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Bed Load Transport by Natural Rivers

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Since stream power ω and sediment transport rate i are different values of the same physical quantity, namely, the time rate of energy supply and dissipation, it is rational to relate one to the other. The experimental relation has been difficult to interpret because of the spurious curvature of log-log plots in which a constant threshold stream power of zero is involved. The substitution of an excess power $\omega - \omega_0$ removes this curvature, and existing data on laboratory bed load transport rate measurements i_b suggest a general empirical relation: $i_b \propto (\omega - \omega_0)[(\omega - \omega_0)/\omega_0]^{1/2}$. Existing laboratory data have also shown clearly that at any given value of $\omega - \omega_0$ the bedload transport rate i_b decreases as an inverse function of the ratio flow depth to grain size Y/D. The East Fork River (Wyoming) project has recently enabled bed load sampling devices to be calibrated, so reasonably reliable measurements can be made in natural rivers. The uncertainties in the measurement of the corresponding river power are discussed, and a simple data reliability test is suggested. Data covering three seasons collected from both Snake and Clearwater rivers appear to be reliable. Though there is much scatter due to day variations in the river conditions, these data, together with data on an imtermediate scale from East Fork River and on a small laboratory scale, conform with startling consistency to the following general empirical relation: $i_b/(\omega - \omega_0) \approx [(\omega - \omega_0)/\omega_0]^{1/2}(Y/D)^{-2/3}$ over a 2 million-fold range of stream discharge. The degree of consistency of the above empirical relation with the theoretical relation deduced previously (Bagnold, 1973) is discussed, as are also some morphological implications of the dependence of i_b on the depth to grain size ratio Y/D.

BASIC IDEAS

In a previous paper [*Bagnold*, 1973] I deduced from simple physical principles the general form of the quantitative relation between the rate of transport of solids, as bed load, over a gravity bed and the appropriately defined flow of the transporting fluid. Bed load is defined as the solid material which is transported in a statistically dispersed state above the bed surface but which is not, however, suspended, i.e., its immersed weight is supported, on average, not by upward currents of fluid turbulence but by a combination of fluid and solid reactive forces exerted at intermittent contacts with the bed solids. The bed load solids 'saltate' just as a bouncing tennis ball, or a stone thrown so as to skim a water surface, may be said to saltate. Statistically, the mean immersed weight stress on the bed due to a saltating load is the same as if the load were sliding over the bed in continuous contact with it. Moreover, experiment [*Francis*, 1973] has shown that the friction coefficient $\tan \alpha$ between a saltating load and the bed approximates to the conventional value found for the simple sliding of the material concerned. Consequently, if the immersed weight of the load per unit bed area is w_b and the mean travel velocity of the load is $\bar{U}_b$, the transport rate $i_b = w_b\bar{U}_b$ and the rate at which the fluid must be doing transporting work per unit of bed area is

$$w_b\bar{U}_b \tan \alpha = i_b \tan \alpha \qquad (1)$$

Regarding the fluid flow as a transporting machine, the appropriate flow quantity on which i_b is dependent is evidently in the nature of a power supply or rate of energy input per unit bed area. In the steady gravity flow of a liquid in a uniform open channel the total supply of kinetic power Ω per unit length of channel is ρQS, where Q is the discharge and S the gravity gradient. The total power supply per unit bed area is therefore

$$\omega = \rho QS/\text{width} \doteq \tau\bar{u} = \rho YS\bar{u} \qquad (2)$$

where τ is the mean boundary shear stress; $\bar{u}$, the mean flow velocity; Y, the flow depth; and ρ, the absolute density mass per volume. Usually, the stream power ω is mostly dissipated in maintaining the flow of the fluid against fluid shear resistance. Some may also be dissipated in doing transporting work in pushing the bed load along against solid-to-solid friction. While ω is the only practical measure of stream power, it is unfortunately not a measure of the power directly available to transport a bed load. That this latter power must be smaller is apparent from the following.

Experiment indicates that in water the saltation of bed load solids is restricted by the inertia of the fluid to a zone within n diameters D above the bed surface, where n is a small number of the order of 10 only. The fluid thrust on the saltating solids is therefore applied at the small distance nD, approximately, by virtue of a fluid velocity u_{nD}. This fluid thrust cannot exceed τ, so the power ω_b directly available for bed load transport is limited to $\omega_b < \tau u_{nD}$, whereas, since the mean flow velocity $\bar{u}$ occurs in turbulent flow at a distance from the bed approximating to $0.4Y$,

$$\omega = \tau\bar{u} = \tau u_{0.4Y}$$

Hence for a given stream power ω the power ω_b directly available for bed load transport is an inverse function of the depth to grain size ratio Y/D.

The dependence of bed load transport rate on the ratio Y/D was clearly shown by Williams' experiments [*Williams*, 1970], in which the transport rates of the same nearly uniform sand, $D = 1.1$ mm, were measured at different constant flow depths Y (Figure 1). Williams' experiments are unique in that he succeeded in isolating and measuring the previously uncertain and spurious effects of wall drag in a laboratory flume. The elimination of this wall drag from his results showed conclusively that the long-known increase of transport with decrease of flow depth, at the same stream power, is not due to wall drag as had often been supposed but to an inherent property of the bed load transport mechanism. The range of flow depths available in laboratories is insufficient to allow reliable extrapolation to greater values of Y/D. Williams himself suggested that the continued decrease of i_b with further increase in Y/D might become insignificant. No theoretical answer to this

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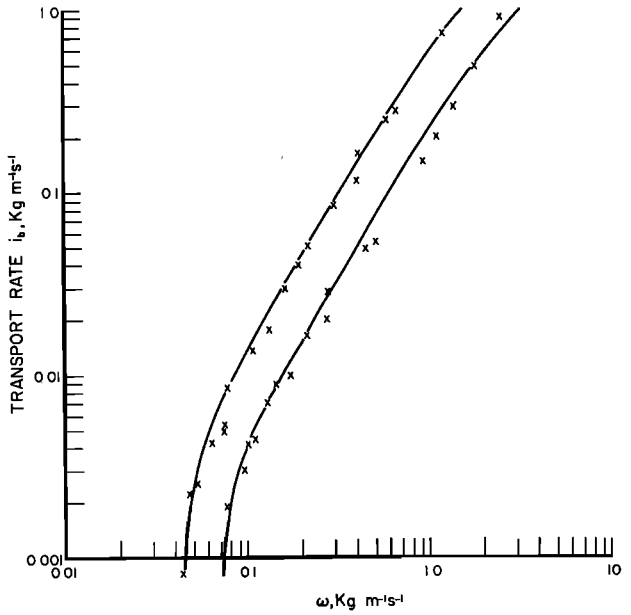

Fig. 1. Williams' experimental data plotted as i_b versus ω, showing reduction in i_b due to increase in Y/D. From left to right, Y/D = 28 for the first curve, and Y/D = 135 for the second curve.

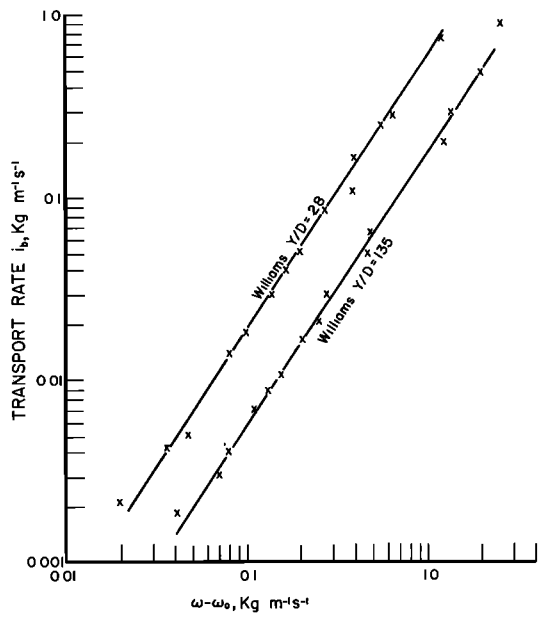

Fig. 2. Data of Figure 1 replotted with $\omega - \omega_0$ substituted for ω. Threshold ω_0 was taken as 0.028 kg m^{-1} s^{-1} for Y/D = 28 and 0.035 for Y/D = 135. D = 1.1 mm.

question is possible at present because of our lack of knowledge both of flow conditions within the saltation zone and of the effect of bed load transport on the mean flow velocity $\bar{u}$. As will emerge later, however, the recent evidence from natural rivers shows conclusively that i_b continues to decrease, in terms of ω, with increase in the ratio Y/D.

Graphical Representation of the Transport Rate: Stream Power Function

Transport rate values i_b in any single set of measurements often extend over a range of 4 or 5 orders of 10, and corresponding values of the stream power ω may extend over a 100-fold range. Consequently, any graphical representation of i_b as a function of ω must necessarily be logarithmic. Now, it is in the nature of the dynamics of fluid-solid systems in which the densities differ that the motion of one phase ceases while that of the other is still finite. This imposes on any log-log plot a spurious curvature which may well mask the relation one is looking for. Figure 1 is an example of this curvature. In general, a plot of log y against log $(x - b)$ in which both x and y vanish together has an inherent curvature due to the constant b. For log y falls ever more rapidly toward minus infinity as x approaches b. This misleading curvature is removed very simply by substituting x' for $(x - b)$ so that x' and y vanish together.

The obvious substitution in the present case is that of replacing ω by $\omega' = (\omega - \omega_0)$, where ω_0 is the threshold value of ω. This is, however, open to serious objections on grounds of precision. Unlike τ_0 or u_{*0}, which are constants of the solid phase, the threshold stream power ω_0 is a variable depending on the flow depth. In a natural river, in which the flow depth varies with stage, ω_0 cannot be measured with any certainty except perhaps at a low stage when conditions are likely to differ considerably from those at higher stages. Nevertheless, at the suggestion of Luna B. Leopold I have reconsidered the feasibility of introducing ω_0 empirically as a constant element of stream power. The physical justification rests on the idea, put forward on page 489, paragraph 2, of the paper by *Bagnold* [1973], that the maintenance of a steady constant transport rate under a steady constant streamflow appears to require the persistence of a residual fluid shear stress τ_0. Evidence for the persistence of such a constant stress element comes from experiments on wind-blown sand. These show that the fluid velocity close to the bed does indeed remain constant as the flow strength above is increased and saltation becomes more intense. Since a constant fluid velocity close to a boundary involves a constant shear stress, the existence of a constant power element seems justified, at any rate notionally.

Let it be assumed arbitrarily that ω_0 is a constant power element having, as a practical definition, a value approximating to the smallest measured ω at which traces of bed load transport are detectable. After allowing for random experimental error and also for the fact that the smallest measured ω may be appreciably larger than ω_0, it is necessary to adjust this initial estimate by trial and error until as nearly as possible the lowest points on a plot such as Figure 2 come into line with the rest of the plot. This adjustment is usually too small to have a significant effect on the general trend. As will appear shortly, it happens that a reasonably precise value for ω_0 can often be found from the data. In Figure 2 the Williams data of Figure 1 are replotted as i_b versus $(\omega - \omega_0)$, where ω_0 is taken as 0.028 kg m^{-1} s^{-1} for Y/D = 28 and as 0.035 kg m^{-1} s^{-1} for Y/D = 135.

The removal of the spurious curvature of the plots by the substitution of $\omega - \omega_0$ for ω so as to make both coordinates vanish together results in unmistakable straight-line plots each giving empirically $i_b \propto (\omega - \omega_0)^{3/2}$. Note that since both plots undoubtedly tend to straight lines, the same technique can be used to arrive at a final ω_0 value as has long been accepted for finding the correct datum for the log-height in plotting turbulent flow velocity profiles.

Uncertainties in Interpretation of Relevant Hydraulic Data From Natural Rivers

It is evident from the foregoing that if the rate of bed load transport by a natural river is to be predicted from hydraulic measurements, reliable values of the following river quantities

are required: (1) effective gravity gradient S and effective flow depth Y, (2) mean velocity $\bar{u}$, (3) effective grain size D, and (4) effective threshold values of u_{*0} and ω_0.

In a uniform laboratory channel all these quantities are readily definable and measurable. S, Y, $\bar{u}$, and D remain constant from section to section along the channel so that both the shear stress τ and the stream power ω are constant along the channel, as is the ratio Y/D. A natural river channel is by no means uniform. In general, the channel shape and cross section fluctuate irregularly along its length, from pool to riffle or from bend to reverse bend. Diverging flow alternates with converging flow, with consequent energy dissipation additional to that due to conventional boundary roughness. Hence none of the above quantities 1–4 are constant along the channel, and a number of ideas which a student would take for granted from experience of laboratory flumes are not necessarily applicable to rivers.

The measured flow depth Y' may well be two or more times the effective depth Y as a factor of shear stress and stream power, e.g., at the cross section of a pool. The effective gravity gradient S cannot in practice be measured at all. Measurement of the local water surface slope S' is but a rough indication of the gravity gradient and requires considerable care in its acceptance. For S' may vary manyfold along the channel, being greatest round a bend or just downstream of a riffle and least at a bend reversal or just upstream of a riffle. The mean of these variations of water surface slope must of course coincide with the overall water surface slope as measured between two sections separated by a number of fluctuation distances. The mean gravity gradient S must, however, be always less than this, maybe considerably less, owing to energy dissipation at each successive change of boundary condition.

It is helpful to regard the effects of river channel fluctuations as similar to those of a succession of broad-crested partly drowned weirs, as shown schematically in Figure 3. The vertical scale is of course very greatly exaggerated, river slopes and their fluctuations being usually far too small to be perceptible.

Existing river data show that if the water surface slope S' is measured over the local surface AA' in Figure 3, i.e., for convenience, over a relatively placid reach, S' may increase up to sevenfold with increase of river stage. This has led to doubt as to whether S' measured thus or overall is the better approximation to the real gravity gradient S as the proper factor of τ and ω. As it is evident from Figure 3 that energy dissipation must be considerably greater, relatively, at low stage than at high and therefore that S must increase with increasing stage, it would follow that S' measured over AA' rather than overall should be the better approximation to S. This question is of basic importance in all attempts to relate transport rate to stream power, for at low stages a possibly gross error in the latter may be involved.

The only practical test which can be made, though necessarily somewhat rough, appears to be the following. It is applicable to any river for which adequate data are available. If the locally measured water surface slope S' approximates to the effective local gravity gradient S over the whole range of river stages, the shear velocity $u_*' = (gYS')^{1/2}$ must approximate to the real shear velocity $u_* = (gYS)^{1/2}$. Now the real mean flow velocity $\bar{u}$ is proportional to u_* and can be written as $\bar{u} = 5.75u_* \log Y/k$, where k is some unknown roughness parameter. By choosing a suitable arbitrary constant value for k we can also write a notional value of $\bar{u}$ as $\bar{u} = 5.75u_*' \log Y/k$. Thus if $S' \simeq S$ so that $u_*' \simeq u_*$, the discrepancy ratio $\bar{u}$ (measured)$/f(u_*')$, where $\bar{u} = f(u_*')$, will show no systematic variation with u_* as the river stage changes. Moreover, provided that the river bed has not changed its character appreciably during a seasonal stage variation, the above ratio will remain constant. On the other hand, if the measurement of S' was wrongly sited, or if S' was wrongly assumed to be constant, the discrepancy ratio will vary systematically with river stage. In this way it seems possible to avoid the more serious errors in the estimation of the unit river power ω. The application of this simple test to data from the Snake River is shown in Figure 4. The near constancy of the discrepancy ratio over three seasons of measurement up to mid-May 1974 is remarkable. An abrupt change of some kind seems then to have resulted from the occurrence of a 200-year Snake River flood.

Regarding the solid phase, we have no firm evidence on which to decide the size D which will be representative of the often very heterogeneous mixture of sizes usually found on river beds. On the principle that the simplest assumption is preferable in such a case I have assumed that the $D_{50\%}$ size is consistently representative. I have further assumed that the $D_{50\%}$ size refers exclusively to the sampled bed load whose transport rate has been measured, whatever may be the actual roughness of the bed.

A final uncertainty must here be mentioned. Assume for a moment that it is now possible to measure the rate at which bed load is being transported past the particular cross section of measurement. This rate is not necessarily the same as the mean rate which is conserved along the channel. For a glance at Figure 3 would suggest that the transport rate of solids also fluctuates along the channel. It is difficult to avoid the inference that the bed of a pool must tend to be scoured during periods of rising stage and progressively filled by deposition during seasonal low-stage periods. Thus the transport rate measured in the neighborhood of A in Figure 3 during a period of rising stage following a seasonal low-stage period might well be excessive because it may include the still unsettled debris from the scour immediately upstream. Such an excess might be exceptionally large when arising from the relatively violent scour due to some artificial flow obstruction. All such fluctuations of transport rate should, however, become averaged out when measurements are made over a sufficient period of time.

It would seem from the foregoing that considerable scatter is to be expected in any plot of river transport rates against stream power. Nevertheless, provided that the data cover a

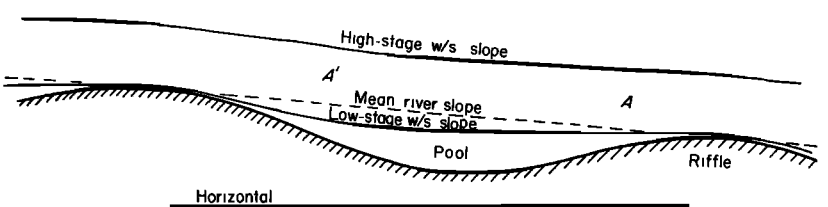

Fig. 3. Schematic profile of a typical river irregularity, showing the variation of local water surface slope with both river stage and location. The vertical scale is greatly exaggerated.

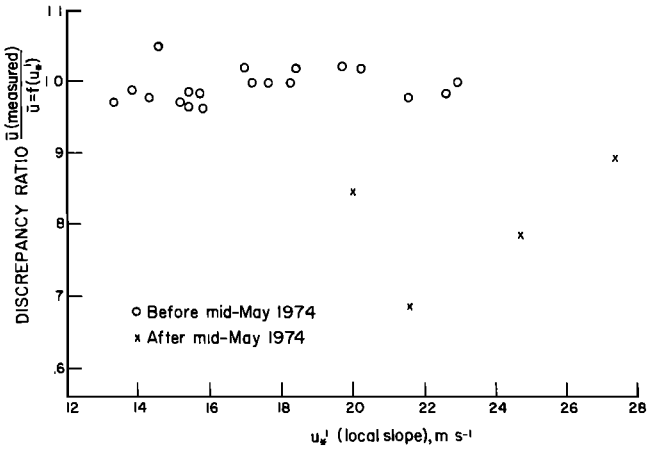

Fig. 4. Discrepancy ratios $\bar{u}$ (measured)/$f(u_*')$, where $\bar{u} = f(u_*')$, for Snake River, 1972–1974, showing the remarkable constancy up to mid-May 1974, when the conditions appear to have changed. Constancy of the discrepancy ratio indicates reliability of the local surface slope as a measure of the local effective gravity gradient.

reasonable range of magnitudes, say, 3 or 4 orders of 10, and also of elapsed time, say, 2 to 3 years, the smoothed plot should give fairly precise evidence of the general relation of the mean along-the-channel bed load transport rate to the concurrent stream power.

Availability of Reliable Bed Load Transport Rate Data in Rivers

Previous to 1973, no reliable measurements were available. Various attempts had been made using improvised sampling methods, but since no direct method had been devised of measuring the total rate of transport by a river, there was no way of checking the reliability of any sampling method. Indeed, the results seemed so randomly variable that the usefulness of sampling methods was in some doubt.

The East Fork River (Wyoming) project was successfully developed in 1973 by the U.S. Geological Survey (USGS) under the direction of Luna B. Leopold. The river (maximum discharge over the study period of 45 $m^3 s^{-1}$; width of 15–20 m) having been temporarily diverted, a concrete channel was in-

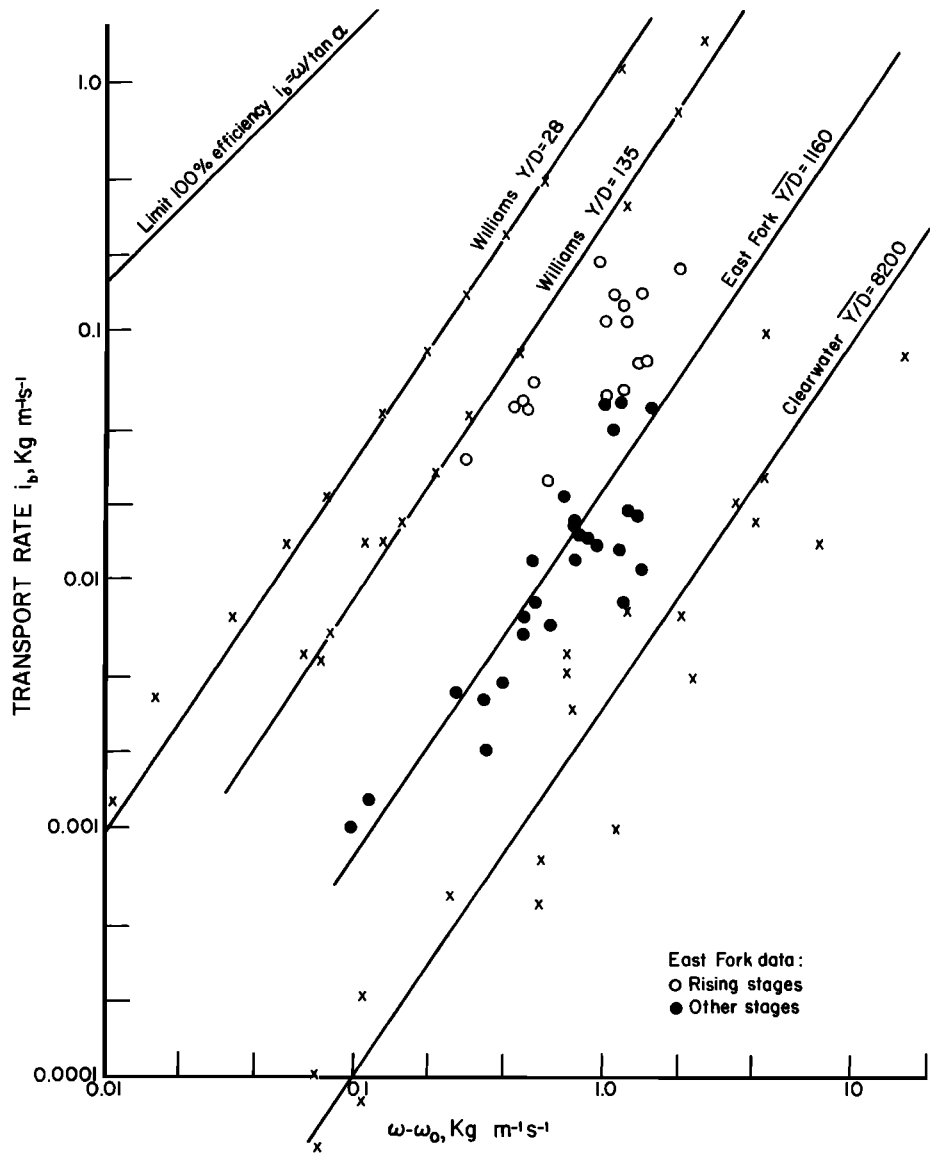

Fig. 5*a*. Comparative plots of measured bed load transport rates i_b against $\omega - \omega_0$ for Williams' data, Figure 2, East Fork River and Clearwater River. Assumed values of ω_0 are 0.1 kg m^{-1} s^{-1} for East Fork and 0.25 kg m^{-1} s^{-1} for Clearwater.

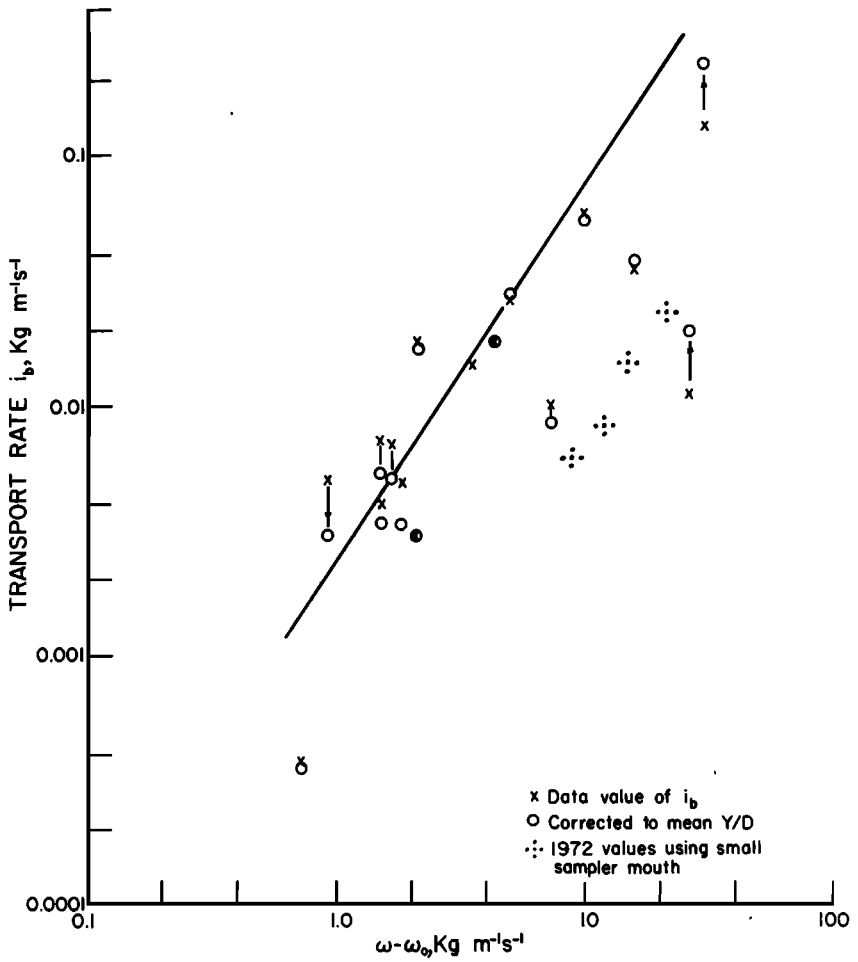

Fig. 5*b*. Same as Figure 5*a* for Snake River data, shown separately because of near coincidence with Clearwater data.

serted transversely into the bed. Hydraulically operated horizontal gates, opening to a 15-cm width and flush with the bed surface, enable the passing bed load to be trapped either in individual sections or across the whole river simultaneously. The trapped material is collected continuously by an endless conveyor belt, raised to the bank by a bucket lift and weighted in batches before being returned to the river. A partial description of the apparatus is given by *Leopold and Emmett* [1976].

Measurements of bed load transport rates and of the concurrent hydraulic data now cover the 3 years 1973–1975 [*Leopold and Emmett*, 1976]. Simultaneous transport rate measurements have been made throughout this period by sampling, using a Helley-Smith bed load sampler developed by the USGS. It was found that when systematic traverses of the river bed have been repeated several times and averaged, the integrated result approximates closely to the result obtained by the direct trapping across the whole river bed. Thus it has been possible for the first time to check the reliability of the far simpler and more practical sampling method.

The reason for the gross and seemingly random variation of individual sampler measurements has also become apparent. The conveyor belt system has disclosed that contrary to laboratory experience, bed load transport by the natural river is unsteady both in time and cross-sectional distribution. Twofold or more variations in the total river transport rate can occur within a several-minute period, and, in addition, discrete streams of solids wander at random laterally over the bed, though the longer-term cross-channel mean of bed load is reasonably well defined. Thus while single sampler measurements may be wholly unreliable, repeated systematic traversing across the river yields reliable average results.

Measurements of bed load transport rate, using the Helley-Smith sampler, have also been made, by W. W. Emmett of the USGS, on the two relatively great rivers Snake and Clearwater (maximum discharges of 6500 and 5000 m^3 s^{-1} and widths of 200 m and 150 m, respectively) near their confluence [*Emmett*, 1976]. Now that the behavior of this sampler is better understood and its measurements have been checked at the East Fork project, the data it has yielded from these two rivers, which cover the 3 years 1972–1974, may be accepted with some confidence provided that the $D_{50\%}$ size of the bed load collected is at most an order of 10 smaller than the sampler mouth. On the majority of occasions the $D_{50\%}$ size in both rivers was between 0.5 and 1 mm. The sampler mouth being 15 cm square, wall obstruction should be negligible. Both rivers, however, have cobble beds, and on some occasions the $D_{50\%}$ size of the material collected was 100-fold larger, at around 50 mm. It seems likely that on these occasions the measured transport rate would have been considerably too small owing to a jostling effect like that of a crowd of people all trying to get through a door. The data for these occasions will be considered separately.

Applying the test of the appropriateness of the river slope measurements, mentioned in the last section, I can find little amiss with the choice of measuring site in either river. Thus the stream power values ω appear to be reliable. This also applies

to the East Fork data. We thus have now five sets of reasonably reliable bed load transport rate data, two referring to small-scale laboratory experiments with coarse uniform sand, and three referring to natural rivers. The range of conditions is very wide, being 2×10^6 in discharge, 360 in the Y/D ratio, and, somewhat less certainly, 100-fold in $D_{50\%}$. Data collection from all three rivers is continuing. The 1975–1976 data from Snake and Clearwater have doubled the previous information, and more data were collected from East Fork in 1976 than during the combined 1973–1975 period.

A General Empirical i_b-ω Relation?

In Figures 5*a* and 5*b* I have repeated the two plots of Figure 2 together with the plots of all the data on the East Fork and Clearwater rivers. The Snake River plot is shown separately as Figure 5*b* for greater clarity, since there is much overlap with the Clearwater plot. There is much scatter, as was to be expected. The mean straight-line trend at a common slope of 3/2 is, however, apparent except in the anomalously high i_b values for East Fork. It is significant that each of these high values is associated with a rapid rise of stage and most pronounced with the initial spring rise. In Figure 6 the appropriate mean values of $(\omega - \omega_0)$ for any given i_b are plotted against the mean ratio Y/D for each set of data. As can be seen $(\omega - \omega_0) \propto (Y/D)^{2/3}$ to a close approximation over a very wide range of Y/D. Thus from Figures 5 and 6 together it appears that i_b can be written empirically as

$$i_b \simeq 1.6\,[(\omega - \omega_0)^{3/2}]/\omega_0^{1/2} \times (\overline{Y/D})^{-2/3} \qquad (3a)$$

The numerical coefficient 1.6 strongly suggests its replacement by $1/\tan\alpha$. Equation (3) can be written more consistently as

$$i_b = 1.6(\omega - \omega_0)[(\omega - \omega_0)/\omega_0]^{1/2} \times (\bar{Y}/\bar{D})^{-2/3} \qquad (3b)$$

and in terms of transport efficiency

$$\frac{i_b}{\omega - \omega_0} = 1.6\left(\frac{\omega - \omega_0}{\omega_0}\right)^{1/2}\left(\frac{Y}{D}\right)^{-2/3} \qquad (3c)$$

Since i_b is very clearly an inverse function of Y/D, some, but by no means all, of the scatter is evidently due to the day-to-day variation of Y/D with river stage. For example, on an occasion when Y/D happens to be higher than average, the measured i_b would be correspondingly lower. In other words, the data for each river occasion must be treated as if they were referring to a different river. Thus some, but by no means all, of the apparent scatter can be reduced to the river average by applying the correcting factor $[(Y/D)/(\overline{Y/D})]^{2/3}$ to the measured i_b values, as shown in Figure 5*b*.

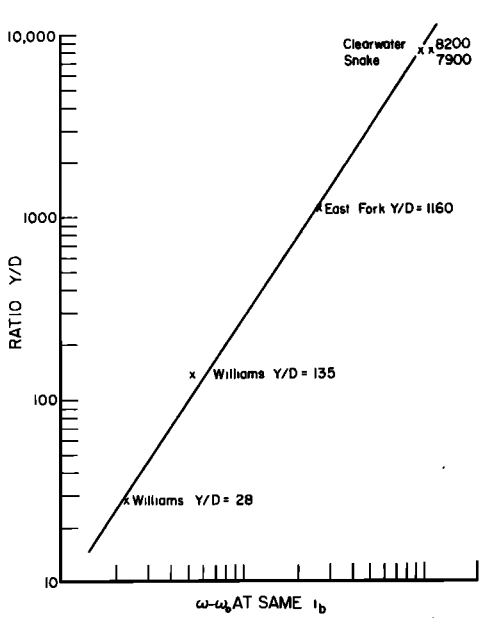

Fig. 6. Increase in stream power $\omega - \omega_0$ needed to maintain a constant i_b as H/D is increased. Empirically, $(\omega - \omega_0) \propto (Y/D)^{2/3}$.

It seems possible to account for the anomalously high East Fork i_b values measured during rapidly rising stages by the existence, 100 m or 70 mean depths upstream, of a road bridge built on timber piers. These piers disturbed the flow to the extent that the scour hole immediately downstream of them was observed to be progressively enlarged as the river stage rose. Thus the measured transport rate just downstream of the scour hole may well have included that of the still unsettled scour debris. This explanation is consistent with the restriction of the excessive transport rate occasions to those of rapidly rising stages. Comparison of the measured volume increase of the scour hole with the integrated transport rate excess during a rising stage period should provide a helpful clue. Under the assumption, suggested in the section on interpreting hydraulic data, that a pool tends to be scoured during a rising stage period and partially refilled by deposition during subsequent low-stage periods, the precise cause of the differential scouring remains uncertain. Some workers appear to hold tacitly that the acceleration associated with increasing discharge must in some way cause a general increase in transport efficiency along the channel. This seems doubtful, however, since the maximum daily velocity increase recorded at East Fork was but 0.2 m s^{-1}, an acceleration of the order of only 10^{-7} g. It has been suggested alternatively that the excess transport rates measured during a rise of stage represent the removal of the finer and more readily transportable material deposited during a long low-stage period. Again, this explanation is doubtful, as it stands, for the grain size has already been taken into account in (3) both in ω_0 and in Y/D. I am inclined to the view that the anomalously high i_b values measured at East Fork are local only and result from strong local scouring due to a local increase in turbulence intensity, this increase being caused basically by the flow divergence in the lee of a significant flow obstruction, in this instance, the piers of the road bridge. Whether the obstruction is artificial or natural, the effect should be potentially self-augmenting. At any rate, initially, the deeper the scour the more rapid the flow divergence and the stronger the local turbulence intensity. Since average continuity along the channel requires the flow divergences to be followed by corresponding convergences, with associated local turbulence suppression and resulting sediment deposition, we have, on this view, the essential elements of an instability such as might, in the first instance, underlie the generally rhythmic character of natural river channels. The question whether the anomalous excess transport rates measured during rising stage periods are only local or are general along the river could be answered by making simultaneous measurements at cross sections differently sited in relation to pool and riffle.

On a number of high-stage occasions the $D_{50\%}$ size of the bed load collected in both Snake and Clearwater rivers changed abruptly from around 0.5 mm to around 50 mm, probably when the gravel or cobble bed happened to become denuded of a sand cover. From the last column of Table 1 it would seem at first sight that the transport rate of this large material fails to conform with (3). However, it is hardly possible that the measured rate of transport of solids 6–9 cm in size, and probably larger, into a sampler mouth but 15 cm square can ap-

TABLE 1. Transport Rates of Cobble-Sized Material in Snake and Clearwater Rivers

Date	$D_{50\%}$,* m	D_{max},* m	Y/D	ω, kg m^{-1} s^{-1}	$\omega - \omega_0$, kg m^{-1} s^{-1}	i_b^1†	i_b^2‡	Discrepancy Ratio i_b^1/i_b^2
				Clearwater River				
May 18, 1972	0.035	045	164	8.13	4.13	0.22	0.073	3.0
June 1, 1972	0.030	045	202	10.60	6.60	0.35	0.059	6.0
May 8, 1974	0.052	064	100	5.15	1.15	0.04	0.039	1.2
June 20, 1974	0.061	090	104	13.20	9.20	1.00	0.026	38.0
				Snake River				
February 6, 1974	0.066	?	55.4	4.45	0.45	0.016	0.016	1.0
March 13, 1974	0.036	064	110	6.35	2.35	0.125	0.058	2.2
March 26, 1974	0.043	064	101	9.00	5.00	0.420	0.077	5.4
April 11, 1974	0.042	090	112	12.60	8.60	0.870	0.153	5.7
April 24, 1974	0.035	090	136	13.00	9.00	0.830	0.095	8.7
May 7, 1974	0.057	090	94	18.00	14.00	2.000	0.093	22.0
May 14, 1974	0.028	090	180	15.20	11.20	0.940	0.105	9.0

*Sampled.
†From equation (3).
‡Measured.

proximate to the real free rate outside. Jostling against the sampler walls and partial blockage of the opening must greatly reduce the measured entry rate. Evidence of this severe wall effect can be seen in Figure 5*b*. In 1972 the only available sampler had a mouth but 7.5 cm square. This being found too small, a scaled up sampler having a 15 × 15 cm mouth was used subsequently. As can be seen, the i_b values as measured in 1972 with the smaller sampler are less by an order of 10 than those measured later with the larger instrument. The transport rate of this large material may therefore conform well to (3). If so the rates, from Table 1, would be as plotted in Figure 7. In this connection, considerable practical difficulty may be found in the reliable measurement of transport rates of large material by the sampler method. For the weight necessary to keep the sampler stationary on the bed must increase as the square of its dimensions, and the amount of loading is limited by the need to minimize interference with the movement of the bed material. Even the 15 × 15 cm sampler had to be loaded to 75 kg.

DISCUSSION

Relation (3) is of course purely empirical. If it is found to be generally applicable, it must result from some broad natural principles we do not at present understand. This relation, be it noted, appears to apply to a mean of river i_b values over an appreciable time period, individual values varying widely according to seasonal and other changes in the bed conditions. It is of interest to compare (3) with an alternative relation deduced by *Bagnold* [1973] purporting to apply to known constant bed conditions.

Pursuing the ideas outlined in the first section of this paper, let it be assumed initially that at a sufficiently high stage of flow, so many solids are saltating over the bed, virtually the whole of the fluid thrust τ is exerted on the saltating solids. Then the directly available transporting power ω_b is

$$\omega_b = \tau u_{nD}$$

The fluid velocity u_{nD} cannot be measured directly. Its value can, however, be inferred on the assumption that the velocity gradient in the body of the fluid above the saltation zone, being unaffected by the presence of a solid phase, should be logarithmic. On the further assumption that the mean velocity $\bar{u}$ coincides with the velocity at a distance $0.4Y$ from the bed, the velocity difference between the distances $0.4Y$ and nD from the bed will approximate to $5.75u_* \log (0.4Y/nD)$, whence

$$\omega_b = \tau[\bar{u} - 5.75u_* \log (0.4Y/nD)]$$

Now a fluid thrust on a solid requires a mean relative, or slip, velocity $\bar{u}_r$, and it can be easily shown by comparing the equation for vertical and tangential equilibrium, respectively, that

$$\bar{u}_r \simeq V_g$$

to a close approximation, where V_g is the terminal relative velocity of fall of a solid through the fluid. Thus at sufficiently high stage,

$$i_b = \frac{\omega}{\tan \alpha}\left[1 - \frac{5.75u_* \log (0.4Y/nD) + V_g}{\bar{u}}\right] \quad (4)$$

Since i_b is zero at the threshold of bed movement while ω is still finite, it was necessary to introduce an arbitrary modifying factor which increases from zero toward unity as the stage increases. I chose the simple function $(u_* - u_{*0})/u_*$. A number of similar functions are of course possible. With this modification,

$$i_b = \frac{\omega}{\tan \alpha}\,\frac{u_* - u_{*0}}{u_*}\left[1 - \frac{5.75u_* \log (0.4Y/nD) + V_g}{\bar{u}}\right] \quad (5)$$

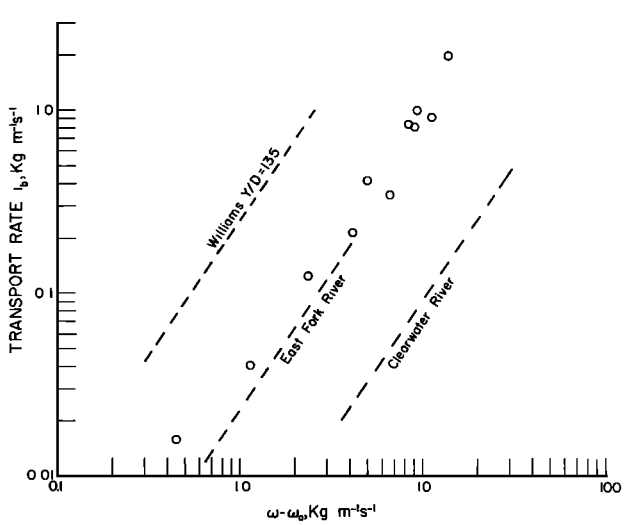

Fig. 7. Transport rate of cobble material in Snake and Clearwater rivers according to (3). The value of ω_0 is assumed to be 4 kg m^{-1} s^{-1}.

Finally, from observation of the saltations of single solids [*Francis*, 1973] it appeared that the effective saltation height nD increases somewhat slowly with increase of u_*/u_{*0}. The relation

$$n = a(u_*/u_{*0})^{0.6} \tag{6}$$

where a is a numerical parameter, suggested itself as a reasonable assumption.

With $1/\tan\alpha$ given its experimental value 1.6 and the parameter a chosen arbitrarily as 1.4, relation (5) was found to fit the Williams data for uniform sand, Figure 1, remarkably well.

The value of $\bar{u}$ in (4) and (5) is unpredictable, being known only by direct measurement in the actual conditions of bed load transport. It is just possible therefore that relations (3) and (5) are approximately equivalent over the limited range of conditions in which bed load transport, as this is understood in the present context, can occur (see below). The two relations seem to some extent to be complementary to one another. While (3) suggests a mean long-term trend, (5), being based on accepted dynamical ideas, throws light on the transport mechanism as operating in known constant bed conditions. It is evident from (5), for instance, that the transport efficiency $i_b \tan\alpha/\omega$ must have an upper limit less than unity, reached when the log term is zero, i.e., when $Y = 2.5nD$ and the flow is so shallow that the top of the saltation zone almost coincides with the water surface. It is perhaps not quite so evident that in the special conditions of the open gravity flow of a liquid, both i_b and ω are also limited.

Unlike other agencies for bed load transport, e.g., atmospheric wind, an abysmal ocean current, or a pressure gradient dp/dx, none of which are directly dependent on a gravity gradient, the stream power ω is defined and measured by $\rho YS\bar{u}$. Since maximum efficiency requires the flow depth Y to be a minimum, ω can only be increased by increasing the gradient S. Now, at appreciable gradients the directly available transporting power ω_b is augmented by an additional element i_bS due to the gravity component acting directly on the solids. Hence the fluid power ω must be replaced in both relations (3) and (5) by an augmented power ω^+. In the river context, however, these limiting conditions are somewhat irrelevant and border on the flow of super-saturated solids. The extreme case is of course that of the simple avalanche where $i_b \tan\alpha = i_bS$. The above carries the practical implication that all plots of i_b versus ω values measured in streams and rivers other than steep mountain torrents, such as those in Figure 1, 2, 5*a*, and 5*b*, must terminate at a discontinuity beyond which extrapolation is unreal.

Owing to the unfortunate sensitivity of (5) to errors in the assumed value of n (errors arising largely from the fact that conditions on a river bed are unobservable by any known means), it is not possible to apply (5) directly to river data. It is, however, possible to gain some insight into river bed conditions indirectly by applying (5) backwards to calculate the effective saltation height as influenced by the bed texture, etc. According to the empirical relation (6), a convenient measure of the effective saltation height is

$$a = n/(u_*/u_{*0})^{0.6}$$

which, being independent of river stage, is indicative of the relative height of saltation rise as this is affected by the texture, mean array, and heterogeneity of the bed surface solids. The value of a found to fit Williams' uniform grain size conditions was 1.4. Table 2 shows the corresponding East Fork River values computed from the river data by (5). As can be seen, the

TABLE 2. Effective Saltation Heights From East Fork River Data

Date	Power Ω, kg s^{-1}	Rate I_b, kg s^{-1}	Y/D	a	nD, cm
May 26, 1975	11.27	0.179	760	2.30	0.60
June 1, 1973	11.27	0.173	2311	3.20	0.39
June 2, 1973	12.46	0.214	1500	2.80	0.50
June 3, 1973	11.62	0.219	1500	1.58	0.25
June 6, 1973	7.91	0.121	1445	2.30	0.31
June 7, 1973	11.13	0.237	1710	3.00	0.455
June 6, 1973	13.44	0.200	1190	2.00	0.44
May 25, 1974	2.62	0.051	900	1.70	0.19
May 26, 1974	6.94	0.749	1254	3.30	0.44
May 27, 1974	15.05	1.600	1233	3.96	0.93
May 28, 1974	20.86	2.050	1143	4.50	1.40
May 29, 1974	29.00	2.650	658	2.36	0.83
May 30, 1974	22.50	0.716	1112	3.20	1.06
May 31, 1974	16.00	0.590	950	2.87	0.84
June 1, 1974	17.00	0.188	1460	2.87	0.65
June 2, 1974	18.00	0.118	1444	2.98	0.70
June 3, 1974	20.80	0.157	1800	3.30	0.75
June 4, 1974	19.50	0.261	1550	3.00	0.69
June 5, 1974	18.50	0.278	1810	3.30	0.69
June 15, 1975	20.30	1.080	1490	3.80	0.94
June 16, 1975	21.20	1.080	1351	3.80	1.07
June 17, 1975	15.50	0.725	955	2.98	0.84
June 18, 1975	8.96	0.097	1490	2.35	0.33
June 14, 1975	7.07	0.088	1030	1.94	0.30
June 21, 1975	5.06	0.029	842	1.50	0.21
June 22, 1975	4.90	0.043	905	1.60	0.20
June 23, 1975	5.66	0.056	944	1.84	0.28
June 24, 1975	7.56	0.177	796	2.00	0.38
June 25, 1975	15.20	0.763	1153	3.10	0.77
June 26, 1975	9.17	0.361	910	2.40	0.49
July 1, 1975	15.20	1.970	822	4.50	1.47
July 8, 1975	15.00	0.289	1400	2.90	0.62

TABLE 3. Effective Saltation Heights From Clearwater River Data

Date	Unit Power ω, kg m^{-1} s^{-1}	Unit Rate i_b, kg m^{-1} s^{-1}	Y/D, $\times 10^3$	a	nD, cm
May 10, 1972	4.20	0.017	10.6	1.80	3.00
June 15, 1972	3.80	0.021	9.2	1.55	2.75
April 17, 1973	0.81	0.0008	6.7	12.23	12.00
April 30, 1973	0.25	0.0088	5.2	12.80	16.00
May 14, 1973	0.42	0.0019	7.4	8.22	9.10
May 16, 1973	1.11	0.02	8.1	3.70	4.90
May 22, 1973	0.97	0.0052	7.8	3.78	5.00
May 24, 1973	0.97	0.0044	7.8	3.78	5.00
June 5, 1973	0.36	0.00008	7.6	9.76	10.10
June 8, 1973	0.50	0.00055	7.0	6.69	8.16
June 12, 1973	0.36	0.00022	7.2	9.30	9.60
June 13, 1973	0.32	0.0001	6.7	9.24	10.20
June 19, 1973	0.53	0.0014	7.0	5.40	6.50
June 20, 1973	0.32	0.00005	7.2	8.65	8.90
February 5, 1974	1.02	0.003	7.9	2.88	3.75
March 13, 1974	1.51	0.0075	6.1	2.12	3.70
March 27, 1974	1.38	0.0014	7.9	2.23	3.20
April 10, 1974	2.50	0.0042	9.1	1.54	2.45
April 23, 1974	2.30	0.0073	8.2	1.69	2.75
May 15, 1974	1.24	0.003	7.6	2.69	3.80
May 23, 1974	0.80	0.0005	8.5	3.09	3.70
June 4, 1974	4.75	0.096	9.7	1.79	3.30
June 6, 1974	7.67	0.014	12.5	1.76	3.00
June 12, 1974	4.55	0.026	10.2	1.65	2.90
June 17, 1974	16.00	0.08	14.1	1.60	3.40

a values are of the same order as in Williams' uniform sand conditions. The mean value is, however, twice as large at 2.8. This is to be expected from the very heterogeneous grain size composition, the $D_{50\%}$ grains having to jump over their larger and slower-moving neighbors. Tables 3 and 4 show the a values found similarly for the much bigger Clearwater and Snake rivers. Both these rivers are reported to have coarse or cobble pavements over which is transported sand of much the same $D_{50\%}$ as that in East Fork. It will be seen in Table 3 that the a values fall into two distinct groups. In 1972 and 1974 the relatively high stages at which measurements were made correspond to a values remarkably similar to those of East Fork. The lower stages of 1973 correspond, however, to considerably larger a values, with few exceptions. We may interpret this as indicating that in 1972 and 1974 the pavement was almost wholly sand covered whereas during 1973 the pavement became partially or wholly exposed. The transport of sand over and round a bed of solids 100 times larger would necessarily involve an average saltation rise much higher in relation to the datum bed level. Interestingly though, there appears to be no correlation between the changes in the a values and the scatter of the i_b values. In Table 4 we see that all the a values are large, from which we may surmise that the coarse pavement of the Snake River is rarely if ever sand covered. Notice also in Tables 3 and 4 that the large distances nD are of the same order as are the sizes of the large pavement material.

Figures 5a and 5b, together with the resulting empirical relation (3), confirm the dependence, already pointed out by

TABLE 4. Effective Saltation Heights From Snake River Data

Date	Unit Power ω, kg m^{-1} s^{-1}	Unit Rate i_b, kg m^{-1} s^{-1}	Y/D, $\times 10^3$	a	nD, cm
May 11, 1972	11.40	0.0059	7.30	5.10	13.6
May 19, 1972	16.80	0.014	10.90	7.20	17.1
June 2, 1972	23.70	0.024	12.10	9.70	25.5
June 14, 1972	14.50	0.0083	10.00	7.00	16.2
June 18, 1973	2.80	0.00004	6.70	9.88	16.7
May 1, 1973	3.30	0.00038	7.50	9.50	15.6
May 15, 1973	4.60	0.0003	7.44	8.60	16.4
May 17, 1973	6.90	0.018	7.10	6.50	15.0
May 21, 1973	7.50	0.027	8.40	7.00	14.5
May 23, 1973	6.20	0.014	8.70	7.80	14.6
June 6, 1973	4.00	0.007	5.10	7.70	18.2
June 7, 1973	4.50	0.018	4.00	6.30	18.4
June 11, 1973	4.20	0.007	4.80	7.20	18.1
June 13, 1973	4.40	0.005	4.10	7.10	18.1
June 18, 1973	4.00	0.004	6.00	8.20	17.6
June 19, 1973	3.50	0.005	3.70	6.65	19.0
May 16, 1974	12.50	0.059	7.20	5.14	14.3
May 22, 1974	9.75	0.010	6.30	5.30	14.7
June 5, 1974	29.70	0.011	19.40	15.9?	32.0?
June 11, 1974	18.20	0.035	8.80	6.40	18.9
June 24, 1974	32.00	0.134	18.30	14.7?	33.0?

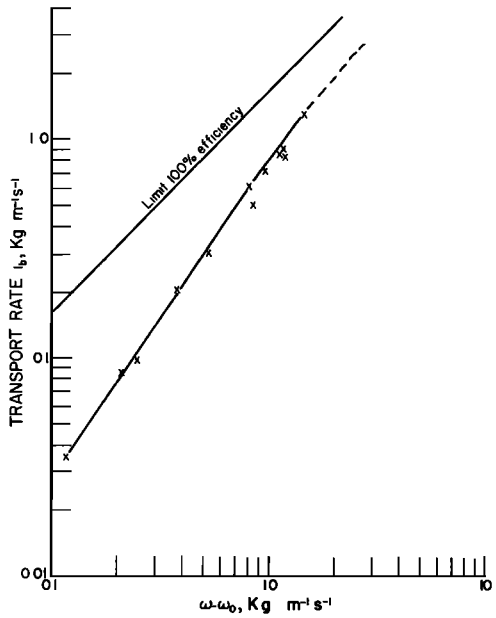

Fig. 8. Gilbert's sand D, 0.78 mm. The value of ω_0 is assumed to be 0.06 kg m^{-1} s^{-1}. Values of i_b are adjusted to a constant Y/D = 50. Note the remarkable reduction of scatter resulting from the adjustment to a common Y/D.

Bagnold [1973], of the bed load transport rate i_b on the depth to grain size ratio Y/D. This dependence has several important implications:

1. Transport rates measured in small laboratory flumes cannot without considerable modification be applied to flows of greater depth.

2. The cause of the gross scatter associated with early laboratory measurements of i_b, such as those of *Gilbert* [1914] or *U.S. Waterways Experimental Station* [1935], in which the stage was varied by varying the flow depth Y, is simply explained. This scatter is largely removed by adjusting the i_b to a common Y/D ratio. In Figure 8 I have plotted a random selection of Gilbert's data for his sand D, including all those for the highest stages, as i_b versus $(\omega - \omega_0)$. I have taken ω_0 as the value corresponding to the smallest i_b in the data. I have also adjusted the i_b values to a common Y/D of 50 by applying the factor $[Y/D/50]^{2/3}$. The reduction in the scatter is striking and indicates the basic accuracy of Gilbert's observations.

3. Since the whole river values I_b and $\Omega = \rho QS$ are both obtained by multiplying i_b and ω, respectively, by the river width, all relations between i_b and ω apply equally between I_b and Ω. Thus at any given discharge Q and gradient S an alluvial river can transport a bed load of a given mean grain size at a greater rate the shallower the flow depth. It follows that if over the years a river is neither aggrading nor degrading its channel, the width/depth ratio must have become so adjusted that the mean sediment transport rate everywhere is just equal to the environmentally controlled rate at which sediment enters the river upstream. The bed is then protected from downcutting by a constant sediment layer. If, however, the mean supply rate of mobile sediment were to fall below the mean transport rate, this protective layer would be eroded away. Downcutting would result; the ratio Y/D would consequently increase, thereby reducing the excess transport capacity until a balance was restored. If, on the other hand, the mean supply rate were to increase, the bed would remain protected by an increasing thickness of deposited sediment leaving only the banks unprotected. The flow would become shallower and the consequent decrease in Y/D and increase in the mean transport rate would again tend to restore a balance.

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