1.82	1.81	1.79	1.77
200	14.06	5.66	3.96	3.26	*2.89*	2.51	2.32	2.18	2.09	2.04	1.99	1.96	1.94	1.90	1.88	1.86	1.84	1.83	1.80	1.79

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.67**	**1.89**	**1.61**	**1.48**	**1.39**	**1.29**	**1.24**	**1.19**	**1.16**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.07**	**1.06**	**1.06**
2	3.46	**2.28**	**1.90**	**1.72**	**1.60**	**1.48**	**1.41**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
3	4.00	**2.53**	**2.08**	**1.86**	**1.73**	**1.58**	**1.50**	**1.44**	**1.40**	**1.38**	**1.36**	**1.34**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**	**1.26**
4	4.43	**2.71**	**2.20**	**1.96**	**1.82**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
5	4.79	**2.87**	**2.31**	**2.04**	**1.89**	**1.72**	**1.62**	**1.55**	**1.51**	**1.48**	**1.46**	**1.44**	**1.42**	**1.40**	**1.39**	**1.38**	**1.37**	**1.36**	**1.35**	**1.34**
8	5.64	3.20	**2.53**	**2.22**	**2.04**	**1.84**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
12	6.48	3.51	**2.72**	**2.37**	**2.17**	**1.95**	**1.83**	**1.74**	**1.69**	**1.65**	**1.62**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.52**	**1.51**	**1.49**	**1.48**
16	7.15	3.75	**2.87**	**2.48**	**2.26**	**2.02**	**1.90**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
20	7.71	3.94	**2.99**	**2.57**	**2.33**	**2.08**	**1.95**	**1.85**	**1.79**	**1.74**	**1.71**	**1.69**	**1.67**	**1.64**	**1.62**	**1.61**	**1.60**	**1.59**	**1.57**	**1.56**
30	8.85	4.31	*3.21*	**2.73**	**2.47**	**2.19**	**2.04**	**1.93**	**1.86**	**1.82**	**1.78**	**1.76**	**1.74**	**1.71**	**1.69**	**1.67**	**1.66**	**1.65**	**1.63**	**1.62**
40	9.76	4.58	3.37	**2.85**	**2.57**	**2.26**	**2.11**	**1.99**	**1.92**	**1.87**	**1.84**	**1.81**	**1.79**	**1.75**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**
50	10.52	4.81	3.50	**2.94**	**2.64**	**2.32**	**2.16**	**2.04**	**1.96**	**1.91**	**1.87**	**1.85**	**1.82**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.71**	**1.70**
60	11.19	5.00	3.61	**3.02**	**2.71**	**2.37**	**2.20**	**2.08**	**2.00**	**1.94**	**1.91**	**1.88**	**1.85**	**1.82**	**1.80**	**1.78**	**1.77**	**1.75**	**1.74**	**1.72**
75	12.05	5.24	3.74	*3.12*	**2.79**	**2.43**	**2.25**	**2.12**	**2.04**	**1.98**	**1.94**	**1.91**	**1.89**	**1.85**	**1.83**	**1.81**	**1.80**	**1.79**	**1.77**	**1.75**
100	13.28	5.57	3.92	*3.25*	**2.89**	**2.51**	**2.32**	**2.18**	**2.09**	**2.04**	**1.99**	**1.96**	**1.94**	**1.90**	**1.87**	**1.85**	**1.84**	**1.83**	**1.80**	**1.78**
125	14.30	5.84	4.07	3.35	**2.97**	**2.57**	**2.37**	**2.23**	**2.14**	**2.08**	**2.03**	**2.00**	**1.97**	**1.93**	**1.91**	**1.88**	**1.87**	**1.85**	**1.84**	**1.82**
150	15.23	6.07	4.19	3.43	**3.04**	**2.62**	**2.42**	**2.26**	**2.17**	**2.11**	**2.06**	**2.03**	**2.00**	**1.96**	**1.93**	**1.91**	**1.90**	**1.89**	**1.86**	**1.85**
175	16.02	6.27	4.30	3.51	*3.09*	**2.67**	**2.45**	**2.30**	**2.20**	**2.14**	**2.09**	**2.06**	**2.03**	**1.99**	**1.96**	**1.94**	**1.92**	**1.91**	**1.88**	**1.87**
200	16.72	6.45	4.38	3.57	*3.14*	**2.71**	**2.48**	**2.32**	**2.23**	**2.16**	**2.11**	**2.08**	**2.05**	**2.01**	**1.98**	**1.96**	**1.94**	**1.93**	**1.90**	**1.89**

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.29**	**1.62**	**1.39**	**1.27**	**1.20**	**1.11**	**1.07**	**1.03**	**1.01**	**0.99**	**0.98**	**0.97**	**0.96**	**0.95**	**0.94**	**0.94**	**0.93**	**0.93**	**0.92**	**0.92**
2	2.99	**1.98**	**1.66**	**1.50**	**1.41**	**1.30**	**1.24**	**1.20**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.07**	**1.06**	**1.06**
3	3.46	**2.21**	**1.82**	**1.64**	**1.53**	**1.41**	**1.34**	**1.29**	**1.26**	**1.23**	**1.22**	**1.20**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.15**	**1.14**	**1.13**
4	3.84	**2.38**	**1.94**	**1.74**	**1.62**	**1.49**	**1.41**	**1.36**	**1.32**	**1.29**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
5	4.15	**2.51**	**2.04**	**1.82**	**1.69**	**1.54**	**1.46**	**1.40**	**1.37**	**1.34**	**1.32**	**1.31**	**1.29**	**1.28**	**1.26**	**1.25**	**1.25**	**1.24**	**1.23**	**1.23**
8	4.90	2.82	**2.25**	**1.98**	**1.83**	**1.67**	**1.58**	**1.51**	**1.46**	**1.44**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.33**	**1.31**	**1.31**
12	5.63	3.10	**2.43**	**2.13**	**1.96**	**1.77**	**1.67**	**1.60**	**1.55**	**1.51**	**1.49**	**1.47**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.39**	**1.38**	**1.37**
16	6.22	3.31	**2.57**	**2.24**	**2.05**	**1.85**	**1.74**	**1.66**	**1.61**	**1.57**	**1.54**	**1.53**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
20	6.71	3.48	*2.68*	**2.32**	**2.12**	**1.90**	**1.79**	**1.70**	**1.65**	**1.61**	**1.59**	**1.57**	**1.55**	**1.53**	**1.51**	**1.50**	**1.49**	**1.48**	**1.46**	**1.46**
30	7.70	3.81	2.88	**2.47**	**2.25**	**2.01**	**1.88**	**1.79**	**1.73**	**1.69**	**1.66**	**1.64**	**1.62**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.52**
40	8.50	4.06	3.03	**2.59**	**2.34**	**2.09**	**1.95**	**1.85**	**1.79**	**1.74**	**1.71**	**1.69**	**1.67**	**1.64**	**1.62**	**1.61**	**1.60**	**1.59**	**1.57**	**1.56**
50	9.16	4.27	3.15	*2.68*	**2.42**	**2.15**	**2.00**	**1.90**	**1.83**	**1.79**	**1.75**	**1.73**	**1.71**	**1.68**	**1.66**	**1.64**	**1.63**	**1.62**	**1.61**	**1.59**
60	9.74	4.44	3.25	*2.75*	**2.48**	**2.19**	**2.04**	**1.93**	**1.86**	**1.82**	**1.79**	**1.76**	**1.74**	**1.71**	**1.69**	**1.67**	**1.66**	**1.65**	**1.63**	**1.62**
75	10.50	4.66	3.38	*2.84*	**2.55**	**2.25**	**2.09**	**1.98**	**1.91**	**1.86**	**1.82**	**1.80**	**1.78**	**1.75**	**1.72**	**1.71**	**1.69**	**1.68**	**1.67**	**1.66**
100	11.57	4.96	3.54	2.97	*2.66*	**2.33**	**2.16**	**2.04**	**1.96**	**1.91**	**1.87**	**1.85**	**1.82**	**1.79**	**1.77**	**1.75**	**1.74**	**1.73**	**1.71**	**1.70**
125	12.50	5.20	3.67	3.06	*2.73*	**2.39**	**2.21**	**2.09**	**2.01**	**1.95**	**1.91**	**1.88**	**1.86**	**1.82**	**1.80**	**1.79**	**1.77**	**1.76**	**1.74**	**1.73**
150	13.28	5.40	3.78	3.14	*2.80*	**2.44**	**2.26**	**2.12**	**2.04**	**1.98**	**1.94**	**1.91**	**1.89**	**1.86**	**1.83**	**1.81**	**1.80**	**1.79**	**1.77**	**1.75**
175	13.96	5.57	3.88	3.21	*2.84*	**2.48**	**2.29**	**2.15**	**2.07**	**2.01**	**1.97**	**1.94**	**1.92**	**1.88**	**1.86**	**1.84**	**1.82**	**1.81**	**1.79**	**1.77**
200	14.65	5.71	3.98	3.27	2.89	**2.51**	**2.32**	**2.19**	**2.10**	**2.04**	**2.00**	**1.96**	**1.93**	**1.90**	**1.87**	**1.85**	**1.84**	**1.83**	**1.81**	**1.79**

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.86	1.94	1.64	1.49	1.40	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
2	3.68	2.34	1.93	1.73	1.61	1.48	1.41	1.35	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
3	4.26	2.59	2.10	1.87	1.74	1.59	1.51	1.44	1.40	1.38	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
4	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
5	5.09	*2.93*	2.33	2.06	1.90	1.72	1.63	1.56	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
8	5.98	3.27	2.55	2.23	2.05	1.84	1.74	1.66	1.61	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
12	6.87	3.59	2.75	2.38	2.17	1.95	1.83	1.74	1.69	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
16	7.58	3.82	*2.90*	2.49	2.27	2.02	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
20	8.18	4.02	3.01	2.58	2.34	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
30	9.38	4.39	3.23	2.74	2.47	2.19	2.04	1.93	1.87	1.82	1.78	1.76	1.74	1.71	1.69	1.67	1.66	1.65	1.63	1.62
40	10.33	4.67	3.40	*2.86*	2.57	2.27	2.11	1.99	1.92	1.87	1.84	1.81	1.79	1.75	1.73	1.72	1.70	1.69	1.68	1.66
50	11.13	4.90	3.53	*2.96*	2.65	2.33	2.16	2.04	1.96	1.91	1.87	1.85	1.82	1.79	1.77	1.75	1.74	1.73	1.71	1.70
60	11.84	5.09	3.63	*3.03*	2.71	2.38	2.20	2.08	2.00	1.94	1.91	1.88	1.85	1.82	1.80	1.78	1.77	1.75	1.73	1.72
75	12.74	5.33	3.77	3.13	2.79	2.44	2.26	2.12	2.04	1.98	1.94	1.91	1.89	1.86	1.83	1.81	1.80	1.79	1.77	1.75
100	14.06	5.66	3.96	3.26	*2.89*	2.51	2.32	2.18	2.09	2.04	1.99	1.96	1.94	1.90	1.87	1.86	1.84	1.83	1.80	1.77
125	15.14	5.96	4.10	3.36	*2.97*	2.58	2.37	2.23	2.14	2.08	2.03	2.00	1.97	1.93	1.90	1.88	1.87	1.85	1.84	1.82
150	16.11	6.18	4.22	3.44	*3.04*	2.62	2.42	2.26	2.17	2.11	2.06	2.03	2.00	1.96	1.93	1.91	1.89	1.89	1.86	1.85
175	16.99	6.40	4.32	3.52	*3.10*	2.67	2.45	2.29	2.20	2.14	2.09	2.06	2.03	1.99	1.96	1.93	1.92	1.91	1.89	1.87
200	17.77	6.54	4.42	3.59	3.15	2.71	2.48	2.33	2.23	2.16	2.11	2.08	2.05	2.01	1.98	1.95	1.94	1.93	1.90	1.89

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.46	2.28	1.90	1.72	1.60	1.48	1.41	1.35	1.32	1.29	1.27	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
2	4.43	2.71	2.20	1.96	1.82	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
3	5.11	2.99	2.39	2.11	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
4	5.64	3.20	2.53	2.22	2.04	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
5	6.09	3.37	2.63	2.30	2.11	1.90	1.79	1.70	1.65	1.61	1.59	1.56	1.55	1.52	1.51	1.50	1.49	1.48	1.46	1.46
8	7.15	3.75	2.87	2.48	2.26	2.02	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
12	8.21	4.10	*3.08*	2.64	2.39	2.13	1.99	1.89	1.82	1.78	1.74	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
16	9.05	4.37	3.24	2.76	2.49	2.20	2.06	1.95	1.88	1.83	1.80	1.77	1.75	1.72	1.70	1.68	1.67	1.66	1.64	1.63
20	9.75	4.58	3.37	2.85	2.57	2.26	2.11	1.99	1.92	1.87	1.83	1.81	1.79	1.75	1.73	1.72	1.70	1.69	1.68	1.66
30	11.18	5.00	3.61	3.02	2.71	2.37	2.20	2.08	2.00	1.94	1.91	1.88	1.85	1.82	1.80	1.78	1.76	1.75	1.74	1.72
40	12.30	5.32	3.78	*3.15*	2.81	2.45	2.27	2.13	2.05	2.00	1.96	1.92	1.90	1.86	1.84	1.82	1.81	1.80	1.78	1.76
50	13.28	5.57	3.92	*3.25*	2.89	2.51	2.32	2.18	2.10	2.04	1.99	1.96	1.94	1.90	1.87	1.85	1.84	1.83	1.80	1.78
60	14.11	5.79	4.04	3.33	2.95	2.56	2.36	2.22	2.13	2.07	2.02	1.99	1.97	1.93	1.90	1.88	1.86	1.85	1.83	1.82
75	15.23	6.07	4.19	3.44	3.03	2.62	2.42	2.26	2.17	2.11	2.06	2.03	2.00	1.96	1.93	1.91	1.90	1.89	1.86	1.85
100	16.80	6.45	4.38	3.57	*3.14*	2.70	2.48	2.33	2.23	2.16	2.11	2.08	2.05	2.01	1.98	1.96	1.94	1.93	1.90	1.89
125	18.07	6.74	4.54	3.67	*3.22*	2.76	2.53	2.37	2.27	2.20	2.15	2.11	2.09	2.04	2.01	1.99	1.97	1.96	1.93	1.92
150	19.14	7.01	4.68	3.77	*3.30*	2.82	2.58	2.41	2.30	2.23	2.19	2.15	2.12	2.07	2.04	2.01	2.00	1.98	1.96	1.94
175	20.12	7.23	4.79	3.85	3.36	2.86	2.61	2.44	2.33	2.26	2.21	2.17	2.14	2.09	2.06	2.03	2.02	2.00	1.98	1.96
200	21.09	7.42	4.88	3.91	3.41	2.91	2.65	2.47	2.36	2.28	2.23	2.19	2.15	2.11	2.08	2.05	2.04	2.02	1.99	1.98

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.99	1.98	1.66	1.50	1.41	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06
2	3.84	2.38	1.94	1.74	1.62	1.49	1.41	1.36	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
3	4.43	2.63	2.12	1.88	1.74	1.59	1.51	1.45	1.41	1.38	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26
4	4.90	2.82	2.25	1.98	1.83	1.67	1.58	1.51	1.46	1.44	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
5	5.29	2.97	2.35	2.06	1.90	1.72	1.63	1.56	1.51	1.48	1.46	1.44	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34
8	6.22	3.31	2.57	2.24	2.05	1.85	1.74	1.66	1.61	1.57	1.54	1.53	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
12	7.14	3.63	*2.77*	2.39	2.18	1.95	1.83	1.74	1.69	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
16	7.88	3.87	2.91	2.50	2.27	2.03	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
20	8.49	4.06	3.03	2.59	2.34	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
30	9.74	4.44	3.25	*2.75*	2.48	2.19	2.04	1.93	1.86	1.82	1.78	1.76	1.74	1.71	1.69	1.67	1.66	1.65	1.63	1.62
40	10.73	4.72	3.41	2.87	2.58	2.27	2.11	1.99	1.92	1.87	1.84	1.81	1.79	1.75	1.73	1.72	1.70	1.69	1.68	1.67
50	11.55	4.96	3.54	2.97	*2.65*	2.33	2.16	2.04	1.96	1.91	1.87	1.85	1.82	1.79	1.77	1.75	1.74	1.73	1.71	1.70
60	12.30	5.14	3.65	3.04	*2.72*	2.38	2.20	2.08	2.00	1.94	1.91	1.88	1.85	1.82	1.80	1.78	1.76	1.75	1.73	1.72
75	13.26	5.40	3.79	3.14	*2.79*	2.44	2.26	2.12	2.04	1.98	1.94	1.91	1.89	1.85	1.83	1.81	1.80	1.79	1.77	1.75
100	14.63	5.74	3.96	3.26	2.90	2.52	2.32	2.18	2.10	2.04	2.00	1.96	1.94	1.90	1.87	1.85	1.83	1.82	1.79	1.77
125	15.72	6.02	4.12	3.37	2.97	2.58	2.38	2.23	2.14	2.08	2.03	2.00	1.97	1.93	1.91	1.88	1.87	1.85	1.82	1.80
150	16.68	6.22	4.24	3.45	3.04	*2.62*	2.42	2.26	2.17	2.11	2.06	2.03	2.00	1.96	1.93	1.91	1.89	1.88	1.86	1.85
175	17.64	6.43	4.34	3.52	3.10	*2.67*	2.45	2.30	2.20	2.14	2.09	2.06	2.03	1.99	1.96	1.94	1.92	1.91	1.88	1.87
200	18.32	6.63	4.44	3.59	3.14	*2.71*	2.49	2.32	2.23	2.16	2.11	2.08	2.05	2.01	1.98	1.96	1.94	1.93	1.91	1.89

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.68	2.34	1.93	1.73	1.61	1.48	1.41	1.35	1.32	1.29	1.28	1.26	1.25	1.23	1.22	1.21	1.21	1.20	1.19	1.19
2	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
3	5.42	3.06	2.42	2.12	1.95	1.77	1.67	1.59	1.55	1.51	1.49	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37
4	5.98	3.27	2.55	2.23	2.04	1.84	1.74	1.66	1.61	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
5	6.46	3.44	2.66	2.31	2.12	1.90	1.79	1.70	1.65	1.61	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.46	1.46
8	7.58	3.82	*2.90*	2.49	2.27	2.03	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
12	8.70	4.18	3.11	2.65	2.40	2.13	1.99	1.89	1.82	1.78	1.75	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
16	9.59	4.45	3.27	2.77	2.50	2.21	2.06	1.95	1.88	1.83	1.80	1.77	1.75	1.72	1.70	1.68	1.67	1.66	1.64	1.63
20	10.33	4.67	3.40	*2.86*	2.57	2.27	2.11	1.99	1.92	1.87	1.84	1.81	1.79	1.75	1.73	1.72	1.70	1.69	1.68	1.67
30	11.84	5.09	3.64	*3.04*	2.71	2.38	2.20	2.08	2.00	1.95	1.91	1.88	1.85	1.82	1.80	1.78	1.77	1.75	1.74	1.72
40	13.06	5.41	3.81	3.16	2.81	2.45	2.27	2.14	2.05	2.00	1.96	1.92	1.90	1.86	1.84	1.82	1.81	1.80	1.78	1.76
50	14.05	5.67	3.95	3.26	*2.89*	2.51	2.32	2.18	2.10	2.04	1.99	1.96	1.94	1.90	1.87	1.85	1.84	1.83	1.80	1.77
60	14.94	5.90	4.07	3.34	*2.96*	2.56	2.36	2.22	2.13	2.07	2.03	1.99	1.97	1.93	1.90	1.88	1.86	1.85	1.82	1.82
75	16.13	6.17	4.22	3.44	*3.04*	2.63	2.42	2.26	2.17	2.11	2.06	2.03	2.00	1.96	1.93	1.91	1.89	1.89	1.86	1.85
100	17.77	6.56	4.41	3.58	3.14	2.70	2.48	2.32	2.23	2.16	2.11	2.08	2.05	2.01	1.98	1.95	1.94	1.93	1.90	1.89
125	19.14	6.87	4.58	3.69	3.23	2.77	2.54	2.37	2.27	2.20	2.15	2.11	2.08	2.04	2.01	1.99	1.97	1.96	1.93	1.92
150	20.23	7.11	4.72	3.78	3.30	*2.82*	2.58	2.41	2.31	2.23	2.18	2.14	2.11	2.07	2.03	2.01	2.00	1.98	1.96	1.94
175	21.33	7.38	4.82	3.86	3.37	*2.86*	2.61	2.44	2.33	2.26	2.21	2.17	2.14	2.09	2.06	2.03	2.01	2.00	1.98	1.96
200	22.42	7.59	4.92	3.93	3.42	*2.91*	2.65	2.47	2.36	2.29	2.23	2.20	2.16	2.11	2.08	2.05	2.03	2.02	1.99	1.98

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (20 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.43	**2.71**	**2.20**	**1.96**	**1.82**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**	**1.31**
2	5.64	3.20	**2.53**	**2.22**	**2.04**	**1.84**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
3	6.48	3.51	**2.72**	**2.37**	**2.17**	**1.95**	**1.83**	**1.74**	**1.69**	**1.65**	**1.62**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.52**	**1.51**	**1.49**	**1.48**
4	7.15	3.75	**2.87**	**2.48**	**2.26**	**2.02**	**1.90**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.60**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
5	7.72	3.94	**2.99**	**2.57**	**2.33**	**2.08**	**1.95**	**1.85**	**1.79**	**1.74**	**1.71**	**1.69**	**1.67**	**1.64**	**1.62**	**1.61**	**1.60**	**1.59**	**1.57**	**1.56**
8	9.05	4.37	3.24	**2.76**	**2.49**	**2.21**	**2.06**	**1.95**	**1.88**	**1.83**	**1.80**	**1.77**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.64**	**1.63**
12	10.37	4.77	3.47	**2.93**	**2.63**	**2.31**	**2.15**	**2.03**	**1.96**	**1.90**	**1.87**	**1.84**	**1.82**	**1.78**	**1.76**	**1.74**	**1.73**	**1.72**	**1.70**	**1.69**
16	11.43	5.07	3.64	**3.05**	**2.73**	**2.39**	**2.22**	**2.09**	**2.01**	**1.96**	**1.92**	**1.89**	**1.86**	**1.83**	**1.81**	**1.79**	**1.77**	**1.76**	**1.74**	**1.73**
20	12.32	5.31	3.78	*3.15*	**2.81**	**2.45**	**2.27**	**2.14**	**2.05**	**2.00**	**1.96**	**1.92**	**1.90**	**1.86**	**1.84**	**1.82**	**1.81**	**1.80**	**1.78**	**1.76**
30	14.12	5.79	4.04	3.33	**2.95**	**2.56**	**2.36**	**2.22**	**2.13**	**2.07**	**2.03**	**1.99**	**1.97**	**1.93**	**1.90**	**1.88**	**1.86**	**1.85**	**1.83**	**1.82**
40	15.55	6.15	4.23	3.46	**3.06**	**2.64**	**2.43**	**2.28**	**2.18**	**2.12**	**2.07**	**2.04**	**2.01**	**1.97**	**1.94**	**1.92**	**1.91**	**1.89**	**1.87**	**1.86**
50	16.75	6.44	4.38	3.57	*3.14*	**2.70**	**2.48**	**2.32**	**2.23**	**2.16**	**2.11**	**2.08**	**2.05**	**2.01**	**1.98**	**1.95**	**1.94**	**1.93**	**1.90**	**1.89**
60	17.77	6.68	4.51	3.66	*3.21*	**2.76**	**2.53**	**2.36**	**2.26**	**2.19**	**2.14**	**2.11**	**2.08**	**2.03**	**2.00**	**1.98**	**1.96**	**1.95**	**1.93**	**1.91**
75	19.14	7.01	4.67	3.77	*3.30*	**2.82**	**2.58**	**2.41**	**2.30**	**2.23**	**2.18**	**2.14**	**2.11**	**2.07**	**2.04**	**2.01**	**2.00**	**1.98**	**1.96**	**1.94**
100	21.05	7.43	4.89	3.91	3.41	**2.90**	**2.65**	**2.47**	**2.36**	**2.29**	**2.23**	**2.19**	**2.16**	**2.11**	**2.08**	**2.05**	**2.04**	**2.02**	**1.99**	**1.98**
125	22.70	7.79	5.06	4.03	3.49	**2.97**	**2.70**	**2.51**	**2.40**	**2.32**	**2.27**	**2.23**	**2.19**	**2.14**	**2.11**	**2.08**	**2.06**	**2.05**	**2.02**	**2.00**
150	24.20	8.07	5.21	4.12	3.57	**3.02**	**2.74**	**2.55**	**2.44**	**2.36**	**2.30**	**2.26**	**2.22**	**2.17**	**2.14**	**2.11**	**2.09**	**2.07**	**2.05**	**2.03**
175	25.43	8.34	5.33	4.20	3.63	**3.06**	**2.79**	**2.58**	**2.47**	**2.38**	**2.33**	**2.29**	**2.24**	**2.19**	**2.16**	**2.13**	**2.11**	**2.10**	**2.07**	**2.05**
200	26.52	8.54	5.43	4.27	3.69	*3.10*	**2.81**	**2.61**	**2.49**	**2.41**	**2.35**	**2.31**	**2.26**	**2.21**	**2.18**	**2.15**	**2.13**	**2.11**	**2.08**	**2.07**

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (40 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.84	**2.38**	**1.94**	**1.74**	**1.62**	**1.49**	**1.41**	**1.36**	**1.32**	**1.29**	**1.28**	**1.26**	**1.25**	**1.23**	**1.22**	**1.21**	**1.21**	**1.20**	**1.19**	**1.19**
2	4.90	2.82	**2.25**	**1.98**	**1.83**	**1.67**	**1.58**	**1.51**	**1.46**	**1.43**	**1.41**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.33**	**1.31**	**1.31**
3	5.63	3.10	**2.43**	**2.13**	**1.96**	**1.77**	**1.67**	**1.60**	**1.55**	**1.51**	**1.49**	**1.47**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.39**	**1.38**	**1.37**
4	6.22	3.31	**2.57**	**2.24**	**2.05**	**1.85**	**1.74**	**1.66**	**1.61**	**1.57**	**1.54**	**1.53**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.44**	**1.43**	**1.42**
5	6.71	3.48	*2.68*	**2.32**	**2.12**	**1.90**	**1.79**	**1.70**	**1.65**	**1.61**	**1.59**	**1.57**	**1.55**	**1.53**	**1.51**	**1.50**	**1.49**	**1.48**	**1.46**	**1.46**
8	7.88	3.87	2.91	**2.50**	**2.27**	**2.03**	**1.90**	**1.80**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.61**	**1.59**	**1.57**	**1.56**	**1.55**	**1.54**	**1.53**
12	9.03	4.23	3.13	*2.66*	**2.40**	**2.13**	**1.99**	**1.89**	**1.82**	**1.78**	**1.74**	**1.72**	**1.70**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.60**	**1.59**
16	9.96	4.50	3.29	*2.78*	**2.50**	**2.21**	**2.06**	**1.95**	**1.88**	**1.83**	**1.80**	**1.77**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.66**	**1.64**	**1.63**
20	10.74	4.72	3.41	2.87	**2.58**	**2.27**	**2.11**	**1.99**	**1.92**	**1.87**	**1.84**	**1.81**	**1.79**	**1.76**	**1.73**	**1.72**	**1.70**	**1.69**	**1.68**	**1.67**
30	12.30	5.15	3.65	3.04	*2.71*	**2.38**	**2.20**	**2.08**	**2.00**	**1.95**	**1.91**	**1.88**	**1.85**	**1.82**	**1.80**	**1.78**	**1.77**	**1.75**	**1.74**	**1.72**
40	13.52	5.47	3.83	3.16	*2.82*	**2.46**	**2.27**	**2.14**	**2.05**	**2.00**	**1.96**	**1.92**	**1.90**	**1.87**	**1.84**	**1.82**	**1.81**	**1.80**	**1.78**	**1.76**
50	14.61	5.74	3.96	3.27	2.90	**2.51**	**2.32**	**2.18**	**2.09**	**2.04**	**1.99**	**1.96**	**1.94**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.81**	**1.79**
60	15.47	5.96	4.08	3.35	2.96	**2.56**	**2.36**	**2.22**	**2.13**	**2.07**	**2.03**	**1.99**	**1.97**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.83**	**1.82**
75	16.72	6.25	4.24	3.46	3.05	*2.63*	**2.42**	**2.27**	**2.17**	**2.11**	**2.06**	**2.03**	**2.00**	**1.96**	**1.93**	**1.91**	**1.90**	**1.88**	**1.87**	**1.85**
100	18.44	6.64	4.43	3.59	3.14	*2.71*	**2.48**	**2.32**	**2.23**	**2.16**	**2.11**	**2.08**	**2.05**	**2.01**	**1.98**	**1.95**	**1.94**	**1.92**	**1.90**	**1.89**
125	19.69	6.95	4.59	3.69	3.24	*2.77*	**2.54**	**2.37**	**2.27**	**2.20**	**2.15**	**2.11**	**2.08**	**2.04**	**2.01**	**1.99**	**1.97**	**1.96**	**1.93**	**1.92**
150	21.25	7.19	4.73	3.79	3.30	*2.81*	*2.58*	**2.40**	**2.30**	**2.23**	**2.18**	**2.14**	**2.11**	**2.07**	**2.04**	**2.01**	**1.99**	**1.98**	**1.96**	**1.94**
175	22.19	7.42	4.84	3.87	3.36	*2.87*	*2.62*	**2.44**	**2.32**	**2.26**	**2.21**	**2.17**	**2.14**	**2.09**	**2.06**	**2.03**	**2.02**	**2.00**	**1.98**	**1.96**
200	23.12	7.66	4.92	3.95	3.42	*2.91*	*2.65*	**2.46**	**2.35**	**2.29**	**2.23**	**2.19**	**2.16**	**2.11**	**2.08**	**2.05**	**2.03**	**2.02**	**1.99**	**1.98**

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.71	2.78	2.23	1.98	1.83	1.66	1.57	1.51	1.46	1.43	1.41	1.40	1.38	1.36	1.35	1.34	1.33	1.32	1.31	1.31
2	5.98	3.27	2.55	2.23	2.04	1.84	1.74	1.66	1.61	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
3	6.87	3.59	2.75	2.38	2.17	1.95	1.83	1.74	1.69	1.65	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.51	1.49	1.48
4	7.58	3.82	*2.90*	2.49	2.27	2.03	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
5	8.17	4.02	3.01	2.58	2.34	2.08	1.95	1.85	1.79	1.74	1.71	1.69	1.67	1.64	1.62	1.61	1.60	1.59	1.57	1.56
8	9.58	4.45	3.27	2.77	2.50	2.21	2.06	1.95	1.88	1.83	1.80	1.77	1.75	1.72	1.70	1.68	1.67	1.66	1.64	1.63
12	10.98	4.85	3.50	*2.94*	2.63	2.32	2.15	2.03	1.96	1.90	1.87	1.84	1.82	1.78	1.76	1.74	1.73	1.72	1.70	1.69
16	12.11	5.16	3.67	*3.06*	2.73	2.39	2.22	2.09	2.01	1.96	1.92	1.89	1.86	1.83	1.81	1.79	1.77	1.76	1.74	1.73
20	13.05	5.41	3.81	3.16	2.81	2.45	2.27	2.14	2.05	2.00	1.96	1.93	1.90	1.87	1.84	1.82	1.81	1.80	1.78	1.76
30	14.92	5.90	4.07	3.34	*2.96*	2.56	2.36	2.22	2.13	2.07	2.02	1.99	1.97	1.93	1.90	1.88	1.87	1.85	1.83	1.82
40	16.48	6.25	4.26	3.48	*3.07*	2.64	2.43	2.28	2.18	2.12	2.07	2.04	2.01	1.97	1.94	1.92	1.91	1.89	1.87	1.86
50	17.73	6.56	4.41	3.58	3.14	2.71	2.48	2.32	2.23	2.16	2.11	2.08	2.05	2.00	1.98	1.96	1.94	1.93	1.90	1.89
60	18.91	6.80	4.55	3.67	3.21	2.75	2.52	2.36	2.26	2.19	2.14	2.10	2.08	2.03	2.00	1.98	1.97	1.95	1.93	1.91
75	20.31	7.11	4.71	3.78	3.30	*2.82*	2.58	2.41	2.30	2.23	2.18	2.14	2.11	2.07	2.04	2.01	2.00	1.98	1.96	1.94
100	22.34	7.58	4.92	3.93	3.42	*2.90*	2.65	2.47	2.36	2.28	2.23	2.19	2.15	2.11	2.08	2.05	2.04	2.02	2.00	1.98
125	24.06	7.89	5.08	4.04	3.50	*2.97*	2.70	2.51	2.39	2.32	2.27	2.22	2.19	2.14	2.11	2.08	2.07	2.05	2.03	2.01
150	25.62	8.20	5.23	4.14	3.57	*3.02*	2.74	2.55	2.43	2.35	2.29	2.25	2.22	2.17	2.14	2.11	2.09	2.08	2.05	2.03
175	26.88	8.44	5.39	4.22	3.63	*3.07*	2.77	2.58	2.46	2.38	2.32	2.28	2.25	2.19	2.16	2.13	2.11	2.10	2.07	2.05
200	28.12	8.75	5.47	4.30	3.69	*3.11*	2.81	2.61	2.48	2.40	2.34	2.29	2.27	2.21	2.18	2.15	2.13	2.11	2.09	2.07

Table 19-15. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 2 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.64	3.20	2.53	2.22	2.04	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42
2	7.15	3.75	2.87	2.48	2.26	2.02	1.90	1.80	1.74	1.70	1.67	1.65	1.63	1.60	1.59	1.57	1.56	1.55	1.54	1.53
3	8.21	4.10	*3.08*	2.64	2.39	2.13	1.99	1.89	1.82	1.78	1.74	1.72	1.70	1.67	1.65	1.64	1.63	1.62	1.60	1.59
4	9.05	4.37	3.24	2.76	2.49	2.21	2.06	1.95	1.88	1.83	1.80	1.77	1.75	1.72	1.70	1.68	1.67	1.66	1.64	1.63
5	9.76	4.58	3.37	2.85	2.57	2.26	2.11	1.99	1.92	1.87	1.84	1.81	1.79	1.76	1.73	1.72	1.70	1.69	1.68	1.67
8	11.43	5.07	3.65	3.05	2.73	2.39	2.22	2.09	2.01	1.96	1.92	1.89	1.86	1.83	1.81	1.79	1.77	1.76	1.74	1.73
12	13.11	5.52	3.90	*3.23*	2.87	2.50	2.31	2.17	2.09	2.03	1.99	1.95	1.93	1.89	1.87	1.85	1.83	1.82	1.80	1.79
16	14.41	5.87	4.08	3.36	2.98	2.58	2.38	2.23	2.14	2.08	2.04	2.00	1.98	1.94	1.91	1.89	1.88	1.86	1.84	1.83
20	15.55	6.15	4.23	3.46	3.06	2.64	2.43	2.28	2.18	2.12	2.07	2.04	2.01	1.97	1.94	1.92	1.91	1.89	1.87	1.86
30	17.81	6.69	4.51	3.66	*3.21*	2.75	2.52	2.36	2.26	2.19	2.14	2.10	2.08	2.03	2.00	1.98	1.96	1.95	1.93	1.91
40	19.61	7.11	4.73	3.80	*3.32*	2.84	2.59	2.42	2.31	2.24	2.19	2.15	2.12	2.08	2.05	2.02	2.00	1.99	1.97	1.95
50	21.09	7.42	4.88	3.92	3.41	2.90	2.65	2.47	2.36	2.28	2.23	2.19	2.16	2.11	2.08	2.05	2.04	2.02	1.99	1.98
60	22.50	7.73	5.03	4.00	3.48	2.95	2.69	2.50	2.39	2.31	2.26	2.22	2.18	2.14	2.10	2.08	2.06	2.05	2.02	2.00
75	24.22	8.09	5.21	4.12	3.56	3.02	2.74	2.55	2.43	2.35	2.29	2.25	2.22	2.17	2.13	2.11	2.09	2.08	2.05	2.03
100	26.56	8.59	5.43	4.28	3.69	*3.10*	2.81	2.61	2.49	2.40	2.34	2.29	2.26	2.21	2.18	2.15	2.13	2.11	2.08	2.07
125	28.75	8.98	5.62	4.39	3.77	*3.16*	2.86	2.66	2.53	2.44	2.38	2.33	2.29	2.25	2.21	2.18	2.16	2.14	2.11	2.09
150	30.62	9.30	5.78	4.49	3.87	*3.22*	2.91	2.70	2.56	2.47	2.41	2.36	2.32	2.27	2.23	2.21	2.18	2.17	2.14	2.12
175	31.88	9.61	5.94	4.59	3.93	*3.26*	2.95	2.72	2.59	2.50	2.43	2.38	2.34	2.29	2.26	2.23	2.21	2.19	2.16	2.14
200	33.75	9.84	6.02	4.69	3.98	*3.30*	2.99	2.75	2.62	2.52	2.46	2.40	2.36	2.31	2.28	2.25	2.22	2.21	2.18	2.15

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.37	0.27	0.22	0.19	0.17	0.14	0.13	0.11	0.11	0.10	0.10	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.07	0.07
2	0.71	0.53	0.45	0.41	0.38	0.34	0.32	0.30	0.29	0.28	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.25	0.25	0.24
3	0.92	0.68	0.58	0.53	0.49	0.45	0.42	0.40	0.39	0.38	0.37	0.37	0.36	0.35	0.35	0.35	0.34	0.34	0.34	0.33
4	1.07	0.79	0.67	0.61	0.57	0.52	0.49	0.47	0.45	0.44	0.44	0.43	0.42	0.42	0.41	0.41	0.40	0.40	0.40	0.39
5	1.20	0.87	0.74	0.67	0.63	0.58	0.54	0.52	0.50	0.49	0.48	0.48	0.47	0.46	0.46	0.45	0.45	0.45	0.44	0.44
8	1.49	1.05	0.89	0.81	0.75	0.69	0.65	0.62	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.54	0.54	0.54	0.53	0.53
12	1.77	1.22	1.02	0.92	0.86	0.78	0.74	0.71	0.69	0.67	0.66	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.61	0.60
16	1.98	1.34	1.11	1.00	0.93	0.85	0.80	0.77	0.74	0.73	0.71	0.70	0.70	0.68	0.68	0.67	0.67	0.66	0.65	0.65
20	2.17	1.43	1.19	1.06	0.99	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.72	0.71	0.70	0.70	0.69	0.69
30	2.52	1.61	1.32	1.18	1.09	0.99	0.93	0.89	0.86	0.84	0.83	0.81	0.81	0.79	0.78	0.78	0.77	0.77	0.76	0.75
40	2.81	1.74	1.42	1.26	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
50	3.04	1.85	1.49	1.32	1.22	1.10	1.04	0.99	0.95	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.83
60	3.25	1.94	1.56	1.37	1.26	1.14	1.07	1.02	0.99	0.96	0.94	0.93	0.92	0.91	0.89	0.89	0.88	0.87	0.86	0.86
75	3.52	2.06	1.63	1.44	1.32	1.19	1.12	1.06	1.02	1.00	0.98	0.97	0.96	0.94	0.93	0.92	0.91	0.91	0.90	0.89
100	3.89	2.21	1.74	1.52	1.39	1.25	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.94	0.93
125	4.21	2.33	1.82	1.58	1.45	1.30	1.21	1.15	1.11	1.08	1.06	1.05	1.04	1.02	1.00	0.99	0.99	0.98	0.97	0.96
150	4.49	2.43	1.89	1.64	1.49	1.34	1.25	1.18	1.14	1.11	1.09	1.08	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99
175	4.73	*2.52*	1.94	1.68	1.53	1.37	1.28	1.21	1.17	1.14	1.12	1.10	1.09	1.07	1.05	1.04	1.04	1.03	1.02	1.01
200	4.96	*2.61*	2.00	1.72	1.57	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.64	0.50	0.43	0.39	0.36	0.33	0.31	0.30	0.29	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.25	0.25	0.24	0.24
2	1.00	0.76	0.66	0.60	0.56	0.51	0.49	0.47	0.45	0.44	0.43	0.43	0.42	0.41	0.41	0.41	0.40	0.40	0.40	0.39
3	1.23	0.91	0.79	0.72	0.67	0.62	0.58	0.56	0.54	0.53	0.52	0.51	0.51	0.50	0.49	0.49	0.48	0.48	0.48	0.47
4	1.41	1.03	0.88	0.80	0.75	0.68	0.65	0.62	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.54	0.53	0.53
5	1.56	1.11	0.95	0.86	0.80	0.74	0.70	0.67	0.65	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
8	1.89	1.31	1.10	0.99	0.92	0.85	0.80	0.76	0.74	0.72	0.71	0.70	0.70	0.68	0.68	0.67	0.67	0.66	0.65	0.65
12	2.22	1.48	1.23	1.11	1.03	0.94	0.88	0.84	0.82	0.80	0.79	0.78	0.77	0.75	0.75	0.74	0.73	0.73	0.72	0.72
16	2.48	1.61	1.33	1.19	1.10	1.00	0.94	0.90	0.87	0.85	0.84	0.83	0.82	0.80	0.79	0.79	0.78	0.78	0.77	0.76
20	2.69	1.72	1.40	1.25	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
30	3.12	1.91	1.54	1.36	1.26	1.14	1.07	1.02	0.99	0.96	0.94	0.93	0.92	0.90	0.89	0.89	0.88	0.87	0.86	0.86
40	3.46	2.06	1.65	1.45	1.33	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
50	3.75	2.18	1.73	1.51	1.39	1.25	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.94	0.93
60	3.99	2.28	1.79	1.57	1.43	1.29	1.21	1.14	1.10	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.96	0.96
75	4.32	2.40	1.88	1.63	1.49	1.33	1.25	1.18	1.14	1.11	1.09	1.08	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99
100	4.77	2.57	1.99	1.72	1.56	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
125	5.16	*2.71*	2.08	1.79	1.62	1.44	1.35	1.27	1.23	1.20	1.17	1.15	1.14	1.12	1.10	1.09	1.08	1.08	1.07	1.06
150	5.47	2.82	2.15	1.84	1.67	1.48	1.38	1.31	1.26	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
175	5.78	2.93	2.21	1.89	1.71	1.52	1.41	1.33	1.28	1.25	1.22	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10
200	6.05	3.02	2.27	1.93	1.75	1.54	1.44	1.36	1.30	1.27	1.25	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.90	0.71	0.62	0.57	0.54	0.50	0.48	0.46	0.45	0.44	0.43	0.42	0.42	0.41	0.41	0.40	0.40	0.40	0.39	0.39
2	1.30	0.98	0.85	0.78	0.73	0.68	0.64	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.54	0.53	0.53
3	1.56	1.14	0.98	0.89	0.84	0.77	0.73	0.70	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60
4	1.76	1.26	1.08	0.98	0.91	0.84	0.79	0.76	0.74	0.72	0.71	0.70	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65
5	1.93	1.36	1.15	1.04	0.97	0.89	0.84	0.81	0.78	0.76	0.75	0.74	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
8	2.32	1.57	1.31	1.17	1.09	0.99	0.94	0.90	0.87	0.85	0.84	0.82	0.82	0.80	0.79	0.79	0.78	0.78	0.77	0.76
12	2.71	1.76	1.44	1.29	1.19	1.08	1.02	0.97	0.94	0.92	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.83
16	3.01	1.90	1.54	1.37	1.27	1.15	1.08	1.03	1.00	0.97	0.95	0.94	0.93	0.91	0.90	0.90	0.89	0.88	0.87	0.87
20	3.26	2.01	1.62	1.44	1.32	1.20	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
30	3.76	2.23	1.77	1.56	1.43	1.28	1.20	1.14	1.10	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.96	0.96
40	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
50	4.50	2.52	1.97	1.71	1.56	1.39	1.30	1.23	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
60	4.79	2.63	2.04	1.76	1.61	1.43	1.34	1.27	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
75	5.18	2.77	2.13	1.83	1.67	1.48	1.38	1.30	1.26	1.22	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
100	5.72	2.96	2.25	1.92	1.74	1.54	1.44	1.35	1.30	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
125	6.17	3.12	2.34	1.99	1.80	1.59	1.48	1.39	1.34	1.30	1.28	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15
150	6.56	3.24	2.42	2.06	1.85	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.27	1.24	1.22	1.21	1.20	1.19	1.18	1.17
175	6.91	3.36	2.49	2.10	1.89	1.66	1.54	1.45	1.39	1.35	1.33	1.31	1.29	1.26	1.25	1.23	1.22	1.21	1.20	1.19
200	7.23	3.46	2.54	2.15	1.93	1.69	1.57	1.47	1.42	1.38	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.22	1.21

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.71	0.53	0.45	0.41	0.38	0.34	0.32	0.30	0.29	0.28	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.25	0.25	0.24
2	1.07	0.79	0.67	0.61	0.57	0.52	0.49	0.47	0.45	0.44	0.44	0.43	0.42	0.42	0.41	0.41	0.40	0.40	0.40	0.39
3	1.31	0.94	0.80	0.73	0.68	0.62	0.59	0.56	0.54	0.53	0.52	0.51	0.51	0.50	0.49	0.49	0.49	0.48	0.48	0.47
4	1.49	1.05	0.89	0.81	0.75	0.69	0.65	0.62	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.54	0.54	0.54	0.53	0.53
5	1.64	1.14	0.96	0.87	0.81	0.74	0.70	0.67	0.65	0.63	0.62	0.62	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
8	1.98	1.34	1.11	1.00	0.93	0.85	0.80	0.77	0.74	0.73	0.71	0.70	0.70	0.68	0.68	0.67	0.67	0.66	0.65	0.65
12	2.32	1.51	1.25	1.11	1.03	0.94	0.89	0.85	0.82	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.73	0.72	0.72
16	2.59	1.64	1.34	1.19	1.10	1.00	0.95	0.90	0.87	0.85	0.84	0.83	0.82	0.80	0.79	0.79	0.78	0.78	0.77	0.76
20	2.81	1.74	1.42	1.26	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
30	3.25	1.94	1.56	1.37	1.26	1.14	1.07	1.02	0.99	0.96	0.94	0.93	0.92	0.91	0.89	0.89	0.88	0.87	0.86	0.86
40	3.60	2.09	1.66	1.45	1.34	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
50	3.89	2.21	1.74	1.52	1.39	1.25	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.94	0.93
60	4.15	2.30	1.80	1.57	1.44	1.29	1.21	1.15	1.11	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.97	0.96
75	4.49	2.43	1.89	1.64	1.49	1.34	1.25	1.18	1.14	1.11	1.09	1.08	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99
100	4.96	*2.61*	2.00	1.72	1.57	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
125	5.35	2.74	2.09	1.79	1.63	1.45	1.35	1.27	1.23	1.20	1.17	1.15	1.14	1.12	1.10	1.09	1.08	1.08	1.06	1.06
150	5.70	2.85	2.16	1.85	1.67	1.48	1.38	1.31	1.26	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
175	6.02	2.97	2.23	1.89	1.72	1.52	1.41	1.33	1.28	1.25	1.23	1.21	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10
200	6.25	3.05	2.28	1.93	1.75	1.54	1.44	1.36	1.30	1.27	1.25	1.23	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.00	0.76	0.66	0.60	0.56	0.51	0.49	0.47	0.45	0.44	0.43	0.43	0.42	0.41	0.41	0.41	0.40	0.40	0.40	0.39
2	1.41	1.03	0.88	0.80	0.75	0.68	0.65	0.62	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.54	0.53	0.53
3	1.68	1.19	1.01	0.91	0.85	0.78	0.74	0.71	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60
4	1.89	1.31	1.10	0.99	0.92	0.85	0.80	0.76	0.74	0.72	0.71	0.70	0.70	0.68	0.68	0.67	0.67	0.66	0.65	0.65
5	2.07	1.40	1.17	1.05	0.98	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
8	2.48	1.61	1.33	1.19	1.10	1.00	0.94	0.90	0.87	0.85	0.84	0.83	0.82	0.80	0.79	0.79	0.78	0.78	0.77	0.76
12	2.88	1.80	1.47	1.30	1.20	1.09	1.03	0.98	0.95	0.92	0.91	0.89	0.88	0.87	0.86	0.85	0.84	0.84	0.83	0.83
16	3.20	1.94	1.57	1.38	1.27	1.15	1.08	1.03	1.00	0.97	0.96	0.94	0.93	0.91	0.90	0.90	0.89	0.88	0.87	0.87
20	3.46	2.06	1.65	1.45	1.33	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
30	3.99	2.28	1.79	1.57	1.43	1.29	1.21	1.14	1.10	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.96	0.96
40	4.41	2.44	1.90	1.65	1.51	1.35	1.26	1.20	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
50	4.77	2.57	1.99	1.72	1.56	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
60	5.08	*2.69*	2.06	1.77	1.61	1.44	1.34	1.27	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
75	5.47	2.82	2.15	1.84	1.67	1.48	1.38	1.31	1.26	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
100	6.05	3.02	2.27	1.93	1.75	1.54	1.44	1.36	1.30	1.27	1.25	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
125	6.52	3.16	2.36	2.00	1.81	1.59	1.48	1.40	1.34	1.30	1.28	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15
150	6.95	3.30	2.44	2.06	1.86	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.26	1.24	1.23	1.21	1.20	1.19	1.18	1.17
175	7.34	3.42	2.50	2.11	1.89	1.67	1.54	1.45	1.40	1.36	1.33	1.30	1.29	1.26	1.25	1.23	1.22	1.21	1.20	1.19
200	7.66	3.52	2.56	2.16	1.93	1.69	1.57	1.47	1.42	1.38	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.22	1.21

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.30	0.98	0.85	0.78	0.73	0.68	0.64	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.54	0.54	0.53	0.53
2	1.76	1.26	1.08	0.98	0.91	0.84	0.79	0.76	0.74	0.72	0.71	0.70	0.69	0.68	0.67	0.67	0.66	0.66	0.65	0.65
3	2.08	1.44	1.21	1.09	1.02	0.93	0.88	0.84	0.82	0.80	0.78	0.77	0.77	0.75	0.74	0.74	0.73	0.73	0.72	0.72
4	2.32	1.57	1.31	1.17	1.09	0.99	0.94	0.90	0.87	0.85	0.84	0.82	0.82	0.80	0.79	0.79	0.78	0.78	0.77	0.76
5	2.53	1.67	1.38	1.24	1.15	1.04	0.99	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
8	3.01	1.90	1.54	1.37	1.27	1.15	1.08	1.03	1.00	0.97	0.95	0.94	0.93	0.91	0.90	0.90	0.89	0.88	0.87	0.87
12	3.48	2.11	1.69	1.49	1.37	1.24	1.16	1.10	1.07	1.04	1.02	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.93
16	3.85	2.26	1.80	1.57	1.44	1.30	1.22	1.15	1.12	1.09	1.07	1.05	1.04	1.02	1.01	1.00	0.99	0.98	0.97	0.97
20	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
30	4.79	2.63	2.04	1.76	1.61	1.43	1.34	1.27	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
40	5.29	2.81	2.15	1.85	1.68	1.49	1.39	1.32	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
50	5.72	2.96	2.25	1.92	1.74	1.54	1.44	1.35	1.30	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
60	6.09	3.09	2.32	1.98	1.79	1.58	1.47	1.39	1.33	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14
75	6.56	3.24	2.42	2.06	1.85	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.27	1.24	1.22	1.21	1.20	1.19	1.18	1.17
100	7.23	3.46	2.54	2.15	1.93	1.69	1.57	1.47	1.42	1.38	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.22	1.21
125	7.81	3.63	2.65	2.23	1.99	1.74	1.61	1.51	1.45	1.41	1.38	1.35	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23
150	8.28	3.77	2.73	2.29	2.04	1.78	1.65	1.54	1.48	1.44	1.41	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
175	8.75	3.91	2.80	2.34	2.08	1.82	1.67	1.57	1.50	1.46	1.43	1.40	1.38	1.35	1.33	1.32	1.31	1.30	1.28	1.27
200	9.14	4.02	*2.87*	2.38	2.12	1.85	1.70	1.59	1.52	1.48	1.45	1.42	1.40	1.37	1.35	1.34	1.32	1.31	1.30	1.29

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.20	0.87	0.74	0.67	0.63	0.58	0.54	0.52	0.50	0.49	0.48	0.48	0.47	0.46	0.46	0.45	0.45	0.45	0.44	0.44
2	1.64	1.14	0.96	0.87	0.81	0.74	0.70	0.67	0.65	0.63	0.62	0.62	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
3	1.93	1.31	1.09	0.98	0.91	0.83	0.79	0.75	0.73	0.71	0.70	0.69	0.68	0.67	0.66	0.66	0.65	0.65	0.64	0.64
4	2.16	1.43	1.19	1.06	0.99	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
5	2.36	1.53	1.26	1.12	1.04	0.95	0.90	0.85	0.83	0.81	0.79	0.78	0.77	0.76	0.75	0.75	0.74	0.74	0.73	0.72
8	2.81	1.74	1.42	1.26	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
12	3.25	1.94	1.56	1.37	1.26	1.14	1.07	1.02	0.99	0.96	0.94	0.93	0.92	0.91	0.89	0.89	0.88	0.87	0.86	0.86
16	3.60	2.09	1.66	1.45	1.34	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
20	3.89	2.21	1.74	1.52	1.39	1.25	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.94	0.93
30	4.48	2.43	1.89	1.64	1.49	1.34	1.25	1.18	1.14	1.11	1.09	1.08	1.06	1.04	1.03	1.02	1.01	1.01	1.00	0.99
40	4.95	*2.60*	2.00	1.72	1.57	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
50	5.34	2.74	2.08	1.79	1.63	1.45	1.35	1.27	1.23	1.20	1.17	1.15	1.14	1.12	1.10	1.09	1.08	1.08	1.07	1.06
60	5.68	2.86	2.16	1.85	1.67	1.48	1.38	1.31	1.26	1.22	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
75	6.13	3.01	2.25	1.92	1.73	1.53	1.42	1.35	1.30	1.26	1.23	1.21	1.20	1.18	1.16	1.15	1.14	1.13	1.12	1.11
100	6.76	3.21	2.37	2.01	1.81	1.59	1.48	1.39	1.34	1.30	1.28	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15
125	7.30	3.37	*2.47*	2.08	1.87	1.64	1.52	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.18	1.17
150	7.77	3.51	*2.55*	2.14	1.92	1.68	1.56	1.46	1.41	1.37	1.34	1.32	1.30	1.27	1.25	1.24	1.23	1.22	1.21	1.20
175	8.16	3.63	*2.62*	2.19	1.96	1.71	1.59	1.49	1.43	1.39	1.36	1.34	1.32	1.29	1.27	1.26	1.25	1.24	1.23	1.22
200	8.55	3.73	2.68	2.24	2.00	1.74	1.61	1.51	1.45	1.41	1.38	1.35	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.56	1.11	0.95	0.86	0.80	0.74	0.70	0.67	0.65	0.63	0.62	0.61	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
2	2.07	1.40	1.17	1.05	0.98	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
3	2.42	1.58	1.31	1.17	1.08	0.99	0.93	0.89	0.86	0.84	0.83	0.81	0.81	0.79	0.78	0.78	0.77	0.77	0.76	0.75
4	2.69	1.72	1.40	1.25	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
5	2.92	1.82	1.48	1.31	1.21	1.10	1.03	0.98	0.95	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.83
8	3.46	2.06	1.65	1.45	1.33	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
12	3.99	2.28	1.79	1.57	1.43	1.29	1.21	1.14	1.11	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.97	0.96
16	4.41	2.44	1.90	1.65	1.51	1.35	1.26	1.20	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
20	4.77	2.57	1.99	1.72	1.57	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
30	5.48	2.83	2.15	1.84	1.67	1.48	1.38	1.31	1.26	1.22	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
40	6.04	3.02	2.27	1.93	1.75	1.54	1.44	1.36	1.30	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
50	6.52	3.17	2.36	2.00	1.81	1.59	1.48	1.39	1.34	1.30	1.28	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15
60	6.93	3.30	2.44	2.06	1.86	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.27	1.24	1.22	1.21	1.20	1.19	1.18	1.17
75	7.48	3.47	2.54	2.14	1.92	1.68	1.56	1.46	1.41	1.37	1.34	1.31	1.30	1.27	1.25	1.24	1.23	1.22	1.21	1.20
100	8.24	3.69	*2.67*	2.23	1.99	1.74	1.61	1.51	1.45	1.41	1.38	1.36	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23
125	8.91	3.88	*2.77*	2.31	2.06	1.79	1.65	1.55	1.49	1.44	1.41	1.39	1.37	1.34	1.32	1.31	1.29	1.29	1.27	1.26
150	9.45	4.03	2.86	2.37	2.11	1.83	1.69	1.58	1.52	1.47	1.44	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
175	9.96	4.17	2.93	2.43	2.15	1.87	1.72	1.61	1.54	1.49	1.46	1.43	1.41	1.38	1.36	1.35	1.34	1.33	1.31	1.30
200	10.39	4.30	3.00	2.47	2.19	1.89	1.74	1.63	1.56	1.51	1.48	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.31

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.93	1.36	1.15	1.04	0.97	0.89	0.84	0.81	0.78	0.76	0.75	0.74	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
2	2.53	1.67	1.38	1.24	1.15	1.04	0.99	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
3	2.94	1.86	1.52	1.35	1.25	1.13	1.07	1.02	0.98	0.96	0.94	0.93	0.92	0.90	0.89	0.89	0.88	0.87	0.86	0.86
4	3.26	2.01	1.62	1.44	1.32	1.20	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
5	3.53	2.13	1.71	1.50	1.38	1.24	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.94	0.93
8	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
12	4.79	2.63	2.04	1.76	1.61	1.43	1.34	1.27	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
16	5.30	2.81	2.15	1.85	1.68	1.49	1.39	1.32	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
20	5.72	2.96	2.24	1.92	1.74	1.54	1.44	1.36	1.30	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
30	6.56	3.24	2.42	2.05	1.85	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.27	1.24	1.22	1.21	1.20	1.19	1.18	1.17
40	7.24	3.45	2.54	2.15	1.93	1.69	1.57	1.47	1.42	1.38	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.21	1.21
50	7.80	3.63	2.65	2.22	1.99	1.74	1.61	1.51	1.45	1.41	1.38	1.36	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23
60	8.30	3.77	2.73	2.29	2.04	1.78	1.64	1.54	1.48	1.44	1.41	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
75	8.95	3.96	*2.84*	2.36	2.10	1.83	1.69	1.58	1.52	1.47	1.44	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
100	9.84	4.22	*2.98*	2.46	2.19	1.89	1.74	1.63	1.56	1.51	1.48	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.31
125	10.62	4.42	3.09	2.54	2.25	1.94	1.78	1.67	1.60	1.55	1.51	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
150	11.29	4.59	3.18	2.61	2.30	1.98	1.82	1.70	1.62	1.57	1.54	1.51	1.49	1.45	1.43	1.41	1.40	1.39	1.37	1.36
175	11.88	4.75	3.27	2.67	2.35	2.02	1.85	1.72	1.65	1.60	1.56	1.53	1.51	1.47	1.45	1.43	1.42	1.41	1.39	1.38
200	12.42	4.88	3.34	2.71	2.39	2.05	1.87	1.75	1.67	1.62	1.58	1.55	1.52	1.49	1.47	1.45	1.44	1.43	1.41	1.39

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.64	1.14	0.96	0.87	0.81	0.74	0.70	0.67	0.65	0.63	0.62	0.62	0.61	0.60	0.59	0.59	0.58	0.58	0.57	0.57
2	2.16	1.43	1.19	1.06	0.99	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
3	2.52	1.61	1.32	1.18	1.09	0.99	0.93	0.89	0.86	0.84	0.83	0.82	0.81	0.79	0.78	0.78	0.77	0.77	0.76	0.75
4	2.81	1.74	1.42	1.26	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
5	3.04	1.85	1.49	1.32	1.22	1.10	1.04	0.99	0.95	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.83
8	3.60	2.09	1.66	1.45	1.34	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
12	4.15	2.31	1.80	1.57	1.44	1.29	1.21	1.15	1.11	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.97	0.96
16	4.58	*2.47*	1.91	1.66	1.51	1.35	1.26	1.20	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
20	4.95	*2.60*	2.00	1.72	1.57	1.40	1.30	1.24	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
30	5.68	2.86	2.16	1.85	1.67	1.48	1.38	1.31	1.26	1.22	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08
40	6.27	3.05	2.28	1.94	1.75	1.55	1.44	1.36	1.30	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
50	6.76	3.21	2.37	2.01	1.81	1.59	1.48	1.39	1.34	1.30	1.28	1.26	1.24	1.22	1.20	1.19	1.18	1.17	1.16	1.15
60	7.19	3.34	*2.45*	2.07	1.86	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.27	1.24	1.22	1.21	1.20	1.19	1.18	1.17
75	7.77	3.51	*2.55*	2.14	1.92	1.68	1.56	1.46	1.41	1.37	1.34	1.32	1.30	1.27	1.25	1.24	1.23	1.22	1.21	1.20
100	8.55	3.73	2.68	2.24	2.00	1.74	1.61	1.51	1.45	1.41	1.38	1.35	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23
125	9.22	3.93	2.78	2.31	2.06	1.79	1.66	1.55	1.49	1.44	1.41	1.39	1.37	1.34	1.32	1.31	1.29	1.28	1.27	1.26
150	9.84	4.08	2.87	2.37	2.11	1.83	1.69	1.58	1.52	1.47	1.44	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
175	10.31	4.22	2.95	*2.43*	2.16	1.87	1.72	1.61	1.54	1.49	1.46	1.44	1.42	1.38	1.36	1.35	1.34	1.33	1.31	1.30
200	10.78	4.34	3.01	*2.48*	2.19	1.89	1.74	1.63	1.56	1.51	1.48	1.46	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.31

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.07	1.40	1.17	1.05	0.98	0.90	0.85	0.81	0.78	0.77	0.75	0.74	0.74	0.72	0.71	0.71	0.70	0.70	0.69	0.69
2	2.69	1.72	1.40	1.25	1.16	1.05	0.99	0.94	0.91	0.89	0.88	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
3	3.12	1.91	1.54	1.36	1.26	1.14	1.07	1.02	0.98	0.96	0.94	0.93	0.92	0.90	0.89	0.89	0.88	0.87	0.86	0.86
4	3.46	2.06	1.65	1.45	1.33	1.20	1.13	1.07	1.04	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
5	3.75	2.18	1.73	1.51	1.39	1.25	1.17	1.11	1.07	1.05	1.03	1.01	1.00	0.98	0.97	0.96	0.96	0.95	0.94	0.93
8	4.41	2.44	1.90	1.65	1.51	1.35	1.26	1.20	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
12	5.08	*2.68*	2.06	1.77	1.61	1.43	1.34	1.27	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.09	1.08	1.07	1.06	1.05
16	5.60	2.87	2.17	1.86	1.69	1.50	1.39	1.32	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
20	6.04	3.02	2.27	1.93	1.75	1.54	1.44	1.36	1.30	1.27	1.24	1.22	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12
30	6.93	3.30	2.44	2.06	1.86	1.63	1.51	1.43	1.37	1.33	1.30	1.28	1.27	1.24	1.22	1.21	1.20	1.19	1.18	1.17
40	7.66	3.52	2.56	2.16	1.93	1.69	1.57	1.47	1.42	1.38	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.21	1.20
50	8.24	3.69	*2.67*	2.23	1.99	1.74	1.61	1.51	1.45	1.41	1.38	1.36	1.34	1.31	1.29	1.28	1.27	1.26	1.24	1.23
60	8.77	3.85	*2.75*	2.29	2.05	1.78	1.65	1.54	1.48	1.44	1.41	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
75	9.45	4.03	2.86	2.37	2.11	1.83	1.69	1.58	1.52	1.47	1.44	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.29	1.28
100	10.39	4.30	3.00	2.47	2.19	1.89	1.74	1.63	1.56	1.51	1.48	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.31
125	11.25	4.49	3.12	2.55	2.26	1.94	1.78	1.67	1.60	1.55	1.51	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
150	11.88	4.67	3.20	*2.62*	2.30	1.98	1.82	1.70	1.63	1.57	1.54	1.51	1.48	1.46	1.43	1.41	1.40	1.39	1.37	1.36
175	12.50	4.84	3.28	*2.68*	2.35	2.02	1.85	1.72	1.65	1.60	1.56	1.53	1.51	1.47	1.45	1.43	1.42	1.41	1.39	1.38
200	13.12	4.96	3.36	*2.73*	2.39	2.05	1.88	1.75	1.67	1.62	1.58	1.55	1.52	1.49	1.46	1.45	1.44	1.43	1.41	1.39

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.53	1.67	1.38	1.24	1.15	1.04	0.99	0.94	0.91	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80
2	3.26	2.01	1.62	1.44	1.32	1.20	1.12	1.07	1.03	1.01	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92	0.91	0.90
3	3.77	2.23	1.77	1.56	1.43	1.28	1.20	1.14	1.10	1.08	1.06	1.04	1.03	1.01	1.00	0.99	0.98	0.98	0.97	0.96
4	4.17	2.39	1.88	1.64	1.50	1.35	1.26	1.19	1.15	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
5	4.50	2.52	1.97	1.71	1.56	1.39	1.30	1.23	1.19	1.16	1.14	1.12	1.11	1.09	1.07	1.06	1.05	1.05	1.04	1.03
8	5.30	2.81	2.15	1.85	1.68	1.49	1.39	1.32	1.27	1.23	1.21	1.19	1.18	1.15	1.14	1.13	1.12	1.11	1.10	1.09
12	6.08	3.08	2.32	1.98	1.79	1.58	1.47	1.39	1.33	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14
16	6.71	3.29	2.45	2.08	1.87	1.64	1.52	1.44	1.38	1.34	1.31	1.29	1.28	1.25	1.23	1.22	1.21	1.20	1.19	1.18
20	7.24	3.45	2.54	2.15	1.93	1.69	1.57	1.47	1.42	1.38	1.35	1.32	1.31	1.28	1.26	1.25	1.24	1.23	1.21	1.21
30	8.30	3.77	2.73	2.29	2.04	1.78	1.64	1.54	1.48	1.44	1.41	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.26	1.25
40	9.14	4.02	*2.87*	2.39	2.12	1.84	1.70	1.59	1.53	1.48	1.45	1.42	1.40	1.37	1.35	1.34	1.32	1.32	1.30	1.29
50	9.84	4.22	*2.98*	2.46	2.19	1.89	1.74	1.63	1.56	1.51	1.48	1.45	1.43	1.40	1.38	1.36	1.35	1.34	1.33	1.31
60	10.47	4.38	3.07	2.53	2.24	1.93	1.77	1.66	1.59	1.54	1.50	1.48	1.46	1.42	1.40	1.39	1.37	1.36	1.35	1.34
75	11.29	4.59	3.18	2.61	2.30	1.98	1.82	1.70	1.62	1.57	1.54	1.51	1.49	1.45	1.43	1.41	1.40	1.39	1.37	1.36
100	12.42	4.88	3.34	2.71	2.39	2.05	1.87	1.75	1.67	1.62	1.58	1.55	1.52	1.49	1.47	1.45	1.44	1.43	1.41	1.39
125	13.44	5.12	3.46	2.80	2.46	2.10	1.91	1.78	1.70	1.65	1.61	1.58	1.55	1.52	1.49	1.47	1.46	1.45	1.43	1.42
150	14.22	5.31	3.56	*2.87*	2.51	2.14	1.95	1.82	1.73	1.67	1.63	1.60	1.58	1.54	1.52	1.50	1.48	1.47	1.45	1.44
175	15.00	5.49	3.65	*2.93*	2.56	2.18	1.98	1.84	1.75	1.69	1.66	1.62	1.60	1.56	1.53	1.51	1.50	1.49	1.46	1.45
200	15.62	5.62	3.73	*2.99*	2.60	2.21	2.01	1.87	1.78	1.71	1.67	1.64	1.62	1.58	1.55	1.53	1.51	1.50	1.48	1.47

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.16**	**1.43**	**1.19**	**1.06**	**0.99**	**0.90**	**0.85**	**0.81**	**0.78**	**0.77**	**0.75**	**0.74**	**0.74**	**0.72**	**0.71**	**0.71**	**0.70**	**0.70**	**0.69**	**0.69**
2	2.81	**1.74**	**1.42**	**1.26**	**1.16**	**1.05**	**0.99**	**0.94**	**0.91**	**0.89**	**0.88**	**0.86**	**0.85**	**0.84**	**0.83**	**0.82**	**0.82**	**0.81**	**0.80**	**0.80**
3	3.25	**1.94**	**1.56**	**1.37**	**1.26**	**1.14**	**1.07**	**1.02**	**0.99**	**0.96**	**0.94**	**0.93**	**0.92**	**0.91**	**0.89**	**0.89**	**0.88**	**0.87**	**0.86**	**0.86**
4	3.60	**2.09**	**1.66**	**1.45**	**1.34**	**1.20**	**1.13**	**1.07**	**1.04**	**1.01**	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.90**
5	3.89	**2.21**	**1.74**	**1.52**	**1.39**	**1.25**	**1.17**	**1.11**	**1.07**	**1.05**	**1.03**	**1.01**	**1.00**	**0.98**	**0.97**	**0.96**	**0.96**	**0.95**	**0.94**	**0.93**
8	4.58	*2.47*	**1.91**	**1.66**	**1.51**	**1.35**	**1.26**	**1.20**	**1.15**	**1.12**	**1.10**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.02**	**1.01**	**1.00**
12	5.27	2.72	**2.07**	**1.78**	**1.62**	**1.44**	**1.34**	**1.27**	**1.22**	**1.19**	**1.17**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.06**	**1.05**
16	5.82	2.90	**2.19**	**1.87**	**1.69**	**1.50**	**1.39**	**1.32**	**1.27**	**1.24**	**1.21**	**1.19**	**1.18**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**
20	6.27	3.05	**2.28**	**1.94**	**1.75**	**1.55**	**1.44**	**1.36**	**1.31**	**1.27**	**1.24**	**1.22**	**1.21**	**1.19**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**	**1.12**
30	7.20	3.34	*2.45*	**2.07**	**1.86**	**1.63**	**1.51**	**1.43**	**1.37**	**1.33**	**1.30**	**1.28**	**1.27**	**1.24**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
40	7.93	3.56	*2.58*	**2.16**	**1.94**	**1.70**	**1.57**	**1.48**	**1.42**	**1.38**	**1.35**	**1.32**	**1.31**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.21**	**1.21**
50	8.54	3.74	2.68	**2.24**	**2.00**	**1.74**	**1.61**	**1.51**	**1.45**	**1.41**	**1.38**	**1.36**	**1.34**	**1.31**	**1.29**	**1.28**	**1.27**	**1.26**	**1.24**	**1.23**
60	9.08	3.88	2.76	**2.30**	**2.05**	**1.78**	**1.64**	**1.54**	**1.48**	**1.44**	**1.41**	**1.38**	**1.36**	**1.34**	**1.32**	**1.30**	**1.29**	**1.28**	**1.26**	**1.25**
75	9.81	4.08	2.87	**2.37**	**2.11**	**1.83**	**1.69**	**1.58**	**1.52**	**1.47**	**1.44**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.29**	**1.28**
100	10.79	4.33	3.02	*2.48*	**2.19**	**1.90**	**1.74**	**1.63**	**1.56**	**1.51**	**1.48**	**1.45**	**1.43**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.32**	**1.32**
125	11.62	4.54	3.12	*2.56*	**2.26**	**1.95**	**1.79**	**1.67**	**1.60**	**1.55**	**1.51**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
150	12.40	4.71	3.22	*2.62*	**2.31**	**1.98**	**1.82**	**1.70**	**1.62**	**1.57**	**1.54**	**1.51**	**1.49**	**1.45**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**
175	13.09	4.88	3.30	*2.69*	**2.36**	**2.02**	**1.85**	**1.73**	**1.65**	**1.60**	**1.56**	**1.53**	**1.51**	**1.47**	**1.45**	**1.43**	**1.42**	**1.41**	**1.39**	**1.38**
200	13.67	5.03	3.37	2.73	**2.39**	**2.05**	**1.87**	**1.75**	**1.67**	**1.62**	**1.57**	**1.55**	**1.53**	**1.49**	**1.46**	**1.45**	**1.43**	**1.42**	**1.41**	**1.39**

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.69	**1.72**	**1.40**	**1.25**	**1.16**	**1.05**	**0.99**	**0.94**	**0.91**	**0.89**	**0.88**	**0.86**	**0.85**	**0.84**	**0.83**	**0.82**	**0.82**	**0.81**	**0.80**	**0.80**
2	3.46	**2.06**	**1.65**	**1.45**	**1.33**	**1.20**	**1.13**	**1.07**	**1.04**	**1.01**	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.90**
3	3.99	**2.28**	**1.79**	**1.57**	**1.43**	**1.29**	**1.21**	**1.14**	**1.11**	**1.08**	**1.06**	**1.04**	**1.03**	**1.01**	**1.00**	**0.99**	**0.98**	**0.98**	**0.97**	**0.96**
4	4.41	**2.44**	**1.90**	**1.65**	**1.51**	**1.35**	**1.26**	**1.20**	**1.15**	**1.12**	**1.10**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.02**	**1.01**	**1.00**
5	4.77	**2.57**	**1.99**	**1.72**	**1.57**	**1.40**	**1.30**	**1.24**	**1.19**	**1.16**	**1.14**	**1.12**	**1.11**	**1.09**	**1.07**	**1.06**	**1.05**	**1.05**	**1.04**	**1.03**
8	5.60	2.87	**2.17**	**1.86**	**1.69**	**1.50**	**1.39**	**1.32**	**1.27**	**1.23**	**1.21**	**1.19**	**1.18**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**
12	6.43	3.14	**2.34**	**1.99**	**1.80**	**1.58**	**1.47**	**1.39**	**1.33**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**
16	7.09	3.35	**2.47**	**2.08**	**1.87**	**1.65**	**1.53**	**1.44**	**1.38**	**1.34**	**1.31**	**1.29**	**1.28**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**
20	7.65	3.52	**2.57**	**2.16**	**1.93**	**1.69**	**1.57**	**1.47**	**1.42**	**1.38**	**1.35**	**1.32**	**1.31**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.21**	**1.21**
30	8.76	3.85	*2.75*	**2.29**	**2.05**	**1.78**	**1.64**	**1.54**	**1.48**	**1.44**	**1.41**	**1.38**	**1.36**	**1.33**	**1.32**	**1.30**	**1.29**	**1.28**	**1.26**	**1.25**
40	9.67	4.09	2.89	**2.39**	**2.13**	**1.84**	**1.70**	**1.59**	**1.53**	**1.48**	**1.45**	**1.42**	**1.40**	**1.37**	**1.35**	**1.34**	**1.32**	**1.32**	**1.30**	**1.29**
50	10.40	4.28	3.00	**2.47**	**2.19**	**1.90**	**1.74**	**1.63**	**1.56**	**1.51**	**1.48**	**1.45**	**1.43**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.31**
60	11.08	4.46	3.09	**2.54**	**2.24**	**1.93**	**1.78**	**1.66**	**1.59**	**1.54**	**1.50**	**1.48**	**1.46**	**1.43**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.34**
75	11.91	4.68	3.21	*2.62*	**2.31**	**1.98**	**1.82**	**1.70**	**1.62**	**1.57**	**1.54**	**1.51**	**1.49**	**1.45**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**
100	13.13	4.96	3.36	*2.73*	**2.39**	**2.05**	**1.87**	**1.75**	**1.67**	**1.61**	**1.58**	**1.55**	**1.52**	**1.49**	**1.47**	**1.45**	**1.43**	**1.43**	**1.41**	**1.39**
125	14.16	5.20	3.49	*2.81*	**2.46**	**2.10**	**1.92**	**1.79**	**1.70**	**1.65**	**1.61**	**1.58**	**1.55**	**1.52**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
150	15.04	5.40	3.59	*2.88*	**2.51**	**2.14**	**1.95**	**1.82**	**1.73**	**1.67**	**1.63**	**1.60**	**1.58**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**
175	15.82	5.57	3.69	2.94	**2.56**	**2.17**	**1.98**	**1.84**	**1.75**	**1.70**	**1.65**	**1.62**	**1.60**	**1.56**	**1.54**	**1.51**	**1.50**	**1.49**	**1.47**	**1.45**
200	16.60	5.74	3.76	3.00	*2.60*	**2.21**	**2.01**	**1.86**	**1.78**	**1.72**	**1.67**	**1.64**	**1.61**	**1.57**	**1.55**	**1.53**	**1.51**	**1.50**	**1.48**	**1.47**

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.26	**2.01**	**1.62**	**1.44**	**1.32**	**1.20**	**1.12**	**1.07**	**1.03**	**1.01**	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.90**
2	4.17	**2.39**	**1.88**	**1.64**	**1.50**	**1.35**	**1.26**	**1.19**	**1.15**	**1.12**	**1.10**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.02**	**1.01**	**1.00**
3	4.80	**2.63**	**2.04**	**1.76**	**1.61**	**1.43**	**1.34**	**1.27**	**1.22**	**1.19**	**1.17**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.06**	**1.05**
4	5.29	**2.81**	**2.15**	**1.85**	**1.68**	**1.49**	**1.39**	**1.32**	**1.27**	**1.23**	**1.21**	**1.19**	**1.18**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**
5	5.72	2.96	**2.24**	**1.92**	**1.74**	**1.54**	**1.44**	**1.36**	**1.30**	**1.27**	**1.24**	**1.22**	**1.21**	**1.19**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**	**1.12**
8	6.71	3.29	**2.45**	**2.08**	**1.87**	**1.64**	**1.52**	**1.44**	**1.38**	**1.34**	**1.31**	**1.29**	**1.28**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**
12	7.70	3.60	**2.63**	**2.21**	**1.98**	**1.73**	**1.60**	**1.51**	**1.45**	**1.40**	**1.37**	**1.35**	**1.33**	**1.30**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**
16	8.48	3.83	**2.76**	**2.31**	**2.06**	**1.79**	**1.66**	**1.55**	**1.49**	**1.45**	**1.41**	**1.39**	**1.37**	**1.34**	**1.32**	**1.31**	**1.30**	**1.29**	**1.27**	**1.26**
20	9.14	4.02	*2.87*	**2.38**	**2.12**	**1.84**	**1.70**	**1.59**	**1.53**	**1.48**	**1.45**	**1.42**	**1.40**	**1.37**	**1.35**	**1.34**	**1.32**	**1.32**	**1.30**	**1.29**
30	10.47	4.38	3.07	**2.53**	**2.24**	**1.93**	**1.78**	**1.66**	**1.59**	**1.54**	**1.50**	**1.48**	**1.46**	**1.43**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.34**
40	11.55	4.66	3.22	**2.63**	**2.32**	**2.00**	**1.83**	**1.71**	**1.63**	**1.58**	**1.54**	**1.52**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
50	12.45	4.88	3.34	**2.72**	**2.39**	**2.05**	**1.87**	**1.75**	**1.67**	**1.61**	**1.58**	**1.55**	**1.52**	**1.49**	**1.47**	**1.45**	**1.44**	**1.43**	**1.41**	**1.39**
60	13.23	5.07	3.44	**2.79**	**2.44**	**2.09**	**1.91**	**1.78**	**1.70**	**1.64**	**1.60**	**1.57**	**1.55**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.41**
75	14.26	5.31	3.56	*2.87*	**2.51**	**2.14**	**1.95**	**1.82**	**1.73**	**1.67**	**1.63**	**1.60**	**1.58**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.43**
100	15.72	5.64	3.74	*2.99*	**2.60**	**2.20**	**2.01**	**1.86**	**1.78**	**1.72**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.50**	**1.48**	**1.46**
125	16.89	5.91	3.87	*3.08*	**2.67**	**2.26**	**2.05**	**1.90**	**1.81**	**1.75**	**1.70**	**1.67**	**1.64**	**1.61**	**1.58**	**1.56**	**1.54**	**1.53**	**1.51**	**1.49**
150	17.97	6.15	3.98	3.15	**2.73**	**2.30**	**2.08**	**1.93**	**1.84**	**1.77**	**1.73**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.56**	**1.55**	**1.53**	**1.51**
175	18.95	6.35	4.08	3.22	**2.78**	**2.33**	**2.11**	**1.96**	**1.86**	**1.79**	**1.75**	**1.72**	**1.68**	**1.64**	**1.62**	**1.60**	**1.58**	**1.57**	**1.54**	**1.53**
200	19.73	6.49	4.15	3.27	**2.82**	**2.37**	**2.14**	**1.98**	**1.88**	**1.81**	**1.77**	**1.73**	**1.70**	**1.66**	**1.63**	**1.61**	**1.60**	**1.58**	**1.56**	**1.54**

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.81	**1.74**	**1.42**	**1.26**	**1.16**	**1.05**	**0.99**	**0.94**	**0.91**	**0.89**	**0.88**	**0.86**	**0.85**	**0.84**	**0.83**	**0.82**	**0.82**	**0.81**	**0.80**	**0.80**
2	3.60	**2.09**	**1.66**	**1.45**	**1.34**	**1.20**	**1.13**	**1.07**	**1.04**	**1.01**	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.90**
3	4.15	**2.31**	**1.80**	**1.57**	**1.44**	**1.29**	**1.21**	**1.15**	**1.11**	**1.08**	**1.06**	**1.04**	**1.03**	**1.01**	**1.00**	**0.99**	**0.98**	**0.98**	**0.97**	**0.96**
4	4.58	*2.47*	**1.91**	**1.66**	**1.51**	**1.35**	**1.26**	**1.20**	**1.15**	**1.12**	**1.10**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.02**	**1.01**	**1.00**
5	4.95	*2.60*	**2.00**	**1.72**	**1.57**	**1.40**	**1.31**	**1.24**	**1.19**	**1.16**	**1.14**	**1.12**	**1.11**	**1.09**	**1.07**	**1.06**	**1.05**	**1.05**	**1.04**	**1.03**
8	5.82	2.90	**2.18**	**1.87**	**1.69**	**1.50**	**1.39**	**1.32**	**1.27**	**1.24**	**1.21**	**1.19**	**1.18**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**
12	6.68	3.18	**2.35**	**2.00**	**1.80**	**1.59**	**1.47**	**1.39**	**1.33**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**
16	7.36	3.39	*2.48*	**2.09**	**1.88**	**1.65**	**1.53**	**1.44**	**1.38**	**1.34**	**1.31**	**1.29**	**1.28**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**
20	7.93	3.56	*2.58*	**2.16**	**1.94**	**1.70**	**1.57**	**1.48**	**1.42**	**1.38**	**1.35**	**1.32**	**1.31**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.21**	**1.21**
30	9.10	3.89	2.76	**2.30**	**2.05**	**1.78**	**1.65**	**1.54**	**1.48**	**1.44**	**1.41**	**1.38**	**1.36**	**1.33**	**1.32**	**1.30**	**1.29**	**1.28**	**1.26**	**1.25**
40	10.02	4.13	2.90	**2.40**	**2.13**	**1.85**	**1.70**	**1.59**	**1.53**	**1.48**	**1.45**	**1.42**	**1.40**	**1.37**	**1.35**	**1.34**	**1.32**	**1.32**	**1.30**	**1.29**
50	10.81	4.34	3.01	*2.48*	**2.19**	**1.90**	**1.74**	**1.63**	**1.56**	**1.51**	**1.48**	**1.45**	**1.43**	**1.40**	**1.38**	**1.36**	**1.35**	**1.34**	**1.33**	**1.31**
60	11.48	4.50	3.11	*2.54*	**2.24**	**1.94**	**1.78**	**1.66**	**1.59**	**1.54**	**1.51**	**1.48**	**1.46**	**1.42**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.33**
75	12.36	4.72	3.22	*2.63*	**2.31**	**1.98**	**1.82**	**1.70**	**1.62**	**1.57**	**1.54**	**1.51**	**1.49**	**1.45**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**
100	13.59	5.01	3.38	2.73	*2.40*	**2.05**	**1.88**	**1.75**	**1.67**	**1.61**	**1.58**	**1.55**	**1.52**	**1.49**	**1.47**	**1.45**	**1.44**	**1.42**	**1.40**	**1.39**
125	14.65	5.24	3.50	2.82	*2.46*	**2.10**	**1.92**	**1.78**	**1.70**	**1.65**	**1.61**	**1.58**	**1.55**	**1.52**	**1.49**	**1.48**	**1.46**	**1.45**	**1.43**	**1.42**
150	15.59	5.45	3.60	2.89	*2.52*	**2.14**	**1.95**	**1.82**	**1.73**	**1.67**	**1.63**	**1.60**	**1.58**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**
175	16.41	5.62	3.69	2.94	*2.56*	**2.18**	**1.98**	**1.84**	**1.75**	**1.70**	**1.66**	**1.62**	**1.60**	**1.56**	**1.53**	**1.52**	**1.50**	**1.49**	**1.47**	**1.45**
200	17.11	5.80	3.76	3.00	*2.61*	**2.20**	**2.01**	**1.86**	**1.78**	**1.71**	**1.67**	**1.64**	**1.61**	**1.57**	**1.55**	**1.53**	**1.52**	**1.51**	**1.48**	**1.47**

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.46	**2.06**	**1.65**	**1.45**	**1.33**	**1.20**	**1.13**	**1.07**	**1.04**	**1.01**	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**	**0.93**	**0.92**	**0.92**	**0.91**	**0.90**
2	4.41	**2.44**	**1.90**	**1.65**	**1.51**	**1.35**	**1.26**	**1.20**	**1.15**	**1.12**	**1.10**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.02**	**1.01**	**1.00**
3	5.08	*2.68*	**2.06**	**1.77**	**1.61**	**1.44**	**1.34**	**1.27**	**1.22**	**1.19**	**1.17**	**1.15**	**1.13**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**	**1.06**	**1.05**
4	5.60	2.87	**2.17**	**1.86**	**1.69**	**1.50**	**1.39**	**1.32**	**1.27**	**1.23**	**1.21**	**1.19**	**1.18**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**
5	6.05	3.02	**2.27**	**1.93**	**1.75**	**1.54**	**1.44**	**1.36**	**1.30**	**1.27**	**1.24**	**1.22**	**1.21**	**1.19**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**	**1.12**
8	7.09	3.35	**2.47**	**2.08**	**1.87**	**1.65**	**1.53**	**1.44**	**1.38**	**1.34**	**1.31**	**1.29**	**1.28**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**
12	8.13	3.66	*2.65*	**2.22**	**1.98**	**1.73**	**1.60**	**1.51**	**1.45**	**1.40**	**1.37**	**1.35**	**1.33**	**1.30**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**
16	8.96	3.90	*2.78*	**2.32**	**2.06**	**1.80**	**1.66**	**1.56**	**1.49**	**1.45**	**1.41**	**1.39**	**1.37**	**1.34**	**1.32**	**1.31**	**1.30**	**1.29**	**1.27**	**1.26**
20	9.66	4.09	2.89	**2.39**	**2.13**	**1.85**	**1.70**	**1.59**	**1.53**	**1.48**	**1.45**	**1.42**	**1.40**	**1.37**	**1.35**	**1.34**	**1.32**	**1.32**	**1.30**	**1.29**
30	11.07	4.46	3.09	**2.54**	**2.24**	**1.93**	**1.78**	**1.66**	**1.59**	**1.54**	**1.50**	**1.48**	**1.46**	**1.43**	**1.40**	**1.39**	**1.37**	**1.36**	**1.35**	**1.34**
40	12.19	4.74	3.24	*2.64*	**2.33**	**2.00**	**1.83**	**1.71**	**1.63**	**1.58**	**1.55**	**1.52**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
50	13.12	4.97	3.36	*2.73*	**2.39**	**2.05**	**1.87**	**1.75**	**1.67**	**1.61**	**1.58**	**1.55**	**1.52**	**1.49**	**1.47**	**1.45**	**1.44**	**1.42**	**1.41**	**1.39**
60	13.95	5.16	3.46	*2.80*	**2.45**	**2.09**	**1.91**	**1.78**	**1.70**	**1.64**	**1.60**	**1.57**	**1.55**	**1.51**	**1.49**	**1.47**	**1.46**	**1.44**	**1.42**	**1.41**
75	15.06	5.41	3.59	*2.88*	**2.52**	**2.14**	**1.95**	**1.81**	**1.73**	**1.67**	**1.63**	**1.60**	**1.58**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.43**
100	16.58	5.74	3.76	3.00	*2.60*	**2.21**	**2.01**	**1.86**	**1.78**	**1.72**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.50**	**1.48**	**1.47**
125	17.81	6.01	3.90	3.09	*2.67*	**2.26**	**2.05**	**1.90**	**1.81**	**1.75**	**1.70**	**1.67**	**1.64**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.51**	**1.49**
150	18.98	6.24	4.00	3.16	*2.73*	**2.30**	**2.08**	**1.93**	**1.84**	**1.77**	**1.73**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.56**	**1.55**	**1.53**	**1.51**
175	19.92	6.45	4.10	3.22	*2.78*	**2.34**	**2.12**	**1.96**	**1.86**	**1.79**	**1.75**	**1.71**	**1.68**	**1.64**	**1.62**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**
200	20.86	6.62	4.19	3.28	*2.83*	**2.37**	**2.14**	**1.98**	**1.88**	**1.82**	**1.77**	**1.73**	**1.70**	**1.66**	**1.63**	**1.61**	**1.60**	**1.58**	**1.56**	**1.55**

Table 19-16. κ-Multipliers for 1-of-3 Intrawell Prediction Limits on Means of Order 2 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.17	**2.39**	**1.88**	**1.64**	**1.50**	**1.35**	**1.26**	**1.19**	**1.15**	**1.12**	**1.10**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.02**	**1.02**	**1.01**	**1.00**
2	5.29	**2.81**	**2.15**	**1.85**	**1.68**	**1.49**	**1.39**	**1.32**	**1.27**	**1.23**	**1.21**	**1.19**	**1.18**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**
3	6.08	3.08	**2.32**	**1.98**	**1.79**	**1.58**	**1.47**	**1.39**	**1.33**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**
4	6.71	3.29	**2.45**	**2.08**	**1.87**	**1.64**	**1.52**	**1.44**	**1.38**	**1.34**	**1.31**	**1.29**	**1.28**	**1.25**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**
5	7.23	3.46	**2.54**	**2.15**	**1.93**	**1.69**	**1.57**	**1.47**	**1.42**	**1.38**	**1.35**	**1.32**	**1.31**	**1.28**	**1.26**	**1.25**	**1.24**	**1.23**	**1.21**	**1.21**
8	8.48	3.83	**2.76**	**2.31**	**2.06**	**1.79**	**1.66**	**1.55**	**1.49**	**1.45**	**1.41**	**1.39**	**1.37**	**1.34**	**1.32**	**1.31**	**1.30**	**1.29**	**1.27**	**1.26**
12	9.72	4.18	*2.96*	**2.45**	**2.17**	**1.88**	**1.73**	**1.62**	**1.55**	**1.51**	**1.47**	**1.45**	**1.43**	**1.40**	**1.37**	**1.36**	**1.35**	**1.34**	**1.32**	**1.31**
16	10.71	4.44	3.10	**2.55**	**2.26**	**1.95**	**1.79**	**1.67**	**1.60**	**1.55**	**1.51**	**1.49**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
20	11.54	4.66	3.22	**2.63**	**2.32**	**2.00**	**1.83**	**1.71**	**1.63**	**1.58**	**1.54**	**1.52**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
30	13.21	5.07	3.44	**2.79**	**2.44**	**2.09**	**1.91**	**1.78**	**1.70**	**1.64**	**1.60**	**1.57**	**1.55**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.41**
40	14.56	5.38	3.60	*2.90*	**2.53**	**2.15**	**1.96**	**1.83**	**1.74**	**1.68**	**1.64**	**1.61**	**1.58**	**1.55**	**1.52**	**1.50**	**1.49**	**1.48**	**1.46**	**1.44**
50	15.70	5.64	3.73	*2.99*	**2.60**	**2.20**	**2.01**	**1.86**	**1.78**	**1.72**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.50**	**1.48**	**1.47**
60	16.67	5.86	3.84	*3.06*	**2.66**	**2.25**	**2.04**	**1.89**	**1.80**	**1.74**	**1.70**	**1.66**	**1.64**	**1.60**	**1.57**	**1.55**	**1.54**	**1.52**	**1.50**	**1.49**
75	17.99	6.14	3.98	3.15	**2.73**	**2.30**	**2.08**	**1.93**	**1.84**	**1.77**	**1.73**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.56**	**1.55**	**1.53**	**1.51**
100	19.80	6.50	4.16	3.27	**2.82**	**2.37**	**2.14**	**1.98**	**1.88**	**1.81**	**1.77**	**1.73**	**1.70**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.54**
125	21.33	6.83	4.31	3.37	*2.89*	**2.42**	**2.18**	**2.02**	**1.92**	**1.85**	**1.80**	**1.76**	**1.73**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.58**	**1.57**
150	22.62	7.09	4.42	3.45	*2.96*	**2.46**	**2.22**	**2.05**	**1.94**	**1.87**	**1.82**	**1.78**	**1.75**	**1.71**	**1.68**	**1.66**	**1.64**	**1.63**	**1.60**	**1.59**
175	23.91	7.29	4.54	3.52	*3.01*	**2.50**	**2.25**	**2.07**	**1.97**	**1.89**	**1.84**	**1.80**	**1.77**	**1.73**	**1.70**	**1.68**	**1.66**	**1.64**	**1.62**	**1.60**
200	24.84	7.50	4.63	3.57	*3.05*	**2.53**	**2.28**	**2.09**	**1.99**	**1.91**	**1.86**	**1.82**	**1.79**	**1.74**	**1.71**	**1.69**	**1.67**	**1.66**	**1.64**	**1.62**

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.25	1.04	0.96	0.91	0.88	0.84	0.82	0.81	0.79	0.79	0.78	0.78	0.77	0.77	0.76	0.76	0.76	0.76	0.75	0.75
2	1.78	1.41	1.27	1.20	1.15	1.09	1.06	1.04	1.02	1.01	1.00	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96
3	2.12	1.63	1.45	1.36	1.30	1.23	1.19	1.16	1.14	1.13	1.12	1.11	1.11	1.10	1.09	1.08	1.08	1.08	1.07	1.07
4	2.39	1.79	1.58	1.47	1.41	1.33	1.28	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.17	1.16	1.16	1.15	1.15	1.14
5	2.62	1.92	1.68	1.56	1.49	1.40	1.35	1.31	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.22	1.21	1.21	1.20	1.20
8	3.14	2.21	1.90	1.75	1.66	1.55	1.49	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
12	3.64	2.47	2.09	1.91	1.80	1.68	1.61	1.56	1.53	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
16	4.05	2.66	2.23	2.03	1.91	1.77	1.69	1.64	1.60	1.57	1.56	1.54	1.53	1.51	1.50	1.49	1.49	1.48	1.47	1.46
20	4.38	2.82	2.34	2.12	1.99	1.84	1.76	1.70	1.66	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
30	5.06	3.11	2.55	2.29	2.13	1.96	1.87	1.80	1.76	1.73	1.70	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60	1.59
40	5.59	3.33	2.70	2.41	2.24	2.05	1.95	1.88	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
50	6.04	3.51	2.82	2.50	2.32	2.12	2.01	1.93	1.88	1.85	1.82	1.80	1.78	1.76	1.75	1.73	1.72	1.72	1.70	1.69
60	6.43	3.67	2.92	2.58	2.39	2.18	2.06	1.98	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73
75	6.95	3.86	*3.04*	2.68	2.47	2.24	2.12	2.03	1.98	1.94	1.91	1.89	1.87	1.85	1.83	1.81	1.80	1.80	1.78	1.77
100	7.66	4.12	3.21	2.81	2.58	2.33	2.20	2.11	2.04	2.00	1.97	1.95	1.93	1.90	1.88	1.87	1.86	1.85	1.84	1.82
125	8.27	4.33	3.34	2.91	2.66	2.40	2.26	2.16	2.10	2.05	2.02	2.00	1.98	1.95	1.93	1.91	1.90	1.89	1.88	1.87
150	8.80	4.51	3.45	2.99	2.73	2.46	2.31	2.21	2.14	2.09	2.06	2.03	2.01	1.98	1.96	1.95	1.94	1.93	1.91	1.90
175	9.27	4.66	3.54	*3.06*	2.79	2.50	2.35	2.24	2.17	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95	1.94	1.93
200	9.70	4.80	3.63	*3.12*	2.84	2.55	2.39	2.28	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.65	1.35	1.24	1.17	1.13	1.08	1.05	1.03	1.01	1.00	1.00	0.99	0.99	0.98	0.97	0.97	0.97	0.96	0.96	0.96
2	2.24	1.74	1.55	1.45	1.39	1.32	1.28	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.15	1.15	1.14
3	2.64	1.98	1.74	1.62	1.54	1.45	1.40	1.36	1.34	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.25	1.24
4	2.96	2.15	1.87	1.73	1.64	1.54	1.49	1.44	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
5	3.22	2.29	1.98	1.82	1.73	1.62	1.55	1.51	1.48	1.45	1.44	1.43	1.42	1.40	1.39	1.38	1.38	1.37	1.37	1.36
8	3.83	2.60	2.21	2.01	1.90	1.76	1.69	1.63	1.60	1.57	1.55	1.54	1.53	1.51	1.50	1.49	1.48	1.48	1.47	1.46
12	4.43	2.89	2.41	2.18	2.04	1.89	1.81	1.74	1.70	1.67	1.65	1.64	1.62	1.60	1.59	1.58	1.57	1.57	1.56	1.55
16	4.91	3.10	2.56	2.30	2.15	1.98	1.89	1.82	1.77	1.74	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.63	1.61	1.61
20	5.31	3.27	2.67	2.39	2.23	2.05	1.95	1.87	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
30	6.11	3.60	2.89	2.57	2.38	2.17	2.06	1.98	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73
40	6.75	3.85	3.05	2.69	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
50	7.28	4.05	3.18	2.79	2.57	2.33	2.20	2.10	2.04	2.00	1.97	1.95	1.93	1.90	1.88	1.87	1.86	1.85	1.84	1.82
60	7.75	4.22	*3.29*	2.87	2.64	2.39	2.25	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
75	8.36	4.43	3.42	2.98	2.73	2.45	2.31	2.20	2.14	2.09	2.06	2.03	2.01	1.98	1.96	1.95	1.94	1.93	1.91	1.90
100	9.22	4.72	3.60	3.11	2.84	2.54	2.39	2.27	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
125	9.95	4.96	3.74	*3.22*	2.92	2.61	2.45	2.33	2.25	2.20	2.17	2.14	2.11	2.08	2.06	2.04	2.03	2.02	2.00	1.99
150	10.58	5.16	3.86	*3.30*	3.00	2.67	2.50	2.37	2.30	2.24	2.20	2.17	2.15	2.12	2.09	2.07	2.06	2.05	2.03	2.02
175	11.14	5.33	3.96	*3.38*	3.06	2.72	2.54	2.41	2.33	2.27	2.23	2.20	2.18	2.14	2.12	2.10	2.09	2.08	2.06	2.04
200	11.65	5.49	4.05	3.44	3.11	2.76	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (1 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.03	1.65	1.50	1.42	1.36	1.30	1.26	1.23	1.22	1.20	1.19	1.18	1.18	1.17	1.16	1.16	1.15	1.15	1.14	1.14
2	2.71	2.06	1.82	1.70	1.62	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
3	3.17	2.32	2.02	1.86	1.77	1.66	1.60	1.55	1.52	1.50	1.48	1.47	1.46	1.44	1.43	1.43	1.42	1.41	1.41	1.40
4	3.53	2.51	2.16	1.98	1.88	1.75	1.68	1.63	1.59	1.57	1.55	1.54	1.53	1.51	1.50	1.49	1.48	1.48	1.47	1.46
5	3.83	2.66	2.27	2.08	1.96	1.82	1.75	1.69	1.65	1.62	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
8	4.54	3.00	2.51	2.27	2.13	1.97	1.88	1.81	1.77	1.74	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.63	1.61	1.61
12	5.24	3.31	2.73	2.45	2.28	2.09	1.99	1.92	1.87	1.83	1.81	1.79	1.77	1.75	1.74	1.72	1.72	1.71	1.70	1.69
16	5.80	3.55	2.88	2.57	2.39	2.18	2.07	1.99	1.94	1.90	1.87	1.85	1.84	1.81	1.79	1.78	1.77	1.76	1.75	1.74
20	6.26	3.74	3.01	2.67	2.47	2.25	2.13	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
30	7.20	4.10	3.24	2.85	2.63	2.38	2.25	2.15	2.08	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
40	7.95	4.38	3.42	2.98	2.74	2.47	2.32	2.22	2.15	2.10	2.07	2.05	2.02	2.00	1.97	1.96	1.95	1.94	1.92	1.91
50	8.58	4.60	3.56	3.09	2.82	2.54	2.39	2.27	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
60	9.12	4.79	3.67	3.17	2.90	2.59	2.43	2.32	2.24	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
75	9.84	5.03	3.81	3.28	2.98	2.66	2.50	2.37	2.29	2.24	2.20	2.17	2.15	2.12	2.09	2.07	2.06	2.05	2.03	2.02
100	10.84	5.35	4.01	3.42	3.10	2.75	2.57	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
125	11.69	5.62	4.16	*3.53*	3.19	2.83	2.63	2.50	2.41	2.35	2.31	2.27	2.25	2.21	2.18	2.17	2.15	2.14	2.12	2.10
150	12.43	5.84	4.29	3.63	3.27	2.88	2.68	2.54	2.45	2.39	2.34	2.31	2.28	2.24	2.22	2.20	2.18	2.17	2.15	2.13
175	13.09	6.03	4.40	3.71	3.33	2.93	2.73	2.58	2.48	2.42	2.37	2.34	2.31	2.27	2.24	2.22	2.21	2.20	2.17	2.16
200	13.68	6.21	4.50	3.78	3.39	2.97	2.76	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.20	2.18

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (2 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.78	1.41	1.27	1.20	1.15	1.09	1.06	1.04	1.02	1.01	1.00	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96
2	2.39	1.79	1.58	1.47	1.41	1.33	1.28	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.17	1.16	1.16	1.15	1.15	1.14
3	2.81	2.03	1.77	1.63	1.55	1.46	1.41	1.37	1.34	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.25	1.24
4	3.14	2.21	1.90	1.75	1.66	1.55	1.49	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
5	3.41	2.35	2.01	1.84	1.74	1.62	1.56	1.51	1.48	1.46	1.44	1.43	1.42	1.40	1.39	1.39	1.38	1.37	1.37	1.36
8	4.05	2.66	2.23	2.03	1.91	1.77	1.69	1.64	1.60	1.57	1.56	1.54	1.53	1.51	1.50	1.49	1.49	1.48	1.47	1.46
12	4.68	2.95	2.44	2.19	2.05	1.89	1.81	1.74	1.70	1.67	1.65	1.64	1.62	1.60	1.59	1.58	1.57	1.57	1.56	1.55
16	5.17	3.16	2.58	2.31	2.16	1.98	1.89	1.82	1.77	1.74	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.63	1.62	1.61
20	5.59	3.33	2.70	2.41	2.24	2.05	1.95	1.88	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
30	6.43	3.67	2.92	2.58	2.39	2.18	2.06	1.98	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73
40	7.10	3.92	*3.08*	2.71	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
50	7.66	4.12	3.21	2.81	2.58	2.33	2.20	2.11	2.04	2.00	1.97	1.95	1.93	1.90	1.88	1.87	1.86	1.85	1.84	1.82
60	8.15	4.29	3.32	2.89	2.65	2.39	2.25	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
75	8.80	4.51	3.45	2.99	2.73	2.46	2.31	2.21	2.14	2.09	2.06	2.03	2.01	1.98	1.96	1.95	1.94	1.93	1.91	1.90
100	9.70	4.80	3.63	*3.12*	2.84	2.55	2.39	2.28	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
125	10.46	5.04	3.77	3.23	2.93	2.62	2.45	2.33	2.25	2.20	2.17	2.14	2.11	2.08	2.06	2.04	2.03	2.02	2.00	1.99
150	11.12	5.24	3.89	3.32	*3.00*	2.67	2.50	2.37	2.30	2.24	2.20	2.17	2.15	2.12	2.09	2.07	2.06	2.05	2.03	2.02
175	11.71	5.42	3.99	3.39	*3.06*	2.72	2.54	2.41	2.33	2.28	2.23	2.20	2.18	2.15	2.12	2.10	2.09	2.08	2.06	2.04
200	12.26	5.58	4.08	3.46	*3.12*	2.76	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (2 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.24	1.74	1.55	1.45	1.39	1.32	1.28	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.16	1.15	1.15	1.15	1.14
2	2.96	2.15	1.87	1.73	1.64	1.54	1.49	1.44	1.42	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
3	3.44	2.41	2.07	1.89	1.79	1.67	1.61	1.56	1.52	1.50	1.48	1.47	1.46	1.45	1.43	1.43	1.42	1.42	1.41	1.40
4	3.83	2.60	2.21	2.01	1.90	1.76	1.69	1.63	1.60	1.57	1.55	1.54	1.53	1.51	1.50	1.49	1.48	1.48	1.47	1.46
5	4.15	2.76	2.32	2.10	1.98	1.83	1.75	1.69	1.65	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
8	4.91	3.10	2.56	2.30	2.15	1.98	1.89	1.82	1.77	1.74	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.63	1.61	1.61
12	5.65	3.42	2.77	2.47	2.30	2.10	2.00	1.92	1.87	1.84	1.81	1.79	1.78	1.75	1.74	1.72	1.72	1.71	1.70	1.69
16	6.25	3.65	2.93	2.59	2.40	2.19	2.08	1.99	1.94	1.90	1.87	1.85	1.84	1.81	1.80	1.78	1.77	1.76	1.75	1.74
20	6.75	3.85	3.05	2.69	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
30	7.75	4.22	*3.29*	2.87	2.64	2.39	2.25	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
40	8.55	4.50	3.46	3.01	2.75	2.47	2.33	2.22	2.15	2.11	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
50	9.22	4.72	3.60	3.11	2.84	2.54	2.39	2.27	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
60	9.81	4.92	3.72	3.20	2.91	2.60	2.44	2.32	2.24	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
75	10.58	5.16	3.86	*3.30*	3.00	2.67	2.50	2.37	2.30	2.24	2.20	2.17	2.15	2.12	2.09	2.07	2.06	2.05	2.03	2.02
100	11.65	5.49	4.05	3.44	3.11	2.76	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
125	12.55	5.76	4.21	3.56	3.20	2.83	2.64	2.50	2.41	2.35	2.31	2.27	2.25	2.21	2.19	2.17	2.15	2.14	2.12	2.10
150	13.33	5.98	4.34	3.65	*3.28*	2.89	2.69	2.54	2.45	2.39	2.34	2.31	2.28	2.24	2.22	2.20	2.18	2.17	2.15	2.13
175	14.04	6.18	4.45	3.73	*3.34*	2.94	2.73	2.58	2.48	2.42	2.38	2.34	2.31	2.27	2.24	2.22	2.21	2.20	2.17	2.16
200	14.69	6.36	4.54	3.80	*3.40*	2.98	2.76	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.20	2.18

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (2 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.71	2.06	1.82	1.70	1.62	1.53	1.48	1.44	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31
2	3.53	2.51	2.16	1.98	1.88	1.75	1.68	1.63	1.59	1.57	1.55	1.54	1.53	1.51	1.50	1.49	1.48	1.48	1.47	1.46
3	4.10	2.79	2.36	2.15	2.02	1.88	1.80	1.74	1.70	1.67	1.65	1.63	1.62	1.60	1.59	1.58	1.57	1.57	1.56	1.55
4	4.54	3.00	2.51	2.27	2.13	1.97	1.88	1.81	1.77	1.74	1.72	1.70	1.69	1.67	1.65	1.64	1.63	1.63	1.61	1.61
5	4.92	3.17	2.63	2.37	2.21	2.04	1.94	1.87	1.82	1.79	1.77	1.75	1.73	1.71	1.70	1.69	1.68	1.67	1.66	1.65
8	5.80	3.55	2.88	2.57	2.39	2.18	2.07	1.99	1.94	1.90	1.87	1.85	1.84	1.81	1.79	1.78	1.77	1.76	1.75	1.74
12	6.67	3.90	3.11	2.75	2.54	2.31	2.18	2.09	2.03	1.99	1.96	1.94	1.92	1.89	1.88	1.86	1.85	1.84	1.83	1.82
16	7.36	4.16	3.28	2.88	2.65	2.40	2.26	2.16	2.10	2.06	2.02	2.00	1.98	1.95	1.93	1.92	1.91	1.90	1.88	1.87
20	7.95	4.38	3.42	2.98	2.74	2.47	2.32	2.22	2.15	2.10	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
30	9.12	4.79	3.67	3.17	2.90	2.59	2.43	2.32	2.24	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
40	10.06	5.10	3.86	3.31	3.01	2.68	2.51	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
50	10.84	5.35	4.01	3.42	3.10	2.75	2.57	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
60	11.53	5.57	4.13	*3.51*	3.17	2.81	2.62	2.49	2.40	2.34	2.30	2.27	2.24	2.20	2.18	2.16	2.14	2.13	2.11	2.10
75	12.43	5.84	4.29	3.63	3.27	2.88	2.68	2.54	2.45	2.39	2.34	2.31	2.28	2.24	2.22	2.20	2.18	2.17	2.15	2.13
100	13.68	6.21	4.50	3.78	3.39	2.97	2.76	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.20	2.18
125	14.75	6.51	4.66	3.89	*3.48*	3.05	2.82	2.66	2.56	2.50	2.45	2.41	2.38	2.34	2.31	2.29	2.27	2.25	2.23	2.22
150	15.66	6.76	4.80	3.99	*3.56*	3.11	2.87	2.71	2.60	2.53	2.48	2.44	2.41	2.37	2.34	2.32	2.30	2.29	2.26	2.24
175	16.49	6.98	4.92	4.08	*3.62*	3.16	2.92	2.74	2.64	2.57	2.51	2.47	2.44	2.40	2.37	2.34	2.32	2.31	2.28	2.27
200	17.24	7.18	5.03	4.15	3.68	3.20	2.95	2.77	2.67	2.59	2.54	2.50	2.47	2.42	2.39	2.36	2.35	2.33	2.31	2.29

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.62**	**1.92**	**1.68**	**1.56**	**1.49**	**1.40**	**1.35**	**1.31**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**	**1.22**	**1.21**	**1.21**	**1.20**	**1.20**
2	3.41	**2.35**	**2.01**	**1.84**	**1.74**	**1.62**	**1.56**	**1.51**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**	**1.39**	**1.38**	**1.37**	**1.37**	**1.36**
3	3.95	**2.62**	**2.20**	**2.00**	**1.88**	**1.75**	**1.68**	**1.62**	**1.58**	**1.56**	**1.54**	**1.53**	**1.51**	**1.50**	**1.49**	**1.48**	**1.47**	**1.47**	**1.46**	**1.45**
4	4.38	**2.82**	**2.34**	**2.12**	**1.99**	**1.84**	**1.76**	**1.70**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
5	4.74	**2.98**	**2.46**	**2.21**	**2.07**	**1.91**	**1.82**	**1.75**	**1.71**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**	**1.60**	**1.59**	**1.58**	**1.58**	**1.56**	**1.56**
8	5.59	3.33	**2.70**	**2.41**	**2.24**	**2.05**	**1.95**	**1.88**	**1.83**	**1.79**	**1.77**	**1.75**	**1.74**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
12	6.43	3.67	**2.92**	**2.58**	**2.39**	**2.18**	**2.06**	**1.98**	**1.92**	**1.89**	**1.86**	**1.84**	**1.82**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.74**	**1.73**
16	7.10	3.92	*3.08*	**2.71**	**2.49**	**2.26**	**2.14**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
20	7.66	4.12	3.21	**2.81**	**2.58**	**2.33**	**2.20**	**2.11**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.85**	**1.84**	**1.82**
30	8.80	4.51	3.45	**2.99**	**2.73**	**2.46**	**2.31**	**2.21**	**2.14**	**2.09**	**2.06**	**2.03**	**2.01**	**1.98**	**1.96**	**1.95**	**1.94**	**1.93**	**1.91**	**1.90**
40	9.70	4.80	3.63	*3.12*	**2.84**	**2.55**	**2.39**	**2.28**	**2.20**	**2.15**	**2.12**	**2.09**	**2.07**	**2.04**	**2.02**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
50	10.46	5.04	3.77	3.23	**2.93**	**2.62**	**2.45**	**2.33**	**2.25**	**2.20**	**2.17**	**2.14**	**2.11**	**2.08**	**2.06**	**2.04**	**2.03**	**2.02**	**2.00**	**1.99**
60	11.12	5.24	3.89	3.32	*3.00*	**2.67**	**2.50**	**2.37**	**2.30**	**2.24**	**2.20**	**2.17**	**2.15**	**2.12**	**2.09**	**2.07**	**2.06**	**2.05**	**2.03**	**2.02**
75	11.99	5.50	4.04	3.43	*3.09*	**2.74**	**2.56**	**2.43**	**2.35**	**2.29**	**2.25**	**2.22**	**2.19**	**2.16**	**2.13**	**2.11**	**2.10**	**2.09**	**2.07**	**2.06**
100	13.21	5.85	4.23	3.57	*3.21*	**2.83**	**2.64**	**2.50**	**2.41**	**2.35**	**2.31**	**2.27**	**2.25**	**2.21**	**2.19**	**2.17**	**2.15**	**2.14**	**2.12**	**2.10**
125	14.23	6.13	4.39	3.68	3.30	**2.90**	**2.70**	**2.55**	**2.46**	**2.40**	**2.35**	**2.32**	**2.29**	**2.25**	**2.24**	**2.19**	**2.19**	**2.18**	**2.15**	**2.14**
150	15.14	6.36	4.52	3.77	3.37	**2.96**	**2.75**	**2.60**	**2.50**	**2.44**	**2.39**	**2.35**	**2.33**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.18**	**2.17**
175	15.92	6.57	4.63	3.85	3.44	*3.01*	**2.79**	**2.63**	**2.53**	**2.47**	**2.42**	**2.38**	**2.36**	**2.31**	**2.28**	**2.26**	**2.25**	**2.23**	**2.21**	**2.19**
200	16.65	6.75	4.73	3.92	3.49	*3.05*	**2.83**	**2.66**	**2.56**	**2.49**	**2.45**	**2.41**	**2.38**	**2.34**	**2.31**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.22	**2.29**	**1.98**	**1.82**	**1.73**	**1.62**	**1.55**	**1.51**	**1.48**	**1.45**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**	**1.38**	**1.38**	**1.37**	**1.37**	**1.36**
2	4.15	**2.76**	**2.32**	**2.10**	**1.98**	**1.83**	**1.75**	**1.69**	**1.65**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
3	4.80	**3.05**	**2.52**	**2.27**	**2.12**	**1.96**	**1.87**	**1.80**	**1.76**	**1.73**	**1.70**	**1.69**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
4	5.31	3.27	**2.67**	**2.39**	**2.23**	**2.05**	**1.95**	**1.87**	**1.83**	**1.79**	**1.77**	**1.75**	**1.74**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
5	5.74	3.45	**2.79**	**2.49**	**2.31**	**2.12**	**2.01**	**1.93**	**1.88**	**1.84**	**1.82**	**1.80**	**1.78**	**1.76**	**1.75**	**1.73**	**1.72**	**1.72**	**1.70**	**1.69**
8	6.75	3.85	**3.05**	**2.69**	**2.49**	**2.26**	**2.14**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
12	7.75	4.22	*3.29*	**2.87**	**2.64**	**2.39**	**2.25**	**2.15**	**2.09**	**2.04**	**2.01**	**1.99**	**1.97**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**
16	8.55	4.50	3.46	**3.01**	**2.75**	**2.47**	**2.33**	**2.22**	**2.15**	**2.11**	**2.07**	**2.05**	**2.03**	**2.00**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
20	9.22	4.72	3.60	**3.11**	**2.84**	**2.54**	**2.39**	**2.27**	**2.20**	**2.15**	**2.12**	**2.09**	**2.07**	**2.04**	**2.02**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
30	10.58	5.16	3.86	*3.30*	**3.00**	**2.67**	**2.50**	**2.37**	**2.30**	**2.24**	**2.20**	**2.17**	**2.15**	**2.12**	**2.09**	**2.07**	**2.06**	**2.05**	**2.03**	**2.02**
40	11.65	5.49	4.05	3.44	**3.11**	**2.76**	**2.58**	**2.44**	**2.36**	**2.30**	**2.26**	**2.23**	**2.21**	**2.17**	**2.14**	**2.13**	**2.11**	**2.10**	**2.08**	**2.07**
50	12.55	5.76	4.21	3.56	**3.20**	**2.83**	**2.64**	**2.50**	**2.41**	**2.35**	**2.31**	**2.27**	**2.25**	**2.21**	**2.19**	**2.17**	**2.15**	**2.14**	**2.12**	**2.10**
60	13.33	5.98	4.34	3.65	*3.28*	**2.89**	**2.69**	**2.54**	**2.45**	**2.39**	**2.34**	**2.31**	**2.28**	**2.24**	**2.22**	**2.20**	**2.18**	**2.17**	**2.15**	**2.13**
75	14.38	6.27	4.50	3.76	*3.37*	**2.96**	**2.75**	**2.59**	**2.50**	**2.44**	**2.39**	**2.35**	**2.33**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.18**	**2.17**
100	15.82	6.67	4.71	3.92	3.49	**3.05**	**2.83**	**2.66**	**2.56**	**2.50**	**2.45**	**2.41**	**2.38**	**2.34**	**2.31**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**
125	17.04	6.98	4.88	4.04	3.59	**3.12**	**2.89**	**2.72**	**2.61**	**2.54**	**2.49**	**2.45**	**2.42**	**2.38**	**2.35**	**2.31**	**2.31**	**2.29**	**2.27**	**2.25**
150	18.12	7.25	5.03	4.14	3.67	**3.18**	**2.94**	**2.76**	**2.65**	**2.58**	**2.53**	**2.49**	**2.46**	**2.41**	**2.37**	**2.35**	**2.34**	**2.32**	**2.30**	**2.28**
175	19.09	7.47	5.15	4.22	3.74	*3.23*	**2.98**	**2.80**	**2.69**	**2.61**	**2.56**	**2.51**	**2.48**	**2.44**	**2.40**	**2.38**	**2.36**	**2.35**	**2.32**	**2.30**
200	19.97	7.67	5.26	4.30	3.79	*3.28*	**3.02**	**2.83**	**2.72**	**2.64**	**2.59**	**2.54**	**2.51**	**2.46**	**2.43**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (5 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.83	**2.66**	**2.27**	**2.08**	**1.96**	**1.82**	**1.75**	**1.69**	**1.65**	**1.62**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
2	4.92	**3.17**	**2.63**	**2.37**	**2.21**	**2.04**	**1.94**	**1.87**	**1.82**	**1.79**	**1.77**	**1.75**	**1.73**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
3	5.67	3.49	**2.85**	**2.54**	**2.36**	**2.16**	**2.06**	**1.97**	**1.92**	**1.89**	**1.86**	**1.84**	**1.82**	**1.80**	**1.78**	**1.77**	**1.76**	**1.75**	**1.74**	**1.73**
4	6.26	3.74	**3.01**	**2.67**	**2.47**	**2.25**	**2.13**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
5	6.76	3.94	**3.14**	**2.77**	**2.56**	**2.32**	**2.20**	**2.10**	**2.04**	**2.00**	**1.97**	**1.95**	**1.93**	**1.90**	**1.88**	**1.87**	**1.86**	**1.85**	**1.83**	**1.82**
8	7.95	4.38	**3.42**	**2.98**	**2.74**	**2.47**	**2.32**	**2.22**	**2.15**	**2.10**	**2.07**	**2.05**	**2.03**	**2.00**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
12	9.12	4.79	3.67	**3.17**	**2.90**	**2.59**	**2.43**	**2.32**	**2.24**	**2.19**	**2.16**	**2.13**	**2.11**	**2.07**	**2.05**	**2.03**	**2.02**	**2.01**	**1.99**	**1.98**
16	10.06	5.10	3.86	**3.31**	**3.01**	**2.68**	**2.51**	**2.39**	**2.31**	**2.26**	**2.22**	**2.19**	**2.16**	**2.13**	**2.10**	**2.09**	**2.07**	**2.06**	**2.04**	**2.03**
20	10.84	5.35	4.01	**3.42**	**3.10**	**2.75**	**2.57**	**2.44**	**2.36**	**2.30**	**2.26**	**2.23**	**2.21**	**2.17**	**2.14**	**2.13**	**2.11**	**2.10**	**2.08**	**2.07**
30	12.43	5.84	4.29	3.63	**3.27**	**2.88**	**2.68**	**2.54**	**2.45**	**2.39**	**2.34**	**2.31**	**2.28**	**2.24**	**2.22**	**2.20**	**2.18**	**2.17**	**2.15**	**2.13**
40	13.68	6.21	4.50	3.78	**3.39**	**2.97**	**2.76**	**2.61**	**2.51**	**2.45**	**2.40**	**2.37**	**2.34**	**2.30**	**2.27**	**2.25**	**2.23**	**2.22**	**2.20**	**2.18**
50	14.75	6.51	4.66	3.89	*3.48*	**3.05**	**2.82**	**2.66**	**2.56**	**2.50**	**2.45**	**2.41**	**2.38**	**2.34**	**2.31**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**
60	15.66	6.76	4.80	3.99	*3.56*	**3.11**	**2.87**	**2.71**	**2.60**	**2.53**	**2.48**	**2.44**	**2.41**	**2.37**	**2.34**	**2.32**	**2.30**	**2.29**	**2.26**	**2.24**
75	16.87	7.08	4.98	4.11	*3.65*	**3.18**	**2.94**	**2.76**	**2.65**	**2.58**	**2.53**	**2.49**	**2.45**	**2.41**	**2.38**	**2.35**	**2.34**	**2.32**	**2.30**	**2.28**
100	18.58	7.52	5.21	4.28	3.78	**3.27**	**3.01**	**2.83**	**2.72**	**2.64**	**2.58**	**2.54**	**2.51**	**2.46**	**2.43**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**
125	20.02	7.86	5.40	4.40	3.88	**3.35**	**3.08**	**2.88**	**2.77**	**2.69**	**2.63**	**2.58**	**2.55**	**2.49**	**2.46**	**2.44**	**2.42**	**2.40**	**2.38**	**2.36**
150	21.29	8.15	5.55	4.51	3.96	**3.41**	**3.13**	**2.93**	**2.81**	**2.73**	**2.66**	**2.62**	**2.58**	**2.53**	**2.49**	**2.47**	**2.45**	**2.43**	**2.40**	**2.39**
175	22.41	8.40	5.69	4.60	4.03	**3.46**	**3.17**	**2.96**	**2.84**	**2.75**	**2.69**	**2.64**	**2.60**	**2.56**	**2.52**	**2.49**	**2.47**	**2.46**	**2.43**	**2.41**
200	23.44	8.64	5.80	4.68	4.10	*3.50*	**3.21**	**2.99**	**2.87**	**2.78**	**2.72**	**2.67**	**2.63**	**2.58**	**2.54**	**2.52**	**2.49**	**2.48**	**2.45**	**2.43**

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (10 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.41	**2.35**	**2.01**	**1.84**	**1.74**	**1.62**	**1.56**	**1.51**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**	**1.39**	**1.38**	**1.37**	**1.37**	**1.36**
2	4.38	**2.82**	**2.34**	**2.12**	**1.99**	**1.84**	**1.76**	**1.70**	**1.66**	**1.63**	**1.61**	**1.59**	**1.58**	**1.56**	**1.55**	**1.54**	**1.53**	**1.53**	**1.52**	**1.51**
3	5.06	3.11	**2.55**	**2.29**	**2.13**	**1.96**	**1.87**	**1.80**	**1.76**	**1.73**	**1.70**	**1.69**	**1.67**	**1.65**	**1.64**	**1.63**	**1.62**	**1.61**	**1.60**	**1.59**
4	5.59	3.33	**2.70**	**2.41**	**2.24**	**2.05**	**1.95**	**1.88**	**1.83**	**1.79**	**1.77**	**1.75**	**1.74**	**1.71**	**1.70**	**1.69**	**1.68**	**1.67**	**1.66**	**1.65**
5	6.04	3.51	**2.82**	**2.50**	**2.32**	**2.12**	**2.01**	**1.93**	**1.88**	**1.85**	**1.82**	**1.80**	**1.78**	**1.76**	**1.75**	**1.73**	**1.72**	**1.72**	**1.70**	**1.69**
8	7.10	3.92	*3.08*	**2.71**	**2.49**	**2.26**	**2.14**	**2.05**	**1.99**	**1.95**	**1.92**	**1.90**	**1.88**	**1.86**	**1.84**	**1.83**	**1.82**	**1.81**	**1.79**	**1.78**
12	8.16	4.29	3.32	**2.89**	**2.65**	**2.39**	**2.25**	**2.15**	**2.09**	**2.04**	**2.01**	**1.99**	**1.97**	**1.94**	**1.92**	**1.90**	**1.89**	**1.88**	**1.87**	**1.86**
16	8.99	4.57	3.49	*3.02*	**2.76**	**2.48**	**2.33**	**2.22**	**2.15**	**2.11**	**2.07**	**2.05**	**2.03**	**2.00**	**1.97**	**1.96**	**1.95**	**1.94**	**1.92**	**1.91**
20	9.70	4.80	3.63	*3.12*	**2.84**	**2.55**	**2.39**	**2.28**	**2.20**	**2.15**	**2.12**	**2.09**	**2.07**	**2.04**	**2.02**	**2.00**	**1.99**	**1.98**	**1.96**	**1.95**
30	11.12	5.24	3.89	3.32	*3.00*	**2.67**	**2.50**	**2.37**	**2.30**	**2.24**	**2.20**	**2.17**	**2.15**	**2.12**	**2.09**	**2.07**	**2.06**	**2.05**	**2.03**	**2.02**
40	12.25	5.57	4.08	3.46	*3.12*	**2.76**	**2.58**	**2.44**	**2.36**	**2.30**	**2.26**	**2.23**	**2.21**	**2.17**	**2.15**	**2.13**	**2.11**	**2.10**	**2.08**	**2.07**
50	13.20	5.84	4.23	3.57	*3.21*	**2.83**	**2.64**	**2.50**	**2.41**	**2.35**	**2.31**	**2.27**	**2.25**	**2.21**	**2.19**	**2.17**	**2.15**	**2.14**	**2.12**	**2.10**
60	14.03	6.07	4.36	3.66	3.28	**2.89**	**2.69**	**2.54**	**2.45**	**2.39**	**2.34**	**2.31**	**2.28**	**2.24**	**2.22**	**2.19**	**2.18**	**2.17**	**2.15**	**2.13**
75	15.12	6.36	4.53	3.78	3.37	**2.96**	**2.75**	**2.59**	**2.50**	**2.44**	**2.39**	**2.35**	**2.33**	**2.29**	**2.26**	**2.24**	**2.22**	**2.21**	**2.18**	**2.17**
100	16.66	6.77	4.74	3.93	3.50	*3.05*	**2.83**	**2.66**	**2.56**	**2.49**	**2.45**	**2.41**	**2.38**	**2.34**	**2.31**	**2.29**	**2.27**	**2.25**	**2.23**	**2.22**
125	17.96	7.09	4.91	4.05	3.59	*3.13*	**2.89**	**2.70**	**2.61**	**2.54**	**2.49**	**2.45**	**2.42**	**2.38**	**2.35**	**2.32**	**2.31**	**2.29**	**2.27**	**2.25**
150	19.09	7.37	5.06	4.15	3.67	*3.18*	**2.94**	**2.76**	**2.65**	**2.58**	**2.53**	**2.49**	**2.46**	**2.41**	**2.38**	**2.35**	**2.34**	**2.32**	**2.30**	**2.28**
175	20.10	7.61	5.18	4.24	3.74	*3.24*	*2.98*	**2.80**	**2.69**	**2.61**	**2.56**	**2.52**	**2.48**	**2.44**	**2.40**	**2.38**	**2.36**	**2.35**	**2.32**	**2.30**
200	21.01	7.82	5.29	4.31	3.80	*3.28*	*3.02*	**2.83**	**2.72**	**2.64**	**2.58**	**2.54**	**2.51**	**2.46**	**2.43**	**2.40**	**2.38**	**2.37**	**2.34**	**2.32**

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.15	2.76	2.32	2.10	1.98	1.83	1.75	1.69	1.65	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
2	5.31	3.27	2.67	2.39	2.23	2.05	1.95	1.87	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
3	6.11	3.60	2.89	2.57	2.38	2.17	2.06	1.98	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73
4	6.75	3.85	3.05	2.69	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
5	7.28	4.05	3.18	2.79	2.57	2.33	2.20	2.10	2.04	2.00	1.97	1.95	1.93	1.90	1.88	1.87	1.86	1.85	1.83	1.82
8	8.55	4.50	3.46	3.01	2.75	2.47	2.33	2.22	2.15	2.11	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
12	9.81	4.92	3.72	3.20	2.91	2.60	2.44	2.32	2.24	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
16	10.81	5.23	3.90	*3.33*	3.02	2.69	2.52	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
20	11.65	5.49	4.05	3.44	3.11	2.76	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
30	13.33	5.98	4.34	3.65	*3.28*	2.89	2.69	2.54	2.45	2.39	2.34	2.31	2.28	2.24	2.22	2.20	2.18	2.17	2.15	2.13
40	14.68	6.36	4.54	3.80	*3.40*	2.98	2.76	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.20	2.18
50	15.82	6.67	4.71	3.91	3.49	3.05	2.83	2.66	2.56	2.50	2.45	2.41	2.38	2.34	2.31	2.29	2.27	2.25	2.23	2.22
60	16.82	6.92	4.85	4.01	3.57	3.11	2.88	2.71	2.60	2.53	2.48	2.44	2.41	2.37	2.34	2.32	2.30	2.29	2.26	2.24
75	18.13	7.25	5.03	4.14	3.67	3.18	2.94	2.76	2.65	2.58	2.53	2.49	2.45	2.41	2.38	2.35	2.34	2.32	2.30	2.28
100	19.98	7.66	5.27	4.30	3.79	*3.28*	3.02	2.83	2.72	2.64	2.59	2.54	2.51	2.46	2.43	2.40	2.38	2.37	2.34	2.32
125	21.50	8.03	5.45	4.42	3.89	*3.35*	3.08	2.88	2.77	2.69	2.63	2.58	2.55	2.50	2.46	2.44	2.42	2.40	2.38	2.36
150	22.85	8.33	5.61	4.53	3.98	*3.41*	3.13	2.93	2.81	2.72	2.66	2.62	2.58	2.53	2.49	2.47	2.45	2.43	2.40	2.39
175	24.02	8.61	5.74	4.63	4.04	*3.46*	3.17	2.96	2.84	2.76	2.69	2.65	2.61	2.56	2.52	2.49	2.47	2.46	2.43	2.41
200	25.20	8.85	5.80	4.70	4.11	*3.51*	*3.21*	3.00	2.87	2.79	2.72	2.67	2.63	2.58	2.54	2.52	2.49	2.48	2.45	2.43

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.92	3.17	2.63	2.37	2.21	2.04	1.94	1.87	1.82	1.79	1.77	1.75	1.73	1.71	1.70	1.69	1.68	1.67	1.66	1.65
2	6.26	3.74	3.01	2.67	2.47	2.25	2.13	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
3	7.20	4.10	3.24	2.85	2.63	2.38	2.25	2.15	2.08	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
4	7.95	4.38	3.42	2.98	2.74	2.47	2.32	2.22	2.15	2.10	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
5	8.57	4.60	3.56	3.09	2.82	2.54	2.39	2.27	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
8	10.06	5.10	3.86	3.31	3.01	2.68	2.51	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
12	11.53	5.57	4.13	*3.51*	3.17	2.81	2.62	2.49	2.40	2.34	2.30	2.27	2.24	2.20	2.18	2.16	2.14	2.13	2.11	2.10
16	12.70	5.92	4.33	3.66	3.29	2.90	2.70	2.56	2.46	2.40	2.36	2.32	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
20	13.68	6.21	4.50	3.78	3.39	2.98	2.76	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.20	2.18
30	15.66	6.76	4.80	3.99	*3.56*	3.11	2.87	2.71	2.60	2.53	2.48	2.44	2.41	2.37	2.34	2.32	2.30	2.29	2.26	2.24
40	17.24	7.18	5.03	4.15	3.68	3.20	2.95	2.77	2.67	2.59	2.54	2.50	2.47	2.42	2.39	2.36	2.35	2.33	2.31	2.29
50	18.57	7.51	5.21	4.28	3.78	3.27	3.01	2.83	2.72	2.64	2.58	2.54	2.51	2.46	2.43	2.40	2.38	2.37	2.34	2.32
60	19.75	7.80	5.36	4.38	3.86	3.33	3.07	2.87	2.76	2.68	2.62	2.57	2.54	2.49	2.46	2.43	2.41	2.40	2.37	2.35
75	21.27	8.16	5.55	4.51	3.97	3.41	3.13	2.92	2.81	2.72	2.66	2.62	2.58	2.53	2.49	2.47	2.45	2.43	2.40	2.39
100	23.44	8.64	5.80	4.68	4.10	*3.50*	3.21	2.99	2.87	2.78	2.72	2.67	2.63	2.58	2.54	2.52	2.49	2.48	2.45	2.43
125	25.25	9.04	5.99	4.82	4.20	*3.58*	3.27	3.05	2.92	2.83	2.76	2.71	2.67	2.62	2.58	2.55	2.53	2.51	2.48	2.46
150	26.84	9.38	6.14	4.94	4.29	*3.64*	3.32	3.11	2.96	2.86	2.80	2.75	2.71	2.65	2.61	2.58	2.56	2.54	2.51	2.49
175	28.24	9.70	6.27	5.03	4.37	*3.70*	3.36	3.14	3.00	2.89	2.83	2.77	2.73	2.67	2.63	2.60	2.58	2.56	2.53	2.51
200	29.53	9.96	6.39	5.11	4.43	*3.74*	3.40	3.18	3.02	2.92	2.85	2.80	2.76	2.70	2.66	2.63	2.60	2.58	2.55	2.53

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (20 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.38	2.82	2.34	2.12	1.99	1.84	1.76	1.70	1.66	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51
2	5.59	3.33	2.70	2.41	2.24	2.05	1.95	1.88	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
3	6.44	3.67	2.92	2.58	2.39	2.18	2.06	1.98	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.77	1.76	1.75	1.74	1.73
4	7.10	3.92	*3.08*	2.71	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
5	7.67	4.12	3.21	2.81	2.58	2.33	2.20	2.11	2.04	2.00	1.97	1.95	1.93	1.90	1.88	1.87	1.86	1.85	1.84	1.82
8	8.99	4.57	3.49	*3.02*	2.76	2.48	2.33	2.22	2.15	2.11	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
12	10.32	5.00	3.74	*3.21*	2.92	2.60	2.44	2.32	2.25	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
16	11.37	5.32	3.93	3.35	*3.03*	2.69	2.52	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
20	12.25	5.58	4.08	3.46	*3.12*	2.76	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
30	14.04	6.08	4.36	3.66	3.28	2.89	2.69	2.54	2.45	2.39	2.34	2.31	2.28	2.24	2.22	2.20	2.18	2.17	2.15	2.13
40	15.47	6.46	4.58	3.81	3.40	*2.98*	2.77	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.19	2.18
50	16.66	6.77	4.74	3.93	3.50	*3.05*	2.83	2.66	2.57	2.50	2.45	2.41	2.38	2.34	2.31	2.29	2.27	2.25	2.23	2.22
60	17.71	7.03	4.88	4.03	3.58	*3.11*	2.88	2.71	2.60	2.54	2.48	2.44	2.41	2.37	2.34	2.32	2.30	2.29	2.26	2.24
75	19.07	7.36	5.05	4.15	3.67	*3.19*	2.94	2.76	2.65	2.58	2.53	2.49	2.46	2.41	2.38	2.35	2.33	2.32	2.30	2.28
100	21.01	7.82	5.30	4.31	3.80	*3.28*	*3.02*	2.83	2.72	2.64	2.58	2.54	2.51	2.46	2.43	2.40	2.38	2.37	2.34	2.32
125	22.68	8.17	5.48	4.44	3.90	3.35	*3.08*	2.88	2.77	2.69	2.63	2.58	2.55	2.50	2.46	2.44	2.42	2.40	2.38	2.36
150	24.08	8.53	5.65	4.55	3.98	3.42	*3.13*	2.93	2.81	2.72	2.66	2.61	2.58	2.53	2.49	2.47	2.45	2.43	2.40	2.38
175	25.31	8.79	5.78	4.64	4.05	3.46	*3.18*	*2.97*	2.84	2.75	2.69	2.65	2.61	2.56	2.52	2.49	2.47	2.46	2.43	2.41
200	26.37	9.05	5.89	4.72	4.11	3.52	*3.21*	*3.00*	2.87	2.78	2.72	2.67	2.64	2.58	2.54	2.52	2.49	2.48	2.45	2.43

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (20 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.31	3.27	2.67	2.39	2.23	2.05	1.95	1.87	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
2	6.75	3.85	3.05	2.69	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
3	7.75	4.22	*3.29*	2.87	2.64	2.39	2.25	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
4	8.55	4.50	3.46	3.01	2.75	2.47	2.33	2.22	2.15	2.11	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
5	9.22	4.72	3.60	3.11	2.84	2.54	2.39	2.27	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
8	10.81	5.23	3.90	*3.34*	3.02	2.69	2.52	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
12	12.39	5.71	4.18	3.54	3.19	2.82	2.63	2.49	2.40	2.34	2.30	2.27	2.24	2.20	2.18	2.16	2.14	2.13	2.11	2.10
16	13.66	6.07	4.38	3.68	*3.30*	2.91	2.70	2.56	2.47	2.40	2.36	2.32	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
20	14.72	6.36	4.55	3.80	*3.40*	2.98	2.77	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.20	2.18
30	16.85	6.93	4.85	4.02	3.57	3.11	2.88	2.71	2.60	2.53	2.48	2.44	2.41	2.37	2.34	2.32	2.30	2.29	2.26	2.24
40	18.54	7.35	5.08	4.17	3.69	3.20	2.96	2.78	2.67	2.59	2.54	2.50	2.47	2.42	2.39	2.36	2.35	2.33	2.31	2.29
50	20.00	7.70	5.26	4.30	3.79	*3.28*	3.02	2.83	2.72	2.64	2.58	2.54	2.51	2.46	2.43	2.40	2.38	2.37	2.34	2.32
60	21.27	8.00	5.42	4.41	3.87	*3.34*	3.07	2.88	2.76	2.68	2.62	2.58	2.54	2.49	2.46	2.43	2.41	2.40	2.37	2.35
75	22.85	8.37	5.60	4.54	3.98	*3.41*	3.13	2.93	2.80	2.72	2.66	2.62	2.58	2.53	2.49	2.47	2.45	2.43	2.40	2.39
100	25.22	8.88	5.87	4.70	4.11	*3.51*	*3.21*	3.00	2.87	2.78	2.72	2.67	2.63	2.58	2.54	2.52	2.49	2.48	2.45	2.43
125	27.07	9.32	6.06	4.84	4.22	3.58	*3.27*	3.05	2.92	2.83	2.76	2.71	2.68	2.62	2.58	2.55	2.53	2.51	2.48	2.46
150	28.83	9.67	6.24	4.95	4.31	3.65	*3.32*	3.10	2.96	2.86	2.80	2.75	2.71	2.65	2.61	2.58	2.56	2.54	2.51	2.49
175	30.23	9.98	6.39	5.05	4.37	3.70	*3.37*	3.13	2.99	2.89	2.82	2.77	2.74	2.68	2.64	2.60	2.58	2.56	2.53	2.51
200	31.64	10.28	6.53	5.14	4.44	3.75	*3.41*	3.16	3.02	2.92	2.85	2.80	2.76	2.70	2.66	2.63	2.60	2.58	2.55	2.53

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (20 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.26	3.74	3.01	2.67	2.47	2.25	2.13	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
2	7.95	4.38	3.42	2.98	2.74	2.47	2.32	2.22	2.15	2.10	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
3	9.12	4.79	3.67	3.17	2.90	2.59	2.44	2.32	2.24	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
4	10.06	5.10	3.86	3.31	3.01	2.68	2.51	2.39	2.31	2.25	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
5	10.85	5.35	4.01	3.42	3.10	2.75	2.57	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
8	12.71	5.92	4.33	3.66	3.29	2.90	2.70	2.56	2.46	2.40	2.36	2.32	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
12	14.57	6.45	4.63	3.87	3.46	3.03	2.81	2.65	2.55	2.49	2.44	2.40	2.37	2.33	2.30	2.28	2.26	2.25	2.22	2.21
16	16.04	6.86	4.85	4.03	*3.59*	3.13	2.89	2.72	2.62	2.55	2.50	2.46	2.43	2.38	2.35	2.33	2.31	2.30	2.27	2.25
20	17.29	7.18	5.03	4.15	3.68	3.20	2.95	2.78	2.67	2.59	2.54	2.50	2.47	2.42	2.39	2.36	2.35	2.33	2.31	2.29
30	19.80	7.81	5.36	4.38	3.86	3.33	3.07	2.87	2.76	2.68	2.62	2.57	2.54	2.49	2.46	2.43	2.41	2.40	2.37	2.35
40	21.80	8.29	5.61	4.55	3.99	3.43	3.14	2.94	2.82	2.74	2.68	2.63	2.59	2.54	2.50	2.48	2.46	2.44	2.41	2.39
50	23.47	8.68	5.81	4.68	4.10	*3.50*	3.21	3.00	2.87	2.78	2.72	2.67	2.63	2.58	2.54	2.51	2.49	2.48	2.45	2.43
60	24.96	9.01	5.98	4.80	4.19	*3.57*	3.26	3.04	2.91	2.82	2.75	2.71	2.67	2.61	2.57	2.54	2.52	2.50	2.48	2.46
75	26.89	9.45	6.19	4.93	4.29	*3.64*	3.32	3.10	2.96	2.86	2.80	2.75	2.71	2.65	2.61	2.58	2.56	2.54	2.51	2.49
100	29.53	10.02	6.46	5.12	4.43	*3.74*	3.41	3.16	3.02	2.92	2.85	2.80	2.76	2.70	2.66	2.63	2.60	2.58	2.55	2.53
125	31.99	10.46	6.68	5.26	4.54	3.82	*3.47*	3.22	3.07	2.97	2.89	2.84	2.80	2.74	2.69	2.66	2.64	2.62	2.58	2.56
150	33.75	10.90	6.88	5.38	4.64	3.89	*3.52*	3.26	3.11	3.00	2.93	2.87	2.83	2.77	2.72	2.69	2.66	2.65	2.61	2.59
175	35.86	11.25	7.03	5.49	4.70	3.94	*3.56*	3.30	3.14	3.03	2.96	2.90	2.86	2.79	2.75	2.71	2.69	2.67	2.63	2.61
200	37.27	11.51	7.16	5.58	4.77	3.99	*3.60*	3.33	3.17	3.06	2.98	2.92	2.88	2.81	2.77	2.73	2.71	2.69	2.65	2.63

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (40 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.59	3.33	2.70	2.41	2.24	2.05	1.95	1.88	1.83	1.79	1.77	1.75	1.74	1.71	1.70	1.69	1.68	1.67	1.66	1.65
2	7.10	3.92	*3.08*	2.71	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
3	8.16	4.29	3.32	2.89	2.65	2.39	2.25	2.15	2.09	2.04	2.01	1.99	1.97	1.94	1.92	1.90	1.89	1.88	1.87	1.86
4	8.99	4.57	3.49	*3.02*	2.76	2.48	2.33	2.22	2.15	2.11	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
5	9.70	4.80	3.63	*3.12*	2.84	2.55	2.39	2.28	2.20	2.15	2.12	2.09	2.07	2.04	2.02	2.00	1.99	1.98	1.96	1.95
8	11.37	5.32	3.93	3.35	*3.03*	2.69	2.52	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
12	13.03	5.80	4.21	3.55	*3.19*	2.82	2.63	2.49	2.40	2.34	2.30	2.27	2.24	2.20	2.18	2.16	2.14	2.13	2.11	2.10
16	14.35	6.16	4.41	3.69	3.31	2.91	2.71	2.56	2.47	2.40	2.36	2.32	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
20	15.46	6.46	4.57	3.81	3.40	*2.98*	2.77	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.19	2.18
30	17.71	7.03	4.88	4.03	3.58	*3.11*	2.88	2.71	2.60	2.53	2.48	2.44	2.41	2.37	2.34	2.32	2.30	2.28	2.26	2.24
40	19.51	7.46	5.11	4.19	3.70	*3.21*	2.96	2.78	2.67	2.59	2.54	2.50	2.47	2.42	2.39	2.36	2.35	2.33	2.31	2.29
50	21.01	7.82	5.30	4.31	3.80	*3.28*	*3.02*	2.83	2.72	2.64	2.58	2.54	2.51	2.46	2.43	2.40	2.38	2.37	2.34	2.32
60	22.32	8.12	5.45	4.42	3.88	3.34	*3.07*	2.87	2.76	2.68	2.62	2.58	2.54	2.49	2.46	2.43	2.41	2.40	2.37	2.35
75	24.08	8.50	5.64	4.54	3.98	3.41	*3.13*	2.93	2.81	2.72	2.66	2.62	2.58	2.53	2.49	2.47	2.45	2.43	2.40	2.39
100	26.48	9.02	5.90	4.72	4.11	3.51	*3.21*	*3.00*	2.87	2.78	2.72	2.67	2.63	2.58	2.54	2.51	2.49	2.48	2.45	2.43
125	28.54	9.43	6.11	4.86	4.22	3.59	*3.27*	*3.05*	2.92	2.83	2.76	2.71	2.67	2.62	2.58	2.55	2.53	2.51	2.48	2.46
150	30.35	9.80	6.28	4.97	4.31	3.65	*3.33*	*3.09*	*2.96*	2.86	2.80	2.75	2.71	2.65	2.61	2.58	2.56	2.54	2.51	2.49
175	31.88	10.11	6.43	5.07	4.38	3.70	3.37	*3.13*	*2.99*	2.89	2.83	2.77	2.73	2.68	2.63	2.60	2.58	2.56	2.53	2.51
200	33.40	10.40	6.56	5.16	4.45	3.75	3.41	*3.16*	*3.02*	2.92	2.85	2.80	2.76	2.70	2.66	2.63	2.60	2.58	2.55	2.53

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	6.75	3.85	3.05	2.69	2.49	2.26	2.14	2.05	1.99	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.82	1.81	1.79	1.78
2	8.55	4.50	3.46	3.01	2.75	2.47	2.33	2.22	2.15	2.11	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
3	9.81	4.92	3.72	3.20	2.91	2.60	2.44	2.32	2.24	2.19	2.16	2.13	2.11	2.07	2.05	2.03	2.02	2.01	1.99	1.98
4	10.81	5.23	3.90	*3.33*	3.02	2.69	2.52	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
5	11.66	5.49	4.05	3.44	3.11	2.76	2.58	2.44	2.36	2.30	2.26	2.23	2.21	2.17	2.14	2.13	2.11	2.10	2.08	2.07
8	13.65	6.07	4.38	3.68	*3.30*	2.91	2.70	2.56	2.47	2.40	2.36	2.32	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
12	15.64	6.61	4.68	3.89	3.47	3.04	2.82	2.65	2.56	2.49	2.44	2.40	2.37	2.33	2.30	2.28	2.26	2.25	2.22	2.21
16	17.23	7.02	4.90	4.05	3.60	3.13	2.89	2.72	2.62	2.55	2.50	2.46	2.43	2.38	2.35	2.33	2.31	2.30	2.27	2.25
20	18.56	7.35	5.08	4.17	3.69	3.20	2.96	2.78	2.67	2.59	2.54	2.50	2.47	2.42	2.39	2.36	2.35	2.33	2.31	2.29
30	21.25	8.00	5.42	4.40	3.87	*3.34*	3.07	2.87	2.76	2.68	2.62	2.58	2.54	2.49	2.46	2.43	2.41	2.40	2.37	2.35
40	23.41	8.49	5.67	4.57	4.00	*3.43*	3.15	2.94	2.82	2.74	2.68	2.63	2.59	2.54	2.51	2.48	2.46	2.44	2.41	2.39
50	25.22	8.89	5.86	4.71	4.11	*3.51*	*3.21*	3.00	2.87	2.78	2.72	2.67	2.63	2.58	2.54	2.51	2.49	2.48	2.45	2.43
60	26.81	9.23	6.03	4.82	4.19	3.57	*3.26*	3.04	2.91	2.82	2.75	2.70	2.67	2.61	2.57	2.54	2.52	2.51	2.48	2.46
75	28.89	9.66	6.24	4.95	4.30	3.65	*3.32*	3.10	2.96	2.86	2.80	2.75	2.71	2.65	2.61	2.58	2.56	2.54	2.51	2.49
100	31.76	10.24	6.53	5.14	4.44	3.75	*3.40*	3.16	3.02	2.92	2.85	2.80	2.76	2.70	2.66	2.63	2.60	2.58	2.55	2.53
125	34.22	10.72	6.75	5.29	4.55	3.82	*3.47*	*3.22*	3.07	2.97	2.89	2.84	2.80	2.74	2.69	2.66	2.64	2.62	2.58	2.56
150	36.33	11.13	6.94	5.41	4.64	3.89	*3.52*	*3.26*	3.11	3.00	2.93	2.87	2.83	2.76	2.72	2.69	2.66	2.65	2.61	2.59
175	38.32	11.48	7.10	5.51	4.72	3.94	*3.56*	*3.30*	3.14	3.03	2.96	2.90	2.86	2.79	2.75	2.71	2.69	2.67	2.63	2.61
200	40.08	11.81	7.25	5.60	4.79	3.99	3.60	*3.33*	3.17	3.06	2.98	2.93	2.88	2.81	2.77	2.73	2.71	2.69	2.65	2.63

Table 19-17. κ-Multipliers for 1-of-1 Intrawell Prediction Limits on Means of Order 3 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	7.95	4.38	3.42	2.98	2.74	2.47	2.32	2.22	2.15	2.10	2.07	2.05	2.03	2.00	1.97	1.96	1.95	1.94	1.92	1.91
2	10.06	5.10	3.86	3.31	3.01	2.68	2.51	2.39	2.31	2.26	2.22	2.19	2.16	2.13	2.10	2.09	2.07	2.06	2.04	2.03
3	11.53	5.57	4.13	*3.51*	3.17	2.81	2.62	2.49	2.40	2.34	2.30	2.27	2.24	2.20	2.18	2.16	2.14	2.13	2.11	2.10
4	12.71	5.92	4.33	3.66	3.29	2.90	2.70	2.56	2.46	2.40	2.36	2.32	2.30	2.26	2.23	2.21	2.19	2.18	2.16	2.14
5	13.70	6.21	4.50	3.78	3.39	2.98	2.76	2.61	2.51	2.45	2.40	2.37	2.34	2.30	2.27	2.25	2.23	2.22	2.19	2.18
8	16.04	6.85	4.85	4.03	*3.59*	3.13	2.89	2.72	2.62	2.55	2.50	2.46	2.43	2.38	2.35	2.33	2.31	2.30	2.27	2.25
12	18.37	7.46	5.18	4.25	3.76	3.26	3.00	2.82	2.71	2.63	2.58	2.53	2.50	2.45	2.42	2.39	2.38	2.36	2.33	2.32
16	20.23	7.92	5.42	4.42	3.89	3.35	3.08	2.89	2.77	2.69	2.63	2.59	2.55	2.50	2.47	2.44	2.42	2.41	2.38	2.36
20	21.80	8.29	5.61	4.55	3.99	3.43	3.15	2.94	2.82	2.74	2.67	2.63	2.59	2.54	2.51	2.48	2.46	2.44	2.41	2.39
30	24.96	9.02	5.97	4.79	4.18	*3.57*	3.26	3.04	2.91	2.82	2.75	2.70	2.67	2.61	2.57	2.54	2.52	2.51	2.48	2.46
40	27.48	9.57	6.24	4.97	4.32	*3.66*	3.34	3.11	2.97	2.88	2.81	2.76	2.72	2.66	2.62	2.59	2.57	2.55	2.52	2.50
50	29.59	10.01	6.46	5.12	4.43	*3.74*	3.40	3.16	3.02	2.92	2.85	2.80	2.76	2.70	2.66	2.63	2.60	2.58	2.55	2.53
60	31.46	10.39	6.64	5.24	4.52	3.81	3.46	3.21	3.06	2.96	2.89	2.83	2.79	2.73	2.69	2.65	2.63	2.61	2.58	2.56
75	33.87	10.88	6.87	5.38	4.63	3.88	*3.52*	3.26	3.11	3.00	2.93	2.87	2.83	2.77	2.72	2.69	2.66	2.65	2.61	2.59
100	37.32	11.53	7.18	5.58	4.78	3.99	*3.60*	3.33	3.17	3.06	2.98	2.93	2.88	2.81	2.77	2.73	2.71	2.69	2.65	2.63
125	40.20	12.07	7.43	5.73	4.89	4.07	*3.67*	3.39	3.22	3.11	3.02	2.96	2.92	2.85	2.80	2.77	2.74	2.72	2.68	2.66
150	42.66	12.51	7.63	5.87	4.99	4.13	*3.72*	3.43	3.26	3.14	3.06	3.00	2.95	2.88	2.83	2.80	2.77	2.75	2.71	2.68
175	45.00	12.92	7.81	5.98	5.07	4.19	*3.76*	*3.47*	3.29	3.18	3.09	3.02	2.98	2.91	2.86	2.82	2.79	2.77	2.73	2.71
200	47.11	13.27	7.97	6.08	5.14	4.24	*3.80*	*3.50*	3.32	3.20	3.11	3.05	3.00	2.93	2.89	2.84	2.81	2.79	2.75	2.73

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (1 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.67	0.53	0.46	0.42	0.40	0.37	0.35	0.34	0.33	0.32	0.31	0.31	0.31	0.30	0.30	0.29	0.29	0.29	0.29	0.29
2	1.04	0.80	0.70	0.64	0.61	0.56	0.54	0.52	0.50	0.49	0.49	0.48	0.47	0.47	0.46	0.46	0.46	0.45	0.45	0.45
3	1.28	0.96	0.84	0.77	0.72	0.67	0.64	0.61	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.55	0.54	0.54	0.54	0.53
4	1.47	1.08	0.93	0.85	0.80	0.74	0.71	0.68	0.66	0.65	0.64	0.63	0.62	0.62	0.61	0.60	0.60	0.60	0.59	0.59
5	1.62	1.17	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.70	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
8	1.97	1.38	1.17	1.06	0.99	0.91	0.87	0.83	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72
12	2.31	1.56	1.31	1.18	1.10	1.01	0.95	0.91	0.89	0.87	0.86	0.84	0.84	0.82	0.81	0.81	0.80	0.80	0.79	0.79
16	2.58	1.69	1.41	1.26	1.17	1.07	1.02	0.97	0.94	0.92	0.91	0.90	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
20	2.80	1.80	1.48	1.33	1.23	1.12	1.06	1.02	0.99	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.89	0.88	0.88	0.87
30	3.24	2.01	1.63	1.45	1.34	1.22	1.15	1.09	1.06	1.04	1.02	1.01	1.00	0.98	0.97	0.96	0.95	0.95	0.94	0.93
40	3.60	2.16	1.74	1.53	1.42	1.28	1.21	1.15	1.11	1.09	1.07	1.05	1.04	1.03	1.01	1.00	1.00	0.99	0.98	0.98
50	3.89	2.28	1.82	1.60	1.47	1.33	1.25	1.19	1.15	1.13	1.11	1.09	1.08	1.06	1.05	1.04	1.03	1.03	1.01	1.01
60	4.15	2.39	1.89	1.66	1.52	1.37	1.29	1.23	1.18	1.16	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03
75	4.48	2.52	1.98	1.73	1.58	1.42	1.33	1.27	1.22	1.19	1.17	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.07	1.07
100	4.95	2.70	2.09	1.82	1.66	1.48	1.39	1.32	1.27	1.24	1.22	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.11
125	5.35	*2.84*	2.19	1.89	1.72	1.53	1.43	1.36	1.31	1.28	1.25	1.24	1.22	1.20	1.18	1.17	1.16	1.16	1.14	1.14
150	5.69	2.96	2.26	1.95	1.77	1.58	1.47	1.39	1.34	1.31	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16
175	6.00	3.07	2.33	2.00	1.81	1.61	1.50	1.42	1.37	1.33	1.31	1.29	1.27	1.25	1.23	1.22	1.21	1.20	1.19	1.18
200	6.28	3.16	2.39	2.04	1.85	1.64	1.53	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (1 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	0.95	0.75	0.67	0.62	0.59	0.55	0.53	0.51	0.50	0.49	0.48	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.45	0.45
2	1.36	1.04	0.91	0.84	0.79	0.73	0.70	0.67	0.66	0.64	0.64	0.63	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
3	1.64	1.21	1.05	0.96	0.90	0.83	0.80	0.76	0.74	0.73	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.66
4	1.85	1.33	1.14	1.04	0.98	0.90	0.86	0.83	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72
5	2.02	1.43	1.22	1.11	1.04	0.96	0.91	0.87	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
8	2.43	1.65	1.38	1.25	1.17	1.07	1.01	0.97	0.94	0.92	0.91	0.90	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
12	2.83	1.85	1.53	1.37	1.27	1.16	1.10	1.05	1.02	1.00	0.98	0.97	0.96	0.94	0.93	0.92	0.92	0.91	0.90	0.90
16	3.14	2.00	1.63	1.46	1.35	1.23	1.16	1.11	1.07	1.05	1.03	1.02	1.01	0.99	0.98	0.97	0.96	0.96	0.95	0.94
20	3.41	2.12	1.72	1.52	1.41	1.28	1.20	1.15	1.11	1.09	1.07	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
30	3.94	2.34	1.87	1.65	1.52	1.37	1.29	1.22	1.18	1.16	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03
40	4.35	2.51	1.98	1.74	1.59	1.43	1.34	1.28	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
50	4.70	2.65	2.07	1.81	1.65	1.48	1.39	1.32	1.27	1.24	1.22	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10
60	5.01	2.76	2.15	1.87	1.70	1.52	1.42	1.35	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13
75	5.41	*2.91*	2.24	1.94	1.76	1.57	1.47	1.39	1.34	1.31	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16
100	5.97	3.11	2.37	2.03	1.84	1.64	1.53	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
125	6.45	3.27	2.46	2.11	1.91	1.69	1.57	1.48	1.43	1.39	1.36	1.34	1.32	1.30	1.28	1.27	1.26	1.25	1.24	1.23
150	6.86	3.40	2.55	2.17	1.96	1.73	1.61	1.52	1.46	1.42	1.39	1.37	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25
175	7.22	3.52	2.62	2.22	2.00	1.76	1.64	1.54	1.48	1.44	1.41	1.39	1.37	1.34	1.33	1.31	1.30	1.29	1.28	1.27
200	7.56	3.63	2.68	2.27	2.04	1.79	1.66	1.57	1.51	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (1 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.22	0.97	0.86	0.80	0.77	0.72	0.69	0.67	0.65	0.64	0.63	0.62	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
2	1.68	1.27	1.11	1.02	0.96	0.89	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
3	1.99	1.45	1.25	1.14	1.07	0.99	0.94	0.91	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.81	0.80	0.80	0.79	0.79
4	2.23	1.58	1.35	1.23	1.15	1.06	1.01	0.96	0.94	0.92	0.90	0.89	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
5	2.43	1.69	1.43	1.29	1.21	1.11	1.05	1.01	0.98	0.96	0.95	0.93	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87
8	2.91	1.93	1.60	1.44	1.34	1.22	1.15	1.10	1.07	1.05	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.94
12	3.37	2.14	1.75	1.56	1.44	1.31	1.24	1.18	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
16	3.73	2.31	1.86	1.65	1.52	1.38	1.30	1.23	1.19	1.17	1.14	1.13	1.12	1.10	1.08	1.07	1.07	1.06	1.05	1.04
20	4.04	2.44	1.95	1.72	1.58	1.43	1.34	1.27	1.23	1.20	1.18	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
30	4.66	2.69	2.12	1.85	1.69	1.52	1.42	1.35	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13
40	5.14	2.87	2.24	1.94	1.77	1.58	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
50	5.55	3.02	2.33	2.02	1.83	1.63	1.52	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
60	5.91	3.15	2.41	2.08	1.89	1.67	1.56	1.47	1.42	1.38	1.35	1.33	1.32	1.29	1.27	1.26	1.25	1.24	1.23	1.22
75	6.38	3.32	2.51	2.15	1.95	1.72	1.60	1.51	1.46	1.42	1.39	1.37	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25
100	7.04	3.53	2.65	2.25	2.03	1.79	1.66	1.56	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28
125	7.59	3.71	2.75	2.33	2.09	1.84	1.70	1.60	1.54	1.50	1.47	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31
150	8.07	3.86	2.84	2.40	2.15	1.88	1.74	1.64	1.57	1.53	1.49	1.47	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.33
175	8.51	4.00	2.92	2.45	2.19	1.92	1.77	1.66	1.60	1.55	1.52	1.49	1.47	1.44	1.42	1.40	1.39	1.38	1.36	1.35
200	8.90	4.11	2.98	2.50	2.23	1.95	1.80	1.69	1.62	1.57	1.53	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (2 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.04	0.80	0.70	0.64	0.61	0.56	0.54	0.52	0.50	0.49	0.49	0.48	0.47	0.47	0.46	0.46	0.46	0.45	0.45	0.45
2	1.47	1.08	0.93	0.85	0.80	0.74	0.71	0.68	0.66	0.65	0.64	0.63	0.62	0.62	0.61	0.60	0.60	0.60	0.59	0.59
3	1.75	1.25	1.07	0.97	0.91	0.84	0.80	0.77	0.75	0.73	0.72	0.71	0.71	0.70	0.69	0.68	0.68	0.68	0.67	0.67
4	1.97	1.38	1.17	1.06	0.99	0.91	0.87	0.83	0.81	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72
5	2.15	1.48	1.24	1.12	1.05	0.96	0.91	0.88	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
8	2.58	1.69	1.41	1.26	1.17	1.07	1.02	0.97	0.94	0.92	0.91	0.90	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
12	2.99	1.89	1.55	1.38	1.28	1.17	1.10	1.05	1.02	1.00	0.98	0.97	0.96	0.94	0.93	0.92	0.92	0.91	0.90	0.90
16	3.32	2.04	1.65	1.47	1.36	1.23	1.16	1.11	1.07	1.05	1.03	1.02	1.01	0.99	0.98	0.97	0.96	0.96	0.95	0.94
20	3.60	2.16	1.74	1.53	1.42	1.28	1.21	1.15	1.11	1.09	1.07	1.05	1.04	1.03	1.01	1.00	1.00	0.99	0.98	0.98
30	4.15	2.39	1.89	1.66	1.52	1.37	1.29	1.23	1.18	1.16	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03
40	4.58	2.56	2.00	1.75	1.60	1.44	1.35	1.28	1.24	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
50	4.95	2.70	2.09	1.82	1.66	1.48	1.39	1.32	1.27	1.24	1.22	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.11
60	5.27	*2.81*	2.17	1.88	1.71	1.53	1.43	1.35	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13
75	5.69	2.96	2.26	1.95	1.77	1.58	1.47	1.39	1.34	1.31	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16
100	6.28	3.16	2.39	2.04	1.85	1.64	1.53	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
125	6.77	3.33	2.48	2.12	1.91	1.69	1.57	1.48	1.43	1.39	1.36	1.34	1.32	1.30	1.28	1.27	1.26	1.25	1.24	1.23
150	7.21	3.46	2.57	2.18	1.96	1.73	1.61	1.52	1.46	1.42	1.39	1.37	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25
175	7.59	3.58	2.64	2.23	2.01	1.77	1.64	1.54	1.48	1.44	1.41	1.39	1.37	1.34	1.33	1.31	1.30	1.29	1.28	1.27
200	7.94	3.69	*2.70*	2.28	2.04	1.79	1.66	1.57	1.51	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (2 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.36	1.04	0.91	0.84	0.79	0.73	0.70	0.67	0.66	0.64	0.64	0.63	0.62	0.61	0.61	0.60	0.60	0.60	0.59	0.59
2	1.85	1.33	1.14	1.04	0.98	0.90	0.86	0.83	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.74	0.73	0.73	0.72	0.72
3	2.18	1.52	1.28	1.16	1.09	1.00	0.95	0.91	0.89	0.87	0.85	0.84	0.84	0.82	0.81	0.81	0.80	0.80	0.79	0.79
4	2.43	1.65	1.38	1.25	1.17	1.07	1.01	0.97	0.94	0.92	0.91	0.90	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
5	2.64	1.76	1.46	1.31	1.22	1.12	1.06	1.01	0.98	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.89	0.88	0.87	0.87
8	3.14	2.00	1.63	1.46	1.35	1.23	1.16	1.11	1.07	1.05	1.03	1.02	1.01	0.99	0.98	0.97	0.96	0.96	0.95	0.94
12	3.64	2.22	1.79	1.58	1.46	1.32	1.24	1.18	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.02	1.01	1.00
16	4.03	2.38	1.90	1.67	1.53	1.38	1.30	1.24	1.20	1.17	1.15	1.13	1.12	1.10	1.08	1.07	1.07	1.06	1.05	1.04
20	4.35	2.51	1.98	1.74	1.59	1.43	1.34	1.28	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
30	5.01	2.76	2.15	1.87	1.70	1.52	1.42	1.35	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13
40	5.53	*2.95*	2.27	1.96	1.78	1.59	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
50	5.97	3.11	2.37	2.03	1.84	1.64	1.53	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
60	6.36	3.24	2.45	2.09	1.89	1.68	1.56	1.48	1.42	1.38	1.35	1.33	1.32	1.29	1.27	1.26	1.25	1.24	1.23	1.22
75	6.86	3.40	2.55	2.17	1.96	1.73	1.61	1.52	1.46	1.42	1.39	1.37	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25
100	7.56	3.63	2.68	2.27	2.04	1.79	1.66	1.57	1.51	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28
125	8.15	3.81	2.79	2.35	2.10	1.84	1.71	1.60	1.54	1.50	1.47	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31
150	8.67	3.96	*2.87*	2.41	2.16	1.88	1.74	1.64	1.57	1.53	1.49	1.47	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.33
175	9.14	4.10	*2.95*	2.47	2.20	1.92	1.77	1.66	1.60	1.55	1.52	1.49	1.47	1.44	1.42	1.40	1.39	1.38	1.36	1.35
200	9.55	4.22	*3.02*	2.52	2.24	1.95	1.80	1.69	1.62	1.57	1.54	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (2 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.68	1.27	1.11	1.02	0.96	0.89	0.85	0.82	0.80	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.73	0.73	0.72	0.72
2	2.23	1.58	1.35	1.23	1.15	1.06	1.01	0.96	0.94	0.92	0.90	0.89	0.89	0.87	0.86	0.86	0.85	0.85	0.84	0.83
3	2.61	1.78	1.49	1.35	1.26	1.15	1.09	1.05	1.02	0.99	0.98	0.97	0.96	0.94	0.93	0.92	0.92	0.91	0.90	0.90
4	2.91	1.93	1.60	1.44	1.34	1.22	1.15	1.10	1.07	1.05	1.03	1.01	1.00	0.99	0.98	0.97	0.96	0.96	0.95	0.94
5	3.15	2.04	1.68	1.50	1.40	1.27	1.20	1.14	1.11	1.08	1.07	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
8	3.73	2.31	1.86	1.65	1.52	1.38	1.30	1.23	1.19	1.17	1.14	1.13	1.12	1.10	1.08	1.07	1.07	1.06	1.05	1.04
12	4.31	2.55	2.03	1.78	1.63	1.47	1.38	1.31	1.26	1.23	1.21	1.19	1.18	1.16	1.14	1.13	1.13	1.12	1.11	1.10
16	4.76	2.73	2.14	1.87	1.71	1.53	1.43	1.36	1.31	1.28	1.26	1.24	1.22	1.20	1.19	1.17	1.17	1.16	1.15	1.14
20	5.14	2.87	2.24	1.94	1.77	1.58	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
30	5.91	3.15	2.41	2.08	1.89	1.67	1.56	1.47	1.42	1.38	1.35	1.33	1.32	1.29	1.27	1.26	1.25	1.24	1.23	1.22
40	6.52	3.36	2.54	2.17	1.97	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.27	1.26
50	7.04	3.53	2.65	2.25	2.03	1.79	1.66	1.56	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28
60	7.49	3.68	2.73	2.32	2.08	1.83	1.70	1.60	1.53	1.49	1.46	1.44	1.42	1.39	1.37	1.35	1.34	1.33	1.32	1.31
75	8.07	3.86	2.84	2.40	2.15	1.88	1.74	1.64	1.57	1.53	1.49	1.47	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.33
100	8.90	4.11	2.98	2.50	2.23	1.95	1.80	1.69	1.62	1.57	1.53	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
125	9.59	4.31	*3.10*	2.58	2.30	2.00	1.84	1.72	1.65	1.60	1.57	1.54	1.52	1.49	1.46	1.45	1.44	1.43	1.41	1.40
150	10.20	4.49	*3.19*	2.65	2.35	2.04	1.88	1.76	1.68	1.63	1.59	1.57	1.54	1.51	1.49	1.47	1.46	1.45	1.43	1.42
175	10.75	4.64	3.28	2.71	2.40	2.08	1.91	1.78	1.71	1.65	1.62	1.59	1.56	1.53	1.51	1.49	1.48	1.47	1.45	1.43
200	11.25	4.77	3.35	2.76	2.44	2.11	1.93	1.81	1.73	1.67	1.64	1.61	1.58	1.55	1.52	1.51	1.49	1.48	1.46	1.45

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (5 COC, Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	1.62	1.17	1.01	0.92	0.86	0.80	0.76	0.73	0.71	0.70	0.68	0.68	0.67	0.66	0.65	0.65	0.64	0.64	0.64	0.63
2	2.15	1.48	1.24	1.12	1.05	0.96	0.91	0.88	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
3	2.51	1.66	1.38	1.24	1.16	1.06	1.00	0.96	0.93	0.91	0.90	0.89	0.88	0.86	0.85	0.85	0.84	0.84	0.83	0.82
4	2.80	1.80	1.48	1.33	1.23	1.12	1.06	1.02	0.99	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.89	0.88	0.88	0.87
5	3.04	1.91	1.56	1.39	1.29	1.17	1.11	1.06	1.03	1.00	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90
8	3.60	2.16	1.74	1.53	1.42	1.28	1.21	1.15	1.11	1.09	1.07	1.05	1.04	1.03	1.01	1.00	1.00	0.99	0.98	0.98
12	4.15	2.39	1.89	1.66	1.52	1.37	1.29	1.23	1.18	1.16	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03
16	4.58	2.56	2.00	1.75	1.60	1.44	1.35	1.28	1.24	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
20	4.95	2.70	2.09	1.82	1.66	1.48	1.39	1.32	1.27	1.24	1.22	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.11
30	5.69	2.96	2.26	1.95	1.77	1.58	1.47	1.39	1.34	1.31	1.28	1.26	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16
40	6.28	3.16	2.39	2.04	1.85	1.64	1.53	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
50	6.78	3.33	2.48	2.12	1.91	1.69	1.57	1.48	1.43	1.39	1.36	1.34	1.32	1.30	1.28	1.27	1.26	1.25	1.24	1.23
60	7.21	3.46	2.57	2.18	1.96	1.73	1.61	1.52	1.46	1.42	1.39	1.37	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25
75	7.77	3.64	2.67	2.26	2.03	1.78	1.65	1.56	1.49	1.45	1.42	1.40	1.38	1.35	1.34	1.32	1.31	1.30	1.29	1.28
100	8.55	3.87	*2.81*	2.35	2.11	1.85	1.71	1.61	1.54	1.50	1.47	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31
125	9.22	4.06	*2.92*	2.44	2.17	1.89	1.75	1.65	1.58	1.53	1.50	1.47	1.45	1.42	1.40	1.39	1.38	1.37	1.35	1.34
150	9.84	4.22	3.01	2.50	2.23	1.94	1.79	1.68	1.61	1.56	1.53	1.50	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
175	10.35	4.37	3.09	2.56	2.27	1.97	1.82	1.70	1.63	1.58	1.55	1.52	1.50	1.47	1.45	1.43	1.42	1.41	1.39	1.38
200	10.82	4.49	3.15	2.61	2.31	2.00	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.49	1.46	1.45	1.44	1.43	1.41	1.40

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (5 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.02	1.43	1.22	1.11	1.04	0.96	0.91	0.87	0.85	0.83	0.82	0.81	0.80	0.79	0.78	0.78	0.77	0.77	0.76	0.76
2	2.64	1.76	1.46	1.31	1.22	1.12	1.06	1.01	0.98	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.89	0.88	0.87	0.87
3	3.07	1.96	1.61	1.44	1.33	1.21	1.14	1.09	1.06	1.04	1.02	1.01	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93
4	3.41	2.12	1.72	1.52	1.41	1.28	1.20	1.15	1.11	1.09	1.07	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
5	3.69	2.24	1.80	1.59	1.47	1.33	1.25	1.19	1.15	1.12	1.11	1.09	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.01
8	4.35	2.51	1.98	1.74	1.59	1.43	1.34	1.28	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
12	5.01	2.76	2.15	1.87	1.70	1.52	1.42	1.35	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13
16	5.53	*2.95*	2.27	1.96	1.78	1.59	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
20	5.97	3.11	2.37	2.03	1.84	1.64	1.53	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
30	6.86	3.40	2.55	2.17	1.96	1.73	1.61	1.51	1.46	1.42	1.39	1.37	1.35	1.32	1.31	1.29	1.28	1.27	1.26	1.25
40	7.56	3.63	2.68	2.27	2.04	1.79	1.66	1.57	1.51	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28
50	8.15	3.81	2.79	2.35	2.10	1.84	1.71	1.61	1.54	1.50	1.47	1.44	1.42	1.39	1.37	1.36	1.35	1.34	1.32	1.31
60	8.67	3.96	*2.88*	2.41	2.16	1.88	1.74	1.64	1.57	1.53	1.49	1.47	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.33
75	9.34	4.16	*2.99*	2.49	2.22	1.94	1.79	1.68	1.61	1.56	1.53	1.50	1.48	1.45	1.43	1.41	1.40	1.39	1.37	1.36
100	10.31	4.42	3.13	2.60	2.31	2.00	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.49	1.46	1.45	1.44	1.42	1.41	1.40
125	11.09	4.64	3.25	2.68	2.37	2.05	1.89	1.77	1.69	1.64	1.60	1.57	1.55	1.52	1.49	1.48	1.46	1.45	1.43	1.42
150	11.80	4.82	3.35	2.75	2.43	2.09	1.92	1.80	1.72	1.67	1.63	1.60	1.57	1.54	1.52	1.50	1.48	1.47	1.46	1.44
175	12.42	4.98	3.44	2.81	2.48	2.13	1.95	1.82	1.74	1.69	1.65	1.62	1.59	1.56	1.54	1.52	1.50	1.49	1.47	1.46
200	12.97	5.12	3.52	*2.86*	2.52	2.16	1.98	1.85	1.76	1.71	1.67	1.64	1.61	1.58	1.55	1.53	1.52	1.51	1.49	1.47

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (5 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.43**	**1.69**	**1.43**	**1.29**	**1.21**	**1.11**	**1.05**	**1.01**	**0.98**	**0.96**	**0.95**	**0.93**	**0.92**	**0.91**	**0.90**	**0.89**	**0.89**	**0.88**	**0.87**	**0.87**
2	3.15	**2.04**	**1.68**	**1.50**	**1.40**	**1.27**	**1.20**	**1.14**	**1.11**	**1.08**	**1.07**	**1.05**	**1.04**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.97**
3	3.65	**2.27**	**1.84**	**1.63**	**1.50**	**1.36**	**1.28**	**1.22**	**1.18**	**1.15**	**1.13**	**1.12**	**1.11**	**1.09**	**1.07**	**1.06**	**1.06**	**1.05**	**1.04**	**1.03**
4	4.04	**2.44**	**1.95**	**1.72**	**1.58**	**1.43**	**1.34**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
5	4.37	**2.57**	**2.04**	**1.79**	**1.64**	**1.48**	**1.39**	**1.32**	**1.27**	**1.24**	**1.22**	**1.20**	**1.19**	**1.17**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.10**
8	5.14	**2.87**	**2.24**	**1.94**	**1.77**	**1.58**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
12	5.91	3.15	**2.41**	**2.08**	**1.89**	**1.67**	**1.56**	**1.47**	**1.42**	**1.38**	**1.35**	**1.33**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
16	6.52	3.36	**2.54**	**2.17**	**1.97**	**1.74**	**1.62**	**1.53**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
20	7.04	3.54	**2.65**	**2.25**	**2.03**	**1.79**	**1.66**	**1.56**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.30**	**1.28**
30	8.08	3.86	**2.84**	**2.40**	**2.15**	**1.88**	**1.74**	**1.64**	**1.57**	**1.53**	**1.49**	**1.47**	**1.45**	**1.42**	**1.40**	**1.38**	**1.37**	**1.36**	**1.35**	**1.33**
40	8.90	4.11	**2.98**	**2.50**	**2.23**	**1.95**	**1.80**	**1.69**	**1.62**	**1.57**	**1.53**	**1.51**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
50	9.59	4.32	*3.10*	**2.58**	**2.30**	**2.00**	**1.84**	**1.72**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.49**	**1.46**	**1.45**	**1.43**	**1.43**	**1.41**	**1.40**
60	10.20	4.49	*3.19*	**2.65**	**2.35**	**2.04**	**1.88**	**1.76**	**1.68**	**1.63**	**1.59**	**1.57**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
75	11.00	4.71	3.32	**2.74**	**2.42**	**2.09**	**1.92**	**1.80**	**1.72**	**1.67**	**1.63**	**1.60**	**1.57**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.46**	**1.44**
100	12.11	5.00	3.48	**2.85**	**2.51**	**2.16**	**1.98**	**1.85**	**1.76**	**1.71**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
125	13.05	5.24	3.60	**2.94**	**2.58**	**2.21**	**2.02**	**1.88**	**1.80**	**1.74**	**1.70**	**1.67**	**1.64**	**1.61**	**1.58**	**1.56**	**1.54**	**1.53**	**1.51**	**1.50**
150	13.91	5.45	3.71	**3.01**	**2.64**	**2.26**	**2.06**	**1.92**	**1.83**	**1.77**	**1.72**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.57**	**1.56**	**1.53**	**1.52**
175	14.61	5.62	3.80	*3.08*	**2.69**	**2.29**	**2.09**	**1.94**	**1.85**	**1.79**	**1.75**	**1.71**	**1.69**	**1.65**	**1.62**	**1.60**	**1.58**	**1.57**	**1.55**	**1.54**
200	15.31	5.78	3.89	*3.13*	**2.73**	**2.32**	**2.12**	**1.97**	**1.88**	**1.81**	**1.77**	**1.73**	**1.71**	**1.67**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (10 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	**2.15**	**1.48**	**1.24**	**1.12**	**1.05**	**0.96**	**0.91**	**0.88**	**0.85**	**0.83**	**0.82**	**0.81**	**0.80**	**0.79**	**0.78**	**0.78**	**0.77**	**0.77**	**0.76**	**0.76**
2	2.80	**1.80**	**1.48**	**1.33**	**1.23**	**1.12**	**1.06**	**1.02**	**0.99**	**0.96**	**0.95**	**0.94**	**0.93**	**0.91**	**0.90**	**0.89**	**0.89**	**0.88**	**0.87**	**0.87**
3	3.24	**2.01**	**1.63**	**1.45**	**1.34**	**1.22**	**1.15**	**1.09**	**1.06**	**1.04**	**1.02**	**1.01**	**0.99**	**0.98**	**0.97**	**0.96**	**0.95**	**0.95**	**0.94**	**0.93**
4	3.60	**2.16**	**1.74**	**1.53**	**1.42**	**1.28**	**1.21**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.98**
5	3.89	**2.28**	**1.82**	**1.60**	**1.47**	**1.33**	**1.25**	**1.19**	**1.15**	**1.13**	**1.11**	**1.09**	**1.08**	**1.06**	**1.05**	**1.04**	**1.03**	**1.03**	**1.01**	**1.01**
8	4.59	**2.56**	**2.00**	**1.75**	**1.60**	**1.44**	**1.35**	**1.28**	**1.23**	**1.20**	**1.18**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
12	5.27	*2.81*	**2.17**	**1.88**	**1.71**	**1.53**	**1.43**	**1.35**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**
16	5.82	3.01	**2.29**	**1.97**	**1.79**	**1.59**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
20	6.28	3.16	**2.39**	**2.04**	**1.85**	**1.64**	**1.53**	**1.44**	**1.39**	**1.35**	**1.33**	**1.31**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
30	7.21	3.46	**2.57**	**2.18**	**1.96**	**1.73**	**1.61**	**1.52**	**1.46**	**1.42**	**1.39**	**1.37**	**1.35**	**1.32**	**1.31**	**1.29**	**1.28**	**1.27**	**1.26**	**1.25**
40	7.95	3.69	*2.70*	**2.28**	**2.04**	**1.79**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.30**	**1.28**
50	8.57	3.87	*2.81*	**2.36**	**2.11**	**1.84**	**1.71**	**1.61**	**1.54**	**1.50**	**1.47**	**1.44**	**1.42**	**1.39**	**1.37**	**1.36**	**1.35**	**1.34**	**1.32**	**1.31**
60	9.11	4.03	*2.90*	**2.42**	**2.16**	**1.89**	**1.74**	**1.64**	**1.57**	**1.53**	**1.49**	**1.47**	**1.45**	**1.42**	**1.40**	**1.38**	**1.37**	**1.36**	**1.35**	**1.33**
75	9.81	4.22	3.01	**2.50**	**2.23**	**1.94**	**1.79**	**1.68**	**1.61**	**1.56**	**1.53**	**1.50**	**1.48**	**1.45**	**1.43**	**1.41**	**1.40**	**1.39**	**1.37**	**1.36**
100	10.82	4.49	3.16	**2.61**	**2.31**	**2.00**	**1.84**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.49**	**1.46**	**1.45**	**1.44**	**1.43**	**1.41**	**1.40**
125	11.67	4.71	3.27	*2.69*	**2.38**	**2.05**	**1.89**	**1.77**	**1.69**	**1.64**	**1.60**	**1.57**	**1.55**	**1.52**	**1.49**	**1.48**	**1.46**	**1.45**	**1.43**	**1.42**
150	12.40	4.90	3.37	*2.76*	**2.44**	**2.10**	**1.92**	**1.80**	**1.72**	**1.67**	**1.63**	**1.60**	**1.57**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.46**	**1.44**
175	13.09	5.05	3.45	*2.82*	**2.48**	**2.13**	**1.95**	**1.82**	**1.74**	**1.69**	**1.65**	**1.62**	**1.59**	**1.56**	**1.54**	**1.52**	**1.50**	**1.49**	**1.47**	**1.46**
200	13.67	5.20	3.53	*2.87*	**2.52**	**2.16**	**1.98**	**1.85**	**1.76**	**1.71**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (10 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.64	1.76	1.46	1.31	1.22	1.12	1.06	1.01	0.98	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.89	0.88	0.87	0.87
2	3.41	2.12	1.72	1.52	1.41	1.28	1.20	1.15	1.11	1.09	1.07	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
3	3.94	2.34	1.87	1.65	1.52	1.37	1.29	1.22	1.18	1.16	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03
4	4.35	2.51	1.98	1.74	1.59	1.43	1.34	1.28	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
5	4.71	2.65	2.07	1.81	1.65	1.48	1.39	1.32	1.27	1.24	1.22	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.10
8	5.53	*2.95*	2.27	1.96	1.78	1.59	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
12	6.36	3.24	2.45	2.09	1.89	1.68	1.56	1.48	1.42	1.38	1.35	1.33	1.32	1.29	1.27	1.26	1.25	1.24	1.23	1.22
16	7.01	3.45	2.58	2.19	1.98	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.27	1.26
20	7.56	3.63	2.68	2.27	2.04	1.79	1.66	1.57	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28
30	8.67	3.96	*2.87*	2.41	2.16	1.88	1.74	1.64	1.57	1.53	1.49	1.47	1.45	1.42	1.40	1.38	1.37	1.36	1.35	1.33
40	9.55	4.22	*3.02*	2.51	2.24	1.95	1.80	1.69	1.62	1.57	1.54	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
50	10.30	4.43	3.13	2.60	2.31	2.00	1.84	1.73	1.65	1.60	1.57	1.54	1.52	1.49	1.46	1.45	1.44	1.43	1.41	1.40
60	10.94	4.60	3.23	2.67	2.36	2.04	1.88	1.76	1.68	1.63	1.59	1.57	1.54	1.51	1.49	1.47	1.46	1.45	1.43	1.42
75	11.79	4.82	3.35	2.75	2.43	2.10	1.92	1.80	1.72	1.66	1.63	1.60	1.57	1.54	1.52	1.50	1.48	1.47	1.45	1.44
100	12.99	5.12	3.51	*2.86*	2.52	2.16	1.98	1.85	1.76	1.71	1.67	1.64	1.61	1.58	1.55	1.53	1.52	1.51	1.49	1.46
125	14.01	5.37	3.64	*2.95*	2.59	2.22	2.02	1.89	1.80	1.74	1.70	1.67	1.64	1.61	1.58	1.56	1.54	1.53	1.51	1.49
150	14.89	5.58	3.75	*3.03*	2.65	2.26	2.06	1.92	1.83	1.77	1.73	1.69	1.67	1.63	1.60	1.58	1.57	1.55	1.54	1.52
175	15.67	5.76	3.85	*3.09*	2.70	2.29	2.09	1.94	1.85	1.79	1.75	1.72	1.69	1.65	1.62	1.60	1.59	1.57	1.55	1.54
200	16.41	5.91	3.92	3.15	2.74	2.33	2.12	1.97	1.87	1.81	1.77	1.73	1.71	1.67	1.64	1.62	1.60	1.59	1.57	1.55

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (10 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.15	2.04	1.68	1.50	1.40	1.27	1.20	1.14	1.11	1.08	1.07	1.05	1.04	1.02	1.01	1.00	1.00	0.99	0.98	0.97
2	4.04	2.44	1.95	1.72	1.58	1.43	1.34	1.27	1.23	1.20	1.18	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
3	4.66	2.68	2.12	1.85	1.69	1.52	1.42	1.35	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13
4	5.14	2.87	2.24	1.94	1.77	1.58	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
5	5.55	3.02	2.33	2.02	1.83	1.63	1.52	1.44	1.39	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.21	1.20
8	6.52	3.36	2.54	2.17	1.97	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.27	1.26
12	7.49	3.68	2.73	2.32	2.08	1.83	1.70	1.60	1.53	1.49	1.46	1.44	1.42	1.39	1.37	1.35	1.34	1.33	1.32	1.31
16	8.25	3.92	2.87	2.42	2.17	1.90	1.75	1.65	1.58	1.54	1.50	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
20	8.90	4.11	2.98	2.50	2.23	1.95	1.80	1.69	1.62	1.57	1.53	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
30	10.21	4.49	*3.20*	2.65	2.35	2.04	1.88	1.76	1.68	1.63	1.59	1.57	1.54	1.51	1.49	1.47	1.46	1.45	1.43	1.42
40	11.23	4.77	3.35	2.76	2.44	2.11	1.93	1.81	1.73	1.67	1.63	1.61	1.58	1.55	1.52	1.51	1.49	1.48	1.46	1.45
50	12.11	5.00	3.48	2.85	2.51	2.16	1.98	1.85	1.76	1.71	1.67	1.64	1.61	1.58	1.55	1.53	1.52	1.51	1.49	1.47
60	12.87	5.19	3.58	2.92	2.57	2.20	2.01	1.88	1.79	1.73	1.69	1.66	1.64	1.60	1.57	1.56	1.54	1.53	1.50	1.50
75	13.87	5.44	3.71	3.01	2.64	2.26	2.06	1.92	1.83	1.77	1.72	1.69	1.67	1.63	1.60	1.58	1.57	1.56	1.53	1.52
100	15.28	5.79	3.88	*3.13*	2.73	2.32	2.12	1.97	1.88	1.81	1.77	1.73	1.71	1.67	1.64	1.62	1.60	1.59	1.57	1.55
125	16.46	6.05	4.03	*3.22*	2.81	2.38	2.16	2.01	1.91	1.85	1.80	1.76	1.74	1.70	1.66	1.64	1.63	1.61	1.59	1.58
150	17.48	6.30	4.14	3.30	2.87	2.42	2.20	2.04	1.94	1.87	1.82	1.79	1.76	1.72	1.69	1.66	1.65	1.63	1.61	1.60
175	18.46	6.49	4.25	3.37	2.92	2.46	2.23	2.06	1.96	1.90	1.85	1.81	1.78	1.73	1.70	1.68	1.66	1.65	1.63	1.61
200	19.24	6.67	4.33	3.43	2.97	2.49	2.26	2.09	1.98	1.92	1.86	1.83	1.80	1.75	1.72	1.70	1.68	1.67	1.64	1.63

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (20 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	2.80	**1.80**	**1.48**	**1.33**	**1.23**	**1.12**	**1.06**	**1.02**	**0.99**	**0.96**	**0.95**	**0.94**	**0.93**	**0.91**	**0.90**	**0.89**	**0.89**	**0.88**	**0.87**	**0.87**
2	3.60	**2.16**	**1.74**	**1.53**	**1.42**	**1.28**	**1.21**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.98**
3	4.15	**2.39**	**1.89**	**1.66**	**1.52**	**1.37**	**1.29**	**1.23**	**1.18**	**1.16**	**1.14**	**1.12**	**1.11**	**1.09**	**1.08**	**1.07**	**1.06**	**1.05**	**1.04**	**1.03**
4	4.58	**2.56**	**2.00**	**1.75**	**1.60**	**1.44**	**1.35**	**1.28**	**1.23**	**1.20**	**1.18**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
5	4.95	**2.70**	**2.09**	**1.82**	**1.66**	**1.48**	**1.39**	**1.32**	**1.27**	**1.24**	**1.22**	**1.20**	**1.19**	**1.17**	**1.15**	**1.14**	**1.13**	**1.12**	**1.11**	**1.11**
8	5.82	3.01	**2.29**	**1.97**	**1.79**	**1.59**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
12	6.68	3.29	**2.47**	**2.10**	**1.90**	**1.68**	**1.56**	**1.48**	**1.42**	**1.38**	**1.36**	**1.33**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
16	7.37	3.51	**2.60**	**2.20**	**1.98**	**1.74**	**1.62**	**1.53**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
20	7.95	3.69	*2.70*	**2.28**	**2.04**	**1.79**	**1.66**	**1.57**	**1.51**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.30**	**1.28**
30	9.11	4.03	*2.89*	**2.42**	**2.16**	**1.89**	**1.74**	**1.64**	**1.57**	**1.53**	**1.49**	**1.47**	**1.45**	**1.42**	**1.40**	**1.38**	**1.37**	**1.36**	**1.35**	**1.33**
40	10.05	4.28	3.04	**2.52**	**2.24**	**1.95**	**1.80**	**1.69**	**1.62**	**1.57**	**1.54**	**1.51**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
50	10.81	4.49	3.16	**2.61**	**2.31**	**2.00**	**1.84**	**1.73**	**1.65**	**1.60**	**1.57**	**1.54**	**1.52**	**1.49**	**1.46**	**1.45**	**1.44**	**1.42**	**1.41**	**1.40**
60	11.48	4.67	3.25	**2.67**	**2.37**	**2.05**	**1.88**	**1.76**	**1.68**	**1.63**	**1.59**	**1.57**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
75	12.42	4.89	3.37	*2.76*	**2.44**	**2.10**	**1.92**	**1.80**	**1.72**	**1.67**	**1.63**	**1.60**	**1.57**	**1.54**	**1.52**	**1.50**	**1.48**	**1.47**	**1.46**	**1.44**
100	13.65	5.20	3.53	*2.87*	**2.52**	**2.16**	**1.98**	**1.85**	**1.77**	**1.71**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.47**
125	14.77	5.45	3.66	*2.96*	**2.59**	**2.22**	**2.03**	**1.89**	**1.80**	**1.74**	**1.70**	**1.67**	**1.64**	**1.60**	**1.58**	**1.56**	**1.54**	**1.53**	**1.51**	**1.50**
150	15.70	5.65	3.78	3.03	**2.65**	**2.26**	**2.06**	**1.92**	**1.83**	**1.77**	**1.72**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**
175	16.41	5.86	3.87	3.11	*2.70*	**2.29**	**2.09**	**1.94**	**1.85**	**1.79**	**1.75**	**1.71**	**1.69**	**1.65**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**	**1.54**
200	17.11	6.01	3.96	3.15	*2.74*	**2.33**	**2.12**	**1.97**	**1.88**	**1.81**	**1.77**	**1.73**	**1.71**	**1.67**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (20 COC, Semi-Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.41	**2.12**	**1.72**	**1.52**	**1.41**	**1.28**	**1.20**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.02**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.97**
2	4.35	**2.51**	**1.98**	**1.74**	**1.59**	**1.43**	**1.34**	**1.28**	**1.23**	**1.20**	**1.18**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
3	5.01	**2.76**	**2.15**	**1.87**	**1.70**	**1.52**	**1.42**	**1.35**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**
4	5.53	*2.95*	**2.27**	**1.96**	**1.78**	**1.59**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
5	5.97	3.11	**2.37**	**2.03**	**1.84**	**1.64**	**1.53**	**1.44**	**1.39**	**1.35**	**1.33**	**1.31**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
8	7.01	3.45	**2.58**	**2.19**	**1.98**	**1.74**	**1.62**	**1.53**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
12	8.04	3.78	**2.77**	**2.33**	**2.09**	**1.83**	**1.70**	**1.60**	**1.53**	**1.49**	**1.46**	**1.44**	**1.42**	**1.39**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	8.86	4.02	*2.91*	**2.43**	**2.18**	**1.90**	**1.75**	**1.65**	**1.58**	**1.54**	**1.50**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
20	9.55	4.22	*3.02*	**2.52**	**2.24**	**1.95**	**1.80**	**1.69**	**1.62**	**1.57**	**1.53**	**1.51**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
30	10.96	4.60	3.23	**2.67**	**2.36**	**2.04**	**1.88**	**1.76**	**1.68**	**1.63**	**1.59**	**1.57**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
40	12.07	4.89	3.39	**2.78**	**2.45**	**2.11**	**1.94**	**1.81**	**1.73**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.52**	**1.51**	**1.49**	**1.48**	**1.46**	**1.45**
50	13.01	5.12	3.51	*2.86*	**2.52**	**2.16**	**1.98**	**1.85**	**1.76**	**1.71**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.51**	**1.49**	**1.46**
60	13.83	5.32	3.62	*2.94*	**2.58**	**2.20**	**2.02**	**1.88**	**1.79**	**1.74**	**1.69**	**1.66**	**1.64**	**1.60**	**1.57**	**1.55**	**1.54**	**1.53**	**1.50**	**1.50**
75	14.88	5.58	3.75	*3.02*	**2.65**	**2.26**	**2.06**	**1.92**	**1.83**	**1.77**	**1.73**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**
100	16.41	5.92	3.93	3.15	**2.74**	**2.33**	**2.12**	**1.97**	**1.88**	**1.81**	**1.77**	**1.73**	**1.71**	**1.67**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**
125	17.70	6.21	4.06	3.24	**2.81**	**2.38**	**2.16**	**2.01**	**1.91**	**1.85**	**1.80**	**1.77**	**1.74**	**1.70**	**1.67**	**1.64**	**1.63**	**1.61**	**1.59**	**1.58**
150	18.75	6.45	4.19	3.33	*2.87*	**2.42**	**2.20**	**2.04**	**1.94**	**1.88**	**1.82**	**1.79**	**1.76**	**1.72**	**1.69**	**1.66**	**1.65**	**1.63**	**1.61**	**1.60**
175	19.69	6.65	4.28	3.38	*2.93*	**2.46**	**2.23**	**2.07**	**1.96**	**1.90**	**1.85**	**1.81**	**1.78**	**1.74**	**1.70**	**1.68**	**1.66**	**1.65**	**1.63**	**1.61**
200	20.62	6.86	4.37	3.44	*2.97*	**2.49**	**2.26**	**2.09**	**1.98**	**1.92**	**1.87**	**1.83**	**1.80**	**1.76**	**1.72**	**1.70**	**1.68**	**1.67**	**1.64**	**1.63**

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (20 COC, Quarterly)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.04	**2.44**	**1.95**	**1.72**	**1.58**	**1.43**	**1.34**	**1.27**	**1.23**	**1.20**	**1.18**	**1.16**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
2	5.14	**2.87**	**2.24**	**1.94**	**1.77**	**1.58**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
3	5.91	3.15	**2.41**	**2.08**	**1.89**	**1.67**	**1.56**	**1.47**	**1.42**	**1.38**	**1.35**	**1.33**	**1.32**	**1.29**	**1.27**	**1.26**	**1.25**	**1.24**	**1.23**	**1.22**
4	6.52	3.36	**2.54**	**2.17**	**1.97**	**1.74**	**1.62**	**1.53**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
5	7.04	3.53	**2.65**	**2.25**	**2.03**	**1.79**	**1.66**	**1.56**	**1.50**	**1.46**	**1.43**	**1.41**	**1.39**	**1.36**	**1.34**	**1.33**	**1.32**	**1.31**	**1.30**	**1.28**
8	8.25	3.92	**2.87**	**2.42**	**2.17**	**1.90**	**1.75**	**1.65**	**1.58**	**1.54**	**1.50**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
12	9.46	4.28	*3.08*	**2.57**	**2.29**	**1.99**	**1.83**	**1.72**	**1.65**	**1.60**	**1.56**	**1.53**	**1.51**	**1.48**	**1.46**	**1.44**	**1.43**	**1.42**	**1.40**	**1.39**
16	10.43	4.55	3.23	**2.68**	**2.37**	**2.05**	**1.89**	**1.77**	**1.69**	**1.64**	**1.60**	**1.57**	**1.55**	**1.52**	**1.50**	**1.48**	**1.47**	**1.45**	**1.44**	**1.42**
20	11.24	4.77	3.35	**2.76**	**2.44**	**2.11**	**1.93**	**1.81**	**1.73**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.52**	**1.51**	**1.49**	**1.48**	**1.46**	**1.45**
30	12.88	5.20	3.58	**2.92**	**2.57**	**2.20**	**2.01**	**1.88**	**1.79**	**1.73**	**1.69**	**1.66**	**1.64**	**1.60**	**1.57**	**1.56**	**1.54**	**1.53**	**1.50**	**1.50**
40	14.18	5.52	3.75	**3.04**	**2.66**	**2.27**	**2.07**	**1.93**	**1.84**	**1.78**	**1.73**	**1.70**	**1.68**	**1.64**	**1.61**	**1.59**	**1.57**	**1.56**	**1.54**	**1.53**
50	15.29	5.79	3.89	*3.13*	**2.73**	**2.32**	**2.12**	**1.97**	**1.88**	**1.81**	**1.77**	**1.73**	**1.71**	**1.67**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**
60	16.23	6.01	4.00	*3.21*	**2.79**	**2.37**	**2.15**	**2.00**	**1.90**	**1.84**	**1.79**	**1.76**	**1.73**	**1.69**	**1.66**	**1.64**	**1.62**	**1.61**	**1.59**	**1.57**
75	17.46	6.30	4.15	3.30	**2.87**	**2.42**	**2.20**	**2.04**	**1.94**	**1.87**	**1.83**	**1.79**	**1.76**	**1.72**	**1.69**	**1.66**	**1.65**	**1.64**	**1.61**	**1.60**
100	19.22	6.68	4.34	3.43	**2.97**	**2.49**	**2.26**	**2.09**	**1.98**	**1.92**	**1.87**	**1.83**	**1.80**	**1.75**	**1.72**	**1.70**	**1.68**	**1.67**	**1.64**	**1.63**
125	20.74	6.97	4.48	3.53	**3.04**	**2.55**	**2.30**	**2.13**	**2.02**	**1.95**	**1.90**	**1.86**	**1.83**	**1.78**	**1.75**	**1.72**	**1.71**	**1.69**	**1.67**	**1.65**
150	22.03	7.27	4.61	3.62	*3.11*	**2.59**	**2.34**	**2.16**	**2.05**	**1.98**	**1.92**	**1.88**	**1.85**	**1.80**	**1.77**	**1.74**	**1.73**	**1.71**	**1.68**	**1.67**
175	23.20	7.50	4.72	3.69	*3.16*	**2.63**	**2.37**	**2.19**	**2.07**	**2.00**	**1.94**	**1.90**	**1.88**	**1.82**	**1.79**	**1.77**	**1.74**	**1.73**	**1.70**	**1.68**
200	24.38	7.68	4.83	3.75	*3.21*	**2.67**	**2.40**	**2.21**	**2.09**	**2.02**	**1.96**	**1.92**	**1.89**	**1.84**	**1.80**	**1.78**	**1.76**	**1.74**	**1.72**	**1.70**

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (40 COC, Annual)

w / n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	3.60	**2.16**	**1.74**	**1.53**	**1.42**	**1.28**	**1.21**	**1.15**	**1.11**	**1.09**	**1.07**	**1.05**	**1.04**	**1.03**	**1.01**	**1.00**	**1.00**	**0.99**	**0.98**	**0.98**
2	4.59	**2.56**	**2.00**	**1.75**	**1.60**	**1.44**	**1.35**	**1.28**	**1.23**	**1.20**	**1.18**	**1.17**	**1.15**	**1.13**	**1.12**	**1.11**	**1.10**	**1.09**	**1.08**	**1.07**
3	5.27	*2.81*	**2.17**	**1.88**	**1.71**	**1.53**	**1.43**	**1.35**	**1.30**	**1.27**	**1.25**	**1.23**	**1.21**	**1.19**	**1.18**	**1.17**	**1.16**	**1.15**	**1.14**	**1.13**
4	5.82	3.01	**2.29**	**1.97**	**1.79**	**1.59**	**1.48**	**1.40**	**1.35**	**1.32**	**1.29**	**1.27**	**1.26**	**1.23**	**1.22**	**1.21**	**1.20**	**1.19**	**1.18**	**1.17**
5	6.28	3.16	**2.39**	**2.04**	**1.85**	**1.64**	**1.53**	**1.44**	**1.39**	**1.35**	**1.33**	**1.31**	**1.29**	**1.27**	**1.25**	**1.24**	**1.23**	**1.22**	**1.21**	**1.20**
8	7.37	3.51	**2.60**	**2.20**	**1.98**	**1.74**	**1.62**	**1.53**	**1.47**	**1.43**	**1.40**	**1.38**	**1.36**	**1.33**	**1.31**	**1.30**	**1.29**	**1.28**	**1.27**	**1.26**
12	8.45	3.84	*2.79*	**2.34**	**2.10**	**1.84**	**1.70**	**1.60**	**1.54**	**1.49**	**1.46**	**1.44**	**1.42**	**1.39**	**1.37**	**1.35**	**1.34**	**1.33**	**1.32**	**1.31**
16	9.31	4.08	*2.93*	**2.44**	**2.18**	**1.90**	**1.76**	**1.65**	**1.58**	**1.54**	**1.50**	**1.48**	**1.46**	**1.43**	**1.41**	**1.39**	**1.38**	**1.37**	**1.35**	**1.34**
20	10.03	4.28	3.04	**2.52**	**2.25**	**1.95**	**1.80**	**1.69**	**1.62**	**1.57**	**1.54**	**1.51**	**1.49**	**1.46**	**1.44**	**1.42**	**1.41**	**1.40**	**1.38**	**1.37**
30	11.50	4.67	3.25	**2.67**	**2.37**	**2.05**	**1.88**	**1.76**	**1.68**	**1.63**	**1.59**	**1.57**	**1.54**	**1.51**	**1.49**	**1.47**	**1.46**	**1.45**	**1.43**	**1.42**
40	12.66	4.96	3.41	*2.79*	**2.45**	**2.11**	**1.94**	**1.81**	**1.73**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.52**	**1.51**	**1.49**	**1.48**	**1.46**	**1.45**
50	13.64	5.20	3.53	*2.87*	**2.52**	**2.16**	**1.98**	**1.85**	**1.76**	**1.71**	**1.67**	**1.64**	**1.61**	**1.58**	**1.55**	**1.53**	**1.52**	**1.50**	**1.48**	**1.46**
60	14.49	5.40	3.64	*2.94*	**2.58**	**2.21**	**2.02**	**1.88**	**1.79**	**1.74**	**1.69**	**1.66**	**1.64**	**1.60**	**1.57**	**1.55**	**1.54**	**1.53**	**1.50**	**1.48**
75	15.65	5.66	3.77	3.03	**2.65**	**2.26**	**2.06**	**1.92**	**1.83**	**1.77**	**1.73**	**1.69**	**1.67**	**1.63**	**1.60**	**1.58**	**1.57**	**1.55**	**1.53**	**1.52**
100	17.23	6.02	3.95	3.15	*2.74*	**2.33**	**2.12**	**1.97**	**1.88**	**1.81**	**1.77**	**1.73**	**1.71**	**1.67**	**1.64**	**1.62**	**1.60**	**1.59**	**1.57**	**1.55**
125	18.59	6.29	4.08	3.25	*2.82*	**2.38**	**2.16**	**2.01**	**1.91**	**1.85**	**1.80**	**1.76**	**1.74**	**1.70**	**1.67**	**1.65**	**1.63**	**1.62**	**1.59**	**1.58**
150	19.69	6.53	4.20	3.33	*2.88*	**2.43**	**2.20**	**2.04**	**1.94**	**1.87**	**1.82**	**1.79**	**1.76**	**1.72**	**1.69**	**1.67**	**1.66**	**1.64**	**1.61**	**1.60**
175	20.78	6.73	4.31	3.40	*2.93*	**2.46**	**2.23**	**2.06**	**1.97**	**1.90**	**1.85**	**1.81**	**1.78**	**1.74**	**1.71**	**1.69**	**1.67**	**1.66**	**1.63**	**1.61**
200	21.60	6.94	4.41	3.45	*2.97*	**2.50**	**2.26**	**2.09**	**1.99**	**1.91**	**1.87**	**1.83**	**1.80**	**1.76**	**1.73**	**1.71**	**1.69**	**1.68**	**1.64**	**1.62**

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (40 COC, Semi-Annual)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	4.35	2.51	1.98	1.74	1.59	1.43	1.34	1.28	1.23	1.20	1.18	1.17	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07
2	5.53	*2.95*	2.27	1.96	1.78	1.59	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
3	6.36	3.24	2.45	2.09	1.89	1.68	1.56	1.48	1.42	1.38	1.36	1.33	1.32	1.29	1.27	1.26	1.25	1.24	1.23	1.22
4	7.01	3.45	2.58	2.19	1.98	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.27	1.26
5	7.56	3.63	2.68	2.27	2.04	1.79	1.66	1.57	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.33	1.32	1.31	1.30	1.28
8	8.86	4.02	*2.91*	2.43	2.18	1.90	1.75	1.65	1.58	1.54	1.50	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
12	10.16	4.38	3.11	2.58	2.29	1.99	1.83	1.72	1.65	1.60	1.56	1.53	1.51	1.48	1.46	1.44	1.43	1.42	1.40	1.39
16	11.19	4.66	3.27	2.69	2.38	2.06	1.89	1.77	1.69	1.64	1.60	1.57	1.55	1.52	1.50	1.48	1.47	1.46	1.44	1.42
20	12.06	4.89	3.39	2.78	2.45	2.11	1.94	1.81	1.73	1.67	1.64	1.61	1.58	1.55	1.52	1.51	1.49	1.48	1.46	1.45
30	13.81	5.32	3.62	*2.94*	2.58	2.20	2.02	1.88	1.79	1.74	1.69	1.66	1.64	1.60	1.57	1.55	1.54	1.53	1.50	1.48
40	15.21	5.65	3.79	*3.05*	2.67	2.27	2.07	1.93	1.84	1.78	1.74	1.70	1.68	1.64	1.61	1.59	1.57	1.56	1.54	1.53
50	16.41	5.92	3.92	3.14	2.74	2.33	2.12	1.97	1.88	1.81	1.77	1.73	1.71	1.67	1.64	1.62	1.60	1.59	1.57	1.55
60	17.43	6.15	4.04	3.22	2.80	2.37	2.15	2.00	1.90	1.84	1.79	1.76	1.73	1.69	1.66	1.64	1.62	1.61	1.59	1.57
75	18.80	6.44	4.18	3.32	*2.88*	2.42	2.20	2.04	1.94	1.87	1.82	1.79	1.76	1.72	1.69	1.66	1.65	1.63	1.61	1.60
100	20.64	6.84	4.38	3.44	*2.97*	2.50	2.26	2.09	1.98	1.92	1.86	1.83	1.80	1.76	1.72	1.70	1.68	1.67	1.64	1.62
125	22.29	7.14	4.53	3.55	*3.05*	2.55	2.30	2.13	2.02	1.95	1.90	1.86	1.83	1.79	1.75	1.72	1.70	1.69	1.67	1.65
150	23.65	7.42	4.66	3.63	*3.11*	2.59	2.34	2.16	2.05	1.98	1.92	1.88	1.85	1.81	1.78	1.75	1.73	1.71	1.69	1.67
175	24.88	7.66	4.77	3.71	3.16	2.63	2.37	2.19	2.07	2.00	1.94	1.91	1.87	1.83	1.80	1.76	1.74	1.73	1.70	1.69
200	25.98	7.86	4.87	3.76	3.21	2.67	2.40	2.21	2.09	2.02	1.97	1.92	1.89	1.85	1.82	1.78	1.76	1.75	1.72	1.70

Table 19-18. κ-Multipliers for 1-of-2 Intrawell Prediction Limits on Means of Order 3 (40 COC, Quarterly)

w/n	4	6	8	10	12	16	20	25	30	35	40	45	50	60	70	80	90	100	125	150
1	5.14	2.87	2.24	1.94	1.77	1.58	1.48	1.40	1.35	1.32	1.29	1.27	1.26	1.23	1.22	1.21	1.20	1.19	1.18	1.17
2	6.52	3.36	2.54	2.17	1.97	1.74	1.62	1.53	1.47	1.43	1.40	1.38	1.36	1.33	1.31	1.30	1.29	1.28	1.27	1.26
3	7.49	3.68	2.73	2.32	2.08	1.83	1.70	1.60	1.53	1.49	1.46	1.44	1.42	1.39	1.37	1.35	1.34	1.33	1.32	1.31
4	8.25	3.92	2.87	2.42	2.17	1.90	1.75	1.65	1.58	1.54	1.50	1.48	1.46	1.43	1.41	1.39	1.38	1.37	1.35	1.34
5	8.90	4.11	2.98	2.50	2.23	1.95	1.80	1.69	1.62	1.57	1.53	1.51	1.49	1.46	1.44	1.42	1.41	1.40	1.38	1.37
8	10.42	4.55	3.23	2.68	2.37	2.06	1.89	1.77	1.69	1.64	1.60	1.57	1.55	1.52	1.50	1.48	1.47	1.45	1.44	1.42
12	11.95	4.96	3.45	2.83	2.50	2.15	1.97	1.84	1.76	1.70	1.66	1.63	1.61	1.57	1.55	1.53	1.51	1.50	1.48	1.47
16	13.16	5.27	3.62	2.95	2.59	2.22	2.03	1.89	1.80	1.74	1.70	1.67	1.64	1.61	1.58	1.56	1.55	1.53	1.52	1.50
20	14.18	5.52	3.75	3.04	2.66	2.27	2.07	1.93	1.84	1.78	1.73	1.70	1.68	1.64	1.61	1.59	1.57	1.56	1.54	1.53
30	16.24	6.01	4.00	*3.21*	2.79	2.37	2.15	2.00	1.90	1.84	1.79	1.76	1.73	1.69	1.66	1.64	1.62	1.61	1.59	1.57
40	17.88	6.37	4.18	3.33	2.89	2.44	2.21	2.05	1.95	1.88	1.83	1.80	1.77	1.72	1.69	1.67	1.66	1.64	1.62	1.60
50	19.28	6.67	4.33	3.43	2.96	2.49	2.26	2.09	1.98	1.92	1.87	1.83	1.80	1.75	1.72	1.70	1.68	1.67	1.64	1.62
60	20.47	6.93	4.46	3.51	3.02	2.54	2.29	2.12	2.01	1.94	1.89	1.85	1.82	1.77	1.74	1.72	1.70	1.69	1.66	1.64
75	22.08	7.26	4.61	3.61	*3.10*	2.59	2.34	2.16	2.05	1.98	1.92	1.88	1.85	1.80	1.77	1.75	1.73	1.71	1.68	1.67
100	24.34	7.69	4.82	3.75	*3.20*	2.67	2.40	2.21	2.09	2.02	1.96	1.92	1.89	1.84	1.80	1.78	1.76	1.75	1.72	1.70
125	26.11	8.07	4.99	3.86	*3.29*	2.72	2.44	2.25	2.13	2.05	2.00	1.95	1.92	1.86	1.83	1.81	1.79	1.77	1.74	1.72
150	27.89	8.37	5.13	3.95	3.35	2.77	2.48	2.28	2.16	2.08	2.02	1.98	1.94	1.89	1.85	1.83	1.81	1.79	1.76	1.74
175	29.26	8.61	5.25	4.02	3.41	2.81	2.51	2.31	2.18	2.10	2.04	2.00	1.96	1.91	1.87	1.85	1.83	1.81	1.77	1.76
200	30.62	8.89	5.37	4.09	3.46	2.85	2.54	2.33	2.20	2.12	2.06	2.01	1.98	1.93	1.90	1.86	1.84	1.83	1.79	1.77

D STATISTICAL TABLES

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D.4.2 PLANS ON MEDIANS

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Table 19-19. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan (PL=X_n)

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.6667-1	0.3571-1	0.2222-1	0.1515-1	0.1099-1	0.6536-2	0.4329-2	0.2849-2	0.2016-2	0.1502-2	0.1161-2
2	0.1190	0.6667-1	0.4242-1	0.2930-1	0.2143-1	0.1287-1	0.8564-2	0.5656-2	0.4011-2	0.2991-2	0.2316-2
3	0.1619	0.9394-1	0.6094-1	0.4258-1	0.3137-1	0.1900-1	0.1271-1	0.8422-2	0.5984-2	0.4468-2	0.3462-2
4	0.1980	0.1183	0.7802-1	0.5509-1	0.4087-1	0.2496-1	0.1677-1	0.1115-1	0.7937-2	0.5934-2	0.4602-2
5	0.2290	0.1402	0.9387-1	0.6691-1	0.4995-1	0.3074-1	0.2075-1	0.1384-1	0.9870-2	0.7388-2	0.5735-2
8	0.3016	0.1954	0.1355	0.9890-1	0.7507-1	0.4717-1	0.3222-1	0.2168-1	0.1555-1	0.1168-1	0.9091-2
10	0.3386	0.2255	0.1594	0.1178	0.9028-1	0.5744-1	0.3952-1	0.2674-1	0.1925-1	0.1449-1	0.1130-1
12	0.3696	0.2519	0.1808	0.1352	0.1045	0.6721-1	0.4656-1	0.3168-1	0.2287-1	0.1726-1	0.1347-1
15	0.4080	0.2859	0.2094	0.1589	0.1241	0.8105-1	0.5667-1	0.3885-1	0.2819-1	0.2134-1	0.1669-1
20	0.4576	0.3320	0.2496	0.1932	0.1532	0.1022	0.7248-1	0.5025-1	0.3673-1	0.2794-1	0.2194-1
25	0.4955	0.3690	0.2830	0.2225	0.1787	0.1214	0.8714-1	0.6102-1	0.4491-1	0.3433-1	0.2704-1
30	0.5259	0.3997	0.3116	0.2482	0.2014	0.1390	0.1008	0.7125-1	0.5276-1	0.4051-1	0.3201-1
35	0.5509	0.4257	0.3363	0.2708	0.2218	0.1551	0.1136	0.8097-1	0.6030-1	0.4649-1	0.3685-1
40	0.5721	0.4483	0.3582	0.2911	0.2403	0.1701	0.1257	0.9024-1	0.6757-1	0.5230-1	0.4157-1
45	0.5904	0.4680	0.3776	0.3095	0.2572	0.1840	0.1370	0.9910-1	0.7458-1	0.5793-1	0.4618-1
50	0.6063	0.4856	0.3951	0.3262	0.2727	0.1970	0.1478	0.1076	0.8134-1	0.6341-1	0.5068-1
60	0.6330	0.5155	0.4256	0.3556	0.3005	0.2208	0.1677	0.1235	0.9422-1	0.7393-1	0.5938-1
70	0.6546	0.5403	0.4512	0.3809	0.3246	0.2420	0.1858	0.1383	0.1063	0.8391-1	0.6770-1
80	0.6726	0.5613	0.4733	0.4029	0.3460	0.2611	0.2024	0.1521	0.1177	0.9340-1	0.7568-1
90	0.6880	0.5795	0.4927	0.4224	0.3651	0.2785	0.2178	0.1650	0.1285	0.1025	0.8334-1
100	0.7012	0.5954	0.5098	0.4399	0.3823	0.2944	0.2320	0.1771	0.1387	0.1111	0.9072-1
120	0.7231	0.6221	0.5389	0.4699	0.4122	0.3226	0.2576	0.1992	0.1577	0.1274	0.1047
140	0.7407	0.6438	0.5629	0.4950	0.4376	0.3471	0.2802	0.2191	0.1751	0.1424	0.1177
160	0.7552	0.6619	0.5832	0.5165	0.4595	0.3686	0.3004	0.2372	0.1910	0.1564	0.1300
180	0.7674	0.6773	0.6007	0.5351	0.4787	0.3877	0.3185	0.2537	0.2057	0.1694	0.1415
200	0.7778	0.6907	0.6160	0.5516	0.4958	0.4049	0.3351	0.2689	0.2194	0.1817	0.1524

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-19. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan (PL=X_n)

w\n	50	60	70	80	90	100	120	140	160	180	200
1	**0.7541-3**	**0.5288-3**	*0.3912-3*	*0.3011-3*	*0.2389-3*	*0.1941-3*	*0.1355-3*	0.9989-4	0.7668-4	0.6071-4	0.4926-4
2	**0.1505-2**	**0.1056-2**	*0.7816-3*	*0.6017-3*	*0.4775-3*	*0.3881-3*	*0.2709-3*	0.1997-3	0.1533-3	0.1214-3	0.9850-4
3	**0.2253-2**	**0.1582-2**	*0.1171-2*	*0.9018-3*	*0.7157-3*	*0.5818-3*	*0.4061-3*	0.2995-3	0.2299-3	0.1821-3	0.1477-3
4	**0.2998-2**	**0.2106-2**	*0.1560-2*	*0.1201-2*	*0.9536-3*	*0.7752-3*	*0.5413-3*	0.3992-3	0.3065-3	0.2427-3	0.1969-3
5	**0.3739-2**	**0.2628-2**	*0.1948-2*	*0.1500-2*	*0.1191-2*	*0.9685-3*	*0.6764-3*	0.4989-3	0.3831-3	0.3033-3	0.2462-3
8	**0.5946-2**	**0.4187-2**	*0.3106-2*	*0.2395-2*	*0.1902-2*	*0.1547-2*	*0.1081-2*	0.7975-3	0.6125-3	0.4851-3	0.3937-3
10	**0.7403-2**	**0.5219-2**	*0.3874-2*	*0.2988-2*	*0.2374-2*	*0.1932-2*	*0.1350-2*	0.9963-3	0.7653-3	0.6062-3	0.4919-3
12	**0.8848-2**	**0.6244-2**	*0.4638-2*	*0.3580-2*	*0.2845-2*	*0.2315-2*	*0.1619-2*	0.1195-2	0.9179-3	0.7271-3	0.5902-3
15	**0.1099-1**	**0.7772-2**	*0.5779-2*	*0.4463-2*	*0.3549-2*	*0.2889-2*	*0.2021-2*	0.1492-2	0.1147-2	0.9084-3	0.7374-3
20	**0.1451-1**	**0.1029-1**	*0.7665-2*	*0.5926-2*	*0.4717-2*	*0.3842-2*	*0.2690-2*	0.1987-2	0.1527-2	0.1210-2	0.9825-3
25	**0.1797-1**	**0.1277-1**	*0.9530-2*	*0.7377-2*	*0.5876-2*	*0.4789-2*	*0.3356-2*	0.2480-2	0.1907-2	0.1511-2	0.1227-2
30	**0.2136-1**	**0.1522-1**	*0.1138-1*	*0.8816-2*	*0.7028-2*	*0.5732-2*	*0.4019-2*	0.2972-2	0.2286-2	0.1812-2	0.1472-2
35	**0.2469-1**	**0.1764-1**	*0.1320-1*	*0.1024-1*	*0.8173-2*	*0.6669-2*	*0.4680-2*	0.3462-2	0.2664-2	0.2112-2	0.1716-2
40	**0.2796-1**	**0.2002-1**	*0.1501-1*	*0.1166-1*	*0.9310-2*	*0.7601-2*	*0.5338-2*	0.3951-2	0.3041-2	0.2412-2	0.1959-2
45	**0.3118-1**	**0.2238-1**	*0.1680-1*	*0.1307-1*	*0.1044-1*	*0.8529-2*	*0.5994-2*	0.4439-2	0.3417-2	0.2711-2	0.2203-2
50	**0.3434-1**	**0.2470-1**	*0.1858-1*	*0.1446-1*	*0.1156-1*	*0.9451-2*	*0.6647-2*	0.4925-2	0.3793-2	0.3009-2	0.2446-2
60	**0.4051-1**	**0.2927-1**	*0.2207-1*	*0.1722-1*	*0.1379-1*	*0.1128-1*	*0.7947-2*	0.5893-2	0.4541-2	0.3605-2	0.2931-2
70	**0.4648-1**	**0.3372-1**	*0.2551-1*	*0.1993-1*	*0.1598-1*	*0.1309-1*	*0.9237-2*	0.6856-2	0.5287-2	0.4199-2	0.3414-2
80	**0.5227-1**	**0.3807-1**	*0.2887-1*	*0.2261-1*	*0.1815-1*	*0.1489-1*	*0.1052-1*	0.7814-2	0.6029-2	0.4790-2	0.3896-2
90	**0.5789-1**	**0.4232-1**	*0.3218-1*	*0.2524-1*	*0.2030-1*	*0.1666-1*	*0.1179-1*	0.8766-2	0.6768-2	0.5380-2	0.4377-2
100	**0.6335-1**	**0.4648-1**	*0.3543-1*	*0.2784-1*	*0.2242-1*	*0.1842-1*	*0.1305-1*	0.9713-2	0.7504-2	0.5967-2	0.4857-2
120	**0.7382-1**	**0.5453-1**	*0.4176-1*	*0.3293-1*	*0.2659-1*	*0.2189-1*	*0.1555-1*	0.1159-1	0.8966-2	0.7136-2	0.5812-2
140	**0.8376-1**	**0.6225-1**	*0.4789-1*	*0.3788-1*	*0.3066-1*	*0.2529-1*	*0.1801-1*	0.1345-1	0.1042-1	0.8297-2	0.6762-2
160	**0.9322-1**	**0.6967-1**	*0.5382-1*	*0.4270-1*	*0.3464-1*	*0.2863-1*	*0.2044-1*	0.1529-1	0.1185-1	0.9451-2	0.7706-2
180	**0.1022**	**0.7682-1**	*0.5957-1*	*0.4741-1*	*0.3854-1*	*0.3191-1*	*0.2284-1*	0.1711-1	0.1328-1	0.1060-1	0.8646-2
200	**0.1109**	**0.8371-1**	*0.6515-1*	*0.5199-1*	*0.4236-1*	*0.3513-1*	*0.2520-1*	0.1892-1	0.1470-1	0.1173-1	0.9580-2

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-19. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan (PL=X_{n-1})

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.2000	0.1071	0.6667-1	0.4545-1	0.3297-1	0.1961-1	0.1299-1	0.8547-2	0.6048-2	0.4505-2	0.3484-2
2	0.3286	0.1905	0.1232	0.8591-1	0.6319-1	0.3818-1	0.2550-1	0.1688-1	0.1199-1	0.8948-2	0.6932-2
3	0.4190	0.2576	0.1720	0.1223	0.9104-1	0.5582-1	0.3758-1	0.2502-1	0.1783-1	0.1333-1	0.1034-1
4	0.4866	0.3130	0.2147	0.1551	0.1168	0.7260-1	0.4924-1	0.3296-1	0.2356-1	0.1766-1	0.1372-1
5	0.5391	0.3598	0.2524	0.1851	0.1408	0.8860-1	0.6052-1	0.4072-1	0.2920-1	0.2193-1	0.1706-1
8	0.6450	0.4651	0.3436	0.2612	0.2038	0.1325	0.9224-1	0.6297-1	0.4556-1	0.3442-1	0.2688-1
10	0.6911	0.5163	0.3914	0.3030	0.2397	0.1588	0.1118	0.7702-1	0.5603-1	0.4248-1	0.3327-1
12	0.7261	0.5577	0.4317	0.3396	0.2719	0.1831	0.1303	0.9048-1	0.6617-1	0.5036-1	0.3953-1
15	0.7653	0.6071	0.4820	0.3867	0.3144	0.2163	0.1561	0.1097	0.8082-1	0.6182-1	0.4871-1
20	0.8099	0.6672	0.5466	0.4499	0.3733	0.2648	0.1950	0.1394	0.1039	0.8006-1	0.6344-1
25	0.8398	0.7103	0.5955	0.4997	0.4214	0.3063	0.2295	0.1665	0.1253	0.9732-1	0.7752-1
30	0.8613	0.7429	0.6340	0.5402	0.4616	0.3424	0.2605	0.1914	0.1454	0.1137	0.9101-1
35	0.8776	0.7686	0.6652	0.5739	0.4957	0.3743	0.2884	0.2145	0.1643	0.1293	0.1039
40	0.8903	0.7893	0.6910	0.6024	0.5252	0.4026	0.3138	0.2358	0.1821	0.1441	0.1164
45	0.9006	0.8064	0.7129	0.6270	0.5510	0.4280	0.3370	0.2558	0.1989	0.1582	0.1283
50	0.9091	0.8208	0.7316	0.6484	0.5738	0.4509	0.3584	0.2744	0.2149	0.1717	0.1398
60	0.9222	0.8438	0.7621	0.6840	0.6123	0.4909	0.3965	0.3084	0.2443	0.1971	0.1616
70	0.9320	0.8613	0.7860	0.7125	0.6438	0.5246	0.4295	0.3385	0.2711	0.2205	0.1820
80	0.9395	0.8751	0.8053	0.7359	0.6700	0.5535	0.4584	0.3656	0.2955	0.2421	0.2010
90	0.9455	0.8864	0.8212	0.7555	0.6923	0.5787	0.4841	0.3901	0.3179	0.2622	0.2189
100	0.9505	0.8957	0.8345	0.7722	0.7115	0.6008	0.5070	0.4124	0.3386	0.2810	0.2358
120	0.9580	0.9102	0.8558	0.7991	0.7430	0.6380	0.5464	0.4515	0.3756	0.3151	0.2668
140	0.9635	0.9211	0.8720	0.8200	0.7678	0.6681	0.5792	0.4849	0.4079	0.3454	0.2947
160	0.9677	0.9296	0.8847	0.8367	0.7879	0.6931	0.6069	0.5138	0.4363	0.3725	0.3201
180	0.9710	0.9364	0.8951	0.8505	0.8046	0.7143	0.6308	0.5391	0.4616	0.3970	0.3432
200	0.9737	0.9419	0.9037	0.8619	0.8186	0.7325	0.6516	0.5615	0.4844	0.4192	0.3644

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-19. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan (PL=X_{n-1})

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.2262-2	0.1586-2	0.1174-2	0.9033-3	0.7167-3	0.5824-3	0.4064-3	0.2997-3	0.2300-3	*0.1821-3*	*0.1478-3*
2	0.4509-2	0.3165-2	0.2343-2	0.1804-2	0.1432-2	0.1164-2	0.8124-3	0.5990-3	0.4599-3	*0.3642-3*	*0.2955-3*
3	0.6740-2	0.4736-2	0.3508-2	0.2702-2	0.2145-2	0.1744-2	0.1218-2	0.8981-3	0.6896-3	*0.5461-3*	*0.4431-3*
4	0.8956-2	0.6299-2	0.4669-2	0.3598-2	0.2857-2	0.2323-2	0.1623-2	0.1197-2	0.9191-3	*0.7279-3*	*0.5907-3*
5	0.1116-1	0.7854-2	0.5825-2	0.4491-2	0.3567-2	0.2901-2	0.2027-2	0.1495-2	0.1149-2	*0.9096-3*	*0.7382-3*
8	0.1767-1	0.1248-1	0.9270-2	0.7155-2	0.5688-2	0.4629-2	0.3237-2	0.2389-2	0.1836-2	*0.1454-2*	*0.1180-2*
10	0.2194-1	0.1552-1	0.1155-1	0.8919-2	0.7094-2	0.5776-2	0.4041-2	0.2984-2	0.2293-2	*0.1817-2*	*0.1475-2*
12	0.2615-1	0.1853-1	0.1380-1	0.1067-1	0.8494-2	0.6918-2	0.4843-2	0.3577-2	0.2749-2	*0.2179-2*	*0.1769-2*
15	0.3237-1	0.2300-1	0.1716-1	0.1328-1	0.1058-1	0.8624-2	0.6041-2	0.4465-2	0.3433-2	*0.2721-2*	*0.2209-2*
20	0.4247-1	0.3032-1	0.2268-1	0.1759-1	0.1403-1	0.1145-1	0.8029-2	0.5939-2	0.4568-2	*0.3622-2*	*0.2942-2*
25	0.5226-1	0.3746-1	0.2811-1	0.2184-1	0.1744-1	0.1424-1	0.1000-1	0.7406-2	0.5700-2	*0.4521-2*	*0.3673-2*
30	0.6176-1	0.4445-1	0.3344-1	0.2603-1	0.2082-1	0.1702-1	0.1197-1	0.8866-2	0.6827-2	*0.5418-2*	*0.4403-2*
35	0.7098-1	0.5129-1	0.3869-1	0.3017-1	0.2415-1	0.1976-1	0.1392-1	0.1032-1	0.7951-2	*0.6311-2*	*0.5130-2*
40	0.7994-1	0.5799-1	0.4384-1	0.3425-1	0.2746-1	0.2249-1	0.1586-1	0.1177-1	0.9070-2	*0.7202-2*	*0.5856-2*
45	0.8866-1	0.6454-1	0.4892-1	0.3828-1	0.3073-1	0.2519-1	0.1778-1	0.1320-1	0.1019-1	*0.8091-2*	*0.6580-2*
50	0.9714-1	0.7096-1	0.5391-1	0.4226-1	0.3396-1	0.2786-1	0.1970-1	0.1464-1	0.1130-1	*0.8977-2*	*0.7303-2*
60	0.1134	0.8342-1	0.6367-1	0.5006-1	0.4033-1	0.3315-1	0.2349-1	0.1748-1	0.1351-1	*0.1074-1*	*0.8743-2*
70	0.1289	0.9540-1	0.7313-1	0.5768-1	0.4658-1	0.3835-1	0.2724-1	0.2030-1	0.1570-1	*0.1250-1*	*0.1018-1*
80	0.1437	0.1069	0.8231-1	0.6512-1	0.5270-1	0.4346-1	0.3094-1	0.2310-1	0.1788-1	*0.1424-1*	*0.1160-1*
90	0.1577	0.1181	0.9123-1	0.7238-1	0.5870-1	0.4849-1	0.3460-1	0.2587-1	0.2004-1	*0.1597-1*	*0.1302-1*
100	0.1712	0.1288	0.9990-1	0.7948-1	0.6459-1	0.5344-1	0.3821-1	0.2861-1	0.2219-1	*0.1770-1*	*0.1444-1*
120	0.1963	0.1492	0.1165	0.9320-1	0.7604-1	0.6310-1	0.4532-1	0.3403-1	0.2645-1	*0.2112-1*	*0.1724-1*
140	0.2196	0.1683	0.1323	0.1063	0.8708-1	0.7248-1	0.5227-1	0.3935-1	0.3064-1	*0.2450-1*	*0.2002-1*
160	0.2410	0.1863	0.1473	0.1189	0.9774-1	0.8158-1	0.5907-1	0.4459-1	0.3478-1	*0.2785-1*	*0.2278-1*
180	0.2610	0.2032	0.1616	0.1310	0.1080	0.9041-1	0.6572-1	0.4973-1	0.3886-1	*0.3116-1*	*0.2551-1*
200	0.2797	0.2192	0.1753	0.1427	0.1180	0.9900-1	0.7222-1	0.5480-1	0.4289-1	*0.3443-1*	*0.2822-1*

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-20. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-3 Plan (PL=X_n)

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.2857-1	0.1190-1	0.6061-2	0.3497-2	0.2198-2	0.1032-2	0.5647-3	0.3053-3	0.1833-3	*0.1185-3*	*0.8103-4*
2	0.5238-1	0.2273-1	0.1179-1	0.6868-2	0.4342-2	0.2051-2	0.1125-2	0.6091-3	0.3661-3	*0.2369-3*	*0.1620-3*
3	0.7283-1	0.3267-1	0.1722-1	0.1013-1	0.6435-2	0.3056-2	0.1681-2	0.9117-3	0.5483-3	*0.3550-3*	*0.2428-3*
4	0.9076-1	0.4187-1	0.2240-1	0.1328-1	0.8481-2	0.4049-2	0.2233-2	0.1213-2	0.7301-3	*0.4728-3*	*0.3235-3*
5	0.1067	0.5045-1	0.2735-1	0.1633-1	0.1048-1	0.5031-2	0.2781-2	0.1513-2	0.9113-3	*0.5905-3*	*0.4041-3*
8	0.1463	0.7320-1	0.4101-1	0.2499-1	0.1624-1	0.7906-2	0.4401-2	0.2405-2	0.1452-2	*0.9422-3*	*0.6453-3*
10	0.1678	0.8643-1	0.4929-1	0.3038-1	0.1990-1	0.9769-2	0.5462-2	0.2994-2	0.1810-2	*0.1176-2*	*0.8056-3*
12	0.1865	0.9846-1	0.5704-1	0.3552-1	0.2342-1	0.1159-1	0.6509-2	0.3577-2	0.2167-2	*0.1408-2*	*0.9654-3*
15	0.2107	0.1147	0.6780-1	0.4279-1	0.2848-1	0.1426-1	0.8054-2	0.4444-2	0.2697-2	*0.1755-2*	*0.1204-2*
20	0.2437	0.1380	0.8386-1	0.5396-1	0.3640-1	0.1853-1	0.1056-1	0.5866-2	0.3573-2	*0.2330-2*	*0.1601-2*
25	0.2704	0.1580	0.9810-1	0.6412-1	0.4376-1	0.2261-1	0.1300-1	0.7262-2	0.4438-2	*0.2900-2*	*0.1995-2*
30	0.2928	0.1754	0.1109	0.7348-1	0.5065-1	0.2652-1	0.1537-1	0.8632-2	0.5292-2	*0.3465-2*	*0.2386-2*
35	0.3121	0.1908	0.1225	0.8215-1	0.5713-1	0.3028-1	0.1767-1	0.9978-2	0.6136-2	*0.4025-2*	*0.2776-2*
40	0.3289	0.2046	0.1332	0.9024-1	0.6326-1	0.3389-1	0.1991-1	0.1130-1	0.6971-2	*0.4581-2*	*0.3162-2*
45	0.3438	0.2172	0.1431	0.9783-1	0.6907-1	0.3738-1	0.2210-1	0.1260-1	0.7796-2	*0.5132-2*	*0.3547-2*
50	0.3573	0.2287	0.1523	0.1050	0.7461-1	0.4075-1	0.2423-1	0.1388-1	0.8612-2	*0.5679-2*	*0.3929-2*
60	0.3805	0.2492	0.1689	0.1181	0.8494-1	0.4717-1	0.2836-1	0.1639-1	0.1022-1	*0.6760-2*	*0.4688-2*
70	0.4001	0.2669	0.1837	0.1301	0.9445-1	0.5323-1	0.3231-1	0.1882-1	0.1179-1	*0.7824-2*	*0.5437-2*
80	0.4171	0.2826	0.1970	0.1410	0.1033	0.5895-1	0.3610-1	0.2118-1	0.1333-1	*0.8874-2*	*0.6179-2*
90	0.4319	0.2966	0.2091	0.1510	0.1115	0.6439-1	0.3975-1	0.2347-1	0.1484-1	*0.9908-2*	*0.6913-2*
100	0.4451	0.3093	0.2202	0.1603	0.1192	0.6957-1	0.4327-1	0.2571-1	0.1632-1	*0.1093-1*	*0.7639-2*
120	0.4677	0.3314	0.2399	0.1771	0.1333	0.7926-1	0.4996-1	0.3003-1	0.1921-1	*0.1293-1*	*0.9069-2*
140	0.4865	0.3502	0.2570	0.1920	0.1459	0.8819-1	0.5624-1	0.3415-1	0.2200-1	*0.1488-1*	*0.1047-1*
160	0.5026	0.3666	0.2722	0.2054	0.1574	0.9648-1	0.6216-1	0.3810-1	0.2470-1	*0.1678-1*	*0.1185-1*
180	0.5166	0.3811	0.2858	0.2175	0.1680	0.1042	0.6777-1	0.4189-1	0.2732-1	*0.1864-1*	*0.1320-1*
200	0.5289	0.3941	0.2980	0.2286	0.1777	0.1115	0.7311-1	0.4554-1	0.2987-1	*0.2045-1*	*0.1453-1*

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-20. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-3 Plan (PL=X_n)

w\n	50	60	70	80	90	100	120	140	160	180	200
1	*0.4269-4*	*0.2518-4*	*0.1608-4*	0.1088-4	0.7706-5	0.5654-5	0.3304-5	0.2096-5	0.1411-5	0.9953-6	0.7280-6
2	*0.8534-4*	*0.5035-4*	*0.3215-4*	0.2177-4	0.1541-4	0.1131-4	0.6609-5	0.4191-5	0.2823-5	0.1991-5	0.1456-5
3	*0.1280-3*	*0.7551-4*	*0.4822-4*	0.3264-4	0.2312-4	0.1696-4	0.9913-5	0.6287-5	0.4234-5	0.2986-5	0.2184-5
4	*0.1706-3*	*0.1007-3*	*0.6429-4*	0.4352-4	0.3082-4	0.2261-4	0.1322-4	0.8382-5	0.5645-5	0.3981-5	0.2912-5
5	*0.2131-3*	*0.1258-3*	*0.8035-4*	0.5440-4	0.3852-4	0.2827-4	0.1652-4	0.1048-4	0.7056-5	0.4976-5	0.3640-5
8	*0.3406-3*	*0.2011-3*	*0.1285-3*	0.8701-4	0.6162-4	0.4522-4	0.2643-4	0.1676-4	0.1129-4	0.7962-5	0.5823-5
10	*0.4255-3*	*0.2513-3*	*0.1606-3*	0.1087-3	0.7701-4	0.5652-4	0.3304-4	0.2095-4	0.1411-4	0.9952-5	0.7279-5
12	*0.5102-3*	*0.3015-3*	*0.1926-3*	0.1305-3	0.9240-4	0.6782-4	0.3964-4	0.2514-4	0.1693-4	0.1194-4	0.8735-5
15	*0.6371-3*	*0.3766-3*	*0.2407-3*	0.1630-3	0.1155-3	0.8476-4	0.4955-4	0.3143-4	0.2117-4	0.1493-4	0.1092-4
20	*0.8480-3*	*0.5016-3*	*0.3207-3*	0.2173-3	0.1539-3	0.1130-3	0.6605-4	0.4190-4	0.2822-4	0.1990-4	0.1456-4
25	*0.1058-2*	*0.6263-3*	*0.4006-3*	0.2715-3	0.1923-3	0.1412-3	0.8255-4	0.5237-4	0.3527-4	0.2488-4	0.1820-4
30	*0.1268-2*	*0.7507-3*	*0.4804-3*	0.3256-3	0.2307-3	0.1694-3	0.9905-4	0.6283-4	0.4232-4	0.2985-4	0.2183-4
35	*0.1476-2*	*0.8749-3*	*0.5600-3*	0.3797-3	0.2691-3	0.1976-3	0.1155-3	0.7330-4	0.4937-4	0.3482-4	0.2547-4
40	*0.1684-2*	*0.9989-3*	*0.6396-3*	0.4337-3	0.3074-3	0.2257-3	0.1320-3	0.8376-4	0.5642-4	0.3980-4	0.2911-4
45	*0.1892-2*	*0.1123-2*	*0.7191-3*	0.4877-3	0.3457-3	0.2539-3	0.1485-3	0.9422-4	0.6347-4	0.4477-4	0.3275-4
50	*0.2098-2*	*0.1246-2*	*0.7984-3*	0.5416-3	0.3840-3	0.2820-3	0.1650-3	0.1047-3	0.7052-4	0.4974-4	0.3639-4
60	*0.2509-2*	*0.1492-2*	*0.9568-3*	0.6493-3	0.4605-3	0.3382-3	0.1979-3	0.1256-3	0.8461-4	0.5968-4	0.4366-4
70	*0.2918-2*	*0.1737-2*	*0.1115-2*	0.7568-3	0.5369-3	0.3944-3	0.2308-3	0.1465-3	0.9870-4	0.6962-4	0.5093-4
80	*0.3324-2*	*0.1981-2*	*0.1272-2*	0.8641-3	0.6131-3	0.4505-3	0.2637-3	0.1674-3	0.1128-3	0.7956-4	0.5820-4
90	*0.3727-2*	*0.2224-2*	*0.1429-2*	0.9712-3	0.6893-3	0.5066-3	0.2966-3	0.1883-3	0.1269-3	0.8950-4	0.6548-4
100	*0.4128-2*	*0.2466-2*	*0.1586-2*	0.1078-2	0.7654-3	0.5626-3	0.3294-3	0.2092-3	0.1409-3	0.9944-4	0.7275-4
120	*0.4922-2*	*0.2948-2*	*0.1898-2*	0.1291-2	0.9172-3	0.6744-3	0.3951-3	0.2509-3	0.1691-3	0.1193-3	0.8728-4
140	*0.5707-2*	*0.3425-2*	*0.2208-2*	0.1504-2	0.1069-2	0.7860-3	0.4607-3	0.2926-3	0.1972-3	0.1392-3	0.1018-3
160	*0.6482-2*	*0.3900-2*	*0.2517-2*	0.1715-2	0.1220-2	0.8974-3	0.5262-3	0.3343-3	0.2253-3	0.1590-3	0.1163-3
180	*0.7249-2*	*0.4370-2*	*0.2825-2*	0.1926-2	0.1370-2	0.1009-2	0.5916-3	0.3759-3	0.2534-3	0.1788-3	0.1309-3
200	*0.8007-2*	*0.4837-2*	*0.3130-2*	0.2136-2	0.1520-2	0.1120-2	0.6569-3	0.4175-3	0.2815-3	0.1987-3	0.1454-3

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-20. Per-Constituent Significance Levels (α) for Non-parametric 1-of-3 Plan (PL=X_{n-1})

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.1143	0.4762-1	0.2424-1	0.1399-1	0.8791-2	0.4128-2	0.2259-2	0.1221-2	0.7331-3	0.4742-3	0.3241-3
2	0.1952	0.8766-1	0.4615-1	0.2710-1	0.1721-1	0.8162-2	0.4487-2	0.2432-2	0.1463-2	0.9468-3	0.6475-3
3	0.2568	0.1221	0.6615-1	0.3944-1	0.2528-1	0.1211-1	0.6686-2	0.3635-2	0.2189-2	0.1418-2	0.9701-3
4	0.3059	0.1523	0.8453-1	0.5111-1	0.3303-1	0.1597-1	0.8856-2	0.4828-2	0.2911-2	0.1887-2	0.1292-2
5	0.3464	0.1791	0.1015	0.6218-1	0.4050-1	0.1975-1	0.1100-1	0.6012-2	0.3630-2	0.2355-2	0.1613-2
8	0.4353	0.2448	0.1460	0.9230-1	0.6139-1	0.3064-1	0.1727-1	0.9512-2	0.5767-2	0.3750-2	0.2572-2
10	0.4780	0.2799	0.1714	0.1103	0.7421-1	0.3757-1	0.2133-1	0.1180-1	0.7175-2	0.4673-2	0.3208-2
12	0.5124	0.3101	0.1942	0.1269	0.8628-1	0.4424-1	0.2529-1	0.1406-1	0.8570-2	0.5590-2	0.3841-2
15	0.5536	0.3485	0.2244	0.1496	0.1032	0.5384-1	0.3107-1	0.1739-1	0.1064-1	0.6955-2	0.4786-2
20	0.6046	0.3997	0.2670	0.1827	0.1286	0.6883-1	0.4031-1	0.2280-1	0.1403-1	0.9202-2	0.6346-2
25	0.6419	0.4399	0.3023	0.2114	0.1512	0.8274-1	0.4910-1	0.2804-1	0.1735-1	0.1142-1	0.7889-2
30	0.6708	0.4728	0.3323	0.2366	0.1716	0.9573-1	0.5749-1	0.3312-1	0.2060-1	0.1360-1	0.9417-2
35	0.6940	0.5004	0.3584	0.2591	0.1901	0.1079	0.6550-1	0.3805-1	0.2378-1	0.1575-1	0.1093-1
40	0.7132	0.5240	0.3814	0.2793	0.2072	0.1194	0.7319-1	0.4285-1	0.2690-1	0.1787-1	0.1243-1
45	0.7294	0.5445	0.4018	0.2977	0.2229	0.1302	0.8058-1	0.4751-1	0.2997-1	0.1997-1	0.1391-1
50	0.7433	0.5626	0.4202	0.3145	0.2375	0.1405	0.8768-1	0.5206-1	0.3298-1	0.2203-1	0.1537-1
60	0.7661	0.5932	0.4521	0.3443	0.2638	0.1596	0.1011	0.6082-1	0.3884-1	0.2608-1	0.1827-1
70	0.7842	0.6182	0.4790	0.3701	0.2871	0.1771	0.1137	0.6917-1	0.4450-1	0.3004-1	0.2110-1
80	0.7989	0.6392	0.5021	0.3927	0.3079	0.1931	0.1255	0.7715-1	0.4998-1	0.3389-1	0.2389-1
90	0.8112	0.6571	0.5223	0.4128	0.3266	0.2080	0.1367	0.8480-1	0.5529-1	0.3766-1	0.2663-1
100	0.8216	0.6727	0.5402	0.4309	0.3437	0.2218	0.1472	0.9214-1	0.6045-1	0.4135-1	0.2932-1
120	0.8385	0.6987	0.5705	0.4621	0.3738	0.2468	0.1667	0.1060	0.7032-1	0.4848-1	0.3457-1
140	0.8518	0.7195	0.5955	0.4885	0.3996	0.2690	0.1844	0.1189	0.7968-1	0.5532-1	0.3966-1
160	0.8625	0.7367	0.6165	0.5111	0.4221	0.2889	0.2006	0.1310	0.8857-1	0.6190-1	0.4459-1
180	0.8713	0.7512	0.6346	0.5308	0.4421	0.3069	0.2156	0.1424	0.9705-1	0.6824-1	0.4937-1
200	0.8788	0.7637	0.6504	0.5482	0.4599	0.3233	0.2295	0.1531	0.1052	0.7436-1	0.5403-1

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-20. Per-Constituent Significance Levels (α) for Non-parametric 1-of-3 Plan (PL=X_{n-1})

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.1708-3	0.1007-3	0.6431-4	0.4353-4	0.3082-4	0.2262-4	0.1322-4	0.8382-5	0.5645-5	0.3981-5	0.2912-5
2	0.3413-3	0.2014-3	0.1286-3	0.8705-4	0.6164-4	0.4523-4	0.2643-4	0.1676-4	0.1129-4	0.7962-5	0.5824-5
3	0.5116-3	0.3020-3	0.1928-3	0.1306-3	0.9245-4	0.6784-4	0.3965-4	0.2515-4	0.1693-4	0.1194-4	0.8735-5
4	0.6817-3	0.4024-3	0.2571-3	0.1740-3	0.1233-3	0.9045-4	0.5286-4	0.3353-4	0.2258-4	0.1592-4	0.1165-4
5	0.8516-3	0.5029-3	0.3212-3	0.2175-3	0.1540-3	0.1130-3	0.6608-4	0.4191-4	0.2822-4	0.1990-4	0.1456-4
8	0.1360-2	0.8037-3	0.5136-3	0.3479-3	0.2464-3	0.1808-3	0.1057-3	0.6704-4	0.4515-4	0.3185-4	0.2329-4
10	0.1698-2	0.1004-2	0.6417-3	0.4347-3	0.3079-3	0.2260-3	0.1321-3	0.8380-4	0.5644-4	0.3981-4	0.2912-4
12	0.2035-2	0.1204-2	0.7697-3	0.5214-3	0.3694-3	0.2711-3	0.1585-3	0.1006-3	0.6773-4	0.4777-4	0.3494-4
15	0.2539-2	0.1503-2	0.9614-3	0.6515-3	0.4616-3	0.3388-3	0.1981-3	0.1257-3	0.8465-4	0.5970-4	0.4367-4
20	0.3375-2	0.2000-2	0.1280-2	0.8679-3	0.6151-3	0.4516-3	0.2641-3	0.1675-3	0.1129-3	0.7960-4	0.5822-4
25	0.4206-2	0.2495-2	0.1598-2	0.1084-2	0.7684-3	0.5642-3	0.3300-3	0.2094-3	0.1411-3	0.9949-4	0.7277-4
30	0.5032-2	0.2989-2	0.1916-2	0.1300-2	0.9215-3	0.6768-3	0.3959-3	0.2512-3	0.1692-3	0.1194-3	0.8732-4
35	0.5853-2	0.3481-2	0.2232-2	0.1515-2	0.1074-2	0.7892-3	0.4618-3	0.2930-3	0.1974-3	0.1393-3	0.1019-3
40	0.6669-2	0.3971-2	0.2548-2	0.1730-2	0.1227-2	0.9015-3	0.5276-3	0.3348-3	0.2256-3	0.1591-3	0.1164-3
45	0.7481-2	0.4459-2	0.2863-2	0.1945-2	0.1380-2	0.1014-2	0.5934-3	0.3766-3	0.2538-3	0.1790-3	0.1310-3
50	0.8288-2	0.4945-2	0.3177-2	0.2159-2	0.1532-2	0.1126-2	0.6592-3	0.4184-3	0.2819-3	0.1989-3	0.1455-3
60	0.9888-2	0.5913-2	0.3804-2	0.2586-2	0.1836-2	0.1350-2	0.7906-3	0.5019-3	0.3382-3	0.2386-3	0.1746-3
70	0.1147-1	0.6874-2	0.4427-2	0.3012-2	0.2140-2	0.1573-2	0.9218-3	0.5854-3	0.3945-3	0.2784-3	0.2037-3
80	0.1304-1	0.7828-2	0.5048-2	0.3437-2	0.2443-2	0.1797-2	0.1053-2	0.6688-3	0.4508-3	0.3181-3	0.2327-3
90	0.1458-1	0.8776-2	0.5665-2	0.3860-2	0.2745-2	0.2019-2	0.1184-2	0.7521-3	0.5070-3	0.3578-3	0.2618-3
100	0.1612-1	0.9717-2	0.6280-2	0.4282-2	0.3046-2	0.2242-2	0.1315-2	0.8354-3	0.5632-3	0.3975-3	0.2908-3
120	0.1914-1	0.1158-1	0.7501-2	0.5122-2	0.3647-2	0.2685-2	0.1576-2	0.1002-2	0.6756-3	0.4768-3	0.3489-3
140	0.2209-1	0.1342-1	0.8712-2	0.5957-2	0.4244-2	0.3128-2	0.1837-2	0.1168-2	0.7878-3	0.5561-3	0.4070-3
160	0.2500-1	0.1524-1	0.9912-2	0.6786-2	0.4840-2	0.3568-2	0.2097-2	0.1334-2	0.8999-3	0.6353-3	0.4650-3
180	0.2784-1	0.1703-1	0.1110-1	0.7610-2	0.5432-2	0.4007-2	0.2357-2	0.1500-2	0.1012-2	0.7145-3	0.5230-3
200	0.3064-1	0.1880-1	0.1228-1	0.8429-2	0.6022-2	0.4445-2	0.2616-2	0.1665-2	0.1124-2	0.7936-3	0.5810-3

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-21. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-4 Plan (PL=X_n)

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.1429-1	0.4762-2	0.2020-2	0.9990-3	0.5495-3	0.2064-3	0.9411-4	0.4210-4	0.2156-4	0.1216-4	0.7366-5
2	0.2655-1	0.9191-2	0.3963-2	0.1975-2	0.1091-2	0.4114-3	0.1879-3	0.8413-4	0.4311-4	0.2431-4	0.1473-4
3	0.3735-1	0.1334-1	0.5835-2	0.2930-2	0.1625-2	0.6151-3	0.2814-3	0.1261-3	0.6463-4	0.3645-4	0.2209-4
4	0.4701-1	0.1725-1	0.7645-2	0.3865-2	0.2152-2	0.8176-3	0.3745-3	0.1680-3	0.8613-4	0.4859-4	0.2945-4
5	0.5578-1	0.2096-1	0.9397-2	0.4781-2	0.2671-2	0.1019-2	0.4674-3	0.2098-3	0.1076-3	0.6072-4	0.3681-4
8	0.7815-1	0.3105-1	0.1435-1	0.7427-2	0.4192-2	0.1615-2	0.7441-3	0.3348-3	0.1719-3	0.9707-4	0.5886-4
10	0.9067-1	0.3710-1	0.1744-1	0.9115-2	0.5176-2	0.2006-2	0.9271-3	0.4179-3	0.2147-3	0.1213-3	0.7355-4
12	0.1018	0.4271-1	0.2038-1	0.1075-1	0.6137-2	0.2394-2	0.1109-2	0.5006-3	0.2574-3	0.1454-3	0.8822-4
15	0.1165	0.5045-1	0.2454-1	0.1310-1	0.7542-2	0.2966-2	0.1380-2	0.6242-3	0.3213-3	0.1817-3	0.1102-3
20	0.1372	0.6193-1	0.3095-1	0.1682-1	0.9790-2	0.3901-2	0.1826-2	0.8290-3	0.4275-3	0.2419-3	0.1468-3
25	0.1545	0.7203-1	0.3680-1	0.2030-1	0.1194-1	0.4812-2	0.2265-2	0.1032-2	0.5331-3	0.3019-3	0.1834-3
30	0.1693	0.8109-1	0.4221-1	0.2358-1	0.1399-1	0.5702-2	0.2698-2	0.1234-2	0.6383-3	0.3618-3	0.2199-3
35	0.1823	0.8931-1	0.4724-1	0.2670-1	0.1597-1	0.6571-2	0.3126-2	0.1434-2	0.7431-3	0.4215-3	0.2563-3
40	0.1939	0.9684-1	0.5195-1	0.2966-1	0.1787-1	0.7423-2	0.3548-2	0.1633-2	0.8474-3	0.4811-3	0.2926-3
45	0.2043	0.1038	0.5639-1	0.3249-1	0.1971-1	0.8256-2	0.3965-2	0.1830-2	0.9512-3	0.5405-3	0.3289-3
50	0.2138	0.1103	0.6059-1	0.3521-1	0.2149-1	0.9074-2	0.4376-2	0.2026-2	0.1055-2	0.5998-3	0.3652-3
60	0.2306	0.1220	0.6837-1	0.4032-1	0.2489-1	0.1066-1	0.5186-2	0.2414-2	0.1260-2	0.7179-3	0.4375-3
70	0.2451	0.1325	0.7547-1	0.4508-1	0.2810-1	0.1220-1	0.5977-2	0.2796-2	0.1464-2	0.8353-3	0.5095-3
80	0.2578	0.1419	0.8201-1	0.4954-1	0.3115-1	0.1368-1	0.6751-2	0.3174-2	0.1667-2	0.9522-3	0.5813-3
90	0.2692	0.1505	0.8807-1	0.5373-1	0.3405-1	0.1512-1	0.7509-2	0.3548-2	0.1867-2	0.1069-2	0.6529-3
100	0.2794	0.1584	0.9374-1	0.5770-1	0.3683-1	0.1651-1	0.8253-2	0.3917-2	0.2067-2	0.1184-2	0.7242-3
120	0.2973	0.1725	0.1041	0.6506-1	0.4204-1	0.1919-1	0.9700-2	0.4642-2	0.2461-2	0.1414-2	0.8662-3
140	0.3125	0.1849	0.1133	0.7177-1	0.4688-1	0.2174-1	0.1110-1	0.5352-2	0.2850-2	0.1642-2	0.1007-2
160	0.3258	0.1959	0.1217	0.7796-1	0.5140-1	0.2416-1	0.1245-1	0.6047-2	0.3234-2	0.1868-2	0.1148-2
180	0.3375	0.2058	0.1294	0.8370-1	0.5564-1	0.2649-1	0.1376-1	0.6728-2	0.3612-2	0.2092-2	0.1287-2
200	0.3480	0.2148	0.1365	0.8907-1	0.5965-1	0.2872-1	0.1503-1	0.7396-2	0.3986-2	0.2313-2	0.1426-2

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-21. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-4 Plan (PL=X_n)

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.3162-5	0.1574-5	0.8691-6	0.5183-6	0.3279-6	0.2175-6	0.1066-6	0.5821-7	0.3442-7	0.2164-7	0.1427-7
2	0.6324-5	0.3148-5	0.1738-5	0.1037-5	0.6558-6	0.4350-6	0.2132-6	0.1164-6	0.6884-7	0.4327-7	0.2855-7
3	0.9485-5	0.4721-5	0.2607-5	0.1555-5	0.9837-6	0.6524-6	0.3198-6	0.1746-6	0.1033-6	0.6491-7	0.4282-7
4	0.1265-4	0.6295-5	0.3476-5	0.2073-5	0.1312-5	0.8699-6	0.4264-6	0.2328-6	0.1377-6	0.8655-7	0.5709-7
5	0.1581-4	0.7868-5	0.4345-5	0.2591-5	0.1640-5	0.1087-5	0.5330-6	0.2911-6	0.1721-6	0.1082-6	0.7137-7
8	0.2528-4	0.1259-4	0.6952-5	0.4146-5	0.2623-5	0.1740-5	0.8527-6	0.4657-6	0.2754-6	0.1731-6	0.1142-6
10	0.3160-4	0.1573-4	0.8689-5	0.5182-5	0.3279-5	0.2175-5	0.1066-5	0.5821-6	0.3442-6	0.2164-6	0.1427-6
12	0.3791-4	0.1888-4	0.1043-4	0.6218-5	0.3935-5	0.2610-5	0.1279-5	0.6985-6	0.4131-6	0.2596-6	0.1713-6
15	0.4738-4	0.2359-4	0.1303-4	0.7772-5	0.4918-5	0.3262-5	0.1599-5	0.8731-6	0.5163-6	0.3245-6	0.2141-6
20	0.6314-4	0.3145-4	0.1737-4	0.1036-4	0.6557-5	0.4349-5	0.2132-5	0.1164-5	0.6884-6	0.4327-6	0.2855-6
25	0.7890-4	0.3931-4	0.2171-4	0.1295-4	0.8196-5	0.5436-5	0.2665-5	0.1455-5	0.8605-6	0.5409-6	0.3568-6
30	0.9464-4	0.4716-4	0.2605-4	0.1554-4	0.9835-5	0.6523-5	0.3198-5	0.1746-5	0.1033-5	0.6491-6	0.4282-6
35	0.1104-3	0.5501-4	0.3039-4	0.1813-4	0.1147-4	0.7610-5	0.3730-5	0.2037-5	0.1205-5	0.7573-6	0.4996-6
40	0.1261-3	0.6285-4	0.3473-4	0.2072-4	0.1311-4	0.8697-5	0.4263-5	0.2328-5	0.1377-5	0.8654-6	0.5709-6
45	0.1418-3	0.7069-4	0.3907-4	0.2331-4	0.1475-4	0.9784-5	0.4796-5	0.2619-5	0.1549-5	0.9736-6	0.6423-6
50	0.1575-3	0.7853-4	0.4340-4	0.2589-4	0.1639-4	0.1087-4	0.5329-5	0.2910-5	0.1721-5	0.1082-5	0.7137-6
60	0.1888-3	0.9420-4	0.5207-4	0.3107-4	0.1966-4	0.1304-4	0.6394-5	0.3492-5	0.2065-5	0.1298-5	0.8564-6
70	0.2201-3	0.1098-3	0.6073-4	0.3624-4	0.2294-4	0.1522-4	0.7460-5	0.4074-5	0.2409-5	0.1515-5	0.9991-6
80	0.2514-3	0.1255-3	0.6939-4	0.4141-4	0.2621-4	0.1739-4	0.8525-5	0.4656-5	0.2754-5	0.1731-5	0.1142-5
90	0.2825-3	0.1411-3	0.7805-4	0.4658-4	0.2949-4	0.1956-4	0.9591-5	0.5238-5	0.3098-5	0.1947-5	0.1285-5
100	0.3137-3	0.1567-3	0.8670-4	0.5175-4	0.3276-4	0.2173-4	0.1066-4	0.5820-5	0.3442-5	0.2164-5	0.1427-5
120	0.3758-3	0.1879-3	0.1040-3	0.6208-4	0.3931-4	0.2608-4	0.1279-4	0.6984-5	0.4130-5	0.2596-5	0.1713-5
140	0.4378-3	0.2191-3	0.1213-3	0.7241-4	0.4585-4	0.3042-4	0.1492-4	0.8147-5	0.4818-5	0.3029-5	0.1998-5
160	0.4996-3	0.2501-3	0.1385-3	0.8273-4	0.5239-4	0.3476-4	0.1705-4	0.9311-5	0.5507-5	0.3462-5	0.2284-5
180	0.5612-3	0.2812-3	0.1558-3	0.9304-4	0.5892-4	0.3910-4	0.1918-4	0.1047-4	0.6195-5	0.3894-5	0.2569-5
200	0.6226-3	0.3122-3	0.1730-3	0.1033-3	0.6546-4	0.4344-4	0.2131-4	0.1164-4	0.6883-5	0.4327-5	0.2854-5

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-21. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-4 Plan (PL=X_{n-1})

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.7143-1	0.2381-1	0.1010-1	0.4995-2	0.2747-2	0.1032-2	0.4705-3	0.2105-3	0.1078-3	0.6079-4	0.3683-4
2	0.1247	0.4462-1	0.1950-1	0.9784-2	0.5423-2	0.2052-2	0.9382-3	0.4204-3	0.2154-3	0.1215-3	0.7364-4
3	0.1669	0.6314-1	0.2831-1	0.1439-1	0.8032-2	0.3060-2	0.1403-2	0.6296-3	0.3229-3	0.1822-3	0.1104-3
4	0.2017	0.7983-1	0.3660-1	0.1882-1	0.1058-1	0.4056-2	0.1865-2	0.8382-3	0.4302-3	0.2428-3	0.1472-3
5	0.2312	0.9503-1	0.4444-1	0.2310-1	0.1307-1	0.5042-2	0.2324-2	0.1046-2	0.5372-3	0.3033-3	0.1839-3
8	0.2991	0.1339	0.6571-1	0.3514-1	0.2021-1	0.7935-2	0.3686-2	0.1666-2	0.8574-3	0.4846-3	0.2940-3
10	0.3333	0.1557	0.7837-1	0.4258-1	0.2474-1	0.9815-2	0.4582-2	0.2077-2	0.1070-2	0.6051-3	0.3673-3
12	0.3619	0.1750	0.9006-1	0.4962-1	0.2909-1	0.1166-1	0.5467-2	0.2485-2	0.1282-2	0.7254-3	0.4404-3
15	0.3972	0.2004	0.1061	0.5954-1	0.3534-1	0.1436-1	0.6778-2	0.3093-2	0.1598-2	0.9054-3	0.5500-3
20	0.4427	0.2357	0.1296	0.7463-1	0.4508-1	0.1870-1	0.8917-2	0.4094-2	0.2123-2	0.1204-2	0.7322-3
25	0.4776	0.2649	0.1500	0.8825-1	0.5411-1	0.2286-1	0.1100-1	0.5083-2	0.2643-2	0.1502-2	0.9138-3
30	0.5057	0.2897	0.1681	0.1007	0.6255-1	0.2686-1	0.1304-1	0.6058-2	0.3159-2	0.1798-2	0.1095-2
35	0.5289	0.3112	0.1843	0.1121	0.7048-1	0.3071-1	0.1504-1	0.7021-2	0.3671-2	0.2092-2	0.1275-2
40	0.5487	0.3301	0.1991	0.1228	0.7795-1	0.3443-1	0.1699-1	0.7973-2	0.4179-2	0.2385-2	0.1455-2
45	0.5658	0.3470	0.2125	0.1327	0.8504-1	0.3803-1	0.1889-1	0.8912-2	0.4684-2	0.2677-2	0.1635-2
50	0.5807	0.3623	0.2250	0.1420	0.9177-1	0.4152-1	0.2077-1	0.9841-2	0.5185-2	0.2968-2	0.1814-2
60	0.6060	0.3888	0.2472	0.1590	0.1043	0.4818-1	0.2440-1	0.1167-1	0.6177-2	0.3545-2	0.2170-2
70	0.6266	0.4114	0.2667	0.1743	0.1158	0.5449-1	0.2791-1	0.1345-1	0.7155-2	0.4117-2	0.2524-2
80	0.6439	0.4309	0.2840	0.1882	0.1265	0.6048-1	0.3130-1	0.1520-1	0.8120-2	0.4684-2	0.2876-2
90	0.6587	0.4480	0.2996	0.2009	0.1364	0.6618-1	0.3457-1	0.1692-1	0.9073-2	0.5247-2	0.3226-2
100	0.6715	0.4633	0.3137	0.2127	0.1457	0.7163-1	0.3775-1	0.1860-1	0.1001-1	0.5805-2	0.3574-2
120	0.6930	0.4895	0.3386	0.2338	0.1626	0.8187-1	0.4384-1	0.2187-1	0.1186-1	0.6907-2	0.4265-2
140	0.7103	0.5113	0.3599	0.2523	0.1778	0.9135-1	0.4960-1	0.2502-1	0.1367-1	0.7993-2	0.4948-2
160	0.7247	0.5300	0.3785	0.2688	0.1916	0.1002	0.5508-1	0.2807-1	0.1543-1	0.9062-2	0.5624-2
180	0.7370	0.5462	0.3950	0.2837	0.2042	0.1085	0.6031-1	0.3103-1	0.1716-1	0.1012-1	0.6293-2
200	0.7476	0.5605	0.4097	0.2972	0.2158	0.1163	0.6531-1	0.3390-1	0.1886-1	0.1115-1	0.6956-2

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-21. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-4 Plan (PL=X_{n-1})

w\n	50	60	70	80	90	100	120	140	160	180	200
1	**0.1581-4**	*0.7869-5*	*0.4345-5*	*0.2591-5*	*0.1640-5*	*0.1087-5*	*0.5330-6*	*0.2911-6*	0.1721-6	0.1082-6	0.7137-7
2	**0.3162-4**	*0.1574-4*	*0.8691-5*	*0.5183-5*	*0.3279-5*	*0.2175-5*	*0.1066-5*	*0.5821-6*	0.3442-6	0.2164-6	0.1427-6
3	**0.4742-4**	*0.2360-4*	*0.1304-4*	*0.7774-5*	*0.4919-5*	*0.3262-5*	*0.1599-5*	*0.8732-6*	0.5163-6	0.3246-6	0.2141-6
4	**0.6321-4**	*0.3147-4*	*0.1738-4*	*0.1036-4*	*0.6558-5*	*0.4349-5*	*0.2132-5*	*0.1164-5*	0.6884-6	0.4327-6	0.2855-6
5	**0.7900-4**	*0.3933-4*	*0.2172-4*	*0.1296-4*	*0.8197-5*	*0.5437-5*	*0.2665-5*	*0.1455-5*	0.8605-6	0.5409-6	0.3568-6
8	**0.1264-3**	*0.6292-4*	*0.3475-4*	*0.2073-4*	*0.1312-4*	*0.8698-5*	*0.4264-5*	*0.2328-5*	0.1377-5	0.8655-6	0.5709-6
10	**0.1579-3**	*0.7864-4*	*0.4344-4*	*0.2591-4*	*0.1639-4*	*0.1087-4*	*0.5329-5*	*0.2910-5*	0.1721-5	0.1082-5	0.7137-6
12	**0.1894-3**	*0.9435-4*	*0.5212-4*	*0.3109-4*	*0.1967-4*	*0.1305-4*	*0.6395-5*	*0.3493-5*	0.2065-5	0.1298-5	0.8564-6
15	**0.2367-3**	*0.1179-3*	*0.6514-4*	*0.3886-4*	*0.2459-4*	*0.1631-4*	*0.7994-5*	*0.4366-5*	0.2582-5	0.1623-5	0.1071-5
20	**0.3153-3**	*0.1572-3*	*0.8684-4*	*0.5180-4*	*0.3278-4*	*0.2174-4*	*0.1066-4*	*0.5821-5*	0.3442-5	0.2164-5	0.1427-5
25	**0.3939-3**	*0.1964-3*	*0.1085-3*	*0.6474-4*	*0.4097-4*	*0.2718-4*	*0.1332-4*	*0.7276-5*	0.4303-5	0.2705-5	0.1784-5
30	**0.4723-3**	*0.2356-3*	*0.1302-3*	*0.7768-4*	*0.4916-4*	*0.3261-4*	*0.1599-4*	*0.8731-5*	0.5163-5	0.3245-5	0.2141-5
35	**0.5506-3**	*0.2747-3*	*0.1519-3*	*0.9061-4*	*0.5735-4*	*0.3804-4*	*0.1865-4*	*0.1019-4*	0.6023-5	0.3786-5	0.2498-5
40	**0.6288-3**	*0.3138-3*	*0.1735-3*	*0.1035-3*	*0.6554-4*	*0.4348-4*	*0.2131-4*	*0.1164-4*	0.6884-5	0.4327-5	0.2855-5
45	**0.7069-3**	*0.3529-3*	*0.1952-3*	*0.1165-3*	*0.7373-4*	*0.4891-4*	*0.2398-4*	*0.1310-4*	0.7744-5	0.4868-5	0.3211-5
50	**0.7849-3**	*0.3920-3*	*0.2168-3*	*0.1294-3*	*0.8191-4*	*0.5434-4*	*0.2664-4*	*0.1455-4*	0.8605-5	0.5409-5	0.3568-5
60	**0.9405-3**	*0.4700-3*	*0.2601-3*	*0.1552-3*	*0.9828-4*	*0.6520-4*	*0.3197-4*	*0.1746-4*	0.1033-4	0.6491-5	0.4282-5
70	**0.1096-2**	*0.5480-3*	*0.3033-3*	*0.1811-3*	*0.1146-3*	*0.7606-4*	*0.3729-4*	*0.2037-4*	0.1205-4	0.7572-5	0.4996-5
80	**0.1250-2**	*0.6258-3*	*0.3464-3*	*0.2069-3*	*0.1310-3*	*0.8691-4*	*0.4262-4*	*0.2328-4*	0.1377-4	0.8654-5	0.5709-5
90	**0.1405-2**	*0.7035-3*	*0.3896-3*	*0.2327-3*	*0.1473-3*	*0.9776-4*	*0.4794-4*	*0.2619-4*	0.1549-4	0.9735-5	0.6423-5
100	**0.1559-2**	*0.7810-3*	*0.4327-3*	*0.2584-3*	*0.1637-3*	*0.1086-3*	*0.5327-4*	*0.2910-4*	0.1721-4	0.1082-4	0.7136-5
120	**0.1865-2**	*0.9359-3*	*0.5188-3*	*0.3100-3*	*0.1963-3*	*0.1303-3*	*0.6391-4*	*0.3491-4*	0.2065-4	0.1298-4	0.8563-5
140	**0.2170-2**	*0.1090-2*	*0.6047-3*	*0.3614-3*	*0.2290-3*	*0.1520-3*	*0.7456-4*	*0.4073-4*	0.2409-4	0.1514-4	0.9990-5
160	**0.2474-2**	*0.1244-2*	*0.6905-3*	*0.4129-3*	*0.2616-3*	*0.1737-3*	*0.8520-4*	*0.4654-4*	0.2753-4	0.1731-4	0.1142-4
180	**0.2775-2**	*0.1398-2*	*0.7761-3*	*0.4642-3*	*0.2942-3*	*0.1953-3*	*0.9583-4*	*0.5236-4*	0.3097-4	0.1947-4	0.1284-4
200	**0.3075-2**	*0.1551-2*	*0.8616-3*	*0.5155-3*	*0.3268-3*	*0.2170-3*	*0.1065-3*	*0.5817-4*	0.3441-4	0.2163-4	0.1427-4

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-22. Per-Constituent Significance Levels (α) for Non-Parametric Modified California Plan (PL=X_n)

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.5714-1	0.2619-1	0.1414-1	0.8492-2	0.5495-2	0.2683-2	0.1506-2	0.8315-3	0.5067-3	0.3313-3	0.2284-3
2	0.9971-1	0.4830-1	0.2684-1	0.1638-1	0.1071-1	0.5289-2	0.2985-2	0.1654-2	0.1010-2	0.6610-3	0.4559-3
3	0.1335	0.6746-1	0.3838-1	0.2377-1	0.1568-1	0.7824-2	0.4438-2	0.2468-2	0.1510-2	0.9892-3	0.6827-3
4	0.1614	0.8438-1	0.4898-1	0.3072-1	0.2043-1	0.1029-1	0.5867-2	0.3274-2	0.2006-2	0.1316-2	0.9088-3
5	0.1852	0.9954-1	0.5879-1	0.3728-1	0.2498-1	0.1270-1	0.7272-2	0.4071-2	0.2499-2	0.1641-2	0.1134-2
8	0.2402	0.1374	0.8449-1	0.5507-1	0.3762-1	0.1958-1	0.1136-1	0.6416-2	0.3959-2	0.2607-2	0.1805-2
10	0.2682	0.1580	0.9925-1	0.6565-1	0.4532-1	0.2391-1	0.1398-1	0.7943-2	0.4916-2	0.3244-2	0.2249-2
12	0.2917	0.1761	0.1126	0.7541-1	0.5255-1	0.2807-1	0.1653-1	0.9442-2	0.5862-2	0.3876-2	0.2691-2
15	0.3211	0.1996	0.1304	0.8878-1	0.6263-1	0.3401-1	0.2023-1	0.1164-1	0.7260-2	0.4813-2	0.3347-2
20	0.3593	0.2319	0.1558	0.1084	0.7777-1	0.4323-1	0.2610-1	0.1519-1	0.9535-2	0.6349-2	0.4428-2
25	0.3891	0.2581	0.1772	0.1255	0.9125-1	0.5173-1	0.3163-1	0.1859-1	0.1175-1	0.7854-2	0.5492-2
30	0.4133	0.2802	0.1958	0.1406	0.1034	0.5962-1	0.3687-1	0.2187-1	0.1390-1	0.9330-2	0.6542-2
35	0.4336	0.2992	0.2121	0.1541	0.1145	0.6701-1	0.4186-1	0.2504-1	0.1600-1	0.1078-1	0.7576-2
40	0.4510	0.3158	0.2267	0.1665	0.1247	0.7395-1	0.4662-1	0.2811-1	0.1805-1	0.1220-1	0.8596-2
45	0.4662	0.3307	0.2399	0.1778	0.1342	0.8051-1	0.5118-1	0.3107-1	0.2006-1	0.1360-1	0.9603-2
50	0.4796	0.3440	0.2520	0.1882	0.1431	0.8672-1	0.5555-1	0.3395-1	0.2201-1	0.1497-1	0.1060-1
60	0.5025	0.3671	0.2732	0.2069	0.1591	0.9825-1	0.6380-1	0.3947-1	0.2580-1	0.1766-1	0.1255-1
70	0.5214	0.3867	0.2916	0.2233	0.1734	0.1088	0.7148-1	0.4470-1	0.2944-1	0.2025-1	0.1445-1
80	0.5374	0.4036	0.3077	0.2379	0.1863	0.1185	0.7868-1	0.4968-1	0.3295-1	0.2278-1	0.1631-1
90	0.5514	0.4185	0.3221	0.2510	0.1981	0.1275	0.8545-1	0.5442-1	0.3633-1	0.2523-1	0.1812-1
100	0.5636	0.4318	0.3350	0.2630	0.2089	0.1359	0.9185-1	0.5897-1	0.3959-1	0.2762-1	0.1990-1
120	0.5842	0.4545	0.3575	0.2841	0.2281	0.1512	0.1037	0.6752-1	0.4581-1	0.3221-1	0.2335-1
140	0.6011	0.4734	0.3765	0.3023	0.2449	0.1649	0.1145	0.7545-1	0.5167-1	0.3658-1	0.2667-1
160	0.6154	0.4896	0.3931	0.3183	0.2598	0.1772	0.1244	0.8286-1	0.5721-1	0.4077-1	0.2986-1
180	0.6277	0.5038	0.4076	0.3324	0.2732	0.1885	0.1335	0.8981-1	0.6248-1	0.4478-1	0.3295-1
200	0.6384	0.5162	0.4206	0.3452	0.2853	0.1988	0.1421	0.9637-1	0.6749-1	0.4863-1	0.3594-1

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-22. Per-Constituent Significance Levels (α) for Non-Parametric Modified California Plan (PL=X_n)

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.1217-3	0.7240-4	0.4650-4	0.3161-4	0.2246-4	0.1653-4	0.9700-5	0.6170-5	0.4165-5	0.2943-5	0.2155-5
2	0.2432-3	0.1447-3	0.9296-4	0.6321-4	0.4492-4	0.3305-4	0.1940-4	0.1234-4	0.8330-5	0.5885-5	0.4311-5
3	0.3645-3	0.2169-3	0.1394-3	0.9479-4	0.6736-4	0.4957-4	0.2910-4	0.1851-4	0.1249-4	0.8827-5	0.6466-5
4	0.4856-3	0.2891-3	0.1858-3	0.1264-3	0.8980-4	0.6608-4	0.3879-4	0.2468-4	0.1666-4	0.1177-4	0.8621-5
5	0.6064-3	0.3611-3	0.2321-3	0.1579-3	0.1122-3	0.8259-4	0.4848-4	0.3084-4	0.2082-4	0.1471-4	0.1078-4
8	0.9674-3	0.5768-3	0.3710-3	0.2524-3	0.1795-3	0.1321-3	0.7755-4	0.4934-4	0.3331-4	0.2354-4	0.1724-4
10	0.1207-2	0.7202-3	0.4633-3	0.3154-3	0.2242-3	0.1651-3	0.9693-4	0.6167-4	0.4164-4	0.2942-4	0.2155-4
12	0.1446-2	0.8632-3	0.5556-3	0.3782-3	0.2690-3	0.1980-3	0.1163-3	0.7400-4	0.4996-4	0.3530-4	0.2586-4
15	0.1802-2	0.1077-2	0.6937-3	0.4724-3	0.3360-3	0.2474-3	0.1453-3	0.9248-4	0.6244-4	0.4412-4	0.3232-4
20	0.2392-2	0.1432-2	0.9231-3	0.6291-3	0.4476-3	0.3297-3	0.1937-3	0.1233-3	0.8324-4	0.5882-4	0.4309-4
25	0.2977-2	0.1785-2	0.1152-2	0.7853-3	0.5590-3	0.4118-3	0.2420-3	0.1541-3	0.1040-3	0.7352-4	0.5386-4
30	0.3556-2	0.2136-2	0.1379-2	0.9411-3	0.6701-3	0.4938-3	0.2903-3	0.1848-3	0.1248-3	0.8821-4	0.6462-4
35	0.4131-2	0.2485-2	0.1606-2	0.1097-2	0.7810-3	0.5757-3	0.3385-3	0.2156-3	0.1456-3	0.1029-3	0.7538-4
40	0.4700-2	0.2832-2	0.1832-2	0.1252-2	0.8918-3	0.6574-3	0.3867-3	0.2463-3	0.1664-3	0.1176-3	0.8615-4
45	0.5266-2	0.3178-2	0.2058-2	0.1406-2	0.1002-2	0.7391-3	0.4348-3	0.2770-3	0.1871-3	0.1323-3	0.9690-4
50	0.5826-2	0.3521-2	0.2282-2	0.1560-2	0.1113-2	0.8206-3	0.4830-3	0.3077-3	0.2079-3	0.1469-3	0.1077-3
60	0.6934-2	0.4203-2	0.2729-2	0.1868-2	0.1333-2	0.9833-3	0.5790-3	0.3690-3	0.2493-3	0.1763-3	0.1292-3
70	0.8025-2	0.4878-2	0.3172-2	0.2173-2	0.1552-2	0.1146-2	0.6750-3	0.4303-3	0.2908-3	0.2056-3	0.1507-3
80	0.9100-2	0.5546-2	0.3612-2	0.2478-2	0.1770-2	0.1307-2	0.7707-3	0.4915-3	0.3322-3	0.2349-3	0.1722-3
90	0.1016-1	0.6207-2	0.4050-2	0.2780-2	0.1988-2	0.1469-2	0.8664-3	0.5526-3	0.3736-3	0.2642-3	0.1936-3
100	0.1120-1	0.6863-2	0.4484-2	0.3081-2	0.2205-2	0.1630-2	0.9618-3	0.6136-3	0.4149-3	0.2935-3	0.2151-3
120	0.1325-1	0.8155-2	0.5344-2	0.3680-2	0.2636-2	0.1950-2	0.1152-2	0.7356-3	0.4975-3	0.3520-3	0.2580-3
140	0.1524-1	0.9424-2	0.6194-2	0.4272-2	0.3064-2	0.2269-2	0.1342-2	0.8572-3	0.5800-3	0.4104-3	0.3009-3
160	0.1719-1	0.1067-1	0.7032-2	0.4860-2	0.3490-2	0.2586-2	0.1531-2	0.9786-3	0.6624-3	0.4688-3	0.3438-3
180	0.1909-1	0.1190-1	0.7861-2	0.5442-2	0.3912-2	0.2902-2	0.1720-2	0.1100-2	0.7446-3	0.5271-3	0.3866-3
200	0.2094-1	0.1310-1	0.8680-2	0.6018-2	0.4332-2	0.3216-2	0.1908-2	0.1221-2	0.8268-3	0.5854-3	0.4293-3

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-22. Per-Constituent Significance Levels (α) for Non-Parametric Modified California Plan (PL=X_{n-1})

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.2000	0.9524-1	0.5253-1	0.3197-1	0.2088-1	0.1032-1	0.5835-2	0.3242-2	0.1984-2	0.1301-2	0.8987-3
2	0.3182	0.1663	0.9619-1	0.6018-1	0.3998-1	0.2014-1	0.1149-1	0.6424-2	0.3944-2	0.2591-2	0.1792-2
3	0.3981	0.2221	0.1334	0.8541-1	0.5760-1	0.2950-1	0.1698-1	0.9550-2	0.5881-2	0.3871-2	0.2680-2
4	0.4567	0.2677	0.1658	0.1082	0.7393-1	0.3846-1	0.2232-1	0.1262-1	0.7795-2	0.5140-2	0.3563-2
5	0.5019	0.3059	0.1943	0.1290	0.8916-1	0.4705-1	0.2751-1	0.1564-1	0.9688-2	0.6400-2	0.4441-2
8	0.5932	0.3920	0.2635	0.1821	0.1295	0.7085-1	0.4230-1	0.2440-1	0.1524-1	0.1012-1	0.7046-2
10	0.6335	0.4343	0.3000	0.2116	0.1528	0.8534-1	0.5157-1	0.3001-1	0.1885-1	0.1255-1	0.8758-2
12	0.6647	0.4689	0.3313	0.2377	0.1739	0.9889-1	0.6042-1	0.3545-1	0.2238-1	0.1495-1	0.1045-1
15	0.7004	0.5107	0.3707	0.2717	0.2022	0.1177	0.7300-1	0.4333-1	0.2754-1	0.1848-1	0.1296-1
20	0.7422	0.5632	0.4228	0.3185	0.2424	0.1458	0.9234-1	0.5576-1	0.3582-1	0.2421-1	0.1705-1
25	0.7715	0.6022	0.4635	0.3566	0.2761	0.1705	0.1100	0.6742-1	0.4373-1	0.2974-1	0.2104-1
30	0.7934	0.6327	0.4964	0.3884	0.3051	0.1926	0.1262	0.7840-1	0.5131-1	0.3510-1	0.2494-1
35	0.8105	0.6574	0.5239	0.4157	0.3304	0.2126	0.1412	0.8880-1	0.5858-1	0.4030-1	0.2874-1
40	0.8244	0.6780	0.5474	0.4393	0.3528	0.2308	0.1552	0.9866-1	0.6558-1	0.4535-1	0.3246-1
45	0.8358	0.6954	0.5677	0.4602	0.3729	0.2475	0.1683	0.1080	0.7232-1	0.5026-1	0.3610-1
50	0.8455	0.7104	0.5855	0.4789	0.3910	0.2630	0.1806	0.1170	0.7882-1	0.5504-1	0.3967-1
60	0.8611	0.7352	0.6155	0.5108	0.4227	0.2907	0.2032	0.1338	0.9119-1	0.6422-1	0.4658-1
70	0.8732	0.7549	0.6400	0.5374	0.4496	0.3150	0.2234	0.1492	0.1028	0.7296-1	0.5322-1
80	0.8829	0.7710	0.6604	0.5600	0.4728	0.3366	0.2419	0.1636	0.1138	0.8130-1	0.5962-1
90	0.8909	0.7845	0.6778	0.5796	0.4931	0.3559	0.2587	0.1770	0.1241	0.8928-1	0.6580-1
100	0.8976	0.7961	0.6929	0.5968	0.5112	0.3735	0.2742	0.1895	0.1339	0.9693-1	0.7177-1
120	0.9083	0.8149	0.7179	0.6257	0.5421	0.4042	0.3020	0.2124	0.1522	0.1114	0.8315-1
140	0.9165	0.8297	0.7379	0.6492	0.5676	0.4304	0.3262	0.2329	0.1689	0.1248	0.9386-1
160	0.9231	0.8417	0.7543	0.6689	0.5893	0.4531	0.3477	0.2514	0.1843	0.1373	0.1040
180	0.9285	0.8516	0.7682	0.6857	0.6079	0.4731	0.3669	0.2684	0.1985	0.1490	0.1136
200	0.9330	0.8601	0.7801	0.7002	0.6243	0.4909	0.3843	0.2839	0.2118	0.1601	0.1227

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-22. Per-Constituent Significance Levels (α) for Non-Parametric Modified California Plan (PL=X_{n-1})

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.4806-3	0.2864-3	0.1842-3	0.1254-3	0.8919-4	0.6568-4	0.3859-4	0.2457-4	0.1659-4	0.1173-4	0.8593-5
2	0.9597-3	0.5723-3	0.3682-3	0.2507-3	0.1783-3	0.1313-3	0.7716-4	0.4913-4	0.3318-4	0.2345-4	0.1718-4
3	0.1437-2	0.8576-3	0.5520-3	0.3759-3	0.2674-3	0.1969-3	0.1157-3	0.7368-4	0.4977-4	0.3518-4	0.2578-4
4	0.1913-2	0.1142-2	0.7355-3	0.5010-3	0.3564-3	0.2625-3	0.1543-3	0.9823-4	0.6635-4	0.4690-4	0.3437-4
5	0.2387-2	0.1426-2	0.9187-3	0.6259-3	0.4454-3	0.3281-3	0.1928-3	0.1228-3	0.8294-4	0.5862-4	0.4296-4
8	0.3801-2	0.2275-2	0.1467-2	0.1000-2	0.7119-3	0.5245-3	0.3084-3	0.1964-3	0.1327-3	0.9379-4	0.6873-4
10	0.4736-2	0.2838-2	0.1831-2	0.1249-2	0.8892-3	0.6553-3	0.3853-3	0.2454-3	0.1658-3	0.1172-3	0.8590-4
12	0.5665-2	0.3399-2	0.2195-2	0.1497-2	0.1066-2	0.7860-3	0.4623-3	0.2945-3	0.1990-3	0.1407-3	0.1031-3
15	0.7048-2	0.4236-2	0.2738-2	0.1869-2	0.1332-2	0.9817-3	0.5776-3	0.3680-3	0.2486-3	0.1758-3	0.1288-3
20	0.9325-2	0.5621-2	0.3639-2	0.2486-2	0.1773-2	0.1307-2	0.7695-3	0.4904-3	0.3314-3	0.2343-3	0.1717-3
25	0.1157-1	0.6992-2	0.4533-2	0.3101-2	0.2212-2	0.1632-2	0.9612-3	0.6127-3	0.4141-3	0.2928-3	0.2146-3
30	0.1378-1	0.8351-2	0.5423-2	0.3713-2	0.2650-2	0.1956-2	0.1153-2	0.7349-3	0.4968-3	0.3513-3	0.2575-3
35	0.1596-1	0.9696-2	0.6306-2	0.4322-2	0.3086-2	0.2279-2	0.1344-2	0.8569-3	0.5794-3	0.4098-3	0.3004-3
40	0.1810-1	0.1103-1	0.7184-2	0.4928-2	0.3522-2	0.2602-2	0.1534-2	0.9789-3	0.6619-3	0.4682-3	0.3432-3
45	0.2022-1	0.1235-1	0.8057-2	0.5531-2	0.3955-2	0.2923-2	0.1725-2	0.1101-2	0.7444-3	0.5266-3	0.3861-3
50	0.2231-1	0.1366-1	0.8924-2	0.6132-2	0.4388-2	0.3244-2	0.1915-2	0.1222-2	0.8269-3	0.5850-3	0.4289-3
60	0.2642-1	0.1625-1	0.1064-1	0.7327-2	0.5248-2	0.3883-2	0.2295-2	0.1465-2	0.9916-3	0.7017-3	0.5145-3
70	0.3041-1	0.1879-1	0.1234-1	0.8511-2	0.6103-2	0.4520-2	0.2673-2	0.1708-2	0.1156-2	0.8182-3	0.6000-3
80	0.3432-1	0.2129-1	0.1402-1	0.9685-2	0.6953-2	0.5153-2	0.3051-2	0.1950-2	0.1320-2	0.9347-3	0.6855-3
90	0.3813-1	0.2375-1	0.1568-1	0.1085-1	0.7798-2	0.5783-2	0.3427-2	0.2192-2	0.1484-2	0.1051-2	0.7709-3
100	0.4185-1	0.2617-1	0.1732-1	0.1200-1	0.8637-2	0.6411-2	0.3802-2	0.2433-2	0.1648-2	0.1167-2	0.8563-3
120	0.4906-1	0.3091-1	0.2055-1	0.1429-1	0.1030-1	0.7657-2	0.4550-2	0.2914-2	0.1975-2	0.1399-2	0.1027-2
140	0.5598-1	0.3551-1	0.2372-1	0.1653-1	0.1195-1	0.8891-2	0.5293-2	0.3393-2	0.2302-2	0.1631-2	0.1197-2
160	0.6262-1	0.3998-1	0.2681-1	0.1875-1	0.1357-1	0.1012-1	0.6032-2	0.3871-2	0.2627-2	0.1862-2	0.1367-2
180	0.6902-1	0.4433-1	0.2985-1	0.2093-1	0.1518-1	0.1133-1	0.6766-2	0.4347-2	0.2951-2	0.2093-2	0.1537-2
200	0.7520-1	0.4856-1	0.3283-1	0.2308-1	0.1677-1	0.1253-1	0.7497-2	0.4821-2	0.3275-2	0.2324-2	0.1707-2

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

D.4.2 PLANS ON MEDIANS OF ORDER 3

Table 19-23. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-1 Plan for Median (PL=X_n)

w/n	4	6	8	10	12	16	20	25	30	35	40
1	0.1429	0.8333-1	0.5455-1	0.3846-1	0.2857-1	0.1754-1	0.1186-1	0.7937-2	0.5682-2	0.4267-2	0.3322-2
2	0.2333	0.1455	0.9890-1	0.7143-1	0.5392-1	0.3377-1	0.2308-1	0.1557-1	0.1120-1	0.8443-2	0.6588-2
3	0.2979	0.1945	0.1361	0.1002	0.7669-1	0.4885-1	0.3372-1	0.2293-1	0.1658-1	0.1253-1	0.9798-2
4	0.3473	0.2347	0.1681	0.1258	0.9735-1	0.6295-1	0.4386-1	0.3003-1	0.2181-1	0.1653-1	0.1296-1
5	0.3867	0.2686	0.1961	0.1487	0.1162	0.7619-1	0.5353-1	0.3690-1	0.2691-1	0.2046-1	0.1606-1
8	0.4704	0.3458	0.2630	0.2057	0.1647	0.1116	0.8012-1	0.5621-1	0.4147-1	0.3178-1	0.2509-1
10	0.5094	0.3841	0.2980	0.2367	0.1918	0.1323	0.9614-1	0.6815-1	0.5062-1	0.3898-1	0.3089-1
12	0.5404	0.4157	0.3277	0.2636	0.2158	0.1512	0.1110	0.7943-1	0.5937-1	0.4593-1	0.3652-1
15	0.5770	0.4544	0.3650	0.2982	0.2473	0.1767	0.1315	0.9527-1	0.7184-1	0.5592-1	0.4467-1
20	0.6218	0.5035	0.4139	0.3448	0.2907	0.2132	0.1618	0.1192	0.9105-1	0.7154-1	0.5756-1
25	0.6543	0.5405	0.4520	0.3821	0.3262	0.2441	0.1882	0.1407	0.1086	0.8604-1	0.6965-1
30	0.6794	0.5698	0.4828	0.4128	0.3560	0.2708	0.2115	0.1601	0.1248	0.9956-1	0.8106-1
35	0.6996	0.5938	0.5084	0.4388	0.3816	0.2943	0.2324	0.1779	0.1397	0.1122	0.9185-1
40	0.7162	0.6139	0.5303	0.4613	0.4039	0.3153	0.2513	0.1942	0.1537	0.1242	0.1021
45	0.7303	0.6312	0.5492	0.4809	0.4236	0.3341	0.2685	0.2092	0.1667	0.1354	0.1118
50	0.7424	0.6462	0.5658	0.4984	0.4413	0.3511	0.2844	0.2232	0.1790	0.1461	0.1211
60	0.7623	0.6711	0.5939	0.5280	0.4716	0.3810	0.3125	0.2486	0.2014	0.1658	0.1385
70	0.7781	0.6912	0.6167	0.5526	0.4970	0.4065	0.3369	0.2710	0.2216	0.1838	0.1545
80	0.7911	0.7079	0.6359	0.5733	0.5187	0.4286	0.3585	0.2910	0.2398	0.2003	0.1693
90	0.8019	0.7220	0.6523	0.5912	0.5375	0.4481	0.3777	0.3091	0.2566	0.2155	0.1831
100	0.8112	0.7342	0.6665	0.6068	0.5540	0.4655	0.3949	0.3256	0.2719	0.2296	0.1959
120	0.8264	0.7542	0.6902	0.6330	0.5820	0.4952	0.4249	0.3547	0.2993	0.2551	0.2194
140	0.8383	0.7702	0.7092	0.6543	0.6049	0.5200	0.4503	0.3797	0.3232	0.2776	0.2404
160	0.8481	0.7833	0.7249	0.6721	0.6242	0.5411	0.4721	0.4015	0.3443	0.2977	0.2593
180	0.8562	0.7944	0.7382	0.6872	0.6407	0.5594	0.4912	0.4207	0.3632	0.3158	0.2764
200	0.8632	0.8039	0.7497	0.7003	0.6550	0.5754	0.5081	0.4380	0.3802	0.3323	0.2922

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-23. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-1 Plan for Median (PL=X_n)

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.2177-2	0.1536-2	0.1142-2	0.8816-3	0.7013-3	0.5711-3	0.3998-3	0.2955-3	0.2272-3	0.1801-3	0.1463-3
2	0.4329-2	0.3059-2	0.2276-2	0.1759-2	0.1400-2	0.1140-2	0.7988-3	0.5905-3	0.4541-3	0.3601-3	0.2925-3
3	0.6456-2	0.4570-2	0.3403-2	0.2632-2	0.2096-2	0.1708-2	0.1197-2	0.8849-3	0.6808-3	0.5399-3	0.4386-3
4	0.8560-2	0.6069-2	0.4524-2	0.3501-2	0.2789-2	0.2273-2	0.1594-2	0.1179-2	0.9071-3	0.7195-3	0.5845-3
5	0.1064-1	0.7555-2	0.5637-2	0.4365-2	0.3479-2	0.2837-2	0.1990-2	0.1472-2	0.1133-2	0.8989-3	0.7304-3
8	0.1675-1	0.1194-1	0.8938-2	0.6935-2	0.5535-2	0.4518-2	0.3174-2	0.2350-2	0.1809-2	0.1436-2	0.1167-2
10	0.2071-1	0.1481-1	0.1111-1	0.8628-2	0.6892-2	0.5630-2	0.3958-2	0.2933-2	0.2259-2	0.1793-2	0.1458-2
12	0.2460-1	0.1764-1	0.1325-1	0.1030-1	0.8239-2	0.6735-2	0.4739-2	0.3513-2	0.2707-2	0.2149-2	0.1748-2
15	0.3028-1	0.2180-1	0.1642-1	0.1279-1	0.1024-1	0.8380-2	0.5904-2	0.4381-2	0.3378-2	0.2683-2	0.2182-2
20	0.3939-1	0.2854-1	0.2158-1	0.1686-1	0.1353-1	0.1109-1	0.7829-2	0.5817-2	0.4489-2	0.3568-2	0.2903-2
25	0.4809-1	0.3505-1	0.2660-1	0.2085-1	0.1676-1	0.1376-1	0.9734-2	0.7241-2	0.5593-2	0.4448-2	0.3621-2
30	0.5642-1	0.4134-1	0.3150-1	0.2475-1	0.1993-1	0.1639-1	0.1162-1	0.8655-2	0.6691-2	0.5325-2	0.4337-2
35	0.6441-1	0.4743-1	0.3627-1	0.2857-1	0.2306-1	0.1898-1	0.1348-1	0.1006-1	0.7782-2	0.6196-2	0.5049-2
40	0.7209-1	0.5334-1	0.4092-1	0.3231-1	0.2612-1	0.2153-1	0.1533-1	0.1145-1	0.8866-2	0.7064-2	0.5758-2
45	0.7948-1	0.5907-1	0.4546-1	0.3598-1	0.2914-1	0.2406-1	0.1716-1	0.1283-1	0.9943-2	0.7927-2	0.6465-2
50	0.8661-1	0.6464-1	0.4990-1	0.3958-1	0.3211-1	0.2654-1	0.1897-1	0.1420-1	0.1101-1	0.8786-2	0.7169-2
60	0.1001	0.7531-1	0.5847-1	0.4659-1	0.3792-1	0.3142-1	0.2253-1	0.1691-1	0.1314-1	0.1049-1	0.8567-2
70	0.1128	0.8544-1	0.6668-1	0.5334-1	0.4355-1	0.3617-1	0.2603-1	0.1957-1	0.1523-1	0.1218-1	0.9955-2
80	0.1247	0.9507-1	0.7456-1	0.5986-1	0.4901-1	0.4080-1	0.2946-1	0.2220-1	0.1731-1	0.1385-1	0.1133-1
90	0.1359	0.1043	0.8213-1	0.6617-1	0.5433-1	0.4532-1	0.3282-1	0.2480-1	0.1936-1	0.1551-1	0.1270-1
100	0.1466	0.1130	0.8942-1	0.7228-1	0.5949-1	0.4974-1	0.3613-1	0.2735-1	0.2138-1	0.1715-1	0.1405-1
120	0.1662	0.1295	0.1032	0.8395-1	0.6943-1	0.5827-1	0.4258-1	0.3236-1	0.2537-1	0.2039-1	0.1674-1
140	0.1841	0.1447	0.1161	0.9496-1	0.7888-1	0.6644-1	0.4881-1	0.3723-1	0.2927-1	0.2358-1	0.1938-1
160	0.2006	0.1588	0.1282	0.1054	0.8789-1	0.7428-1	0.5484-1	0.4198-1	0.3309-1	0.2670-1	0.2198-1
180	0.2157	0.1719	0.1397	0.1153	0.9651-1	0.8181-1	0.6068-1	0.4661-1	0.3683-1	0.2978-1	0.2455-1
200	0.2297	0.1843	0.1504	0.1247	0.1048	0.8905-1	0.6635-1	0.5113-1	0.4050-1	0.3280-1	0.2708-1

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-23. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-1 Plan for Median (PL=X_{n-1})

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.3714	0.2262	0.1515	0.1084	0.8132-1	0.5057-1	0.3444-1	0.2320-1	0.1668-1	0.1257-1	0.9805-2
2	0.5381	0.3636	0.2587	0.1923	0.1480	0.9501-1	0.6589-1	0.4497-1	0.3259-1	0.2469-1	0.1933-1
3	0.6336	0.4570	0.3394	0.2597	0.2041	0.1345	0.9476-1	0.6546-1	0.4781-1	0.3639-1	0.2860-1
4	0.6957	0.5251	0.4026	0.3152	0.2520	0.1699	0.1214	0.8481-1	0.6238-1	0.4771-1	0.3762-1
5	0.7395	0.5771	0.4537	0.3620	0.2935	0.2019	0.1461	0.1031	0.7634-1	0.5866-1	0.4640-1
8	0.8173	0.6796	0.5622	0.4672	0.3913	0.2821	0.2107	0.1527	0.1151	0.8950-1	0.7144-1
10	0.8474	0.7232	0.6120	0.5183	0.4410	0.3257	0.2474	0.1820	0.1385	0.1086	0.8714-1
12	0.8689	0.7559	0.6509	0.5595	0.4822	0.3634	0.2801	0.2088	0.1604	0.1266	0.1021
15	0.8916	0.7922	0.6957	0.6087	0.5327	0.4115	0.3232	0.2450	0.1907	0.1519	0.1234
20	0.9158	0.8329	0.7483	0.6685	0.5962	0.4754	0.3827	0.2971	0.2353	0.1899	0.1559
25	0.9311	0.8600	0.7847	0.7115	0.6432	0.5251	0.4310	0.3410	0.2741	0.2238	0.1854
30	0.9416	0.8792	0.8115	0.7439	0.6796	0.5653	0.4713	0.3788	0.3083	0.2542	0.2122
35	0.9493	0.8937	0.8321	0.7694	0.7087	0.5984	0.5054	0.4117	0.3387	0.2817	0.2369
40	0.9552	0.9051	0.8484	0.7900	0.7326	0.6264	0.5348	0.4407	0.3660	0.3068	0.2596
45	0.9599	0.9141	0.8618	0.8071	0.7527	0.6503	0.5604	0.4665	0.3907	0.3298	0.2807
50	0.9637	0.9216	0.8729	0.8214	0.7697	0.6711	0.5831	0.4896	0.4131	0.3509	0.3003
60	0.9694	0.9332	0.8903	0.8443	0.7972	0.7054	0.6213	0.5295	0.4526	0.3887	0.3357
70	0.9736	0.9417	0.9034	0.8617	0.8185	0.7328	0.6523	0.5628	0.4862	0.4215	0.3670
80	0.9767	0.9482	0.9137	0.8755	0.8356	0.7551	0.6782	0.5912	0.5153	0.4503	0.3948
90	0.9792	0.9535	0.9219	0.8867	0.8495	0.7737	0.7002	0.6156	0.5409	0.4759	0.4198
100	0.9812	0.9577	0.9286	0.8960	0.8612	0.7895	0.7191	0.6370	0.5635	0.4988	0.4425
120	0.9843	0.9642	0.9391	0.9105	0.8797	0.8149	0.7500	0.6726	0.6018	0.5382	0.4819
140	0.9864	0.9689	0.9468	0.9213	0.8936	0.8346	0.7743	0.7012	0.6331	0.5711	0.5153
160	0.9881	0.9726	0.9527	0.9297	0.9045	0.8502	0.7940	0.7248	0.6594	0.5990	0.5440
180	0.9894	0.9754	0.9575	0.9365	0.9134	0.8630	0.8103	0.7447	0.6818	0.6230	0.5690
200	0.9904	0.9777	0.9613	0.9420	0.9206	0.8737	0.8241	0.7616	0.7011	0.6440	0.5910

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-23. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-1 Plan for Median (PL=X_{n-1})

w/n	50	60	70	80	90	100	120	140	160	180	200
1	0.6446-2	0.4558-2	0.3393-2	0.2623-2	0.2088-2	0.1702-2	0.1193-2	0.8822-3	0.6788-3	0.5385-3	0.4375-3
2	0.1277-1	0.9053-2	0.6750-2	0.5225-2	0.4163-2	0.3395-2	0.2381-2	0.1762-2	0.1356-2	0.1076-2	0.8744-3
3	0.1897-1	0.1349-1	0.1007-1	0.7806-2	0.6225-2	0.5079-2	0.3565-2	0.2639-2	0.2032-2	0.1613-2	0.1311-2
4	0.2507-1	0.1786-1	0.1336-1	0.1037-1	0.8273-2	0.6754-2	0.4745-2	0.3514-2	0.2707-2	0.2148-2	0.1746-2
5	0.3105-1	0.2218-1	0.1662-1	0.1291-1	0.1031-1	0.8420-2	0.5920-2	0.4387-2	0.3380-2	0.2683-2	0.2181-2
8	0.4837-1	0.3480-1	0.2620-1	0.2041-1	0.1634-1	0.1337-1	0.9419-2	0.6989-2	0.5390-2	0.4282-2	0.3483-2
10	0.5942-1	0.4295-1	0.3243-1	0.2531-1	0.2029-1	0.1662-1	0.1173-1	0.8713-2	0.6723-2	0.5343-2	0.4347-2
12	0.7012-1	0.5090-1	0.3854-1	0.3015-1	0.2420-1	0.1985-1	0.1403-1	0.1043-1	0.8050-2	0.6401-2	0.5210-2
15	0.8553-1	0.6247-1	0.4750-1	0.3726-1	0.2998-1	0.2462-1	0.1744-1	0.1298-1	0.1003-1	0.7980-2	0.6499-2
20	0.1097	0.8088-1	0.6189-1	0.4878-1	0.3937-1	0.3242-1	0.2304-1	0.1719-1	0.1330-1	0.1060-1	0.8635-2
25	0.1321	0.9829-1	0.7566-1	0.5989-1	0.4850-1	0.4003-1	0.2855-1	0.2134-1	0.1654-1	0.1319-1	0.1076-1
30	0.1531	0.1148	0.8887-1	0.7063-1	0.5738-1	0.4747-1	0.3396-1	0.2544-1	0.1975-1	0.1576-1	0.1286-1
35	0.1727	0.1305	0.1015	0.8102-1	0.6601-1	0.5473-1	0.3928-1	0.2949-1	0.2292-1	0.1831-1	0.1496-1
40	0.1911	0.1454	0.1137	0.9107-1	0.7441-1	0.6183-1	0.4451-1	0.3348-1	0.2606-1	0.2084-1	0.1704-1
45	0.2085	0.1596	0.1254	0.1008	0.8259-1	0.6877-1	0.4966-1	0.3743-1	0.2917-1	0.2335-1	0.1910-1
50	0.2249	0.1732	0.1367	0.1103	0.9056-1	0.7556-1	0.5472-1	0.4132-1	0.3225-1	0.2584-1	0.2115-1
60	0.2552	0.1987	0.1582	0.1283	0.1059	0.8872-1	0.6460-1	0.4897-1	0.3832-1	0.3076-1	0.2521-1
70	0.2826	0.2222	0.1782	0.1454	0.1206	0.1013	0.7419-1	0.5644-1	0.4427-1	0.3560-1	0.2922-1
80	0.3075	0.2439	0.1969	0.1616	0.1345	0.1135	0.8348-1	0.6373-1	0.5011-1	0.4037-1	0.3318-1
90	0.3304	0.2641	0.2146	0.1769	0.1479	0.1252	0.9251-1	0.7085-1	0.5585-1	0.4507-1	0.3709-1
100	0.3514	0.2830	0.2312	0.1915	0.1607	0.1364	0.1013	0.7782-1	0.6148-1	0.4970-1	0.4095-1
120	0.3889	0.3172	0.2619	0.2187	0.1848	0.1577	0.1181	0.9129-1	0.7244-1	0.5875-1	0.4853-1
140	0.4214	0.3475	0.2895	0.2436	0.2070	0.1776	0.1341	0.1042	0.8302-1	0.6754-1	0.5593-1
160	0.4500	0.3747	0.3146	0.2665	0.2277	0.1963	0.1492	0.1166	0.9326-1	0.7609-1	0.6315-1
180	0.4754	0.3992	0.3375	0.2876	0.2470	0.2138	0.1636	0.1285	0.1032	0.8441-1	0.7021-1
200	0.4981	0.4214	0.3586	0.3072	0.2651	0.2304	0.1774	0.1400	0.1128	0.9251-1	0.7711-1

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-24. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan for Median (PL=X_n)

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.5238-1	0.2121-1	0.1019-1	0.5495-2	0.3221-2	0.1321-2	0.6385-3	0.3002-3	0.1592-3	0.9207-4	0.5690-4
2	0.8898-1	0.3853-1	0.1918-1	0.1055-1	0.6265-2	0.2605-2	0.1266-2	0.5976-3	0.3174-3	0.1838-3	0.1137-3
3	0.1171	0.5324-1	0.2725-1	0.1526-1	0.9157-2	0.3853-2	0.1884-2	0.8923-3	0.4749-3	0.2753-3	0.1703-3
4	0.1400	0.6608-1	0.3461-1	0.1966-1	0.1191-1	0.5069-2	0.2492-2	0.1185-2	0.6315-3	0.3664-3	0.2268-3
5	0.1593	0.7747-1	0.4138-1	0.2381-1	0.1455-1	0.6255-2	0.3092-2	0.1474-2	0.7873-3	0.4573-3	0.2832-3
8	0.2035	0.1057	0.5901-1	0.3502-1	0.2187-1	0.9651-2	0.4839-2	0.2329-2	0.1250-2	0.7280-3	0.4517-3
10	0.2259	0.1210	0.6910-1	0.4167-1	0.2633-1	0.1180-1	0.5965-2	0.2888-2	0.1555-2	0.9071-3	0.5634-3
12	0.2448	0.1344	0.7820-1	0.4781-1	0.3052-1	0.1386-1	0.7062-2	0.3438-2	0.1856-2	0.1085-2	0.6746-3
15	0.2683	0.1518	0.9038-1	0.5623-1	0.3638-1	0.1682-1	0.8661-2	0.4248-2	0.2304-2	0.1350-2	0.8406-3
20	0.2991	0.1758	0.1078	0.6863-1	0.4521-1	0.2143-1	0.1121-1	0.5562-2	0.3036-2	0.1786-2	0.1115-2
25	0.3232	0.1954	0.1226	0.7947-1	0.5312-1	0.2571-1	0.1363-1	0.6834-2	0.3753-2	0.2216-2	0.1387-2
30	0.3430	0.2120	0.1354	0.8913-1	0.6030-1	0.2972-1	0.1594-1	0.8067-2	0.4456-2	0.2640-2	0.1656-2
35	0.3597	0.2264	0.1468	0.9786-1	0.6690-1	0.3349-1	0.1815-1	0.9265-2	0.5145-2	0.3059-2	0.1922-2
40	0.3741	0.2391	0.1571	0.1058	0.7301-1	0.3706-1	0.2028-1	0.1043-1	0.5821-2	0.3472-2	0.2186-2
45	0.3867	0.2505	0.1664	0.1132	0.7871-1	0.4044-1	0.2233-1	0.1157-1	0.6486-2	0.3879-2	0.2448-2
50	0.3980	0.2608	0.1749	0.1200	0.8404-1	0.4367-1	0.2430-1	0.1268-1	0.7139-2	0.4282-2	0.2707-2
60	0.4173	0.2787	0.1901	0.1323	0.9382-1	0.4972-1	0.2806-1	0.1482-1	0.8413-2	0.5074-2	0.3219-2
70	0.4334	0.2941	0.2034	0.1432	0.1026	0.5530-1	0.3160-1	0.1687-1	0.9649-2	0.5849-2	0.3723-2
80	0.4473	0.3075	0.2151	0.1530	0.1106	0.6049-1	0.3494-1	0.1884-1	0.1085-1	0.6607-2	0.4220-2
90	0.4593	0.3194	0.2257	0.1620	0.1180	0.6535-1	0.3812-1	0.2074-1	0.1202-1	0.7349-2	0.4708-2
100	0.4700	0.3300	0.2352	0.1702	0.1248	0.6993-1	0.4115-1	0.2257-1	0.1315-1	0.8078-2	0.5190-2
120	0.4882	0.3484	0.2520	0.1847	0.1371	0.7835-1	0.4683-1	0.2606-1	0.1535-1	0.9495-2	0.6133-2
140	0.5034	0.3640	0.2665	0.1975	0.1480	0.8598-1	0.5207-1	0.2934-1	0.1744-1	0.1086-1	0.7051-2
160	0.5163	0.3775	0.2792	0.2087	0.1578	0.9296-1	0.5695-1	0.3245-1	0.1945-1	0.1219-1	0.7947-2
180	0.5274	0.3893	0.2904	0.2189	0.1666	0.9941-1	0.6151-1	0.3540-1	0.2138-1	0.1347-1	0.8822-2
200	0.5373	0.3999	0.3006	0.2281	0.1748	0.1054	0.6582-1	0.3822-1	0.2324-1	0.1472-1	0.9677-2

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-24. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan for Median (PL=X_n)

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.2513-4	0.1276-4	0.7145-5	0.4307-5	0.2749-5	0.1835-5	0.9090-6	*0.5001-6*	*0.2974-6*	*0.1878-6*	*0.1243-6*
2	0.5024-4	0.2550-4	0.1429-4	0.8613-5	0.5497-5	0.3671-5	0.1818-5	*0.1000-5*	*0.5949-6*	*0.3756-6*	*0.2487-6*
3	0.7531-4	0.3825-4	0.2143-4	0.1292-4	0.8244-5	0.5506-5	0.2727-5	*0.1500-5*	*0.8923-6*	*0.5634-6*	*0.3730-6*
4	0.1004-3	0.5098-4	0.2856-4	0.1722-4	0.1099-4	0.7340-5	0.3636-5	*0.2000-5*	*0.1190-5*	*0.7511-6*	*0.4973-6*
5	0.1254-3	0.6370-4	0.3570-4	0.2153-4	0.1374-4	0.9175-5	0.4545-5	*0.2501-5*	*0.1487-5*	*0.9389-6*	*0.6216-6*
8	0.2003-3	0.1018-3	0.5709-4	0.3443-4	0.2198-4	0.1468-4	0.7271-5	*0.4001-5*	*0.2379-5*	*0.1502-5*	*0.9946-6*
10	0.2501-3	0.1272-3	0.7134-4	0.4303-4	0.2747-4	0.1835-4	0.9088-5	*0.5001-5*	*0.2974-5*	*0.1878-5*	*0.1243-5*
12	0.2998-3	0.1526-3	0.8557-4	0.5162-4	0.3296-4	0.2201-4	0.1090-4	*0.6001-5*	*0.3569-5*	*0.2253-5*	*0.1492-5*
15	0.3741-3	0.1905-3	0.1069-3	0.6451-4	0.4119-4	0.2751-4	0.1363-4	*0.7501-5*	*0.4461-5*	*0.2817-5*	*0.1865-5*
20	0.4975-3	0.2537-3	0.1424-3	0.8596-4	0.5489-4	0.3667-4	0.1817-4	*0.1000-4*	*0.5948-5*	*0.3755-5*	*0.2486-5*
25	0.6203-3	0.3167-3	0.1779-3	0.1074-3	0.6859-4	0.4583-4	0.2271-4	*0.1250-4*	*0.7434-5*	*0.4694-5*	*0.3108-5*
30	0.7424-3	0.3794-3	0.2133-3	0.1288-3	0.8228-4	0.5498-4	0.2725-4	*0.1500-4*	*0.8921-5*	*0.5633-5*	*0.3729-5*
35	0.8640-3	0.4420-3	0.2486-3	0.1502-3	0.9596-4	0.6413-4	0.3179-4	*0.1750-4*	*0.1041-4*	*0.6571-5*	*0.4351-5*
40	0.9849-3	0.5045-3	0.2839-3	0.1715-3	0.1096-3	0.7327-4	0.3632-4	*0.1999-4*	*0.1189-4*	*0.7510-5*	*0.4972-5*
45	0.1105-2	0.5667-3	0.3191-3	0.1929-3	0.1233-3	0.8241-4	0.4086-4	*0.2249-4*	*0.1338-4*	*0.8448-5*	*0.5594-5*
50	0.1225-2	0.6288-3	0.3542-3	0.2142-3	0.1369-3	0.9155-4	0.4539-4	*0.2499-4*	*0.1487-4*	*0.9387-5*	*0.6215-5*
60	0.1463-2	0.7525-3	0.4244-3	0.2568-3	0.1642-3	0.1098-3	0.5446-4	*0.2998-4*	*0.1784-4*	*0.1126-4*	*0.7458-5*
70	0.1699-2	0.8755-3	0.4943-3	0.2992-3	0.1914-3	0.1280-3	0.6352-4	*0.3497-4*	*0.2081-4*	*0.1314-4*	*0.8701-5*
80	0.1933-2	0.9979-3	0.5639-3	0.3416-3	0.2186-3	0.1463-3	0.7257-4	*0.3997-4*	*0.2378-4*	*0.1502-4*	*0.9943-5*
90	0.2164-2	0.1120-2	0.6334-3	0.3839-3	0.2458-3	0.1645-3	0.8162-4	*0.4495-4*	*0.2675-4*	*0.1689-4*	*0.1119-4*
100	0.2394-2	0.1241-2	0.7026-3	0.4261-3	0.2729-3	0.1826-3	0.9067-4	*0.4994-4*	*0.2972-4*	*0.1877-4*	*0.1243-4*
120	0.2848-2	0.1481-2	0.8405-3	0.5103-3	0.3270-3	0.2190-3	0.1087-3	*0.5991-4*	*0.3565-4*	*0.2252-4*	*0.1491-4*
140	0.3295-2	0.1720-2	0.9775-3	0.5941-3	0.3810-3	0.2552-3	0.1268-3	*0.6988-4*	*0.4159-4*	*0.2627-4*	*0.1740-4*
160	0.3735-2	0.1956-2	0.1114-2	0.6776-3	0.4348-3	0.2914-3	0.1449-3	*0.7984-4*	*0.4752-4*	*0.3002-4*	*0.1988-4*
180	0.4169-2	0.2189-2	0.1249-2	0.7608-3	0.4885-3	0.3275-3	0.1629-3	*0.8979-4*	*0.5345-4*	*0.3377-4*	*0.2236-4*
200	0.4596-2	0.2421-2	0.1384-2	0.8437-3	0.5421-3	0.3635-3	0.1809-3	*0.9974-4*	*0.5938-4*	*0.3752-4*	*0.2485-4*

Footnote. PL = Prediction Limit; X_n = Maximum order statistic

Table 19-24. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan for Median (PL=X_{n-1})

w\n	4	6	8	10	12	16	20	25	30	35	40
1	0.2048	0.8874-1	0.4429-1	0.2448-1	0.1460-1	0.6125-2	0.3001-2	0.1427-2	0.7629-3	0.4439-3	0.2755-3
2	0.3138	0.1515	0.8008-1	0.4576-1	0.2786-1	0.1195-1	0.5917-2	0.2832-2	0.1519-2	0.8852-3	0.5500-3
3	0.3849	0.1996	0.1101	0.6462-1	0.4003-1	0.1751-1	0.8754-2	0.4216-2	0.2267-2	0.1324-2	0.8234-3
4	0.4363	0.2383	0.1360	0.8158-1	0.5129-1	0.2283-1	0.1152-1	0.5579-2	0.3009-2	0.1760-2	0.1096-2
5	0.4758	0.2705	0.1586	0.9698-1	0.6178-1	0.2794-1	0.1421-1	0.6922-2	0.3745-2	0.2194-2	0.1367-2
8	0.5558	0.3429	0.2135	0.1363	0.8957-1	0.4215-1	0.2193-1	0.1084-1	0.5913-2	0.3480-2	0.2175-2
10	0.5914	0.3785	0.2425	0.1582	0.1057	0.5085-1	0.2679-1	0.1337-1	0.7329-2	0.4326-2	0.2708-2
12	0.6192	0.4079	0.2674	0.1776	0.1204	0.5902-1	0.3146-1	0.1583-1	0.8722-2	0.5163-2	0.3238-2
15	0.6515	0.4437	0.2990	0.2031	0.1401	0.7045-1	0.3814-1	0.1942-1	0.1077-1	0.6402-2	0.4025-2
20	0.6902	0.4892	0.3413	0.2386	0.1684	0.8764-1	0.4852-1	0.2514-1	0.1409-1	0.8427-2	0.5319-2
25	0.7178	0.5237	0.3748	0.2678	0.1925	0.1030	0.5810-1	0.3057-1	0.1729-1	0.1040-1	0.6592-2
30	0.7389	0.5511	0.4024	0.2925	0.2135	0.1169	0.6702-1	0.3574-1	0.2039-1	0.1233-1	0.7844-2
35	0.7557	0.5736	0.4257	0.3140	0.2321	0.1296	0.7538-1	0.4068-1	0.2339-1	0.1422-1	0.9076-2
40	0.7694	0.5926	0.4459	0.3329	0.2487	0.1413	0.8324-1	0.4542-1	0.2631-1	0.1607-1	0.1029-1
45	0.7810	0.6090	0.4635	0.3498	0.2638	0.1522	0.9067-1	0.4997-1	0.2914-1	0.1789-1	0.1149-1
50	0.7910	0.6232	0.4792	0.3650	0.2775	0.1623	0.9771-1	0.5436-1	0.3190-1	0.1967-1	0.1266-1
60	0.8072	0.6471	0.5061	0.3915	0.3019	0.1808	0.1108	0.6268-1	0.3721-1	0.2312-1	0.1497-1
70	0.8201	0.6665	0.5283	0.4139	0.3229	0.1973	0.1228	0.7047-1	0.4227-1	0.2646-1	0.1722-1
80	0.8306	0.6827	0.5473	0.4334	0.3414	0.2121	0.1339	0.7781-1	0.4711-1	0.2969-1	0.1942-1
90	0.8394	0.6965	0.5637	0.4504	0.3579	0.2257	0.1441	0.8475-1	0.5175-1	0.3281-1	0.2156-1
100	0.8469	0.7085	0.5781	0.4656	0.3727	0.2381	0.1537	0.9134-1	0.5621-1	0.3584-1	0.2365-1
120	0.8592	0.7283	0.6025	0.4917	0.3985	0.2604	0.1712	0.1036	0.6464-1	0.4166-1	0.2770-1
140	0.8688	0.7443	0.6224	0.5135	0.4204	0.2797	0.1868	0.1148	0.7252-1	0.4717-1	0.3158-1
160	0.8767	0.7575	0.6392	0.5320	0.4393	0.2969	0.2009	0.1252	0.7993-1	0.5242-1	0.3532-1
180	0.8832	0.7687	0.6536	0.5481	0.4560	0.3123	0.2138	0.1349	0.8692-1	0.5743-1	0.3892-1
200	0.8888	0.7783	0.6662	0.5623	0.4708	0.3263	0.2257	0.1439	0.9354-1	0.6223-1	0.4240-1

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

Table 19-24. Per-Constituent Significance Levels (α) for Non-Parametric 1-of-2 Plan for Median (PL=X_{n-1})

w\n	50	60	70	80	90	100	120	140	160	180	200
1	0.1225-3	0.6242-4	0.3507-4	0.2119-4	0.1354-4	0.9057-5	0.4495-5	0.2477-5	0.1475-5	0.9321-6	0.6175-6
2	0.2447-3	0.1248-3	0.7011-4	0.4236-4	0.2708-4	0.1811-4	0.8991-5	0.4954-5	0.2950-5	0.1864-5	0.1235-5
3	0.3667-3	0.1870-3	0.1051-3	0.6353-4	0.4062-4	0.2717-4	0.1349-4	0.7431-5	0.4425-5	0.2796-5	0.1853-5
4	0.4884-3	0.2493-3	0.1401-3	0.8470-4	0.5416-4	0.3622-4	0.1798-4	0.9908-5	0.5900-5	0.3728-5	0.2470-5
5	0.6099-3	0.3114-3	0.1751-3	0.1058-3	0.6769-4	0.4527-4	0.2247-4	0.1239-4	0.7374-5	0.4660-5	0.3088-5
8	0.9731-3	0.4975-3	0.2799-3	0.1693-3	0.1083-3	0.7241-4	0.3595-4	0.1981-4	0.1180-4	0.7456-5	0.4940-5
10	0.1214-2	0.6212-3	0.3497-3	0.2115-3	0.1353-3	0.9050-4	0.4494-4	0.2477-4	0.1475-4	0.9320-5	0.6175-5
12	0.1454-2	0.7447-3	0.4194-3	0.2537-3	0.1623-3	0.1086-3	0.5392-4	0.2972-4	0.1770-4	0.1118-4	0.7410-5
15	0.1813-2	0.9294-3	0.5237-3	0.3169-3	0.2028-3	0.1357-3	0.6739-4	0.3715-4	0.2212-4	0.1398-4	0.9262-5
20	0.2406-2	0.1236-2	0.6972-3	0.4222-3	0.2702-3	0.1808-3	0.8983-4	0.4952-4	0.2949-4	0.1864-4	0.1235-4
25	0.2994-2	0.1541-2	0.8702-3	0.5272-3	0.3376-3	0.2260-3	0.1123-3	0.6189-4	0.3686-4	0.2330-4	0.1544-4
30	0.3577-2	0.1845-2	0.1043-2	0.6320-3	0.4048-3	0.2710-3	0.1347-3	0.7426-4	0.4423-4	0.2795-4	0.1852-4
35	0.4155-2	0.2147-2	0.1215-2	0.7367-3	0.4720-3	0.3161-3	0.1571-3	0.8663-4	0.5160-4	0.3261-4	0.2161-4
40	0.4728-2	0.2448-2	0.1386-2	0.8411-3	0.5391-3	0.3610-3	0.1795-3	0.9899-4	0.5896-4	0.3727-4	0.2469-4
45	0.5297-2	0.2747-2	0.1557-2	0.9453-3	0.6061-3	0.4060-3	0.2019-3	0.1114-3	0.6633-4	0.4192-4	0.2778-4
50	0.5861-2	0.3045-2	0.1727-2	0.1049-2	0.6730-3	0.4509-3	0.2243-3	0.1237-3	0.7369-4	0.4658-4	0.3087-4
60	0.6976-2	0.3636-2	0.2067-2	0.1257-2	0.8066-3	0.5406-3	0.2690-3	0.1484-3	0.8841-4	0.5589-4	0.3704-4
70	0.8074-2	0.4223-2	0.2404-2	0.1464-2	0.9398-3	0.6302-3	0.3137-3	0.1731-3	0.1031-3	0.6520-4	0.4321-4
80	0.9156-2	0.4804-2	0.2740-2	0.1670-2	0.1073-2	0.7196-3	0.3583-3	0.1978-3	0.1178-3	0.7451-4	0.4938-4
90	0.1022-1	0.5380-2	0.3074-2	0.1875-2	0.1205-2	0.8088-3	0.4030-3	0.2224-3	0.1326-3	0.8381-4	0.5554-4
100	0.1128-1	0.5951-2	0.3406-2	0.2079-2	0.1338-2	0.8980-3	0.4475-3	0.2471-3	0.1473-3	0.9311-4	0.6171-4
120	0.1334-1	0.7080-2	0.4065-2	0.2486-2	0.1601-2	0.1076-2	0.5366-3	0.2964-3	0.1767-3	0.1117-3	0.7404-4
140	0.1535-1	0.8190-2	0.4717-2	0.2891-2	0.1864-2	0.1253-2	0.6254-3	0.3456-3	0.2060-3	0.1303-3	0.8637-4
160	0.1731-1	0.9284-2	0.5362-2	0.3292-2	0.2125-2	0.1429-2	0.7141-3	0.3947-3	0.2354-3	0.1489-3	0.9870-4
180	0.1923-1	0.1036-1	0.6001-2	0.3691-2	0.2385-2	0.1605-2	0.8027-3	0.4439-3	0.2647-3	0.1675-3	0.1110-3
200	0.2111-1	0.1142-1	0.6634-2	0.4087-2	0.2643-2	0.1781-2	0.8911-3	0.4929-3	0.2941-3	0.1860-3	0.1233-3

Footnote. PL = Prediction Limit; X_{n-1} = 2nd largest order statistic

D STATISTICAL TABLES

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s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	-4.435	-3.437	-3.047	-2.849	-2.730	-2.653	-2.598	-2.558	-2.527	-2.503	-2.484	-2.467
0.20	-3.720	-3.089	-2.819	-2.677	-2.590	-2.534	-2.494	-2.465	-2.442	-2.425	-2.411	-2.400
0.30	-3.260	-2.836	-2.646	-2.544	-2.482	-2.441	-2.413	-2.393	-2.378	-2.366	2.357	-2.350
0.40	-2.943	-2.649	-2.514	-2.442	-2.399	-2.371	-2.353	-2.340	-2.330	-2.324	-2.319	-2.315
0.50	-2.714	-2.508	-2.414	-2.364	-2.337	-2.320	-2.309	-2.302	-2.298	-2.295	-2.294	-2.293
0.60	-2.544	-2.402	-2.338	-2.307	-2.292	-2.283	-2.279	-2.278	-2.278	-2.279	-2.281	-2.283
0.70	-2.415	-2.321	-2.282	-2.266	-2.261	-2.260	-2.262	-2.265	-2.269	-2.274	-2.278	-2.283
0.80	-2.317	-2.260	-2.242	-2.238	-2.241	-2.247	-2.255	-2.262	-2.270	-2.277	-2.284	-2.291
0.90	-2.242	-2.216	-2.214	-2.221	-2.232	-2.244	-2.256	-2.268	-2.279	-2.289	-2.298	-2.308
1.00	-2.185	-2.184	-2.196	-2.214	-2.232	-2.249	-2.265	-2.280	-2.295	-2.308	-2.320	-2.331
1.25	-2.099	-2.147	-2.189	-2.227	-2.260	-2.290	-2.316	-2.339	-2.361	-2.380	-2.398	-2.414
1.50	-2.069	-2.153	-2.220	-2.275	-2.322	-2.362	-2.397	-2.428	-2.456	-2.481	-2.504	-2.525
1.75	-2.075	-2.190	-2.277	-2.348	-2.407	-2.457	-2.501	-2.540	-2.574	-2.605	-2.633	-2.659
2.00	-2.106	-2.247	-2.355	-2.440	-2.511	-2.571	-2.623	-2.668	-2.709	-2.746	-2.778	-2.809
2.50	-2.217	-2.408	-2.552	-2.665	-2.758	-2.836	-2.904	-2.964	-3.017	-3.064	-3.107	-3.147
3.00	-2.371	-2.610	-2.788	-2.927	-3.042	-3.140	-3.223	-3.296	-3.361	-3.419	-3.472	-3.521
3.50	-2.553	-2.839	-3.050	-3.216	-3.352	-3.467	-3.566	-3.652	-3.729	-3.799	-3.861	-3.918
4.00	-2.756	-3.087	-3.331	-3.523	-3.680	-3.812	-3.926	-4.026	-4.115	-4.195	-4.267	-4.333
4.50	-2.973	-3.349	-3.626	-3.842	-4.020	-4.170	-4.299	-4.412	-4.513	-4.603	-4.685	-4.760
5.00	-3.202	-3.622	-3.930	-4.171	-4.370	-4.537	-4.681	-4.808	-4.920	-5.021	-5.112	-5.195
6.00	-3.683	-4.189	-4.559	-4.850	-5.089	-5.291	-5.465	-5.618	-5.754	-5.875	-5.986	-6.087
7.00	-4.185	-4.775	-5.208	-5.548	-5.827	-6.064	-6.267	-6.446	-6.605	-6.748	-6.877	-6.995
8.00	-4.700	-5.374	-5.868	-6.258	-6.577	-6.847	-7.081	-7.286	-7.468	-7.632	-7.780	-7.916
9.00	-5.223	-5.980	-6.536	-6.975	-7.334	-7.639	-7.902	-8.133	-8.339	-8.523	-8.690	-8.843
10.00	-5.753	-6.593	-7.211	-7.698	-8.098	-8.437	-8.730	-8.987	-9.215	-9.420	-9.607	-9.776

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-1. Land's Factors ($H_{.01}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

s_y \ n	15	16	17	18	19	21	23	25	28	31	36
0.10	-2.454	-2.442	-2.432	-2.424	-2.416	-2.404	-2.395	-2.386	-2.377	-2.369	-2.361
0.20	-2.390	-2.383	-2.376	-2.370	-2.365	-2.357	-2.351	-2.346	-2.340	-2.336	-2.331
0.30	-2.344	-2.339	-2.335	-2.332	-2.329	-2.325	-2.322	2.320	-2.317	-2.316	-2.315
0.40	-2.312	-2.310	-2.308	-2.307	-2.306	-2.306	-2.305	-2.305	-2.306	-2.308	-2.310
0.50	-2.293	-2.293	-2.294	-2.294	-2.295	-2.298	-2.300	-2.302	-2.306	-2.310	-2.316
0.60	-2.285	-2.288	-2.290	-2.292	-2.295	-2.300	-2.305	-2.309	-2.316	-2.322	-2.330
0.70	-2.287	-2.292	-2.296	-2.300	-2.304	-2.312	-2.319	-2.325	-2.334	-2.342	-2.354
0.80	-2.298	-2.304	-2.310	-2.315	-2.321	-2.331	-2.341	-2.349	-2.361	-2.373	-2.386
0.90	-2.316	-2.324	-2.332	-2.339	-2.346	-2.358	-2.370	-2.380	-2.394	-2.406	-2.425
1.00	-2.341	-2.351	-2.360	-2.369	-2.377	-2.392	-2.406	-2.418	-2.434	-2.449	-2.470
1.25	-2.429	-2.443	-2.456	-2.468	-2.479	-2.500	-2.519	-2.535	-2.558	-2.578	-2.606
1.50	2.545	-2.563	-2.579	-2.595	-2.609	-2.635	-2.659	-2.680	-2.709	-2.734	-2.769
1.75	-2.682	-2.704	-2.724	-2.743	-2.760	-2.792	-2.821	-2.847	-2.881	-2.911	-2.954
2.00	-2.836	-2.862	-2.886	-2.908	-2.929	-2.966	-3.000	-3.030	-3.070	-3.105	-3.155
2.50	-3.183	-3.216	-3.247	-3.275	-3.302	-3.351	-3.394	-3.434	-3.486	-3.531	-3.569
3.00	-3.564	-3.605	-3.643	-3.679	-3.711	-3.771	-3.825	-3.873	-3.936	-3.992	-4.071
3.50	-3.970	-4.019	-4.063	-4.105	-4.144	-4.215	-4.279	-4.335	-4.410	-4.476	-4.570
4.00	-4.393	-4.449	-4.500	-4.549	-4.593	-4.676	-4.749	-4.814	-4.901	-4.977	-5.086
4.50	-4.828	-4.891	-4.950	-5.005	-5.055	-5.148	-5.231	-5.305	-5.404	-5.491	-5.614
5.00	-5.272	-5.343	-5.408	-5.469	-5.526	-5.630	-5.723	-5.805	-5.916	-6.012	-6.150
6.00	-6.179	-6.264	-6.343	-6.418	-6.486	-6.612	-6.724	-6.824	-6.958	-7.075	-7.241
7.00	-7.104	-7.204	-7.297	-7.383	-7.465	-7.611	-7.742	-7.860	-8.017	-8.154	-8.348
8.00	-8.040	-8.154	-8.261	-8.360	-8.453	-8.621	-8.772	-8.906	-9.086	-9.244	-9.467
9.00	-8.983	-9.113	-9.232	-9.344	-9.449	-9.640	-9.809	-9.961	-10.160	-10.340	-10.590
10.00	-9.932	-10.080	-10.210	-10.330	-10.450	-10.660	-10.850	-11.020	-11.250	-11.440	-11.720

Table 21-2. Land's Factors ($H_{.025}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	-2.988	-2.504	-2.314	-2.215	-2.157	-2.117	-2.090	-2.070	-2.055	-2.042	-2.032	-2.025
0.20	-2.639	-2.316	-2.183	-2.113	-2.071	-2.044	-2.025	-2.012	-2.001	-1.994	-1.987	-1.982
0.30	-2.396	-2.176	-2.083	-2.034	-2.006	-1.988	-1.976	-1.968	-1.962	-1.958	-1.954	-1.952
0.40	-2.220	-2.070	-2.007	-1.975	-1.958	-1.948	-1.941	-1.938	-1.935	-1.934	-1.933	-1.933
0.50	-2.090	-1.989	-1.950	-1.932	-1.923	-1.919	-1.918	-1.918	-1.919	-1.920	-1.922	-1.924
0.60	-1.992	-1.929	-1.908	-1.901	-1.900	-1.902	-1.905	-1.908	-1.913	-1.917	-1.920	-1.924
0.70	-1.919	-1.885	-1.879	-1.882	-1.887	-1.894	-1.901	-1.908	-1.914	-1.921	-1.926	-1.932
0.80	-1.864	-1.854	-1.860	-1.871	-1.830	-1.894	-1.904	-1.914	-1.923	-1.932	-1.939	-1.946
0.90	-1.823	-1.833	-1.850	-1.869	-1.885	-1.901	-1.915	-1.927	-1.939	-1.949	-1.958	-1.967
1.00	-1.794	-1.820	-1.848	-1.873	-1.894	-1.913	-1.931	-1.946	-1.959	-1.972	-1.983	-1.993
1.25	-1.759	-1.819	-1.867	-1.907	-1.939	-1.967	-1.992	-2.013	-2.032	-2.049	-2.064	-2.079
1.50	-1.761	-1.849	-1.914	-1.966	-2.009	-2.045	-2.076	-2.104	-2.128	-2.150	-2.169	-2.187
1.75	-1.789	-1.899	-1.981	-2.045	-2.097	-2.141	-2.179	-2.212	-2.242	-2.268	-2.291	-2.313
2.00	-1.834	-1.965	-2.062	-2.138	-2.200	-2.252	-2.296	-2.335	-2.369	-2.400	-2.428	-2.452
2.50	-1.960	-2.132	-2.259	-2.357	-2.438	-2.505	-2.562	-2.612	-2.656	-2.696	-2.731	-2.764
3.00	-2.118	-2.331	-2.487	-2.607	-2.706	-2.788	-2.858	-2.919	-2.973	-3.022	-3.065	-3.105
3.50	-2.299	-2.552	-2.736	-2.879	-2.994	-3.091	-3.174	-3.246	-3.310	-3.367	-3.418	-3.465
4.00	-2.496	-2.789	-3.001	-3.164	-3.298	-3.409	-3.505	-3.588	-3.661	-3.727	-3.786	-3.840
4.50	-2.706	-3.037	-3.276	-3.461	-3.612	-3.738	-3.846	-3.940	-4.023	-4.097	-4.164	-4.226
5.00	-2.925	-3.294	-3.560	-3.766	-3.934	-4.074	-4.194	-4.300	-4.392	-4.475	-4.550	-4.618
6.00	-3.382	-3.826	-4.145	-4.393	-4.594	-4.763	-4.908	-5.035	-5.147	-5.247	-5.337	-5.419
7.00	-3.856	-4.372	-4.744	-5.033	-5.269	-5.467	-5.637	-5.785	-5.916	-6.033	-6.139	-6.235
8.00	-4.341	-4.929	-5.354	-5.685	-5.955	-6.181	-6.375	-6.545	-6.695	-6.829	-6.950	-7.060
9.00	-4.832	-5.492	-5.971	-6.343	-6.646	-6.901	-7.120	-7.311	-7.480	-7.631	-7.768	-7.892
10.00	-5.328	-6.061	-6.592	-7.006	-7.343	-7.626	-7.869	-8.082	-8.270	-8.438	-8.590	-8.728

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-2. Land's Factors ($H_{.025}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

s_y\n	15	16	17	18	19	21	23	25	28	31	36
0.10	-2.018	-2.012	-2.008	-2.003	-2.000	-1.993	-1.989	-1.985	-1.980	-1.977	-1.972
0.20	-1.978	-1.974	-1.972	-1.969	-1.967	-1.964	-1.961	-1.959	-1.957	-1.956	-1.954
0.30	-1.950	-1.949	-1.947	-1.946	-1.946	-1.945	-1.945	-1.945	-1.945	-1.945	-1.946
0.40	-1.933	-1.934	-1.934	-1.935	-1.935	-1.936	-1.938	-1.940	-1.942	-1.944	-1.948
0.50	-1.926	-1.928	-1.930	-1.932	-1.933	-1.937	-1.941	-1.944	-1.948	-1.952	-1.958
0.60	-1.928	-1.931	-1.934	-1.938	-1.940	-1.946	-1.951	-1.956	-1.962	-1.968	-1.976
0.70	-1.937	-1.942	-1.946	-1.951	-1.955	-1.906	-1.969	-1.975	-1.983	-1.991	-2.001
0.80	-1.953	-1.959	-1.965	-1.971	-1.976	-1.985	-1.993	-2.001	-2.011	-2.020	-2.032
0.90	-1.975	-1.983	-1.990	-1.996	-2.003	-2.014	-2.023	-2.003	-2.044	-2.055	-2.069
1.00	-2.003	-2.012	-2.109	-2.027	-2.024	-2.047	-2.059	-2.069	-2.083	-2.095	-2.112
1.25	-2.091	-2.104	-2.114	-2.125	-2.134	-2.151	-2.167	-2.181	-2.199	-2.215	-2.237
1.50	-2.203	-2.218	-2.232	-2.245	-2.257	-2.278	-2.298	-2.315	-2.338	-2.358	-2.386
1.75	2.332	-2.351	-2.367	-2.383	-2.396	-2.423	-2.446	-2.467	-2.495	-2.518	-2.552
2.00	-2.476	-2.496	-2.516	-2.534	-2.551	-2.581	-2.608	-2.633	-2.665	-2.693	-2.733
2.50	-2.793	-2.821	-2.845	-2.869	-2.890	-2.930	-2.956	-2.997	-3.038	-3.074	-3.125
3.00	-3.141	-3.174	-3.205	-3.233	-3.260	-3.308	-3.351	-3.389	-3.440	-3.484	-3.547
3.50	-3.508	-3.547	-3.583	-3.617	-3.649	-3.706	-3.757	-3.802	-3.862	-3.914	-3.988
4.00	-3.889	-3.935	-3.976	-4.015	-4.052	-4.118	-4.176	-4.229	-4.298	-4.358	-4.444
4.50	-4.281	-4.332	-4.380	-4.424	-4.465	-4.539	-4.606	-4.665	-4.744	-4.812	-4.910
5.00	-4.680	-4.738	-4.790	-4.840	-4.886	-4.969	-5.043	-5.110	-5.197	-5.273	-5.382
6.00	-5.494	-5.564	-5.628	-5.687	-5.743	-5.844	-5.933	-6.013	-6.119	-6.212	-6.343
7.00	-6.324	-6.404	-6.480	-6.549	-6.614	-6.732	-6.837	-6.931	-7.056	-7.164	-7.318
8.00	-7.161	-7.254	-7.340	-7.420	-7.495	-7.630	-7.750	-7.858	-8.001	-8.125	-8.301
9.00	-8.006	-8.111	-8.208	-8.298	-8.382	-8.535	-8.670	-8.791	-8.952	-9.092	-9.292
10.00	-8.855	-8.972	-9.079	-9.179	-9.273	-9.443	-9.594	-9.729	-9.908	-10.060	-10.290

Table 21-3. Land's Factors ($H_{.05}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	-2.130	-1.898	-1.806	-1.759	-1.731	-1.712	-1.699	-1.690	-1.683	-1.677	-1.673	-1.669
0.20	-1.969	-1.791	-1.729	-1.697	-1.678	-1.667	-1.658	-1.653	-1.649	-1.646	-1.644	-1.642
0.30	-1.816	-1.710	-1.669	-1.650	-1.639	-1.633	-1.629	-1.627	-1.626	-1.625	-1.625	-1.624
0.40	-1.717	-1.650	-1.625	-1.615	-1.611	-1.610	-1.610	-1.611	-1.612	-1.613	-1.614	-1.615
0.50	-1.644	-1.605	-1.594	-1.592	-1.594	-1.596	-1.599	-1.603	-1.606	-1.609	-1.612	-1.615
0.60	-1.589	-1.572	-1.573	-1.578	-1.584	-1.591	-1.597	-1.602	-1.608	-1.612	-1.617	-1.621
0.70	-1.549	-1.550	-1.560	-1.572	-1.582	-1.592	-1.600	-1.608	-1.615	-1.622	-1.628	-1.633
0.80	-1.521	-1.537	-1.555	-1.572	-1.586	-1.599	-1.610	-1.620	-1.629	-1.636	-1.644	-1.651
0.90	-1.502	-1.530	-1.556	-1.577	-1.595	-1.611	-1.625	-1.637	-1.647	-1.656	-1.665	-1.673
1.00	-1.490	-1.530	-1.562	-1.588	-1.610	-1.628	-1.644	-1.658	-1.670	-1.681	-1.690	-1.699
1.25	-1.486	-1.549	-1.596	-1.632	-1.662	-1.687	-1.708	-1.727	-1.743	-1.758	-1.770	-1.782
1.50	-1.508	-1.590	-1.650	-1.696	-1.733	-1.764	-1.791	-1.814	-1.834	-1.853	-1.869	-1.883
1.75	-1.547	-1.647	-1.719	-1.774	-1.819	-1.857	-1.889	-1.916	-1.940	-1.962	-1.981	-1.998
2.00	-1.598	-1.714	-1.799	-1.864	-1.917	-1.960	-1.998	-2.029	-2.058	-2.083	-2.106	-2.126
2.50	-1.727	-1.877	-1.986	-2.070	-2.138	-2.193	-2.241	-2.283	-2.319	-2.351	-2.380	-2.406
3.00	-1.880	-2.065	-2.199	-2.301	-2.384	-2.452	-2.510	-2.560	-2.604	-2.644	-2.679	-2.711
3.50	-2.051	-2.272	-2.429	-2.550	-2.647	-2.727	-2.795	-2.855	-2.907	-2.953	-2.995	-3.033
4.00	-2.237	-2.491	-2.672	-2.810	-2.922	-3.015	-3.093	-3.161	-3.221	-3.275	-3.323	-3.366
4.50	-2.434	-2.720	-2.924	-3.080	-3.206	-3.310	-3.399	-3.476	-3.544	-3.605	-3.659	-3.708
5.00	-2.638	-2.957	-3.183	-3.356	-3.497	-3.613	-3.712	-3.798	-3.873	-3.941	-4.001	-4.056
6.00	-3.062	-3.444	-3.715	-3.923	-4.092	-4.231	-4.351	-4.455	-4.546	-4.627	-4.700	-4.766
7.00	-3.499	-3.943	-4.260	-4.502	-4.699	-4.862	-5.002	-5.123	-5.230	-5.325	-5.411	-5.488
8.00	-3.945	-4.451	-4.812	-5.090	-5.315	-5.502	-5.661	-5.800	-5.922	-6.031	-6.129	-6.218
9.00	-4.397	-4.965	-5.371	-5.684	-5.936	-6.146	-6.326	-6.482	-6.620	-6.742	-6.853	-6.954
10.00	-4.852	-5.483	-5.933	-6.280	-6.560	-6.795	-6.994	-7.168	-7.321	-7.458	-7.581	-7.592

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-3. Land's Factors ($H_{.05}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

s_y \ n	15	16	17	18	19	21	23	25	28	31	36
0.10	-1.666	-1.663	-1.661	-1.659	-1.658	-1.655	-1.653	-1.651	-1.649	-1.648	-1.647
0.20	-1.640	-1.639	-1.638	-1.638	-1.637	-1.636	-1.636	-1.635	-1.636	-1.636	-1.636
0.30	-1.625	-1.625	-1.625	-1.626	-1.626	-1.627	-1.628	-1.629	-1.630	-1.632	-1.633
0.40	-1.617	-1.618	-1.620	-1.622	-1.622	-1.625	-1.627	-1.629	-1.632	-1.635	-1.639
0.50	-1.618	-1.620	-1.622	-1.625	-1.627	-1.631	-1.634	-1.638	-1.642	-1.646	-1.651
0.60	-1.625	-1.629	-1.632	-1.635	-1.638	-1.643	-1.648	-1.652	-1.658	-1.662	-1.659
0.70	-1.638	-1.643	-1.647	-1.651	-1.654	-1.661	-1.667	-1.672	-1.679	-1.686	-2.694
0.80	-1.656	-1.662	-1.667	-1.672	-1.677	-1.685	-1.691	-1.698	-1.706	-1.714	-1.724
0.90	-1.680	-1.686	-1.692	-1.698	-1.703	-1.713	-1.721	-1.728	-1.738	-1.747	-1.759
1.00	-1.707	-1.715	-1.722	-1.728	-1.734	-1.745	-1.755	-1.763	-1.774	-1.784	-1.798
1.25	-1.793	-1.803	-1.812	-1.820	-1.828	-1.842	-1.854	-1.866	-1.880	-1.893	-1.911
1.50	-1.896	-1.909	-1.920	-1.930	-1.940	-1.958	-1.973	-1.987	-2.005	-2.020	-2.043
1.75	-2.015	-2.029	-2.043	-2.055	-2.067	-2.088	-2.107	-2.123	-2.145	-2.164	-2.190
2.00	-2.144	-2.162	-2.177	-2.192	-2.205	-2.230	-2.251	-2.271	-2.269	-2.318	-2.349
2.50	-2.430	-2.452	-2.472	-2.491	-2.508	-2.540	-2.568	-2.593	-2.625	-2.654	-2.694
3.00	-2.740	-2.767	2.792	-2.815	-2.836	-2.874	-2.908	-2.939	-2.979	-3.014	-3.063
3.50	-3.067	-3.099	3.128	-3.155	-3.180	-3.226	-3.266	-3.302	-3.349	-3.391	-3.448
4.00	-3.406	-3.443	3.476	-3.507	-3.536	-3.589	-3.635	-3.677	-3.731	-3.779	-3.846
4.50	-3.753	-3.794	3.833	-3.868	-3.901	-3.960	-4.013	-4.060	-4.122	-4.176	-4.252
5.00	-4.107	-4.153	4.195	-4.235	-4.272	-4.338	-4.397	-4.449	-4.518	-4.579	-4.664
6.00	-4.827	-4.882	4.934	-4.981	-5.026	-5.106	-5.177	-5.241	-5.325	-5.397	-5.500
7.00	-5.559	-5.624	5.685	-5.741	-5.793	-5.886	-5.970	-6.045	-6.142	-6.227	-6.348
8.00	-6.300	-6.374	6.443	-6.507	-6.566	-6.674	-6.770	-6.855	-6.968	-7.066	-7.204
9.00	-7.045	-7.129	7.207	-7.278	-7.346	-7.468	-7.575	-7.672	-7.798	-7.909	-8.064
10.00	-7.794	-7.888	7.974	-8.054	-8.129	-8.264	-8.385	-8.491	-8.632	-10.060	-8.928

Table 21-4. Land's Factors ($H_{.10}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	-1.431	-1.351	-1.320	-1.305	-1.296	-1.291	-1.287	-1.285	-1.283	-1.281	-1.281	-1.280
0.20	-1.350	-1.299	-1.281	-1.273	-1.268	-1.267	-1.266	-1.266	-1.266	-1.266	-1.266	-1.266
0.30	-1.289	-1.260	-1.252	-1.251	-1.250	-1.251	-1.253	-1.254	-1.255	-1.257	-1.258	-1.259
0.40	-1.245	-1.233	-1.233	-1.236	-1.239	-1.243	-1.246	-1.249	-1.252	-1.254	-1.257	-1.258
0.50	-1.213	-1.214	-1.221	-1.228	-1.234	-1.240	-1.245	-1.250	-1.254	-1.257	-1.261	-1.264
0.60	-1.190	-1.202	-1.215	-1.226	-1.235	-1.243	-1.250	-1.256	-1.261	-1.266	-1.270	-1.274
0.70	-1.176	-1.197	-1.215	-1.229	-1.241	-1.251	-1.259	-1.266	-1.273	-1.278	-1.283	-1.288
0.80	-1.168	-1.197	-1.219	-1.237	-1.251	-1.262	-1.272	-1.280	-1.288	-1.294	-1.301	-1.306
0.90	-1.165	-1.201	-1.227	-1.248	-1.264	-1.277	-1.289	-1.298	-1.307	-1.314	-1.321	-1.327
1.00	-1.166	-1.208	-1.239	-1.262	-1.281	-1.296	-1.309	-1.320	-1.329	-1.337	-1.345	-1.353
1.25	-1.184	-1.240	-1.280	-1.310	-1.334	-1.353	-1.370	-1.384	-1.396	-1.407	-1.471	-1.426
1.50	-1.217	-1.285	-1.334	-1.371	-1.400	-1.424	-1.444	-1.462	-1.477	-1.491	-1.503	-1.514
1.75	-1.260	-1.341	-1.398	-1.442	-1.477	-1.505	-1.530	-1.551	-1.569	-1.585	-1.599	-1.612
2.00	-1.310	-1.403	-1.470	-1.521	-1.562	-1.595	-1.623	-1.647	1.669	-1.688	-1.704	-1.719
2.50	-1.426	-1.547	-1.634	-1.700	-1.751	-1.794	-1.830	-1.862	-1.889	-1.913	-1.934	-1.953
3.00	-1.560	-1.712	-1.817	-1.897	-1.960	-2.013	-2.057	-2.095	-2.128	-2.157	-2.183	-2.207
3.50	-1.710	-1.889	-2.014	-2.108	-2.183	-2.244	-2.296	-2.341	-2.380	-2.415	-2.446	-2.473
4.00	-1.871	-2.078	-2.221	-2.329	-2.415	-2.485	-2.545	-2.596	-2.641	-2.681	-2.717	-2.749
4.50	-2.041	-2.274	-2.435	-2.557	-2.653	-2.733	-2.801	-2.858	-2.910	-2.955	-2.995	-3.031
5.00	-2.217	-2.475	-2.654	-2.789	-2.897	-2.986	-3.061	-3.126	-3.183	-3.233	-3.278	-3.319
6.00	-2.581	-2.889	-3.104	-3.267	-3.396	-3.503	-3.593	-3.671	-3.740	-3.800	-3.855	-3.904
7.00	-2.955	-3.314	-3.564	-3.753	-3.904	-4.029	-4.135	-4.226	-4.306	-4.377	-4.441	-4.498
8.00	-3.336	-3.744	-4.030	-4.246	-4.418	-4.561	-4.683	-4.787	-4.879	-4.960	-5.033	-5.099
9.00	-3.721	-4.180	-4.500	-4.742	-4.937	-5.098	-5.234	-5.352	-5.455	-5.547	-5.629	-5.703
10.00	-4.109	-4.618	-4.973	-5.243	-5.459	-5.638	-5.789	-5.920	-6.035	-6.137	-6.228	-6.311

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-4. Land's Factors ($H_{.10}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y\n	15	16	17	18	19	21	23	25	28	31	36
0.10	-1.279	-1.278	-1.278	-1.278	-1.278	-1.277	-1.277	-1.277	-1.277	-1.277	-1.277
0.20	-1.266	-1.267	-1.267	-1.267	-1.268	-1.268	-1.270	-1.270	-1.271	-1.272	-1.272
0.30	-1.260	-1.261	-1.262	-1.263	-1.265	-1.266	-1.268	-1.269	-1.271	-1.272	-1.275
0.40	-1.261	-1.262	-1.264	-1.266	-1.267	-1.270	-1.272	-1.274	-1.277	-1.279	-1.282
0.50	-1.266	-1.269	-1.271	-1.273	-1.275	-1.279	-1.281	-1.284	-1.288	-1.291	-1.295
0.60	-1.277	-1.280	-1.283	-1.286	-1.288	-1.292	-1.296	-1.299	-1.304	-1.307	-1.313
0.70	-1.292	-1.296	-1.299	-1.302	-1.305	-1.310	-1.315	-1.319	-1.324	-1.329	-1.336
0.80	-1.311	-1.315	-1.319	-1.323	-1.326	-1.332	-1.338	-1.342	-1.349	-1.354	-1.361
0.90	-1.333	-1.338	-1.342	-1.346	-1.351	-1.358	-1.364	-1.369	-1.377	-1.383	-1.391
1.00	-1.358	-1.364	-1.369	-1.374	-1.378	-1.387	-1.393	-1.399	-1.408	-1.414	-1.424
1.25	-1.434	-1.441	-1.448	-1.455	-1.460	-1.470	-1.479	-1.487	-1.498	-1.507	-1.519
1.50	-1.523	-1.533	-1.541	-1.548	-1.555	-1.568	-1.579	-1.589	-1.602	-1.613	-1.629
1.75	-1.624	-1.634	-1.645	-1.654	-1.662	-1.677	-1.690	-1.703	-1.718	-1.732	-1.750
2.00	-1.733	-1.746	-1.757	-1.767	-1.777	-1.795	-1.810	-1.825	-1.843	-1.859	-1.881
2.50	-1.971	-1.987	-2.002	-2.016	-2.029	-2.051	-2.072	-2.090	-2.113	-2.133	-2.161
3.00	-2.229	-2.248	2.266	-2.283	-2.298	-2.326	-2.351	-2.373	-2.402	-2.427	-2.461
3.50	-2.499	-2.522	2.544	-2.563	-2.581	-2.615	-2.644	-2.670	-2.704	-2.733	-2.775
4.00	-2.778	-2.805	2.830	-2.853	-2.874	-2.913	-2.946	-2.976	-3.015	-3.050	-3.097
4.50	-3.064	-3.095	3.123	-3.149	-3.173	-3.217	-3.255	-3.288	-3.333	-3.372	-3.426
5.00	-3.356	-3.390	3.421	-3.450	-3.477	-3.525	-3.567	-3.605	-3.655	-3.698	-3.759
6.00	-3.949	-3.989	4.027	-4.062	-4.094	-4.153	-4.204	-4.250	-4.311	-4.363	-4.436
7.00	-4.549	-4.599	4.642	-4.683	-4.721	-4.790	-4.850	-4.604	-4.975	-5.037	-5.122
8.00	-5.159	-5.213	5.264	-5.311	-5.354	-5.433	-5.002	-5.564	-5.645	-5.715	-5.815
9.00	-5.771	-5.833	5.890	-5.942	-5.992	-6.080	-6.158	-6.228	-6.319	-6.399	-6.510
10.00	-6.386	-6.455	6.518	-6.578	-6.632	-6.730	-6.817	-6.894	-6.996	-8.755	-7.208

Table 21-5. Land's Factors ($H_{.90}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	1.686	1.506	1.438	1.403	1.381	1.367	1.356	1.349	1.343	1.338	1.334	1.330
0.20	1.885	1.620	1.522	1.472	1.442	1.422	1.407	1.396	1.387	1.380	1.374	1.369
0.30	2.156	1.763	1.627	1.558	1.517	1.489	1.469	1.453	1.441	1.432	1.424	1.417
0.40	2.521	1.942	1.755	1.662	1.607	1.569	1.543	1.523	1.507	1.494	1.483	1.474
0.50	2.990	2.160	1.907	1.785	1.712	1.664	1.630	1.604	1.583	1.567	1.553	1.542
0.60	3.542	2.417	2.084	1.926	1.834	1.773	1.729	1.696	1.671	1.650	1.633	1.619
0.70	4.136	2.708	2.284	2.085	1.970	1.894	1.849	1.800	1.768	1.743	1.722	1.705
0.80	4.742	3.023	2.503	2.260	2.119	2.027	1.962	1.914	1.876	1.845	1.820	1.799
0.90	5.349	3.353	2.736	2.447	2.280	2.171	2.094	2.036	1.992	1.955	1.926	1.901
1.00	5.955	3.691	2.980	2.644	2.450	2.324	2.234	2.167	2.115	2.073	2.038	2.010
1.25	7.466	4.558	3.617	3.167	2.904	2.732	2.610	2.518	2.448	2.391	2.344	2.305
1.50	8.973	5.436	4.276	3.713	3.383	3.166	3.012	2.896	2.806	2.733	2.674	2.623
1.75	10.480	6.319	4.944	4.273	3.877	3.615	3.429	3.289	3.180	3.092	3.109	2.959
2.00	11.980	7.206	5.619	4.842	4.380	4.075	3.857	3.693	3.564	3.461	3.376	3.305
2.50	14.990	8.986	6.979	5.990	5.401	5.010	4.730	4.518	4.353	4.220	4.110	4.017
3.00	18.000	10.770	8.346	7.147	6.434	5.958	5.617	5.359	5.157	4.994	4.860	4.746
3.50	21.000	12.560	9.717	8.312	7.473	6.913	6.511	6.208	5.970	5.778	5.619	5.486
4.00	24.000	14.340	11.090	9.480	8.516	7.873	7.411	7.062	6.788	6.566	6.384	6.299
4.50	27.010	16.130	12.470	10.650	9.562	8.836	8.314	7.919	7.610	7.360	7.154	6.978
5.00	30.010	17.920	13.840	11.820	10.610	9.800	9.219	8.779	8.434	8.155	7.924	7.729
6.00	36.020	21.490	16.600	14.170	12.710	11.740	11.030	10.500	10.090	9.751	9.473	9.238
7.00	42.020	25.070	19.350	16.510	14.810	13.670	12.850	12.230	11.750	11.350	11.030	10.750
8.00	48.030	28.650	22.110	18.860	16.910	15.610	14.670	13.960	13.410	12.960	12.580	12.270
9.00	54.030	32.230	24.870	21.210	19.020	17.550	16.500	15.700	15.070	14.560	14.140	13.790
10.00	60.040	35.810	27.630	23.560	21.120	19.490	18.320	17.430	16.730	16.170	15.700	15.310

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-5. Land's Factors ($H_{.90}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

s_y \ n	15	16	17	18	19	21	23	25	28	31	36
0.10	1.328	1.325	1.323	1.322	1.320	1.317	1.315	1.313	1.310	1.308	1.306
0.20	1.365	1.361	1.358	1.355	1.353	1.348	1.345	1.342	1.338	1.335	1.332
0.30	1.411	1.406	1.402	1.398	1.394	1.388	1.383	1.379	1.374	1.370	1.364
0.40	1.467	1.460	1.455	1.449	1.444	1.437	1.430	1.425	1.417	1.412	1.404
0.50	1.532	1.524	1.516	1.509	1.503	1.494	1.485	1.478	1.469	1.462	1.452
0.60	1.606	1.596	1.586	1.578	1.570	1.558	1.548	1.539	1.528	1.519	1.507
0.70	1.690	1.766	1.666	1.655	1.646	1.631	1.618	1.607	1.594	1.583	1.568
0.80	1.781	1.765	1.752	1.739	1.728	1.710	1.695	1.682	1.667	1.654	1.636
0.90	1.880	1.861	1.845	1.831	1.819	1.797	1.779	1.764	1.745	1.731	1.710
1.00	1.985	1.963	1.945	1.929	1.914	1.889	1.868	1.851	1.830	1.812	1.789
1.25	2.271	2.242	2.217	2.195	2.174	2.141	2.113	2.089	2.060	2.036	2.005
1.50	2.581	2.544	2.512	2.483	2.458	2.415	2.379	2.349	2.312	2.282	2.242
1.75	2.907	2.862	2.823	2.788	2.757	2.705	2.662	2.625	2.579	2.543	2.494
2.00	3.244	3.191	3.145	3.104	3.069	3.005	2.954	2.911	2.858	2.814	2.758
2.50	3.938	3.870	3.810	3.757	3.710	3.629	3.562	3.506	3.463	3.380	3.305
3.00	4.650	4.565	4.492	4.427	4.369	4.270	4.188	4.119	4.033	3.964	3.872
3.50	5.370	5.271	5.184	5.107	5.039	4.921	4.825	4.743	4.641	4.559	4.450
4.00	6.097	5.983	5.883	5.794	5.715	5.580	5.468	5.374	5.257	5.161	5.036
4.50	6.829	6.699	6.586	6.485	6.396	6.243	6.116	6.009	5.876	5.769	5.626
5.00	7.563	7.418	7.292	7.179	7.080	6.909	6.767	6.648	6.500	6.379	6.219
6.00	9.037	8.862	8.710	8.575	8.454	8.248	8.076	-7.933	7.753	7.607	7.415
7.00	10.520	10.310	10.130	9.975	9.833	9.592	9.391	9.222	9.013	8.842	8.616
8.00	12.000	11.770	11.560	11.380	11.220	10.940	10.710	10.520	10.280	10.080	9.821
9.00	13.480	13.220	12.990	12.780	12.600	12.290	12.030	11.810	11.540	11.320	11.030
10.00	14.970	14.680	14.420	14.190	13.990	13.640	13.350	13.110	12.810	12.560	12.240

Table 21-6. Land's Factors ($H_{.95}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	2.750	2.222	2.035	1.942	1.886	1.849	1.822	1.802	1.787	1.775	1.763	1.756
0.20	3.295	2.463	2.198	2.069	1.992	1.943	1.908	1.881	1.860	1.843	1.830	1.818
0.30	4.109	2.777	2.402	2.226	2.125	2.058	2.011	1.977	1.949	1.927	1.909	1.894
0.40	5.220	3.175	2.651	2.415	2.282	2.195	2.134	2.089	2.054	2.026	2.003	1.984
0.50	6.495	3.658	2.947	2.638	2.465	2.354	2.277	2.220	2.176	2.141	2.112	2.088
0.60	7.807	4.209	3.287	2.892	2.673	2.534	2.439	2.368	2.314	2.271	2.235	2.206
0.70	9.120	4.801	3.662	3.173	2.904	2.735	2.618	2.532	2.466	2.414	2.371	2.336
0.80	10.430	5.414	4.062	3.477	3.155	2.952	2.813	2.710	2.632	2.570	2.520	2.479
0.90	11.740	6.038	4.478	3.796	3.420	3.184	3.021	2.902	2.810	2.738	2.679	2.631
1.00	13.050	6.669	4.905	4.127	3.698	3.426	3.239	3.103	2.998	2.915	2.848	2.792
1.25	16.330	8.265	6.001	4.990	4.426	4.069	3.820	3.639	3.500	3.389	3.300	3.226
1.50	19.600	9.874	7.120	5.880	5.184	4.741	4.433	4.207	4.033	3.896	3.784	3.691
1.75	22.870	11.490	8.250	6.786	5.960	5.432	5.065	4.795	4.587	4.422	4.288	4.176
2.00	26.140	13.110	9.387	7.701	6.747	6.135	5.710	5.396	5.154	4.962	4.805	4.675
2.50	32.690	16.350	11.670	9.546	8.339	7.563	7.021	6.621	6.312	6.067	5.866	5.698
3.00	39.230	19.600	13.970	11.400	9.945	9.006	8.350	7.864	7.489	7.191	6.947	6.743
3.50	45.770	22.850	16.270	13.270	11.560	10.460	9.688	9.118	8.677	8.326	8.039	7.799
4.00	52.310	26.110	18.580	15.140	13.180	11.920	11.030	10.380	9.872	9.469	9.140	8.864
4.50	58.850	29.360	20.880	17.010	14.800	13.380	12.380	11.640	11.070	10.620	10.240	9.933
5.00	65.390	32.620	23.190	18.880	16.430	14.840	13.730	12.910	12.270	11.770	11.350	11.010
6.00	78.470	39.130	27.810	22.630	19.680	17.780	16.440	15.450	14.690	14.080	13.580	13.160
7.00	91.550	45.650	32.430	26.390	22.940	20.720	19.160	18.000	17.100	16.390	15.810	15.320
8.00	104.600	52.160	37.060	30.140	26.200	23.660	21.870	20.550	19.530	18.710	18.040	17.480
9.00	117.700	58.680	41.680	33.900	29.460	26.600	24.590	23.100	21.950	21.030	20.280	19.650
10.00	130.800	65.200	46.310	37.660	32.730	29.540	27.310	25.660	24.380	23.350	22.510	21.820

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-6. Land's Factors ($H_{.95}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

s_y \ n	15	16	17	18	19	21	23	25	28	31	36
0.10	1.749	1.743	1.738	1.733	1.729	1.722	1.716	1.711	1.706	1.701	1.695
0.20	1.809	1.800	1.793	1.787	1.781	1.771	1.763	1.756	1.749	1.742	1.734
0.30	1.882	1.871	1.861	1.853	1.845	1.833	1.822	1.813	1.802	1.793	1.783
0.40	1.968	1.954	1.942	1.931	1.921	1.905	1.892	1.881	1.867	1.856	1.841
0.50	2.068	2.050	2.035	2.021	2.009	1.989	1.973	1.959	1.942	1.928	1.910
0.60	2.181	2.160	2.141	2.124	2.110	2.085	2.065	2.048	2.027	2.010	1.988
0.70	2.306	2.280	2.258	2.238	2.221	2.191	2.167	2.147	2.122	2.102	2.075
0.80	2.443	2.412	2.386	2.362	2.342	2.307	2.279	2.255	2.225	2.202	2.171
0.90	2.589	2.554	2.523	2.496	2.472	2.432	2.399	2.371	2.337	2.310	2.273
1.00	2.744	2.704	2.669	2.638	2.611	2.564	2.526	2.495	2.456	2.423	2.383
1.25	3.163	3.109	3.062	3.021	2.984	2.923	2.873	2.830	2.779	2.737	2.682
1.50	3.612	3.544	3.485	3.434	3.388	3.311	3.248	3.195	3.130	3.077	3.008
1.75	4.081	4.000	3.929	3.867	3.812	3.719	3.643	3.579	3.501	3.437	3.355
2.00	4.564	4.470	4.387	4.314	4.251	4.141	4.052	3.977	3.886	3.812	3.715
2.50	5.557	5.435	5.328	5.236	5.153	5.013	4.898	4.802	4.683	4.588	4.463
3.00	6.570	6.422	6.293	6.179	6.078	5.907	5.766	5.649	5.504	5.388	5.234
3.50	7.596	7.422	7.269	7.136	7.016	6.815	6.649	6.510	6.340	6.201	6.020
4.00	8.630	8.429	8.254	8.100	7.963	7.731	7.540	7.380	7.184	7.024	6.816
4.50	9.669	9.442	9.244	9.070	8.916	8.652	8.437	8.257	8.034	7.854	7.618
5.00	10.710	10.460	10.240	10.040	9.872	9.579	9.338	9.137	8.889	8.688	8.424
6.00	12.810	12.500	12.230	12.000	11.790	11.440	11.150	10.910	10.610	10.360	10.050
7.00	14.900	14.550	14.240	13.960	13.720	13.310	12.970	12.680	12.330	12.050	11.680
8.00	17.010	16.600	16.240	15.930	15.650	15.180	14.790	14.470	14.060	13.740	13.310
9.00	19.110	18.650	18.250	17.900	17.590	17.050	16.620	16.250	15.800	15.430	14.950
10.00	21.220	20.710	20.260	19.870	19.520	18.930	18.440	18.040	17.530	12.560	16.590

Table 21-7. Land's Factors ($H_{.975}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	4.367	3.100	2.703	2.513	2.403	2.330	2.879	2.242	2.212	2.190	2.169	2.155
0.20	5.849	3.571	2.987	2.723	2.573	2.476	2.409	2.359	2.321	2.291	2.265	2.245
0.30	8.166	4.210	3.348	2.982	2.781	2.653	2.565	2.501	2.451	2.411	2.380	2.353
0.40	10.860	5.031	3.794	3.296	3.030	2.864	2.750	2.667	2.604	2.554	2.514	2.480
0.50	13.590	5.989	4.322	3.664	3.319	3.107	2.963	2.859	2.780	2.718	2.668	2.626
0.60	16.310	7.019	4.914	4.081	3.647	3.382	3.204	3.076	2.979	2.903	2.842	2.791
0.70	19.040	8.083	5.548	4.534	4.005	3.684	3.469	3.314	3.198	3.106	3.033	2.973
0.80	21.760	9.164	6.208	5.014	4.389	4.009	3.754	3.572	3.434	3.327	3.240	3.169
0.90	24.490	10.250	6.885	5.512	4.791	4.351	4.056	3.844	3.685	3.561	3.461	3.379
1.00	27.210	11.350	7.572	6.024	5.206	4.707	4.371	4.130	3.949	3.807	3.693	3.599
1.25	34.020	14.110	9.320	7.339	6.285	5.636	5.199	4.884	4.647	4.461	4.312	4.189
1.50	40.830	16.880	11.090	8.684	7.397	6.602	6.064	5.676	5.383	5.153	4.968	4.815
1.75	47.630	19.650	12.880	10.050	8.528	7.588	6.951	6.490	6.142	5.869	5.648	5.466
2.00	54.440	22.430	14.670	11.420	9.671	8.588	7.853	7.320	6.916	6.599	6.344	6.133
2.50	68.050	28.000	18.270	14.180	11.980	10.610	9.681	9.006	8.493	8.091	7.765	7.497
3.00	81.660	33.580	21.870	16.960	14.300	12.650	11.530	10.710	10.090	9.605	9.210	8.884
3.50	95.270	39.160	25.490	19.740	16.640	14.710	13.390	12.430	11.700	11.130	10.670	10.290
4.00	108.900	44.740	29.110	22.530	18.980	16.770	15.260	14.160	13.320	12.670	12.140	11.700
4.50	122.500	50.320	32.730	25.320	21.320	18.830	17.130	15.890	14.950	14.210	13.610	13.110
5.00	136.100	55.900	36.350	28.120	23.670	20.890	19.000	17.630	16.580	15.750	15.090	14.540
6.00	163.300	67.070	43.590	33.710	28.370	25.030	22.760	21.100	19.850	18.850	18.050	17.390
7.00	190.600	78.240	50.840	39.310	33.070	29.180	26.520	24.590	23.120	21.960	21.020	20.250
8.00	217.800	89.410	58.100	44.910	37.770	33.330	30.280	28.080	26.390	25.070	23.990	23.110
9.00	245.000	100.600	65.350	50.510	42.480	37.470	34.050	31.570	29.670	28.180	26.970	25.970
10.00	272.200	111.800	72.600	56.110	47.190	41.620	37.820	35.060	32.950	31.290	29.950	28.840

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-7. Land's Factors ($H_{.975}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

$s_y \backslash n$	15	16	17	18	19	21	23	25	28	31	36
0.10	2.141	2.130	2.120	2.112	2.104	2.091	2.081	2.072	2.062	2.053	2.043
0.20	2.227	2.212	2.199	2.188	2.178	2.161	2.147	2.135	2.121	2.110	2.096
0.30	2.331	2.311	2.295	2.280	2.267	2.246	2.228	2.213	2.194	2.180	2.161
0.40	2.452	2.428	2.407	2.388	2.372	2.345	2.323	2.305	2.281	2.263	2.239
0.50	2.592	2.562	2.536	2.513	2.493	2.460	2.432	2.409	2.381	2.359	2.329
0.60	2.749	2.712	2.681	2.653	2.630	2.588	2.555	2.528	2.494	2.467	2.432
0.70	2.922	2.879	2.841	2.808	2.780	2.731	2.692	2.659	2.619	2.587	2.545
0.80	3.109	3.059	3.015	2.976	2.943	2.886	2.840	2.802	2.755	2.717	2.668
0.90	3.310	3.251	3.200	3.157	3.117	3.052	2.999	2.955	2.901	2.858	2.801
1.00	3.521	3.454	3.397	3.347	3.302	3.227	3.167	3.116	3.056	3.007	2.943
1.25	4.086	3.998	3.922	3.856	3.798	3.700	3.621	3.555	3.474	3.410	3.327
1.50	4.688	4.579	4.485	4.402	4.330	4.209	4.109	4.027	3.927	3.847	3.743
1.75	5.314	5.183	5.070	4.972	4.887	4.740	4.622	4.524	4.404	4.307	4.183
2.00	5.956	5.804	5.674	5.559	5.461	5.289	5.151	5.037	4.897	4.784	4.639
2.50	7.271	7.078	6.911	6.765	6.636	6.419	6.243	6.096	5.916	5.772	5.585
3.00	8.610	8.376	8.174	7.996	7.840	7.576	7.361	7.182	6.963	6.787	6.559
3.50	9.964	9.689	9.451	9.242	9.058	8.748	8.495	8.284	8.027	7.820	7.551
4.00	11.330	11.010	10.740	10.500	10.290	9.930	9.639	9.397	9.101	8.863	8.554
4.50	12.700	12.340	12.030	11.760	11.520	11.120	10.790	10.520	10.180	9.913	9.564
5.00	14.070	13.670	13.330	13.030	12.760	12.310	11.950	11.640	11.270	10.970	10.580
6.00	16.830	16.350	15.930	15.570	15.250	14.710	14.270	13.900	13.450	13.090	12.620
7.00	19.590	19.030	18.550	18.130	17.750	17.120	16.600	16.170	15.650	15.220	14.670
8.00	22.360	21.720	21.170	20.680	20.250	19.530	18.940	18.450	17.840	17.360	16.730
9.00	25.130	24.410	23.790	23.240	22.760	21.940	21.280	20.720	20.050	19.500	18.790
10.00	27.900	27.100	26.410	25.800	25.270	24.360	23.620	23.000	22.250	17.130	20.850

Table 21-8. Land's Factors ($H_{.99}$) for Confidence Bounds on Lognormal Arithmetic Mean for *n* = 3(1)19(2)25(3)31(5)36

s_y \ n	3	4	5	6	7	8	9	10	11	12	13	14
0.10	8.328	4.665	3.760	3.360	3.137	2.994	2.897	2.825	2.770	2.727	2.691	2.663
0.20	13.940	5.768	4.310	3.731	3.422	3.231	3.101	3.006	2.935	2.878	2.833	2.796
0.30	20.880	7.336	5.035	4.199	3.775	3.519	3.348	3.225	3.132	3.060	3.002	2.955
0.40	27.850	9.244	5.934	4.771	4.199	3.862	3.640	3.482	3.364	3.273	3.200	3.140
0.50	34.820	11.290	6.966	5.434	4.691	4.258	3.976	3.778	3.631	3.517	3.426	3.353
0.60	41.780	13.390	8.077	6.167	5.240	4.702	4.353	4.109	3.929	3.790	3.680	3.590
0.70	48.750	15.520	9.231	6.947	5.831	5.183	4.764	4.471	4.255	4.089	3.958	3.851
0.80	55.710	17.650	10.410	7.757	6.452	5.693	5.201	4.858	4.604	4.110	4.256	4.131
0.90	62.580	19.800	11.600	8.856	7.095	6.225	5.659	5.264	4.973	4.750	4.572	4.428
1.00	69.650	21.950	12.810	9.430	7.753	6.772	6.133	5.686	5.357	5.103	4.903	4.740
1.25	87.060	27.350	15.850	11.580	9.442	8.186	7.365	6.789	6.363	6.036	5.775	5.564
1.50	104.500	32.770	18.920	13.760	11.170	9.641	8.640	7.936	7.414	7.102	6.693	6.432
1.75	121.900	38.190	22.010	15.950	12.920	11.120	9.940	9.109	8.492	8.016	7.638	7.330
2.00	139.300	43.610	25.100	18.160	14.680	12.610	11.260	10.300	9.587	9.039	8.602	8.245
2.50	174.100	54.470	31.290	22.600	18.220	15.630	13.920	12.710	11.810	11.120	10.560	10.110
3.00	208.900	65.340	37.500	27.050	21.790	18.660	16.600	15.140	14.060	13.220	12.540	12.010
3.50	243.800	76.210	43.720	31.520	25.360	21.710	19.300	17.590	16.320	15.340	14.560	13.910
4.00	278.600	87.080	49.940	35.980	28.940	24.760	22.000	20.050	18.590	17.470	16.570	15.840
4.50	313.400	97.960	56.160	40.450	32.530	27.820	24.710	22.510	20.870	19.600	18.590	17.760
5.00	348.200	108.800	62.380	44.930	36.120	30.880	27.420	24.980	23.150	21.740	20.620	19.700
6.00	417.900	130.600	74.840	53.880	43.300	37.010	32.860	29.920	27.730	26.030	24.680	23.570
7.00	487.500	152.300	87.290	62.840	50.490	43.140	38.300	34.870	32.310	30.330	28.750	27.450
8.00	557.200	174.100	99.750	71.790	57.680	49.280	43.740	39.820	36.890	34.630	32.820	31.340
9.00	626.900	195.900	112.200	80.750	64.870	55.430	49.190	44.770	41.480	38.930	36.900	35.230
10.00	696.500	217.600	124.700	89.720	72.070	61.570	54.640	49.730	46.070	43.240	40.980	39.130

Source: Land (1975)
Footnote. Notation n = 3(1)19(2)25(3)31(5)36 is shorthand for n from 3 to 19 by unit steps, from 19 to 25 by 2's, from 25 to 31 by 3's, and from 31 to 36 by 5's

Table 21-8. Land's Factors ($H_{.99}$) for Confidence Bounds on Lognormal Arithmetic Mean for n = 3(1)19(2)25(3)31(5)36

s_y \ n	15	16	17	18	19	21	23	25	28	31	36
0.10	2.638	2.618	2.600	2.584	2.571	2.548	2.529	2.514	2.495	2.480	2.462
0.20	2.764	2.737	2.714	2.694	2.676	2.647	2.623	2.602	2.579	2.559	2.534
0.30	2.914	2.880	2.851	2.826	2.803	2.767	2.735	2.710	2.679	2.655	2.623
0.40	3.090	3.047	3.011	2.979	2.951	2.904	2.867	2.836	2.798	2.767	2.729
0.50	3.291	3.239	3.194	3.155	3.121	3.064	3.017	2.979	2.933	2.896	2.849
0.60	3.515	3.453	3.398	3.351	3.311	3.242	3.186	3.141	3.085	3.041	2.984
0.70	3.762	3.687	3.623	3.567	3.519	3.438	3.372	3.318	3.253	3.200	3.134
0.80	4.027	3.940	3.865	3.800	3.744	3.649	3.573	3.510	3.434	3.373	3.296
0.90	4.309	4.209	4.123	4.049	3.983	3.875	3.787	3.716	3.628	3.559	3.471
1.00	4.605	4.491	4.394	4.309	4.235	4.112	4.013	3.931	3.833	3.755	3.655
1.25	5.388	5.240	5.114	5.004	4.908	4.749	4.620	4.513	4.385	4.283	4.143
1.50	6.217	6.034	5.878	5.743	5.625	5.426	5.267	5.136	4.978	4.852	4.691
1.75	7.074	6.857	6.671	6.510	6.369	6.134	5.944	5.788	5.599	5.449	5.256
2.00	7.949	7.699	7.483	7.297	7.134	6.861	6.641	6.460	6.241	6.066	5.842
2.50	9.735	9.415	9.145	8.907	8.700	8.353	8.073	7.842	7.562	7.339	7.052
3.00	11.550	11.170	10.840	10.550	10.300	9.875	9.536	9.256	8.916	8.645	8.269
3.50	13.380	12.930	12.540	12.210	11.910	11.420	11.020	10.690	10.290	9.970	9.560
4.00	15.230	14.710	14.260	13.880	13.540	12.970	12.510	12.130	11.670	11.310	10.840
4.50	17.070	16.490	15.990	15.550	15.170	14.350	14.010	13.590	13.070	12.660	12.120
5.00	18.930	18.280	17.720	17.240	16.810	16.100	15.520	15.050	14.470	14.010	13.420
6.00	22.650	21.870	21.190	20.610	20.100	19.240	18.550	17.980	17.280	16.730	16.010
7.00	26.380	25.460	24.680	24.000	23.400	22.390	21.580	20.920	20.100	19.450	18.620
8.00	30.110	29.060	28.170	27.390	26.700	25.550	24.630	23.860	22.930	22.190	21.230
9.00	33.840	32.670	31.660	30.780	30.010	28.720	27.670	26.810	25.760	24.930	23.800
10.00	37.580	36.280	35.150	34.180	33.320	31.880	30.720	29.770	28.600	21.640	26.470

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Table 21-9. Factors (τ) for Parametric Upper Conf. Bounds on Percentiles (P)

	P = 0.80					P = 0.90				
n\(1−α)	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**
2	3.417	6.987	14.051	28.140	70.376	5.049	10.253	20.581	41.201	103.029
3	2.016	3.039	4.424	6.343	10.111	2.871	4.258	6.155	8.797	13.995
4	1.675	2.295	3.026	3.915	5.417	2.372	3.188	4.162	5.354	7.380
5	1.514	1.976	2.483	3.058	3.958	2.145	2.742	3.407	4.166	5.362
6	1.417	1.795	2.191	2.621	3.262	2.012	2.494	3.006	3.568	4.411
7	1.352	1.676	2.005	2.353	2.854	1.923	2.333	2.755	3.206	3.859
8	1.304	1.590	1.875	2.170	2.584	1.859	2.219	2.582	2.960	3.497
9	1.266	1.525	1.779	2.036	2.391	1.809	2.133	2.454	2.783	3.240
10	1.237	1.474	1.703	1.933	2.246	1.770	2.066	2.355	2.647	3.048
11	1.212	1.433	1.643	1.851	2.131	1.738	2.011	2.275	2.540	2.898
12	1.192	1.398	1.593	1.784	2.039	1.711	1.966	2.210	2.452	2.777
13	1.174	1.368	1.551	1.728	1.963	1.689	1.928	2.155	2.379	2.677
14	1.159	1.343	1.514	1.681	1.898	1.669	1.895	2.109	2.317	2.593
15	1.145	1.321	1.483	1.639	1.843	1.652	1.867	2.068	2.264	2.521
16	1.133	1.301	1.455	1.603	1.795	1.637	1.842	2.033	2.218	2.459
17	1.123	1.284	1.431	1.572	1.753	1.623	1.819	2.002	2.177	2.405
18	1.113	1.268	1.409	1.543	1.716	1.611	1.800	1.974	2.141	2.357
19	1.104	1.254	1.389	1.518	1.682	1.600	1.782	1.949	2.108	2.314
20	1.096	1.241	1.371	1.495	1.652	1.590	1.765	1.926	2.079	2.276
21	1.089	1.229	1.355	1.474	1.625	1.581	1.750	1.905	2.053	2.241
22	1.082	1.218	1.340	1.455	1.600	1.572	1.737	1.886	2.028	2.209
23	1.076	1.208	1.326	1.437	1.577	1.564	1.724	1.869	2.006	2.180
24	1.070	1.199	1.313	1.421	1.556	1.557	1.712	1.853	1.985	2.154
25	1.065	1.190	1.302	1.406	1.537	1.550	1.702	1.838	1.966	2.129
26	1.060	1.182	1.291	1.392	1.519	1.544	1.691	1.824	1.949	2.106
27	1.055	1.174	1.280	1.379	1.502	1.538	1.682	1.811	1.932	2.085
28	1.051	1.167	1.271	1.367	1.486	1.533	1.673	1.799	1.917	2.065
29	1.047	1.160	1.262	1.355	1.472	1.528	1.665	1.788	1.903	2.047
30	1.043	1.154	1.253	1.344	1.458	1.523	1.657	1.777	1.889	2.030
31	1.039	1.148	1.245	1.334	1.445	1.518	1.650	1.767	1.877	2.014
32	1.035	1.143	1.237	1.325	1.433	1.514	1.643	1.758	1.865	1.998
33	1.032	1.137	1.230	1.316	1.422	1.510	1.636	1.749	1.853	1.984
34	1.029	1.132	1.223	1.307	1.411	1.506	1.630	1.740	1.843	1.970
35	1.026	1.127	1.217	1.299	1.400	1.502	1.624	1.732	1.833	1.957
36	1.023	1.123	1.211	1.291	1.391	1.498	1.618	1.725	1.823	1.945
37	1.020	1.118	1.205	1.284	1.381	1.495	1.613	1.717	1.814	1.934
38	1.017	1.114	1.199	1.277	1.372	1.492	1.608	1.710	1.805	1.922
39	1.015	1.110	1.194	1.270	1.364	1.489	1.603	1.704	1.797	1.912
40	1.013	1.106	1.188	1.263	1.356	1.486	1.598	1.697	1.789	1.902
41	1.010	1.103	1.183	1.257	1.348	1.483	1.593	1.691	1.781	1.892
42	1.008	1.099	1.179	1.251	1.341	1.480	1.589	1.685	1.774	1.883
43	1.006	1.096	1.174	1.246	1.333	1.477	1.585	1.680	1.767	1.874
44	1.004	1.092	1.170	1.240	1.327	1.475	1.581	1.674	1.760	1.865
45	1.002	1.089	1.165	1.235	1.320	1.472	1.577	1.669	1.753	1.857
46	1.000	1.086	1.161	1.230	1.314	1.470	1.573	1.664	1.747	1.849
47	0.998	1.083	1.157	1.225	1.308	1.468	1.570	1.659	1.741	1.842
48	0.996	1.080	1.154	1.220	1.302	1.465	1.566	1.654	1.735	1.835
49	0.994	1.078	1.150	1.216	1.296	1.463	1.563	1.650	1.730	1.828
50	0.993	1.075	1.146	1.211	1.291	1.461	1.559	1.646	1.724	1.821
55	0.985	1.063	1.130	1.191	1.266	1.452	1.545	1.626	1.700	1.790
60	0.978	1.052	1.116	1.174	1.245	1.444	1.532	1.609	1.679	1.764
65	0.972	1.043	1.104	1.159	1.226	1.437	1.521	1.594	1.661	1.741
70	0.967	1.035	1.094	1.146	1.210	1.430	1.511	1.581	1.645	1.722
75	0.963	1.028	1.084	1.135	1.196	1.425	1.503	1.570	1.630	1.704
80	0.959	1.022	1.076	1.124	1.183	1.420	1.495	1.559	1.618	1.688
85	0.955	1.016	1.068	1.115	1.171	1.415	1.488	1.550	1.606	1.674
90	0.951	1.011	1.061	1.106	1.161	1.411	1.481	1.542	1.596	1.661
95	0.948	1.006	1.055	1.098	1.151	1.408	1.475	1.534	1.586	1.650
100	0.945	1.001	1.049	1.091	1.142	1.404	1.470	1.527	1.578	1.639

Source: Hahn & Meeker (1991)

Table 21-9. Factors (τ) for Parametric Upper Conf. Bounds on Percentiles (P)

	P = 0.95					P = 0.99				
n\(1−α)	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**
2	6.464	13.090	26.260	52.559	131.426	9.156	18.500	37.094	74.234	185.617
3	3.604	5.311	7.656	10.927	17.370	5.010	7.340	10.553	15.043	23.896
4	2.968	3.957	5.144	6.602	9.083	4.110	5.438	7.042	9.018	12.387
5	2.683	3.400	4.203	5.124	6.578	3.711	4.666	5.741	6.980	8.939
6	2.517	3.092	3.708	4.385	5.406	3.482	4.243	5.062	5.967	7.335
7	2.407	2.894	3.399	3.940	4.728	3.331	3.972	4.642	5.361	6.412
8	2.328	2.754	3.187	3.640	4.285	3.224	3.783	4.354	4.954	5.812
9	2.268	2.650	3.031	3.424	3.972	3.142	3.641	4.143	4.662	5.389
10	2.220	2.568	2.911	3.259	3.738	3.078	3.532	3.981	4.440	5.074
11	2.182	2.503	2.815	3.129	3.556	3.026	3.443	3.852	4.265	4.829
12	2.149	2.448	2.736	3.023	3.410	2.982	3.371	3.747	4.124	4.633
13	2.122	2.402	2.671	2.936	3.290	2.946	3.309	3.659	4.006	4.472
14	2.098	2.363	2.614	2.861	3.189	2.914	3.257	3.585	3.907	4.337
15	2.078	2.329	2.566	2.797	3.102	2.887	3.212	3.520	3.822	4.222
16	2.059	2.299	2.524	2.742	3.028	2.863	3.172	3.464	3.749	4.123
17	2.043	2.272	2.486	2.693	2.963	2.841	3.137	3.414	3.684	4.037
18	2.029	2.249	2.453	2.650	2.905	2.822	3.105	3.370	3.627	3.960
19	2.016	2.227	2.423	2.611	2.854	2.804	3.077	3.331	3.575	3.892
20	2.004	2.208	2.396	2.576	2.808	2.789	3.052	3.295	3.529	3.832
21	1.993	2.190	2.371	2.544	2.766	2.774	3.028	3.263	3.487	3.777
22	1.983	2.174	2.349	2.515	2.729	2.761	3.007	3.233	3.449	3.727
23	1.973	2.159	2.328	2.489	2.694	2.749	2.987	3.206	3.414	3.681
24	1.965	2.145	2.309	2.465	2.662	2.738	2.969	3.181	3.382	3.640
25	1.957	2.132	2.292	2.442	2.633	2.727	2.952	3.158	3.353	3.601
26	1.949	2.120	2.275	2.421	2.606	2.718	2.937	3.136	3.325	3.566
27	1.943	2.109	2.260	2.402	2.581	2.708	2.922	3.116	3.300	3.533
28	1.936	2.099	2.246	2.384	2.558	2.700	2.909	3.098	3.276	3.502
29	1.930	2.089	2.232	2.367	2.536	2.692	2.896	3.080	3.254	3.473
30	1.924	2.080	2.220	2.351	2.515	2.684	2.884	3.064	3.233	3.447
31	1.919	2.071	2.208	2.336	2.496	2.677	2.872	3.048	3.213	3.421
32	1.914	2.063	2.197	2.322	2.478	2.671	2.862	3.034	3.195	3.398
33	1.909	2.055	2.186	2.308	2.461	2.664	2.852	3.020	3.178	3.375
34	1.904	2.048	2.176	2.296	2.445	2.658	2.842	3.007	3.161	3.354
35	1.900	2.041	2.167	2.284	2.430	2.652	2.833	2.995	3.145	3.334
36	1.895	2.034	2.158	2.272	2.415	2.647	2.824	2.983	3.131	3.315
37	1.891	2.028	2.149	2.262	2.402	2.642	2.816	2.972	3.116	3.297
38	1.888	2.022	2.141	2.251	2.389	2.637	2.808	2.961	3.103	3.280
39	1.884	2.016	2.133	2.241	2.376	2.632	2.800	2.951	3.090	3.264
40	1.880	2.010	2.125	2.232	2.364	2.627	2.793	2.941	3.078	3.249
41	1.877	2.005	2.118	2.223	2.353	2.623	2.786	2.932	3.066	3.234
42	1.874	2.000	2.111	2.214	2.342	2.619	2.780	2.923	3.055	3.220
43	1.871	1.995	2.105	2.206	2.331	2.615	2.773	2.914	3.044	3.206
44	1.868	1.990	2.098	2.198	2.321	2.611	2.767	2.906	3.034	3.193
45	1.865	1.986	2.092	2.190	2.312	2.607	2.761	2.898	3.024	3.180
46	1.862	1.981	2.086	2.183	2.303	2.604	2.756	2.890	3.014	3.168
47	1.859	1.977	2.081	2.176	2.294	2.600	2.750	2.883	3.005	3.157
48	1.857	1.973	2.075	2.169	2.285	2.597	2.745	2.876	2.996	3.146
49	1.854	1.969	2.070	2.163	2.277	2.594	2.740	2.869	2.988	3.135
50	1.852	1.965	2.065	2.156	2.269	2.590	2.735	2.862	2.980	3.125
55	1.841	1.948	2.042	2.128	2.233	2.576	2.713	2.833	2.943	3.078
60	1.832	1.933	2.022	2.103	2.202	2.564	2.694	2.807	2.911	3.038
65	1.823	1.920	2.005	2.082	2.176	2.554	2.677	2.785	2.883	3.004
70	1.816	1.909	1.990	2.063	2.153	2.544	2.662	2.765	2.859	2.974
75	1.810	1.899	1.976	2.047	2.132	2.536	2.649	2.748	2.838	2.947
80	1.804	1.890	1.964	2.032	2.114	2.528	2.638	2.733	2.819	2.924
85	1.799	1.882	1.954	2.019	2.097	2.522	2.627	2.719	2.802	2.902
90	1.794	1.874	1.944	2.006	2.082	2.516	2.618	2.706	2.786	2.883
95	1.790	1.867	1.935	1.995	2.069	2.510	2.609	2.695	2.772	2.866
100	1.786	1.861	1.927	1.985	2.056	2.505	2.601	2.684	2.759	2.850

Table 21-10. Factors (τ) for Parametric Lower Conf. Bounds on Percentiles (P)

	P = 0.80					P = 0.90				
n\(1−α)	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**
2	0.288	-0.084	-0.521	-1.229	-3.204	0.737	0.403	0.138	-0.143	-0.707
3	0.377	0.111	-0.127	-0.380	-0.792	0.799	0.535	0.334	0.159	-0.072
4	0.432	0.209	0.021	-0.158	-0.405	0.847	0.617	0.444	0.298	0.123
5	0.470	0.272	0.110	-0.038	-0.227	0.883	0.675	0.519	0.389	0.238
6	0.499	0.319	0.173	0.043	-0.117	0.911	0.719	0.575	0.455	0.319
7	0.522	0.355	0.220	0.103	-0.040	0.933	0.755	0.619	0.507	0.381
8	0.540	0.384	0.258	0.150	0.020	0.952	0.783	0.655	0.550	0.431
9	0.556	0.408	0.290	0.188	0.067	0.968	0.808	0.686	0.585	0.472
10	0.569	0.428	0.316	0.220	0.107	0.981	0.828	0.712	0.615	0.508
11	0.580	0.446	0.339	0.247	0.140	0.993	0.847	0.734	0.642	0.538
12	0.591	0.461	0.359	0.271	0.169	1.004	0.863	0.754	0.665	0.565
13	0.599	0.475	0.376	0.292	0.194	1.013	0.877	0.772	0.685	0.589
14	0.608	0.487	0.392	0.310	0.216	1.022	0.890	0.788	0.704	0.610
15	0.615	0.498	0.406	0.327	0.236	1.029	0.901	0.802	0.721	0.629
16	0.621	0.508	0.419	0.342	0.254	1.036	0.912	0.815	0.736	0.647
17	0.627	0.518	0.430	0.356	0.271	1.043	0.921	0.827	0.750	0.663
18	0.633	0.526	0.441	0.369	0.286	1.049	0.930	0.839	0.763	0.678
19	0.638	0.534	0.451	0.380	0.299	1.054	0.939	0.849	0.775	0.692
20	0.643	0.541	0.460	0.391	0.312	1.059	0.946	0.858	0.786	0.705
21	0.647	0.548	0.468	0.401	0.324	1.064	0.953	0.867	0.796	0.716
22	0.651	0.554	0.476	0.410	0.335	1.068	0.960	0.876	0.806	0.728
23	0.655	0.560	0.484	0.419	0.345	1.073	0.966	0.884	0.815	0.738
24	0.659	0.565	0.491	0.427	0.355	1.076	0.972	0.891	0.823	0.748
25	0.662	0.570	0.497	0.435	0.364	1.080	0.978	0.898	0.831	0.757
26	0.665	0.575	0.503	0.442	0.373	1.084	0.983	0.904	0.839	0.766
27	0.669	0.580	0.509	0.449	0.381	1.087	0.988	0.911	0.846	0.774
28	0.671	0.584	0.515	0.456	0.388	1.090	0.993	0.917	0.853	0.782
29	0.674	0.588	0.520	0.462	0.396	1.093	0.997	0.922	0.860	0.790
30	0.677	0.592	0.525	0.468	0.403	1.096	1.002	0.928	0.866	0.797
31	0.679	0.596	0.530	0.473	0.409	1.099	1.006	0.933	0.872	0.804
32	0.682	0.600	0.534	0.479	0.416	1.101	1.010	0.938	0.878	0.810
33	0.684	0.603	0.539	0.484	0.422	1.104	1.013	0.942	0.883	0.817
34	0.686	0.606	0.543	0.489	0.427	1.106	1.017	0.947	0.888	0.823
35	0.688	0.610	0.547	0.494	0.433	1.108	1.020	0.951	0.893	0.828
36	0.690	0.613	0.551	0.498	0.438	1.111	1.024	0.955	0.898	0.834
37	0.692	0.616	0.554	0.502	0.443	1.113	1.027	0.959	0.903	0.839
38	0.694	0.618	0.558	0.507	0.448	1.115	1.030	0.963	0.907	0.844
39	0.696	0.621	0.561	0.511	0.453	1.117	1.033	0.967	0.911	0.849
40	0.698	0.624	0.565	0.514	0.457	1.119	1.036	0.970	0.916	0.854
41	0.699	0.626	0.568	0.518	0.462	1.120	1.038	0.974	0.920	0.859
42	0.701	0.629	0.571	0.522	0.466	1.122	1.041	0.977	0.923	0.863
43	0.702	0.631	0.574	0.525	0.470	1.124	1.044	0.980	0.927	0.867
44	0.704	0.633	0.577	0.529	0.474	1.126	1.046	0.983	0.931	0.872
45	0.705	0.635	0.579	0.532	0.478	1.127	1.048	0.986	0.934	0.876
46	0.707	0.637	0.582	0.535	0.481	1.129	1.051	0.989	0.938	0.880
47	0.708	0.640	0.585	0.538	0.485	1.130	1.053	0.992	0.941	0.883
48	0.709	0.642	0.587	0.541	0.488	1.132	1.055	0.995	0.944	0.887
49	0.711	0.643	0.590	0.544	0.492	1.133	1.057	0.997	0.947	0.891
50	0.712	0.645	0.592	0.547	0.495	1.134	1.059	1.000	0.950	0.894
55	0.718	0.654	0.603	0.559	0.510	1.141	1.069	1.012	0.964	0.910
60	0.723	0.661	0.612	0.571	0.523	1.146	1.077	1.022	0.976	0.924
65	0.727	0.668	0.621	0.581	0.535	1.151	1.085	1.032	0.987	0.937
70	0.731	0.674	0.628	0.589	0.545	1.156	1.091	1.040	0.997	0.948
75	0.735	0.679	0.635	0.597	0.554	1.160	1.097	1.048	1.006	0.958
80	0.738	0.684	0.641	0.605	0.563	1.163	1.103	1.054	1.014	0.968
85	0.741	0.689	0.647	0.611	0.571	1.167	1.108	1.061	1.021	0.976
90	0.743	0.693	0.652	0.618	0.578	1.170	1.112	1.066	1.028	0.984
95	0.746	0.697	0.657	0.623	0.584	1.172	1.116	1.072	1.034	0.991
100	0.748	0.700	0.661	0.628	0.591	1.175	1.120	1.077	1.040	0.998

Source: Adapted from Hahn & Meeker (1991)

Table 21-10. Factors (τ) for Parametric Lower Conf. Bounds on Percentiles (P)

	P = 0.95					P = 0.99				
n\(1−α)	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**
2	1.077	0.717	0.475	0.273	0.000	1.672	1.225	0.954	0.761	0.564
3	1.126	0.840	0.639	0.478	0.295	1.710	1.361	1.130	0.958	0.782
4	1.172	0.922	0.743	0.601	0.443	1.760	1.455	1.246	1.088	0.924
5	1.209	0.982	0.818	0.687	0.543	1.801	1.525	1.331	1.182	1.027
6	1.238	1.028	0.875	0.752	0.618	1.834	1.578	1.396	1.256	1.108
7	1.261	1.065	0.920	0.804	0.678	1.862	1.622	1.449	1.315	1.173
8	1.281	1.096	0.958	0.847	0.727	1.885	1.658	1.493	1.364	1.227
9	1.298	1.122	0.990	0.884	0.768	1.904	1.688	1.530	1.406	1.273
10	1.313	1.144	1.017	0.915	0.804	1.922	1.715	1.563	1.442	1.314
11	1.325	1.163	1.041	0.943	0.835	1.937	1.738	1.591	1.474	1.349
12	1.337	1.180	1.062	0.967	0.862	1.950	1.758	1.616	1.502	1.381
13	1.347	1.196	1.081	0.989	0.887	1.962	1.776	1.638	1.528	1.409
14	1.356	1.210	1.098	1.008	0.909	1.973	1.793	1.658	1.551	1.434
15	1.364	1.222	1.114	1.026	0.929	1.983	1.808	1.677	1.572	1.458
16	1.372	1.234	1.128	1.042	0.948	1.992	1.822	1.694	1.591	1.479
17	1.379	1.244	1.141	1.057	0.965	2.000	1.834	1.709	1.608	1.499
18	1.385	1.254	1.153	1.071	0.980	2.008	1.846	1.724	1.625	1.517
19	1.391	1.263	1.164	1.084	0.995	2.015	1.857	1.737	1.640	1.534
20	1.397	1.271	1.175	1.095	1.008	2.022	1.867	1.749	1.654	1.550
21	1.402	1.279	1.184	1.107	1.021	2.028	1.876	1.761	1.667	1.565
22	1.407	1.286	1.193	1.117	1.033	2.034	1.885	1.772	1.680	1.579
23	1.412	1.293	1.202	1.127	1.044	2.039	1.893	1.782	1.691	1.592
24	1.416	1.300	1.210	1.136	1.054	2.045	1.901	1.791	1.702	1.605
25	1.420	1.306	1.217	1.145	1.064	2.049	1.908	1.801	1.713	1.616
26	1.424	1.311	1.225	1.153	1.074	2.054	1.915	1.809	1.723	1.627
27	1.427	1.317	1.231	1.161	1.083	2.058	1.922	1.817	1.732	1.638
28	1.431	1.322	1.238	1.168	1.091	2.063	1.928	1.825	1.741	1.648
29	1.434	1.327	1.244	1.175	1.099	2.067	1.934	1.833	1.749	1.658
30	1.437	1.332	1.250	1.182	1.107	2.070	1.940	1.840	1.757	1.667
31	1.440	1.336	1.255	1.189	1.114	2.074	1.945	1.846	1.765	1.676
32	1.443	1.341	1.261	1.195	1.121	2.078	1.951	1.853	1.773	1.684
33	1.446	1.345	1.266	1.201	1.128	2.081	1.956	1.859	1.780	1.692
34	1.449	1.349	1.271	1.206	1.135	2.084	1.960	1.865	1.787	1.700
35	1.451	1.352	1.276	1.212	1.141	2.087	1.965	1.871	1.793	1.708
36	1.453	1.356	1.280	1.217	1.147	2.090	1.970	1.876	1.799	1.715
37	1.456	1.360	1.284	1.222	1.153	2.093	1.974	1.882	1.806	1.722
38	1.458	1.363	1.289	1.227	1.158	2.096	1.978	1.887	1.811	1.728
39	1.460	1.366	1.293	1.232	1.164	2.098	1.982	1.892	1.817	1.735
40	1.462	1.369	1.297	1.236	1.169	2.101	1.986	1.896	1.823	1.741
41	1.464	1.372	1.300	1.241	1.174	2.103	1.989	1.901	1.828	1.747
42	1.466	1.375	1.304	1.245	1.179	2.106	1.993	1.905	1.833	1.753
43	1.468	1.378	1.308	1.249	1.183	2.108	1.996	1.910	1.838	1.758
44	1.470	1.381	1.311	1.253	1.188	2.110	2.000	1.914	1.843	1.764
45	1.472	1.383	1.314	1.257	1.192	2.112	2.003	1.918	1.847	1.769
46	1.473	1.386	1.317	1.260	1.197	2.114	2.006	1.922	1.852	1.774
47	1.475	1.389	1.321	1.264	1.201	2.116	2.009	1.925	1.856	1.779
48	1.477	1.391	1.324	1.267	1.205	2.118	2.012	1.929	1.860	1.784
49	1.478	1.393	1.327	1.271	1.209	2.120	2.015	1.933	1.865	1.789
50	1.480	1.396	1.329	1.274	1.212	2.122	2.018	1.936	1.869	1.793
55	1.487	1.406	1.343	1.289	1.230	2.131	2.031	1.952	1.887	1.815
60	1.493	1.415	1.354	1.303	1.245	2.138	2.042	1.966	1.903	1.833
65	1.498	1.424	1.364	1.315	1.259	2.145	2.052	1.979	1.918	1.850
70	1.503	1.431	1.374	1.326	1.272	2.151	2.061	1.990	1.931	1.865
75	1.508	1.438	1.382	1.335	1.283	2.156	2.069	2.000	1.943	1.879
80	1.512	1.444	1.390	1.344	1.293	2.161	2.077	2.010	1.954	1.891
85	1.515	1.449	1.397	1.352	1.302	2.166	2.083	2.018	1.964	1.903
90	1.519	1.454	1.403	1.360	1.311	2.170	2.090	2.026	1.973	1.913
95	1.522	1.459	1.409	1.367	1.319	2.174	2.095	2.033	1.981	1.923
100	1.525	1.463	1.414	1.373	1.326	2.177	2.101	2.040	1.989	1.932

Table 21-11. Achievable Conf. Levels for One-Sided Non-Parametric Conf. Bounds Around Median, Upper 95th Percentile, and Upper 99th Percentile (n ≤ 20)

	Rank of	Confidence Level					
n	**Bound**	**UCL 50th**	**LCL 50th**	**UCL 95th**	**LCL 95th**	**UCL 99th**	**LCL 99th**
4	**4**	0.9375	0.0625	0.1855	0.8145	0.0394	0.9606
4	**3**	0.6875	0.3125	0.0140	0.9860	0.0006	0.9994
4	**2**	0.3125	0.6875	0.0005	0.9995	0.0000	1.0000
4	**1**	0.0625	0.9375	0.0000	1.0000	0.0000	1.0000
5	**5**	0.9688	0.0312	0.2262	0.7738	0.0490	0.9510
5	**4**	0.8125	0.1875	0.0226	0.9774	0.0010	0.9990
5	**3**	0.5000	0.5000	0.0012	0.9988	0.0000	1.0000
5	**2**	0.1875	0.8125	0.0000	1.0000	0.0000	1.0000
5	**1**	0.0312	0.9688	0.0000	1.0000	0.0000	1.0000
6	**6**	0.9844	0.0156	0.2649	0.7351	0.0585	0.9415
6	**5**	0.8906	0.1094	0.0328	0.9672	0.0015	0.9985
6	**4**	0.6562	0.3438	0.0022	0.9978	0.0000	1.0000
6	**3**	0.3438	0.6562	0.0001	0.9999	0.0000	1.0000
6	**2**	0.1094	0.8906	0.0000	1.0000	0.0000	1.0000
6	**1**	0.0156	0.9844	0.0000	1.0000	0.0000	1.0000
7	**7**	0.9922	0.0078	0.3017	0.6983	0.0679	0.9321
7	**6**	0.9375	0.0625	0.0444	0.9556	0.0020	0.9980
7	**5**	0.7734	0.2266	0.0038	0.9962	0.0000	1.0000
7	**4**	0.5000	0.5000	0.0002	0.9998	0.0000	1.0000
7	**3**	0.2266	0.7734	0.0000	1.0000	0.0000	1.0000
7	**2**	0.0625	0.9375	0.0000	1.0000	0.0000	1.0000
7	**1**	0.0078	0.9922	0.0000	1.0000	0.0000	1.0000
8	**8**	0.9961	0.0039	0.3366	0.6634	0.0773	0.9227
8	**7**	0.9648	0.0352	0.0572	0.9428	0.0027	0.9973
8	**6**	0.8555	0.1445	0.0058	0.9942	0.0001	0.9999
8	**5**	0.6367	0.3633	0.0004	0.9996	0.0000	1.0000
8	**4**	0.3633	0.6367	0.0000	1.0000	0.0000	1.0000
8	**3**	0.1445	0.8555	0.0000	1.0000	0.0000	1.0000
8	**2**	0.0352	0.9648	0.0000	1.0000	0.0000	1.0000
8	**1**	0.0039	0.9961	0.0000	1.0000	0.0000	1.0000
9	**9**	0.9980	0.0020	0.3698	0.6302	0.0865	0.9135
9	**8**	0.9805	0.0195	0.0712	0.9288	0.0034	0.9966
9	**7**	0.9102	0.0898	0.0084	0.9916	0.0001	0.9999
9	**6**	0.7461	0.2539	0.0006	0.9994	0.0000	1.0000
9	**5**	0.5000	0.5000	0.0000	1.0000	0.0000	1.0000
9	**4**	0.2539	0.7461	0.0000	1.0000	0.0000	1.0000
9	**3**	0.0898	0.9102	0.0000	1.0000	0.0000	1.0000
9	**2**	0.0195	0.9805	0.0000	1.0000	0.0000	1.0000
9	**1**	0.0020	0.9980	0.0000	1.0000	0.0000	1.0000
10	**10**	0.9990	0.0010	0.4013	0.5987	0.0956	0.9044
10	**9**	0.9893	0.0107	0.0861	0.9139	0.0043	0.9957
10	**8**	0.9453	0.0547	0.0115	0.9885	0.0001	0.9999
10	**7**	0.8281	0.1719	0.0010	0.9990	0.0000	1.0000
10	**6**	0.6230	0.3770	0.0001	0.9999	0.0000	1.0000
10	**5**	0.3770	0.6230	0.0000	1.0000	0.0000	1.0000
10	**4**	0.1719	0.8281	0.0000	1.0000	0.0000	1.0000
10	**3**	0.0547	0.9453	0.0000	1.0000	0.0000	1.0000
10	**2**	0.0107	0.9893	0.0000	1.0000	0.0000	1.0000
10	**1**	0.0010	0.9990	0.0000	1.0000	0.0000	1.0000

Footnote. LCL = lower confidence limit; UCL = upper confidence limit; 50th = median

Table 21-11. Achievable Conf. Levels for One-Sided Non-Parametric Conf. Bounds Around Median, Upper 95th Percentile, and Upper 99th Percentile (n ≤ 20)

	Rank of	Confidence Level					
n	**Bound**	**UCL 50th**	**LCL 50th**	**UCL 95th**	**LCL 95th**	**UCL 99th**	**LCL 99th**
11	**11**	0.9995	0.0005	0.4312	0.5688	0.1047	0.8953
11	**10**	0.9941	0.0059	0.1019	0.8981	0.0052	0.9948
11	**9**	0.9673	0.0327	0.0152	0.9848	0.0002	0.9998
11	**8**	0.8867	0.1133	0.0016	0.9984	0.0000	1.0000
11	**7**	0.7256	0.2744	0.0001	0.9999	0.0000	1.0000
11	**6**	0.5000	0.5000	0.0000	1.0000	0.0000	1.0000
11	**5**	0.2744	0.7256	0.0000	1.0000	0.0000	1.0000
11	**4**	0.1133	0.8867	0.0000	1.0000	0.0000	1.0000
11	**3**	0.0327	0.9673	0.0000	1.0000	0.0000	1.0000
11	**2**	0.0059	0.9941	0.0000	1.0000	0.0000	1.0000
11	**1**	0.0005	0.9995	0.0000	1.0000	0.0000	1.0000
12	**12**	0.9998	0.0002	0.4596	0.5404	0.1136	0.8864
12	**11**	0.9968	0.0032	0.1184	0.8816	0.0062	0.9938
12	**10**	0.9807	0.0193	0.0196	0.9804	0.0002	0.9998
12	**9**	0.9270	0.0730	0.0022	0.9978	0.0000	1.0000
12	**8**	0.8062	0.1938	0.0002	0.9998	0.0000	1.0000
12	**7**	0.6128	0.3872	0.0000	1.0000	0.0000	1.0000
12	**6**	0.3872	0.6128	0.0000	1.0000	0.0000	1.0000
12	**5**	0.1938	0.8062	0.0000	1.0000	0.0000	1.0000
12	**4**	0.0730	0.9270	0.0000	1.0000	0.0000	1.0000
12	**3**	0.0193	0.9807	0.0000	1.0000	0.0000	1.0000
12	**2**	0.0032	0.9968	0.0000	1.0000	0.0000	1.0000
12	**1**	0.0002	0.9998	0.0000	1.0000	0.0000	1.0000
13	**13**	0.9999	0.0001	0.4867	0.5133	0.1225	0.8775
13	**12**	0.9983	0.0017	0.1354	0.8646	0.0072	0.9928
13	**11**	0.9888	0.0112	0.0245	0.9755	0.0003	0.9997
13	**10**	0.9539	0.0461	0.0031	0.9969	0.0000	1.0000
13	**9**	0.8666	0.1334	0.0003	0.9997	0.0000	1.0000
13	**8**	0.7095	0.2905	0.0000	1.0000	0.0000	1.0000
13	**7**	0.5000	0.5000	0.0000	1.0000	0.0000	1.0000
13	**6**	0.2905	0.7095	0.0000	1.0000	0.0000	1.0000
13	**5**	0.1334	0.8666	0.0000	1.0000	0.0000	1.0000
13	**4**	0.0461	0.9539	0.0000	1.0000	0.0000	1.0000
13	**3**	0.0112	0.9888	0.0000	1.0000	0.0000	1.0000
13	**2**	0.0017	0.9983	0.0000	1.0000	0.0000	1.0000
13	**1**	0.0001	0.9999	0.0000	1.0000	0.0000	1.0000
14	**14**	0.9999	0.0001	0.5123	0.4877	0.1313	0.8687
14	**13**	0.9991	0.0009	0.1530	0.8470	0.0084	0.9916
14	**12**	0.9935	0.0065	0.0301	0.9699	0.0003	0.9997
14	**11**	0.9713	0.0287	0.0042	0.9958	0.0000	1.0000
14	**10**	0.9102	0.0898	0.0004	0.9996	0.0000	1.0000
14	**9**	0.7880	0.2120	0.0000	1.0000	0.0000	1.0000
14	**8**	0.6047	0.3953	0.0000	1.0000	0.0000	1.0000
14	**7**	0.3953	0.6047	0.0000	1.0000	0.0000	1.0000
14	**6**	0.2120	0.7880	0.0000	1.0000	0.0000	1.0000
14	**5**	0.0898	0.9102	0.0000	1.0000	0.0000	1.0000
14	**4**	0.0287	0.9713	0.0000	1.0000	0.0000	1.0000
14	**3**	0.0065	0.9935	0.0000	1.0000	0.0000	1.0000
14	**2**	0.0009	0.9991	0.0000	1.0000	0.0000	1.0000
14	**1**	0.0001	0.9999	0.0000	1.0000	0.0000	1.0000

Table 21-11. Achievable Conf. Levels for One-Sided Non-Parametric Conf. Bounds Around Median, Upper 95th Percentile, and Upper 99th Percentile (n ≤ 20)

	Rank of	Confidence Level					
n	**Bound**	**UCL 50th**	**LCL 50th**	**UCL 95th**	**LCL 95th**	**UCL 99th**	**LCL 99th**
15	**15**	1.0000	0.0000	0.5367	0.4633	0.1399	0.8601
15	**14**	0.9995	0.0005	0.1710	0.8290	0.0096	0.9904
15	**13**	0.9963	0.0037	0.0362	0.9638	0.0004	0.9996
15	**12**	0.9824	0.0176	0.0055	0.9945	0.0000	1.0000
15	**11**	0.9408	0.0592	0.0006	0.9994	0.0000	1.0000
15	**10**	0.8491	0.1509	0.0001	0.9999	0.0000	1.0000
15	**9**	0.6964	0.3036	0.0000	1.0000	0.0000	1.0000
15	**8**	0.5000	0.5000	0.0000	1.0000	0.0000	1.0000
15	**7**	0.3036	0.6964	0.0000	1.0000	0.0000	1.0000
15	**6**	0.1509	0.8491	0.0000	1.0000	0.0000	1.0000
15	**5**	0.0592	0.9408	0.0000	1.0000	0.0000	1.0000
15	**4**	0.0176	0.9824	0.0000	1.0000	0.0000	1.0000
15	**3**	0.0037	0.9963	0.0000	1.0000	0.0000	1.0000
15	**2**	0.0005	0.9995	0.0000	1.0000	0.0000	1.0000
15	**1**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
16	**16**	1.0000	0.0000	0.5599	0.4401	0.1485	0.8515
16	**15**	0.9997	0.0003	0.1892	0.8108	0.0109	0.9891
16	**14**	0.9979	0.0021	0.0429	0.9571	0.0005	0.9995
16	**13**	0.9894	0.0106	0.0070	0.9930	0.0000	1.0000
16	**12**	0.9616	0.0384	0.0009	0.9991	0.0000	1.0000
16	**11**	0.8949	0.1051	0.0001	0.9999	0.0000	1.0000
16	**10**	0.7728	0.2272	0.0000	1.0000	0.0000	1.0000
16	**9**	0.5982	0.4018	0.0000	1.0000	0.0000	1.0000
16	**8**	0.4018	0.5982	0.0000	1.0000	0.0000	1.0000
16	**7**	0.2272	0.7728	0.0000	1.0000	0.0000	1.0000
16	**6**	0.1051	0.8949	0.0000	1.0000	0.0000	1.0000
16	**5**	0.0384	0.9616	0.0000	1.0000	0.0000	1.0000
16	**4**	0.0106	0.9894	0.0000	1.0000	0.0000	1.0000
16	**3**	0.0021	0.9979	0.0000	1.0000	0.0000	1.0000
16	**2**	0.0003	0.9997	0.0000	1.0000	0.0000	1.0000
16	**1**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
17	**17**	1.0000	0.0000	0.5819	0.4181	0.1571	0.8429
17	**16**	0.9999	0.0001	0.2078	0.7922	0.0123	0.9877
17	**15**	0.9988	0.0012	0.0503	0.9497	0.0006	0.9994
17	**14**	0.9936	0.0064	0.0088	0.9912	0.0000	1.0000
17	**13**	0.9755	0.0245	0.0012	0.9988	0.0000	1.0000
17	**12**	0.9283	0.0717	0.0001	0.9999	0.0000	1.0000
17	**11**	0.8338	0.1662	0.0000	1.0000	0.0000	1.0000
17	**10**	0.6855	0.3145	0.0000	1.0000	0.0000	1.0000
17	**9**	0.5000	0.5000	0.0000	1.0000	0.0000	1.0000
17	**8**	0.3145	0.6855	0.0000	1.0000	0.0000	1.0000
17	**7**	0.1662	0.8338	0.0000	1.0000	0.0000	1.0000
17	**6**	0.0717	0.9283	0.0000	1.0000	0.0000	1.0000
17	**5**	0.0245	0.9755	0.0000	1.0000	0.0000	1.0000
17	**4**	0.0064	0.9936	0.0000	1.0000	0.0000	1.0000
17	**3**	0.0012	0.9988	0.0000	1.0000	0.0000	1.0000
17	**2**	0.0001	0.9999	0.0000	1.0000	0.0000	1.0000
17	**1**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000

Table 21-11. Achievable Conf. Levels for One-Sided Non-Parametric Conf. Bounds Around Median, Upper 95th Percentile, and Upper 99th Percentile (n ≤ 20)

	Rank of	Confidence Level					
n	**Bound**	**UCL 50th**	**LCL 50th**	**UCL 95th**	**LCL 95th**	**UCL 99th**	**LCL 99th**
18	**18**	1.0000	0.0000	0.6028	0.3972	0.1655	0.8345
18	**17**	0.9999	0.0001	0.2265	0.7735	0.0138	0.9862
18	**16**	0.9993	0.0007	0.0581	0.9419	0.0007	0.9993
18	**15**	0.9962	0.0038	0.0109	0.9891	0.0000	1.0000
18	**14**	0.9846	0.0154	0.0015	0.9985	0.0000	1.0000
18	**13**	0.9519	0.0481	0.0002	0.9998	0.0000	1.0000
18	**12**	0.8811	0.1189	0.0000	1.0000	0.0000	1.0000
18	**11**	0.7597	0.2403	0.0000	1.0000	0.0000	1.0000
18	**10**	0.5927	0.4073	0.0000	1.0000	0.0000	1.0000
18	**9**	0.4073	0.5927	0.0000	1.0000	0.0000	1.0000
18	**8**	0.2403	0.7597	0.0000	1.0000	0.0000	1.0000
18	**7**	0.1189	0.8811	0.0000	1.0000	0.0000	1.0000
18	**6**	0.0481	0.9519	0.0000	1.0000	0.0000	1.0000
18	**5**	0.0154	0.9846	0.0000	1.0000	0.0000	1.0000
18	**4**	0.0038	0.9962	0.0000	1.0000	0.0000	1.0000
18	**3**	0.0007	0.9993	0.0000	1.0000	0.0000	1.0000
18	**2**	0.0001	0.9999	0.0000	1.0000	0.0000	1.0000
18	**1**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
19	**19**	1.0000	0.0000	0.6226	0.3774	0.1738	0.8262
19	**18**	1.0000	0.0000	0.2453	0.7547	0.0153	0.9847
19	**17**	0.9996	0.0004	0.0665	0.9335	0.0009	0.9991
19	**16**	0.9978	0.0022	0.0132	0.9868	0.0000	1.0000
19	**15**	0.9904	0.0096	0.0020	0.9980	0.0000	1.0000
19	**14**	0.9682	0.0318	0.0002	0.9998	0.0000	1.0000
19	**13**	0.9165	0.0835	0.0000	1.0000	0.0000	1.0000
19	**12**	0.8204	0.1796	0.0000	1.0000	0.0000	1.0000
19	**11**	0.6762	0.3238	0.0000	1.0000	0.0000	1.0000
19	**10**	0.5000	0.5000	0.0000	1.0000	0.0000	1.0000
19	**9**	0.3238	0.6762	0.0000	1.0000	0.0000	1.0000
19	**8**	0.1796	0.8204	0.0000	1.0000	0.0000	1.0000
19	**7**	0.0835	0.9165	0.0000	1.0000	0.0000	1.0000
19	**6**	0.0318	0.9682	0.0000	1.0000	0.0000	1.0000
19	**5**	0.0096	0.9904	0.0000	1.0000	0.0000	1.0000
19	**4**	0.0022	0.9978	0.0000	1.0000	0.0000	1.0000
19	**3**	0.0004	0.9996	0.0000	1.0000	0.0000	1.0000
19	**2**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
19	**1**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000

Table 21-11. Achievable Conf. Levels for One-Sided Non-Parametric Conf. Bounds Around Median, Upper 95th Percentile, and Upper 99th Percentile (n ≤ 20)

	Rank of	Confidence Level					
n	**Bound**	**UCL 50th**	**LCL 50th**	**UCL 95th**	**LCL 95th**	**UCL 99th**	**LCL 99th**
20	**20**	1.0000	0.0000	0.6415	0.3585	0.1821	0.8179
20	**19**	1.0000	0.0000	0.2642	0.7358	0.0169	0.9831
20	**18**	0.9998	0.0002	0.0755	0.9245	0.0010	0.9990
20	**17**	0.9987	0.0013	0.0159	0.9841	0.0000	1.0000
20	**16**	0.9941	0.0059	0.0026	0.9974	0.0000	1.0000
20	**15**	0.9793	0.0207	0.0003	0.9997	0.0000	1.0000
20	**14**	0.9423	0.0577	0.0000	1.0000	0.0000	1.0000
20	**13**	0.8684	0.1316	0.0000	1.0000	0.0000	1.0000
20	**12**	0.7483	0.2517	0.0000	1.0000	0.0000	1.0000
20	**11**	0.5881	0.4119	0.0000	1.0000	0.0000	1.0000
20	**10**	0.4119	0.5881	0.0000	1.0000	0.0000	1.0000
20	**9**	0.2517	0.7483	0.0000	1.0000	0.0000	1.0000
20	**8**	0.1316	0.8684	0.0000	1.0000	0.0000	1.0000
20	**7**	0.0577	0.9423	0.0000	1.0000	0.0000	1.0000
20	**6**	0.0207	0.9793	0.0000	1.0000	0.0000	1.0000
20	**5**	0.0059	0.9941	0.0000	1.0000	0.0000	1.0000
20	**4**	0.0013	0.9987	0.0000	1.0000	0.0000	1.0000
20	**3**	0.0002	0.9998	0.0000	1.0000	0.0000	1.0000
20	**2**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
20	**1**	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000

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D STATISTICAL TABLES

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TABLE 22-7 Minimum *n* to Detect kp_0 Incr. over Percentile 1-p_0 , with 1-β and α, $k > 1$.....D-265

TABLE 22-8 Minimum *n* to Detect kp_0 Incr. over Percentile 1-p_0 , with 1-β and α, $k < 1$.....D-267

Table 22-1. Combinations of n (≤ 40) and α (≤ .20) Achieving (1–β) Power to Detect Increases of 1.5 × GWPS

1–β = 0.50		1–β = 0.60		1–β = 0.70		1–β = 0.80		1–β = 0.90		1–β = 0.95		1–β = 0.99	
n	α	n	α	n	α	n	α	n	α	n	α	n	α
4	0.177	6	0.179	8	0.197	12	0.195	19	0.191	26	0.188		
5	0.149	7	0.156	9	0.175	13	0.177	20	0.177	27	0.176		
6	0.127	8	0.136	10	0.156	14	0.160	21	0.163	28	0.164		
7	0.108	9	0.119	11	0.139	15	0.146	22	0.151	29	0.153		
8	0.093	10	0.104	12	0.124	16	0.132	23	0.139	30	0.143		
9	0.080	11	0.092	13	0.111	17	0.120	24	0.129	31	0.133		
10	0.069	12	0.081	14	0.099	18	0.109	25	0.119	32	0.124		
11	0.060	13	0.071	15	0.089	19	0.099	26	0.110	33	0.116		
12	0.052	14	0.063	16	0.079	20	0.090	27	0.101	34	0.108		
13	0.045	15	0.056	17	0.071	21	0.082	28	0.093	35	0.101		
14	0.039	16	0.049	18	0.064	22	0.074	29	0.086	36	0.094		
15	0.034	17	0.043	19	0.057	23	0.068	30	0.079	37	0.087		
16	0.030	18	0.038	20	0.051	24	0.061	31	0.073	38	0.081		
17	0.026	19	0.034	21	0.046	25	0.056	32	0.067	39	0.076		
18	0.023	20	0.030	22	0.041	26	0.051	33	0.062	40	0.070		
19	0.020	21	0.027	23	0.037	27	0.046	34	0.057				
20	0.018	22	0.024	24	0.033	28	0.042	35	0.053				
21	0.015	23	0.021	25	0.030	29	0.038	36	0.049				
22	0.014	24	0.019	26	0.027	30	0.034	37	0.045				
23	0.012	25	0.017	27	0.024	31	0.031	38	0.041				
24	0.010	26	0.015	28	0.022	32	0.029	39	0.038				
25	0.009	27	0.013	29	0.020	33	0.026	40	0.035				
26	0.008	28	0.012	30	0.018	34	0.023						
27	0.007	29	0.010	31	0.016	35	0.021						
28	0.006	30	0.009	32	0.014	36	0.019						
29	0.006	31	0.008	33	0.013	37	0.018						
30	0.005	32	0.007	34	0.011	38	0.016						
31	0.004	33	0.007	35	0.010	39	0.015						
32	0.004	34	0.006	36	0.009	40	0.013						
33	0.003	35	0.005	37	0.008								
34	0.003	36	0.005	38	0.007								
35	0.003	37	0.004	39	0.007								
36	0.002	38	0.004	40	0.006								
37	0.002	39	0.003										
38	0.002	40	0.003										
39	0.002												
40	0.002												

Table 22-2. Combinations of n (≤ 40) and α (≤ .20) Achieving (1–β) Power to Detect Increases of 2 × GWPS

1–β = 0.50		1–β = 0.60		1–β = 0.70		1–β = 0.80		1–β = 0.90		1–β = 0.95		1–β = 0.99	
n	**α**	**n**	**α**	**n**	**α**	**n**	**α**	**n**	**α**	**n**	**α**	**n**	**α**
3	0.091	3	0.123	3	0.164	4	0.163	5	0.199	7	0.183	11	0.180
4	0.057	4	0.080	4	0.113	5	0.119	6	0.152	8	0.144	12	0.148
5	0.037	5	0.054	5	0.079	6	0.086	7	0.116	9	0.113	13	0.121
6	0.024	6	0.036	6	0.055	7	0.063	8	0.088	10	0.089	14	0.099
7	0.016	7	0.025	7	0.039	8	0.046	9	0.067	11	0.069	15	0.080
8	0.011	8	0.017	8	0.027	9	0.034	10	0.051	12	0.054	16	0.065
9	0.007	9	0.012	9	0.019	10	0.024	11	0.039	13	0.042	17	0.053
10	0.005	10	0.008	10	0.014	11	0.018	12	0.029	14	0.033	18	0.043
11	0.003	11	0.006	11	0.010	12	0.013	13	0.022	15	0.025	19	0.034
12	0.002	12	0.004	12	0.007	13	0.010	14	0.017	16	0.020	20	0.027
13	0.002	13	0.003	13	0.005	14	0.007	15	0.013	17	0.015	21	0.022
14	0.001	14	0.002	14	0.004	15	0.005	16	0.010	18	0.012	22	0.018
≥15	<0.001	15	0.001	15	0.003	16	0.004	17	0.007	19	0.009	23	0.014
		≥16	<0.001	16	0.002	17	0.003	18	0.005	20	0.007	24	0.011
				17	0.001	18	0.002	19	0.004	21	0.005	25	0.009
				≥18	<0.001	19	0.002	20	0.003	22	0.004	26	0.007
						20	0.001	21	0.002	23	0.003	27	0.006
						≥21	<0.001	22	0.002	24	0.002	28	0.004
								23	0.001	25	0.002	29	0.004
								≥24	<0.001	26	0.002	30	0.003
										27	0.001	31	0.002
										≥28	<0.001	32	0.002
												33	0.002
												34	0.001
												≥35	<0.001

Table 22-3. Minimum Individual Test α Meeting Power Criteria Given *n* and *CV*

	50% Power at *R* = 1.5					**80% Power at *R* = 2**				
CV	**n=4**	**n=6**	**n=8**	**n=10**	**n=12**	**n=4**	**n=6**	**n=8**	**n=10**	**n=12**
0.1	*0.003*	*0.000*	*0.000*	*0.000*	*0.000*	*0.001*	*0.000*	*0.000*	*0.000*	*0.000*
0.2	0.022	*0.005*	*0.001*	*0.000*	*0.000*	0.014	*0.002*	*0.000*	*0.000*	*0.000*
0.3	0.056	0.021	*0.008*	*0.003*	*0.001*	0.050	0.013	*0.003*	*0.001*	*0.000*
0.4	0.097	0.048	0.025	0.014	*0.007*	0.113	0.043	0.017	*0.007*	*0.003*
0.5	0.137	0.082	0.051	0.032	0.021	0.191	0.093	0.047	0.024	0.013
0.6	0.174	0.116	0.080	0.056	0.040	0.270	0.156	0.094	0.057	0.035
0.7	0.206	0.148	0.110	0.083	0.064	0.342	0.222	0.149	0.101	0.069
0.8	0.233	0.177	0.139	0.110	0.088	0.402	0.284	0.206	0.151	0.112
0.9	0.256	0.203	0.165	0.136	0.113	0.451	0.339	0.261	0.203	0.158
1.0	0.276	0.226	0.189	0.160	0.136	0.492	0.386	0.310	0.251	0.205
1.2	0.309	0.263	0.229	0.201	0.178	**0.553**	0.462	0.393	0.337	0.291
1.4	0.333	0.293	0.261	0.235	0.214	**0.596**	**0.517**	0.456	0.406	0.362
1.6	0.352	0.316	0.287	0.263	0.243	**0.626**	**0.558**	**0.505**	0.459	0.420
1.8	0.368	0.335	0.308	0.286	0.267	**0.650**	**0.590**	**0.542**	**0.502**	0.466
2.0	0.380	0.350	0.326	0.305	0.288	**0.667**	**0.614**	**0.572**	**0.536**	**0.504**
2.2	0.391	0.363	0.341	0.322	0.305	**0.682**	**0.634**	**0.596**	**0.564**	**0.534**
2.4	0.400	0.374	0.353	0.335	0.320	**0.693**	**0.650**	**0.616**	**0.586**	**0.560**
2.6	0.407	0.383	0.364	0.347	0.333	**0.703**	**0.664**	**0.632**	**0.605**	**0.581**
2.8	0.414	0.391	0.373	0.358	0.344	**0.711**	**0.675**	**0.646**	**0.621**	**0.599**
3.0	0.419	0.398	0.381	0.367	0.354	**0.718**	**0.685**	**0.658**	**0.635**	**0.614**

Table 22-4. Minimum *n* (≥ 4) to Detect Decreases of .75 x GWPS for Given *CV*, Power (1–β), & Error Rate (α)

CV = 0.2							CV = 0.4						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	4	**0.20**	4	4	4	6	8	11
0.15	4	4	4	4	4	5	**0.15**	4	4	5	7	10	12
0.10	4	4	4	4	4	5	**0.10**	4	5	6	8	11	15
0.05	4	4	4	4	5	6	**0.05**	6	8	9	11	15	18
0.01	6	6	6	7	8	9	**0.01**	11	13	15	18	22	26
CV = 0.6							**CV = 0.8**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	5	7	11	16	22	**0.20**	6	8	12	18	28	38
0.15	5	7	9	13	19	25	**0.15**	8	11	15	22	33	43
0.10	7	9	12	16	23	30	**0.10**	11	15	20	28	40	51
0.05	11	14	17	22	30	37	**0.05**	18	23	29	38	51	65
0.01	21	25	30	36	45	54	**0.01**	35	42	50	61	78	94
CV = 1.0							**CV = 1.2**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	8	12	18	27	42	58	**0.20**	11	17	25	38	60	82
0.15	11	16	23	33	50	67	**0.15**	16	23	33	47	71	95
0.10	17	23	31	42	61	79	**0.10**	23	32	44	60	87	113
0.05	27	35	44	58	79	100	**0.05**	37	49	63	82	113	143
0.01	52	63	76	93	120	145	**0.01**	74	90	109	133	172	208
CV = 1.4							**CV = 1.6**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	14	22	34	51	81	111	**0.20**	18	29	44	67	106	144
0.15	21	31	44	64	96	129	**0.15**	26	40	58	83	125	168
0.10	31	43	59	81	118	153	**0.10**	40	56	77	106	153	199
0.05	50	66	85	111	153	193	**0.05**	65	85	110	144	199	252
0.01	99	121	147	180	233	281	**0.01**	128	157	190	234	303	366

Table 22-4. Minimum *n* (≥ 4) to Detect Decreases of .75 x GWPS for Given *CV*, Power (1–β), & Error Rate (α)

CV = 1.8							CV = 2.0						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	22	36	56	84	133	182	**0.20**	27	44	68	103	164	225
0.15	33	50	72	104	158	212	**0.15**	40	61	89	128	195	261
0.10	50	70	97	133	193	252	**0.10**	61	87	119	164	238	310
0.05	81	107	139	182	252	318	**0.05**	100	132	171	225	310	392
0.01	162	198	240	296	383	463	**0.01**	199	243	296	364	472	571
CV = 2.2							**CV = 2.4**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	32	54	83	125	198	271	**0.20**	38	63	98	148	235	323
0.15	48	74	108	155	236	315	**0.15**	57	88	128	184	280	375
0.10	73	104	144	198	288	375	**0.10**	87	124	171	235	342	446
0.05	120	159	207	271	375	474	**0.05**	143	189	246	323	446	563
0.01	239	293	357	440	570	690	**0.01**	284	348	425	523	678	821
CV = 2.6							**CV = 2.8**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	44	74	115	174	276	378	**0.20**	51	86	133	201	320	438
0.15	67	103	150	216	329	439	**0.15**	77	119	173	250	381	509
0.10	102	145	200	276	402	523	**0.10**	118	168	232	320	465	606
0.05	167	221	288	378	523	661	**0.05**	193	256	334	438	606	766
0.01	333	408	498	614	795	963	**0.01**	386	473	577	711	922	1116

Table 22-5. Minimum *n* (≥ 4) to Detect Decreases of .5 x GWPS for Given *CV*, Power (1–β), & Error Rate (α)

CV = 0.2

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	4	4	4	4
0.15	4	4	4	4	4	4
0.10	4	4	4	4	4	4
0.05	4	4	4	4	4	4
0.01	4	4	4	4	4	4

CV = 0.4

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	4	4	4	4
0.15	4	4	4	4	4	4
0.10	4	4	4	4	4	4
0.05	4	4	4	4	4	4
0.01	4	4	5	5	5	6

CV = 0.6

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	4	4	4	4
0.15	4	4	4	4	4	5
0.10	4	4	4	4	4	5
0.05	4	4	4	4	5	6
0.01	6	6	6	7	8	9

CV = 0.8

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	4	4	5	6
0.15	4	4	4	4	5	7
0.10	4	4	4	5	6	8
0.05	4	5	5	6	8	9
0.01	7	8	9	10	11	13

CV = 1.0

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	4	4	6	8
0.15	4	4	4	5	7	9
0.10	4	4	5	6	9	11
0.05	5	6	7	8	11	13
0.01	9	10	11	13	16	19

CV = 1.2

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	4	6	8	11
0.15	4	4	5	7	10	12
0.10	4	5	6	8	11	15
0.05	6	8	9	11	15	18
0.01	11	13	15	18	22	26

CV = 1.4

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	4	5	7	11	14
0.15	4	5	6	8	12	16
0.10	5	6	8	11	15	19
0.05	8	9	11	14	19	24
0.01	14	17	19	23	29	34

CV = 1.6

α\1-β	0.50	0.60	0.70	0.80	0.90	0.95
0.20	4	5	6	9	13	18
0.15	4	6	8	11	16	20
0.10	6	8	10	13	19	24
0.05	9	11	14	18	24	30
0.01	18	21	24	29	36	44

Table 22-5. Minimum *n* (≥ 4) to Detect Decreases of .5 x GWPS for Given *CV*, Power (1–β), & Error Rate (α)

CV = 1.8							CV = 2.0						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	5	7	11	16	22	**0.20**	4	6	9	13	20	27
0.15	5	7	9	13	19	25	**0.15**	6	8	11	16	23	31
0.10	7	9	12	16	23	30	**0.10**	9	11	15	20	28	36
0.05	11	14	17	22	30	37	**0.05**	13	17	21	27	36	46
0.01	21	25	30	36	45	54	**0.01**	25	30	36	43	55	66
CV = 2.2							**CV = 2.4**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	5	7	10	15	23	32	**0.20**	6	8	12	18	28	38
0.15	7	10	13	19	28	37	**0.15**	8	11	15	22	33	43
0.10	10	13	17	23	34	44	**0.10**	11	15	20	28	40	51
0.05	16	20	25	32	44	55	**0.05**	18	23	29	38	51	65
0.01	30	36	43	52	66	79	**0.01**	35	42	50	61	78	94
CV = 2.6							**CV = 2.8**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	6	9	14	21	32	44	**0.20**	7	11	16	24	37	51
0.15	9	13	18	25	38	51	**0.15**	10	15	21	29	44	58
0.10	13	18	24	32	46	60	**0.10**	15	20	27	37	53	69
0.05	21	27	34	44	60	76	**0.05**	24	30	39	51	69	87
0.01	40	48	58	71	91	110	**0.01**	46	56	67	82	105	127

Table 22-6. Minimum *n* (≥ 4) to Detect Decreases of .25 x GWPS for Given *CV*, Power (1–β), & Error Rate (α)

CV = 0.2							CV = 0.4						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	4	**0.20**	4	4	4	4	4	4
0.15	4	4	4	4	4	4	**0.15**	4	4	4	4	4	4
0.10	4	4	4	4	4	4	**0.10**	4	4	4	4	4	4
0.05	4	4	4	4	4	4	**0.05**	4	4	4	4	4	4
0.01	4	4	4	4	4	4	**0.01**	4	4	4	4	4	4
CV = 0.6							**CV = 0.8**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	4	**0.20**	4	4	4	4	4	4
0.15	4	4	4	4	4	4	**0.15**	4	4	4	4	4	4
0.10	4	4	4	4	4	4	**0.10**	4	4	4	4	4	4
0.05	4	4	4	4	4	4	**0.05**	4	4	4	4	4	4
0.01	4	4	4	4	4	4	**0.01**	4	4	4	4	4	4
CV = 1.0							**CV = 1.2**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	4	**0.20**	4	4	4	4	4	4
0.15	4	4	4	4	4	4	**0.15**	4	4	4	4	4	4
0.10	4	4	4	4	4	4	**0.10**	4	4	4	4	4	4
0.05	4	4	4	4	4	4	**0.05**	4	4	4	4	4	4
0.01	4	4	4	4	5	5	**0.01**	4	4	5	5	5	6
CV = 1.4							**CV = 1.6**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	4	**0.20**	4	4	4	4	4	4
0.15	4	4	4	4	4	4	**0.15**	4	4	4	4	4	4
0.10	4	4	4	4	4	4	**0.10**	4	4	4	4	4	5
0.05	4	4	4	4	4	5	**0.05**	4	4	4	4	5	6
0.01	5	5	5	5	6	7	**0.01**	5	5	6	6	7	8

Table 22-6. Minimum *n* (≥ 4) to Detect Decreases of .25 x GWPS for Given *CV*, Power (1–β), & Error Rate (α)

CV = 1.8							CV = 2.0						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	4	**0.20**	4	4	4	4	4	5
0.15	4	4	4	4	4	5	**0.15**	4	4	4	4	4	5
0.10	4	4	4	4	4	5	**0.10**	4	4	4	4	5	6
0.05	4	4	4	4	5	6	**0.05**	4	4	4	5	6	7
0.01	6	6	6	7	8	9	**0.01**	6	6	7	8	9	10
CV = 2.2							**CV = 2.4**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	4	6	**0.20**	4	4	4	4	5	6
0.15	4	4	4	4	5	6	**0.15**	4	4	4	4	5	7
0.10	4	4	4	4	6	7	**0.10**	4	4	4	5	6	8
0.05	4	4	5	6	7	8	**0.05**	4	5	5	6	8	9
0.01	7	7	8	9	10	12	**0.01**	7	8	9	10	11	13
CV = 2.6							**CV = 2.8**						
α\1-β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**α\1-β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**
0.20	4	4	4	4	5	7	**0.20**	4	4	4	4	6	8
0.15	4	4	4	4	6	8	**0.15**	4	4	4	5	7	8
0.10	4	4	4	5	7	9	**0.10**	4	4	5	6	8	10
0.05	5	5	6	7	9	11	**0.05**	5	6	6	8	10	12
0.01	8	8	9	11	13	15	**0.01**	8	9	10	12	14	17

Table 22-7. Minimum n to Detect kp_0 Exceedances Over Percentile ($1-p_0$) with Power ($1-\beta$) and Error Rate (α)

Percentile = 90th								**Percentile = 95th**							
				k = 2								**k = 2**			
α\1−β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**	**α\1−β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**
0.20	7	13	22	35	59	83	140	**0.20**	14	27	47	77	130	184	311
0.10	15	24	36	53	81	109	173	**0.10**	32	51	77	114	177	239	382
0.05	25	26	50	69	102	133	203	**0.05**	52	76	107	150	221	291	447
0.02	38	52	69	91	128	163	240	**0.02**	81	110	147	197	277	355	525
0.01	49	64	83	108	147	184	266	**0.01**	103	136	177	231	318	401	581
				k = 2.5								**k = 2.5**			
α\1−β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**	**α\1−β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**
0.20	3	6	11	17	29	42	71	**0.20**	6	13	23	38	66	95	162
0.10	7	11	17	25	40	54	87	**0.10**	14	24	37	56	88	121	196
0.05	11	17	24	33	49	65	101	**0.05**	23	35	51	73	109	145	227
0.02	17	24	32	43	61	79	118	**0.02**	36	51	69	94	136	175	264
0.01	22	29	39	51	70	89	130	**0.01**	46	63	83	110	155	197	290
				k = 3								**k = 3**			
α\1−β	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**	**α\1−β**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**
0.20	2	4	7	11	18	26	44	**0.20**	4	8	14	24	42	60	103
0.10	4	7	10	15	24	33	53	**0.10**	8	14	22	34	55	76	124
0.05	7	10	14	20	30	39	61	**0.05**	13	21	30	44	67	90	142
0.02	10	14	19	26	37	47	71	**0.02**	21	29	41	56	82	108	164
0.01	13	17	23	30	42	53	78	**0.01**	26	36	49	66	94	120	179

Table 22-7. Minimum n to Detect kp_0 Exceedences Over Percentile ($1-p_0$) with Power ($1-\beta$) and Error Rate (α)

Percentile = 98th

k = 2

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	35	71	122	200	341	485	823
0.10	81	132	200	297	464	630	1009
0.05	133	196	278	391	580	764	1178
0.02	207	285	381	512	726	930	1382
0.01	266	353	459	602	832	1050	1528

k = 3

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	9	20	37	64	112	162	281
0.10	21	36	58	90	147	204	335
0.05	34	53	79	116	179	241	383
0.02	52	76	107	149	219	288	442
0.01	67	94	127	173	249	321	482

k = 4

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	4	10	19	34	61	89	156
0.10	9	18	29	47	78	109	183
0.05	15	25	39	59	93	128	207
0.02	23	36	52	74	113	150	235
0.01	30	44	61	86	126	166	255

k = 5

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	3	6	12	22	40	59	104
0.10	6	11	18	30	50	71	121
0.05	9	15	24	37	60	82	135
0.02	13	21	31	46	71	96	152
0.01	17	26	37	53	79	105	164

Percentile = 99th

k = 2

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	71	143	247	407	693	987	1677
0.10	163	266	404	602	943	1281	2054
0.05	268	397	563	793	1178	1552	2395
0.02	418	576	772	1038	1473	1889	2810
0.01	536	713	930	1221	1689	2133	3105

k = 3

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	18	41	75	130	229	332	578
0.10	41	73	118	184	300	417	688
0.05	67	107	161	236	366	494	786
0.02	105	154	216	303	448	588	904
0.01	134	189	258	352	507	656	987

k = 4

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	8	20	39	69	125	184	324
0.10	19	35	59	96	160	225	379
0.05	30	51	79	120	192	263	427
0.02	47	72	105	152	231	309	485
0.01	60	88	125	175	259	341	525

k = 5

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	5	13	25	45	83	123	219
0.10	11	21	37	61	104	148	252
0.05	17	30	49	76	123	171	282
0.02	27	43	64	94	147	198	317
0.01	34	52	75	108	164	218	341

Table 22-8. Minimum n to Detect kp_0 Exceedences Over Percentile ($1-p_0$) with Power ($1-\beta$) and Error Rate (α)

Percentile = 90th

k = .50

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	26	38	54	77	114	150	231
0.10	60	78	100	130	177	221	318
0.05	98	121	148	184	239	291	401
0.02	152	181	214	256	321	380	505
0.01	195	227	264	311	382	447	581

k = .25

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	12	16	20	27	37	47	68
0.10	27	32	39	48	61	74	100
0.05	44	51	59	70	86	101	131
0.02	68	77	87	100	119	136	171
0.01	87	97	109	123	144	163	201

Percentile = 95th

k = .50

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	54	80	113	159	236	311	479
0.10	125	163	209	270	368	460	661
0.05	206	254	311	384	500	606	834
0.02	321	380	449	537	672	794	1052
0.01	412	478	555	653	800	934	1212

k = .25

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	24	32	42	55	76	96	139
0.10	56	68	82	99	127	152	206
0.05	92	107	124	146	179	209	271
0.02	143	161	182	209	248	283	355
0.01	183	204	228	257	300	339	417

Percentile = 98th

k = .50

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	139	205	290	407	602	793	1221
0.10	322	419	537	693	943	1178	1689
0.05	531	653	798	987	1281	1552	2133
0.02	827	979	1154	1379	1723	2036	2694
0.01	1061	1232	1428	1677	2054	2395	3105

k = .25

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	62	82	107	140	193	244	354
0.10	144	173	209	254	324	388	525
0.05	236	274	318	373	458	533	692
0.02	368	415	469	535	635	724	907
0.01	472	525	585	659	770	868	1067

Percentile = 99th

k = .50

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	281	413	584	820	1213	1597	2457
0.10	651	846	1083	1397	1900	2373	3402
0.05	1072	1319	1611	1990	2582	3129	4297
0.02	1671	1976	2330	2782	3475	4106	5430
0.01	2144	2487	2883	3384	4144	4830	6259

k = .25

α\1−β	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0.20	125	166	215	282	389	490	711
0.10	290	350	420	512	653	782	1056
0.05	477	553	641	753	922	1075	1393
0.02	743	838	945	1080	1281	1460	1827
0.01	953	1060	1181	1330	1553	1749	2149

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