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The Jackknife, the Bootstrap and Other Resampling Plans

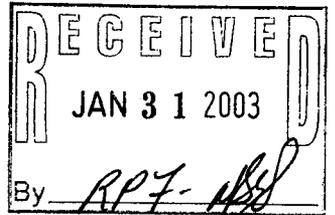
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◀ Previous page

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Next page ▶

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[Back to book details](#)

[Look inside the book](#)

[Conti](#)

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3615

Copyrighted material; sample page 2 of 7

Contents

Preface	vii
Chapter 1	1
INTRODUCTION	
Chapter 2	5
THE JACKKNIFE ESTIMATE OF BIAS	
2.1. Quenouille's bias estimate	5
2.2. The grouped jackknife	7
2.3. A picture	7
2.4. Aitken acceleration	8
2.5. The law school data	9
2.6. What does BIAS really estimate?	10
Chapter 3	13
THE JACKKNIFE ESTIMATE OF VARIANCE	
3.1. The expectation	13
3.2. The unbiased estimate of variance	14
3.3. Trimmed means	14
3.4. The sample median	16
3.5. Ratio estimation	16
3.6. Functions of the expectation	17
3.7. The law school data	17
3.8. Linear regression	18
Chapter 4	21
BIAS OF THE JACKKNIFE VARIANCE ESTIMATE	
4.1. ANOVA decomposition of $\hat{\theta}$	22
4.2. Proof of the main result	22
4.3. Influence functions	24
4.4. Quadratic functionals	24
4.5. Sample size modification	26
Chapter 5	27
THE BOOTSTRAP	
5.1. Monte Carlo evaluation of \hat{SD}	29
5.2. Parametric bootstrap	30

iii

[◀ Previous page](#)

[+ Zoom in](#)

[Next page ▶](#)

[See all 7](#)

[Back to book details](#)

[Look inside the book](#)

[Co](#)

Look for similar books by subject:

Copyrighted material; sample page 3 of 7

CONTENTS	
5.3. Smoothed bootstrap	30
5.4. Bootstrap methods for more general problems	31
5.5. The bootstrap estimate of bias	33
5.6. Finite sample spaces	34
5.7. Regression models	35
 Chapter 6	 37
THE INFINITESIMAL JACKKNIFE, THE DELTA METHOD AND THE INFLUENCE FUNCTION	
6.1. Resampling procedures	37
6.2. Relation between the jackknife and bootstrap estimates of standard deviation	39
6.3. Jaekel's infinitesimal jackknife	39
6.4. Influence function estimates of standard deviation	42
6.5. The delta method	42
6.6. Estimates of bias	44
6.7. More general random variables	45
 Chapter 7	 49
CROSS VALIDATION, JACKKNIFE AND BOOTSTRAP	
7.1. Excess error	49
7.2. Bootstrap estimate of expected excess error	52
7.3. Jackknife approximation to the bootstrap estimate	53
7.4. Cross-validation estimate of excess error	54
7.5. Relationship between the cross-validation and jackknife estimates	57
7.6. A complicated example	58
 Chapter 8	 61
BALANCED REPEATED REPLICATIONS (HALF-SAMPLING)	
8.1. Bootstrap estimate of standard deviation	62
8.2. Half-sample estimate of standard deviation	62
8.3. Balanced repeated replications	64
8.4. Complementary balanced half-samples	65
8.5. Some possible alternative methods	66
 Chapter 9	 69
RANDOM SUBSAMPLING	
9.1. m -estimates	69
9.2. The typical value theorem	70
9.3. Random subsampling	71
9.4. Resampling asymptotics	72
9.5. Random subsampling for other problems	73

[◀ Previous page](#)

[+ Zoom in](#)

[Next page ▶](#)

[See all 7](#)

[Back to book details](#)

[Look inside the book](#)

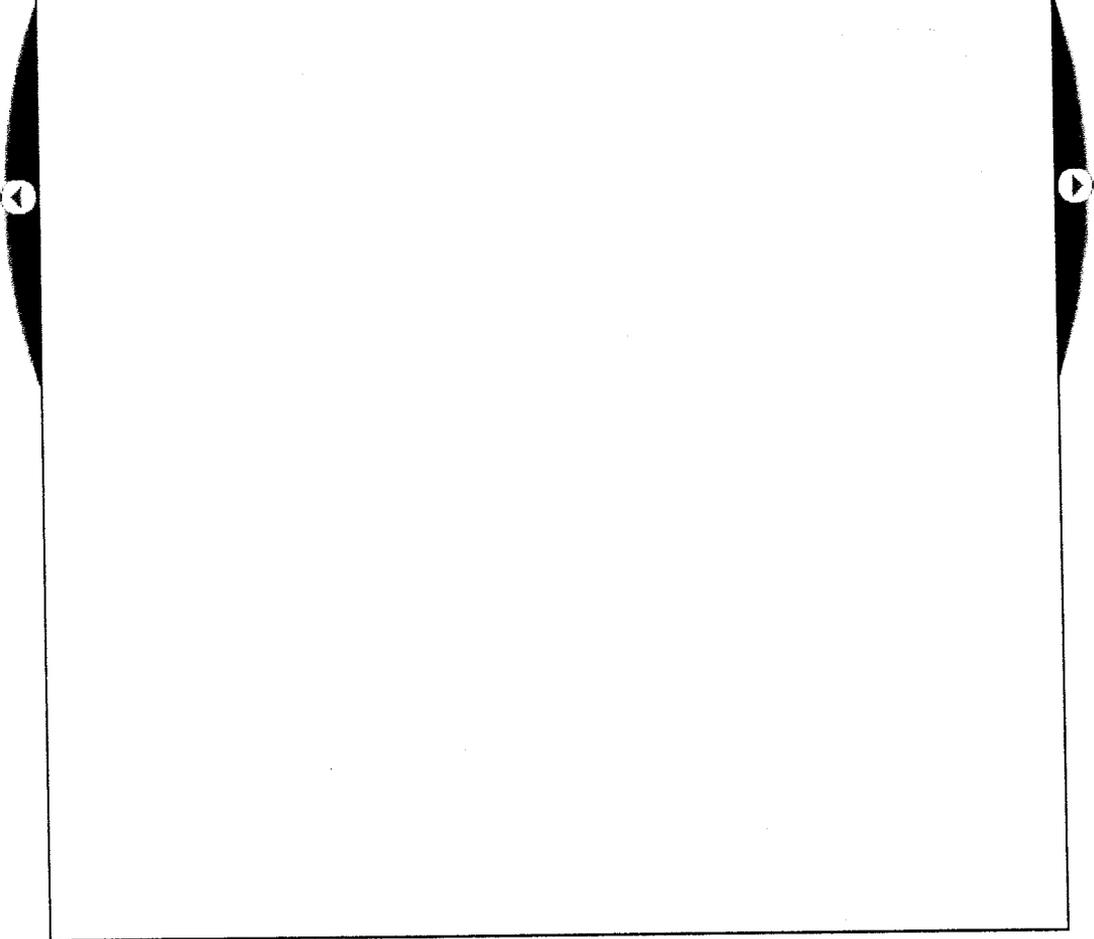
[Co](#)

Look for similar books by subject:

Copyrighted material; sample page 4 of 7

CONTENTS

Chapter 10	75
NONPARAMETRIC CONFIDENCE INTERVALS	
10.1 The median	75
10.2. Typical value theorem for the median	76
10.3. Bootstrap theory for the median	77
10.4. The percentile method	78
10.5. Percentile method for the median	80
10.6. Bayesian justification of the percentile method	81
10.7. The bias-corrected percentile method	82
10.8. Typical value theory and the percentile method	84
10.9. The percentile method for m -estimates	87
10.10. Bootstrap τ and tilting	87
References	91



[◀ Previous page](#)

[+ Zoom in](#)

[Next page ▶](#)

[See all 7](#)

[Back to book details](#)

[Look inside the book](#)

[Co](#)

Look for similar books by subject:

Copyrighted material; sample page 5 of 7

CHAPTER 1**Introduction**

Our goal is to understand a collection of ideas concerning the nonparametric estimation of bias, variance and more general measures of error. Historically the subject begins with the Ouenouille–Tukey jackknife, which is where we will begin also. In fact it would be more logical to begin with the bootstrap, which most clearly exposes the simple idea underlying all of these methods. (And in fact underlies many common parametric methods as well, such as Fisher's information theory for assigning a standard error to a maximum likelihood estimate.) Good simple ideas, of which the jackknife is a prime example, are our most precious intellectual commodity, so there is no need to apologize for the easy mathematical level. The statistical ideas run deep, sometimes over our head at the current level of understanding. Chapter 10, on nonparametric confidence intervals, is particularly speculative in nature.

Some material has been deliberately omitted for these notes. This includes most of the detailed work on the jackknife, especially the asymptotic theory. Miller (1974a) gives an excellent review of the subject.

What are the jackknife and the bootstrap? It is easiest to give a quick answer in terms of a problem where neither is necessary, that of estimating the standard deviation of a sample average. The data set consists of an independent and identically distributed (i.i.d.) sample of size n from an unknown probability distribution F on the real line.

$$(1.1) \quad X_1, X_2, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} F.$$

Having observed values $X_1 = x_1, X_2 = x_2, \dots, X_n = x_n$, we compute the sample average $\bar{x} = \sum_{i=1}^n x_i/n$, for use as an estimate of the expectation of F .

An interesting fact, and a crucial one for statistical applications, is that the data set provides more than the estimate \bar{x} . It also gives an estimate of the accuracy of \bar{x} , namely,

$$(1.2) \quad \hat{\sigma} = \left[\frac{1}{n(n-1)} \sum_{i=1}^n (x_i - \bar{x})^2 \right]^{1/2};$$

$\hat{\sigma}$ is the estimated standard deviation of \bar{X} , the root mean square error of estimation.

The trouble with formula (1.2) is that it doesn't, in any obvious way, extend to estimators other than \bar{X} , for example the sample median. The jackknife and

[◀ Previous page](#)[+ Zoom in](#)[Next page ▶](#)[See all 7](#)[Back to book details](#)[Look inside the book](#)[Co](#)

Look for similar books by subject:

Copyrighted material; sample page 6 of 7

2

CHAPTER 1

the bootstrap are two ways of making this extension. Let

$$(1.3) \quad \bar{x}_{(i)} = \frac{n\bar{x} - x_i}{n-1} = \frac{1}{n-1} \sum_{j \neq i} x_j,$$

the sample average of the data set deleting the i th point. Also, let $\bar{x}_{(i)} = \sum_{j=1}^n x_{(i)j} / n$, the average of these deleted averages. Actually $\bar{x}_{(i)} = \bar{x}$, but we need the dot notation below. The jackknife estimate of standard deviation is

$$(1.4) \quad \hat{\sigma}_{\text{JACK}} = \left(\frac{n-1}{n} \sum_{i=1}^n (\bar{x}_{(i)} - \bar{x}_{(i)})^2 \right)^{1/2}.$$

The reader can verify that this is the same as (1.3). The advantage of (1.4) is that it can be generalized to any estimator $\hat{\theta} = \hat{\theta}(X_1, X_2, \dots, X_n)$. The only change is to substitute $\hat{\theta}_{(i)} = \hat{\theta}(X_1, X_2, \dots, X_{i-1}, X_{i+1}, \dots, X_n)$ for $\bar{x}_{(i)}$ and $\hat{\theta}_{(i)}$ for $\bar{x}_{(i)}$.

The bootstrap generalizes (1.3) in an apparently different way. Let \hat{F} be the empirical probability distribution of the data, putting probability mass $1/n$ on each x_i , and let $X_1^*, X_2^*, \dots, X_n^*$ be an i.i.d. sample from \hat{F} .

$$(1.5) \quad X_1^*, X_2^*, \dots, X_n^* \stackrel{\text{i.i.d.}}{\sim} \hat{F}.$$

In other words, the X_i^* are a random sample drawn with replacement from the observed values x_1, x_2, \dots, x_n . Then $\bar{X}^* = \sum X_i^* / n$ has variance

$$(1.6) \quad \text{Var}_* \bar{X}^* = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2.$$

Var_* indicating variance under sampling scheme (1.5). The bootstrap estimate of standard deviation for an estimator $\hat{\theta}(X_1, X_2, \dots, X_n)$ is

$$(1.7) \quad \hat{\sigma}_{\text{BOOT}} = [\text{Var}_* \hat{\theta}(X_1^*, X_2^*, \dots, X_n^*)]^{1/2}.$$

Comparing (1.7) with (1.2), we see that $\hat{\sigma}_{\text{BOOT}} = [(n-1)/n]^{1/2} \hat{\sigma}$ for $\hat{\theta} = \bar{X}$. We could make $\hat{\sigma}_{\text{BOOT}}$ exactly equal to $\hat{\sigma}$, for $\hat{\theta} = \bar{X}$, simply by adjusting definition (1.7) with the factor $[n/(n-1)]^{1/2}$, but there turns out to be no advantage in doing so. A simple algorithm described in Chapter 4 allows the statistician to compute $\hat{\sigma}_{\text{BOOT}}$ no matter how complicated $\hat{\theta}$ may be.

Many other generalizations of (1.3) have been put forth. All such methods turn out to be closely related in theory, but not necessarily in their numerical consequences for a specific data set. We shall be investigating the theoretical and practical aspects of this collection of methods. Other measures of statistical error besides standard deviation are also considered: bias, prediction error and confidence intervals.

From a traditional point of view, all of the methods discussed here are prodigious computational spendthrifts. We blithely ask the reader to consider techniques which require the usual statistical calculations to be multiplied a thousand times over. None of this would have been feasible twenty-five years

[◀ Previous page](#)

[+ Zoom in](#)

[Next page ▶](#)

[See all 7](#)

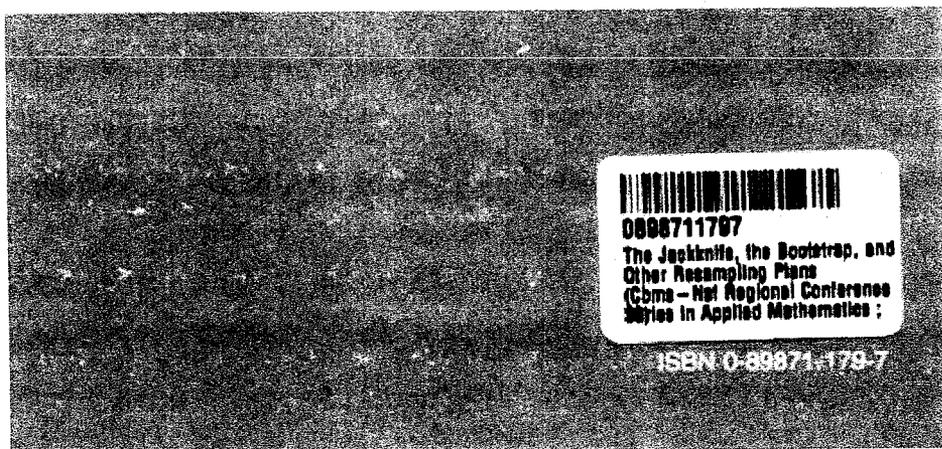
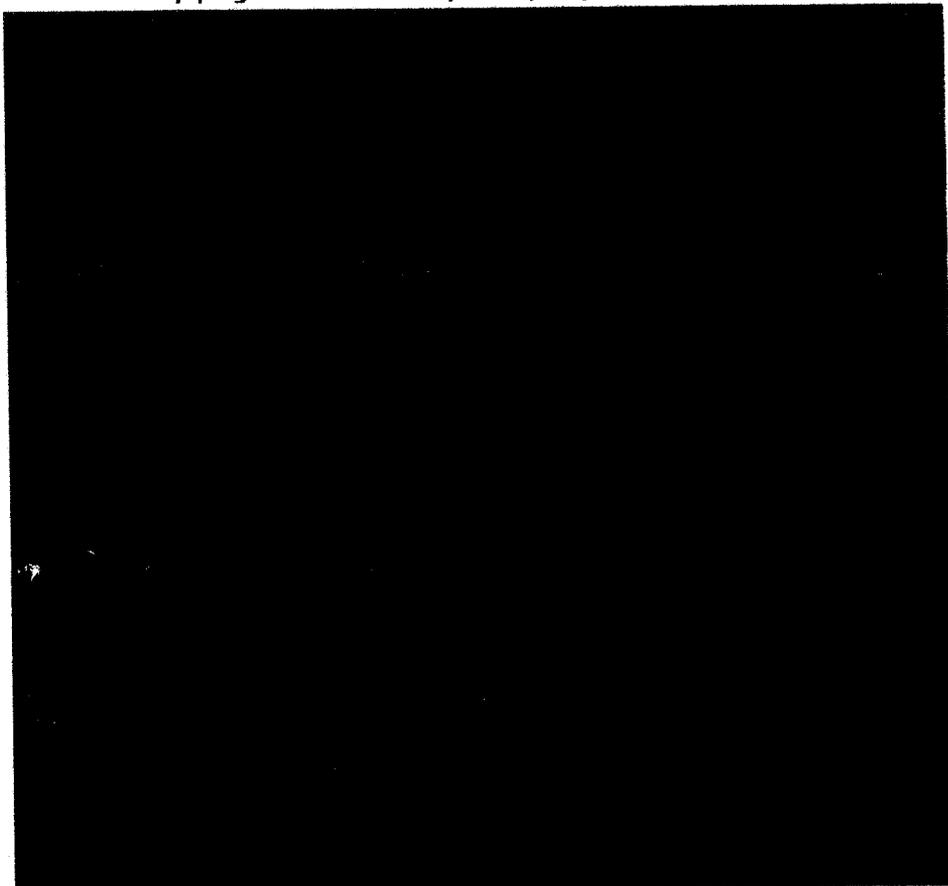
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[◀ Previous page](#)

[+ Zoom in](#)

[Next page ▶](#)

[See all 7 sa](#)

[Back to book details](#)

[Look inside the book](#)

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