This is Volume 45 in
INTERNATIONAL GEOPHYSICS SERIES
A series of monographs and textbooks
Edited by RENATA DMOWSKA and JAMES R. HOLTON



# GEOPHYSICAL DATA ANALYSIS: DISCRETE INVERSE THEORY

## Revised Edition

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ACADEMIC PRESS, INC.
*A Division of Harcourt Brace & Company*
San Diego New York Boston
London Sydney Tokyo Toronto

ACADEMIC PRESS
525 B Street, Suite 1900
San Diego, California 92101-4495

United Kingdom Edition published by
Academic Press Limited
24–28 Oval Road, London NW1 7DX

Library of Congress Cataloging-in-Publication Data

Menke, William
Geophysical data analysis : discrete inverse theory / William Menke. -- Rev. ed.
p. cm.
Bibliography: p.
Includes index.
ISBN 0-12-490921-3 (alk. paper)
1. Geophysics—Measurement. 2. Oceanography--Measurement.
3. Inverse problems (Differential equations)—Numerical solutions.
I. Title. II. Series.
QC802.A1M46 1989
551--dc20 89-31224
CIP

PRINTED IN THE UNITED STATES OF AMERICA
94 95 96 97 BB 9 8 7 6 5 4 3 2

# CONTENTS

## 11.4 Tomography and Continuous Inverse Theory

The term "tomography" has come to be used in geophysics almost synonymously with the term "inverse theory." Tomography is derived from the Greek word *tomos*, that is, slice, and denotes forming an image of an object from measurements made from slices (or rays) through it. We consider tomography a subset of inverse theory, distinguished by a special form of the data kernel that involves measurements made along rays. The model function in tomography is a function of two or more variables and is related to the data by

$$d_i = \int_{C_i} m[x(s), y(s)]\, ds \tag{11.16}$$

Here the model function is integrated along a ray $C_i$ having arc length $s$. This integral is equivalent to the one in a standard continuous problem [Eq. (11.2)] when the data kernel is $G_i(x,y) = \delta(x(s) - x_i[y(s)])\, ds/dy$, where $\delta(x)$ is the Dirac delta function:

$$d_i = \int\int m(x,y)\delta(x(s) - x_i[y(s)])\frac{ds}{dy}\,dx\,dy$$
$$= \int_{C_i} m[x(s),y(s)]\, ds \tag{11.17}$$

Here $x$ is supposed to vary with $y$ along the curve $C_i$ and $y$ is supposed to vary with arc length $s$.

While the tomography problem is a special case of a continuous inverse problem, several factors limit the applicability of the formulas of the previous sections. First, the Dirac delta functions in the data kernel are not square integrable, so that the $S_{ij}$ ["overlap" integrals; see Eq. (11.6)] have nonintegrable singularities at points where rays intersect. Furthermore, in three-dimensional cases the rays may not intersect at all, so that all the $S_{ij}$ may be identically zero. Neither of these problems is insurmountable, and they can be overcome by replacing the rays with tubes of finite cross-sectional width. (Rays are often an idealization of a finite-width process anyway, as in acoustic wave propagation, where they are an infinitesimal wavelength approximation.) Since this approximation is equivalent to some statement about the smoothness of the model function $m(x,y)$, it often suffices to discretize the continuous problem by dividing it into constant $m$ subregions, where the subregions are large enough to guarantee a reasonable number containing more than one ray. The discrete inverse problem is then of the form $d_i = \Sigma_j G_{ij} m_j$, where the data kernel $G_{ij}$ gives the arc length of the $i$th ray in the $j$th subregion. The concepts of resolution and variance, now interpreted in the discrete fashion of Chapter 4, are still applicable and of considerable importance.





## 11.5 Tomography and the Radon Transform

The simplest tomography problem involves straight-line rays and a two-dimensional model function $m(x,y)$ and is called Radon's problem. By historical convention, the straight-line rays $C_i$ in Eq. (11.6) are parametrized by their perpendicular distance $u$ from the origin and the angle $\theta$ (Fig. 11.2) that the perpendicular makes with the $x$ axis. Position $(x,y)$ and ray coordinates $(u,s)$, where $s$ is arc length, are related by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} u \\ s \end{pmatrix}$$
$$\begin{pmatrix} u \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \tag{11.18}$$

The tomography problem is then

$$d(u,\theta) = \int_{-\infty}^{+\infty} m(x = u\cos\theta - s\sin\theta,\ y = u\sin\theta + s\cos\theta)\, ds \tag{11.19}$$

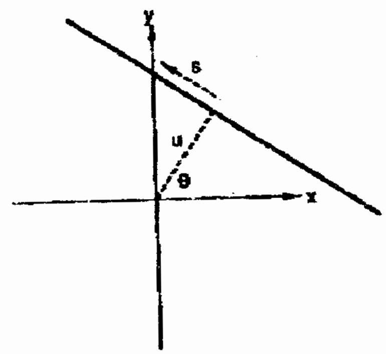


Fig. 11.2. The Radon transform is performed by integrating a function of $(x,y)$ along straight lines (bold) parameterized by their arc length $s$, perpendicular distance $u$, and angle $\theta$.