This worksheet calculates the steady-state advection, diffusion and decay of radon-222 in the vadose zone. Radon-222 is transported by diffusion in both the aqueous and vapor phases and by advection (Darcy infiltration) in the aqueous phase. Local thermodynamic equilibrium is assumed so that the partitioning of radon between the aqueous and gas phases can be expressed using a constant partitioning coefficient. Radon-222 undergoes radioactive decay with a half-life of 3.82 days. This solution is derived from first principles.

Clifford K. Ho
Sandia National Laboratories
(505) 844-2384, ckho@sandia.gov
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This worksheet solves the radon transport in a three-region vadose zone: (1) a clean overburden region, (2) a source region with radon-222 generation from radium-226, and (3) a clean region beneath the source region.

Radon-222 inhalation dose factor (rem/pCi): $DCF_{222} := 1.44 \cdot 10^{-8}$

from FRAMES (Federal Guidance Report #11, EPA/520/1-88-020, Eckerman, 1988)

Input parameters:

Define number of realizations: $n := 100$

$i := 1 .. n$

Total porosity: $\phi := \text{runif}(n, .302, .445)$

Moisture Content: $\theta_w := \text{runif}(n, .053, .225)$

Gas Content: $\theta_{g_i} := \phi_i - \theta_{w_i}$

Water/gas partition coefficient (20C): $k := 0.26$ From Rogers et al. (1984)

This is equal to Cw/Cg

Radon-222 Half-life (3.82 days): $t_{half} := 3.82 \cdot 24 \cdot 3600 \cdot s$

Radium-226 Inventory (Ci): $Ci_226 := 6$

Specific Activity of radium-226 (Ci/g): $SA_{226} := 0.99$

Specific Activity of radon-222 (Ci/g): $SA_{222} := 1.54 \cdot 10^5$

Waste zone length: $length := runif(n, 10 \cdot .3048, 430 \cdot .3048) \cdot m$

Waste zone width: $width := runif(n, 10 \cdot .3048, 300 \cdot .3048) \cdot m$

Waste zone thickness: $L_s := runif(n, 10 \cdot .3048, 27 \cdot .3048) \cdot m$

L = 22 ft

from maps of MWL

Thickness of clean overburden: $L_1 := runif(n, 0 \cdot .3048, 16 \cdot .3048) \cdot m$

Distance to bottom of waste zone: $L_2 := L_1 + L_s$

Distance from bottom of waste zone to aquifer: $L_3 := runif(n, 133, 143) \cdot m$

Darcy Infiltration:

q = 10^(-11) m/s $q := runif\left(n, 1.18 \cdot 10^{-11}, 6.12 \cdot 10^{-11}\right) \cdot \frac{m}{s}$

$$\lambda_{226} := \frac{\ln(2)}{1600 \cdot 365.25 \cdot 24 \cdot 3600} \cdot s^{-1}$$

$$\lambda_{226} = 1.373 \times 10^{-11} s^{-1}$$

Calculated Parameters

Volume of waste zone: $V_i := length_i \cdot width_i \cdot L_{s_i}$

Decay constant: $\lambda_{222} := \frac{\ln(2)}{t_{half}}$ $\lambda_{222} = 2.1 \times 10^{-6} s^{-1}$

Emanation factor for sealed source: $logE := runif(n, -6, 0)$

This accounts for the containment of the radium-226. Ranges from 1E-12 (calibrated to ~1 pCi/m^2/s) to 1 (no containment)

$E_i := 10^{logE_i}$

In the previous version I used the decay constant for radon-222, but I should have used the decay constant for radium-226, as shown in the Qi term below.

Radon volumetric generation: $Q_i := \frac{E_i \cdot Ci_226}{SA_{226} \cdot 1000} \cdot \frac{\lambda_{226}}{V_i} \cdot kg$

Liquid saturation, S: $S_i := \frac{\theta_{w_i}}{\phi_i}$

Effective diffusion coefficient: $D_{eff_i} := \frac{0.07}{10^4} \cdot e^{-4 \cdot \left[S_i - S_i \cdot (\phi_i)^2 + (S_i)^5 \right]} \cdot \frac{m^2}{s}$

(for both liquid and gas phases, Rogers et al., 1984, NUREG/CR-3533)

Solution:

$$r_{1_i} := \frac{k \cdot q_i + \sqrt{(k \cdot q_i)^2 + 4 \cdot D_{eff_i} \cdot \lambda_{222} \cdot \left(\theta_{g_i} + k \cdot \theta_{w_i}\right)}}{2 \cdot D_{eff_i}}$$

$$r_{2_i} := \frac{k \cdot q_i - \sqrt{(k \cdot q_i)^2 + 4 \cdot D_{eff_i} \cdot \lambda_{222} \cdot \left(\theta_{g_i} + k \cdot \theta_{w_i}\right)}}{2 \cdot D_{eff_i}}$$

$$Q_{source_i} := \frac{Q_i}{\lambda_{222} \cdot \left(\theta_{g_i} + k \cdot \theta_{w_i}\right)}$$

$$c_{3_i} := \frac{r_{2_i} \cdot Q_{source_i}}{e^{r_{1_i} \cdot L_{2_i}} \cdot \left(r_{1_i} - r_{2_i}\right)} \qquad c_{2_i} := \frac{c_{3_i} \cdot e^{r_{1_i} \cdot L_{1_i}} \cdot \left(r_{1_i} - r_{2_i}\right) - r_{2_i} \cdot Q_{source_i}}{e^{r_{1_i} \cdot L_{1_i}} \cdot \left(r_{2_i} - r_{1_i}\right)}$$

$$c_{4_i} := \frac{c_{2_i} \cdot \left(e^{r_{2_i} \cdot L_{1_i}} - e^{r_{1_i} \cdot L_{1_i}}\right) - c_{3_i} \cdot e^{r_{1_i} \cdot L_{1_i}} - Q_{source_i}}{e^{r_{2_i} \cdot L_{1_i}}}$$

$$c_{6_i} := \frac{c_{3_i} \cdot e^{r_{1_i} \cdot L_{2_i}} + c_{4_i} \cdot e^{r_{2_i} \cdot L_{2_i}} + Q_{source_i}}{e^{r_{2_i} \cdot L_{2_i}}}$$

Combined Gas Concentration:

$ntot := 100$

$j := 1 .. ntot$

$x_j := \frac{j}{ntot} \cdot L3_i$

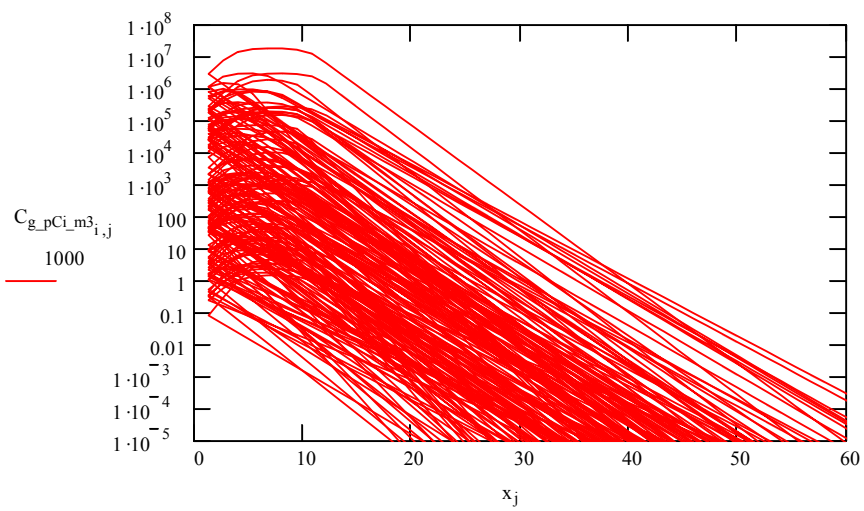
x_j denotes the spatial position, whereas "i" denotes the realization number.

$$C_{g_{i,j}} := if\left[x_j < L1_i, c2_i \cdot \left(e^{r2_i \cdot x_j} - e^{r1_i \cdot x_j}\right), if\left(x_j < L2_i, c3_i \cdot e^{r1_i \cdot x_j} + c4_i \cdot e^{r2_i \cdot x_j} + Q_{source_i}, c6_i \cdot e^{r2_i \cdot x_j}\right)\right]$$

Gas concentration in pCi/m^3

$$C_{g_pCi_m3_{i,j}} := C_{g_{i,j}} \cdot 1000 \cdot SA_{222} \cdot 10^{12}$$

Below is a plot of the surface gas concentration in pCi/L as a function of depth for 100 realizations



1.351
2.701
4.052
5.403

x

358.725	927.806	$2.036 \cdot 10^3$
$1.835 \cdot 10^8$	$4.972 \cdot 10^8$	$1.161 \cdot 10^9$
$2.139 \cdot 10^6$	$5.712 \cdot 10^6$	$1.312 \cdot 10^7$
$4.475 \cdot 10^4$	$7.776 \cdot 10^4$	$8.443 \cdot 10^4$

$C_{g_pCi_m3}$

Surface Gas Flux of Radon-222 (pCi/m²/s):

$$q_{s_i} := D_{eff_i} \cdot c_{2_i} \cdot \left(r_{2_i} - r_{1_i}\right) \cdot 1000 \cdot SA_{222} \cdot 10^{12}$$

Water Flux (pCi/s):

$$q_{w_i} := q_i \cdot length_i \cdot width_i \cdot k \cdot c_{6_i} \cdot e^{r_{2_i} \cdot L_{3_i}} \cdot 1000 \cdot SA_{222} \cdot 10^{12}$$

Aquifer Concentration (pCi/L):

$$C_{w_i} := \frac{q_{w_i}}{q_i \cdot length_i \cdot width_i \cdot 1000}$$

Output:

q_s
$3.44 \cdot 10^{-4}$
150.686
2.289
0.051
28.888

C_w
0
0
0
0
0

Input:

ϕ
0.302
0.33
0.386
0.352

θ_w
0.128
0.152
0.161
0.14

L_1
3.969
3.971
4.379
1.314

L_s
3.765
5.9
5.449
4.606
4.823

E
$2.803 \cdot 10^{-6}$
0.308
0.034
$4.077 \cdot 10^{-6}$
0.012

137.123
139.011
137.758
142.513
141.349

L_3

$1.419 \cdot 10^{-6}$
$1.238 \cdot 10^{-6}$
$1.604 \cdot 10^{-6}$
$1.675 \cdot 10^{-6}$

D_{eff}

$4.167 \cdot 10^{-11}$
$2.965 \cdot 10^{-11}$
$6.002 \cdot 10^{-11}$

q

$1.832 \cdot 10^{4}$
$3.775 \cdot 10^{3}$
$2.331 \cdot 10^{4}$

V

length =

	1
1	72.486
2	9.936
3	70.214
4	15.168

m

width =

	1
1	67.125
2	64.394
3	60.935

m