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Statistical Considerations and Survey of Plutonium Concentration Variability in Some Terrestrial Ecosystem Components

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Statistical Considerations and Survey of Plutonium Concentration Variability in Some Terrestrial Ecosystem Components[1]

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ABSTRACT

Statistical aspects of the use of the coefficient of variation (c) in the design of environmental plutonium studies are considered. Small sample properties of the estimate of c are studied for both normal and log-normal populations with Monte Carlo simulations. A minimum sample size of 50 is necessary to estimate c with much precision, and an estimate of c from a sample size of less than 5 is nearly worthless. The difficulty of achieving good estimates of c increases as the true value of c increases.

In addition, some of the available data on the variability of plutonium concentrations in terrestrial ecosystem components is summarized. The data were pooled to achieve better estimates of the coefficient of variation than might be available from individual studies. Pooled estimates of c are: soils, 1.35; vegetation, 0.70; and rodent tissues, 1.10. The estimates of c presented should be useful in designing a field sampling program for plutonium.

Additional Index Words: **^{238}Pu, ^{239,240}Pu, radioactive wastes, coefficient of variation, terrestrial ecosystems.**

The large variability associated with environmental plutonium data is a major consideration in designing field studies for this element because of the large number of samples required for acceptable statistical conclusions. The high cost of sample analyses severely restricts the kinds of experiments that can be performed economically and still produce conclusive results.

The purpose of this paper is to present some statistical considerations concerning the coefficient of variation, summarize some of the available data on variability of plutonium concentrations in terrestrial ecosystem components, and identify the potential sources of this variability toward providing guidelines for designing studies that will efficiently achieve acceptable experimental results.

The number of observations (n) required for acceptable results is dependent on (i) the desired precision of the estimate (confidence interval length) or the power of the test to be obtained, and (ii) the variance of the data. The relationship between sample size (n), precision of the estimate, and the calculated variability expressed as the coefficient of variation ($c = \sigma/\mu$, where σ is the standard deviation and μ is the mean), is given in Fig. 1 for the case $c = 0.7$. Ninety-five percent confidence intervals are given as the percent of the mean for various sample sizes. It is apparent from Fig. 1 that large sample sizes are necessary to obtain acceptable confidence limits on the mean. In addition, note that reducing the length of the confidence interval by half requires quadrupling the sample size.

Deming (1950) discusses how c can be used to determine the necessary sample size (n) to estimate a mean with some standard error $\gamma\overline{X}$, when the standard error is expressed as a percent of the mean. Deming denotes γ as the coefficient of variation of the estimate, that is

$$\gamma = SE(\overline{X})/\overline{X}.$$

Without any knowledge of the mean ($\overline{X}$), the sample size required to estimate $\overline{X}$ with standard error $\gamma\overline{X}$ is

$$n = (c/\gamma)^2.$$

The above equation ignores the finite population correction factor used when sampling a large proportion of a finite population. From this equation, we see that n increases as the square of c, or that as c doubles, n quadruples.

The above exercise demonstrates the importance of good estimates of the expected variability in the experiment. Hence an efficient experimental design is contingent on the best available estimate of c. Before large sums of money are expended, we should consider the expected precision of the results to determine whether the experiment is worth the cost.

Plutonium data from 7 geographical regions, representing 15 terrestrial study sites, were selected for summarization based on the availability of the data, source of plutonium in the study area, sampling methodology, and climate. The intent in selecting these study areas was to present plutonium concentration variability estimates from diverse conditions.

We have pooled the data to produce better estimates of the coefficient of variation than might be available.

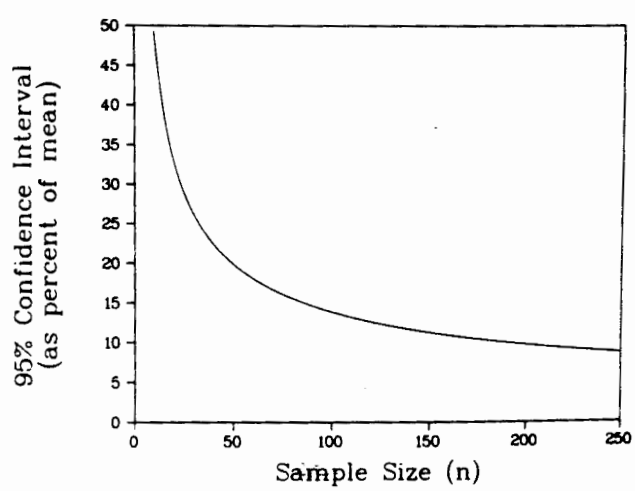

Fig. 1—Ninety-five % confidence intervals (as % of mean) achieved using various numbers of observations (n) for a coefficient of variation (c) equal to 0.7.

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The coefficient of variation appears to be constant over a wide range of concentrations, and hence σ is proportional to μ. We recognize that pooling values from diverse origins results in the risk of combining values that are actually different. However the approach of not pooling is likewise dangerous, as the investigator may use a poorly estimated c based on relatively little data to design a study. In general, c is poorly estimated for any particular study. Given that different estimates from different locations actually represent the same c, we believe that the pooled estimate is much preferred to the individual values. The applicability of the pooled estimates we obtain are a matter of judgment of the investigator designing a study. If prior information on the plutonium concentration is lacking for a particular study, then the pooled estimate will be particularly useful.

STATISTICAL ASPECTS OF THE COEFFICIENT OF VARIATION

Before discussing the coefficient of variation of the plutonium concentrations found at selected sites, we need to consider the precision associated with c for a set of data. Cramér (1946, p. 358) developed an approximate equation for the variance of $\hat{c}$, $\hat{\mathrm{Var}}(\hat{c})$, based on the true value of c (see also Sokal and Rohlf, 1969). The caret (^) signifies an estimate of the quantity. Substituting the estimate of c, we obtained an approximate estimate of the variance of the estimated coefficient of variation:

$$\hat{\mathrm{Var}}(\hat{c}) = \hat{c}^2(1 + 2\hat{c}^2)/2n$$

where $\hat{c} = [\sum(X_i - \overline{X})^2/(n-1)]^{1/2}/\overline{X}$. Approximate 95% confidence intervals are then constructed for c as $\hat{c} \pm t(0.05, n-1)[\hat{\mathrm{Var}}(\hat{c})]^{1/2}$ where $t(0.05, n-1)$ is the t-statistic with $n-1$ degrees of freedom.

Cramér's variance estimate assumes the sample is from a normal distribution truncated at zero and that c is "fairly small," (Cramér, 1946, p. 358). Neither of these assumptions are commonly met with plutonium data. However, we want an easily calculated approximation of $\mathrm{Var}(\hat{c})$ instead of an exact result. Other methods of calculating confidence intervals are summarized by Iglewicz[1] and Owen (1963). However, the more precise methods require a great deal more calculation and often still assume a normally distributed sample. We have opted for Cramér's estimate so that the researcher can quickly put a confidence interval around c for his data and as a result have some feeling for the variability.

The use of the t distribution to provide a confidence interval for c implies c is normally distributed. Iglewicz[1] discusses the asymptotic convergence of the distribution of c to normality. We are well aware of the problems of assuming the normal distribution of c with small sample sizes. However, we emphasize, again, the advantages of an easily calculated confidence interval at the expense of statistical rigor.

The achieved coverage (proportion of time the confidence interval includes the true value of c) of the above approximate confidence interval was compared to the expected 95% for small sample sizes using Monte Carlo simulation techniques. One thousand replications of populations with $\mu = 100$, $c = 0.1$ to 5.0, and $n = 5, 10, 20, 30, 50, 100$ were simulated for a normal population using Bell's (1968) algorithm. The achieved coverages for some values of n are plotted in Fig. 2. The achieved coverage was at least 85% for $c \leq 2$, even for $n = 5$. For small samples with a confidence interval derived from an asymptotic approximation, this value was much closer to the expected 95% than we had anticipated. For values of $c > 2$, however, the achieved coverage drops, even for $n = 100$. With $c = 5$, the achieved coverage for $n = 5, 30, 100$ is 68, 84, and 90%, respectively. For $n = 100$, the achieved coverage was always >92% for $c \leq 2$. In almost all cases where the true value of c was not covered, the confidence interval constructed was low, i.e., $\hat{c} + t_{(0.05, n-1)}[\hat{\mathrm{Var}}(\hat{c})]^{1/2}$ was less than c. This would be expected based on the estimator of $\mathrm{Var}(\hat{c})$. A low value of c results in a very low value of $\hat{\mathrm{Var}}(\hat{c})$.

There is an additional aspect of this confidence interval which is worth noting. The lower confidence bound is more likely to include zero for large c than for small c. Assume that we want to find the value of $\hat{c}$ for a particular sample size for which the lower confidence bound (LCB) is greater than zero, i.e., the confidence interval does not include the value zero. Let

$$\mathrm{LCB} = \hat{c} - t_{(0.05, n-1)}[\hat{\mathrm{Var}}(\hat{c})]^{1/2}.$$

Setting LCB to zero and substituting for $\hat{\mathrm{Var}}(\hat{c})$, we see

$$\hat{c} = t_{(0.05, n-1)}[\hat{c}^2(1 + 2\hat{c})/2n]^{1/2}.$$

Rearranging to solve for $\hat{c}$,

$$\hat{c} = [n/t^2_{(0.05, n-1)} - 1/2]^{1/2}.$$

From this equation, we find that the confidence interval for sample sizes of four and less all include the value of zero. That is, no matter what the value of c, a sample size of four always estimates c so poorly that LCB <0. Also, for a particular value of n, a small c is more likely to have a LCB >0 than a large c. In Fig. 3, we have plotted for a range of n a lower limit of c such that any greater value of c will result in LCB <0 for fixed n. We conclude that (i) $\hat{c}$ based on an n of less than five is nearly worthless; and (ii) for c in the range 0.1 to 2.0, an $n = 50$ is the minimum sample size for which much confidence can be associated with $\hat{c}$.

Another aspect of estimating the coefficient of variation that should be mentioned is that the estimate is always less than $n^{1/2}$ for a sample

[1] B. Inglewicz. 1967. Some properties of the sample coefficient of variation. Ph.D. Diss. Virginia Polytech. Inst. and State Univ., Blacksburg, Va. 147 p.

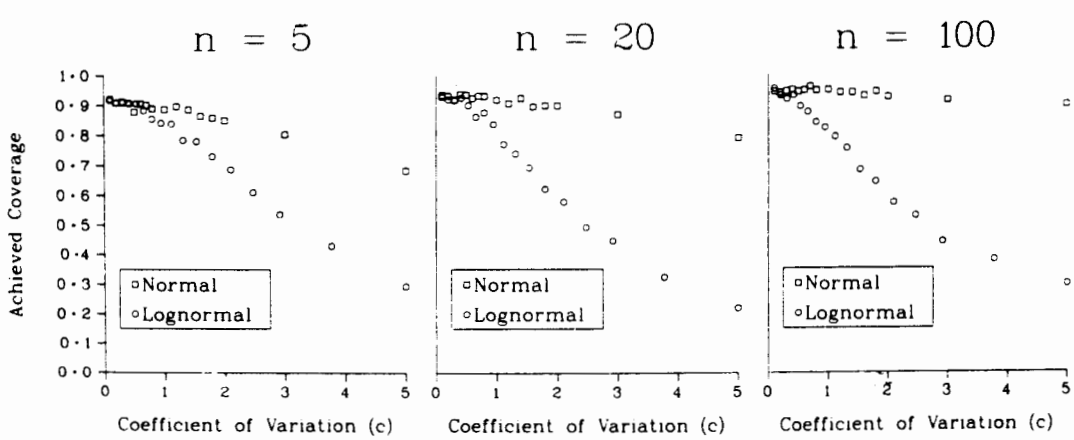

Fig. 2—The achieved coverage of the confidence interval plotted as a function of $\hat{c}$ for various sample sizes.

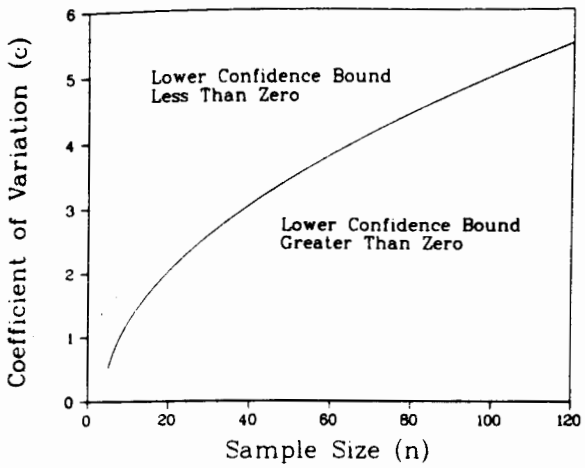

Fig. 3—Sample size (n) and the associated $\hat{c}$ such that the lower confidence bound on $\hat{c}$ is zero.

from a population which takes only positive values (Cramér, 1946). Hence, if $n = 5$, the largest value that $\hat{c}$ can have is $5^{1/2} = 2.24$. Therefore, a biased estimate of c will be obtained for $c > n^{1/2}$.

A criticism of the Monte Carlo study is that plutonium concentrations are not normally distributed, such as the simulated values. We cannot conclusively resolve this criticism since we cannot determine the true distribution of plutonium concentrations. Something can be learned here, however, by studying the coverage of the confidence interval suggested above when the distribution is skewed. Hence, we reran the Monte Carlo simulation with the lognormal distribution. In this case

$$c = [\exp(\sigma^2) - 1]^{1/2},$$

where σ^2 is the variance of the normally distributed log-transformed variable. More efficient estimators (i.e., maximum likelihood estimator) of c are available than the one simulated, but researchers seldom "know" the distribution they are working with. Hence the estimator we simulated is generally used. Aitchison and Brown (1969) compared the efficiency of moment and maximum likelihood estimators of E(X) and Var(X) for the lognormal distribution, and often found large differences. Therefore, we did not expect the confidence intervals constructed for c from lognormal data to be as good as those constructed for normal data. Koopmans et al. (1964) provide a method of putting a confidence interval on c when the sample is from a lognormal population.

The estimate $s/\overline{X}$ was simulated for 1,000 replications of $\sigma = 0.1$ to 1.5 in increments of 0.1, 1.65, and 1.805, providing a range of c from 0.1 to 5.0. Additionally, simulations with $n = 5$, 10, 20, 30, 50, and 100 were used with each value of σ. The mean of the normal distribution was 10.0 for all values of σ. Results of some of the simulations are presented in Fig. 2. The coverage reached the expected 95% only for the case $\sigma = 0.1$ and $n = 100$. Generally the sample size, n, did not have much effect on coverage. For $c = 0.947$ ($\sigma = 0.8$), the achieved coverage was approximately 80%; for $c = 2.102$ ($\sigma = 1.3$), approximately 58%; and for $c = 5.000$ ($\sigma = 1.805$), approximately 25%. The poor performance of the confidence interval can be attributed largely to the bias of the estimate of c. The average relative bias [RB = 100 ($\hat{c} - c$)/c] of $\hat{c}$ was −43% for $c = 5.0$, $n = 100$. The average relative bias decreased as c was reduced, but was still serious for $c = 0.947$ (relative bias = −2.8%, $n = 100$). As with the normal distribution, the upper confidence bound tended to be less than c when the confidence interval did not cover the true value.

As a result of this simulation, we believe many of the c values given in the following sections are biased low because $s/\overline{X}$ was used to estimate c. Plutonium concentrations are known to be skewed, and often the lognormal distribution is considered a reasonable probability density function (Eberhardt, 1976). In addition, the coverage of the confidence interval described is much less than expected. However, researchers have traditionally used the methods simulated, and the results reported in the literature provide the only data on plutonium concentration variability.

Researchers are often tempted to regress $\hat{c}$ with the observed concentration $\overline{X}$. We feel this practice is generally unpractical as $\hat{c}$ is a function of $\overline{X}$, and hence negative slope is expected, so that a significant but inapplicable relationship is present.

Additionally, we would discourage testing values of $\hat{c}$ by analysis of variance and linear regression techniques without correcting for the heterogeneity of variance. As demonstrated above, the $\hat{\text{Var}}(\hat{c})$ is at least proportional to $\hat{c}^2$, so that these methods are invalid unless properly weighted. Likewise, many nonparametric tests (cf., Kruskal-Wallis, Friedman) require constant dispersion (Hollander and Wolfe, 1973) so that these tests will not be valid.

The basic conclusion we can draw from the material discussed above is that $\hat{c} = s/\overline{X}$ has a large variance and is biased with poor coverage, and that the difficulty increases as c increases. Thus to design an experiment with a $\hat{c}$ based on a small sample will often give unsatisfactory results. A good estimate of c needs to be used to achieve a research design meeting the investigators need.

In the remainder of this paper, to roughly compare values of $\hat{c}$, we check pairwise for overlap in the 95% confidence intervals. This method is approximate. However, keep in mind the poor quality of the estimates, and don't be misled into thinking that we have a more precise estimate than we do. Pair-wise checking will provide us with guidelines to use in interpreting the data.

Regression analyses are done with the weight of the observation $\hat{c}$ being $\hat{\text{Var}}(\hat{c})^{-1}$. This procedure is described clearly by Frank (1971).

DESCRIPTION OF SOME ENVIRONMENTAL PLUTONIUM STUDY AREAS

The seven sites chosen to represent the varied conditions under which plutonium is presently distributed in the environment are listed in Table 1. Sources of plutonium in these areas include industrial liquid effluents (Los Alamos), accidental releases from industrial and military sources (Rocky Flats, Thule), localized fallout from both single and multiple weapons tests (Trinity Site, Glenn, and Janet), nonfission explosive tests with plutonium devices (Nevada Test Site [NTS]), and world-wide fallout from nuclear weapons testing (northern Alaska).

The physical size of the study plots varied from 0.01 ha in northern Alaska to 124 ha of Strata 1, Area 13, at the Nevada Test Site. How-

Table 1—Environmental plutonium study areas

Location	Source of Pu	Mean annual precipitation (cm)	References
Los Alamos			
Mortandad Canyon	Industrial liquid effluent	46	Nyhan et al. (1976b)
DP-Los Alamos Canyon	Industrial liquid effluent	46	Nyhan et al. (1976b)
Acid-Pueblo Canyon	Industrial liquid effluent	46	Hakonson and Bostick (1976)
Trinity Site	Single Weapons Test:		Nyhan et al. (1976a)
GZ	1 km from ground zero	15	Nyhan et al. (1976a)
Area 21	44 km from ground zero	20	Hakonson and Johnson (1974)
Enewetak Atoll	Weapons Test:		USAEC (1971)
Janet	Multiple ground zeros and multiple fallout	145	
Glenn	Multiple ground zeros and multiple fallout	145	
Rocky Flats	Unintentional release from leaking drums		Little (1976)
Macroplot 1	Unintentional release from leaking drums	40	
Macroplot 2	Unintentional release from leaking drums	40	
Nevada Test Site			
Area 13			Dunaway and White (1974)
Strata 1	Safety test shots	8	White and Dunaway (1974)
Strata 6	Safety test shots	8	Dunaway (1974)
Area 5 (GMX)	Safety test shots		
Strata 1	Safety test shots	8	
Strata 4	Safety test shots	8	
Northern Alaska	Worldwide fallout	20	Hanson (1975)
Thule, Greenland	Aircraft accident	13	Hanson (1975)

ever, the size of the study area has little bearing on sample size requirements in the absence of a plutonium concentration gradient (Eberhardt, 1976).

Annual precipitation in the study areas varied from about 8 cm in the desert at Nevada Test Site to 145 cm on tropical island Janet in the central Pacific Ocean.

The predominant plutonium isotope present in most study areas is $^{239,240}Pu$, with the exception of the Mortandad Canyon Site at Los Alamos, where the principal radioisotope is ^{238}Pu. Little is known concerning the present physical and chemical forms of the plutonium at the sites in Table 1 or what changes in form may occur as the material weathers. The "age" of the plutonium at these sites varies from less than 10 years to nearly 35 years.

Results from the Rocky Flats Site suggest that the $^{239,240}Pu$ particles in study plot soils are very small, probably < 1 μm in diameter (Little, 1976). Preliminary studies at Los Alamos indicate that as much as 60% of the plutonium in liquid effluent is associated with particles less than 0.45 μm diam, although the amount of plutonium which passes through a 0.45-μm membrane filter varies greatly, presumably due to variable amounts of chelates in the effluent. The physical characteristics of the plutonium probably varies considerably within and among the sites, depending on circumstances related to the particular source.

The predominant chemical form of plutonium at most of the study sites is likely the oxide (PuO_2) because plutonium metal rapidly oxidizes in the air (Coleman, 1965). However, little data supporting this view are available. Some recent work (Dunaway and White, 1974) indicates that $^{239,240}PuO_2$ is the predominant chemical form in the soils at the safety shot areas of NTS.

The data to be presented from the study areas listed in Table 1 include variance components due to within sample or aliquot variability, between samples collected at different locations and soil depths and hence possibly including a gradient effect, and between samples collected at the same location. However, none of the data include variability attributable to time. By pooling estimates of c from these areas, we have combined the variance components listed above to give an estimate of the expected c for a "typical study."

VARIABILITY OF PLUTONIUM CONCENTRATIONS IN STUDY AREA SOILS

The variability in soil plutonium concentrations, as expressed by $\hat{c}$, from several study locations is presented in Table 2 along with the average concentrations and sample sizes from which the $\hat{c}$'s were calculated. Based on the pairwise overlapping confidence interval method for testing the equality of two estimates of c as described earlier in the paper, we found that most of the interval estimates for data in Table 2 overlapped, with the exception of the two Rocky Flats samples, If the Rocky Flats data are excluded, then the only confidence intervals not overlapping are the samples from Janet and Glenn Islands in the Enewetak Atoll.

Table 2—Plutonium concentration and variability estimates in some study area soils.

Location	n	Depth (cm)	Mean (pCi/g)	$\hat{c}$
Los Alamos				
Mortandad	15	30	90	0.79
DP-Los Alamos	21	30	0.73	1.6
Acid-Pueblo	21	30	21	1.7
Trinity Site				
Ground Zero	8	25	0.07	0.68
Area 21	8	33	0.14	0.68
Enewetak Atoll				
Janet	138	15	16	1.3
Glenn	29	15	0.11	0.62
Nevada Test Site				
Area 13				
Stratum 1	39	5	36	1.4
Stratum 6	47	5	14,000	3.1
Area 5				
Stratum 1	41	5	59	1.4
Stratum 4	23	5	7,300	1.1
Thule, Greenland	6	5	0.16	1.5
Rocky Flats				
Macroplot 1	381	21	1,263	2.7
Macroplot 2	550	21	48.4	8.4

Because much of the variability in flora and fauna may be attributed to soil variability, additional emphasis has been made here in determining the components of variability. We examined three sets of data using weighted linear least squares to determine the significance of the plutonium concentration coefficient of variation vs. depth and soil particle size fraction. The assumptions of least squares are a linear, additive model with independent and normally distributed error terms. The regression equation fitted to the data was

$$\hat{c}_i = \beta_0 + \beta_1 d_i + \beta_2 p_i + e_i$$

where

d_i = the depth of the ith sample (cm),
p_i = the midrange of the size fraction of the ith sample (μm),
e_i = the error term,
$\hat{c}_i$ = the coefficient of variation of the ith sample, and
β_j = the jth parameter to be estimated from the data.

The three equations resulting from the data given in Nyhan et al. (1976b) and Little (1976) are given in Table 3.

Note that all three data sets produce the same conclusions about the depth of the sample and the soil particle size fraction, i.e., $\hat{c}$ is reduced on the average by 0.03 for each centimeter of depth, but is relatively unaffected by the particle size fraction. Visual examination of the residuals in each data set showed the equations adequately fit the data.

A lingering question about the regression analysis is whether the soil particle size fraction is nonlinearly related to $\hat{c}$. We checked for a more general relationship by treating particle size as a discrete variable (classification variable) and doing an analysis of covariance for the Los Alamos data set. No significant differences were found between the size fractions. Hence we conclude that c is significantly reduced with depth, but there are no apparent differences due to different soil particle size classes. This result implies that the variability is large in all soil particle size fractions.

In the preceding paragraphs we have identified a variety of factors which we feel should affect plutonium concentration variability. However, as shown in the section on the statistical aspects of $\hat{c}$, the estimation problems are severe and the values of $\hat{c}$ are seldom statistically different. Our calculations show that the Rocky Flats samples are generally more variable. Consequently, we have excluded them from the following calculations. A pooled estimate was made of the remaining values of $\hat{c}$ from Table 2 to provide an estimate of the coefficient of variation based upon field sampling experience. One possible approach is the mean (weighted by sample size) of the $\hat{c}$ values given in the right-hand column of Table 2 as recommended in Ziegler (1973).

Table 3—Results of weighted least squares analysis of $\hat{c}$ vs. depth and soil particle size (Anova table).

Source	df	SS	F	Significance	Complete equation	Reduced equation
			Los Alamos, Mortandad Canyon†			
Depth (d)	1	0.4164	55.3	0.000	$\hat{c}_i = 0.6860 - 0.0314d_i + 0.6086 \times 10^{-5}p_i + e_i$	$\hat{c}_i = 0.7124 - 0.0325d_i + e_i$
Particle size (p)	1	0.0078	0.91	0.324		
Regression	2	0.4714	31.0	0.000		
Residual	21	0.1595				
			Rocky Flats, Macroplot 1‡			
Depth (d)	1	1.66	28.4	0.000	$\hat{c}_i = 0.7472 - 0.0251d_i + 0.2155 \times 10^{-3}p_i + e_i$	$\hat{c}_i = 0.7906 - 0.023d_i + e_i$
Particle size (p)	1	0.17	2.97	0.091		
Regression	2	1.68	14.5	0.000		
Residual	53	3.07				
			Rocky Flats, Macroplot 2‡			
Depth (d)	1	2.34	25.8	0.000	$\hat{c}_i = 1.413 - 0.0326d_i + 0.1326 \times 10^{-4}p_i + e_i$	$\hat{c}_i = 1.4189 - 0.0327d_i + e_i$
Particle size (p)	1	0.00	0.02	0.888		
Regression	2	2.38	13.1	0.000		
Residual	53	4.82				

† From Nyhan et al. (1976).
‡ From Little (1976).

However, we have pointed out earlier the ad hoc nature of our methods to test between values of $\hat{c}$, and thus would suspect that some of the values presented in Table 2 may actually be significantly different, but our methods lack the power to detect these differences. An estimator of the central value of $\hat{c}$ is desired which is robust to outliers, and such an estimator is the median. For the values of $\hat{c}$ in Table 2 (excluding the Rocky Flats samples), the median is 1.35 with a 95% confidence interval of 0.68 − 1.6. Methods for determining confidence intervals for a median and $n < 25$ are given by Hollander and Wolfe (1973, p. 48–49). If the soil sampling is strafied by depth, then the value of $\hat{c}$ used to determine sample size should be estimated from the appropriate regression equation given in Table 3.

PLUTONIUM CONCENTRATION VARIABILITY IN NATIVE VEGETATION

Plutonium concentrations and the associated $\hat{c}$ for native vegetation are presented in Table 4. Average concentrations of plutonium in or on various species ranged over five orders of magnitude (0.003–870 pCi/g) while $\hat{c}$ varied by a factor of 11 (0.36–4.05). The range in concentrations and $\hat{c}$ in vegetation are of the same magnitude as observed in soils (Table 2). As with soils, the consistency of $\hat{c}$ between species and sites is remarkable considering the diversity of ecosystems, climat sources of plutonium, and sampling methodologies.

All of the interval estimates for the data in Table 4 overlapped one another, with the exception of the

Table 4—Variability of Pu concentrations in native vegetation samples.

Location	Sampling technique	Vegetation type	n	Mean (pCi/g dry)	$\hat{c}$
Los Alamos					
Mortandard	Grab	Grass	5	17.0	0.64
DP-Los Alamos	Grab	Grass	9	0.09	1.00
Acid-Pueblo	Grab	Grass	3	0.33	0.70
Trinity Site					
Ground Zero	1 m² quadrat	Grass	7	0.003	0.40
Area 21	1 m² quadrat	Grass	8	0.160	0.36
Rocky Flats					
Macroplot 1	0.5 m² quadrat	Grass	76	140.7	2.02
Macroplot 2	0.5 m² quadrat	Grass	78	2.78	4.05
Los Alamos					
Mortandad	Grab	Tree	6	0.18	0.82
DP-Los Alamos	Grab	Tree	13	0.05	2.00
Acid-Pueblo	Grab	Tree	8	0.006	0.70
Enewetak Atoll					
Janet Island	Grab	Lichen	7	0.007	0.70
Northern Alaska	0.25 m² quadrat	Lichen	8	0.15	0.52
Thule, Greenland	0.25 m² quadrat	Lichen	10	0.26	0.49
Nevada Test Site					
Area 5, Stratum 4	Random grab	Shrub	9	280	0.50
Area 13, Stratum 1	Random grab	Shrub	31	4.7	0.66
Area 13, Stratum 6	Random grab	Shrub	11	870	0.91
Area 5, Stratum 1	Random grab	Shrub	27	11	1.42
Los Alamos					
Mortandad	Grab	Shrub	13	0.71	1.70
DP-Los Alamos	Grab	Shrub	8	0.02	0.82
Acid-Pueblo	Grab	Shrub	10	0.009	0.70
Enewetak Atoll					
Janet Island		Shrub	6	0.008	0.6

Table 5—Plutonium concentration ($\overline{X}$) and variability ($\hat{c}$) in native animal tissues.

Location	Lung			Liver			Pelt			Viscera			Carcass		
	n	$\overline{X}$	$\hat{c}$	n	$\overline{X}$	$\hat{c}$	n	$\overline{X}$	$\hat{c}$	n	$\overline{X}$	$\hat{c}$	n	$\overline{X}$	$\hat{c}$
	pCi/g														
Los Alamos															
Mortandad	15	1.30	1.6	5	0.12	0.65	5	1.9	0.79				5	0.07	1.1
DP-Los Alamos	19	0.08	1.0	19	0.04	2.0	19	0.11	1.1				19	0.01	1.0
Acid-Pueblo	5	0.08	0.91	5	0.08	2.0	5	0.03	1.0				5	0.004	0.7
Trinity Site															
Ground Zero	7	0.30	1.0	7	0.01	2.0	7	0.03	2.0	7	0.11	1.6			
Area 21	2	0.01	1.0				2	0.01	0.10	2	0.02	0.40			
Rocky Flats															
Macroplot 1	47	3.6	1.9	46	8.4	5.4	47	1.5	1.8	40	7.1	2.5			
Macroplot 2	27	45.0	2.8	27	1.3	1.6	27	2.6	4.0	22	2.2	1.9			
Enewetak															
Janet Island	5	0.90	0.45							5	0.67	0.64	5	0.007	0.60

Rocky Flats Macroplot 1 sample. Hence, we eliminated the Rocky Flats data and pooled the remaining data in Table 4 to estimate an overall coefficient of variation. The median was 0.70 with a 95% confidence interval of 0.64–0.82.

Variable amounts of soil associated with the external plant surface may result in a major portion of the plutonium sampling variability in vegetation samples from the field. Development of a method for measuring plant surface area as a means of expressing plutonium concentrations in spatial units (i.e., pCi/cm² plant surface) may reduce the variability.

With the large relative variability shown in Table 4, research questions such as the change in plutonium availability to plants through time can only be answered at the expense of considerable cost and time.

PLUTONIUM CONCENTRATION VARIABILITY IN NATIVE ANIMALS

Plutonium concentrations and variability of data for rodents from four study sites are given in Table 5. Plutonium concentrations in the various species ranged from 0.004 to 45.0 pCi/g dry tissue while the values of $\hat{c}$ varied from 0.10 to 5.4.

The relationship of $\hat{c}$ to the plutonium source was similar to that observed in soils and vegetation, with the Rocky Flats data being the most variable. There was a tendency for lower values of c to be associated with fallout and liquid effluent sources of plutonium, although all of the interval estimates for the data in Table 5 overlapped.

A central value of $\hat{c}$ was obtained by pooling the data in Table 5. The median was 1.1 with a 95% confidence interval of 1.0–1.9 and 1.0 with confidence interval of 0.79–1.1 when the Rocky Flats data are excluded. The median was much higher than the estimate for vegetation. Analytical uncertainties are a large component of $\hat{c}$ for rodent tissues due to the low plutonium concentrations. However, the Rocky Flats data had both the largest concentrations and variability, which would not be expected if detection problems were the cause of the increased $\hat{c}$. Until a better understanding of plutonium concentration variability in rodents is achieved, we believe there are very few research questions that can be asked for which conclusive results can be obtained.

CONCLUSIONS

The usual estimator of the coefficient of variation, $s/\overline{X}$, has a large negative bias for lognormal distributions. The variance of $\hat{c}$, derived by Cramer (1946) is useful in constructing approximate confidence intervals on $\hat{c}$ for normally distributed populations, but performs poorly when the population is a sample from a lognormal distribution. Estimation of c requires large sample sizes.

Coefficient of variation for plutonium concentrations in terrestrial ecosystem components from seven geographical regions ranged from 0.62–8.4 in whole soil samples, 0.36 to 4.05 in vegetation, and 0.10 to 5.4 in rodent tissues. From these data pooled estimates and confidence intervals were obtained for use in the design of future studies: soil, 1.35(CI = 0.68 − 1.6); vegetation, 0.70(CI = 0.64 − 0.82); and rodent tissues, 1.1(CI = 1.0 − 1.9).

The range in $\hat{c}$ in ecosystem components was generally consistent between sample types and between study areas, although lower c's were generally associated with fallout and effluent sources of plutonium. However, the large uncertainty associated with $\hat{c}$ precludes any meaningful interpretation of results.

In the absence of adequate site specific data, we believe that the estimates of $\hat{c}$ presented in this paper will be useful in designing a field sampling program for plutonium.

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