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A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils[1]

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ABSTRACT

A new and relatively simple equation for the soil-water content-pressure head curve, $\theta(h)$, is described in this paper. The particular form of the equation enables one to derive closed-form analytical expressions for the relative hydraulic conductivity, K_r, when substituted in the predictive conductivity models of N.T. Burdine or Y. Mualem. The resulting expressions for $K_r(h)$ contain three independent parameters which may be obtained by fitting the proposed soil-water retention model to experimental data. Results obtained with the closed-form analytical expressions based on the Mualem theory are compared with observed hydraulic conductivity data for five soils with a wide range of hydraulic properties. The unsaturated hydraulic conductivity is predicted well in four out of five cases. It is found that a reasonable description of the soil-water retention curve at low water contents is important for an accurate prediction of the unsaturated hydraulic conductivity.

Additional Index Words: soil-water diffusivity, soil-water retention curve.

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The use of numerical models for simulating fluid flow and mass transport in the unsaturated zone has become increasingly popular the last few years. Recent literature indeed demonstrates that much effort is put into the development of such models (Reeves and Duguid, 1975; Segol, 1976; Vauclin et al., 1979). Unfortunately, it appears that the ability to fully characterize the simulated system has not kept pace with the numerical and modeling expertise. Probably the single most important factor limiting the successful application of unsaturated flow theory to actual field problems is the lack of information regarding the parameters entering the governing transfer equations. Reliable estimates of the unsaturated hydraulic conductivity are especially difficult to obtain, partly because of its extensive variability in the field, and partly because measuring this parameter is time-consuming and expensive. Several investigators have, for these reasons, used models for calculating the unsaturated conductivity from the more easily measured soil-water retention curve. Very popular among these models has been the Millington-Quirk method (Millington and Quirk, 1961), various forms of which have been applied with some success in a number of studies (cf. Jackson et al., 1965; Jackson, 1972; Green and Corey, 1971; Bruce, 1972). Unfortunately, this method has the disadvantage of producing tabular results which, for example when applied to nonhomogeneous soils in multidimensional unsaturated flow models, are quite tedious to use.

Closed-form analytical expressions for predicting the unsaturated hydraulic conductivity have also been developed. For example, Brooks and Corey (1964) and Jeppson (1974) each used an analytical expression for the conductivity based on the Burdine theory (Burdine, 1953). Brooks and Corey (1964, 1966) obtained fairly accurate predictions with their equations, even though a discontinuity is present in the slope of both the soil-water retention curve and the unsaturated hydraulic conductivity curve at some negative value of the pressure head (this point is often referred to as the bubbling pressure). Such a discontinuity sometimes prevents rapid convergence in numerical saturated-unsaturated flow problems. It also appears that predictions based on the Brooks and Corey equations are somewhat less accurate than those obtained with various forms of the (modified) Millington-Quirk method.

Recently Mualem (1976a) derived a new model for predicting the hydraulic conductivity from knowledge of the soil-water retention curve and the conductivity at saturation. Mualem's derivation leads to a simple integral formula for the unsaturated hydraulic conductivity which enables one to derive closed-form analytical expressions, provided suitable equations for the soil-water retention curves are available. It is the purpose of this paper to derive such expressions using an equation for the soil-water retention curve which is both continuous and has a continuous slope. The resulting conductivity models generally contain three independent parameters which may be obtained by matching the proposed soil-water retention curve to experimental data. Results obtained with the closed-form equations based on the Mualem theory will be compared with observed data for a few soils having widely varying hydraulic properties.

THEORETICAL

Equations Based on Mualem's Model

The following equation was derived by Mualem (1976a) for predicting the relative hydraulic conductivity (K_r) from knowledge of the soil-water retention curve

$$K_r = \Theta^{1/2}\left[\int_0^{\Theta}\frac{1}{h(x)}\,dx \Big/ \int_0^{1}\frac{1}{h(x)}\,dx\right]^2 \qquad [1]$$

where h is the pressure head, given here as a function of the dimensionless water content:

$$\Theta = \frac{\theta-\theta_r}{\theta_s-\theta_r}. \qquad [2]$$

In this equation, s and r indicate saturated and residual values of the soil-water content (θ), respectively. To solve Eq. [1], an expression relating the dimensionless water content to the pressure head is needed. An attractive class of $\Theta(h)$-functions, adopted in this study, is given by the following general equation

$$\Theta = \left[\frac{1}{1+(\alpha h)^n}\right]^m \qquad [3]$$

where α, n, and m are as yet undetermined parameters. To simplify notation later, h in Eq. [3] is assumed to be positive. Equation [3] with $m=1$ has been successfully used in many studies to describe soil-water retention data (Ahuja and Swart-

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zendruber, 1972; Endelman et al., 1974; Haverkamp et al., 1977). A typical θh curve based on Eq. [2] and [3] is shown in Figure 1. Note that a nearly symmetrical "S"-shaped curve is obtained, and that the slope $(d\theta/dh)$ becomes zero when θ approaches both its saturated and residual values.

Simple, closed-form expressions for $K_r(\Theta)$ can be derived when certain restrictions are imposed upon the values of m and n in Eq. [3]. Solving this equation for $h=h(\Theta)$ and substituting the resulting expression into Eq. [1] gives

$$K_r(\Theta) = \Theta^{1/2}\left[\frac{f(\Theta)}{f(1)}\right]^2 \qquad [4]$$

where

$$f(\Theta) = \int_0^{\Theta}\left[\frac{x^{1/m}}{1-x^{1/m}}\right]^{1/n} dx. \qquad [5]$$

Substitution of $x=y^m$ into Eq. [5] leads to

$$f(\Theta) = m\int_0^{\Theta^{1/m}} y^{m-1+1/n}\,(1-y)^{-1/n}\,dy. \qquad [6]$$

Equation [6] represents a particular form of the Incomplete Beta-function and, in its most general case, no closed-form expression can be derived. However, it is easily shown that for all integer values of $k=m-1+1/n$ the integration can be carried out without difficulties. For the particular case when $k=0$ (i.e., $m=1-1/n$) integration of Eq. [6] yields

$$f(\Theta) = 1-(1-\Theta^{1/m})^m, \qquad (m=1-1/n) \qquad [7]$$

and because $f(1) = 1$, Eq. [4] becomes

$$K_r(\Theta) = \Theta^{1/2}\left[1-(1-\Theta^{1/m})^m\right]^2. \qquad \begin{matrix}(m=1-1/n)\\(0 < m < 1)\end{matrix} \qquad [8]$$

The relative hydraulic conductivity can also be expressed in terms of the pressure head by substituting Eq. [3] into Eq. [8], i.e.,

$$K_r(h) = \frac{\{1-(\alpha h)^{n-1}\,[1+(\alpha h)^n]^{-m}\}^2}{[1+(\alpha h)^n]^{m/2}}. \qquad (m=1-1/n) \qquad [9]$$

From the hydraulic conductivity and the soil-water retention curve one may also derive an expression for the soil-water diffusivity, which is defined as

$$D(\theta) = K(\theta)\left|\frac{dh}{d\theta}\right|. \qquad [10]$$

This leads to the following equation for $D(\Theta)$:

$$D(\Theta) = \frac{(1-m)K_s}{\alpha m(\theta_s-\theta_r)}\,\Theta^{1/2-1/m}\,\left[(1-\Theta^{1/m})^{-m} + (1-\Theta^{1/m})^{m}-2\right] \qquad [11]$$

where K_s $(= K/K_r)$ is the hydraulic conductivity at saturation.

The soil-hydraulic properties derived above were obtained by assuming that $k=m-1+1/n=0$ in Eq. [6]. One can also derive closed-form expressions for other integer values of k. For $k=1$, for example, the conductivity becomes

$$K_r(\Theta) = \Theta^{1/2}\left[1-m(1-\Theta^{1/m})^{m-1} + (m-1)(1-\Theta^{1/m})^m\right]^2.$$

$$(m=2-1/n) \qquad [12]$$

While this particular model is not only more complicated than Eq. [8], it also represents only a slight pertubation of the earlier function. Hence, Eq. [12] does not present an attractive alternative for Eq. [8], and will not be discussed further.

Equations Based on Burdine's Model

Similar results as above for the Mualem theory can also be obtained when the Burdine theory is taken as a point of departure. The equation given by Burdine (1953) is:

$$K_r(\Theta) = \Theta^2\int_0^{\Theta}\frac{1}{h^2(x)}\,dx \Big/ \int_0^1\frac{1}{h^2(x)}\,dx. \qquad [13]$$

The analysis proceeds in a similar way as before. Equation [3] is inverted to give $h=h(\Theta)$ and substitution of this expression into Eq. [13] yields:

$$K_r(\Theta) = \Theta^2 f(\Theta)/f(1) \qquad [14]$$

where

$$f(\Theta) = \int_0^{\Theta}\left[\frac{x^{1/m}}{1-x^{1/m}}\right]^{2/n} dx. \qquad [15]$$

Substituting $x=y^m$ into Eq. [15] gives

$$f(\Theta) = m\int_0^{\Theta^{1/m}} y^{m-1+2/n}(1-y)^{-2/n}dy. \qquad [16]$$

Again it is assumed that the exponent of y in Eq. [16] vanishes. Hence $m=1-2/n$, and Eq. [16] reduces to

$$f(\Theta) = 1-(1-\Theta^{1/m})^m. \qquad [17]$$

The relative hydraulic conductivity therefore becomes

$$K_r(\Theta) = \Theta^2\left[1-(1-\Theta^{1/m})^m\right] \qquad \begin{matrix}(m=1-2/n)\\(0<m<1;\ n>2)\end{matrix} \qquad [18]$$

or, in terms of the pressure head,

$$K_r(h) = \frac{1-(\alpha h)^{n-2}\,[1+(\alpha h)^n]^{-m}}{[1+(\alpha h)^n]^{2m}}. \qquad [19]$$

The soil-water diffusivity for this case is

$$D(\Theta) = \frac{(1-m)K_s}{2\alpha m(\theta_s-\theta_r)}\,\Theta^{(3-1/m)/2}\left[(1-\Theta^{1/m})^{-(m+1)/2}-(1-\Theta^{1/m})^{(m-1)/2}\right]. \qquad [20]$$

GRAPHICAL INTERPRETATION AND PARAMETER ESTIMATION

Equations [9] and [11], based on the Mualem theory, are shown graphically in Fig. 2 and 3, respectively, using the same values of α, n and $m(=1-1/n)$ as in Fig. 1. As can be seen from Fig. 2, the relative hydraulic conductivity starts out with a zero slope at pressure head values near zero, but then falls off increasingly rapid as h decreases. The soil-water diffusivity, on the other hand, attains (as does the soil-water retention curve) a fairly symmetrical "S"-shaped curve. Note that $D(\theta)$ becomes infinite when θ equals θ_s. Only at intermediate values of θ (approximately between 0.25 and 0.45 cm^3/cm^3 in Fig. 3) does the diffusivity acquire the often assumed exponential dependency on the water content. Similar features of the soil-water diffusivity were also obtained by Ahuja and Swartzendruber (1972) and by Murali et al. (1979).

Equations [19] and [20], based on the Burdine model, generate conductivity and diffusivity curves which closely resemble those shown in Fig. 3 and 4. Preliminary tests indicated that the Burdine-based equations were, in most cases, in lesser agreement with experimental data than the Mualem-based expressions. Through an extensive series of comparisons, Mualem (1976a) also concluded that predictions based on his theory, i.e., based directly on Eq. [1] by means of numerical approximations, were generally more accurate than those based on various forms of the Burdine theory (including the Millington-Quirk method). Because of this, the Burdine-based equations will not be considered further, and attention is focused only on the Mualem-based expressions.

The soil-water content as a function of the pressure head is given by Eq. [2] and [3], i.e., by

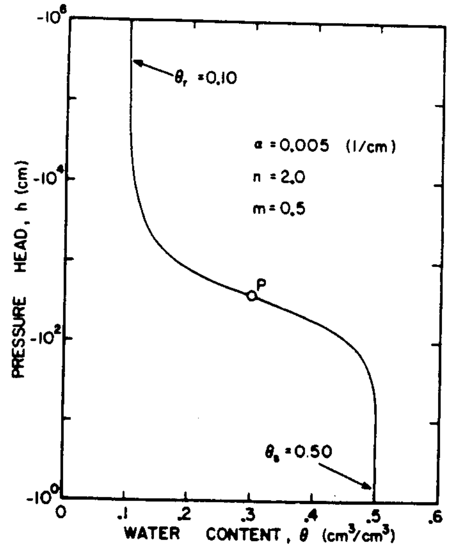

Fig. 1—Typical plot of the soil-water retention curve based on Eq. [3]. The point *P* on the curve is located halfway between θ_r (=0.10) and θ_s(=0.50).

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha h)^n]^m} \qquad [21]$$

where, as before, it is understood that h is positive, and where for the Mualem model

$$m = 1 - 1/n. \qquad [22]$$

Equation [21] contains four independent parameters (θ_r, θ_s, α, and n), which have to be estimated from observed soil-water retention data. Of these four, the saturated water content (θ_s) is probably always available as it is easily obtained experimentally. Also the residual water content (θ_r) may be measured experimentally, for example, by determining the water content on very dry soil. Unfortunately, θ_r measurements are not always made routinely, in which case they have to be estimated by extrapolating available soil-water retention data towards lower water contents. The residual water content is defined here as the wa-

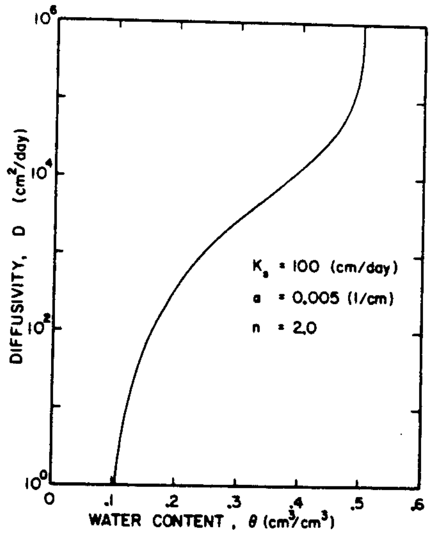

Fig. 3—Plot of the soil-water diffusivity vs. water content as predicted from knowledge of the soil-water retention curve shown in Fig. 1, and the hydraulic conductivity at saturation.

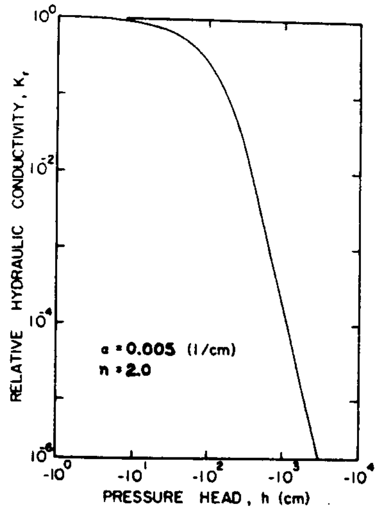

Fig. 2—Plot of the relative hydraulic conductivity vs. pressure head as predicted from knowledge of the soil-water retention curve shown in Fig. 1.

ter content for which the gradient ($d\theta/dh$) becomes zero (excluding the region near θ_s which also has a zero gradient). From a practical point of view it seems sufficient to define θ_r as the water content at some large negative value of the pressure head, e.g., at the permanent wilting point ($h = -15{,}000$ cm). Even in that case, however, significant decreases in h are likely to result in further desorption of water, especially in fine-textured soils. It seems that such further changes in θ are fairly unimportant for most practical field problems. In fact, they would be inconsistent with the general shape of the $\theta(h)$-curve defined by Eq. [21] and probably invalidate the concept of a residual soil-water content itself.

Assuming for the moment that θ_r is a well-defined parameter and that sufficiently accurate estimates of both θ_r and θ_s are available, then the following procedure can be used to obtain estimates of the remaining parameters α and n.

Differentiation of Eq. [21] gives

$$\frac{d\theta}{dh} = \frac{-\alpha\, m(\theta_s - \theta_r)}{1-m} \Theta^{1/m} (1 - \Theta^{1/m})^m \qquad [23]$$

where the right-hand side is expressed in terms of Θ, rather than h. Solving Eq. [3] for α gives furthermore

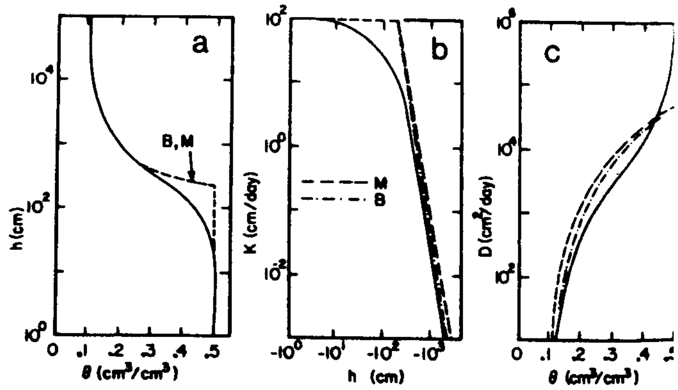

Fig. 4—Comparison of the proposed soil hydraulic functions (solid lines) with curves obtained by applying either the Mualem theory (*M*; dashed lines) or the Burdine theory (*B*; dashed-dotted lines) to the Brooks and Corey model of the soil-water retention curve.

$$\alpha = \frac{1}{h}\,(\Theta^{-1/m}-1)^{1/n}. \qquad [24]$$

Substituting Eq. [24] into Eq. [23] results in

$$h\,\frac{d\theta}{dh} = \frac{-m(\theta_s-\theta_r)}{1-m}\,\Theta(1-\Theta^{1/m}). \qquad [25]$$

The right-hand side of this equation contains only the unknown parameter m (both θ_s and θ_r are assumed to be known). Hence it is possible to obtain an estimate of m by determining the product of the slope $(d\theta/dh)$ and the pressure head (h) at some point on the $\theta(h)$-curve. Soil-water retention data are often plotted on a semilogarithmic scale. One may take advantage of this fact by noting that

$$\frac{d\theta}{d(\log h)} = (\ln 10)\,h\,\frac{d\theta}{dh}. \qquad [26]$$

Let S be the absolute value of the slope of Θ with respect to $\log h$, i.e.,

$$S = \left|\frac{d\Theta}{d(\log h)}\right| \qquad [27a]$$

or, equivalently,

$$S = \frac{1}{(\theta_s-\theta_r)}\left|\frac{d\theta}{d(\log h)}\right|. \qquad [27b]$$

Combining Eq. [25], [26], and [27b] leads to the following expression for S

$$S = 2.303\,\frac{m}{1-m}\,\Theta(1-\Theta^{1/m}). \qquad [28]$$

The best location on the $\theta(h)$-curve for evaluating the slope S is about halfway between θ_r and θ_s. Let P be the point on the soil-water retention curve for which $\Theta=\frac{1}{2}$ (see Fig. 1). From Eq. [2] and [24] it follows that the coordinates of P are given by

$$\theta_P = (\theta_s+\theta_r)/2 \qquad [29a]$$

$$h_P = \frac{1}{\alpha}\,(2^{1/m}-1)^{1-m}, \qquad [29b]$$

and Eq. [28] reduces to

$$S_P(m) = 1.151\,\frac{m}{1-m}\,(1-2^{-1/m}). \qquad [30]$$

The subscript P in these equations indicated evaluation at P. Equation [30] may be used to obtain an estimate for m once the slope S_P is determined graphically from the experimental soil-water retention curve. For this it is more convenient to express m as a function of S_P. The following empirical inversion formula can be used for that purpose:

$$m = \begin{cases} 1-\exp(-0.8\,S_P) & (0<S_P\leq 1) \\ 1-\dfrac{0.5755}{S_P}+\dfrac{0.1}{S_P^2}+\dfrac{0.025}{S_P^3} & (S_P>1) \end{cases} \qquad [31]$$

As an illustrative example, let us apply the foregoing procedure to the hypothetical "experimental" curve in Fig. 1. The point P on this curve is located halfway between θ_r and θ_s (the residual water content is assumed to be known). One may verify graphically that the slope of the curve at P is about 0.34. From Eq. [27b] it follows then that the dimensionless slope S_P is about 0.85. Hence from Eq. [31] we have $m \sim 0.5$, and from Eq. [22] $n \sim 2.0$. To obtain an estimate for α in Eq. [9] and [11], it is further necessary to have an estimate for h_P. From Fig. 1 it follows that $\log(h_P) \sim 2.55$, and hence $h_P \sim 355$. Finally, from Eq. [29b] one obtains $\alpha \sim 0.005$.

In some cases, no clearly defined or measured value for the residual soil-water content will be available. In that case θ_r must be estimated by extrapolating measured soil-water retention data to the lower water contents. One possible way for doing this is to apply the graphical method discussed above using different values for θ_r, and subsequently select that value of θ_r which gives the best fit of Eq. [21] to the experimental data. It must be clear that this procedure can become quite elaborate when only a small portion of the soil-water retention curve is measured. An alternative approach would be to use a least-squares curve-fitting technique, thereby allowing one to make simultaneous estimates of θ_r, α, and n. An additional advantage of this approach, actually used for this study, is that now the entire measured curve can be used in the parameter-estimation procedure. A detailed description and listing of the nonlinear least-squares curve-fitting program used for this purpose is given by van Genuchten (1978).

COMPARISON WITH THE BROOKS AND COREY MODEL

It is not the intent of this paper to give accuracy comparisons between various closed-form analytical conductivity expressions. Only a brief discussion of the equations derived by Brooks and Corey (1964) will be given here, since their model of the soil-water retention curve represents a limiting case of the retention model discussed in this study.

Brooks and Corey (1964, 1966) concluded from comparisons with a large number of experimental data that the soil-water retention curve could be described reasonably well with the following general equation

$$\Theta = (h/h_b)^{-\lambda} \qquad (\Theta\leq 1) \qquad [32]$$

where h_b is the bubbling pressure and λ a soil characteristic parameter. Comparing Eq. [32] and [3], one sees that Eq. [3] reduces to Eq. [32] for large values of the pressure head, i.e.,

$$\Theta = (\alpha h)^{-mn}. \qquad [33]$$

For the Mualem theory one has $m=1-1/n$, and hence $\lambda=n-1$, while for the Burdine theory $(m=1-2/n)$ one finds that $\lambda=n-2$. The parameter α, furthermore, is inversely related to the bubbling pressure, h_b. Brooks and Corey used the Burdine theory to predict the relative hydraulic conductivity and the soil-water diffusivity. They derived the following expressions

$$K_r(\Theta) = \Theta^{3+2/\lambda} \qquad [34a]$$

$$K_r(h) = (\alpha h)^{-2-3\lambda} \qquad [34b]$$

$$D(\Theta) = \frac{K_S}{\alpha\lambda(\theta_s-\theta_r)}\,\Theta^{2+1/\lambda}. \qquad [35]$$

Through substitution of Eq. [32] into [1], similar equations can be derived for the Mualem theory:

$$K_r(\Theta) = \Theta^{5/2+2/\lambda} \quad [36a]$$

$$K_r(h) = (\alpha h)^{-2-5\lambda/2} \quad [36b]$$

$$D(\Theta) = \frac{K_s}{\alpha\lambda(\theta_s-\theta_r)}\Theta^{3/2+1/\lambda}. \quad [37]$$

Figure 4 compares the different expressions given above with the earlier obtained relations for the conductivity and diffusivity (Eq. [3], [9], and [11]). The parameters α and n are again the same as before (α=0.005) and n=2), while λ is equal to (n−1). The soil-water retention curves for all three cases become then identical for sufficiently low values of θ. Figure 4a shows that the Brooks and Corey model of the $\theta(h)$-curve approaches the curve based on Eq. [3] asymptotically when θ decreases. However, large deviations between the two models occur when θ approaches saturation. The curve based on Eq. [32] reaches θ_s at a much lower value of h (−200 cm) than the curve based on Eq. [21]. The most important deviations between the conductivity curves are also present at or near the bubbling pressure (Fig. 4b). Differences between the three curves at the lower K-values are relatively small and of no importance for most practical field situations. The diffusivity curves (Fig. 4c) show their most important differences at both the intermediate and higher values of the water content. Note that the diffusivity curves based on Eq. [32] remain finite (D_s=50,000 cm²/day) when θ approaches θ_s, while the solid line (Eq. [10]) becomes infinite at saturation. It is to be emphasized here that Fig. 4 was included only to demonstrate typical properties of the various conductivity and diffusivity models, and that the figure should not be viewed as an accuracy evaluation of any one model.

COMPARISON WITH EXPERIMENTAL DATA

In this section, comparisons are given between observed and calculated conductivity curves for five soils. The observed data for each case, with the exception of the last one, were taken from the soils catalogue of Mualem (1976b). Table 1 summarizes some of the soil-physical properties of the five soils. Estimates of the parameters θ_r, α, and n are also included in this table.

Results for Hygiene Sandstone (Brooks and Corey, 1964) are shown in Fig. 5. This soil has a rather narrow pore-size distribution, causing the soil-water retention curve to become very steep at around h=−125 cm. Table 1 shows that a relatively high value of 10.4 for n was obtained for this soil, a direct consequence of the steep curve (n is an increasing function of the slope S_P). The value of α was found to be 0.079 (1/cm), approximately the inverse of the pressure head at which the retention curve becomes the steepest (Fig. 5). This, of course, follows directly from Eq. [29a] which, for values of m close to one (i.e., for n large) reduces to $h_P = 1/\alpha$. In that case h_P becomes identical to the bubbling pressure, h_b, used in the Brooks and Corey equation (Eq. [32]). Figure 4 shows a nearly exact prediction of the relative hydraulic conductivity, with only some minor deviations occurring at the higher conductivity values.

Results obtained for Touchet Silt Loam G.E.3 (Brooks and Corey, 1964), shown in Fig. 6, are very similar to those of Hygiene Sandstone. The curves in this case are also very steep (n=7.09, Table 1), and again a good prediction of the relative hydraulic conductivity is obtained.

Figure 7 presents results obtained for Silt Loam G.E.3 (Reisenauer, 1963). Note that only a limited portion of the soil-water retention curve was measured. The calculated value of 0.131 for θ_r (Table 1) hence may not be very accurate. Yet it is the best-fit value as "seen" by Eq. [21] when matched against the experimental data, and apparently still results in an accurate prediction of the unsaturated hydraulic conductivity. The predicted curve in Fig. 7 was found to change only slightly when θ_r was forced to vary between 0.05 and 0.15. Note that the curves in Fig. 7 are less steep than for the previous two examples, resulting in a much smaller value of n (Table 1).

The first three examples each showed excellent agreement between observed and predicted conductivity curves. Predictions obtained for Beit Netofa clay (Rawitz, 1965)[3] were found to be less accurate (Fig. 8). The higher conductivity values are seriously

[3] E. Rawitz. 1965. The influence of a number of environmental factors on the availability of soil moisture to plants (in Hebrew). Ph.D. thesis. Hebrew Univ., Rehovot, Israel.

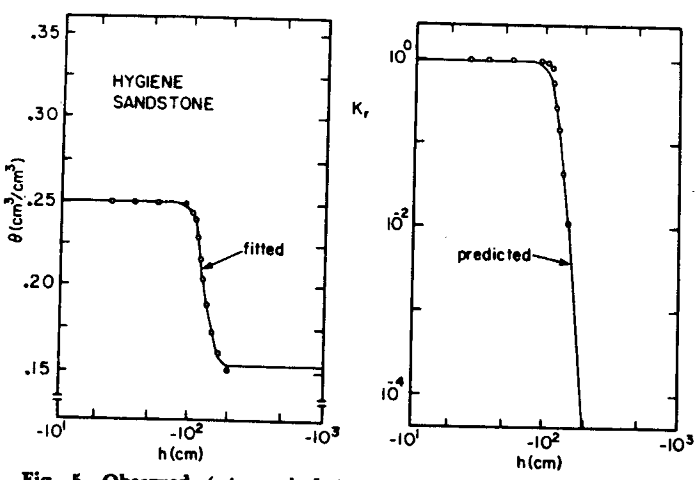

Fig. 5—Observed (*open circles*) and calculated curves (*solid lines*) of the soil hydraulic properties of Hygiene Sandstone.

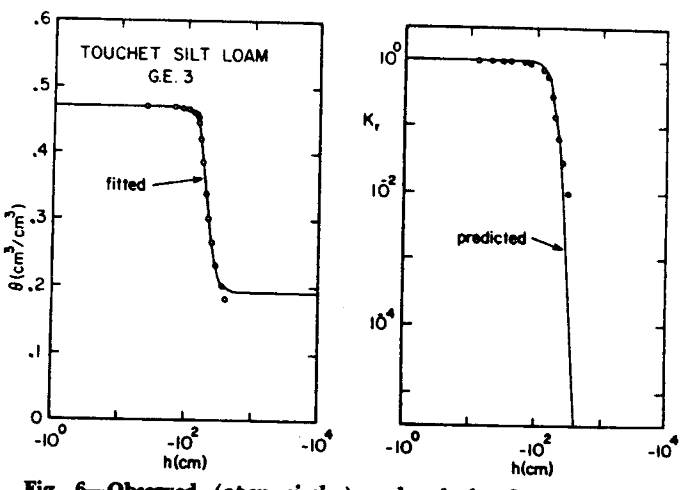

Fig. 6—Observed (*open circles*) and calculated curves (*solid lines*) of the soil hydraulic properties of Touchet Silt Loam G. E. 3.

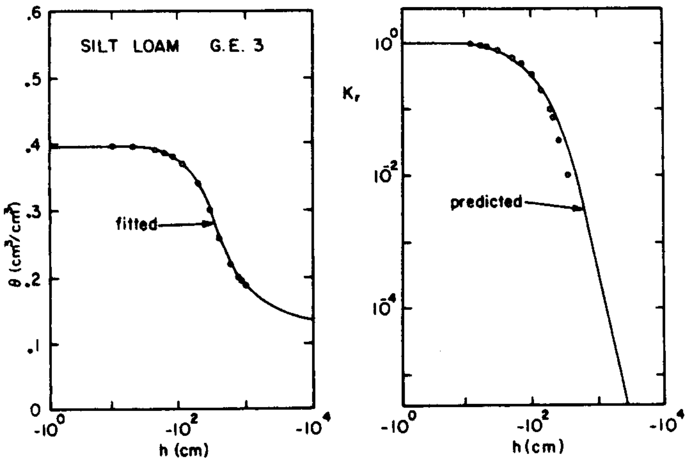

Fig. 7—Observed (*open circles*) and calculated curves (*solid lines*) of the soil hydraulic properties of Silt Loam G.E.3.

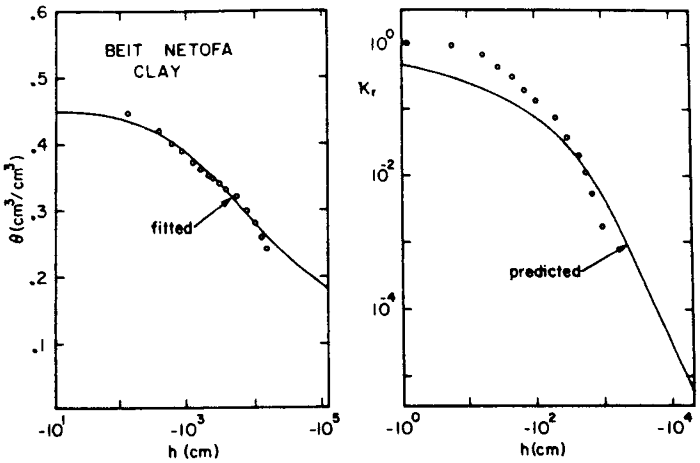

Fig. 8—Observed (*open circles*) and calculated curves (*solid lines*) of the soil hydraulic properties of Beit Netofa clay.

underpredicted, and also the general shape of the predicted curve is different from the observed one. It seems that much of the poor predictions can be traced back to the inability of Eq. [21] to match the experimental soil-water retention data. For example, the residual water content was estimated to be zero (Table 1), a rather surprising result since clay soils have generally higher θ_r-values than coarser soils (the saturated hydraulic conductivity of this soil is only 0.082 cm/day). Limited data at the lower water contents also leaves some doubt about the accuracy of the fitted θ_r-value. This case clearly demonstrates the importance of having some independent procedure for estimating the residual water content.

Results for Guelph Loam (Elrick and Bowman, 1964) are given in Fig. 9. This example represents a case in which hysteresis is present in the soil-water retention curve. The observed data of this example were taken directly from the original study (Fig. 2 and 3 of Elrick and Bowman, 1964). For the wetting branch a maximum ("saturated") value of 0.434 was used for θ, being the highest measured value. Also the wetting branch of the hydraulic conductivity curve was matched to the highest value of K_r measured during wetting (Fig. 9). The value of θ_r, furthermore, was assumed to be the same for drying and wetting, and was obtained from the drying branch of the curve. Both the drying and wetting branches of the retention curve are adequately described by Eq. [21]. Note that some hysteresis is predicted in the relative hydraulic conductivity. Although this is generally to be expected when two different retention curves are present, Eq. [8] also shows that different retention curves may generate the same conductivity curve as long as θ_r, θ_s, and n remain the same (α may be different).

Table 1—Soil-physical properties of the five example soils.

Soil name	θ_s	θ_r	K_s	α	n
	— cm³/cm³ —		cm/day	cm⁻¹	
Hygiene sandstone	0.250	0.153	108.0	0.0079	10.4
Touchet Silt Loam G.E. 3	0.469	0.190	303.0	0.0050	7.09
Silt Loam G.E. 3	0.396	0.131	4.96	0.00423	2.06
Guelph Loam (drying)	0.520	0.218	31.6	0.0115	2.03
(wetting)	0.434	0.218	–	0.0200	2.76
Beit Netofa Clay	0.446	0.0	0.082	0.00152	1.17

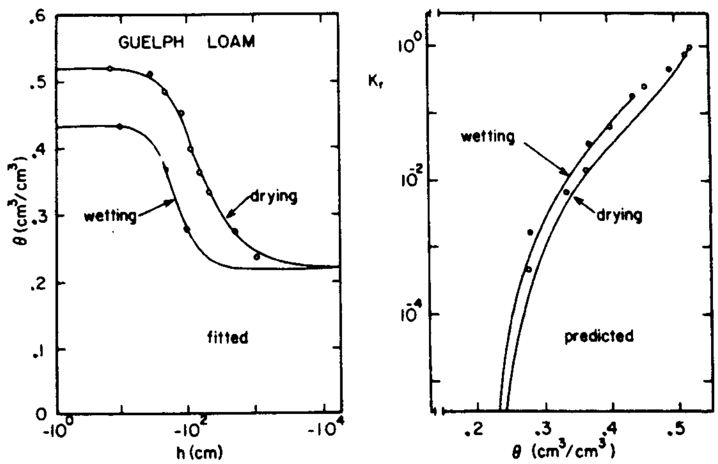

Fig. 9—Observed (*circles*) and calculated curves (*solid lines*) of the soil hydraulic properties of Guelph Loam. The drying and wetting branches of the relative hydraulic conductivity curve were predicted from knowledge of the curve-fitted branches of the soil-water retention curve.

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Nitrate Movement with Zero-order Denitrification in a Soil Profile[1]

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ABSTRACT

A theoretical analysis of the movement of nitrates in an unsaturated soil with zero-order denitrification is presented. The transport equation and boundary conditions are established to represent a field situation (i.e., application of N to the surface of a soil that already has nitrogen present in its profile). An analytical solution is derived. For given values of the diffusivity, pore solute velocity, and the rate coefficients for denitrification, the relative concentration profiles are shown at different times. Comparison with published data is made and it seems that the theoretical relationship fits the data reasonably well.

Additional Index Words: first-order and zero-order reaction, boundary conditions, N transformations.

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Large quantities of *N* fertilizers are being used for crop production. An understanding of the mechanisms of transport and transformations of these fertilizers in soil systems is necessary to develop and implement fertilizer management programs for efficient N use with minimum environmental hazard. The transport of fertilizer N in soils below the root zone of plants, usually in the form of NO_3-N, is an economic loss to the farmer as well as possibly degrading the quality of water resources (both ground and surface, depending on hydrologic factors).

The fate of N at and below the surface is governed by a variety of interrelated and complex processes. The various inorganic (NH_4^+, NO_3^-, NO_2^-, and N_2) and organic forms exist simultaneously and undergo reversible and/or irreversible transformations depending on chemical and microbiological processes. Simultaneously, the physical processes of leaching, diffusion, and possibly ion exchange also are occurring. McLauren (1969, 1970, 1973) has presented analysis for steady and transient states for predicting the distribution of NH_4^+, NO_3^-, and NO_2^- ions in a soil that had been continuously leached with an ammonium salt solution in the absence of ion exchange and ionic diffusion. Cho (1971) presented the theory of convective transport of ammonium ions to include not only simultaneously occurring nitrification and denitrification, but also the ion exchange reactions and ion diffusion. Misra et al. (1974) presented experimental evidence to support his theoretical considerations for N transformations in soils during leaching.

One of the N transformations that is not well understood is denitrification. Denitrification has traditionally been considered an undesirable process by agriculturists. The process has received attention in recent years and substantial information is available on soil factors influencing denitrification (Broadbent and Clark, 1965). Relatively little data are available, however, to assess the significance of denitrification beneath the rooting depth in reducing the quantity of residual NO_3^- moving into receiving waters. Soil properties influenced by drainage are important to denitrification. In addition to a possible direct effect on denitrification (Bremner and Shaw, 1958), soil moisture has an indirect influence on other interacting related factors. Where poor soil drainage results in a

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